MASTER

Scheduling of continuous multi-grade, multi-stage, finite intermediate buffer processes with application in chemical industry

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Scheduling of Continuous Multi-grade, Multi-stage, Finite Intermediate Buffer Processes with Application in Chemical Industry

by

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Abstract

An improved scheduling methodology will be developed for a two-stage continuous HDPE production process with a limited intermediate buffer at The Dow Chemical Company in Tessenderlo, Belgium. The problem under consideration is in fact less strict than a hard limit on the intermediate buffer; excess storage is allowed, but at a cost. The problem is characterized by significant differences in processing rate over the two processing stages, which appears to be one of the causes of the problem.

Preface and acknowledgments

The Master's Thesis you are about to read is a result of a seven-months internship at the Dow Chemical company in Tessenderlo, Belgium and is written in partial fulfillment for the title of Master of Science in Operations Management and Logistics.

I would like to particularly thank my university supervisor Jan Fransoo for his commitment to the project and valuable help and advice when needed, and Nico Dellaert for his help and assistance, especially with programming which turned out to be quite a challenge.

Thanks go to all involved people at Dow Tessenderlo and Terneuzen, especially Marc Celen and Ralph Croes for their support, discussions, and comments, to Donnie Fruijtier for giving me the opportunity to do this project, and many others who made this project not only a very good learning experience but also a very enjoyable one: Miranda, Marjoleine, Monica, Joris, Ludo, and Niels.

As this project is the last feat of my time as a student, many thanks go to my friends and everybody who has been involved the past years. A special word of thanks goes to Serena who received less attention than she deserves, especially during the last period of this project. Also many thanks go to my parents Ria and Vincent for providing me lots of support during my studies, not only financially.

And last but not least, since you were just about to start reading this thesis, thank you for your interest in this thesis and enjoy!

Management Summary

Background

A two-stage multi-grade continuous polymer production process consisting of multiple machines per stage faces high costs associated with the storage of intermediate products. A limited amount of intermediate storage facilities is available on-site free of charge, but excess intermediate inventory has to be stored externally at a cost.

Research methods

As production rates of machines at both stages are rather unequal, material balance and hence intermediate inventory levels are affected by the production schedule. Scheduling in a certain fashion can therefore optimize usage of intermediate storage facilities.

At present, a mixed make-to-order (MTO)/make-to-stock (MTS) planning concept is utilized. A MTS concept means that all demand is fulfilled from final goods inventory. The opposite is MTO in which products are produced once orders arrive. In the actual case, mixed MTS/MTO means that schedule disturbances may occur due to demand fluctuations, in other words sudden demand changes may lead to sudden schedule changes. A statistical analysis is carried out on the feasibility of implementing a *pure* make-to-stock (MTS) planning concept, determining finished goods inventory (FGI) levels required to cope with demand fluctuations. The main advantage of implementing a pure MTS concept is the possibility to schedule the plant optimally in terms of maximizing throughput and minimizing intermediate storage costs while eliminating schedule disturbances (alterations) due to demand fluctuations. The statistical analysis led to the insight that only slight increases in FGI levels would be necessary for implementing a pure MTS concept (a fill rate of 99% requires on average 7260 tons, while in 2007 FGI amounts have varied between 6000 and 8000 t).

The next step, scheduling optimally in terms of throughput and intermediate storage costs, is accomplished by means of a scheduling model based on linear programming techniques. This linear programming formulation takes into account operational constraints that are critical in forming feasible schedules, such as production rates and sequence dependent setup times. Inputs for the scheduling model are the desired production quantities per grade, the schedule horizon length, machine setups, and available raw and intermediate material quantities at the beginning and at the end of a month.

The developed scheduling model is able to distinguish between on-site intermediate storage and costly external storage. The model is also used in investigating possible benefits of extending the on-site storage space by altering the modeled number of silos and assessing the results for the production schedule.

As the model does include transporting costs but is unable to take external holding costs into account due to modeling issues, a different model is used to determine the optimal production cycle lengths, which is mainly a tradeoff between setup costs and holding costs.

Major findings

Based on the scheduling model output using January, February and March demand data as input and corrected by machine failures as they actually have taken place during these months, it is found that there is room to increase plant throughput considerably by optimizing the scheduling processes. Results (displayed in Table 1.1) are obtained by documented metrics (column 'Manual') and scheduling model output ('Model'). The last column represents a scheduling model output, corrected with machine failures as they have actually taken place, in order to fairly compare model results with reality. Although transport and silo rental costs are inflated a bit, it is concluded that the possibility of a minor increase in intermediate storage costs is outweighed by extra revenues due to increased plant throughput. Moreover, in this comparison the value of throughput (the margin of 1 t of product) is chosen very conservatively at [redacted] €/t. In reality this value may even be [redacted] €/t, vastly increasing the value of the model schedule.

Table 1.1. Comparison manual schedule vs. schedule by model, January data. Numbers have been altered to protect confidentiality.

Metric	**Manual**	**Model**	**Model (corrected with machine failures)**
Throughput [t]	13,977	15,195	14,739 (97% asset utilization)
Transported qty [trucks]	321	180	unknown
Transport cost [€]	45,477	21,600	45,477 (estimated)
Silo rental cost [€]	50,000 (approx.)	46,043	50,000 (estimated)
Relative value (approx.)	100%	110.5%	105.4%

The key conclusions and recommendations for Dow Tessenderlo are listed below. All recommendations can be applied individually and are not dependent of each other.

- Do not let demand dynamics lead to schedule disturbances, thereby retaining schedule efficiency and avoiding costly disturbances. This can be realized by slightly increasing the final goods inventory to a level that is sufficient to cope with demand uncertainty. This means that in essence a make-to-stock policy is applied and the customer-order decoupling point is placed at the finished goods inventory level.
- Currently the production frequency for the most grades is below the optimum. Most grades have an optimal production cycle of at least six weeks (cost curves are depicted in Figure A). Optimal production cycle lengths depend on a number of factors, but setup costs and holding costs play the largest role. While holding costs will increase when cycle length is increased as a result of the increased average cycle stock, losses in production time due to less changeovers make up for this and can lead to significant cost reductions. This especially concerns production runs on the first production step as long changeover times are involved. Optimal production lengths correspond to low values of Total Relevant Costs (TRC). Cost reductions can be high as the cost curves are especially steep at the relevant left side. Note that the shown TRC values must be interpreted as approximations.
- Using the developed scheduling model for monthly scheduling will optimize the schedule value in terms of throughput and intermediate storage cost and thus provide a schedule which can be used as a good starting point.
- On extending the intermediate storage space: although gains are to be expected on the area of material handling ease and safety, investments in extra on-site intermediate storage space cannot be justified using the scheduling model. While the number of silos is a model parameter that can be easily altered, no proof is found that the attainable savings will be significantly higher than the related investment costs.

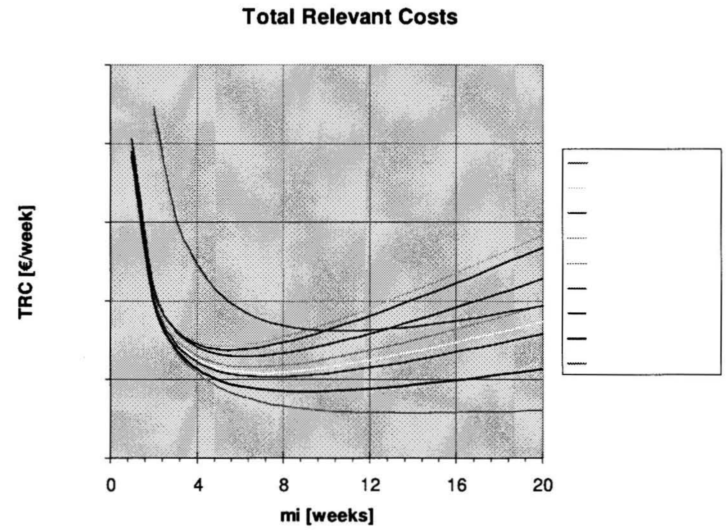

Figure A. Total Relevant Costs as a function of production cycle length m_i per grade i.

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Chapter 1. Introduction

This chapter introduces the study that is performed at the Tessenderlo site of The Dow Chemical Company on scheduling of multi-stage finite intermediate buffer continuous processes. First a general lead-up to the company where the study is carried out is given with a brief description of its current relevant manufacturing and planning processes. This leads to the formulation of the problem statement in section 1.4.1 , followed by the research methodology and an overview of the structure of this report.

1.1 Study context

The Tessenderlo (Belgium) site of The Dow Chemical Company produces polystyrene and high-density polyethylene (HDPE). The focus of this project is on the HDPE plant. HDPE is made in a two-step production process; ethylene gas is converted into HDPE powder, which in the second step is extruded to HDPE pellets. High costs are associated with intermediate storage and logistics. The plant also occasionally suffered from losses in extrusion capacity due to not having the required intermediate products available at the right time. An opportunity lies in the reduction of intermediate storage costs while maximizing production throughput.

A main cause for high intermediate storage costs lies in the limitedness of the internal storage capacity. This means that the two process steps are highly interdependent with the consequence that intermediate product quantities –and thereby storage and handling costs– depend heavily on the production sequence of both steps. An opportunity lies therefore in the development of an optimal production schedule that minimizes costs of intermediates while maximizing the overall plant throughput.

To this end a model is to be developed that takes fundamental operational constraints, particularly the limited intermediate storage space, into account and translates given demand figures into a feasible (nearly) optimal production schedule. This model will also facilitate investment recommendations on extension of intermediate storage space, depending on extension cost versus logistics costs.

1.2 Problem description

1.2.1 Present problems

During the past decennia, improvements have led to asymmetrical maximum production rates over the two stages. This will be covered in more detail later, but the essence is that having different production rates over the stages has consequences for the intermediate product inventory. A higher production rate at the first stage leads to an increasing intermediate inventory over time, and vice versa.

However, the on-site intermediate storage space is limited. This leads to a high dependency of the two process steps. The amount of intermediate storage space that would be needed to decouple the two steps is higher than the amount available. Nevertheless, external off-site storage space can be rented, but this leads to high logistic costs due to the transport of intermediate product to external storage facilities of partner companies and vice versa. An additional disadvantage of this transport which is less easily expressed in monetary value is the increased safety risk as public roads are involved with this transport. Decoupling is desirable because the throughput of the individual processors differs greatly (see Table 2.1). More intermediate storage space makes longer runtimes possible, hence less time-consuming setups. Minimizing external storage costs can ultimately lead to a lower cost-to-serve while customer service should remain unaffected. A low cost-to-serve is a main strategic direction of The Dow Chemical Company.

Another issue is the risk of lost extrusion capacity due to not having the required intermediate product type available in time. This risk is not completely unavoidable, as the combined capacity of the first production stage is 22 t/h, which is less than the combined capacity of the second stage: approximately 24 t/h. Taking setup times into account however, the throughput of the two stages do not differ much. All in all it is key to ensure that always sufficient intermediate product is available for the second stage to keep running. Only this way the utilization and thus the plant throughput can be maximized.

1.3 The issue of reducing intermediate storage usage and handling costs

There are a couple of motives behind the desire to reduce the amounts of intermediate inventory. Currently the amount of intermediate storage space is 6 silos of 150 tons each which makes a total of 900 t on-site. When also utilizing external third-party off-site storage, the maximum storage amounts to 6000–7000 t. Most of the time however, the actual stored quantity is approximately 3000–4500 t. Hence a large share of this is stored externally. A significant amount of money is involved in this external storage (details are given in section 2.1.5 below). This is in fact an important motive behind this research.

Apart from that, having large amounts of intermediate product (powder) also means a certain risk for the business. In essence, oil is converted into powder (via a number of processes). Suppose the oil price drops considerably, the business carries expensive powder for quite some time, as the throughput of reaction and extrusion are rather similar. It would be more desirable to have more end-inventory instead, as relatively high amounts of pellets can be sold quickly via so-called spot-deals. Spot-deals do in fact not happen much but at least there is the possibility.

A last advantage of reducing transports between the Tessenderlo site and the external storage facilities is a reduction in safety risks.

1.4 Project approach

The project will be carried out in two main phases. The first part comprises a survey of all relevant processes: the actual production processes including all related (physical) constraints, site logistics, procurement, sales, order acceptance, demand forecasting, and the like. Costs that are incurred by these processes, above all the costs of powder-transport and external storage will also be investigated. The second part addresses the research question. In order to do so, a planning model that reflects the Tessenderlo situation will be developed. This model will be an adaptation and extension of existing approaches from the scheduling literature, and will form a major part of the project.

An algorithm will be applied to the planning model, seeking to minimize the sum of both work-in-process and end-inventory holding costs, transportation costs, and setup costs, resulting in a (near-) optimal solution. The corresponding cost figure of this solution can be estimated. Inefficiencies are possibly present in the current manual planning processes but are to a certain degree unavoidable. The extent to which they are avoidable can perhaps be extended by the assistance of an automated planning system.

A planning model can serve another purpose: changes in system behavior that result from changing system parameters can be simulated. Altering critical constraints such as the quantity of available internal intermediate storage space generates other optimal schedules with other cost figures. Assessments can be made for possible investment opportunities: do benefits of investments weigh up against costs? It has however to be noted that cost savings as computed by a model will not automatically be fully achieved in practice: applied planning practices determine whether these savings are realized or not. Comparisons of cost figures of different situations do make estimates possible about the *upper limits* of attainable savings, providing a basis for justification of potential investment opportunities.

Integration of scheduling decisions in the supply chain

Tousain and Bosgra (2006) studied market-oriented scheduling and economic optimization of continuous multi-grade chemical processes. The studied problem shows similarities with the scheduling problem of Tessenderlo; the focus of their work is on demand-driven market-oriented scheduling, although restricted to a single machine only. Particular aspects of Tousain and Bosgra (2006) seem very suitable for application in the Tessenderlo scheduling problems: grade change penalties are studied extensively and will show much resemblance.

The market-oriented approach of Tousain and Bosgra is also interesting for Tessenderlo, as their demand for HDPE is often ample. It can therefore be interesting to integrate the decision making or negotiation of sales orders with the scheduling of the plant. One sales order might look less attractive than another when calculated individually, but when taking the existing order portfolio into account this figure may look different, for instance due to the (un)necessity of grade changes. Other benefits of integrating scheduling decisions with both procurement and sales order decisions may be to make better use of procurement opportunities. A general problem that arises with this type of integration is that order acceptance decisions have to be taken instantaneously; it is not realistic to put a customer on hold for some time while a computer determines feasibility and attractiveness of a particular order.

The effect of this integration in this type of industry is questionable though: all products are predominantly made from the same raw material; ethylene. The ethylene demand is also rather constant as Tessenderlo HDPE generally runs at 100% of its capacity, regardless of market conditions: sales prices are adapted instead in order to match supply with demand. Procurement opportunities are mainly a function of inventory holding costs; the higher holding costs are, the less attractive it is to buy large amounts of raw materials in advance. Holding costs for ethylene don't apply for physical storage, as it is being delivered by pipes, but rather for costs of advance payments. Focusing on integrating scheduling with procurement however need not become too strict: scheduling cannot change much in the ethylene usage, because the usage rate does not depend much on the production sequence. Physical storage limitations only apply for other raw materials that are kept on stock. Usage rates for these materials (mainly additives) are much lower. Other long term objectives also play a role in real life: important customers should get a higher priority when not all demand can be fulfilled.

1.4.1 Problem definition and discussion of research questions

The basis of the problem lies in the fact that the throughput of different machines at the first and second stage differ much. Would these have been identical, then the stages need not be decoupled and only little intermediate storage is needed. Planning processes then would have been much more straightforward and could in essence be regarded as a single stage process.

The different production rates however are a given. Decoupling is therefore necessary. As relatively long changeover times are involved, long runtimes are desirable to avoid wasting too much time on changeovers. To realize long runtimes, a considerable amount of intermediate storage space is needed as production rates of the two stages are rather different. The present amount of on-site intermediate storage is however very limited; therefore external off-site space is used, in spite of the costliness. Because of the involved costs the amounts of off-site storage should be kept to a minimum, yet the amount of changeovers should also be minimized as discussed above. Another factor that plays a role in runtime or lotsizing decisions is the holding cost of finished products: longer runtimes lead to higher average amounts of finished goods inventory.

The relevant aspects that are impacted by scheduling are setup costs and both intermediate and finished goods holding costs. The objective of the project is therefore to develop a scheduling process that minimizes average costs per unit time. Costs include transportation, setup, work-in-process inventory holding costs and finished goods inventory holding costs per unit time. In the flowshop scheduling literature, makespan minimization is often applied. As discussed above, this is not necessarily the best objective in the Tessenderlo case and in some other applications in practice. While minimizing makespan will to a certain degree undoubtedly correspond with economic benefits, these economic gains are only guaranteed when specifically aimed for like in ELSP models. The research question is therefore formulated as follows:

How to schedule continuous multi-grade, multi-stage, finite intermediate buffer chemical processes, minimizing the total sum of setup costs, work-in-process inventory holding costs, and finished goods inventory holding costs?

Work-in-process inventory holding costs do in this project mainly consist of the costs that are involved with storing (and retrieving) excess intermediate inventory to external storage sites, and will form an important part of the total relevant costs.

While in fact the Tessenderlo production situation can initially be regarded as a *finite* intermediate buffer situation, in reality off-site storage space can be rented. The distinction between off-site storage and on-site storage without charge also raises a question.

Is investing in additional on-site intermediate storage space an attractive investment opportunity, and if so, in what storage quantity?

1.5 Literature analysis

Classical makespan minimization models do not deal with machine assignment and lot sizing issues (Akrami, Karimi, and Hosseini, 2006). In certain applications therefore, or at least in the practice problem considered in this project and discussed in the next chapter, practical usability of ELSP approaches is likely to be higher. A large field of application in practice encompasses multi-stage, multi-processor plants, typically denoted as hybrid flowshops or flexible flowshops. These flowshops oftentimes use intermediate storage space in between stages. Reasons to use intermediate buffers are, among others, to decouple operation of adjacent machines, and to reduce the effects of variation of process parameters and machine breakdowns (Ku and Karimi, 1988). When in practice the amount of intermediate storage space is limited, it is important to address this in a scheduling model in order not to end up with infeasible schedules.

A couple of contributions can be found in the literature on multi-stage, multi-processor ELSP models, of which Linn and Zhang (1999) presented a comprehensive overview. The majority of flexible flowshop articles deals with restricted problems such as a single machine at one of the stages, reducing both the complexity of the problem and the flexibility of the solution approach. While recently the demand for incorporation of more realistic assumptions is recognized, the number of articles that take limitedness of intermediate storage into account is according to Akrami et al. (2006) still very limited. The model that is proposed in this article also cannot cope with all assumptions that may apply in Tessenderlo or other applications: the authors for instance assume identical parallel machines. In actual production situations parallel machines are often nonidentical: if one stage is a bottleneck, typically a newer machine is added to lessen this bottleneck. Commonly newer machines have greater processing speeds when compared with existing machines (Ruiz, Şerifoğlu, & Urlings, 2008).

For the Tessenderlo case, the most important mismatch with the approach of Akrami et al. (2006) is the assumption that production in the second stage can only start after the first stage has finished processing. In the chemical industry, but also in numerous other process businesses like the pharmaceutical or food industries, it is not uncommon that processing at stage 2 can already begin once product starts flowing out of stage 1. Therefore, a very important aspect of the Tessenderlo situation – and also for many other process industries in general – is the *continuous* nature of the plant: most of the scheduling literature focuses on *batch* scheduling. Continuous scheduling gained less attention.

In the literature this continuous nature is frequently addressed by *lot-splitting*, meaning that lots are split into sublots. As a result, production at the second stage does not have to wait for the first stage to finish a lot, but only has to wait for the first sublot to finish. In Tessenderlo the extrusion stage certainly doesn't have to wait for the powder production stage to finish producing a batch. In fact, production on the second stage can already start once powder starts coming out of the first stage (or out of a silo). The time this takes should be more or less equal to the size of a sublot to get an accurate representation in a model. Unfortunately the computing time of a model increases greatly when the sublot-size becomes small compared to the lotsize.

The aforementioned inadequacies make the approach of Akrami et al. (2006) inadequate for usage in some continuous process industries without extending this model. This extending is what will be attempted in this research in an effort to solve issues at a field site of Dow, described in the next chapter.

A different approach by Giannelos & Georgiadis (2002) did not employ a discrete fine time grid for scheduling continuous processes as this quickly becomes computationally prohibitive in most realistic manufacturing processes, but utilized a continuous-time representation to model continuous flowshop scheduling. Their mixed-integer linear program formulation can handle flexible limited intermediate storage requirements and sequence-dependent changeovers with reasonable computational performance for medium- to large-scaled problems. This model seems suitable to serve as a basis for the Tessenderlo scheduling problem.

1.6 Academic contribution

In the scheduling literature, many approaches have been developed based on theoretical problems, and are sometimes also verified and tested with (randomly generated) synthetic problem data. Assessing a model using synthetic problems makes it hard to draw conclusions on real life applicability. Synthetic problem data can be adjusted to suit the model, although in reality this has to be done in the opposite way; the model should be tailored to suit the problem.

In this research, contributions to the scheduling research will be twofold: insights that come forth by the actual implementation of scheduling techniques in a real-world manufacturing setting, and by further developing solutions that fill in gaps in present-day academic literature. A problem that does not seem to be addressed in the scheduling literature is the distinction between a limited free of charge amount of storage space, and the possibility of charged excess space. This will be dealt with in a scheduling model, as well as investigating the attractiveness of extending the free of charge storage space.

1.7 Report structure

In chapter 1, the problem analysis is presented. The case company is described in chapter 2. The modeling phase starts in chapter 3 with an analysis of the feasibility of a make-to-stock policy. Chapter 4 continues with the actual production schedule modeling of which results are given in chapter 5. The validity of the model is furthermore assessed in chapter 6, followed by conclusions and recommendations.

Chapter 2. Case company description

This chapter contains an overview of all relevant production, planning, and scheduling processes that currently apply to the Tessenderlo HDPE plant of the Dow Chemical company. A general outline of the production plant will be given first, after which more detailed information will be given in the sections Production processes and Planning processes.

The thermoplastic HDPE is the most common plastic in the world and comes in different grades, each with other specific fields of application. Tessenderlo produces HDPE pellets (also called resin) which are further processed elsewhere to make final (consumer-) products. Examples of final products are

- bottles for packaging of consumer goods like milk products, juices, detergents, shampoos, etcetera;
- films for bags such as carrier bags and garbage bags;
- drums up to 60 liters for packaging of industrial goods such as motor oil;
- pipes for transport and distribution of water and gas.

The HDPE plant was built in 1974 with at that time a capacity of 65,000 metric tons per year. Continuous improvements of technologies increased the annual production to 167,000 t in 2007, and an estimated 175,000 t in 2008.

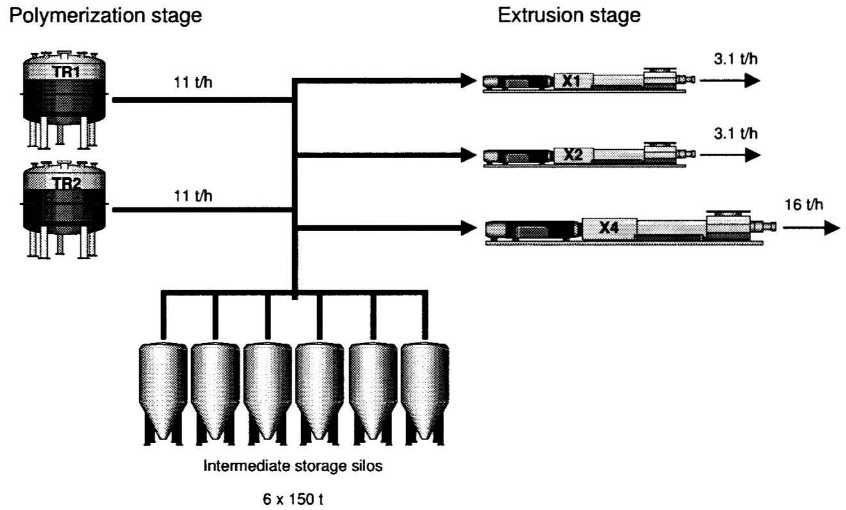


Figure 2.1. Schematic representation of present production units.

In Tessenderlo, HDPE is produced in basically two manufacturing steps (see Figure 2.1). The first step is the production of polyethylene powder, mainly using ethylene as raw material. This powder is altogether with additives in the second step extruded which then yields the final product: HDPE resin, or pellets. The powder-production stage, also called polymerization, consists of two parallel lines with identical production rates; both are capable of producing powder at a rate of approximately 11 t/h. They are indicated as TR1 and TR2. In the second stage, three extruders are installed of which the two smaller ones (X1 and X2) are identical. A third extruder, X4, is much newer, larger and faster than X1 and X2 and utilizing this machine to the maximum is therefore key. Between the two stages, a limited amount of intermediate storage is present on-site. The present capacity, six silos of 150 t each, has demonstrated to be insufficient; therefore additional third-party storage is rented.

The manufacturing layout, including the intermediate storage facilities which will be discussed in the next section, is schematically depicted in Figure 2.1. Production rates of all machines are also shown in Table 2.1. In the next section, production processes will be discussed in more detail.

Table 2.1. Powder production trains and extruders: approximate maximum throughput. Actual maximum throughput is dependent on product type.

Name	Maximum Throughput [t/h]	Remark
Train T1	11.0	
Train T2	11.0	
Extruder X1	3.1	Exclusively for black products
Extruder X2	3.1	
Extruder X4	16.5	

2.1 Production processes

Tessenderlo currently produces 11 different grades of HDPE powder, which are used to produce 8 different extruded grades. Some product types are not extruded but sold as powder to another Dow plant. The majority however is extruded. Extrusion grades (xE) are generally made from the corresponding reaction grade (xP). An overview is given in Table 2.5. Note that at the extrusion stage several additives are added to the feed. The proportion powder in relation to the total feed amount ranges from approximately [redacted] % for some grades to nearly 100 % for most grades. It is not possible to determine the bottleneck stage without taking these recipes into account.

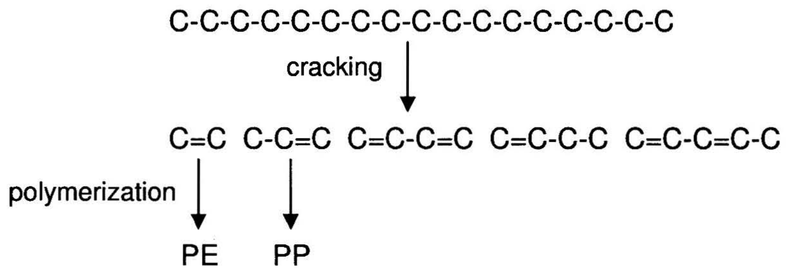
A little background on polymer production

Polymers exist in many varieties and are all around. One characteristic that all polymers share is the size of their molecules, with a molecular mass of up to 10^4 or sometimes even 10^6 g/mol (van der Vegt, 1999). Non-polymers often have a lower molecular mass of order 10^2 g/mol (for example water, 18 g/mol, or sugar, 342 g/mol).

The number of synthetic polymers has been increasing greatly during the last century, but some polymers have been around for a long period of time in nature. Examples are wood, cotton, cork, wool, silk, etcetera. Semi-synthetic polymers originate from nature but have undergone treatment. Examples are leather and rubber.

Yet a fully synthetic polymer is the subject of this research. Production generally starts with distilling crude oil, of which a fuel named naphtha is a product. By cracking and fractioning, naphtha can be broken down to small hydrocarbon monomers such as ethylene (C_2H_2, and propylene, C_3H_6) and others.

Ethylene is then polymerized to very long polymer chains $-[CH_2-CH_2]-$ out of thousands of monomers.



2.1.1 Polymerization stage

The polymerization stage consists of two reaction trains. For the greatest part they are identical, except for the possibility of train 2 to have its reactors run in parallel mode, resulting in a higher throughput for powder grades EP, FP, GP and HP (train 1: 6–7 t/h, train 2: 9 t/h). These grades are therefore generally only produced on train 2. All other characteristics are identical for the two trains. Both trains are in principle able to produce all eight different powder grades.

Not all train grades do correspond one-on-one with extrusion grades. In Table 2.5 the extrusion grades are shown with corresponding train grades and powder fractions. Product types EP, FP, GP, and HP are left out as these products are sold as powder and are thus not extruded.

The polymerization trains have significant transition times when changing grades. While some transitions can be executed instantaneously due to grade similarities, most transitions take about 7 to 10 hours. During a transition production continues, but material that is not within specifications of either one of the grades is produced. At an average throughput of 10 t/h, the amounts of off-grade material produced during a transition are significant. Typical transition times and off-grade production amounts are given in Table 2.2 and Table 2.3, respectively.

In general it is preferred to keep runlengths on the reaction trains for each grade at a minimum of five days. Shorter runlengths lead to relatively long cumulative transition times and an accordingly high production of off-grade material. Profit is still made on off-grade material, but less than on prime material. Changeovers are thus to be kept to a minimum. Also from an operational point of view fewer changeovers are more convenient.

Table 2.2. Train 1 transition times [h], * denotes uncommon transition.

		TO							
		AP	BP	CP	DP	EP	FP	GP	HP
FROM	AP		7	7	7	10	10*	10*	10*
	BP	7		7	7	10	10*	10*	10*
	CP	7	7		7	10	10*	10*	10*
	DP	7	7	7		10	10*	10*	10*
	EP	10	10	10	10		8	8	0
	FP	10	10	10	10	8*		6	8
	GP	10	10	10	10	8*	6*		8
	HP	10	10	10	10	0	8	8	

Table 2.3. Train 1 transition offgrade production [t], at rate 10.5 t/h.

		TO							
		AP	BP	CP	DP	EP	FP	GP	HP
FROM	AP		74	74	74	105	105	105	105
	BP	74		74	74	105	105	105	105
	CP	74	74		74	105	105	105	105
	DP	74	74	74		105	105	105	105
	EP	105	105	105	105		84	84	0
	FP	105	105	105	105	84		63	84
	GP	105	105	105	105	84	63		84
	HP	105	105	105	105	0	84	84	

2.1.2 Extrusion stage

Powder is converted into resin that can be processed by customers by three extruders.

Distinct features of the three extruders are the large differences in throughput and the dedication of X1 to black materials. This dedication is a critical constraint and cannot be ignored as cleaning is a very time-consuming task. Regarding the material balance between the reaction stage (approximately $10.5+10.5=21$ t/h) and the extrusion stage (approximately $3.1+3.1+16=22.2$ t/h) and given the demand figures for the natural and black grades, overruling this constraint should – when the plant runs normally – not be necessary.

In Table 2.4 the production rates of the extruders are given. Note that in these figures recipes are not taken into account and therefore an extrusion production of 3.2 t/h may not correspond to a powder consumption of 3.2 t/h.

X4 in principle always is fed by a reaction train. However, an additional ~5 t/h is needed to operate X4 at its full capacity. This additional powder can be blown back (at a maximum rate of 8 t/h, thus nonrestrictive) from CPE silos through pipelines, which makes X4 able to run at maximum rate. When retrieving powder from external silos however, logistic restrictions worsen the performance: powder can only be moved by trucks during daytime and the feedhopper of extruder X4 only has a capacity of 50 t. This difference is represented by the column 'additional source' in Table 2.4. Note that X4 always runs on hot powder (column hot/cold powder), at least for the greatest part.

In the present situation, one of the identical extruders (X1) is dedicated to producing only black materials to avoid black contamination in the other extruders. Would another extruder be used to produce black HDPE, then this extruder basically has to be disassembled to clean it internally before a white or transparent product can be made. This cleaning obviously is a very time-consuming activity, and therefore in general avoided.

Table 2.4. Maximum throughput of extruders X1, X2 and X4 [t/h]

Product	Hot/cold powder	Max rate X1 [t/h]	Max rate X2 [t/h]	Additional source	Max rate X4 [t/h]
AE	Cold		3.2		
	Hot		3.4	Internal	16
				External	13.5
BE	Hot			Internal	15.5
				External	14
IE	Cold	3.1			
	Hot	3.4			
DE	Cold		2.9		
	Hot		3.1	Internal	15.5
				External	13.3
JE	Hot			Internal	14.7
				External	12.5
KE	Cold	3.6			
	Hot	3.9			
CE	Cold	2.9			
	Hot	2.9			

Table 2.5. Extrusion grades with corresponding powder fractions.

Grade	Powder fraction	Powder type
IE	[redacted] %	CP
KE	[redacted] %	KP
AE	[redacted] %	AP
BE	[redacted] %	BP
DE	[redacted] %	DP
LE	[redacted] %	GP
ME	[redacted] %	MP
NE	[redacted] %	NP
offgrade	Various	Various

2.1.3 Intermediate storage and production routings

Six silos of 150 tons each function as intermediate buffer storage. Each of these six silos can hold at most one product type at a time. It is not necessary to make use of this intermediate buffer; powder can also be directly routed from a powder train to the feedhopper of an extruder. In reality many different material routings are possible, but there certainly are several restrictions. A detailed analysis will be given in below and in section 2.1.4

In addition to the six silos, making use of additional buffer space is also possible. External off-site storage can be rented, yet this is a more costly option since rental and transport costs are involved, not to mention safety issues due to public road transport. The amount of available buffer space is so limited it has severe consequences for the planning and sequencing of the plant. This can be illustrated by an example: suppose extruder X4 is used to produce product type AE. It can do so at a rate of 18 t/h. Powder production train TR1 can supply only at a rate of 11 t/h. An hourly average of 7 t thus has to be pulled out of the intermediate storage. There are obviously limits to the length this course of action can continue: when inventory runs out, a changeover is required. To illustrate this example; when powder production and extrusion start at (more or less) the same time and an intermediate powder inventory of 140 t is present at the beginning, problems start to occur after 20 hours of production.

In addition to the intermediate buffer storage, each extruder has a small 50 tons buffer placed directly before the machine to make a steady material flow possible and to avoid starving of the extruder. This small buffer will not be incorporated in the problem formulation and will be considered as part of the extruders at the second stage.

Another reason for the desire to reduce off-site powder inventory is that extrusion can attain higher throughput rates when powder is sourced from internal silos, or even better, from reaction. This has to do with the powder temperature; hot powder is extruded more easily. The same goes for the important extruder X4, which always needs stored powder in addition to a train source, because of its higher extrusion rate than the reaction rate of a train. Powder temperature is not the main issue here, but the fact that during 8 hours of daytime powder is brought in and stored in a 50 t feedhopper. This 50 t is the only additional powder that is available during the nightshifts, due to which a lower average throughput can be realized per twenty-four hours.

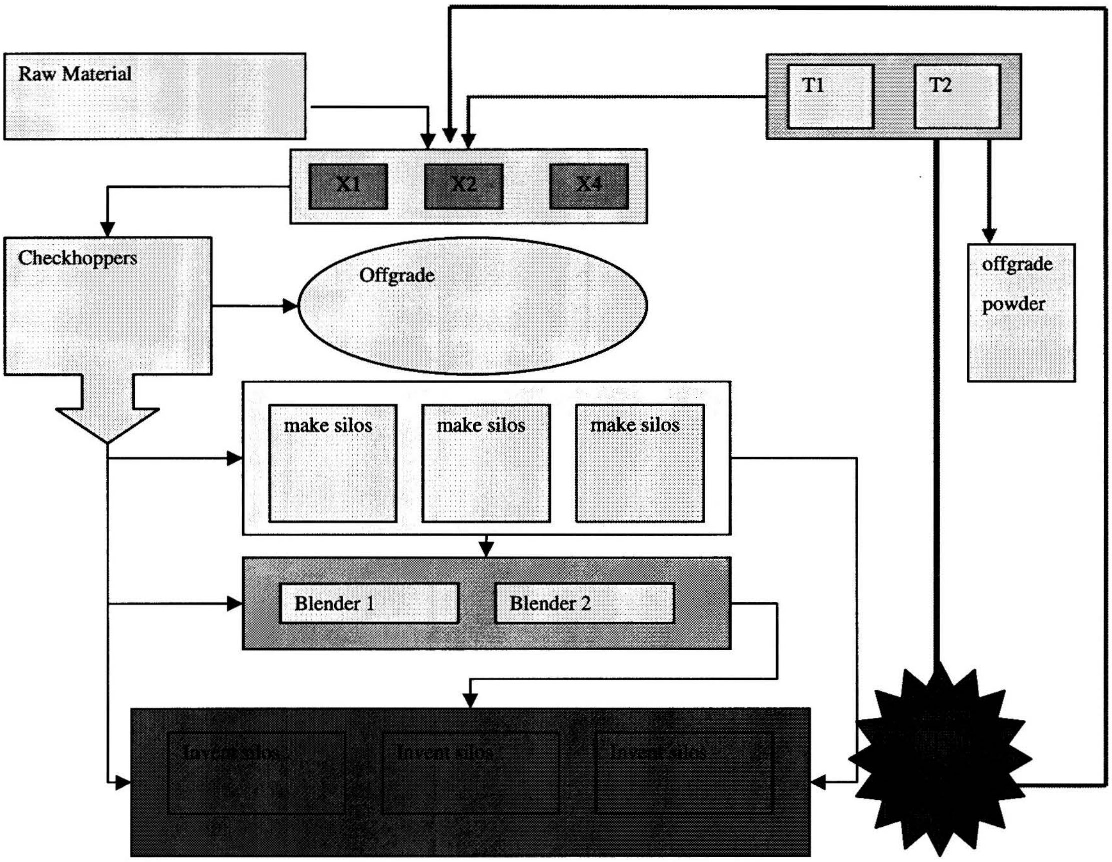


Figure 2.2. Detailed scheme of production units and routings.

Silos and intermediate storage

Various types of silos are present at the plant. Raw materials, intermediate products (powder), and end inventory each have their own type of silo and cannot be used for other purposes. Mostly this is due to technical reasons. The (non-)existence of physical pipelines for example, or the necessity of systems that fill powder (intermediate storage) silos up with nitrogen instead of air to prevent dust explosions. The physical linkage of equipment and silos is depicted in Figure 2.2.

As discussed above, longer runlengths on the reaction trains are desirable to keep changeover times to a minimum. However, longer runlengths lead to higher average intermediate and finished goods inventory levels. Having funds tied up in inventory also comes at a cost. Another disadvantage of longer runlengths is the inability to react swiftly to sudden changes in demand.

Six **intermediate storage silos** are used for storing powder and have a capacity of 150 t each. One silo preferably has to remain empty to avoid wasting prime powder when extruders unexpectedly have mechanical failures. Powder from only one of these six silos can be routed to the extruder stage at a time.

Usually the capacity of these silos is insufficient and external silo storage is rented. The intermediate storage silos are in effect the only silos that fall within the scope of this project.

Additives and other raw materials are stored in **raw material silos**. They are however not strictly used for raw materials, also off-grade powder and rework powder is stored in these silos. Raw materials will be left out-of-scope in this project and thus assumed to be always readily available at any time. Especially when a production schedule for the next week is not changed anymore, this should be always the case.

All extruded materials pass through a **checkhopper**. Samples are taken from the material of which the properties are tested. Should these properties, such as the melt index, fall outside the specification limits, the material is routed to an off-grade silo. So-called **make silos** act as temporary intermediate storage when both blenders are occupied. A number of **finished goods inventory silos** are available on-site. When needed, external storage is rented. In order to get homogeneity in material properties such as the melt index, pellets are routed via a **blender** when necessary.

2.1.4 Past changes in production routings

Over the past decennia, production routings have been changed and improved. Routings as used in the past will first be briefly described in order to understand why certain routings have been changed.

Initial process

Polymerization was always directly fed to extrusion (see Figure 2.3). This was possible when the throughput of both stages was identical. During a grade change, off-grade powder was also extruded. Depending on the quality of the extruded material, it is routed to either off-grade, rework or prime silos. Rework pellets don't qualify for prime specifications but could nonetheless, in small proportions, re-enter an extruder along with prime powder. Off-grade pellets could not be used for re-extrusion and were routed to an off-grade silo.

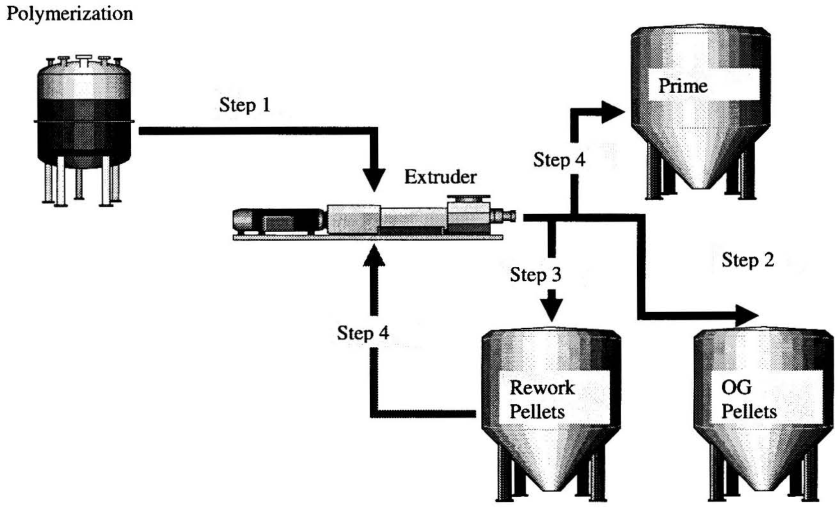


Figure 2.3. Routing initial process.

Improved process

Later on it was recognized that extruding off-grade powder does not add value, but does in fact take up extruder utilization. This was in particular unfavorable when extrusion became a bottleneck stage. By only directing prime powder to extrusion, extrusion capacity is saved for extruding the more profitable prime powder. This improved process is schematically depicted in Figure 2.4.

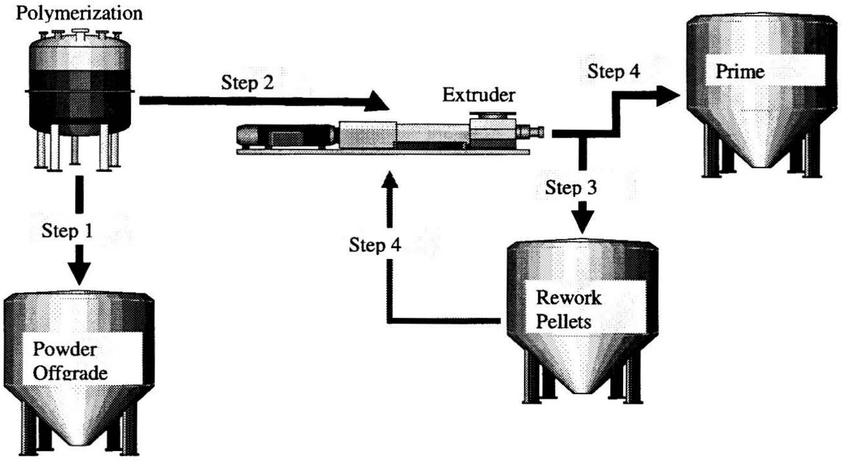


Figure 2.4. Routing improved process.

Actual process

Due to technological innovations throughput was enhanced, both of the polymerization stage and the extrusion stage. Since improvements have not been applied to both stages at the same time, a need to decouple operations at the two stages arose. This need was satisfied by making it possible to store prime powder, thus not having to extrude all powder immediately. It is possible to directly extrude powder, send it to storage, or both (intermittently). A schematic representation of the actual production routings is shown in Figure 2.5.

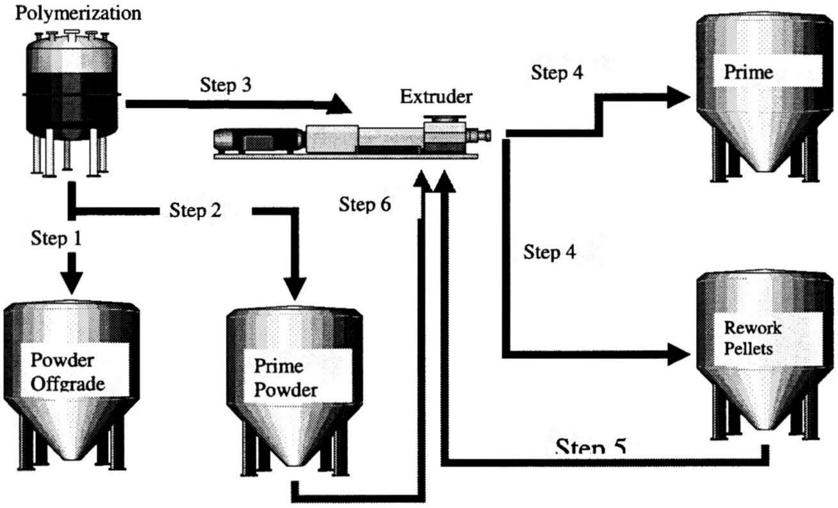


Figure 2.5. Actual production process.

2.1.5 Site Logistics

Each year a large amount of money is spent on site logistics, greatly due to the limitedness of on-site intermediate storage space. External storage space is rented from third parties, partly with long-term contracts and partly with monthly varying capacity contracts. Intermediate storage transport, from Tessenderlo to the external silos and vice versa, and handling count for approximately half of the expenses, the other half is the rental costs. The total yearly amount is in the order of 1–1.2 million Euros. Note that this is not the total amount that is spent on site logistics, but only the relevant powder transport, handling and storage costs.

Variable costs involved with transport and handling are circa [redacted] Euros per ton. Silo rental costs depend on the silo size, but currently totals to approximately [redacted] €/year for an average amount of 5500 t of silo capacity. Only 60% is in effect used, because of loading/unloading, the obvious restriction to have only one grade in one silo at a time, and due to the fact that capacity has to reserved in advance. External powder storage is on average around 2500–3000 t; both internal and external storage in total 3000–3500 t.

The average amount of end-inventory storage in 2007 has varied between 6000 and 8000 t.

2.2 Planning Processes

Responsibilities for the planning processes of HDPE are spread out over multiple departments at two Dow locations: Terneuzen and Tessenderlo. Customers place orders at the sales office in Terneuzen. These orders are entered into the SAP system. The business planner, also in Terneuzen, checks if the order complies with the forecast and inventory, and revises the productplan. The raw material inventory position is compared with the productplan, and procurement orders are placed if necessary. Raw materials are delivered in Tessenderlo. The plant planner uses the productplan as input to build the production plan, which is communicated to Tessenderlo, where products are produced according to the production plan and product specifications. A more detailed analysis of planning and scheduling processes will be part of the project.

2.2.1 Position of scheduling in supply chain

The planning and schedules processes are schematically depicted in Figure 2.6. A long-term strategic plan that deals with the strategy of the next 1–5 years lies at the basis of all shorter-term decisions. Issues such as setting up new markets or new plants, launching or discontinuing product types, and other capital decisions are addressed here.'

This strategic plan serves as a basis for a 1-year sales plan and a 1-year product plan. This sales plan depends on market conditions and customer demand forecasts. Contracts with customers on demand commitment are arranged for a year to come. Dow is not committed to deliver more than 1/10th of a year's contract in one month when a customers asks, to avoid service problems for other customers. Temporarily increasing production output is not possible. The intention is to always use all available production capacity, as planning idle time has turned out to be costly and not necessary to guarantee a satisfactory service level.

Each month an allocation plan is made. Inputs for this plan are planned orders as far as available, and on a forecast based on a statistical forecast of historical sales data which is in turn revised by the sales department and the sales director, and inventory levels. Allocation refers to the quantity of each product that may be sold to a customer.

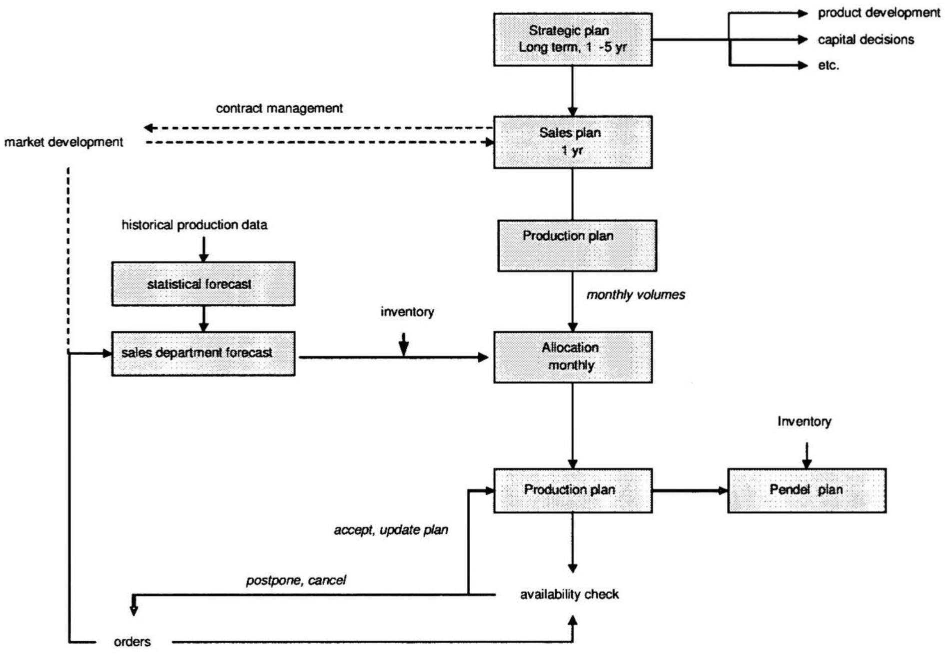


Figure 2.6. Planning processes.

2.2.2 Production planning

The production plan is derived from the monthly allocation and has a horizon of approximately five to six weeks. The longer in advance, the lower the proportion of actual orders that is already entered. Approximately 60% of all customer orders for a given month is known just before the month starts. This proportion keeps rising quite rapidly during the start of the month, and by definition reaches 100% at or just before the end of a month.

During a month, the fact that parts of the allocated amounts are already sold should decrease demand uncertainty for the rest of the month, at least theoretically speaking: the proportion of demand that is already ordered increases during a month. In practice, it has appeared hard to really gain advantage of this decrease in uncertainty. Some customers place orders more often than others. Some want a delivery each three days, some once per month. Suppose in a given month a customer with an allocation of 150 tons wants three deliveries of 50 tons each; on the 1st, the 3rd and the 5th, and nothing for the rest of the month. Nevertheless it cannot be concluded on the 6th that demand for this product during the rest of this month is going to be higher than expected.

Customer orders are entered into SAP first with volume and due date. Projected raw material and end-inventory levels are automatically updated. Should problems such as negative end-inventory levels arise, then the plan is changed, or when this unfortunately turns out to be impossible, orders can be delayed or cancelled. Changing the plan can be done in various ways; a planner can be very creative in working this out. Think for example of unpacking products from end-inventory when a bulk order causes problems. A good mutual understanding between customer and planner can also be beneficial in this respect. Customers generally have several days or weeks of inventory to avoid having to interrupt production when delivery problems come about. A planner that knows the customers well is able to for instance delay a certain delivery to a customer who commonly has more inventory to cope with delivery problems. After all customer orders are entered, the remaining production capacity is filled with forecast orders. In the present situation, intuition and experience play a large role in this.

Given that market conditions are demanding, the main incentive is to maximize production. Transition times should therefore be limited to a minimum, transition costs are of secondary importance (Tousain and Bosgra, 2006). In the opposite case of moderate demand, the need for minimizing setup times would be less high. During a grade change, a certain amount of off-grade material is produced. These off-spec products are sold for lower prices, leaving very little or no margin. The most important aspect of changeovers is therefore the loss of extrusion capacity.

2.2.3 Production reliability

An issue that may or may not be influential on the planning processes is the (mechanical) reliability of the production assets. In the Tessenderlo situation, the reliability has been of a high level. Sometimes months can pass by without any failure, but multiple failures per week can also occur. Most of these failures are however resolved within a couple of hours, making the effect only minor. Due to this high reliability, planning does not take potential future failures into account. When a breakdown occurs, the decrease in production yield is taken care of by the sales department.

Chapter 3. Analysis of MTS planning/scheduling

Current planning practices allow that a certain part of customer orders lead to schedule alterations. Changes in orders, or orders with a short leadtime in particular are a cause of changes in a production schedule as the finished goods inventory is limited and sometimes insufficient to handle some (changes in) customer orders. This corresponds to a make-to-stock (MTS) planning system. Advantages are the greater freedom to schedule optimally in terms of the most relevant objectives. In this project, reducing intermediate storage amounts and handling costs is a key issue (see section 1.3) and off-line scheduling can be optimized for this, or implicitly via minimizing relevant costs. A requirement for optimality is not to change the schedule during a month when actual orders come in and demand uncertainty decreases. Fluctuations in demand should thus be fully covered by end-inventory. This means that if a schedule is not revised when the actual order load differs from the allocated amounts such as in a full MTS system, higher levels of end-inventory may be needed to cover demand fluctuations, ensuring satisfactory customer service levels.

Questions arise when considering the appropriateness of an MTS planning concept. Are the monthly allocations reliable enough to hinge the scheduling on? How much reduction in powder inventory (from the current 3000–4500 t of which 600 t on-site) and handling expenses can be expected by scheduling optimally? How much –if any– increase in pellet inventory is needed to cover demand fluctuations without changing the off-line schedule that is generated prior to the beginning of a month? These questions will be considered in the next paragraphs.

3.1 Allocation reliability

If the extra amount of end-inventory that is needed to fulfill demand during each month and to give scheduling the desired freedom is higher than the to-be-expected reduction in intermediate inventory, the MTS concept will possibly not be viable, as higher amounts of total (both intermediate and end-) inventory are undesirable for several stakeholders. Before deciding to let the monthly planning rely on allocations only, the reliability of these allocations is a key issue, and will therefore be examined.

3.1.1 A worst-case illustration

The replenishment (production) quantities for all products (i.e., the product mix) are determined just before the beginning of a month. Suppose in month n the customer demand turned out to be higher than allocated and the replenishment (production run) takes place at the end of this month x. Now the net finished goods inventory level will, due to the higher demand, be lower than expected. The replenishment quantity for month $n+1$ will be chosen accordingly and equals the allocated quantity for next month plus the excess demand of previous month. This replenishment however may also take place at the end of month $n+1$. This is a worst-case illustration, but when giving scheduling total freedom in determining the production sequence, production runs may take place at the end of a month. Therefore it can be concluded that the order quantities should be sufficient to cover a month of demand plus two months of demand uncertainty. Constraining the freedom of scheduling, for instance by prescribing a certain production sequence, can reduce the amount of safety stock because during allocation the approximate expected replenishment date can be taken into account. The starting point here is total freedom though.

In this situation there is no fixed order-up-to level. Order quantities depend on forecast demand, hence it is uneconomical to order up to a certain level when it is known that demand for next month will not be as high as usual.

A fictional illustrative example is depicted in Figure 3.1. The red line indicates the inventory subject to the expected (allocated) demand, and the blue line indicates the inventory that resulted due to actual demand. In this example, actual demand turned out to be higher than expected in order to represent a worst-case scenario. At the end of month n, the replenishment (production) quantity for month $n+1$ is determined. This equals the allocated demand for month $n+1$, plus the unexpected extra demand that has occurred in month n. Order quantities are equal to

The assumption of a replenishment lead time L of 1 month, so $L+R=2$, is an extreme case. The other extreme case would be $L=0$ months, representing a production run immediately at the beginning of a month. The case $L+R=2$ can also be read from Figure 3.1; the inventory position then corresponds to the net inventory level. The amount of stocks that have to be present to cover demand uncertainty during $L+R$ is lower. The averages of the net inventory levels are approximated by taking the mean value between the net inventory level just after and just before a replenishment (production run) takes place.

Now there are two values, one for $L+R=1$ and one for $L+R=2$, which represent the two extreme cases of producing at the beginning and at the end of a month. Averaging these values gives a more or less representative overall stock level that would on average have been present during 2007 using a MTS concept. The approximate numbers are shown in Figure 3.1 and Figure 3.2.

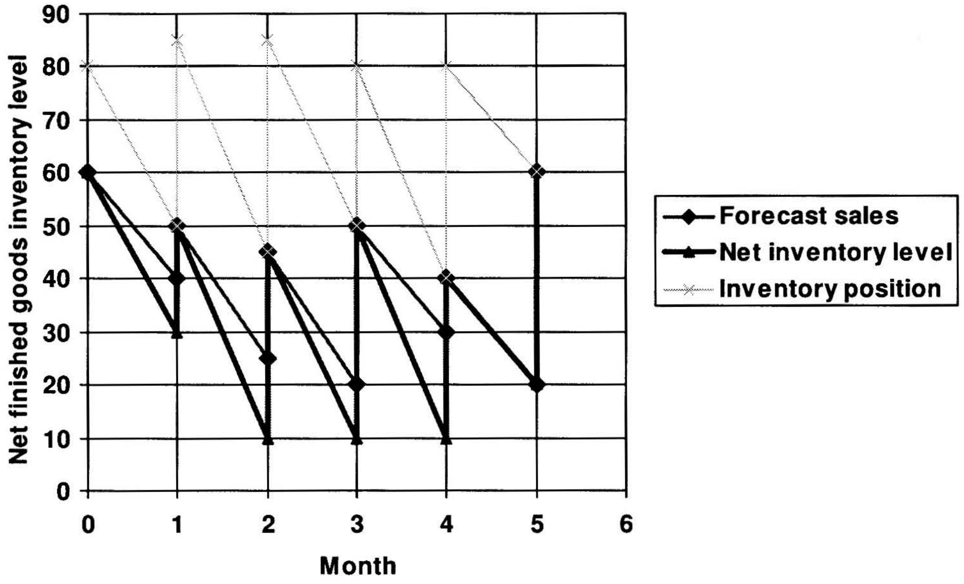


Figure 3.1. Illustration of a worst-case scenario in which actual demand is higher than allocated (forecast) demand during month 1, 2, 3, and 4. During month 5, actual demand equals forecast demand and the net inventory level restores as a result. Production runs take place at the end of each month.

3.1.2 Allocation quantities versus actual sales quantities

If orders are to be fulfilled from the finished goods inventory, order quantities have to be such that a certain proportion, say 99%, of the customer orders is readily available in the FGI. It is however hard to draw reliable conclusions from the limited amount of available allocation data. 2006 data is not available, and would not have been of much use, as the product portfolio has been changed much since, and also the person that was responsible for the allocations has been replaced.

The calculation will be demonstrated for grade AE only. Because of the effect on average inventory levels, two extreme cases are considered: production runs taking place at the begin of a month and at the end of a month.

Let us define a different target stock level for each month, depending on the allocation for the next month.

Case 1: a production run takes place immediately at the begin of a month (replenishment lead time $L = 0$ months). The review period R is chosen at 1 month. This means $L + R = 1$. Now the quantities on-hand can be described as follows:

On-hand begin of month n = target stock level month n = allocation month n + one month of allocation uncertainty

On-hand end of month = on-hand begin of month n − sales month n

Average on-hand = $\sum_n 0.5 \cdot$ on-hand begin of month n + $0.5 \cdot$ on-hand end of month n

Case 2: the other extreme case is when a production run takes place at the end of a month. This means $L + R = 2$, leading to the following expressions:

On-hand begin of month n = target stock level month $[n-1]$ − sales month $[n-1]$ = allocation month $[n-1]$ + 2 months of allocation uncertainty − sales month $[n-1]$.

On-hand end of month = on-hand begin of month n − sales month n.

Average on-hand = $\sum_n 0.5 \cdot$ on-hand begin of month n + $0.5 \cdot$ on-hand end of month n.

3.1.3 Bootstrap confidence intervals

In order to accurately determine required finished goods inventory levels the allocation uncertainty amounts, both for one month and for two consecutive months, have to be determined with reasonably high accuracy. Samples from January 2007 – November 2007 are available, which means 11 –or even 10 in the two consecutive months case– measurements. This number of data samples is rather limited for obtaining accurate inferences. Therefore, in order to properly infer properties from the limited amount of data, bootstrap techniques are utilized. Bootstrapping is a data-based simulation method for statistical inference, which can be used to produce inferences like confidence intervals (Efron & Tibshirani, 1993).

For the two consecutive months case, 5 independent samples or 10 dependent samples can be taken from the January to November data. Although bootstrap techniques assume independent samples, it is chosen to use 10 dependent samples in which the differences between allocated quantities and sold quantities over a period of two months are added up (Jan+Feb, Feb+Mar, Mar+Apr and so on). Inferences based on dependent samples can have reduced accuracy (Efron & Tibshirani, 1993). Using only five samples however would reduce the accuracy of the inferences even more.

From the 10 samples that are assumed to be drawn from a population with probability distribution $f(x;\theta)$, n bootstrap samples are taken randomly, *with replacement*. The mean $\bar{\theta}^*$ of these n bootstrap samples is then determined by

$$\bar{\theta}^* = \tfrac{1}{B}\sum_{i=1}^{B}\hat{\theta}^* . \qquad (3.1)$$

This has been repeated $B = 5000$ times. A histogram of these $B = 5000$ bootstrap samples is shown in Figure 3.2. The most interesting aspect of this analysis is the confidence interval. A confidence interval of α for example corresponds to the $100(1-\frac{\alpha}{2})$ percentile of this distribution. This means that when α is chosen at $\alpha = 0.01$, only in 1% of all months a stockout would occur.

Historical sales and allocation data is analyzed. Service issues arise for underallocations only: when actual demand during a month turns out to be higher than the allocated amount. To be more precise, underallocations are here defined as follows: the average of the positive parts of the difference between actual sales per grade on the last measurement date of a month, and the expected sales per grade at this date based on interpolated allocations. This is in essence represented by the x-axis values in Figure 3.2.

Since historical data is used, the monthly aggregate demand matches the aggregate production, so underallocations occur for half of the demand, volume-wise. Still, there are grades that have been underallocated every month, while others have never been underallocated this year. The mean underallocated quantity per grade and its variance are derived from the 2007 data (Jan – Nov).

Average stock levels resulting from the analysis are shown in Table 3.1 and Table 3.2. Total average on-hand levels are shown in the down-right sides of the tables. These numbers are dependent on the α-level, and are for $\alpha = 0.01$ slightly higher than the average levels over 2007. Graphical representations for all grades are given in Appendix I.

Table 3.1. Inventory levels after and before a replenishment for $L + R = 1$, and average on-hand

Product type	After repl.	Before repl.	Avg on-hand
AE	5563	1057	2779
BE	2650	524	1325
IE	410	290	452
DE	1921	409	971
KE	635	229	319
JE	1252	574	626
LE	338	104	170
NE	78	22	48
Sum	12846	3209	6690

Table 3.2. Inventory levels after and before a replenishment for $L + R = 2$, and average on-hand

Product type	After repl.	Before repl	Avg on-hand
AE	6469	459	3274
BE	3295	320	1678
IE	954	319	492
DE	2172	431	1129
KE	628	179	328
JE	1284	313	666
LE	375	122	199
NE	115	81	65
Sum	15293	2225	7830

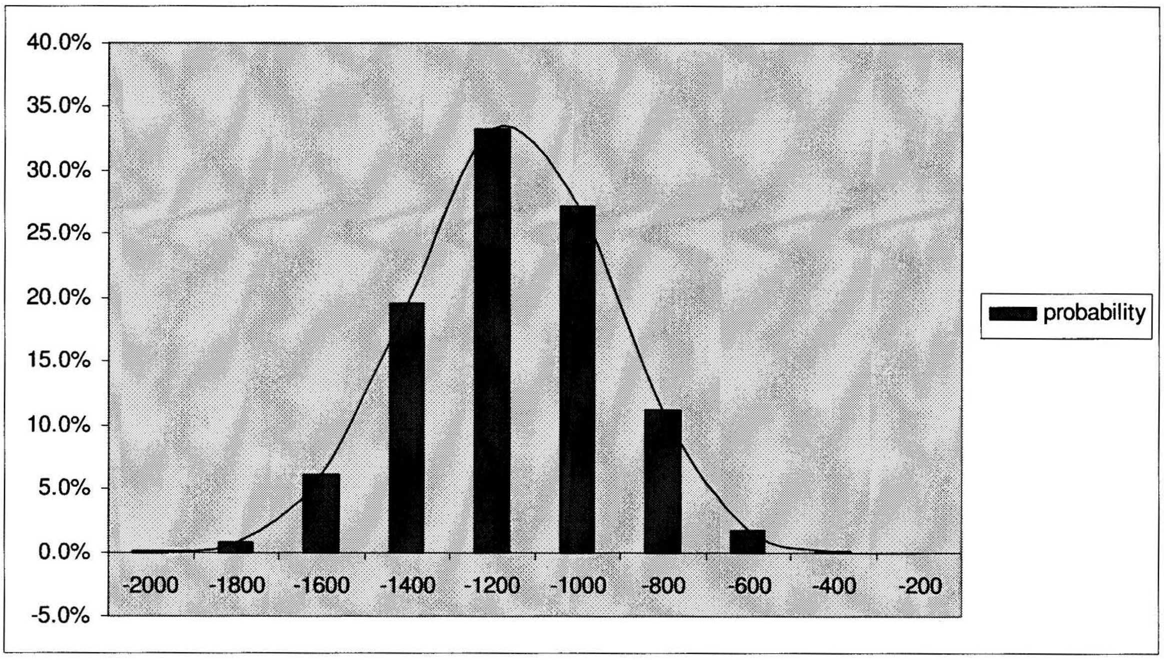


Figure 3.2. Difference between allocations and actual sales of two consecutive months, product type AE

From the extreme cases for required average inventory quantities to cover demand fluctuations (see Table 3.1 and Table 3.2), it can be deduced that on average $(6690+7830)/2=7260$ t of finished goods inventory should be present. This number is at most slightly higher than the amount that has been present during 2007, which varied between 6000 and 8000 t.

Table 3.3. Confidence interval of the mean bootstrap, product type AE.

α	difference allocation and actual sales
0.01	4983.0
0.99	5565.3

3.1.4 Decoupling FGI and manufacturing

A conclusion of the analysis of demand uncertainties was that required amounts of finished goods to cope with demand uncertainties are only slightly higher than the presently average available amounts. The main consequence of this insight is a make-to-stock production environment should be feasible which means that scheduling can take place before the starting of a month and does not have to be altered during the month, when changes in demand with respect to the forecast amounts undoubtedly take place. Not having to adapt the production schedule to sudden changes in demand enables more freedom in optimizing the production schedule, particularly in maximizing throughput and minimizing the intermediate storage quantities that are needed to avoid starving of any extruder, regarding the high costs of powder transport and handling.

Chapter 4. Modeling

4.1 Scheduling model development

An approach that is typically used to deal with large realistically-sized scheduling problems is bottleneck planning or scheduling. First the bottleneck stage is scheduled, and then this schedule is rolled out to other stages. This way of breaking the problem down intro smaller subproblems can be an effective approach in reducing the overall problem complexity. A very elaborate explanation of how this procedure is used in a flexible flowshop setting is given in Quadt (2004).

In this project however, the applicability of this approach is limited. The matter of lotsizing for both stages for example turns out to be highly dependent due to the very limited amount of intermediate storage. Efficiently lotsizing and scheduling the bottleneck stage may result in long runtimes and thereby high handling cost of the intermediates when external storage capacity is used. Furthermore, identifying the bottleneck stage is not unambiguous, as different grades have different production rates and the aggregate production rates of the two stages are close.

In view of the high dependency of the two stages, the only practical way to deal with the scheduling problem under study is to consider both stages at the same time. Only considering both stages together makes optimizing the resulting intermediate powder levels possible.

In order to guarantee feasibility of the schedule, quite a few mainly physical constraints have to be incorporated in the model. Only the essential constraints and a simple data set are included in the early developing phase however. Once a more straightforward model is able to produce correct results, more constraints and real-life data are added step by step. This way of working makes it easier to monitor and verify all changes.

4.1.1 Batch vs. continuous scheduling methods

Scheduling of continuous manufacturing processes can be approximated by using batch scheduling methods. The time horizon is then subdivided into a number of periods of equal length. In order to achieve a reasonable accuracy, the number of time periods has to be high which shows to be computationally impractical. A small example to illustrate this approximation: suppose a time grid with a resolution of 1 day is chosen. In the first time period with a length of one day, a polymerization train produces a 'batch' of 252 tons. Following the batch logic, it is then only until time period 2 (day 2) that even a small extruder with a production rate that is far lower than the polymerization train, can start producing. In a real-life continuous production plant however, it would be possible to start the two stages almost simultaneously. Clearly a higher resolution of the time grid is then needed to achieve higher accuracy, but at a cost: solving times quickly become prohibitively long. In Figure 4.1 an example is shown: a resolution of 1 hour gives a higher accuracy but in order to model a planning horizon of 1 month, at least $24 \cdot 30 = 720$ timeslots are needed.

This drawback only holds for modeling continuous production situations using batch scheduling logic: when considering discrete or batch production, processing at stage $n+1$ can only start after stage n has finished a batch. Therefore the minimum timeslot length for discrete manufacturing is equal to the greatest common divisor (or greatest common factor) of all lotsizes. In continuous manufacturing, a desirable minimum timeslot length is the greatest common divisor of the *flowtimes* of stages $1,\ldots,n-1$ where n is the number of stages.

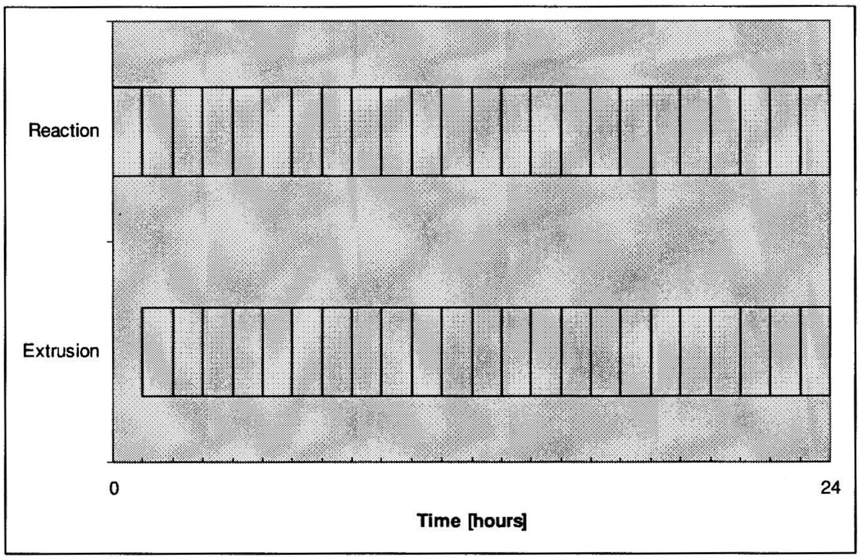


Figure 4.1. Example of discrete-time accuracy with timeslot length of 1 hour.

For this reason continuous-time approaches have been developed to accurately represent continuous processes at reasonable computation cost. Examples of continuous-time flexible flowshop formulations are Karimi & McDonald (1997), Ierapetritou & Floudas (1998), and Giannelos & Georgiadis (2002, 2003). The latter will serve as a basis for the Tessenderlo scheduling model.

4.1.2 State-task network (STN)

Contemporary approaches for the scheduling of continuous multi-product plants, including Giannelos & Georgiadis (2002, 2003), are often based on state-task network (STN) representations. The state-task network is described by Kondili, Pantelides & Sargent (1993) and is a graphical representation that relates processing tasks to material states. A part of the STN representation of Tessenderlo is depicted in Figure 4.2. In this figure, states are represented by circles and tasks by rectangles. Note that a task is defined by both the processing unit and the material type. This prevents the need for variables with more than two indices.

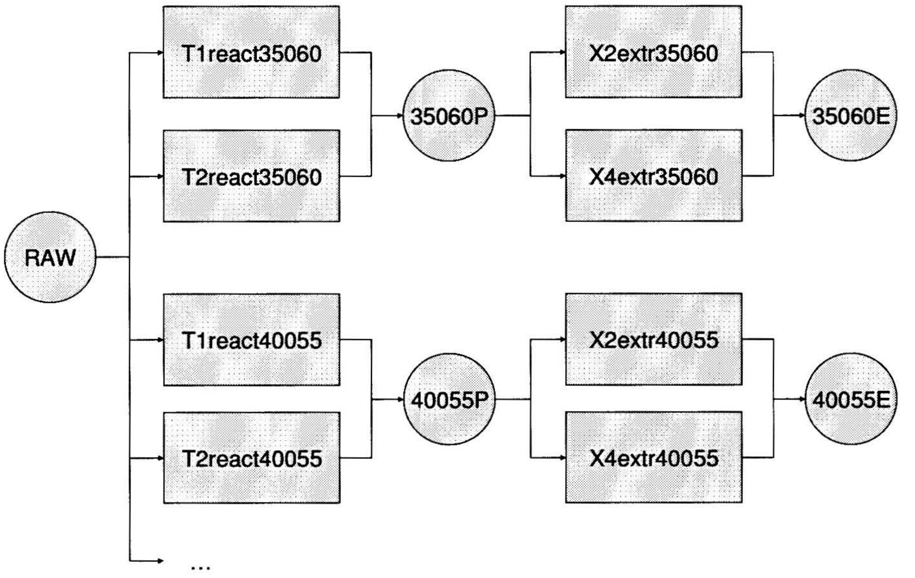


Figure 4.2. Part of the STN representation of the Tessenderlo production process.

4.1.3 Linear programming

For the Tessenderlo situation a model will be developed and formulated as a linear program (LP). Linear programming is a tool that is typically used in solving constrained optimization problems. In operations research it is also widely used: planning and scheduling problems can be optimized using LPs, but also in several other fields of application they are used, such as for example in economics and telecommunications. In linear programming problems, a linear objective function is to be minimized or maximized, subject to linear equality and inequality constraints. Informally and generally speaking, one might say that many linear programs aim to make the best use of a limited set of resources, for example by minimizing cost or maximizing profit.

An important remark here is that developing a linear programming formulation for the plant is not necessarily a project goal in itself, but rather a means to gain insights into optimal planning schemes and optimal amounts of internal storage space. The developed linear program may or may not result in a practical tool for actual scheduling purposes on a monthly basis: practicality in terms of running time is hard to predict. The time required to solve a particular scheduling problem depends of course on the model size, but also heavily on the actual formulation and unfortunately even on formulation aspects such as equation order. A general description of linear programming in general is given in Appendix III.

To solve LP models, several software packages are available which incorporate numerous solvers to efficiently solve LP's. Examples of these software packages are GAMS (General Algebraic Modeling System), AMPL (A Mathematical Programming Language), AIMMS, LINGO, ILOG OPL and many others. In this project GAMS is used to model the Tessenderlo manufacturing setting. The choice for GAMS is mainly due to the reason that Dow already had a license for this software.

4.2 The Tessenderlo scheduling model

As a starting point, the initial formulation of the mixed-integer linear program is largely based on Giannelos and Georgiadis (2002). They describe an event-driven formulation for short-term scheduling of multipurpose continuous processes, which closely matches the manufacturing situation of Tessenderlo. Parts of this formulation are used in unchanged form, but are nevertheless repeated in the section below for comprehensiveness reasons. The limited intermediate storage situation is the main characteristic that has to be added to the formulation in a later stage.

The formulation of the mathematical model that lies at the basis of the Tessenderlo is the following (see also Giannelos and Georgiadis (2002)):

Sets/Indices

I / i = tasks
S / s = states
T / t = event points
U / u = units
I_u = tasks performed in unit u, $I_u \subseteq I$
I^c = continuous process tasks, $I^c \subseteq I$
I_s^c = continuous tasks engaging state s, $I_s^c \subseteq I^c$
I_s^{st} = storage tasks engaging state s, $I_s^{st} \subseteq I^{st}$
$I_{ii'}^{ch}$ = pair of continuous tasks i and i' with changeover requirements
S^{fis} = states of finite intermediate storage

Parameters

$\theta_i^{\max}$ = maximum allowable task duration of task $i \in I^c$
$\theta_i^{\min}$ = minimum allowable task duration of task $i \in I^c$
$R_i^{\max}$ = maximum allowable task rate of task $i \in I^c$
$R_i^{\min}$ = minimum allowable task rate of task $i \in I^c$
$ST_s^{\max}$ = maximum allowable dedicated storage for state s
ST_s^0 = initial amount of state s
v_s = value/price of state s
λ_{si} = fraction of state s engaged in task recipe i
$\theta_{ii'}$ = changeover time from task i to i'
$C^{\max}$ = time horizon
C_u^{ch} = minimum cumulative changeover time in unit u

Variables

θ_{it} = duration of task $i \in I^c$ for event t

θ_{st} = duration of task $i \in I_s^c$ for event t

τ_{it} = ending time of task $i \in I$ for event t

τ_{st} = ending time of task $i \in I_s^c$ for event t

ξ_{it} = total extent (rate × duration) of task $i \in I^c$ for event t

x_{it} = 1 if task $i \in I$ *terminates* at event point t and 0 otherwise

ST_{st} = amount of state s at the end of event point t

y_{st} = 1 if $ST_{st} \neq 0$ and 0 otherwise

Constraints

$$x_{it}\theta_i^{\min} \leq \theta_{it} \leq x_{it}\theta_i^{\max}, \quad \forall i \in I^c, t \in T \tag{4.1}$$

In equation (4.1), bounds are specified between which the task durations are allowed to vary.

$$\theta_{it}R_i^{\min} \leq \xi_{it} \leq \theta_{it}R_i^{\max}, \quad \forall i \in I^c, t \in T \tag{4.2}$$

Task rates are allowed to vary within preset bounds in constraint (4.2).

$$\tau_{it} - \tau_{st} \leq C^{\max}\left(1 - x_{it}\right), \quad \forall s \in S, i \in I_s^c, t \in T \tag{4.3}$$

$$\tau_{it} - \tau_{st} \geq -C^{\max}\left(1 - x_{it}\right), \quad \forall s \in S, i \in I_s^c, t \in T \tag{4.4}$$

$$\theta_{it} - \theta_{st} \leq C^{\max}\left(1 - x_{it}\right), \quad \forall s \in S, i \in I_s^c, t \in T \tag{4.5}$$

$$\theta_{it} - \theta_{st} \geq -C^{\max}\left(1 - x_{it}\right), \quad \forall s \in S, i \in I_s^c, t \in T \tag{4.6}$$

These timing constraints (4.3)–(4.6) equate the left-hand timing variables to each other when task i is performed before event point t, thus $x_{it} = 1$.

$$\tau_{it} - \theta_{it} \geq \tau_{i,t-1}, \quad \forall i \in I_s^c, t \in T \tag{4.7}$$

$$\tau_{st} - \theta_{st} \geq \tau_{s,t-1}, \quad \forall s \in S, t \in T \tag{4.8}$$

Time monotonicity constraints are defined by equations (4.7) and (4.8).

$$ST_{st} = ST_{s,t-1} + \sum_{i \in I_s^c} \lambda_{si}\xi_{it}, \quad \forall s \in S, t \in T \tag{4.9}$$

Constraint (4.9) is a material balance constraint. The amount of state s at the end of event point $t-1$ is changed by execution of tasks producing or consuming this state.

$$\sum_{i \in I_u} x_{it} \leq 1, \quad \forall u \in U, t \in T \tag{4.10}$$

$$\tau_{it} = \tau_{i't}, \quad \forall u \in U, i, i' \in I_u, i = \text{HEAD}(I_u), i' \neq i, t \in T \tag{4.11}$$

Equations (4.10) and (4.11) take care of unit allocation; the $\text{HEAD}(I_u)$ operator indicates the first element of the set I_u.

$$\tau_{i't'} - \theta_{i't'} - \tau_{it} \geq \theta_{ii'} - C^{\max}\left(2 - x_{it} - x_{i't'} + \sum_{i'' \in I_u} \sum_{t'' > t}^{t'' < t'} x_{i''t''}\right), \quad \forall u \in U, i, i' \in I_u, I_{ii'}^{\text{ch}}, t, t' \in T, t < t' \tag{4.12}$$

$$\sum_{i \in I^c} \sum_{t \in T} \theta_{it} \leq C^{\max} - C_u^{\text{ch}}, \quad \forall u \in U \tag{4.13}$$

Equation (4.12) takes care of changeover times when a different grades i and i' are produced in subsequent timeslots t and t'. In constraint (4.13), the search space is reduced by constraining the sum of all production lengths by a maximum of the planning horizon length minus the minimum cumulative changeover time in a production unit to produce all grades.

Limited intermediate storage modeling

$$ST_{st} = INT_{st} + EXT_{st}, \quad \forall s \in S^{\text{fis}}, t \in T \tag{4.14}$$

$$INT_{st} \leq silosInUse_{st} \cdot V^{\max}, \quad \forall s \in S^{\text{fis}}, t \in T \tag{4.15}$$

$$\sum_{s} silosInUse_{st} \leq NS \quad \forall s \in S^{\text{fis}}, t \in T \tag{4.16}$$

$$m \cdot TR_{st}^{\text{OUT}} \geq EXT_{st} - EXT_{s,t-1} \quad \forall s \in S^{\text{fis}}, t \in T \tag{4.17}$$

$$m \cdot TR_{st}^{\text{IN}} \geq EXT_{s,t-1} - EXT_{st} \quad \forall s \in S^{\text{fis}}, t \in T \tag{4.18}$$

$$TR^{\text{TOT}} = \sum_{t \in T} \sum_{s \in S} TR_{st}^{\text{OUT}} + TR_{st}^{\text{IN}} \tag{4.19}$$

Constraint (4.14) ensures that the amount of state s at the end of event point t equals the internally stored amount of state s, INT_{st}, plus the externally stored amount of state s at the end of event point t, EXT_{st}. Equation (4.15) limits the amount of state s that is internally stored to the maximum silo space allocated to state s at the end of event point t. Constraint (4.16) ensures that for internal storage no more silos are used than the number of silos present on-site.

In equations (4.17) – (4.19) the difference in external storage usage between consecutive event points is translated into a number of transports TR_{st} using the parameter m, which represents the storage capacity of a single truck. TR_{st}^{IN} and TR_{st}^{OUT} are positive integer values. The translation of the variable EXT into TR can best be explained by an example, shown in Figure 4.3, indicating powder quantities over three time periods. In time period 1, 250 tons of powder of a certain grade is stored. 2 silos of 150 tons are allocated to this grade, which leaves $(2 \cdot 150) - 250 = 50$ tons of storage space empty. In time period 2, the stored amount increases by 130 tons. Again, 2 internal silos are allocated to this grade, which means that 50 more tons can be stored on-site and 80 tons has to be stored externally. In the last time period no powder is stored externally and the allocated internal storage space is entirely filled.

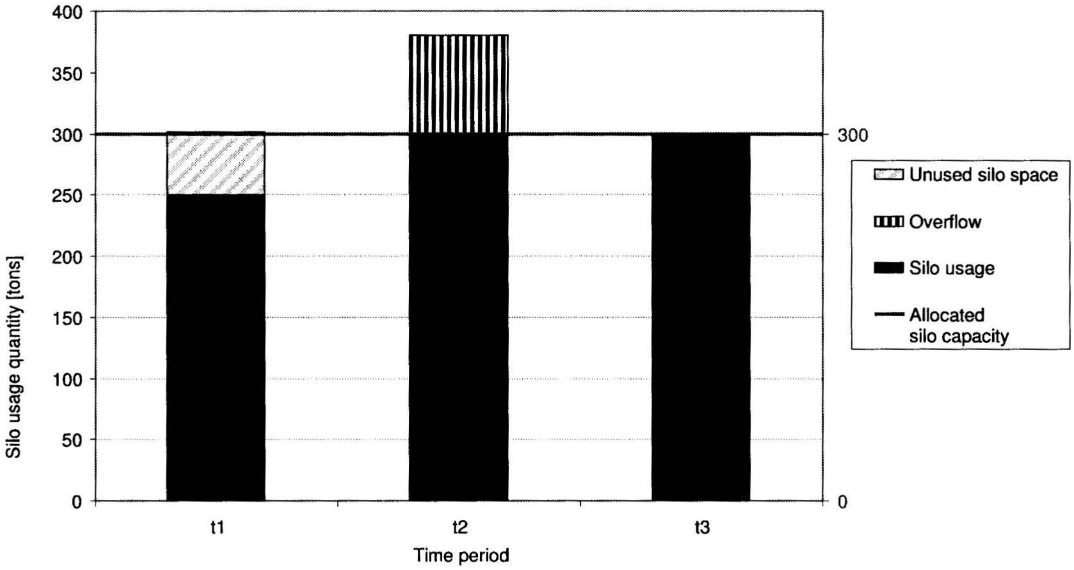


Figure 4.3. Example of internal and external intermediate storage usage.

Objective function

$$\max z = \sum_{s \in S} v_s ST_{s,t_n} - k \cdot TR^{\mathrm{TOT}} \tag{4.20}$$

The objective function (4.20) is of an economic nature. Value of production minus the costs of transportation is maximized.

Silo rental costs

Issues arise when including silo rental costs in the formulation, as multiplying the timeslot length with rental costs per unit time leads to a nonlinear formulation. Therefore rental costs are not included in the scheduling model, but are calculated in retrospect from the resulting schedule. Silo rental costs are approximated by multiplying the external storage quantities between two consecutive event points by the time slot length, and then by multiplying this with a cost factor. This cost factor is derived from survey data as presented in chapter Site Logistics:

$$\text{storage cost} = \frac{\text{yearly rental costs [€/yr]}}{\text{silo utilization } [-] \cdot \text{avg rented storage qty [tons]}} \tag{4.21}$$

The approximate storage cost figure of € [redacted] $\text{ton}^{-1}\text{day}^{-1}$ is based on 2007 data and dependent on the fill percentage of rented silos as silos are rented per unit, regardless of the actual usage. Usage has in the past been around 60%, due to loading, unloading and contractual commitments, hence the factor 0.6

4.2.1 Operational constraints

Several physical and operational constraints are added to the initial formulation to guarantee practical feasibility.

- 6 silos of 150 tons of which 1 always has to remain empty for emergency backup reasons. This is incorporated in the model in the parameters NS and $V^{\max}$, representing the number of silos and the storage space of each silo, respectively.
- Setup times are highly sequence dependent, some transitions yield no off-grade production because of similarities in characteristics while others yield 60 – 100 tons of off-grade product. This is an important characteristic that cannot be ignored: the optimal production sequence is to a certain extent dependent on transition times. This aspect is modeled by means of the transition time from task i to task i', $\theta_{ii'}$, for which the data is entered in tabular form. Note that transition times can be, but not necessarily are symmetric, thus $\theta_{ii'} \neq \theta_{i'i}$.
- Reaction can continue as long as sufficient raw material is available. While this is of course true in reality, here the assumption of infinite availability of raw materials is made. The most important raw material, ethylene, is supplied by pipeline from Terneuzen. Supply of ethylene is not one of the main issues in this project, therefore the initial amount of raw material ethylene is set to an ample monthly amount of $ST_{st} = ST_{raw,t=0} = 20{,}000$ t. Other raw materials like additives generally form a small proportion of the recipe and are not explicitly modeled. This does not mean that they are ignored: extrusion of 0.9[redacted] tons of AP powder yields 1.000000 ton of AE resin due to the addition of additives. The state-task network representation allows for straightforward modeling of the bill-of-materials: each task consumes a certain amount of some state and produces a certain amount of another sate.
- Extrusion rates are dependent on powder temperature. While this is certainly true in reality, this is not modeled for model complexity and thereby computational reasons. The highest extrusion rates can be achieved when warm powder flows freshly from reaction to extrusion. Bringing powder in from external storage facilities leads to lower extrusion rates as the powder has cooled down.
- Extrusion rates are allowed to vary from approximately 70% to 100% of the maximum rate for most machine/product combinations. This is modeled by equation (4.2), which simplifies to $\xi_{it} = \theta_{it} R_i, \forall t \in T$ when $R_i^{\min} = R_i^{\max} = R_i$. Allowing the machine rate to be lower than its nominal rate gives the scheduling algorithm more freedom to develop feasible schedules.
- Some (for instance, black) materials have to be made on a specific extruder to avoid contamination or for other technical reasons (HDPE IE and KE on X1, BE on X4. Tasks that can run on a certain unit are entered in the set I_u (see paragraph Sets/Indices).
- Once end inventory silos are full, external storage capacity is rented. This incurs extra handling costs, as material has to be transported to an external storage location, and from there on to the customer instead of directly from internal storage to the customer. This is less an issue than the high costs of external *intermediate* storage, and is therefore not focused on.

4.3 Determining optimal cycle times

Not every grade does necessarily have to be produced in the same frequency. In other words, the production cycle for certain products is allowed to differ. As lotsizing forms an integral part of the scheduling model formulation, the scheduling model will be able to optimize lotsizes but only within its given scheduling horizon length.

As the scheduling model does include transporting costs but is unable to take external holding costs into account due to modeling issues, in this section a different model will be used to determine the optimal production cycle lengths, which is mainly a tradeoff between setup costs and holding costs. This is done make sure that the scheduling horizon length is chosen larger or equal to the optimal cycle length to avoid suboptimal schedules with more changeovers than optimal on the long run. The entire scheduling model structure then encompasses two phases, as described by Cooke, Rohleder & Silver (2004). The first phase involves lotsizing which can be done using all known problem data such as demand rates, setup times and costs, production rates, etcetera. This lotsizing model determines lotsizes which serve as inputs for the scheduling model. Lotsizes that cannot result in feasible schedules are revised; hence the feedback loop. The process is schematically depicted in Figure 4.4.

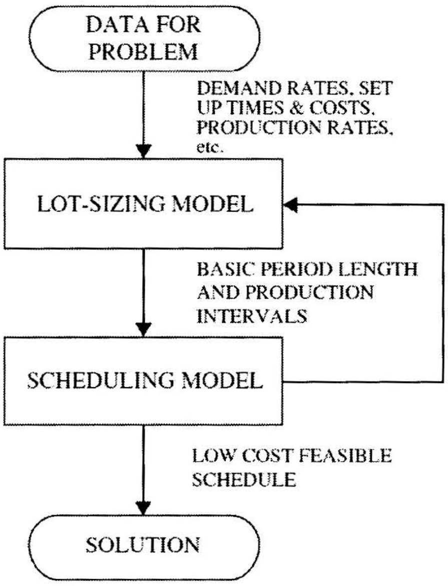


Figure 4.4. ELSP solution structure. From: Cooke, Rohleder & Silver (2004).

Resulting optimal lotsizes or cycle lengths do not have to be input into the scheduling model, but one has to make sure that the scheduling horizon length is long enough to accommodate the optimal cycle lengths of all grades. The scheduling model for the Tessenderlo plant will therefore have the following inputs:

- scheduling horizon length,
- number of event points,
- desired production quantities per grade.

Note that since a make-to-stock policy is applicable (see Chapter 3.), the scheduling model input for desired production quantities does not necessarily needs to resemble the monthly expected customer demand, or allocation. Customer orders will in principle be fulfilled from the finished goods inventory.

Determinants of optimal cycle length

A scheduling approach in which each product is produced exactly once in each cycle is called a *pure rotation* schedule. Other methods are the *cyclic schedule*, in which the entire system behaves in a periodic manner, and the *base period* approach in which the production interval of each item can be integer (often powers of two) multiples of a certain base period (Silver, Pyke, & Peterson, 1998). A case of a pure rotation schedule will be considered here, in which each product shares a common order interval *T*.

Optimal cycle lengths depend on a number of variables. Order cycle lengths certainly are different from production cycle lengths due to the production capacity restriction that does only apply in a production environment, but there are analogies that provide insights. The concept of economic order quantity (*EOQ*) is discussed in length by Silver, Pyke & Peterson (1998). Although the expression for the economic order quantity does not directly apply to a production environment, it gives the insight that the optimal time interval in fact depends on setup costs, demand rates and holding costs:

$$T_{EOQ} = \frac{EOQ}{D} = \sqrt{\frac{2A}{Dvr}} \qquad (4.22)$$

where

$A =$ setup cost [€]

$D =$ demand rate [units/unit time]

$v =$ unit variable cost [€/unit]

$r =$ carrying charge [€/€/unit time]

In a production environment capacity restrictions apply which is often not the case in an order replenishment system as described with formula (4.22). A finite replenishment rate can be incorporated which gives the equation for the economic production quantity (*EPQ*):

$$EPQ = EOQ \cdot \frac{1}{\sqrt{1 - D/p}} \qquad (4.23)$$

where

$p =$ production rate [units/unit time]

Minimizing the total relevant costs will determine the optimal cycle time and thereby the optimal production quantities. The total relevant costs *TRC* per unit time are determined by the setup costs per unit time plus the capital costs and rental costs for the inventory.

When a production run takes place every once in $m_i T$ time units, the setup costs are $\frac{A_i}{m_i T}$.

The average amount of inventory during a cycle is

$$\bar{q}_i = \frac{D_i m_i T \left(1 - \frac{D_i}{p_i}\right)}{2} \qquad (4.24)$$

In Figure 4.5 the expression for $\overline{q}_i$ is explained with an example. Parameter values are listed below the graph.

Both capital costs and silo rental costs are incurred when stocking finished goods inventory. The total relevant costs *TRC* are therefore

$$TRC = \frac{A_i}{m_i T} + \left(Rv_i + k\right)\overline{q}_i \qquad (4.25)$$

where

R = capital cost charge [€/€/unit time]
k = silo rental costs [€/ton/unit time]

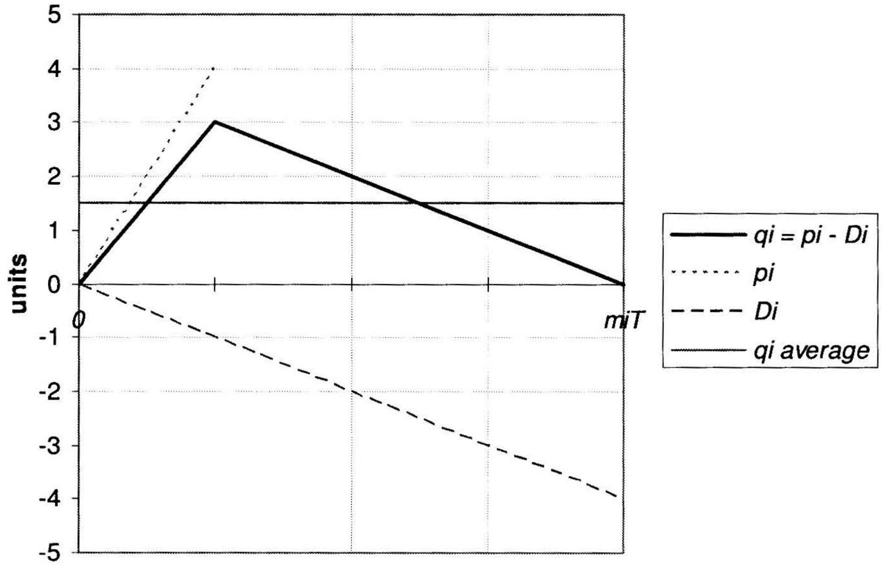


Figure 4.5. Finished goods inventory q_i over time. Parameter values are $p_i = 4$, $D_i = 1$, q_i *average* = 1.5.

In order to use Equation (4.25) to calculate the optimal cycle for all grades, the production situation has to be simplified. Changeover times of the first stage (reaction) are taken as these are much longer. *TRC* results of production rates of 11 t/h (one reaction train) and 19 t/h (extruders X2 + X4) are shown in Figure 4.6 and Figure 4.7. In can be seen that grades needing higher changeover time have longer cycles. Grades EP/FP/GP/HP, which is a production run of a family of grades, needs a longer cumulative changeover time and therefore displays a different cycle time behavior. A table with relevant input parameters of the computation is given in Appendix VI.

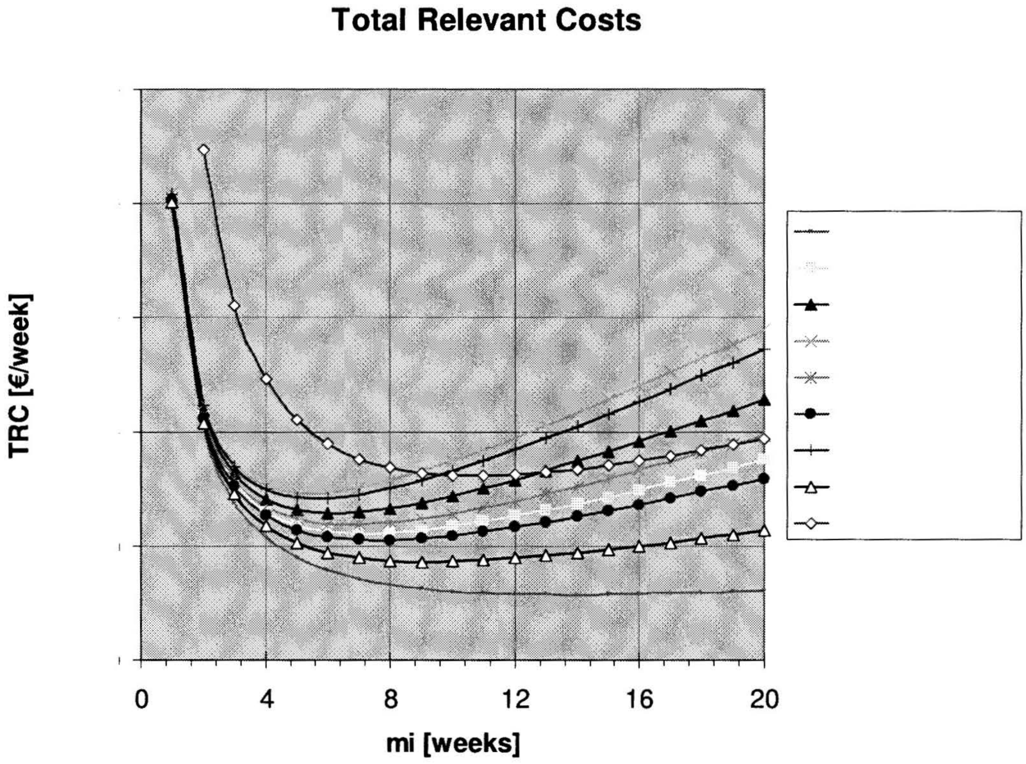


Figure 4.6. Total relevant costs per week *TRC* on reaction trains, production rate $p_i = 11$ t/h.

Results of determining the optimal cycle times are depicted in Figure 4.6 and Figure 4.7. Currently the production frequency for the most grades is below the optimum. Most grades have an optimal production cycle of at least six weeks (cost curves are depicted in Figure 4.6 and Figure 4.7). Optimal production cycle lengths depend on a number of factors, but setup costs and holding costs play the largest role. While holding costs will increase when cycle length is increased as a result of the increased average cycle stock, losses in production time due to less changeovers make up for this and can lead to significant cost reductions. This especially concerns production runs on the first production step as long changeover times are involved. Optimal production lengths correspond to low values of Total Relevant Costs (TRC). Cost reductions can be high as the cost curves are especially steep at the relevant left side. Note that the shown TRC values must be interpreted as approximations.

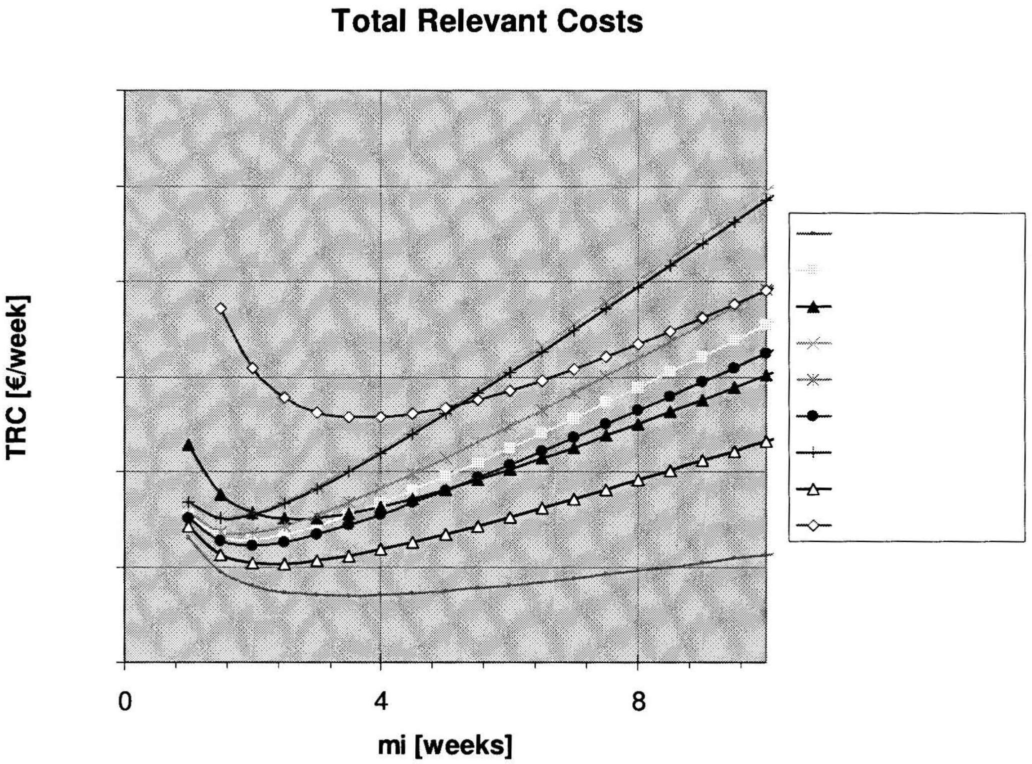


Figure 4.7. Total relevant costs per week *TRC* on X1 or X2, production rate p_i = 3 t/h.

4.4 Benefits of silo extension

The amount of intermediate storage space available on-site is very limited which has serious consequences on the plant scheduling. Investment costs for extending the number of silos are a significant estimated €[redacted] to €[redacted] per 300 t silo, heavily depending on the specifications. Anyhow, benefits should always outweigh the investment costs. Extra silo space will probably decrease powder transport and handling costs, and might also yield an increase in plant throughput as a result of a greater flexibility.

As benefits of silo extension are hard to estimate, the developed model can be a valuable tool in assessing consequences of expanding on-site silo space. The number of intermediate storage silos present on-site is a parameter in the model that can be altered to predict effects on the schedule and throughput, hence potential savings can be weighed up against investment costs.

The research is set up as follows: schedules for January and February are made using a base setting of *NS* = 6 internal silos. These three schedules are then redetermined for 8 and 10 silos. Maximum computation time is set to 300 seconds, which turned out to be long enough to ensure that more silos do not yield schedules that are worse in terms of the objective function. When allowing short computation times, the model may come up with worse schedules when more silos are available, due to the increased solution space. Used parameter values are as follows: transport cost [redacted] €/truck, production value [redacted] €/t.

Table 4.1. Silo extension performance, January schedule constraints.

January	6 silos	8 silos	10 silos	12 silos	16 silos
Number of trucks [-]	180	98	226	161	33
Transporting costs [€]	21,600	11,760	27,120	19,320	3,960
Average amount stored externally [t]	5,755	5,372	4,641	4,927	5,383
Rental costs [€]	46,043	42,983	37,134	39,413	41,391
Production [t]	15,194	15,250	15,394	15,383	15,413
Relative value (approx.)	100.0%	100.9%	101.0%	101.4%	102.4%

Table 4.2. Silo extension performance, February schedule constraints.

February	6 silos	8 silos	10 silos
Number of trucks [-]	303	291	182
Transporting costs [€]	36,360	34,290	21,840
Average amount stored externally [t]	5,472	4,901	4,846
Rental costs [€]	43,777	39,209	38,773
Production [t]	15,940	16,055	16,065
Relative value (approx.)	100.0%	100.8%	101.5%

Table 4.3. Silo extension performance, March schedule constraints.

February	6 silos	8 silos	10 silos
Number of trucks [-]	242	223	254
Transporting costs [€]	29,040	26,760	30,480
Average amount stored externally [t]	8,656	8,238	8,444
Rental costs [€]	69,247	65,904	67,552
Production [t]	15,870	15,887	15,931
Relative value (approx.)	100.0%	100.4%	100.4%

Reductions in the number of intermediate product transports from and to the production site do not always occur when silo space is increased from 6 to 8 or 10 silos. This is because reducing intermediate transport cost is not the primary objective of the scheduling model. In fact, the model tries to maximize the schedule value. Results show that the schedule value is indeed never lower when more free silo space is allowed. Benefits on the other hand do not seem high.

A quick analysis of the involved figures also gives insight. Considering only (intermediate) powders, yearly spent amounts are approximately € [redacted] on transporting cost and € [redacted] on silo rental costs.

Suppose that silo utilization on average remains unchanged at 60%, savings on holding cost correspond to 60% of the extra installed tons. In the case of 2 extra silos, this yields a minor cost reduction of

$$60\% \cdot 2\ [\text{silos}] \cdot 150\ [\text{t/silo}] \cdot X\ [\text{EUR/t/month}] = X\ [\text{EUR/month}]\,. \tag{4.26}$$

Reductions in transporting costs are hard to estimate. Cost of capital is also an issue when storing powder, but it does not matter whether powder is stored on-site or externally, and should therefore not be taken into consideration regarding silo-extension investments.

Most likely, the first added silo yields the most in terms of cost reduction, and every next added silo yields a lesser amount of savings. Therefore relatively small numbers of extra silos can be considered. Suppose that three additional silos are built at a total cost of around €[redacted], which makes the internal capacity grow from 900 t to 1200 t. Total stored powder levels are on average around 3000-4000 t. It is therefore likely that not more than a quarter of the transporting and rental costs of external silos are eliminated due to the extra internal silo space. This corresponds to a cost reduction of €[redacted] per year. Weighing the investment costs against maximum attainable cost savings makes the investment opportunity not seem highly attractive.

Chapter 5. Results

The developed GAMS scheduling model is able to produce feasible production schedules within a few minutes of computing time on a regular desktop computer. Input parameter values are imported from an Excel spreadsheet, and results are exported to Microsoft Excel afterwards.

5.1 Schedule example

An example of a schedule as proposed by the model will be given. First parameter values are listed, then Gantt charts and a number of performance metrics are presented.

Input data is given in Table 5.1, Table 5.2, and Table 5.3, for desired production numbers, available powder quantities at the beginning of the period and desired powder quantities at the end of the period, and machine setups, respectively. Further parameter settings are scheduling horizon $C^{\max} = 30$ days, and available number of internal intermediate storage silos $NS = 6$ silos. Resulting Gantt charts for the reaction trains and the extruders are displayed in Figure 5.1 and Figure 5.2, respectively.

Table 5.1. Desired production quantities. CPE grades are aggregated as production runs take place in one sequence.

Extruded grade	Desired production quantity [t]
AE	8023
BE	1850
IE	621
DE	876
KE	1034
NE	1148
ME	0
EP (powder)	1683
FP (powder)	
GP (powder)	
HP (powder)	

Table 5.2. Powder quantities at beginning and end of month.

Powder grade	Powder qty at t [t]	
	$t = 0$	$t = C^{\max}$
AP	2087	2011

BP	994	860
DP	1513	1752
KP P	0	453
NP	395	395
MP	0	0
EP	182	572
FP	1	281
GP	110	20
HP	340	0

Table 5.3. Machine setups at $t = 0$.

Machine	Setup at $t = 0$
T1	BP
T2	DP
X1	IE
X2	BE
X4	BE

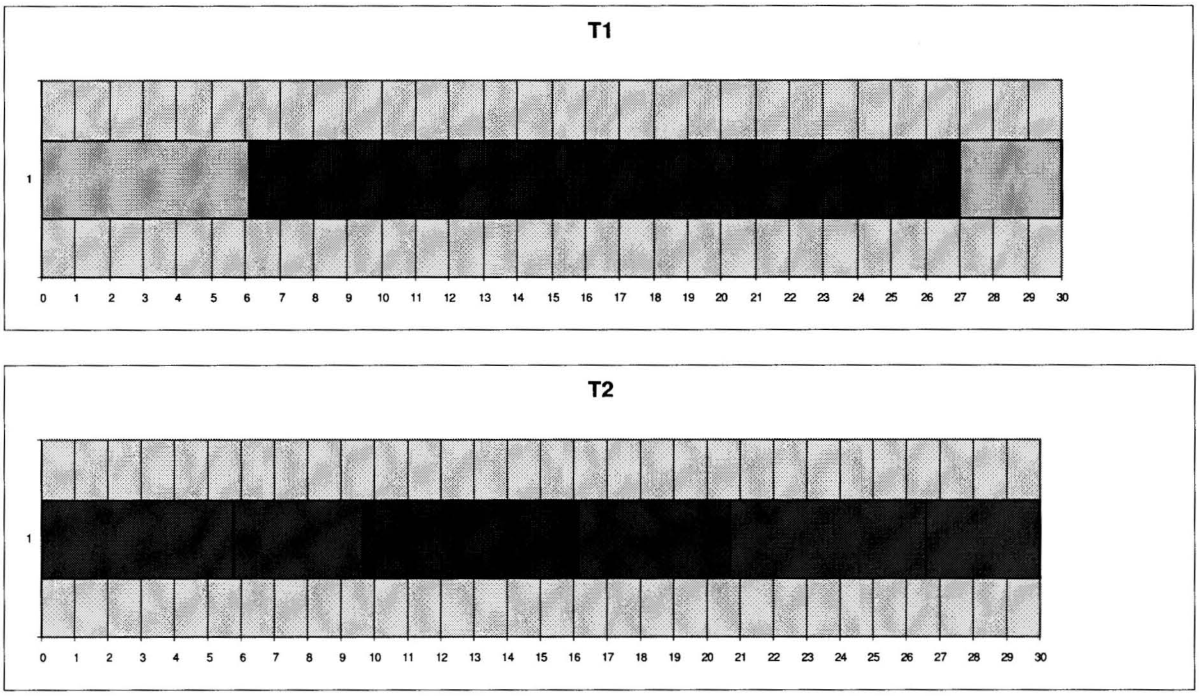


Figure 5.1. Gantt charts for reaction trains T1 and T2. Horizontal axis displays the number of days.

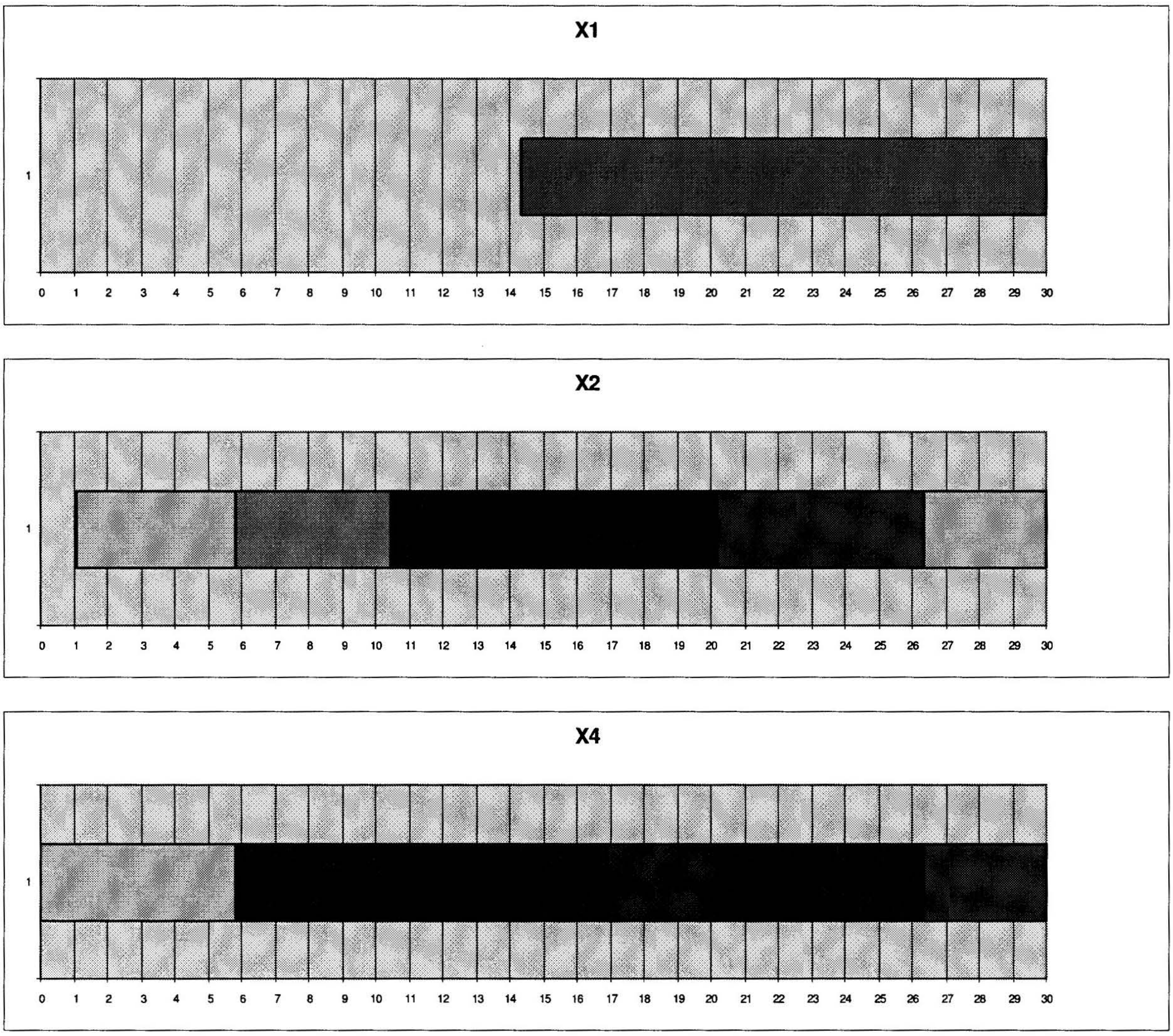


Figure 5.2. Gantt charts for extruders X1, X2, and X4. Horizontal axis displays the number of days.

Chapter 6. Model validity

Gass (1983) suggests a framework for assessing the validity of (decision-aiding) models. Following the dimensions of Gass as a guideline helps evaluating the validity of the Tessenderlo scheduling model. The three main concepts are technical validity, operational validity and dynamic validity. Each will be discussed below and applied to the Tessenderlo model. The most significant issues will be highlighted and implications will be discussed in the last section.

6.1 Technical validity

When developing a model, numerous assumptions have to be made. Technical validity deals with assumptions on different areas: model validity, data validity and logical or mathematical validity.

Model validity deals with the question whether the model corresponds with reality. When a certain scope and detail level is chosen for modeling a real-world situation, it must be made sure that relevant physical constraints that are present in reality are included in the model. In Tessenderlo it is essential to ensure that certain grades are only produced on certain equipment. Reality is properly emulated when more specific constraints are included: transporting costs are higher during weekends, and only one train-changeover can take place at a time.

Data validity is concerned with the accuracy, reliability and validity of data that is used to set model parameters. Input parameters are production rates of all equipment, recipes, and starting values for powder quantities. These parameters are all known with a high degree of certainty as they are perfectly measurable. Changeover times vary somewhat more, but a typical error margin of 1 hour in changeover time does not play a large role when viewed on a scale of one month, especially as under- and overestimated changeover times cancel each other out on the long run. No problems are thus to be expected due to inaccurate data.

Logical validity or *mathematical validity* deals with the computational correctness of the model. This type of validity can be verified by computing production quantities according to the production schedule by hand. All produced tons must correspond to the consumption of raw material for instance, taking recipes into account. The number of days that certain grades are produced, multiplied by the corresponding production rate should yield the production quantity, which in turn should match the quantity as given by the model. No actual inaccuracies are found in the Tessenderlo model.

Predictive validity relates to the corresponding of predicted outcomes with actual outcomes. Developing a production schedule for Tessenderlo is always predictive. Predictive validity issues are only revealed after the actual production according to the schedule has taken place. Values such as the profit of a production schedule are implicitly predicted in the model as this is the value to be maximized by the model. Derivations from actual profits are inevitable as only the profit margin of HDPE pellets and intermediate storage costs are taken into account. This should however not be a problem as long as this value is not used for any other purpose than to maximize the schedule value.

6.2 Operational validity

As the technical validity reveals that certain inconsistencies exist between the model and reality, the impact of these inconsistencies are assessed. A model is by definition unable to exactly reproduce or predict real-world behavior, but often a reasonable approximation is sufficient. Errors should thus remain acceptable. So, are the results appropriate and does the model produce information that enables a decision-maker to judge the appropriateness of the model's results?

Usage of a model is only justified when the cost savings of implementing the model's results appear substantial.

A sensitivity analysis reveals whether input parameters need to be highly accurate. In Tessenderlo, the schedule value is rather dependent on the production rates of the different machines, and also on changeover times and recipes. However, the reaction and extrusion rates and other relevant parameters are known with a high level of certainty.

Implementation validity deals with the issue whether the system will be able to behave like the solution as recommended by the model. This brings no new issues to light as the plant does not have to operate in a different manner when producing according to a model solution.

6.3 Dynamic validity

How will the model be supported in the future to ensure that it remains representative for the real-world production situation? This question is addressed in the element dynamic validity. Two items play a role: updating and reviewing.

Several changes to the production environment can take place. Examples can be introducing or terminating a certain HDPE grade, improving the production rate of a reaction train or extruder, changes in the cost of external storage and handling, etcetera. Many of these potential changes should be relatively simple to implement in the model. Updating procedures are therefore designed to assist users of the model in implementing changes without any support from the model developer.

Reviewing the model periodically can be helpful in keeping the validity adequate. All three validity dimensions should be assessed from time to time.

6.4 Verification

Actual realized production figures of the months January, February and March 2008 are compared to scheduling model output. Total plant throughput, transporting costs and holding costs are the most interesting metrics to compare. Inputs for the model are desired production quantities for all grades, and the machine setup and powder quantities at the start of the particular month.

One important factor that should be taken in consideration when comparing the model with reality is that in reality several breakdowns probably have taken place, which may have impacted the planning, throughput, and transporting and holding costs. When an extruder breaks down for example, powder production is not stopped and therefore (external) powder inventory rises, leading to higher transporting and holding costs than planned beforehand. This behavior cannot be taken into account in the scheduling model and is also not taken into account when manually planning a future situation, except for planned outages.

In Table 6.1, Table 6.2, and Table 6.3 comparisons are shown between the production schedules of January, February and March 2008 and schedules as suggested by the model. Clear differences can be seen which are not surprising: the model performs significantly better as it does not contain any unplanned outages such as machine failures. Seemingly these outages play too big a role in the production environment under consideration to properly compare the production according to the manually made schedule with the a priori model schedule. Therefore the last columns are corrected with the percentage of machine outages that took place in the respective months which shows that throughput and schedule value are still higher when using a modeled schedule that is not revised during a month, except for outages.

Table 6.1. Comparison manual schedule vs. a priori schedule by model, January data.

Metric	Manual	Model	Model (corrected with machine failures)
Throughput [t]	13,977	15,195	14,739 (97.0% asset utilization)
Transported qty [trucks]	321	180	unknown
Transport cost [€]	45,477	21,600	45,477 (estimated)
Silo rental cost [€]	unknown	46,043	50,000 (estimated)
Relative value (approx.)	100%	110.5%	105.4%

Table 6.2. Comparison manual schedule vs. a priori schedule by model, February data.

Metric	Manual	Model	Model (corrected with machine failures)
Throughput [t]	14,487	16,016	15,968 (99.7% asset utilization)
Transported qty [trucks]	264	206	unknown
Transport cost [€]	39,685	24,720	39,685 (estimated)
Silo rental cost [€]	unknown	43,414	50,000 (estimated)
Relative value (approx.)	100%	112.0%	110.6%

Table 6.3. Comparison manual schedule vs. a priori schedule by model, March data.

Metric	Manual	Model	Model (corrected with machine failures)
Throughput [t]	14,832	15,870	15,061 (94.9% asset utilization)
Transported qty [trucks]	301	242	unknown
Transport cost [€]	52,332	29,040	(estimated)
Silo rental cost [€]	unknown	69,247	70,000 (estimated)
Relative value (approx.)	100%	108.5%	101.6%

Chapter 7. Conclusions

A number of conclusions and recommendations can be drawn for this project. All recommendations can be applied individually; however the combination will be most beneficial.

- Do not let demand dynamics lead to schedule disturbances, thereby retaining schedule efficiency and avoiding costly disturbances. This can be realized by slightly increasing the final goods inventory to a level that is sufficient to cope with demand uncertainty. This means that in essence a make-to-stock policy is applied and the customer-order decoupling point is placed at the finished goods inventory level.
- Currently the production frequency for the most grades is below the optimum. Most grades have an optimal production cycle of at least six weeks (cost curves are shown in Figure 4.6 and Figure 4.7 on page 37). Optimal production cycle lengths depend on a number of factors, but setup costs and holding costs play the largest role. While holding costs will increase when cycle length is increased as a result of the increased average cycle stock, losses in production time due to less changeovers make up for this and can lead to significant cost reductions. This especially concerns production runs on the first production step as long changeover times are involved. Optimal production lengths correspond to low values of Total Relevant Costs (TRC). Cost reductions can be high as the cost curves are especially steep at the relevant left side. Note that the shown TRC values must be interpreted as approximations.
- Using the developed scheduling model for monthly scheduling will optimize the schedule value in terms of throughput and intermediate storage cost and thus provide a schedule which can be used a good starting point.

On extending the intermediate storage space:

- although gains are to be expected on the area of material handling ease and safety, investments in extra on-site intermediate storage space cannot be justified using the scheduling model. While the number of silos is a model parameter that can be easily altered, no proof is found that the attainable savings will be significantly higher than the related investment costs.

7.1 Scientific recommendations

When production machines on two consecutive stages have different production rates and intermediate storage is limited, problems may arise due to the dependency between the two stages.

First of all, it is important to investigate the real costs of (extending) intermediate storage, and to weigh this up against the value of extra production that is attainable when ignoring the intermediate storage costs and focusing only on maximizing production. When the orders of magnitude between these two figures are comparable, a modeled approach as described in this work is a good way to maximize overall value. When intermediate storage costs can be ignored and production should be maximized, the model approach can still be attractive for maximizing throughput. In practice, models turn out to be hard to implement and to maintain by someone else than the designer. This is it is important to keep in mind when facing a similar challenge.

The developed model is able to distinguish between free on-site storage and costly external storage, a situation that may well be present in other situations in practice. While it produces decent results within a few minutes, the produced schedules continue to improve when the model is running for hours. Factors that are not easy to grasp seem to play a role in the model efficiency, such as the order of the equations. It seems there is room for improvement on this area.

7.2 Practical recommendations

As implementing the scheduling model is a rather radical change, two other changes should be considered first. Increasing the production cycle length on the reaction step to at least six weeks for most grades will turn a significant amount of changeover time into valuable production time. This will mean a higher amount of cycle stock but benefits of extra production greatly weighs up to this.

Furthermore a make-to-stock policy should be implemented. This means the customer-order decoupling point is placed on the finished goods inventory level, and changes in customer order are handled by finished goods inventory instead of schedule changes. Applying this policy will save on costly changeovers while only a slightly higher level of finished goods inventory is necessary to cope with the customer demand uncertainties.

The third step is to utilize the developed scheduling model in the monthly scheduling. This increases the schedule value even more. One should however realize that maintaining the schedule needs care, and that radical changes in the production environment may require significant changes in the model.

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List of Symbols

Symbol	Meaning	Unit
A	setup cost	€
D	demand rate	units/unit time
v	unit variable cost	€/unit
r	carrying charge	€/€/unit time
p	production rate	units/unit time
q	inventory	units
i	product type	-
m_i	production run interval	-
EOQ	economic order quantity	units
TRC	total relevant costs	€
T	common order interval	unit time
R	capital cost charge	€/€/unit time
k	silo rental costs	€/ton/unit time
I	tasks	-
S	states	-
T	event points	-
U	units	-
I_u	tasks performed in unit *u*	-
I^c	continuous process tasks	-
I_c^s	continuous tasks engaging state *s*	-
I_c^{st}	storage tasks engaging state *s*	-
$I_{ii'}^{ch}$	pair of continuous tasks *i* and *i'* with changeover requirements	-
S^{fis}	states of finite intermediate storage	-
$\theta_i^{\max}$	maximum allowable task duration of task *i*	days
$\theta_i^{\min}$	minimum allowable task duration of task *i*	days
$R_i^{\max}$	maximum allowable task rate of task *i*	t/day
$R_i^{\min}$	minimum allowable task rate of task *i*	t/day

$ST_s^{\max}$	maximum allowable dedicated storage for state *s*	t
ST_s^0	initial amount of state *s*	t
v_s	value/price of state *s*	€
λ_{si}	fraction of state *s* engaged in task recipe *i*	-
$\theta_{ii'}$	changeover time from task *i* to task *i'*	days
$C^{\max}$	time horizon	days
C_u^{ch}	minimum cumulative changeover time in unit *u*	days
θ_{it}	duration of task $i \in I^c$ for event *t*	days
θ_{st}	duration of task $i \in I_s^c$ for event *t*	days
τ_{it}	ending time of task $i \in I$ for event *t*	days
τ_{st}	ending time of task $i \in I_s^c$ for event *t*	
ξ_{it}	total extent (rate×duration) of task $i \in I^c$ for event *t*	t
x_{it}	1 if task *i terminates* at event point *t* and 0 otherwise	-
ST_{st}	amount of state *s* at the end of event point *t*	t
y_{st}	1 if $ST_{st} \neq 0$ and 0 otherwise	-
m	storage capacity of a single truck	t
$silosInUse_{st}$	number of silos used by state *s* at the end of event point *t*	-
TR_{st}	number of truck transports for state *s* at the end of event point *t*	-
INT_{st}	internal intermediate inventory for state *s* at the end of event point *t*	t
EXT_{st}	external intermediate inventory for state *s* at the end of event point *t*	t
z	objective value	€

Appendices

Appendix I. Linear programming

A general (matrix) formulation of a linear program is

$$\begin{aligned} &\text{maximize } C^T x \\ &\text{subject to } Ax \leq b \\ &\text{where } x \geq 0. \end{aligned} \tag{6.1}$$

Vanderbei (2001) describes a linear program as a problem in which *decision variables* have to be chosen in some optimal fashion. These decision variables are

$x_j, \quad j = 1, 2, \ldots, n$

A linear function of these decision variables, the objective function ζ, is then to be maximized or minimized:

$$\zeta = c_1x_1 + c_2x_2 + \ldots + c_nx_n \tag{6.2}$$

Furthermore there are constraints associated to the decision variables:

$$a_1x_1 + a_2x_2 + \ldots + a_nx_n \begin{Bmatrix} \leq \\ = \\ \geq \end{Bmatrix} b \tag{6.3}$$

In short, a linear program can be formulated in this general fashion (Vanderbei, 2001):

$$\begin{aligned} \text{maximize } & c_1x_1 + c_2x_2 + \ldots + c_nx_n \\ \text{subject to } & a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \leq b_1 \\ & a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \leq b_2 \\ & \qquad \vdots \\ & a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \leq b_m \\ & \qquad\qquad x_1, x_2, \ldots, x_n \geq 0 \end{aligned} \tag{6.4}$$

Appendix II. The Tessenderlo scheduling model

Sets/Indices

I / i = tasks

S / s = states

T / t = event points

U / u = units

I_u = tasks performed in unit u, $I_u \subseteq I$

I^c = continuous process tasks, $I^c \subseteq I$

I_s^c = continuous tasks engaging state s, $I_s^c \subseteq I^c$

I_s^{st} = storage tasks engaging state s, $I_s^{st} \subseteq I^{st}$

$I_{ii'}^{ch}$ = pair of continuous tasks i and i' with changeover requirements

S^{fis} = states of finite intermediate storage

Parameters

$\theta_i^{\max}$ = maximum allowable task duration of task $i \in I^c$

$\theta_i^{\min}$ = minimum allowable task duration of task $i \in I^c$

$R_i^{\max}$ = maximum allowable task rate of task $i \in I^c$

$R_i^{\min}$ = minimum allowable task rate of task $i \in I^c$

$ST_s^{\max}$ = maximum allowable dedicated storage for state s

ST_s^0 = initial amount of state s

v_s = value/price of state s

λ_{si} = fraction of state s engaged in task recipe i

$\theta_{ii'}$ = changeover time from task i to i'

$C^{\max}$ = time horizon

C_u^{ch} = minimum cumulative changeover time in unit u

Variables

θ_{it} = duration of task $i \in I^{\mathrm{c}}$ for event t

θ_{st} = duration of task $i \in I_s^{\mathrm{c}}$ for event t

τ_{it} = ending time of task $i \in I$ for event t

τ_{st} = ending time of task $i \in I_s^{\mathrm{c}}$ for event t

ξ_{it} = total extent (rate × duration) of task $i \in I^{\mathrm{c}}$ for event t

$x_{it} = 1$ if task $i \in I$ *terminates* at event point t and 0 otherwise

ST_{st} = amount of state s at the end of event point t

$y_{st} = 1$ if $ST_{st} \neq 0$ and 0 otherwise

Constraints

$$x_{it}\theta_i^{\min} \le \theta_{it} \le x_{it}\theta_i^{\max}, \quad \forall i \in I^{\mathrm{c}}, t \in T$$

$$\theta_{it}R_i^{\min} \le \xi_{it} \le \theta_{it}R_i^{\max}, \quad \forall i \in I^{\mathrm{c}}, t \in T$$

$$\tau_{it} - \tau_{st} \le C^{\max}\left(1 - x_{it}\right), \quad \forall s \in S, i \in I_s^{\mathrm{c}}, t \in T$$

$$\tau_{it} - \tau_{st} \ge -C^{\max}\left(1 - x_{it}\right), \quad \forall s \in S, i \in I_s^{\mathrm{c}}, t \in T$$

$$\theta_{it} - \theta_{st} \le C^{\max}\left(1 - x_{it}\right), \quad \forall s \in S, i \in I_s^{\mathrm{c}}, t \in T$$

$$\theta_{it} - \theta_{st} \ge -C^{\max}\left(1 - x_{it}\right), \quad \forall s \in S, i \in I_s^{\mathrm{c}}, t \in T$$

$$\tau_{it} - \theta_{it} \ge \tau_{i,t-1}, \quad \forall i \in I_s^{\mathrm{c}}, t \in T$$

$$\tau_{st} - \theta_{st} \ge \tau_{s,t-1}, \quad \forall s \in S, t \in T$$

$$ST_{st} = ST_{s,t-1} + \sum_{i \in I_s^{\mathrm{c}}} \lambda_{si}\xi_{it}, \quad \forall s \in S, t \in T$$

$$\sum_{i \in I_u} x_{it} \le 1, \quad \forall u \in U, t \in T$$

$$\tau_{it} = \tau_{i't}, \quad \forall u \in U, i, i' \in I_u, i = \mathrm{HEAD}\left(I_u\right), i' \neq i, t \in T$$

$$\tau_{i't'} - \theta_{i't'} - \tau_{it} \ge \theta_{ii'} - C^{\max}\left(2 - x_{it} - x_{i't'} + \sum_{i'' \in I_u} \sum_{t''>t}^{t''<t'} x_{i''t''}\right), \quad \forall u \in U, i, i' \in I_u, I_{ii'}^{\mathrm{ch}}, t, t' \in T, t < t'$$

$$\sum_{i \in I^c} \sum_{t \in T} \theta_{it} \le C^{\max} - C_u^{\mathrm{ch}}, \quad \forall u \in U$$

$$ST_{st} = INT_{st} + EXT_{st}, \quad \forall s \in S^{\mathrm{fis}}, t \in T$$

$$INT_{st} \le silosInUse_{st} \cdot V^{\max}, \quad \forall s \in S^{\mathrm{fis}}, t \in T$$

$$\sum_{s} silosInUse_{st} \leq NS \quad \forall s \in S^{\text{fis}}, t \in T$$

$$m \cdot TR_{st}^{\text{OUT}} \geq EXT_{st} - EXT_{s,t-1} \quad \forall s \in S^{\text{fis}}, t \in T$$

$$m \cdot TR_{st}^{\text{IN}} \geq EXT_{s,t-1} - EXT_{st} \quad \forall s \in S^{\text{fis}}, t \in T$$

$$TR^{\text{TOT}} = \sum_{t \in T} \sum_{s \in S} TR_{st}^{\text{OUT}} + TR_{st}^{\text{IN}}$$

$$\max z = \sum_{s \in S} v_s ST_{s,t_n} - k \cdot TR^{\text{TOT}}$$