MASTER

Optimal procurement and hedging in flour milling

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Optimal procurement and hedging in flour milling

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Abstract

Procurement policies in commodity markets are crucial to the flour milling industry which operates under a thin profit margin while facing various major uncertainties. This thesis explores the optimal procurement policies of a firm-value-maximizing flour miller, under demand volatilities and stochastic prices. It turns out that myopic base-stock policies are optimal. Meanwhile, the value of downward substitution in commodity markets is also developed, and the result shows that it depends on the convenience yield, the difference between the forward and spot transportation costs, and the quality premium. According to the numerical results, the % benefit of downward substitution exponentially increases with quality requirement uncertainty.

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This thesis report serves as the result of my graduation project for the MSc program Operations Management and Logistics at the Eindhoven University of Technology, and most gladly this project led to the answer toward the very beginning question which serves as the motivation of this whole project.

It all started with a conjecture from my first supervisor dr. Fehmi Tanrisever who got a vision based on a previous master's student's graduation project. It was a long and lucky story how I met Fehmi and gained this precious opportunity to explore the existence of the might-or-might-not-be "treasure" – the answer to the conjecture. It was a challenging and sometimes frustrating groping process. But the curiosity of mine aroused by Fehmi and his strong guidance and support turned the tough journey into such a joyful and unforgettable process. Unlimited discussion time, sharing of his experience and intelligence, trying hard for my attendance chance of a significant PhD level inventory control course and helping me with further literatures… The list can keep going on forever if I have to enumerate everything Fehmi has helped me with developing enough knowledge base and skills to conquer the obstacles on the way to the ultimate answer.

After acquiring some staged outcomes, another important person came into the project – my second supervisor dr. Zumbul Atan. Zumbul voluntarily took the crucial but also time consuming and exhausting step: checking whether the proofs are correct or not. It is a long list of proofs and together they compose a sort of intricate structure. It was her confirmation of the proofs that made me realize we have indeed dug out something that we were looking for. Besides, Zumbul introduced her ideas about how to fix the existing weak points in the thesis draft together with Fehmi, e.g. the kind of messy structure in the draft (especially the proofs section) and the lack of depth in the discussion sections. Thanks for Zumbul's help, I saw the interesting points missed before and the quality of the thesis was improved significantly.

I would also like to thank Dr. Dorothee Honhon who opened the door of the PhD course to me, after Fehmi's request. It was from that course, I gained some crucial missing knowledge and speculation skills jigsaw pieces to find the answer to my question.

Thank my parents, many family relatives and friends. I could not switch into my current master's program and keep pursuing what I truly love without your support.

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Summary

Motivated by the flour milling industry, this thesis considers the procurement problem of a flour miller who procures different types of wheat from upstream wheat providers and earns revenue by trading a specific type of flour to the downstream industries (e.g. bakeries) and aims at maximizing the firm value, while facing an uncertain business environment.

The flour miller procures different types of wheat, and convert it into specific types of flour to sale. The milling process in general includes grinding and sifting which contain further detailed technological processes (usually accompanied with weight loss during the process, prior wheat blending or latter flour blending to achieve flour quality requirement from industrial customers. The grinding process usually also produces by-product).

Though the revenue is considerable, the indeterminacy in the business environment faced by the flour millers is impairing their profit: volatile wheat purchase price, uncertain wheat quality, and not entirely fixed wheat supply amount and delivery date, from the upstream wheat suppliers; uncertain flour demand and uncertain flour quality requirement from downstream customers, together with uncertain conversion ratio during the flour producing processes, etc..

The miller can procure the wheat by forward contracts and on the spot market. Both markets provide abundant amount of wheat in all types, but the spot market ensures immediate delivery and the forward market delivers the wheat one period later. However, the transportation fee on the spot market is higher than the forward market. Because under forward contracts, the logistics provider clearly knows what type of wheat should be delivered from which wheat supplier to which location on which future date, and in what amount, which enables advanced logistical planning for the logistics provider; while on the spot market these information usually would not be confirmed until the last minute, which as a result restrains any advanced logistical planning.

The sales of flour are based on pre-agreed sales contracts between the miller and the downstream food industry. The customers denote their demand and quality requirement (e.g. particle size index, dough volume and falling number) on the contracts which will be mature in one period after signing. But these sales contracts do not possess legal power, so on the mature date the customers can adjust their demand and quality requirement for their own benefits.

The thesis would explore the problem in a uncertainty-complexity increasing way. First, only flour demand uncertainty would be taken into account, to explore the corresponding optimal procurement policy. Even though the sales contracts are signed, the flour demand of the next period is still random. So it is highly likely that the miller observes either wheat shortage or excess one period later, when the wheat procured by forward contracts arrives and the real flour

demand is observed. The miller would purchase the missing amount of wheat from the spot market if wheat shortage happens, or hold the inventory into the next period if wheat turns out to be excessive. No flour backlog is allowed.

Then except for the demand uncertainty, the quality requirement is also assumed to be uncertain. The miller cannot observe the real quality requirement, until the mature dates of the sales contracts. Every type of wheat has its own constant quality characteristics, thus a certain flour quality requirement defines a certain wheat blending ratio.

Next, the thesis also considers the downward substitution problem. Downward substitution means the miller can substitute the shortage of normal wheat with excessive good wheat (if there is any), but not the other way around: After receiving the wheat purchased by forward and observing the flour demand and quality requirement, the miller may face a situation in which the good wheat is more than enough but the normal wheat is in a shortage. Except for procuring (all) the missing amount of normal, the miller can choose to substitute with excessive good wheat.

In each case, the thesis models the decision making process of the miller as a stochastic dynamic program, and every model has a multiple-period time scope. The multiple-period assumption is more realistic to the flour miller instead of one period, because the decision in current period would always be influenced by the one made in the previous period in the form of inventory or backlog, and so is the decision next period by the current period. Moreover, the research results of multiple-period case can also be applied to single-period one.

It turns out that the optimal procurement polices fall into the Newsvendor pattern, and the higher the ratio of (extra expenses on the spot market compared with the forward market)/(the difference between the forward market price plus expected holding cost and the expected forward market price next period), the miller would utilize the forward market more, which is more beneficial compared with the spot market. Another observation is that the convenience yield would not always necessary show up in the optimal procurement policies in the commodity markets, when the spot procurement does not face uncertainties.

The downward substitution value in the commodity markets depends on the convenience yield, the difference between the forward and spot transportation costs, and the negative effects of losing the quality premium. The stochastic price brings dynamic to the value of downward substitution in the commodity markets, resulting in value increase or decrease. When the value is positive, downward substitution would be beneficial; when the value becomes negative, downward substitution should be avoided.

In the simulation section, we simulated how the value of downward substitution would change according to the blending ratio volatility, under a stochastic price process and different marginal profits. According to the numerical experiment result, the % benefit of downward substitution increases in a speeding up style along the rising of quality requirement uncertainty, and such a % is higher for the case with lower marginal profit, on the same quality requirement uncertainty level.

Contents

1. Introduction

According to the European Flour Millers' association (2011), the European flour milling industry represented a turnover of 15 billion euros per year, producing 35 million tons of flour annually; IBISWorld (2012) pointed out that flour milling industry in U.S. would create 21 billion dollars revenue annually, and the revenue was estimated to keep growing in the following five years. Given the huge economic size of the flour milling industry, it is essential to optimize the operating and hedging decisions of the millers.

Flour milling industry procures different types of wheat, and converts it into specific types of flour to sale. The milling process in general includes grinding and sifting which contain further detailed technological processes, usually accompanied with weight loss during the process. The grinding process also produces by-products. Here is an example of the business scenario about flour milling industry in Europe, based on the introduction of the European Flour Millers' association (2012): the millers procure different types of wheat (e.g. common wheat, spelt wheat and durum wheat etc., sometimes even other types of grains instead of wheat) from upstream farms, merchants and grain stores or by import. EU flour millers produce up to 600 different types of flour most of which can be classified into white, whole meal or brown category. The millers convert the wheat into specific types of flour and sell it to downstream bakeries, retailers, caterers or other food manufacturers, together with the by-product being sold to animal feed companies usually. The flour would be consumed as bread or similar baked products (70% in total), and also used in a great variety of other foods including biscuits, cakes, pies, pizzas, coatings, confectionery etc.

Though the revenue is considerable, the indeterminacy in the business environment faced by the flour millers is impairing their profit: uncertain demand, volatile wheat price, uncertain flour quality requirement, besides (in some cases) uncertainties from wheat quality and milling processes (e.g. how much weight would be lost) etc. For instance, Figure 1 depicts the fluctuations of European commodity prices (in euros per metric ton).

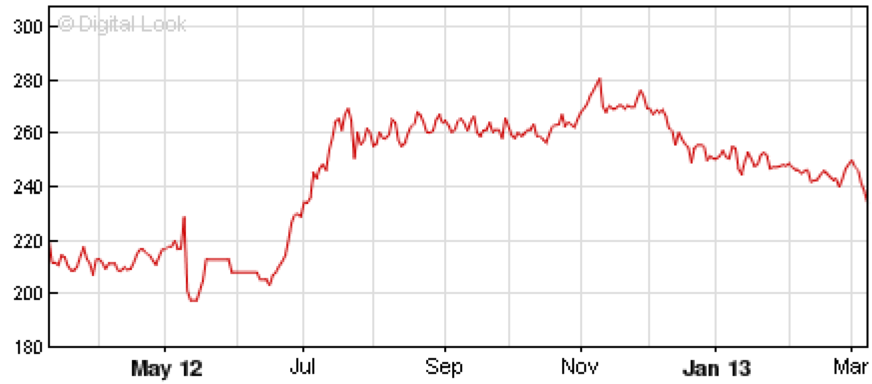

Figure 1: milling wheat price fluctuations (Smale,2013)

Motivated by the flour milling industry, this thesis considers the problem of a flour miller who procures different types of wheat from upstream wheat providers and earns revenue by trading a specific type of flour to the downstream industries (e.g. bakeries) and aims at maximizing the firm value, while facing an uncertain business environment similar to the one described generally above .

The miller can procure the wheat by forward contracts and on the spot market. Both markets provide abundant amount of wheat in all types, but the spot market ensures immediate delivery and the forward market delivers the wheat one period later. As a result, the transportation fee on the spot market is higher than the forward market, as discussed by Goel and Tanrisever (2011).

The sales of flour are based on pre-agreed sales contracts between the miller and the downstream food industry. The customers denote their demand and quality requirement (e.g. particle size index, dough volume and falling number mentioned by Hayta and Çakmakli (2001)) on the contracts which will be mature in one period after signing. But these sales contracts do not possess legal power, so on the maturity date the customers can adjust their demand and quality requirement for their own benefits.

Clearly, the miller faces flour demand uncertainty. Even though the sales contracts are signed, the flour demand of the next period is still random. So it is highly likely that the miller observes either wheat shortage or excess one period later, when the wheat procured by forward contracts arrives and the real flour demand is observed. The miller would purchase the missing amount of wheat from the spot market if wheat shortage happens, or hold the inventory into the next period if wheat turns out to be excessive. No flour backlog is allowed.

Except for the demand uncertainty, the quality requirement is also uncertain. The miller cannot observe the real quality requirement, until the maturity dates of the sales contracts. Every type of wheat has its own constant quality characteristics, thus a certain flour quality requirement defines a certain wheat blending ratio.

The miller also faces a downward substitution problem. After receiving the wheat purchased by forward contracts and observing the flour demand and quality requirement, the miller may face a situation in which the good wheat is more than enough but the normal wheat is in shortage. The miller can substitute the shortage of normal wheat with good wheat. But what is the value of downward substitution, and what would be the best downward substitution policy?

The main purpose of this thesis is to explore the optimal procurement policy of the miller. The miller examined in this thesis is an example of a broader class of integrated operational and financial risks management problems, including firm-value-maximizing companies from textile industry, food manufacturing industry, chemical reagent industry, dye industry and drug manufacturing industry etc which need different primary inputs to generate a single type of output while facing demand uncertainties.

The procurement prices in the wheat forward market and spot market are assumed to evolve under a risk-neutral measure, and the flour sales price is based on the wheat spot price, in order to eliminate arbitrary behaviors.

The thesis finds the optimal procurement policies for a flour miller who faces demand and quality requirement uncertainties in commodity markets, as well as a downward substitution problem, and then compares the results with the existing literatures. Meanwhile, the extra value of downward substitution in the commodity markets is also explored in this thesis. It turns out that the optimal procurement polices fall into the Newsvendor pattern. The value of downward substitution in commodity markets depends on the convenience yield, the difference between the forward and spot transportation costs, and the quality premium. According to the numerical results, the % benefit of downward substitution exponentially increases with quality requirement uncertainty.

The thesis proceeds as follows: In §2, it reviews the relevant literatures and describes the difference of this thesis generally. In §3, a basic case with only demand uncertainty is introduced and explored. The quality requirement is deterministic, and downward substitution is not taken into account. In §4, on the basis of the case in §3, the quality requirement uncertainty is taken into account, and the optimal procurement policy is developed for this new case. In §5, based on the case in §4, the downward substitution problem comes into the scope, and the optimal procurement policy together with the optimal substitution policy are developed. The extra value brought by downward substitution in commodity markets is also examined in this case. In §6, a stochastic price simulation model is explained, and then we simulate how the value of downward substitution changes according to the blending ratio volatility, under a stochastic price process and different marginal profits. Section 7 contains the concluding remarks.

2. Literature Review

There are three streams of literature relevant to the research in this thesis: a) optimal commodity procurement portfolio of contracts including forward and spot; b) downward substitution in supply chain management; c) quality requirement uncertainty.

Recently, a growing body of literature considers forward contracting in commodity procurement (since they can help to mitigate price and demand risks), along with spot market. Wu and Kleindorfer (2005) build an one-period model of a market which has multiple sellers and one buyer who trades by options, forward and on spot market while being risk neutral, to explore the optimal contracting and spot transactions portfolios for them. Dong and Liu (2007) check the determination of an equilibrium forward contract on a non-storable commodity, between two firms in an one-period model. They find that the forward contract would affect inventory policies because of its hedging effect. In a related research, Seifert et al. (2004) consider the benefits of applying spot market in procurement in single period models, compared with exclusive utilization of forward contracts. Their results indicate that significant profit improvements can be achieved when a moderate fraction of commodity is procured on the spot markets. In addition, they point it out that spot markets can offer a higher expected service level to companies, but they may bring a higher variability.

Yi and Scheller-Wolf (2003) study a multi-period inventory management problem of a buyer who faces random demand. Their objective was to minimize the total expected replenishment costs. They assume commodities can be procured by forward contracts with known price and on the spot market with random price. Based on a new closure property of K-convexity, they figure that optimal inventory decisions have a structure similar to the classic (s, S) policy. Martínez-de-Albéniz and Simchi-Levi (2005) solve the optimal supply contracts portfolio (including spot market) problem of a buyer (manufacturer) who can procure by forward contracts, options and from the spot market, based on the “flexibility-price trade-off” of the potential procurement choices. They model the problem in a multi-period environment, assuming supply contracts, spot market costs and inventory holding costs to be convex. They find that the optimal replenishment policy follows a modified base-stock policy for every option. They also figure that the contract selection problem is a concave maximization problem for which they provide closed-form solution for the single period version. However, as pointed out by Haksöz and Seshadri (2007), this paper does not model the connection of forward and/or futures prices to the spot prices. The long-term contract price is therefore assumed to evolve independent of the spot market price. Haksöz and Seshadri (2007) themselves take a literature review on the work in the supply chain operations literature, including plentiful papers which cover the topic of optimal procurement in the presence of forward (or future) and spot markets.

Goel and Gutierrez (2011) examine how the price information and flexibility on the commodity market (which consists of spot and forward markets) lead to significant retrenchment in inventory costs in a periodic review system. They model a two echelon zero-lead-time distributive supply chain with one center location and multiple nonhomogeneous downstream retailers in a multiple-period setting. In the model, the supply chain faces random demand (even for spot procurement) and price, and the holding cost is determined by the spreads between spot and forward prices. Goel and Gutierrez (2012) develope a multi-period and periodic-review inventory model with stochastic demand to study the optimal forward and spot procurement policies of a processor who provides a single type of product. They find that it is possible to reduce inventory related costs significantly by combining spot and futures price information when making the procurement decision. In their paper, optimal forward procurement policies are characterized by a fixed band. This paper considers the procurement problem in a very similar setting to the one in this thesis, and thus the results in this thesis are compared with theirs, especially on the perspective of convenience yield.

The first stream of literature cited above seems similar to another stream which builds models allowing multiple modes of supply. For instance Chiang and Gutierrez (1998) and Tagaras and Vlachos (2001) both explore a periodic review inventory system where regular replenishment and emergency replenishment (with shorter lead time and higher ordering cost) both exist. But the latter stream assumes uncertain demand even in the emergency replenishment, and it usually does not take the price volatility into account.

Except for the optimal procurement policies, this thesis also considers the downward substitution problem. Downward substitution among the final products has already been studied by plentiful operations management papers. Hsu and Bassok (1999) consider a single-period, one input, multi-product, full downward substitution model, in a setting where demand is uncertain and the yield is random. They develop three solution methods to find the optimal production decision. Uday et al. (2004) model a multi-product inventory system with downward substitution and setup costs as a one-period stochastic program with recourse. They exploit structural properties of the model and utilize a combination of optimization techniques to develop effective inventory planning heuristics. Bassok et al. (1999) study a single period multi-product inventory problem with stochastic demands, downward substitution and proportional costs and revenues. They show that a greedy substitution allocation policy was optimal. Nagarajan and Rajagopalan (2008) consider a system where individual demands are negatively correlated and partially downwardly substitutable. They find state independent base-stock policies turn out to be optimal inventory policies. Liu et al. (2009) consider an one-period downward substitution problem for two components, with more subtle exploration of the substitution rule. They present a dynamic substitution rule which is a threshold policy. For systematic literature review on the classic downward substitution problem, the work by Uday et al. (2004) is recommended. But it is very rare, if not at all, to see literatures which examine the value of downward substitution under a setting where commodity markets are involved.

Quality requirement uncertainty is another problem to be considered in this thesis. A similar problem is considered by some remanufacturing literatures. In remanufacturing, the manufacture usually can procure new components or attain good ones from used products to fulfill the demand, so the manufacture faces the challenge of addressing uncertain yields problem. Some relevant papers include: Ferrer and Michael (2004), Ferrer and Whybark (2009), Galbreth and Blackburn (2006), Bakal and Akcali (2006), Mukhopadhyay and Ma (2009), Xu (2010).

This thesis differs from (most of) the above studies mainly in two ways. First, in the thesis the value of downward substitution is examined in commodity markets, so the value of downward substitution is explored not only under demand uncertainty, but also under price volatility under a multiple-periods setting. Second, the thesis takes demand uncertainty, price uncertainty, quality requirement uncertainty and downward substitution into account, in the presence of forward contracts and spot market. Compared with the main body of relevant papers that only takes demand uncertainty and price volatility into account when exploring the optimal procurement policies in commodity markets, this thesis also considers another main uncertainty from the downstream of the industry chain and the possible influence from producing processes on the procurement policies (i.e. quality requirement uncertainty and downward substitution). Such additions contribute to enriching the integrated operational-financial interface focused research, as well as dismantling the partition between producing processes and procurement policies.

3. Case with demand uncertainty

In this thesis, we assume the miller procures two types of wheat- good and normal (hereafter referred to as type 1 and type 2 respectively), and the quality premium for per unit of good wheat is denoted as γ. Since there are two types of wheat, any given quality requirement can be converted to specific blending ratio of wheat.

In this section, from the perspective of the miller, the flour demand is uncertain, while the quality requirement is deterministic. Assume the demand of period $t-1$ would arrive at time t (where $t=1,\ldots,T$), i.e. the end of the period, and it follows a general distribution, i.e. $\xi_t \sim \psi(\bullet)$ (where ξ_t stands for the unrealized flour demand at time t, and $\psi(\bullet)$ stands for a general probability density function, and the corresponding cumulative distribution function is denoted as $\Psi(\bullet)$). Denote the wheat blending ratio at time t as α_t, where $\alpha_t \in [0,1]$ and it means the proportion of type 1 wheat in the flour (so $1-\alpha_t$ for type 2 wheat accordingly). α_t is abbreviated as α hereafter.

The time-line of decision making is as follows: at time t the miller (1) observes the wheat inventory levels from time $t-1$. Denote the wheat inventory levels from last period as $I_{t-1,1}$ and $I_{t-1,2}$ for type 1 and type 2 respectively, (2) receives the wheat purchased by forward at time $t-1$, with which the amounts of type 1 and type 2 wheat accumulate to the order-up-to points of time t-1 (denoted as $z_{t-1,1}$ and $z_{t-1,2}$ accordingly), on the basis of $I_{t-1,1}$ and $I_{t-1,2}$, (3) observes the exact flour demand Q_t, (4) The miller observes the wheat spot price s_t. The delivery cost per unit wheat on the spot market is fixed and denoted as η^s. If there is not enough wheat to meet the flour demand, the miller will procure exactly necessary amount of wheat from the spot market immediately, denoted as $y_{t,1}$ and $y_{t,2}$ respectively for type 1 and type 2 wheat, so there is no possible backlog or lost sale; if the wheat is more than enough, the miller will just hold it into the next period, under fixed holding cost (denoted as h_1 and h_2 per unit respectively for type 1 and 2 wheat) or take a recourse if it is the final period already, (5) converts the wheat into flour according to the blending ratio, and receives the revenue. In order to eliminate possible speculative behavior from the customers, the flour sales price is based on the wheat spot price at time t . The marginal profit is fixed and denoted as λ, (6) observes the forward price F_t, and makes a decision how much to purchase for type 1 and 2 by forward (i.e. $z_{t,1}-I_{t,1}$ and $z_{t,2}-I_{t,2}$ respectively), to fulfill the demand of next period, and pay for them at time t, under a fixed forward delivery cost per unit (denoted as η^f).

Note: (1) in this model we make a general assumption in which $s_t+\vec{h}-F_t>0$, i.e. forward dominates spot, so it is always more valuable to fulfill the demand from the forward market than the spot market. (2) $P_t(F_t, s_t)$ contains the information of forward price and spot prices at time t, and it is abbreviated as P_t hereafter. (3) the expectation is taken under the risk-netural measure, as denoted by E^Q. (4) $\beta = e^{-r\triangle t}$, where r is a risk free discounting rate and $\triangle t = 1$.

We denote the present value of cash flows at time t by V_t.

$$V_t(\vec{z}_{t-1}, P_t, Q_t) = max_{\vec{z}_t \geq \vec{I}_t} v_t(\vec{z}_t | \vec{z}_{t-1}, P_t, Q_t)$$

where

$$v_t(\vec{z}_t | \vec{z}_{t-1}, P_t, Q_t) = -y_{t,1}(s_t + \gamma + \eta^s) - y_{t,2}(s_t + \eta^s) + Q_t[\alpha(s_t + \gamma) + (1-\alpha)s_t + \gamma]$$

$$-I_{t,1}h_1 - I_{t,2}h_2 - \left(z_{t,1} - I_{t,1}\right)(F_t + \gamma + \eta^f) - \left(z_{t,2} - I_{t,2}\right)(F_t + \eta^f)$$

$$+\beta E^Q_{\tilde{Q}_{t+1}, \tilde{P}_{t+1}} V_{t+1}(\vec{z}_t, \tilde{P}_{t+1}, \tilde{Q}_{t+1})$$

where

$$y_{t,1} = [Q_t\alpha - z_{t-1,1}]^+, \;\; y_{t,2} = [Q_t(1-\alpha) - z_{t-1,2}]^+$$

$$I_{t,1} = [Q_t\alpha - z_{t-1,1}]^-, \;\; I_{t,2} = [Q_t(1-\alpha) - z_{t-1,2}]^-$$

The first and second terms in the stochastic dynamic program are the cost of spot procurement. The third term is the revenue from satisfying the demand. The fourth and fifth terms are the holding cost of excessive wheat. The sixth and seventh terms are the cost of forward procurement. The eighth term is the discounted cost to go function.

In the following part of this section, we will rebuild the cash flow function above from the perspective of cost instead of profit, and then develop the (possible) optimal procurement policy step by step.

3.1 The optimal order-up-to point

We rebuild the present value function of cash flow from the perspective of cost, in order to ease the exploration of the optimal order-up-to point in case it exists

$$f_t\left(z_{t-1,1} - Q_t\alpha, z_{t-1,2} - Q_t(1-\alpha)\right) = min_{\vec{z}_t \geq \vec{I}_t}\{y_{t,1}(s_t + \gamma + \eta^s) + I_{t,1}h_1$$

$$+y_{t,2}(s_t + \eta^s) + I_{t,2}h_2 + \left(z_{t,1} - I_{t,1}\right)(F_t + \gamma + \eta^f) + \left(z_{t,2} - I_{t,2}\right)(F_t + \eta^f)$$

$$+\beta E^Q_{\tilde{Q}_{t+1}, \tilde{P}_{t+1}} f_{t+1}(z_{t,1} - \tilde{Q}_{t+1}\alpha, z_{t,2} - \tilde{Q}_{t+1}(1-\alpha))\}$$

$$= min_{\vec{z}_t \geq \vec{I}_t}\{-(F_t + \gamma + \eta^f)I_{t,1} - (F_t + \eta^f)I_{t,2} + I_{t,1}h_1 + I_{t,2}h_2 + y_{t,1}(s_t + \gamma + \eta^s)$$

$$+y_{t,2}(s_t + \eta^s) + G_t(\vec{z}_t)\}$$

where

$$G_t(\vec{z}_t) = (F_t + \gamma + \eta^f)z_{t,1} + (F_t + \eta^f)z_{t,2} + \beta E^{Q}_{\tilde{Q}_{t+1},\tilde{P}_{t+1}} f_{t+1}(z_{t,1} - \tilde{Q}_{t+1}\alpha, z_{t,2} - \tilde{Q}_{t+1}(1-\alpha))$$

The salvage value function is accordingly modified into:

$$f_T\left(z_{T-1,1} - Q_T\alpha, z_{T-1,2} - Q_T(1-\alpha)\right) = y_{T,1}(s_T + \gamma + \eta^s) + y_{T,2}(s_T + \eta^s)$$

$$+I_{T,1}(h_1 - s_T - \gamma - \eta^f) + I_{T,2}(h_2 - s_T - \eta^f)$$

Theorem 1. *A base-stock policy is optimal in each period of a finite-horizon problem.*

Proof: See Section TA3.

Theorem 2. *The base-stock policy defined as follows is optimal in each period of a finite-horizon problem:*

$$\Psi\left(\frac{S_{t,1}}{\alpha}\right) = \frac{\beta(F_t+\gamma+\eta^s)-(F_t+\gamma+\eta^f)}{\beta(\eta^s+h_1-\eta^f)},\ t = 1,\dots,T-1$$

$$\Psi\left(\frac{S_{t,2}}{1-\alpha}\right) = \frac{\beta(F_t+\eta^s)-(F_t+\eta^f)}{\beta(\eta^s+h_2-\eta^f)},\ t = 1,\dots,T-1$$

As defined in Section TA2, $\vec{S}_t$ denote a minimizer of $G_t(\vec{z}_t)$ over all real value $\vec{z}_t$.

Proof: See Section TA7.

An example of Theorem 2 would be the optimal order up to points for the last period.

Result 1. *The optimal order up to points for the last period*

$$\Psi\left(\frac{S_{T-1,1}}{\alpha}\right) = \frac{\beta(F_{T-1}+\gamma+\eta^s)-(F_{T-1}+\gamma+\eta^f)}{\beta(\eta^s+h_1-\eta^f)}$$

$$\Psi\left(\frac{S_{T-1,2}}{1-\alpha}\right) = \frac{\beta(F_{T-1}+\eta^s)-(F_{T-1}+\eta^f)}{\beta(\eta^s+h_2-\eta^f)}$$

Proof: See Section TA4.

The optimal base-stock policy above resembles the result of classic Newsvendor model, which seems contradict with the multiple-period setting of the problem discussed in this thesis: the miller can make the optimal procurement decision as if without considering the influence on the following periods, just like in the single period Newsvendor model. Actually, the prolonged influence is taken into account in the optimal policy already. The cost of per unit understock in this model would be the extra expenses on the spot market compared with the forward market, instead of the lost profit suggested by the Newsvendor model; the cost of per unit overstock here would be the difference between the forward market price plus expected holding cost and the expected forward market price next period, instead of the value loss after salvage in the Newsvendor model (except for the last period). So the optimal base-stock policy here is built based on the tradeoff between the costs of now and future.

Another interesting observation is that the optimal policy is myopic. Such a characteristic may conceal the essence that the result considers the prolonged influence of each procurement decision up to the final period, instead of only up to the next period. Actually, such a result is consistent with how the problem is modeled in a nesting way in Section 3.1: the expected influence of a given procurement decision on all the following periods is wrapped up in one function which would expand at the following time point (i.e. the beginning of the next period).

According to the theorem above, the higher the ratio of (extra expenses on the spot market compared with the forward market)/(the difference between the forward market price plus expected holding cost and the expected forward market price next period), the miller would utilize the forward market more, which is more beneficial compared with the spot market. Though the numerator is always positive, the denominator may be negative when the transportation fee on the spot market is not bigger enough than the one on the forward market, or the discount rate β is too small. In such a case, the ratio would also be negative, which means the miller should not order from the forward market at all, since it is more beneficial to procure from the spot market.

4. Case with quality requirement uncertainty

On the basis of Section 3, in this section, the miller also faces quality requirement uncertainty. The quality requirement in each period would realize at the same time as the demand on the time-line (developed in Section 3). We accordingly assume the blending ratio α is independent and identically distributed in each period. The corresponding probability density function and cumulative distribution function are denoted as ø(•) and Ø(•) accordingly.

Here is the present value function of cash flows (from the perspective of cost) which takes the expectation of α into account:

$$f_t\left(z_{t-1,1} - Q_t\alpha, z_{t-1,2} - Q_t(1-\alpha)\right) = min_{\vec{z}_t \geq \vec{I}_t}\{y_{t,1}(s_t + \gamma + \eta^s) + I_{t,1}h_1$$

$$+y_{t,2}(s_t + \eta^s) + I_{t,2}h_2 + \left(z_{t,1} - I_{t,1}\right)(F_t + \gamma + \eta^f) + \left(z_{t,2} - I_{t,2}\right)(F_t + \eta^f)$$

$$+\beta E^Q_{\tilde{Q}_{t+1},\tilde{P}_{t+1},\tilde{\alpha}} f_{t+1}(z_{t,1} - \tilde{Q}_{t+1}\tilde{\alpha}, z_{t,2} - \tilde{Q}_{t+1}(1-\tilde{\alpha}))\}$$

$$= min_{\vec{z}_t \geq \vec{I}_t}\{-(F_t + \gamma + \eta^f)I_{t,1} - (F_t + \eta^f)I_{t,2} + I_{t,1}h_1 + I_{t,2}h_2 + y_{t,1}(s_t + \gamma + \eta^s)$$

$$+y_{t,2}(s_t + \eta^s) + G_t(\vec{z}_t)\}$$

where

$$y_{t,1} = [Q_t\alpha - z_{t-1,1}]^+,\ y_{t,2} = [Q_t(1-\alpha) - z_{t-1,2}]^+$$

$$I_{t,1} = [Q_t\alpha - z_{t-1,1}]^-,\ I_{t,2} = [Q_t(1-\alpha) - z_{t-1,2}]^-$$

$$G_t(\vec{z}_t) = (F_t + \gamma + \eta^f)z_{t,1} + (F_t + \eta^f)z_{t,2} + \beta E^Q_{\tilde{Q}_{t+1},\tilde{P}_{t+1},\tilde{\alpha}} f_{t+1}(z_{t,1} - \tilde{Q}_{t+1}\tilde{\alpha}, z_{t,2}$$

$$-\tilde{Q}_{t+1}(1-\tilde{\alpha}))$$

The salvage value function is accordingly modified into:

$$f_T\left(z_{T-1,1} - Q_T\alpha, z_{T-1,2} - Q_T(1-\alpha)\right) = y_{T,1}(s_T + \gamma + \eta^s) + y_{T,2}(s_T + \eta^s)$$

$$+I_{T,1}(h_1 - s_T - \gamma - \eta^f) + I_{T,2}(h_2 - s_T - \eta^f)$$

We will develop the (possible) optimal procurement policy step by step in the following part of this section.

Theorem 3. *A base-stock policy is optimal in each period of a finite-horizon problem.*

Proof: See Section TA10.

Theorem 4. *The base-stock policy defined as follows is optimal in each period of a finite-horizon problem:*

$$\int_0^1 \Psi\left(\frac{S_{t,1}}{\alpha}\right) ø(\alpha)d(\alpha) = \frac{\beta(F_t+\gamma+\eta^s)-(F_t+\gamma+\eta^f)}{\beta(\eta^s+h_1-\eta^f)}, t = 1, \ldots, T-1$$

$$\int_0^1 \Psi\left(\frac{S_{t,2}}{1-\alpha}\right) ø(\alpha)d(\alpha) = \frac{\beta(F_t+\eta^s)-(F_t+\eta^f)}{\beta(\eta^s+h_2-\eta^f)}, t = 1, \ldots, T-1$$

Proof: See Section TA14.

An example of Theorem 4 would be the optimal order up to points for the last period.

Result 2. *The optimal order up to points for the last period would be*

$$\int_0^1 \Psi\left(\frac{S_{T-1,1}}{\alpha}\right) ø(\alpha)d(\alpha) = \frac{\beta(F_{T-1}+\gamma+\eta^s)-(F_{T-1}+\gamma+\eta^f)}{\beta(\eta^s+h_1-\eta^f)}$$

$$\int_0^1 \Psi\left(\frac{S_{T-1,2}}{1-\alpha}\right) ø(\alpha)d(\alpha) = \frac{\beta(F_{T-1}+\eta^s)-(F_{T-1}+\eta^f)}{\beta(\eta^s+h_2-\eta^f)}$$

Proof: See Section TA11.

The theorem above shows the introduction of quality requirement uncertainty would not change the optimal procurement policy structure as obtained in Theorem 2 in which only demand uncertainty exists.

The effect of convenience yield: up to now, the problem seems to be very similar to the one considered in the paper by Goel and Gutierrez (2012), but the distinction in the business scenario assumptions leads to an important difference between the results. In the paper from Goel and Gutierrez (2012), the marginal convenience yield $s_t + h - \beta F_{t+1}$ shows up in the optimal spot procurement policy, as the unit cost of overstocking, but in this thesis, as shown in Section 3 and 4, the spot price does not show up, even though the spot procurement option exists in each period.

In the paper from Goel and Gutierrez (2012), when purchasing from the spot market, the demand is still not observed, so the decision maker has to evaluate the tradeoff between the spot procurement and forward procurement, thus the spot price s_t shows up in the optimal procurement policy together with forward price and holding cost.

However, in this thesis, when purchasing from the spot market, the demand and quality requirement are both realized, so the decision maker does not compare the tradeoff between the spot procurement and the forward one, and she/he just needs to order exactly necessary amount from the spot market to avoid backlogs.

When the decision maker in this thesis procures by forward, she/he indeed has to weigh between purchasing by current forward and spot market one period later, thus the decision maker needs to consider the current forward price and the expected spot price of next period which however is equal to the current forward price (i.e. $F_t = E^Q(\tilde{s}_{t+1})$), so the spot price does not show up in the optimal forward procurement policies either.

Based on the comparison, the convenience yield would not always necessary show up in the optimal procurement policies in the commodity markets, when the spot procurement does not face uncertainties.

5. Case with downward substitution

On the basis of Section 4, the normal wheat can be replaced by the good wheat (i.e. downward substitution). Assume the downward substitution ratio is always $1:1$. After adopting the modification in Section 4, the time-line developed in Section 3 would further takes the following updates:

If there is not enough type 1 wheat to meet the flour demand, the miller will procure exactly necessary amount of the type 1 wheat from the spot market immediately, and if there is not enough type 2 wheat, the miller will either procure from the spot market or substitute with excessive type 1 wheat (if possible, and denote the substitution amount as x_t) or take both methods to fulfill exactly the demand of type 2 wheat immediately, so there is no possible backlog or lost sale.

The present value function of cash flows from the cost perspective would update into:

$$f_t\left(z_{t-1,1} - Q_t\alpha, z_{t-1,2} - Q_t(1-\alpha)\right)$$

$$= min_{0\leq x_t \leq \min\{[Q_t\alpha - z_{t-1,1}]^-, [Q_t(1-\alpha) - z_{t-1,2}]^+\}, \vec{z}_t \geq \vec{I}_t}\{y_{t,1}(s_t + \gamma + \eta^s) + I_{t,1}h_1$$

$$+y_{t,2}(s_t + \eta^s) + I_{t,2}h_2 + \left(z_{t,1} - I_{t,1}\right)(F_t + \gamma + \eta^f) + \left(z_{t,2} - I_{t,2}\right)(F_t + \eta^f)$$

$$+\beta E^Q_{\tilde{Q}_{t+1}, \tilde{P}_{t+1}, \tilde{\alpha}} f_{t+1}(z_{t,1} - \tilde{Q}_{t+1}\tilde{\alpha}, z_{t,2} - \tilde{Q}_{t+1}(1-\tilde{\alpha}))\}$$

$$= min_{0\leq x_t \leq \min\{[Q_t\alpha - z_{t-1,1}]^-, [Q_t(1-\alpha) - z_{t-1,2}]^+\}, \vec{z}_t \geq \vec{I}_t}\{-(F_t + \gamma + \eta^f)I_{t,1} - (F_t + \eta^f)I_{t,2} + I_{t,1}h_1$$

$$+ I_{t,2}h_2 + y_{t,1}(s_t + \gamma + \eta^s) + y_{t,2}(s_t + \eta^s) + G_t(\vec{z}_t)\}$$

where

$$y_{t,1} = [Q_t\alpha - z_{t-1,1}]^+,\ y_{t,2} = [Q_t(1-\alpha) - z_{t-1,2}]^+ - x_t$$

$$I_{t,1} = [Q_t\alpha - z_{t-1,1}]^- - x_t,\ I_{t,2} = [Q_t(1-\alpha) - z_{t-1,2}]^-$$

$$G_t(\vec{z}_t) = (F_t + \gamma + \eta^f)z_{t,1} + (F_t + \eta^f)z_{t,2} + \beta E^Q_{\tilde{Q}_{t+1}, \tilde{P}_{t+1}, \tilde{\alpha}} f_{t+1}(z_{t,1} - \tilde{Q}_{t+1}\tilde{\alpha}, z_{t,2}$$

$$-\tilde{Q}_{t+1}(1-\tilde{\alpha}))$$

The salvage value function is:

$$f_T\left(z_{T-1,1} - Q_T\alpha, z_{T-1,2} - Q_T(1-\alpha)\right) = y_{T,1}(s_T + \gamma + \eta^s) + y_{T,2}(s_T + \eta^s)$$

$$+I_{T,1}(h_1 - s_T - \gamma - \eta^f) + I_{T,2}(h_2 - s_T - \eta^f)$$

We assume the optimal procurement policies stay the same as Section 4. The value of downward substitution and the optimal downward substitution policy are developed as follows:

Theorem 5. *At* $t = 1, \ldots, T-1$, *when* $s_t + \eta^s - F_t - \gamma - \eta^f + h_1 > 0$, *the miller would try to slove the shortage issue of type* 2 *wheat by substitution as much as possible, i.e.* $x_t = min\{[Q_t\alpha - z_{t-1,1}]^-, [Q_t(1-\alpha) - z_{t-1,2}]^+\}$; *when* $s_t + \eta^s - F_t - \gamma - \eta^f + h_1 \le 0$, *the miller would not substitute at all, i.e.* $x_t = 0$.

At the final time point $t = T$, *when* $\eta^s - \gamma - \eta^f + h_1 > 0$, *the miller would try to slove the shortage issue of type 2 wheat by substitution as much as possible, i.e.* $x_T = min\{[Q_T\alpha - z_{T-1,1}]^-, [Q_T(1-\alpha) - z_{T-1,2}]^+\}$; *when* $\eta^s - \gamma - \eta^f + h_1 \le 0$, *the miller would not substitute at all, i.e.* $x_T = 0$.

Proof: See Section TA15.

At $\mathrm{t} = 1, \ldots, \mathrm{T} - 1$, when $\mathrm{s_t} + \eta^s - \mathrm{F_t} - \gamma - \eta^f + \mathrm{h_1} > 0$, the per unit cost of purchasing the missing type 2 wheat is higher than the expected per unit value of excessive type 1 wheat (so it is reasonable to substitute the missing part of type 2 wheat with excessive type 1 wheat as much as possible) ; when $\mathrm{s_t} + \eta^s - \mathrm{F_t} - \gamma - \eta^f + \mathrm{h_1} \le 0$ it is the other way around.

At t=T , when $\eta^s - \gamma - \eta^f + \mathrm{h_1} > 0$, the per unit cost of purchasing the missing type 2 wheat is higher than the per unit salvage value of excessive type 1 wheat; when $\eta^s - \gamma - \eta^f + \mathrm{h_1} \le 0$ it is the other way around.

Value of downward substitution in the commodity markets: The main body, if not all, of the relevant existing papers, exclusively focuses on the value of downward substitution against the negative effects of demand uncertainty (backlog penalty e.g.). However, that is not the complete picture of the appealing benefits brought by such an operation mechanism. As reveal by the above exploration, in the commodity markets, downward substitution may very likely bring extra economic benefits when the quality premium is not very high compared with the values of other parameters.

The convenience yield, $s_t - F_t + h_1$, is assumed to be positive by Goel and Gutierrez (2012), and the spot transporation cost on the spot market is always higher than the forward market, as discussed by Goel and Tanrisever (2011), which means $\eta^s - \eta^f > 0$. So when the quality premium is smaller than the sum of convenience yield and the difference between spot and forward transportation costs, i.e. $\gamma < (s_t - F_t + h_1) + (\eta^s - \eta^f)$, the value of downward substitution in the commodity markets, defined as follows, would be positive.

$$V(downward\ substitution)_t = (s_t - F_t + h_1) + (\eta^s - \eta^f) - \gamma, t = 1, \ldots, T-1$$

The stochastic price brings dynamic to the value of downward substitution in commodity markets, resulting in value increase or decrease. When the value is positive, downward substitution would be beneficial; when the value becomes negative, downward substitution should be avoided. In the following section, we are going to simulate how the value of downward substitution changes according to the blending ratio volatility, under a stochastic price process and different marginal profits.

6 Numerical simulation

6.1 Commodity price

The commodity prices are simulated based on the two-factor stochastic model developed by Schwartz and Smith (2000), which takes both short-term volatility and long-term equilibrium level in prices into consideration. In this model, the commodity price at a given time t, s_t is decomposed into two stochastic factors: $\ln(s_t) = \chi_t + \omega_t$, where χ_t stands for the short-term deviation and ω_t represents the equilibrium price level. The short-term deviation χ_t is revert to zero, following an Ornstein-Uhlen-beck process

$$d\chi_t = -(\kappa\chi_t + \lambda_\chi)dt + \sigma_\chi dz_\chi$$

The long-term factor is assumed to evolve following a Brownian motion process

$$d\omega_t = (\mu_\omega - \lambda_\omega)dt + \sigma_\omega dz_\omega$$

where dz_χ and dz_ω are increments of standard Brownian motion processes and they are correlated as $dz_\chi dz_\omega = \rho_{\chi\omega}dt$. The parameter κ is the rate of the mean reversion of the short-term factor χ_t, i.e. the rate at which the short-term deviations are expected to disappear. Parameters σ_χ and σ_ω describe the uncertainty for the short-term deviation factor and long-term equilibrium factor accordingly, λ_χ and λ_ω is the risk premium on short-term and long-term factor respectively, and μ_ω is the drift rate associated with the long-term factor.

Chockalingam and Muthuraman (2007) describes how to simulate the increments of standard Brownian Motion under given correlation coefficient. The value of the correlation coefficient together with other necessary parameters for the numerical simulation are collected from the paper of Goel and Gutierrez (2011) where the two-factor model is applied to explore the effect of term structure model of futures price on procurement policies for gasoline and wheat.

6.2 Downward substitution value

In this section we implement numerical experiments by altering operational parameters to explore how the value of downward substitution would change when the business environment becomes more challenging to the decision maker. We first describe the demand and quality requirement models, then we will explain the stochastic price process parameters for wheat before performing the numerical analysis. We assume the flour miller works on a bi-weekly schedule, and in each period (i.e. two weeks) the demand follows a normal distribution with mean equal to 600 bushels and the standard deviation of 50 bushels. Wheat procured from the forward market has a lower transportation cost compared with spot market (Goel and Tanrisever, 2011). We assume the transportation cost of 10 $cents/bushel$ from the forward market and 40 $cents/bushel$ from the spot market. The quality premium of type 1 (i.e. the good one) wheat is assumed to be 5 $cents/bushel$. The holding cost is assumed to be 12 $cents/bushel/period$ for type 1 wheat and 10 $cents/bushel/period$ for type 2 wheat. We assume the marginal profit of flour to be 17.5 $cents/bushel$. The quality requirement per period (i.e. the blending ratio) is assumed to follow a normal distribution with its mean equal to 0.55 and standard deviation equal to 0.05. The following table illustrates the price process parameters and the operational parameters for the base case. The price process parameters come from the paper by Goel and Gutierrez (2011).

Table 1: parameters value of base case

Parameters	Value	Item remarks
μ_ξ	600 bushels/period	demand mean
σ_ξ	50 bushels/period	demand standard deviation
η^s	40 cents/bushel	transportation cost from spot market
η^f	10 cents/bushel	transportation cost from forward market
h_1	12 cents/bushel/period	holding cost of type 1 wheat
h_2	10 cents/bushel/period	holding cost of type 2 wheat
γ	5 cents/bushel	quality premium
λ	17.5 cents/bushel	marginal profit
μ_α	0.55	blending ratio mean
σ_α	0.05	blending ratio standard deviation
χ_1	-0.081318691	initial short-term factor
k	0.9204	mean reversion rate of the short-term factor
λ_χ	-0.1473	risk premium on short-term factor
σ_χ	0.2826	short-term factor standard deviation
$\rho_{\chi\omega}$	0.1288	correlation ratio
ω_1	6.450380057	initial long-term factor
μ^*	0.0679	$\mu^* = \mu_\omega - \lambda_\omega$
σ_ω	0.1735	long-term factor standard deviation
β	0.998001999	discount rate
dt	0.04	fraction of two weeks (i.e. one period) in one year

We change the quality requirement uncertainty level (i.e. the blending ratio standard deviation) and marginal profit parameters while keeping the other parameters in the base case detailed in the table above, which results in three case groups as shown in the table below. Each case group has a fixed marginal profit, while the requirement uncertainty level increases.

Table 2: different business environments

case group 1	$(\lambda = 17.5\ cents/bushel)$	case group 2	$(\lambda = 22.5\ cents/bushel)$	case group 3	$(\lambda = 27.5\ cents/bushel)$
case 1	$\sigma_\alpha = 0.05$	case 6	$\sigma_\alpha = 0.05$	case 11	$\sigma_\alpha = 0.05$
case 2	$\sigma_\alpha = 0.075$	case 7	$\sigma_\alpha = 0.075$	case 12	$\sigma_\alpha = 0.075$
case 3	$\sigma_\alpha = 0.1$	case 8	$\sigma_\alpha = 0.1$	case 13	$\sigma_\alpha = 0.1$
case 4	$\sigma_\alpha = 0.125$	case 9	$\sigma_\alpha = 0.125$	case 14	$\sigma_\alpha = 0.125$
case 5	$\sigma_\alpha = 0.15$	case 10	$\sigma_\alpha = 0.15$	case 15	$\sigma_\alpha = 0.15$

We now compare the value of the model with downward substitution as discussed in Section 5 and the one without it described in Section 4. We apply Montel Carlo method and assign 10,000 sample paths to each of the 15 cases, under each model, with a planning horizon of $T = 4\,weeks$. The result is shown in Figure 2. The y-axis computes $\frac{Firm\ value\ with\ downward\ substitution - Firm\ value\ without\ downward\ substitution}{Firm\ value\ without\ downward\ substitution} * 100$, which suggests % benefit of downward substitution. The x-axis shows the blending ratio standard deviation, and each curve has a corresponding fixed marginal profit.

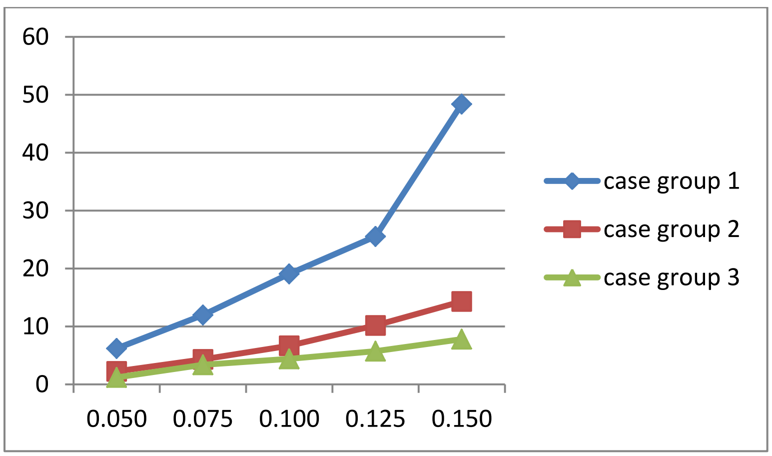

Figure 2: downward substitution value

As we can observe from Figure 2, whatever marginal profit it is in the simulation, the % benefit of downward substitution increases in a speeding up style along the rising of quality requirement uncertainty. Furthermore, the figure indicates that on the same quality requirement uncertainty level, % benefit of downward substitution is higher for the case with lower marginal profit.

7 Conclusion

As shown in the three cases above, myopic base-stock policies are optimal, because the G_t function is jointly convex. G_t function contains the tradeoff when ordering wheat for the next period: if the miller does not order enough, he/she is expected to pay for higher transportation fee per unit wheat on the spot market, i.e. $\eta^s - \eta^f$; if the miller orders more than enough, he/she is expected to lose $\vec{h}$ in per unit wheat value.

The higher the ratio of (extra expenses on the spot market compared with the forward market)/(the difference between the forward market price plus expected holding cost and the expected forward market price next period), the miller would utilize the forward market more. Though the numerator is always positive, the denominator may be negative when the transportation fee on the spot market is not bigger enough than the one on the forward market, or the discount rate β is too small. In such a case, the ratio would also be negative, and the miller should not order from the forward market at all, since it is more beneficial to procure from the spot market.

This thesis also examines the value of downward substitution in the commodity markets, and it turns out it would indeed bring extra value which depends on the convenience yield (Goel and Gutierrez, 2012), the difference between the forward and spot transportation costs (Goel and Tanrisever (2011) and the negative effects of losing the quality premium. The possibility of downward substitution brings more flexibility to the miller. But it seems the decision maker feels no incentives to change the procurement policy inherited from the previous case that does not perform downward substitution.

According to the numerical experiment result, the % benefit of downward substitution increases in a speeding up style along the rising of quality requirement uncertainty, and such a % is higher for the case with lower marginal profit, under the same quality requirement uncertainty level.

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Technical Appendix

The Technical Appendix section consists of three parts conceptually, and each part contains proofs for the theorems and results in Section 3, 4 and 5 accordingly. Part 1 includes TA1, TA2 up to TA7, and it can be further split into two sub-parts : TA1, TA2 and TA3 together serve as the step-by-step proofs of Theorem 1 which aims at exploring the existence of optimal policy qualitatively for the problem with demand uncertainty in Section 3 ; TA4 up to TA7 together established the explicitly optimal policy quantitatively, which leads to Theorem 2.

Part One

TA1. Lemma 1 and proof

This section acts as a preliminary step of Section TA2.

Lemma 1. Suppose that $f(z_1, z_2)$ is a convex function defined on R_+^2 and the real valued function M_1 is defined on R_+^2 by $M_1(\vec{z}) = E_D f(z_1 - \theta D, z_2 - (1-\theta)D)$, where D is a random variable following a general probability density function $\psi(\bullet)$, and $0 \le \theta \le 1$. Then M_1 is convex on R_+^2.

Proof: A, $\bar{A}$, B and $\bar{B}$ are four arbitrary none negative real values and $0 \le \kappa \le 1$ and $\bar{\kappa} = 1 - \kappa$, then

$\kappa \mathrm{M}_1(A, \bar{A}) + \bar{\kappa} M_1(B, \bar{B}) = \kappa \mathrm{E}_D f(A - \theta D, \bar{A} - (1-\theta)D) + \bar{\kappa} \mathrm{E}_D f(B - \theta D, \bar{B} - (1-\theta)D)$

$= \mathrm{E}_D \kappa f(A - \theta D, \bar{A} - (1-\theta)D) + \mathrm{E}_D \bar{\kappa} f(B - \theta D, \bar{B} - (1-\theta)D)$

$= \mathrm{E}_D(\kappa f(A - \theta D, \bar{A} - (1-\theta)D) + \bar{\kappa} f(B - \theta D, \bar{B} - (1-\theta)D))$

$\ge \mathrm{E}_D f(\kappa(A - \theta D) + \bar{\kappa}(B - \theta D), \kappa(\bar{A} - (1-\theta)D) + \bar{\kappa}(\bar{B} - (1-\theta)D))$ [because f is a convex function defined on R_+^2]

$= \mathrm{E}_D f(\kappa A + \bar{\kappa} B - \theta D, \kappa \bar{A} + \bar{\kappa} \bar{B} - (1-\theta)D)$

$= M_1(\kappa A + \bar{\kappa} B, \kappa \bar{A} + \bar{\kappa} \bar{B})$

So M_1 is convex on (z_1, z_2), i.e. convex on R_+^2.

TA2. Lemma 2 and proof

This section finishes the preparation of Section TA3.

Lemma 2. If f_{t+1} is convex on R_+^2, then the following hold:

a) G_t is convex on $(z_{t,1}, z_{t,2})$.

b) A base-stock policy is optimal in period t . Indeed, any minimizer of G_t is an optimal base-stock level.

c) f_t is convex on R_+^2.

Proof: a) if f_{t+1} is convex on R_+^2, according to Lemma 1, $E^Q_{\tilde{Q}_{t+1},\tilde{P}_{t+1}} f_{t+1}(z_{t,1} - \tilde{Q}_{t+1}\alpha, z_{t,2} - \tilde{Q}_{t+1}(1-\alpha))$ is convex on $(z_{t,1}, z_{t,2})$. $(F_t + \gamma + \eta^f)z_{t,1}$ and $(F_t + \eta^f)z_{t,2}$ are also convex on $(z_{t,1}, z_{t,2})$, thus G_t is the positive linear combinations of three convex functions. Hence G_t is also convex on $(z_{t,1}, z_{t,2})$.

b) Let $\vec{S}_t$ denote a minimizer of $G_t(\vec{z}_t)$ over all real value $\vec{z}_t$. If $\vec{I}_t < \vec{S}_t$, then the minimizing $\vec{z}_t \geq \vec{I}_t$ is at $\vec{z}_t = \vec{S}_t$, whereas, if $\vec{I}_t \geq \vec{S}_t$, then the minimizing $\vec{z}_t$ is at $\vec{z}_t = \vec{I}_t$. That is, a base-stock policy with base-stock level $\vec{S}_t$ is optimal for period t.

c) All the possible value of $\vec{z}_{t-1}$ constitues a convex set R_+^2, and all the possible value of $\vec{z}_t(\vec{z}_{t-1})$ constitutea nonempty set $A_1 = \{(z_{t,1}, \mathrm{z}_{t,2}) | z_{t,1} \geq [Q_t\alpha - z_{t-1,1}]^-, z_{t,2} \geq [Q_t(1-\alpha) - z_{t-1,2}]^-\}$ for every given $\vec{z}_{t-1}$, the set $\mathrm{C} = \{(\vec{z}_{t-1}, \vec{z}_t) | \vec{z}_{t-1} \in R_+^2, \vec{z}_t \in A_1\}$ is a convex set. Moreover, since $-(F_t + \gamma + \eta^f)I_{t,1}$, $-(F_t + \eta^f)I_{t,2}$, $I_{t,1}h_1$, $I_{t,2}h_2$, $y_{t,1}(s_t + \gamma + \eta^s)$, $y_{t,2}(s_t + \eta^s)$ and $G_t(\vec{z}_t)$ are all convex on C , their positive combination $-(F_t + \gamma + \eta^f)I_{t,1} - (F_t + \eta^f)I_{t,2} + I_{t,1}h_1 + I_{t,2}h_2 + y_{t,1}(s_t + \gamma + \eta^s) + y_{t,2}(s_t + \eta^s) + G_t(\vec{z}_t)$ is convex on C too and $f_t\left(z_{t-1,1} - Q_t\alpha, z_{t-1,2} - Q_t(1-\alpha)\right) = inf_{\vec{z}_t \in A_1}\{-(F_t + \gamma + \eta^f)I_{t,1} - (F_t + \eta^f)I_{t,2}$

$+I_{t,1}h_1 + I_{t,2}h_2 + y_{t,1}(s_t + \gamma + \eta^s) + y_{t,2}(s_t + \eta^s) + G_t(\vec{z}_t)\}$ and $f_t\left(z_{t-1,1} - Q_t\alpha, z_{t-1,2} - Q_t(1-\alpha)\right) > -\infty$ for every $\vec{z}_{t-1} \in \mathrm{R}_+^2$ constitues a convex set since the cost is always none negative, according to Theorem A.4 in Porteus (2002), f_t is a convex function on $(z_{t-1,1}, z_{t-1,2})$, i.e. convex on R_+^2.

TA3. Proof of Theorem 1

According to the definition of terminal value function in Section 3.1, the function is convex on R_+^2 . Thus, by Lemma 2 (a) and (b), G_{T-1} is jointly convex and a base-stock policy is optimal for period $T-1$. By Lemma 2 (c), f_{T-1} is convex as well. Thus, the argument iterates backward through the periods in the sequence $t = T-2, T-3, \ldots, 1$.

Up to here, the qualitative exploration about the existence of optimal policy is finished, and in the following sections (i.e. the second sub-part of part one, as mentioned at the beginning of Technique Appendix) the quantitative expression of the optimal policy is going to be developed.

TA4. Proof of Result 1

Before jumping into the main body of quantitative exploration, an one-period problem is solved first. The result is going to be referred to when establishing the optimal policy expression for each period in the multiple periods case studied in Section 3.

Let's examine the one-period problem at the end of the time horizon. The expected ordering, holding, and shortage cost, less any expected salvage value, in that period, starting with zero inventory and ordering up to $\vec{z}_{T-1}$ units can be written as

$$G_{T-1}(\vec{z}_{T-1}) = (F_{T-1} + \gamma + \eta^f) z_{T-1,1} + (F_{T-1} + \eta^f) z_{T-1,2}$$

$$+\beta E^{Q}_{\tilde{Q}_T,\tilde{P}_T} f_T(z_{T-1,1} - \tilde{Q}_T\alpha, z_{T-1,2} - \tilde{Q}_T(1-\alpha))$$

$$= (F_{T-1} + \gamma + \eta^f) z_{T-1,1} + (F_{T-1} + \eta^f) z_{T-1,2} + \beta E^{Q}_{\tilde{Q}_T,\tilde{P}_T}\{\left[\tilde{Q}_T\alpha - z_{T-1,1}\right]^{+}(s_T + \gamma + \eta^s)$$

$$+\left[\tilde{Q}_T(1-\alpha) - z_{T-1,2}\right]^{+}(s_T + \eta^s) + \left[\tilde{Q}_T\alpha - z_{T-1,1}\right]^{-}(h_1 - s_T - \gamma - \eta^f)$$

$$+\left[\tilde{Q}_T(1-\alpha) - z_{T-1,2}\right]^{-}(h_2 - s_T - \eta^f)\}$$

$$= (F_{T-1} + \gamma + \eta^f) z_{T-1,1} + (F_{T-1} + \eta^f) z_{T-1,2}$$

$$+\beta(F_{T-1} + \gamma + \eta^s)\int_{z_{T-1,1}/\alpha}^{\infty}\left(\xi_{T-1}\alpha - z_{T-1,1}\right)\psi(\xi_{T-1})d(\xi_{T-1})$$

$$+\beta(F_{T-1} + \eta^s)\int_{z_{T-1,2}/(1-\alpha)}^{\infty}\left(\xi_{T-1}(1-\alpha) - z_{T-1,2}\right)\psi(\xi_{T-1})d(\xi_{T-1})$$

$$+\beta(h_1 - F_{T-1} - \gamma - \eta^f)\int_{0}^{z_{T-1,1}/\alpha}\left(z_{T-1,1} - \xi_{T-1}\alpha\right)\psi(\xi_{T-1})d(\xi_{T-1})$$

$$+\beta(h_2 - F_{T-1} - \eta^f)\int_{0}^{z_{T-1,2/(1-\alpha)}}\left(z_{T-1,2} - \xi_{T-1}(1-\alpha)\right)\psi(\xi_{T-1})d(\xi_{T-1})$$

Let $\vec{S}_{T-1}$ denote a solution to

$$G'_{T-1}(\vec{z}_{T-1}) = 0$$

Then

$$G'_{T-1}(z_{T-1,1}) = F_{T-1} + \gamma + \eta^f - \beta(F_{T-1} + \gamma + \eta^s)\int_{z_{T-1,1}/\alpha}^{\infty} \psi(\xi_{T-1})d(\xi_{T-1})$$

$$+\beta(h_1 - F_{T-1} - \gamma - \eta^f)\int_0^{z_{T-1,1}/\alpha} \psi(\xi_{T-1})d(\xi_{T-1}) = 0$$

$$G'_{T-1}(z_{T-1,2}) = F_{T-1} + \eta^f - \beta(F_{T-1} + \eta^s)\int_{z_{T-1,2/(1-\alpha)}}^{\infty} \psi(\xi_{T-1})d(\xi_{T-1})$$

$$+\beta(h_2 - F_{T-1} - \eta^f)\int_0^{z_{T-1,2/(1-\alpha)}} \psi(\xi_{T-1})d(\xi_{T-1}) = 0$$

i.e.

$$\Psi\left(\frac{S_{T-1,1}}{\alpha}\right) = \frac{\beta(F_{T-1} + \gamma + \eta^s) - (F_{T-1} + \gamma + \eta^f)}{\beta(\eta^s + h_1 - \eta^f)}$$

$$\Psi\left(\frac{S_{T-1,2}}{1-\alpha}\right) = \frac{\beta(F_{T-1} + \eta^s) - (F_{T-1} + \eta^f)}{\beta(\eta^s + h_2 - \eta^f)}$$

TA5. Result 4 and proof

The result in this section functions in a similar way as the result of Section TA4: waiting to be referred to when establishing the ultimate optimal policy expression later.

Result 4. At time point $T-2$

$$E^Q_{\tilde{Q}_{T-1},\tilde{P}_{T-1}}\left\{\frac{d\left\{f_{T-1}\left(z_{T-2,1} - \tilde{Q}_{T-1}\alpha, z_{T-2,2} - \tilde{Q}_{T-1}(1-\alpha)\right)\right\}}{dz_{T-2,1}}\right\}$$

$= -(F_{T-2} + \gamma + \eta^f)\Psi\left(\frac{z_{T-2,1}}{\alpha}\right) + h_1\Psi\left(\frac{z_{T-2,1}}{\alpha}\right) - (F_{T-2} + \gamma + \eta^s)(1 - \Psi\left(\frac{z_{T-2,1}}{\alpha}\right))$, when $E(I_{T-1,1}) \leq E(S_{T-1,1})$

$$E^Q_{\tilde{Q}_{T-1},\tilde{P}_{T-1}}\left\{\frac{d\left\{f_{T-1}\left(z_{T-2,1} - \tilde{Q}_{T-1}\alpha, z_{T-2,2} - \tilde{Q}_{T-1}(1-\alpha)\right)\right\}}{dz_{T-2,2}}\right\}$$

$= -(F_{T-2} + \eta^f)\Psi\left(\frac{z_{T-2,2}}{1-\alpha}\right) + h_2\Psi\left(\frac{z_{T-2,2}}{1-\alpha}\right) - (F_{T-2} + \eta^s)(1 - \Psi\left(\frac{z_{T-2,2}}{1-\alpha}\right))$, when $E(I_{T-1,2}) \leq E(S_{T-1,2})$

Proof: Let's examine f_{T-1}, by plugging in the optimal decision for each state:

- $f_{T-1}\left(z_{T-2,1}-Q_{T-1}\alpha, z_{T-2,2}-Q_{T-1}(1-\alpha)\right)=\{-(F_{T-1}+\gamma+\eta^f)[Q_{T-1}\alpha- z_{T-2,1}]^- -(F_{T-1}+\eta^f)[Q_{T-1}(1-\alpha)-z_{T-2,2}]^- +[Q_{T-1}\alpha-z_{T-2,1}]^- h_1+ [Q_{T-1}(1-\alpha)-z_{T-2,2}]^- h_2+[Q_{T-1}\alpha-z_{T-2,1}]^+(s_{T-1}+\gamma+\eta^s)+[Q_{T-1}(1-\alpha)- z_{T-2,2}]^+(s_{T-1}+\eta^s)+G_{T-1}(S_{T-1,1},S_{T-1,2})\}$, when $I_{T-1,1}\le S_{T-1,1}$ and $I_{T-1,2}\le S_{T-1,2}$
- $f_{T-1}\left(z_{T-2,1}-Q_{T-1}\alpha, z_{T-2,2}-Q_{T-1}(1-\alpha)\right)=\{-(F_{T-1}+\gamma+\eta^f)[Q_{T-1}\alpha- z_{T-2,1}]^- -(F_{T-1}+\eta^f)[Q_{T-1}(1-\alpha)-z_{T-2,2}]^- +[Q_{T-1}\alpha-z_{T-2,1}]^- h_1+ [Q_{T-1}(1-\alpha)-z_{T-2,2}]^- h_2+[Q_{T-1}\alpha-z_{T-2,1}]^+(s_{T-1}+\gamma+\eta^s)+[Q_{T-1}(1-\alpha)- z_{T-2,2}]^+(s_{T-1}+\eta^s)+G_{T-1}(S_{T-1,1},I_{T-1,2})\}$, when $I_{T-1,1}\le S_{T-1,1}$ and $I_{T-1,2}> S_{T-1,2}$
- $f_{T-1}\left(z_{T-2,1}-Q_{T-1}\alpha, z_{T-2,2}-Q_{T-1}(1-\alpha)\right)=\{-(F_{T-1}+\gamma+\eta^f)[Q_{T-1}\alpha- z_{T-2,1}]^- -(F_{T-1}+\eta^f)[Q_{T-1}(1-\alpha)-z_{T-2,2}]^- +[Q_{T-1}\alpha-z_{T-2,1}]^- h_1+ [Q_{T-1}(1-\alpha)-z_{T-2,2}]^- h_2+[Q_{T-1}\alpha-z_{T-2,1}]^+(s_{T-1}+\gamma+\eta^s)+[Q_{T-1}(1-\alpha)- z_{T-2,2}]^+(s_{T-1}+\eta^s)+G_{T-1}(I_{T-1,1},S_{T-1,2})\}$, when $I_{T-1,1}> S_{T-1,1}$ and $I_{T-1,2}\le S_{T-1,2}$
- $f_{T-1}\left(z_{T-2,1}-Q_{T-1}\alpha, z_{T-2,2}-Q_{T-1}(1-\alpha)\right)=\{-(F_{T-1}+\gamma+\eta^f)[Q_{T-1}\alpha- z_{T-2,1}]^- -(F_{T-1}+\eta^f)[Q_{T-1}(1-\alpha)-z_{T-2,2}]^- +[Q_{T-1}\alpha-z_{T-2,1}]^- h_1+ [Q_{T-1}(1-\alpha)-z_{T-2,2}]^- h_2+[Q_{T-1}\alpha-z_{T-2,1}]^+(s_{T-1}+\gamma+\eta^s)+[Q_{T-1}(1-\alpha)- z_{T-2,2}]^+(s_{T-1}+\eta^s)+G_{T-1}(I_{T-1,1},I_{T-1,2})\}$, when $I_{T-1,1}> S_{T-1,1}$ and $I_{T-1,2}> S_{T-1,2}$

So at time point $T-2$

$$E^{Q}_{\tilde{Q}_{T-1},\tilde{P}_{T-1}}\left\{\frac{d\left\{f_{T-1}\left(z_{T-2,1}-\tilde{Q}_{T-1}\alpha, z_{T-2,2}-\tilde{Q}_{T-1}(1-\alpha)\right)\right\}}{dz_{T-2,1}}\right\}$$

$=-(F_{T-2}+\gamma+\eta^f)\Psi\left(\frac{z_{T-2,1}}{\alpha}\right)+h_1\Psi\left(\frac{z_{T-2,1}}{\alpha}\right)-(F_{T-2}+\gamma+\eta^s)(1-\Psi\left(\frac{z_{T-2,1}}{\alpha}\right))$, when $E(I_{T-1,1})\le E(S_{T-1,1})$

$$E^{Q}_{\tilde{Q}_{T-1},\tilde{P}_{T-1}}\left\{\frac{d\left\{f_{T-1}\left(z_{T-2,1}-\tilde{Q}_{T-1}\alpha, z_{T-2,2}-\tilde{Q}_{T-1}(1-\alpha)\right)\right\}}{dz_{T-2,1}}\right\}$$

$=-(F_{T-2}+\gamma+\eta^f)\Psi\left(\frac{z_{T-2,1}}{\alpha}\right)+h_1\Psi\left(\frac{z_{T-2,1}}{\alpha}\right)-(F_{T-2}+\gamma+\eta^s)\left(1-\Psi\left(\frac{z_{T-2,1}}{\alpha}\right)\right)+ E^{Q}_{\tilde{Q}_{T-1},\tilde{P}_{T-1}}\{\frac{dG_{T-1}(I_{T-1,1},W)}{dz_{T-2,1}}\}$, when $E(I_{T-1,1})\le E(S_{T-1,1})$ (here W stands for $I_{T-1,2}$ or $S_{T-1,2}$)

$$E^{Q}_{\tilde{Q}_{T-1},\tilde{P}_{T-1}}\left\{\frac{d\left\{f_{T-1}\left(z_{T-2,1}-\tilde{Q}_{T-1}\alpha, z_{T-2,2}-\tilde{Q}_{T-1}(1-\alpha)\right)\right\}}{dz_{T-2,2}}\right\}$$

$= -(F_{T-2}+\eta^f)\Psi\left(\frac{z_{T-2,2}}{1-\alpha}\right)+h_2\Psi\left(\frac{z_{T-2,2}}{1-\alpha}\right)-(F_{T-2}+\eta^s)(1-\Psi\left(\frac{z_{T-2,2}}{1-\alpha}\right))$, when $E(I_{T-1,2}) \leq E(S_{T-1,2})$

$$E^{Q}_{\tilde{Q}_{T-1},\tilde{P}_{T-1}}\left\{\frac{d\left\{f_{T-1}\left(z_{T-2,1}-\tilde{Q}_{T-1}\alpha, z_{T-2,2}-\tilde{Q}_{T-1}(1-\alpha)\right)\right\}}{dz_{T-2,2}}\right\}$$

$$= -(F_{T-2}+\eta^f)\Psi\left(\frac{z_{T-2,2}}{1-\alpha}\right)+h_2\Psi\left(\frac{z_{T-2,2}}{1-\alpha}\right)-(F_{T-2}+\eta^s)\left(1-\Psi\left(\frac{z_{T-2,2}}{1-\alpha}\right)\right)$$

$+E^{Q}_{\tilde{Q}_{T-1},\tilde{P}_{T-1}}\{\frac{dG_{T-1}(W,I_{T-1,2})}{dz_{T-2,2}}\}$, when $E(I_{T-1,2}) \leq E(S_{T-1,2})$ (here W stands for $I_{T-1,1}$ or $S_{T-1,1}$)

TA6. Lemma 3 and proof

This section develops the backward reasoning which is essential to develop the ultimate optimal policy quantitatively.

Lemma 3. If f_{t+1} is convex on R^2_+, and $E^{Q}_{\tilde{Q}_{t+1},\tilde{P}_{t+1}}\left\{\frac{d\left\{f_{t+1}\left(z_{t,1}-\tilde{Q}_{t+1}\alpha, z_{t,2}-\tilde{Q}_{t+1}(1-\alpha)\right)\right\}}{dz_{t,1}}\right\}$ $= -(F_t+\gamma+\eta^f)\Psi\left(\frac{z_{t,1}}{\alpha}\right)+h_1\Psi\left(\frac{z_{t,1}}{\alpha}\right)-(F_t+\gamma+\eta^s)(1-\Psi\left(\frac{z_{t,1}}{\alpha}\right))$, when $E(I_{t+1,1}) \leq E(S_{t+1,1})$ and $E^{Q}_{\tilde{Q}_{t+1},\tilde{P}_{t+1}}\left\{\frac{d\left\{f_{t+1}\left(z_{t,1}-\tilde{Q}_{t+1}\alpha, z_{t,2}-\tilde{Q}_{t+1}(1-\alpha)\right)\right\}}{dz_{t,2}}\right\}$ $= -(F_t+\eta^f)\Psi\left(\frac{z_{t,2}}{1-\alpha}\right)+h_2\Psi\left(\frac{z_{t,2}}{1-\alpha}\right)-(F_t+\eta^s)(1-\Psi\left(\frac{z_{t,2}}{1-\alpha}\right))$, when $E(I_{t+1,2}) \leq E(S_{t+1,2})$, where $\Psi\left(\frac{S_{t+1,1}}{\alpha}\right) = \frac{\beta(F_{t+1}+\gamma+\eta^s)-(F_{t+1}+\gamma+\eta^f)}{\beta(\eta^s+h_1-\eta^f)}$ and $\Psi\left(\frac{S_{t+1,2}}{1-\alpha}\right) = \frac{\beta(F_{t+1}+\eta^s)-(F_{t+1}+\eta^f)}{\beta(\eta^s+h_2-\eta^f)}$, then the following holds:

(a) The minimizer $\vec{S}_t$ which minimizes $G_t(\vec{z}_t)$ over all real value $\vec{z}_t$ fulfills $\Psi\left(\frac{S_{t,1}}{\alpha}\right) = \frac{\beta(F_t+\gamma+\eta^s)-(F_t+\gamma+\eta^f)}{\beta(\eta^s+h_1-\eta^f)}$ and $\Psi\left(\frac{S_{t,2}}{1-\alpha}\right) = \frac{\beta(F_t+\eta^s)-(F_t+\eta^f)}{\beta(\eta^s+h_2-\eta^f)}$.

(b) The optimal base-stock level in period t is also $\vec{S}_t$.

(c) f_t is jointly convex on R^2_+ and $E^{Q}_{\tilde{Q}_t,\tilde{P}_t}\left\{\frac{d\left\{f_t\left(z_{t-1,1}-\tilde{Q}_t\alpha, z_{t-1,2}-\tilde{Q}_t(1-\alpha)\right)\right\}}{dz_{t-1,1}}\right\}$

$= -(F_{t-1}+\gamma+\eta^f)\Psi\left(\frac{z_{t-1,1}}{\alpha}\right)+h_1\Psi\left(\frac{z_{t-1,1}}{\alpha}\right)-(F_{t-1}+\gamma+\eta^s)(1-\Psi\left(\frac{z_{t-1,1}}{\alpha}\right))$, when $E(I_{t,1})\le E(S_{t,1})$ and $E^Q_{\tilde{Q}_t,\tilde{P}_t}\left\{\frac{d\left\{f_t\left(z_{t-1,1}-\tilde{Q}_t\alpha, z_{t-1,2}-\tilde{Q}_t(1-\alpha)\right)\right\}}{dz_{t-1,2}}\right\} = -(F_{t-1}+\eta^f)\Psi\left(\frac{z_{t-1,2}}{1-\alpha}\right)+ h_2\Psi\left(\frac{z_{t-1,2}}{1-\alpha}\right)-(F_{t-1}+\eta^s)(1-\Psi\left(\frac{z_{t-1,2}}{1-\alpha}\right))$, when $E(I_{t,2})\le E(S_{t,2})$.

Proof: (a) As in Lemma 2 (a), G_t is jointly convex on $(z_{t,1}, z_{t,2})$. To see $\vec{S}_t$ fulfills $\Psi\left(\frac{S_{t,1}}{\alpha}\right)=\frac{\beta(F_t+\gamma+\eta^s)-(F_t+\gamma+\eta^f)}{\beta(\eta^s+h_1-\eta^f)}$ and $\Psi\left(\frac{S_{t,2}}{1-\alpha}\right)=\frac{\beta(F_t+\eta^s)-(F_t+\eta^f)}{\beta(\eta^s+h_2-\eta^f)}$

$$G_t(\vec{z}_t)=(F_t+\gamma+\eta^f)z_{t,1}+(F_t+\eta^f)z_{t,2}+\beta E^Q_{\tilde{Q}_{t+1},\tilde{P}_{t+1}} f_{t+1}(z_{t,1}-\tilde{Q}_{t+1}\alpha, z_{t,2}-\tilde{Q}_{t+1}(1-\alpha))$$

When $\vec{z}_t=\vec{S}_t$, according to the assumption in Lemma 2 above

$$\frac{dG_t\left(z_{t,1},z_{t,2}\right)}{dz_{t,1}}=(F_t+\gamma+\eta^f)+\beta E^Q_{\tilde{Q}_{t+1},\tilde{P}_{t+1}}\{\frac{d\{f_{t+1}(z_{t,1}-\tilde{Q}_{t+1}\alpha, z_{t,2}-\tilde{Q}_{t+1}(1-\alpha))\}}{dz_{t,1}}\}$$

$$=(F_t+\gamma+\eta^f)-\beta(F_t+\gamma+\eta^f)\Psi\left(\frac{z_{t,1}}{\alpha}\right)-\beta(F_t+\gamma+\eta^s)(1-\Psi\left(\frac{z_{t,1}}{\alpha}\right))+\beta h_1\Psi\left(\frac{z_{t,1}}{\alpha}\right)$$

$$=0$$

and

$$\frac{dG_t\left(z_{t,1},z_{t,2}\right)}{dz_{t,2}}=(F_t+\eta^f)+\beta E^Q_{\tilde{Q}_{t+1},\tilde{P}_{t+1}}\{\frac{d\{f_{t+1}(z_{t,1}-\tilde{Q}_{t+1}\alpha, z_{t,2}-\tilde{Q}_{t+1}(1-\alpha))\}}{dz_{t,2}}\}$$

$$=(F_t+\eta^f)-\beta(F_t+\eta^f)\Psi\left(\frac{z_{t,2}}{1-\alpha}\right)-\beta(F_t+\eta^s)(1-\Psi\left(\frac{z_{t,2}}{1-\alpha}\right))+\beta h_2\Psi\left(\frac{z_{t,1}}{1-\alpha}\right)$$

$$=0$$

So $\vec{S}_t$ indeed fulfills $\Psi\left(\frac{S_{t,1}}{\alpha}\right)=\frac{\beta(F_t+\gamma+\eta^s)-(F_t+\gamma+\eta^f)}{\beta(\eta^s+h_1-\eta^f)}$ and $\Psi\left(\frac{S_{t,2}}{1-\alpha}\right)=\frac{\beta(F_t+\eta^s)-(F_t+\eta^f)}{\beta(\eta^s+h_2-\eta^f)}$

(b) supported by Lemma 2 (b) directly.

(c) Lemma 2 (c) ensures that f_t is jointly convex on R^2_+. By calculating the consequences of using the optimal base-stock level in period t, we get:

$E^Q_{\tilde{Q}_t,\tilde{P}_t}\left\{\frac{d\left\{f_t\left(z_{t-1,1}-\tilde{Q}_t\alpha, z_{t-1,2}-\tilde{Q}_t(1-\alpha)\right)\right\}}{dz_{t-1,1}}\right\}=-(F_{t-1}+\gamma+\eta^f)\Psi\left(\frac{z_{t-1,1}}{\alpha}\right)+h_1\Psi\left(\frac{z_{t-1,1}}{\alpha}\right)-(F_{t-1}+\gamma+\eta^s)(1-\Psi\left(\frac{z_{t-1,1}}{\alpha}\right))$, when $E(I_{t,1})\le E(S_{t,1})$

$E^{Q}_{\tilde{Q}_t,\tilde{P}_t}\left\{\frac{d\left\{f_t\left(z_{t-1,1}-\tilde{Q}_t\alpha,z_{t-1,2}-\tilde{Q}_t(1-\alpha)\right)\right\}}{dz_{t-1,2}}\right\} = -(F_{t-1}+\eta^f)\Psi\left(\frac{z_{t-1,2}}{1-\alpha}\right) + h_2\Psi\left(\frac{z_{t-1,2}}{1-\alpha}\right) - (F_{t-1} + \eta^s)(1-\Psi\left(\frac{z_{t-1,2}}{1-\alpha}\right))$, when $E(I_{t,2}) \leq E(S_{t,2})$.

In the following section, the reader is going to see how the ultimate optimal policy expressions are speculated based on the backward reasoning results and all the preliminary work done in the previous sections.

TA7. Proof of Theorem 2

As proven in Section TA3, f_{T-1} is jointly convex on R^2_+, and $E^{Q}_{\tilde{Q}_{T-1},\tilde{P}_{T-1}}\left\{\frac{d\left\{f_{T-1}\left(z_{T-2,1}-\tilde{Q}_{T-1}\alpha,z_{T-2,2}-\tilde{Q}_{T-1}(1-\alpha)\right)\right\}}{dz_{T-2,1}}\right\} = -(F_{T-2}+\gamma+\eta^f)\Psi\left(\frac{z_{T-2,1}}{\alpha}\right) + h_1\Psi\left(\frac{z_{T-2,1}}{\alpha}\right) - (F_{T-2}+\gamma+\eta^s)(1-\Psi\left(\frac{z_{T-2,1}}{\alpha}\right))$, when $E(I_{T-1,1}) \leq E(S_{T-1,1})$, and

$E^{Q}_{\tilde{Q}_{T-1},\tilde{P}_{T-1}}\left\{\frac{d\left\{f_{T-1}\left(z_{T-2,1}-\tilde{Q}_{T-1}\alpha,z_{T-2,2}-\tilde{Q}_{T-1}(1-\alpha)\right)\right\}}{dz_{T-2,2}}\right\} = -(F_{T-2}+\eta^f)\Psi\left(\frac{z_{T-2,2}}{1-\alpha}\right) + h_2\Psi\left(\frac{z_{T-2,2}}{1-\alpha}\right) - (F_{T-2}+\eta^s)(1-\Psi\left(\frac{z_{T-2,2}}{1-\alpha}\right))$, when $E(I_{T-1,2}) \leq E(S_{T-1,2})$, as proven in result 4, where $\Psi\left(\frac{S_{T-1,1}}{\alpha}\right) = \frac{\beta(F_{T-1}+\gamma+\eta^s)-(F_{T-1}+\gamma+\eta^f)}{\beta(\eta^s+h_1-\eta^f)}$ and $\Psi\left(\frac{S_{T-1,2}}{1-\alpha}\right) = \frac{\beta(F_{T-1}+\eta^s)-(F_{T-1}+\eta^f)}{\beta(\eta^s+h_2-\eta^f)}$ according to Result 1.

So according to Lemma 3, in each period, the optimal order up to points exist and can be defined as follows:

$$\Psi\left(\frac{S_{t,1}}{\alpha}\right) = \frac{\beta(F_t+\gamma+\eta^s)-(F_t+\gamma+\eta^f)}{\beta(\eta^s+h_1-\eta^f)},\ t = 1,\ldots,T-1.$$

$$\Psi\left(\frac{S_{t,2}}{1-\alpha}\right) = \frac{\beta(F_t+\eta^s)-(F_t+\eta^f)}{\beta(\eta^s+h_2-\eta^f)},\ t = 1,\ldots,T-1.$$

Up to here, part one is over. As proved in this part, the problem studied in Section 3 has a structured optimal procurement policy for each period. The proofs associated with Section 4 and Section 5 are presented in Part two and three respectively. Part two covers TA8 up to TA14, with TA8 up to TA10 constitutes in a sub-part and TA11 up to TA14 together act as the second sub-part. In part 3, we develop the optimal downward substitution policy.

Part two

TA8. Lemma 4 and proof

Lemma 4. Suppose that $f(z_1, z_2)$ is a convex function defined on R_+^2 and the real valued function M_2 is defined on R_+^2 by $M_2(\vec{z}) = E_{D,\theta} f(z_1 - \theta D, z_2 - (1-\theta)D)$, where θ and D are random variables following a probability density function ø(•) and a general probability density function $\psi(\bullet)$ accordingly, and $0 \le \theta \le 1$. Then M_2 is convex on R_+^2.

Proof: A, $\bar{A}$, B and $\bar{B}$ are four arbitrary none negative real values and $0 \le \kappa \le 1$ and $\bar{\kappa} = 1 - \kappa$, then

$\kappa \mathrm{M}_2(A, \bar{A}) + \bar{\kappa} M_2(B, \bar{B}) = \kappa \mathrm{E}_{D,\theta} f(A - \theta D, \bar{A} - (1-\theta)D) + \bar{\kappa} \mathrm{E}_{D,\theta} f(B - \theta D, \bar{B} - (1-\theta)D)$

$= \mathrm{E}_{D,\theta} \kappa f(A - \theta D, \bar{A} - (1-\theta)D) + \mathrm{E}_{D,\theta} \bar{\kappa} f(B - \theta D, \bar{B} - (1-\theta)D)$

$= \mathrm{E}_{D,\theta}(\kappa f(A - \theta D, \bar{A} - (1-\theta)D) + \bar{\kappa} f(B - \theta D, \bar{B} - (1-\theta)D))$

$\ge \mathrm{E}_{D,\theta} f(\kappa(A - \theta D) + \bar{\kappa}(B - \theta D), \kappa(\bar{A} - (1-\theta)D) + \bar{\kappa}(\bar{B} - (1-\theta)D))$ [because f is a convex function defined on R_+^2]

$= \mathrm{E}_{D,\theta} f(\kappa A + \bar{\kappa} B - \theta D, \kappa \bar{A} + \bar{\kappa} \bar{B} - (1-\theta)D)$

$= M_2(\kappa A + \bar{\kappa} B, \kappa \bar{A} + \bar{\kappa} \bar{B})$

So M_2 is convex on (z_1, z_2), i.e. convex on R_+^2.

TA9. Lemma 5 and proof

Lemma 5. If f_{t+1} is convex on R_+^2, then the following hold:

a) G_t is convex on $(z_{t,1}, z_{t,2})$.

b) A base-stock policy is optimal in period t . Indeed, any minimizer of G_t is an optimal base-stock level.

c) f_t is convex on R_+^2.

Proof: a) if f_{t+1} is convex on R_+^2, according to Lemma 1, $E^{Q}_{\tilde{Q}_{t+1}, \tilde{P}_{t+1}, \tilde{\alpha}} f_{t+1}(z_{t,1} - \tilde{Q}_{t+1}\tilde{\alpha}, z_{t,2} - \tilde{Q}_{t+1}(1 - \tilde{\alpha}))$ is convex on $(z_{t,1}, z_{t,2})$. $(F_t + \gamma + \eta^f) z_{t,1}$ and $(F_t + \eta^f) z_{t,2}$ are also convex on $(z_{t,1}, z_{t,2})$, thus G_t is the positive linear combinations of three convex functions. Hence G_t is also convex on $(z_{t,1}, z_{t,2})$.

b) Let $\vec{S}_t$ denote a minimizer of $G_t(\vec{z}_t)$ over all real value $\vec{z}_t$. If $\vec{I}_t < \vec{S}_t$, then the minimizing $\vec{z}_t \geq \vec{I}_t$ is at $\vec{z}_t = \vec{S}_t$, whereas, if $\vec{I}_t \geq \vec{S}_t$, then the minimizing $\vec{z}_t$ is at $\vec{z}_t = \vec{I}_t$. That is, a base-stock policy with base-stock level $\vec{S}_t$ is optimal for period t.

c) All the possible value of $\vec{z}_{t-1}$ constitues a convex set R_+^2, and all the possible value of $\vec{z}_t(\vec{z}_{t-1})$ constitute a nonempty set $A_2 = \{(z_{t,1}, z_{t,2}) | z_{t,1} \geq [Q_t\alpha - z_{t-1,1}]^-, z_{t,2} \geq [Q_t(1-\alpha) - z_{t-1,2}]^-\}$ for every given $\vec{z}_{t-1}$, the set $C = \{(\vec{z}_{t-1}, \vec{z}_t) | \vec{z}_{t-1} \in R_+^2, \vec{z}_t \in A_2\}$ is a convex set. Moreover, since $-(F_t + \gamma + \eta^f)I_{t,1}$, $-(F_t + \eta^f)I_{t,2}$, $I_{t,1}h_1$, $I_{t,2}h_2$, $y_{t,1}(s_t + \gamma + \eta^s)$, $y_{t,2}(s_t + \eta^s)$ and $G_t(\vec{z}_t)$ are all convex on C, their positive combination $-(F_t + \gamma + \eta^f)I_{t,1} - (F_t + \eta^f)I_{t,2} + I_{t,1}h_1 + I_{t,2}h_2 + y_{t,1}(s_t + \gamma + \eta^s) + y_{t,2}(s_t + \eta^s) + G_t(\vec{z}_t)$ is convex on C too and

$$f_t\left(z_{t-1,1} - Q_t\alpha, z_{t-1,2} - Q_t(1-\alpha)\right) = inf_{\vec{z}_t \in A_2}\{-(F_t + \gamma + \eta^f)I_{t,1} - (F_t + \eta^f)I_{t,2}$$

$$+I_{t,1}h_1 + I_{t,2}h_2 + y_{t,1}(s_t + \gamma + \eta^s) + y_{t,2}(s_t + \eta^s) + G_t(\vec{z}_t)\}$$

and $f_t\left(z_{t-1,1} - Q_t\alpha, z_{t-1,2} - Q_t(1-\alpha)\right) > -\infty$ for every $\vec{z}_{t-1} \in R_+^2$ constitues a convex set since the cost is always none negative, according to Theorem A.4 in Porteous (2000), f_t is a convex function on $(z_{t-1,1}, z_{t-1,2})$, i.e. convex on R_+^2.

TA10. Proof of Theorem 3

According to the definition of terminal value function in Section 4, the function is convex on R_+^2. Thus, by Lemma 5 (a) and (b), G_{T-1} is jointly convex and a base-stock policy is optimal for period $T-1$. By Lemma 5 (c), f_{T-1} is convex as well. Thus, the argument iterates backward through the periods in the sequence $t = T-2, T-3, \ldots, 1$.

TA11. Proof of Result 2

Let's examine the one-period problem at the end of the time horizon. The expected ordering, holding, and shortage cost, less any expected salvage value, in that period, starting with zero inventory and ordering up to $\vec{z}_{T-1}$ units can be written as

$$G_{T-1}(\vec{z}_{T-1}) = (F_{T-1} + \gamma + \eta^f)z_{T-1,1} + (F_{T-1} + \eta^f)z_{T-1,2}$$

$$+\beta E^Q_{\tilde{Q}_T, \tilde{P}_T, \tilde{\alpha}} f_T(z_{T-1,1} - \tilde{Q}_T\tilde{\alpha}, z_{T-1,2} - \tilde{Q}_T(1-\tilde{\alpha}))$$

$$= (F_{T-1} + \gamma + \eta^f)z_{T-1,1} + (F_{T-1} + \eta^f)z_{T-1,2} + \beta E^Q_{\tilde{Q}_T, \tilde{P}_T, \tilde{\alpha}}\{[\tilde{Q}_T\tilde{\alpha} - z_{T-1,1}]^+(s_T + \gamma + \eta^s)$$

$$+[\tilde{Q}_T(1-\tilde{\alpha}) - z_{T-1,2}]^+(s_T + \eta^s) + [\tilde{Q}_T\tilde{\alpha} - z_{T-1,1}]^-(h_1 - s_T - \gamma - \eta^f)$$

$$+\left[\tilde{Q}_T(1-\tilde{\alpha}) - z_{T-1,2}\right]^{-}(h_2 - s_T - \eta^f)\}$$

$$= (F_{T-1} + \gamma + \eta^f)z_{T-1,1} + (F_{T-1} + \eta^f)z_{T-1,2}$$

$$+\beta(F_{T-1} + \gamma + \eta^s)\int_0^1 \int_{\frac{z_{T-1,1}}{\alpha}}^{\infty} \left(\xi_{T-1}\alpha - z_{T-1,1}\right)\psi(\xi_{T-1})ø(\alpha)d(\xi_{T-1})\, d(\alpha)$$

$$+\beta(F_{T-1} + \eta^s)\int_0^1 \int_{z_{T-1,2}/(1-\alpha)}^{\infty} \left(\xi_{T-1}(1-\alpha) - z_{T-1,2}\right)\psi(\xi_{T-1})ø(\alpha)d(\xi_{T-1})d(\alpha)$$

$$+\beta(h_1 - F_{T-1} - \gamma - \eta^f)\int_0^1 \int_0^{z_{T-1,1}/\alpha} \left(z_{T-1,1} - \xi_{T-1}\alpha\right)\psi(\xi_{T-1})ø(\alpha)d(\xi_{T-1})\, d(\alpha)$$

$$+\beta(h_2 - F_{T-1} - \eta^f)\int_0^1 \int_0^{z_{T-1,2/(1-\alpha)}} \left(z_{T-1,2} - \xi_{T-1}(1-\alpha)\right)\psi(\xi_{T-1})ø(\alpha)d(\xi_{T-1})d(\alpha)$$

Let $\vec{S}_{T-1}$ denote a solution to

$$G'_{T-1}(\vec{z}_{T-1}) = 0$$

Then

$$G'_{T-1}\left(z_{T-1,1}\right) = F_{T-1} + \gamma + \eta^f - \beta(F_{T-1} + \gamma + \eta^s)\int_0^1 \int_{z_{T-1,1}/\alpha}^{\infty} \psi(\xi_{T-1})ø(\alpha)d(\xi_{T-1})d(\alpha)$$

$$+\beta(h_1 - F_{T-1} - \gamma - \eta^f)\int_0^1 \int_0^{z_{T-1,1}/\alpha} \psi(\xi_{T-1})ø(\alpha)d(\xi_{T-1})d(\alpha) = 0$$

$$G'_{T-1}\left(z_{T-1,2}\right) = F_{T-1} + \eta^f - \beta(F_{T-1} + \eta^s)\int_0^1 \int_{z_{T-1,2/(1-\alpha)}}^{\infty} \psi(\xi_{T-1})ø(\alpha)d(\xi_{T-1})d(\alpha)$$

$$+\beta(h_2 - F_{T-1} - \eta^f)\int_0^1 \int_0^{z_{T-1,2/(1-\alpha)}} \psi(\xi_{T-1})ø(\alpha)d(\xi_{T-1})d(\alpha) = 0$$

i.e.

$$\int_0^1 \Psi\left(\frac{S_{T-1,1}}{\alpha}\right)ø(\alpha)d(\alpha) = \frac{\beta(F_{T-1} + \gamma + \eta^s) - (F_{T-1} + \gamma + \eta^f)}{\beta(\eta^s + h_1 - \eta^f)}$$

$$\int_0^1 \Psi\left(\frac{S_{T-1,2}}{1-\alpha}\right)ø(\alpha)d(\alpha) = \frac{\beta(F_{T-1} + \eta^s) - (F_{T-1} + \eta^f)}{\beta(\eta^s + h_2 - \eta^f)}$$

TA12. Result 5 and proof

Result 5. At time point $T-2$

$$E^{Q}_{\tilde{Q}_{T-1},\tilde{P}_{T-1},\tilde{\alpha}}\left\{\frac{d\left\{f_{T-1}\left(z_{T-2,1}-\tilde{Q}_{T-1}\tilde{\alpha},z_{T-2,2}-\tilde{Q}_{T-1}(1-\tilde{\alpha})\right)\right\}}{dz_{T-2,1}}\right\}$$

$= -(F_{T-2}+\gamma+\eta^{f})\int_0^1 \Psi\left(\frac{z_{T-2,1}}{\alpha}\right)ø(\alpha)d(\alpha) + h_1\int_0^1 \Psi\left(\frac{z_{T-2,1}}{\alpha}\right)ø(\alpha)d(\alpha) - (F_{T-2}+\gamma+\eta^{s})\int_0^1 (1-\Psi\left(\frac{z_{T-2,1}}{\alpha}\right))ø(\alpha)d(\alpha)$, when $E(I_{T-1,1}) \le E(S_{T-1,1})$

$$E^{Q}_{\tilde{Q}_{T-1},\tilde{P}_{T-1},\tilde{\alpha}}\left\{\frac{d\left\{f_{T-1}\left(z_{T-2,1}-\tilde{Q}_{T-1}\tilde{\alpha},z_{T-2,2}-\tilde{Q}_{T-1}(1-\tilde{\alpha})\right)\right\}}{dz_{T-2,2}}\right\}$$

$= -(F_{T-2}+\eta^{f})\int_0^1 \Psi\left(\frac{z_{T-2,2}}{1-\alpha}\right)ø(\alpha)d(\alpha) + h_2\int_0^1 \Psi\left(\frac{z_{T-2,2}}{1-\alpha}\right)ø(\alpha)d(\alpha) - (F_{T-2}+\eta^{s})\int_0^1 (1-\Psi\left(\frac{z_{T-2,2}}{1-\alpha}\right))ø(\alpha)d(\alpha)$, when $E(I_{T-1,2}) \le E(S_{T-1,2})$

Proof: Let's examine f_{T-1}, by plugging in the optimal decision for each state:

- $f_{T-1}\left(z_{T-2,1}-Q_{T-1}\alpha, z_{T-2,2}-Q_{T-1}(1-\alpha)\right) = \{-(F_{T-1}+\gamma+\eta^{f})\left[Q_{T-1}\alpha - z_{T-2,1}\right]^{-} - (F_{T-1}+\eta^{f})\left[Q_{T-1}(1-\alpha)-z_{T-2,2}\right]^{-} + \left[Q_{T-1}\alpha - z_{T-2,1}\right]^{-}h_1 + \left[Q_{T-1}(1-\alpha)-z_{T-2,2}\right]^{-}h_2 + \left[Q_{T-1}\alpha - z_{T-2,1}\right]^{+}(s_{T-1}+\gamma+\eta^{s}) + \left[Q_{T-1}(1-\alpha)-z_{T-2,2}\right]^{+}(s_{T-1}+\eta^{s}) + G_{T-1}\left(S_{T-1,1},S_{T-1,2}\right)\}$, when $I_{T-1,1} \le S_{T-1,1}$ and $I_{T-1,2} \le S_{T-1,2}$
- $f_{T-1}\left(z_{T-2,1}-Q_{T-1}\alpha, z_{T-2,2}-Q_{T-1}(1-\alpha)\right) = \{-(F_{T-1}+\gamma+\eta^{f})\left[Q_{T-1}\alpha - z_{T-2,1}\right]^{-} - (F_{T-1}+\eta^{f})\left[Q_{T-1}(1-\alpha)-z_{T-2,2}\right]^{-} + \left[Q_{T-1}\alpha - z_{T-2,1}\right]^{-}h_1 + \left[Q_{T-1}(1-\alpha)-z_{T-2,2}\right]^{-}h_2 + \left[Q_{T-1}\alpha - z_{T-2,1}\right]^{+}(s_{T-1}+\gamma+\eta^{s}) + \left[Q_{T-1}(1-\alpha)-z_{T-2,2}\right]^{+}(s_{T-1}+\eta^{s}) + G_{T-1}\left(S_{T-1,1},I_{T-1,2}\right)\}$, when $I_{T-1,1} \le S_{T-1,1}$ and $I_{T-1,2} > S_{T-1,2}$
- $f_{T-1}\left(z_{T-2,1}-Q_{T-1}\alpha, z_{T-2,2}-Q_{T-1}(1-\alpha)\right) = \{-(F_{T-1}+\gamma+\eta^{f})\left[Q_{T-1}\alpha - z_{T-2,1}\right]^{-} - (F_{T-1}+\eta^{f})\left[Q_{T-1}(1-\alpha)-z_{T-2,2}\right]^{-} + \left[Q_{T-1}\alpha - z_{T-2,1}\right]^{-}h_1 + \left[Q_{T-1}(1-\alpha)-z_{T-2,2}\right]^{-}h_2 + \left[Q_{T-1}\alpha - z_{T-2,1}\right]^{+}(s_{T-1}+\gamma+\eta^{s}) + \left[Q_{T-1}(1-\alpha)-z_{T-2,2}\right]^{+}(s_{T-1}+\eta^{s}) + G_{T-1}\left(I_{T-1,1},S_{T-1,2}\right)\}$, when $I_{T-1,1} > S_{T-1,1}$ and $I_{T-1,2} \le S_{T-1,2}$
- $f_{T-1}\left(z_{T-2,1}-Q_{T-1}\alpha, z_{T-2,2}-Q_{T-1}(1-\alpha)\right) = \{-(F_{T-1}+\gamma+\eta^{f})\left[Q_{T-1}\alpha - z_{T-2,1}\right]^{-} - (F_{T-1}+\eta^{f})\left[Q_{T-1}(1-\alpha)-z_{T-2,2}\right]^{-} + \left[Q_{T-1}\alpha - z_{T-2,1}\right]^{-}h_1 +$

$\left[Q_{T-1}(1-\alpha)-z_{T-2,2}\right]^{-}h_2+\left[Q_{T-1}\alpha-z_{T-2,1}\right]^{+}(s_{T-1}+\gamma+\eta^s)+\left[Q_{T-1}(1-\alpha)-z_{T-2,2}\right]^{+}(s_{T-1}+\eta^s)+G_{T-1}\left(I_{T-1,1},I_{T-1,2}\right)\}$, when $I_{T-1,1}>S_{T-1,1}$ and $I_{T-1,2}>S_{T-1,2}$

So at time point $T-2$

$$E^{Q}_{\tilde{Q}_{T-1},\tilde{P}_{T-1},\tilde{\alpha}}\left\{\frac{d\left\{f_{T-1}\left(z_{T-2,1}-\tilde{Q}_{T-1}\tilde{\alpha},z_{T-2,2}-\tilde{Q}_{T-1}(1-\tilde{\alpha})\right)\right\}}{dz_{T-2,1}}\right\}$$

$=-(F_{T-2}+\gamma+\eta^f)\int_0^1\Psi\left(\frac{z_{T-2,1}}{\alpha}\right)\o(\alpha)d(\alpha)+h_1\int_0^1\Psi\left(\frac{z_{T-2,1}}{\alpha}\right)\o(\alpha)d(\alpha)-(F_{T-2}+\gamma+\eta^s)\int_0^1(1-\Psi\left(\frac{z_{T-2,1}}{\alpha}\right))\o(\alpha)d(\alpha)$, when $E(I_{T-1,1})\le E(S_{T-1,1})$

$$E^{Q}_{\tilde{Q}_{T-1},\tilde{P}_{T-1},\tilde{\alpha}}\left\{\frac{d\left\{f_{T-1}\left(z_{T-2,1}-\tilde{Q}_{T-1}\tilde{\alpha},z_{T-2,2}-\tilde{Q}_{T-1}(1-\tilde{\alpha})\right)\right\}}{dz_{T-2,1}}\right\}$$

$=-(F_{T-2}+\gamma+\eta^f)\int_0^1\Psi\left(\frac{z_{T-2,1}}{\alpha}\right)\o(\alpha)d(\alpha)+h_1\int_0^1\Psi\left(\frac{z_{T-2,1}}{\alpha}\right)\o(\alpha)d(\alpha)-(F_{T-2}+\gamma+\eta^s)\int_0^1(1-\Psi\left(\frac{z_{T-2,1}}{\alpha}\right))\o(\alpha)d(\alpha)+E^{Q}_{\tilde{Q}_{T-1},\tilde{P}_{T-1},\tilde{\alpha}}\{\frac{dG_{T-1}(I_{T-1,1},W)}{dz_{T-2,1}}\}$, when $E(I_{T-1,1})\le E(S_{T-1,1})$ (here W stands for $I_{T-1,2}$ or $S_{T-1,2}$)

$$E^{Q}_{\tilde{Q}_{T-1},\tilde{P}_{T-1},\tilde{\alpha}}\left\{\frac{d\left\{f_{T-1}\left(z_{T-2,1}-\tilde{Q}_{T-1}\tilde{\alpha},z_{T-2,2}-\tilde{Q}_{T-1}(1-\tilde{\alpha})\right)\right\}}{dz_{T-2,2}}\right\}$$

$=-(F_{T-2}+\eta^f)\int_0^1\Psi\left(\frac{z_{T-2,2}}{1-\alpha}\right)\o(\alpha)d(\alpha)+h_2\int_0^1\Psi\left(\frac{z_{T-2,2}}{1-\alpha}\right)\o(\alpha)d(\alpha)-(F_{T-2}+\eta^s)\int_0^1(1-\Psi\left(\frac{z_{T-2,2}}{1-\alpha}\right))\o(\alpha)d(\alpha)$, when $E(I_{T-1,2})\le E(S_{T-1,2})$

$$E^{Q}_{\tilde{Q}_{T-1},\tilde{P}_{T-1},\tilde{\alpha}}\left\{\frac{d\left\{f_{T-1}\left(z_{T-2,1}-\tilde{Q}_{T-1}\tilde{\alpha},z_{T-2,2}-\tilde{Q}_{T-1}(1-\tilde{\alpha})\right)\right\}}{dz_{T-2,2}}\right\}$$

$=-(F_{T-2}+\eta^f)\int_0^1\Psi\left(\frac{z_{T-2,2}}{1-\alpha}\right)\o(\alpha)d(\alpha)+h_2\int_0^1\Psi\left(\frac{z_{T-2,2}}{1-\alpha}\right)\o(\alpha)d(\alpha)-(F_{T-2}+\eta^s)\int_0^1(1-\Psi\left(\frac{z_{T-2,2}}{1-\alpha}\right))\o(\alpha)d(\alpha)+E^{Q}_{\tilde{Q}_{T-1},\tilde{P}_{T-1},\tilde{\alpha}}\{\frac{dG_{T-1}(W,I_{T-1,2})}{dz_{T-2,2}}\}$, when $E(I_{T-1,2})\le E(S_{T-1,2})$ (here W stands for $I_{T-1,1}$ or $S_{T-1,1}$)

TA13. Lemma 6 and proof

Lemma 6. If f_{t+1} is convex on R_+^2, and $E^{Q}_{\tilde{Q}_{t+1},\tilde{P}_{t+1},\tilde{\alpha}}\left\{\frac{d\left\{f_{t+1}\left(z_{t,1}-\tilde{Q}_{t+1}\tilde{\alpha},z_{t,2}-\tilde{Q}_{t+1}(1-\tilde{\alpha})\right)\right\}}{dz_{t,1}}\right\}$

$= -(F_t + \gamma + \eta^f) \int_0^1 \Psi\left(\frac{z_{t,1}}{\alpha}\right) ø(\alpha)d(\alpha) + h_1 \int_0^1 \Psi\left(\frac{z_{t,1}}{\alpha}\right) ø(\alpha)d(\alpha) - (F_t + \gamma + \eta^s) \int_0^1 (1 - \Psi\left(\frac{z_{t,1}}{\alpha}\right)) ø(\alpha)d(\alpha)$, when $E(I_{t+1,1}) \le E(S_{t+1,1})$ and $E^Q_{\tilde{Q}_{t+1},\tilde{P}_{t+1},\tilde{\alpha}} \left\{\frac{d\left\{f_{t+1}\left(z_{t,1} - \tilde{Q}_{t+1}\tilde{\alpha}, z_{t,2} - \tilde{Q}_{t+1}(1-\tilde{\alpha})\right)\right\}}{dz_{t,2}}\right\} = -(F_t + \eta^f) \int_0^1 \Psi\left(\frac{z_{t,2}}{1-\alpha}\right) ø(\alpha)d(\alpha) + h_2 \int_0^1 \Psi\left(\frac{z_{t,2}}{1-\alpha}\right) ø(\alpha)d(\alpha) - (F_t + \eta^s) \int_0^1 (1 - \Psi\left(\frac{z_{t,2}}{1-\alpha}\right)) ø(\alpha)d(\alpha)$, when $E(I_{t+1,2}) \le E(S_{t+1,2})$, where $\int_0^1 \Psi\left(\frac{S_{t+1,1}}{\alpha}\right) ø(\alpha)d(\alpha) = \frac{\beta(F_{t+1}+\gamma+\eta^s)-(F_{t+1}+\gamma+\eta^f)}{\beta(\eta^s+h_1-\eta^f)}$ and $\int_0^1 \Psi\left(\frac{S_{t+1,2}}{1-\alpha}\right) ø(\alpha)d(\alpha) = \frac{\beta(F_{t+1}+\eta^s)-(F_{t+1}+\eta^f)}{\beta(\eta^s+h_2-\eta^f)}$, then the following holds:

(a) The minimizer $\vec{S}_t$ which minimizes $G_t(\vec{z}_t)$ over all real value $\vec{z}_t$ fulfills $\int_0^1 \Psi\left(\frac{S_{t,1}}{\alpha}\right) ø(\alpha)d(\alpha) = \frac{\beta(F_t+\gamma+\eta^s)-(F_t+\gamma+\eta^f)}{\beta(\eta^s+h_1-\eta^f)}$ and $\int_0^1 \Psi\left(\frac{S_{t,2}}{1-\alpha}\right) ø(\alpha)d(\alpha) = \frac{\beta(F_t+\eta^s)-(F_t+\eta^f)}{\beta(\eta^s+h_2-\eta^f)}$.

(b) The optimal base-stock level in period t is also $\vec{S}_t$.

(c) f_t is jointly convex on R_+^2 and $E^Q_{\tilde{Q}_t,\tilde{P}_t,\tilde{\alpha}} \left\{\frac{d\left\{f_t\left(z_{t-1,1} - \tilde{Q}_t\tilde{\alpha}, z_{t-1,2} - \tilde{Q}_t(1-\tilde{\alpha})\right)\right\}}{dz_{t-1,1}}\right\} = -(F_{t-1} + \gamma + \eta^f) \int_0^1 \Psi\left(\frac{z_{t-1,1}}{\alpha}\right) ø(\alpha)d(\alpha) + h_1 \int_0^1 \Psi\left(\frac{z_{t-1,1}}{\alpha}\right) ø(\alpha)d(\alpha) - (F_{t-1} + \gamma + \eta^s) \int_0^1 (1 - \Psi\left(\frac{z_{t-1,1}}{\alpha}\right)) ø(\alpha)d(\alpha)$, when $E(I_{t,1}) \le E(S_{t,1})$ and $E^Q_{\tilde{Q}_t,\tilde{P}_t,\tilde{\alpha}} \left\{\frac{d\left\{f_t\left(z_{t-1,1} - \tilde{Q}_t\tilde{\alpha}, z_{t-1,2} - \tilde{Q}_t(1-\tilde{\alpha})\right)\right\}}{dz_{t-1,2}}\right\} = -(F_{t-1} + \eta^f) \int_0^1 \Psi\left(\frac{z_{t-1,2}}{1-\alpha}\right) ø(\alpha)d(\alpha) + h_2 \int_0^1 \Psi\left(\frac{z_{t-1,2}}{1-\alpha}\right) ø(\alpha)d(\alpha) - (F_{t-1} + \eta^s) \int_0^1 (1 - \Psi\left(\frac{z_{t-1,2}}{1-\alpha}\right)) ø(\alpha)d(\alpha)$, when $E(I_{t,2}) \le E(S_{t,2})$.

Proof: (a) As in Lemma 5 (a), G_t is jointly convex on $(z_{t,1}, z_{t,2})$. To see $\vec{S}_t$ fulfills $\int_0^1 \Psi\left(\frac{S_{t,1}}{\alpha}\right) ø(\alpha)d(\alpha) = \frac{\beta(F_t+\gamma+\eta^s)-(F_t+\gamma+\eta^f)}{\beta(\eta^s+h_1-\eta^f)}$ and $\int_0^1 \Psi\left(\frac{S_{t,2}}{1-\alpha}\right) ø(\alpha)d(\alpha) = \frac{\beta(F_t+\eta^s)-(F_t+\eta^f)}{\beta(\eta^s+h_2-\eta^f)}$

$G_t(\vec{z}_t) = (F_t + \gamma + \eta^f)z_{t,1} + (F_t + \eta^f)z_{t,2} + \beta E^Q_{\tilde{Q}_{t+1},\tilde{P}_{t+1},\tilde{\alpha}} f_{t+1}(z_{t,1} - \tilde{Q}_{t+1}\tilde{\alpha}, z_{t,2}$

$-\tilde{Q}_{t+1}(1 - \tilde{\alpha}))$

When $\vec{z}_t = \vec{S}_t$, according to the assumption in Lemma 4 above

$$\frac{dG_t(z_{t,1}, z_{t,2})}{dz_{t,1}} = (F_t + \gamma + \eta^f) + \beta E^Q_{\tilde{Q}_{t+1},\tilde{P}_{t+1},\tilde{\alpha}} \{\frac{d\{f_{t+1}(z_{t,1} - \tilde{Q}_{t+1}\tilde{\alpha}, z_{t,2} - \tilde{Q}_{t+1}(1 - \tilde{\alpha}))\}}{dz_{t,1}}\}$$

$$= (F_t + \gamma + \eta^f) - \beta(F_t + \gamma + \eta^f)\int_0^1 \Psi\left(\frac{z_{t,1}}{\alpha}\right) ø(\alpha)d(\alpha)$$

$$-\beta(F_t + \gamma + \eta^s)\int_0^1 (1 - \Psi\left(\frac{z_{t,1}}{\alpha}\right)) ø(\alpha)d(\alpha) + \beta h_1 \int_0^1 \Psi\left(\frac{z_{t,1}}{\alpha}\right) ø(\alpha)d(\alpha)$$

$$= 0$$

and

$$\frac{dG_t\left(z_{t,1}, z_{t,2}\right)}{dz_{t,2}} = (F_t + \eta^f) + \beta E^Q_{\tilde{Q}_{t+1},\tilde{P}_{t+1},\tilde{\alpha}}\{\frac{d\{f_{t+1}(z_{t,1} - \tilde{Q}_{t+1}\tilde{\alpha}, z_{t,2} - \tilde{Q}_{t+1}(1-\tilde{\alpha}))\}}{dz_{t,2}}\}$$

$$= (F_t + \eta^f) - \beta(F_t + \eta^f)\int_0^1 \Psi\left(\frac{z_{t,2}}{1-\alpha}\right) ø(\alpha)d(\alpha)$$

$$-\beta(F_t + \eta^s)\int_0^1 (1 - \Psi\left(\frac{z_{t,2}}{1-\alpha}\right)) ø(\alpha)d(\alpha) + \beta h_2 \int_0^1 \Psi\left(\frac{z_{t,1}}{1-\alpha}\right) ø(\alpha)d(\alpha)$$

$$= 0$$

So $\vec{S}_t$ indeed fulfills $\int_0^1 \Psi\left(\frac{S_{t,1}}{\alpha}\right) ø(\alpha)d(\alpha) = \frac{\beta(F_t+\gamma+\eta^S)-(F_t+\gamma+\eta^f)}{\beta(\eta^S+h_1-\eta^f)}$ and $\int_0^1 \Psi\left(\frac{S_{t,2}}{1-\alpha}\right) ø(\alpha)d(\alpha) = \frac{\beta(F_t+\eta^S)-(F_t+\eta^f)}{\beta(\eta^S+h_2-\eta^f)}$

(b) supported by Lemma 5 (b) directly.

(c) Lemma 5 (c) ensures that f_t is jointly convex on R^2_+. By calculating the consequences of using the optimal base-stock level in period t, we get:

$E^Q_{\tilde{Q}_t,\tilde{P}_t,\tilde{\alpha}}\left\{\frac{d\left\{f_t\left(z_{t-1,1}-\tilde{Q}_t\tilde{\alpha}, z_{t-1,2}-\tilde{Q}_t(1-\tilde{\alpha})\right)\right\}}{dz_{t-1,1}}\right\} = -(F_{t-1} + \gamma + \eta^f)\int_0^1 \Psi\left(\frac{z_{t-1,1}}{\alpha}\right) ø(\alpha)d(\alpha) +$
$h_1 \int_0^1 \Psi\left(\frac{z_{t-1,1}}{\alpha}\right) ø(\alpha)d(\alpha) - (F_{t-1} + \gamma + \eta^s)\int_0^1 (1 - \Psi\left(\frac{z_{t-1,1}}{\alpha}\right)) ø(\alpha)d(\alpha)$, when $E(I_{t,1}) \leq E(S_{t,1})$

$E^Q_{\tilde{Q}_t,\tilde{P}_t,\tilde{\alpha}}\left\{\frac{d\left\{f_t\left(z_{t-1,1}-\tilde{Q}_t\tilde{\alpha}, z_{t-1,2}-\tilde{Q}_t(1-\tilde{\alpha})\right)\right\}}{dz_{t-1,2}}\right\}$
$= -(F_{t-1} + \eta^f)\int_0^1 \Psi\left(\frac{z_{t-1,2}}{1-\alpha}\right) ø(\alpha)d(\alpha) + h_2 \int_0^1 \Psi\left(\frac{z_{t-1,2}}{1-\alpha}\right) ø(\alpha)d(\alpha) - (F_{t-1} + \eta^s)\int_0^1 (1 -$
$\Psi\left(\frac{z_{t-1,2}}{1-\alpha}\right)) ø(\alpha)d(\alpha)$, when $E(I_{t,2}) \leq E(S_{t,2})$.

TA14. Proof of Theorem 4

As proven in Section TA10, f_{T-1} is jointly convex on R^2_+, and $E^{Q}_{\tilde{Q}_{T-1},\tilde{P}_{T-1},\tilde{\alpha}}\left\{\frac{d\left\{f_{T-1}\left(z_{T-2,1}-\tilde{Q}_{T-1}\tilde{\alpha},z_{T-2,2}-\tilde{Q}_{T-1}(1-\tilde{\alpha})\right)\right\}}{dz_{T-2,1}}\right\} =$

$-(F_{T-2}+\gamma+\eta^f)\int_0^1 \Psi\left(\frac{z_{T-2,1}}{\alpha}\right)ø(\alpha)d(\alpha)+h_1\int_0^1 \Psi\left(\frac{z_{T-2,1}}{\alpha}\right)ø(\alpha)d(\alpha)-(F_{T-2}+\gamma+\eta^s)\int_0^1(1-\Psi\left(\frac{z_{T-2,1}}{\alpha}\right))ø(\alpha)d(\alpha)$, when $E(I_{T-1,1})\le E(S_{T-1,1})$, and $E^{Q}_{\tilde{Q}_{T-1},\tilde{P}_{T-1},\tilde{\alpha}}\left\{\frac{d\left\{f_{T-1}\left(z_{T-2,1}-\tilde{Q}_{T-1}\tilde{\alpha},z_{T-2,2}-\tilde{Q}_{T-1}(1-\tilde{\alpha})\right)\right\}}{dz_{T-2,2}}\right\}$

$=-(F_{T-2}+\eta^f)\int_0^1 \Psi\left(\frac{z_{T-2,2}}{1-\alpha}\right)ø(\alpha)d(\alpha)+h_2\int_0^1 \Psi\left(\frac{z_{T-2,2}}{1-\alpha}\right)ø(\alpha)d(\alpha)-(F_{T-2}+\eta^s)\int_0^1(1-\Psi\left(\frac{z_{T-2,2}}{1-\alpha}\right))ø(\alpha)d(\alpha)$, when $E(I_{T-1,2})\le E(S_{T-1,2})$, as proven in result 5, where $\int_0^1 \Psi\left(\frac{S_{T-1,1}}{\alpha}\right)ø(\alpha)d(\alpha)=\frac{\beta(F_{T-1}+\gamma+\eta^s)-(F_{T-1}+\gamma+\eta^f)}{\beta(\eta^s+h_1-\eta^f)}$ and $\int_0^1 \Psi\left(\frac{S_{T-1,2}}{1-\alpha}\right)ø(\alpha)d(\alpha)=\frac{\beta(F_{T-1}+\eta^s)-(F_{T-1}+\eta^f)}{\beta(\eta^s+h_2-\eta^f)}$ according to Result 2.

So according to Lemma 6, in each period, the optimal order up to points exist and can be defined as follows:

$$\int_0^1 \Psi\left(\frac{S_{t+1,1}}{\alpha}\right)ø(\alpha)d(\alpha)=\frac{\beta(F_{t+1}+\gamma+\eta^s)-(F_{t+1}+\gamma+\eta^f)}{\beta(\eta^s+h_1-\eta^f)},\ t=1,\dots,T-1.$$

$$\int_0^1 \Psi\left(\frac{S_{t+1,2}}{1-\alpha}\right)ø(\alpha)d(\alpha)=\frac{\beta(F_{t+1}+\eta^s)-(F_{t+1}+\eta^f)}{\beta(\eta^s+h_2-\eta^f)},\ t=1,\dots,T-1.$$

Part three

TA15. Proof of Theorem 5

At time $t = 1, \dots, T-1$

$$\frac{d\left\{f_t\left(z_{t-1,1} - Q_t\alpha, z_{t-1,2} - Q_t(1-\alpha)\right)\right\}}{dx_t} = (F_t + \gamma + \eta^f) - h_1 - (s_t + \eta^s)$$

So, when $s_t + \eta^s - F_t - \gamma - \eta^f + h_1 > 0$, the miller would try to slove the shortage issue of type 2 wheat by substitution as much as possible, i.e. $x_t = min\{[\ Q_t\alpha - z_{t-1,1}]^-, [Q_t(1-\alpha) - z_{t-1,2}]^+\};$; when $s_t + \eta^s - F_t - \gamma - \eta^f + h_1 \leq 0$, the miller would not substitute at all, i.e. $x_t = 0$.

At time $t = T$

$$\frac{d\left\{f_T\left(z_{T-1,1} - Q_T\alpha, z_{T-1,2} - Q_T(1-\alpha)\right)\right\}}{dx_T} = (\gamma + \eta^f) - h_1 - \eta^s$$

So, when $\eta^s - \gamma - \eta^f + h_1 > 0$, the miller would try to slove the shortage issue of type 2 wheat by substitution as much as possible, i.e. $x_T = min\{[\ Q_T\alpha - z_{T-1,1}]^-, [Q_T(1-\alpha) - z_{T-1,2}]^+\};$; when $\eta^s - \gamma - \eta^f + h_1 \leq 0$, the miller would not substitute at all, i.e. $x_T = 0$.