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MASTER

Forecast-based inventory policies
an exact analysis based on an optimal forecast

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Forecast-Based Inventory Policies: An exact analysis based on an optimal forecast

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in partial fulfilment of the requirements for the degree of

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Chapter 1

Abstract

This master thesis proposes a forecast-based inventory policy able to cope with stochastic non-stationary demand when a lead time greater than zero is present in the production system. By deciding upon the forecast technique utilized, the model proposed suits situations in which a k–th Exponential Weighted Moving Average is an appropriate forecast model. For comparison, a forecast-based inventory policy that approximates the total demand variability faced by the inventory policy is presented. This approximation accounts for both demand uncertainty and the increment in variability derived from the misspecification of the forecast technique. In order to incorporate this approximation into the buffer calculation, literature on production systems facing random yield is studied. Besides formulating both models, this master thesis also analyzes the two policies numerically in order to gain insight into the impact of each model input parameter on the average total cost. Policy results, obtained through a simulation model, show that the policy proposed outperforms, in terms of costs, forecast-based inventory policies making use of forecasting techniques assuming stationary demand patterns.

Chapter 2

Preface

This master thesis represents the conclusion of my master Operations Management and Logistics at Eindhoven University of Technology. In this preface, I would like to thank to the people who supported me throughout my master thesis.

First, I would like to thank my supervisor Nico Dellaert for his excellent guidance during this project. Second, I would like also to thank Matthew Reindorp for being part of my graduation committee. Third, I would like to thank the people in room N-19 for these pleasant six months. Fourth, I would like to thank my parents, brother and sister for their unconditional love and support. And finally, but not less important, I would like to thank to all my friends in Eindhoven, thank you for have made of this time living abroad a memorable episode in my life.

Chapter 3

Summary

3.1 Introduction

Nowadays, increasing competition has made product life cycles shorter, incrementing the randomness and unpredictability of their demand processes. As a result, managers are challenged with providing accurate forecasts when demand is stochastic and non-stationary, and a lead time greater than zero is present in the production system.

With regard to literature on inventory management, a classification between Standard and Forecast-Based policies is considered. In the Standard inventory management approach, inventory policies assume a stationary demand pattern where the policy parameters are computed at the beginning of the time horizon and account for a constant buffer that guarantees a desired service level (Axsater, 2006). However, when the production system faces stochastic non-stationary demand, the selection of a standard policy seems inappropriate since this type of policy fixes a probability distribution at the beginning that remains constant along the horizon. Different from Standard policies, Forecast-Based policies determine the policy parameters based on predictions (Babai and Dallery, 2005). In the scenario of stochastic non-stationary demand, these forecasts are time dependent in order to satisfy demand.

Furthermore, of literature on forecasting techniques, only a few approaches were found that provide an exact demand uncertainty when demand is stochastic and follows a non-stationary process. The majority of the methods assume that demand will follow a stationary pattern based on information from past sales, opinions from experts in the field, hypothesized relationships, etc. As a consequence, when the forecast technique is not correctly specified and hence unable to provide an exact demand uncertainty, the forecast-based inventory policy is in need of accounting for both demand uncertainty and the increased variability of the expected mean of the demand distribution.

Moreover, within the literature on forecast-based inventory policies, none of the approaches found specifically assumes non-stationary demand and makes use of a forecast technique that provides an exact demand uncertainty or accounts for the increment in variability derived from the misspecification of the forecast technique within the buffer calculation. The majority of the approaches found assume stationary demand and rely on the Martingale Model of Forecast Evolution (MMFE) for diminishing the demand uncertainty.

3.2 Research

Based on the research gap described above, the research assignment is defined as follows:

"Design a forecast-based inventory policy able to cope with the exact demand uncertainty

when demand is stochastic and follows a non-stationary process, and a lead time greater than zero is present in the production system. And, as a final goal, minimize the total holding and backordering costs."

To carry out the research assignment, two steps were taken, with the first step consisting of two parts. The first step involved the development of two inventory policies: a forecast-based inventory policy capable of accounting for the exact demand uncertainty (Exact Model) and a forecast-based inventory policy that approximates this uncertainty when an increment in variability is caused by a biased prediction (Approximation Model). Both policies made use of an adaptive base stock level and assumed stochastic non-stationary demand, a review period equal to one, a constant lead time greater than zero and that demand not satisfied is backordered.

For the Exact Model, a k–th Exponential Weighted Moving Average forecast technique is utilized. Through this forecast technique an unbiased forecast is obtained, where the remaining demand uncertainty is equal to the random noise for time period t. In each period, the base-stock level is adjusted based on changes in the forecast and the demand noise at time t, plus a constant safety stock. The safety stock is determined based on the variance of the end inventory.

For the Approximation Model, literature of production planning under yield uncertainty was studied. This review aimed to come up with ideas for considering the bias in the forecast within the buffer calculation. From the literature on this subject, the approach presented in Inderfurth and Gotzel (2003) is considered, but instead of yield risk, the safety stock takes into account the increment in variability derived from the misspecification of the forecast technique. In this latter regard, the Moving Average and Exponential Smoothing forecasting techniques, and the scenario where partial demand information is available are considered. As in the Exact Model, the base-stock level is adjusted based on the forecasts over the lead time plus the safety stock.

In the second step, both models were simulated and results were analyzed numerically. A base case scenario is presented followed by a sensitivity analysis. In addition to both policies, the continuous review policy (s, S), is used for comparison.

3.3 Results

In the base case scenario analysis, the Exact Model showed the less smooth base-stock levels. The perceptible change among them is because of an order quantity that accounts for the exact demand uncertainty and changes in the forecast, plus a constant safety stock. With regard to the Approximation Model, the base-stock levels were less smooth the higher the percentage of information utilized. Unsurprinsingly, a quick response to changes in demand showed a decreasing cost the higher the percentage of information.

The sensitivity analysis was carried out with respect to the following input parameters: demand variance (σ_ε^2), level of non-stationary demand (α) and service level (z). With regard to both models' sensitivity to the value of σ_ε^2, both policies showed a substantial increase in the total cost when σ_ε^2 was varied from 0 to 1. Also, both models showed that after a steep increase at the beginning, the cost continued to grow but non-linearly, where the Exact Model showed the smallest cost increase as σ_ε^2 was augmented. Moreover, contrary to what was expected, the Exact Model did not always outperform the Approximation Model. The explanation for this phenomenon was found in the impact of the service level on the safety stock. Results obtained varying the safety factor show that, for all values of σ_ε^2, a service level greater than 95% was achieved even when a desired service level equal to 90% was specified.

With regard to the level of non-stationary demand, both models showed an increment in the average cost as α increased. As when σ_ε^2 was varied, the Exact Model did not always outperform the Approximation Model. Again, the explanation of this event was found in the impact of the service level on the safety stock. Results obtained varying the safety factor show that a 95% service level was achieved, for all values of α, even when a desired service equal to 90% was specified. With regard to the Approximation Model making use of demand information, this model did not show any increment or decrement with respect to the value of demand information along the different values of α.

Finally, when the service level was varied, both models showed a higher inventory cost with increasing values of z. As expected, the impact of this input parameter was greater for the Exact Model.

3.4 Conclusions & Discussion

The following conclusions are drawn from the results obtained:

- *When demand is stochastic and non-stationary, forecast-based inventory policies that cope with the exact demand uncertainty outperform forecast-based inventory policies making use of forecasting techniques assuming stationary demand patterns.* It is more beneficial to make use of a forecast technique that enables the Forecast-Based inventory policy to cope with the exact demand uncertainty than utilizing forecasting techniques that assume stationary demand patterns. However, with regard to the Exact Model, a remark needs to be made with respect to the impact of high service levels on the safety stock. It is expected that for high values of service level, the Approximation model will perform better than the Exact model due to an increment in holding inventory in the latter approach.

- *Recent demand observations provide a better prediction for future demand.* Slightly lower costs are obtained under the Approximation Model making use of 0% demand information in comparison with the well known Moving Average (except when T=1) and Exponential Smoothing (except when α_E=1) forecasting techniques.

- *The presence of demand variance is more important than the level of demand variance.* The numerical analysis shows that for both models the presence of demand variability caused the largest increase in cost. Although the cost kept growing as σ_ε^2 was enlarged, this increment was non-linearly. The Exact Model showed the smallest cost increase as σ_ε^2 was augmented.

- *The value of information increases with the level of demand variability.* There is a substantial cost difference along the values of σ_ε^2 across the forecasting techniques making use of demand information, where it is more beneficial to have information when demand is more variable.

- *Forecast-based inventory policies outperform Standard policies.* The utilization of a forecast-based inventory, regardless the forecast technique utilized, achieves a lower total cost in comparison with standard policies.

The limitations of the Exact model are the following: a model that depends upon a specific forecast technique and demand pattern, a model which operates with a periodic review equal to one where an ordering cost equal to zero is assumed and a model that assumes normal distribution and independence among the demand noise terms. With regard to the Approximation model, a remark should be made where the approximation does not account for deviations above demand.

Chapter 4

Introduction

This master thesis is motivated by the challenge faced by manufacturers when demand is stochastic and follows a non-stationary process, and a lead time greater than zero is present in the production system. In order to cope with non-stationary demand, forecast-based inventory policies allow the adjustment of the base-stock level based on the forecast demand over the lead time period. However, in the literature on forecast techniques, only a few approaches were found that provide an exact demand uncertainty when demand is stochastic and follows a non-stationary process. As a consequence, when the forecast technique is not correctly specified and hence unable to provide an exact demand uncertainty, the forecast-based inventory policy is in need of accounting for both demand uncertainty and the increased variability of the expected mean of the demand distribution. Therefore, the research assignment derived from this problem statement is:

"Design a forecast-based inventory policy able to cope with the exact demand uncertainty when demand is stochastic and follows a non-stationary process, and a lead time greater than zero is present in the production system. And, as a final goal, minimize the total holding and backordering costs."

In order to fulfill the research assignment, a literature study of forecasting techniques was carried out. This study aimed to derive methods that are capable of providing an exact demand uncertainty when demand is stochastic and non-stationary. Furthermore, for comparison, a forecast-based inventory policy where the forecast technique does not provide an exact demand uncertainty is presented. This policy makes use of an approximation to account for the increment in demand variability derived from the misspecification of the forecast technique. In order to do so, literature on production systems facing random yield was studied. This study aimed to come up with ideas for incorporating the increment in demand variability in the buffer calculation and as a final goal, approximate the total demand variability faced by the production system. It is expected that forecast-based inventory policies will perform better using an approximation of the demand variability than they would without such an approximation; however, they are not expected to outperform forecast-based inventory policies that are able to cope with the exact demand uncertainty. This master thesis aims to provide a solution when demand is stochastic and non-stationary, the lead time is greater than zero and the forecast technique utilized guarantees an exact demand uncertainty.

This report is organized as follows: In Chapter 5 a literature review on forecasting techniques and inventory management is carried out. Chapter 6 provides the problem description from which the research assigment and research questions are derived. In order to answer the research assigment, a forecast-based inventory policy is proposed in Chapter 7. In Chapter 8 this policy is analyzed numerically. First, a base case scenario is presented. Second, a sensitivity analysis is perfomed for the model input parameters. Finally, in Chapter 9, this master thesis is concluded and its limitations are discussed.

Chapter 5

Literature Review

In this chapter, two relevant fields in the academic literature are studied: Forecasting Techniques and Inventory Management. The purpose for studying both fields is with the aim of identifying gaps in the literature and point out existing approaches that are considered to be relevant to the development of this master thesis. Further, in Chapter 6, existing literature of production planning under yield uncertainty is studied.

5.1 Forecasting Techniques

Literature on forecasting techniques has focused on diffentiating between qualitative and quantitative approaches. In the case of qualitative techniques, also known as judgmental techniques, the forecast demand is based on opinions (Jobber and Lancaster, 2009). Examples of this type of technique are the Jury and the Delphi methods. Although both methods rely on the opinions of experts in the field in order to forecast demand, the Delphi method differs from the Jury Method by avoiding interaction among the experts in order to prevent group-thinking.

Quantitative forecasting methods are further classified into time series and causal techniques. Time series techniques emphasize past events as the input parameter in order to predict the demand. The Moving Average and Exponential Smoothing techniques are two examples of time series forecasting techniques: the former assumes a horizontal trend and the latter allows the specification of weights in the data in order to capture seasonality.

Chen et al. (1999) studied the performance of the Exponential Smoothing forecasting technique when the demand process is correlated and when it follows a linear trend. Regarding the former demand process, results presented shown that negatively correlated demand leads to a higher variability than positively correlated demands. For the scenario where demand follows a linear trend, no relationship was found between the increase in variability and the magnitude of the linear trend. For comparison, the authors utilized forecasts generated via the Moving Average technique. Although for certain demand processes the Exponential Smoothing technique showed larger increases in variability, both forecast methods showed a greater variability when the demand process follows a linear trend than when demand is independent.

Moreover, within the classification of time series, research has been done oriented to non-stationary time series. According to Tsokos (2010), this type of time series are suitable for specific demand processes in order to obtain an unbiased forecast when demand follows a non-stationary process. Examples of these methods are the Weighted Moving Average and the Exponential-Weighted Moving Average approaches. Both techniques emphasize recent observations through the specification of weigths: the former decreases the weight consistently and the latter decreases the weight exponentially.

Furthermore, causal techniques look for existing relationships between independent variables and the forecast dependent variable (Jobber and Lancaster, 2009). As a result, this type of technique establishes hypothesized relationships in order to predict future events. The Linear Regression and Simulation methods are examples of causal techniques. For these techniques, specification of independent variables (Linear Regression) or event probabilities (Simulation) is needed in advance.

So far, the majority of the forecasting techniques assume that demand will follow a stationary pattern based on information from past sales, opinions from experts in the field, hypothesized relationships, etc. However, in the scenario where demand is stochastic and follows a non-stationary process, the possibility of obtaining an exact demand uncertainty is restricted to a limited number of forecasting techniques that fits specific demand processes. Throught this literature study, two non-stationary time series were found: the Weighted Moving Average and the Exponential-Weighted Moving Average techniques.

5.2 Inventory Management

With regard to literature on inventory management, a classification between Standard and Forecast-Based policies is considered. In the Standard inventory management approach, inventory policies assume a stationary demand pattern (Axsater, 2006). Examples of Standard policies are the well known Continuous and Periodic review, (s, S) and (R, S), approaches where the policy parameters are computed at the beginning of the time horizon and account for a constant buffer that guarantees a desired service level. However, when the production system faces non-stationary demand, the selection of a standard policy seems inappropriate since this type of policy establishes a probability distribution at the beginning that remains constant along the horizon. Tunc and Eksioglu (2011) analyzed the cost performance of stationary inventory policies when demand is non-stationary. Making use of the (s, S) policy as a frame for comparison, the authors concluded that the cost efficiency of the standard policies decreases with increasing magnitude of demand variability.

In contrast to Standard policies, Forecast-Based policies determine the policy parameters based on predictions (Babai and Dallery, 2005). In the scenario of non-stationary demand, these forecasts are time dependent in order to satisfy demand. This leads to a periodic calculation of the policy parameters to account for the forecast demand over the lead time period.

In the context of single stage and multiple periods, approaches found in the literature specifically account for the forecast uncertainty. The majority of these policies make use of dynamic forecast updates utilizing the Martingale Model of Forecast Evolution (MMFE) presented in Heath and Jackson (1994). This forecast evolution model was developed for stationary processes, where changes in the prediction occur from one period to the next one as new information becomes available. Heath and Jackson (1994) presented the additive and multiplicative models, where the former assumes that each error vector is independent, with mean equal to zero and stationary, the latter approach makes the same assumptions but applied to the log transformation of the error vector.

Gullu (1996) considered a capacitated single-item system in which demand is assumed random and stationary. Gullu (1996) used the MMFE to generate forecast updates, where the author considered that for each period the forecast demand for the next two periods is available. Due to the updated forecast, the author presented a dynamic order-up-to level where the order quantity is defined by the difference between the inventory position and this level. The order quantity is further restricted by the maximum between itself and production capacity. Results presented shown that the system performs better when the demand forecast for one period is employed.

As Gullu (1996), Toktay and Wein (2001) also studied a capacitated production system where demand is assumed to be stationary and dynamic forecast updates are carried out via the MMFE. The authors proposed a modified forecast-corrected base-stock policy where the release policy is given by the maximum between the forecast-corrected base-stock level and the sum of production capacity and a forecast-corrected inventory level. Toktay and Wein (2001) developed an approximation for obtaining the optimal forecast-corrected base-stock level, where the aspects related to planning horizon, forecast quality, safety stock and production capacity are considered for its derivation. The forecast-corrected inventory level is equal to the inventory on hand minus the total forecasted demand over the forecast horizon. Toktay and Wein (2001) concluded that by using the correct demand structure where the forecast model is correctly specified, the release rule presented achieves the best performance within the classification of base-stock policies.

Altug and Muharremoglu (2011) considered a single-item single-stage inventory system under stochastic capacity where advance demand information is available. The forecast about future capacity is assumed to be external and is modeled via the MMFE. In this approach the state-dependent base-stock level is given by a constant plus a pre-determined function based on the forecast vector. Their results show that state-dependent base-stock levels are optimal when advance demand information is available.

Different from approaches that use MMFE, Babai and Dallery (2005) assumed non-stationary demand. As in Altug and Muharremoglu (2011), Babai and Dallery (2005) made use of advance demand information and assumed that the probability distributions of the demand are known at the beginning. The authors presented two forecast-based inventory control policies: the (r_t, q) is a dynamic re-order point policy and the (T, S_t) is a dynamic order-up-to-level policy. For both policies, the reorder point r_t and replenishment level S_t account for the forecast demand plus the cumulative forecast uncertainty over the lead-time period. The forecast uncertainty is assumed to be proportional to the forecast values and is considered for computing the safety stock. For both approaches, the safety stock is dynamic and protects against the forecast uncertainty over the lead time period, where the forecast uncertainty is assumed to be independent and normally distributed with parameters $(0, \sigma_k)$.

As previously discussed, the cost-efficiency of standard policies decreases with increasing demand variability, making these policies inadequate for production systems facing non-stationary demand. Different from the standard inventory approach, where a demand distribution is fixed at the beginning, forecast-based inventory policies allow for the consideration of different demand distributions along the horizon. From the approaches presented, the majority assumes stationary demand and relies on the Martingale Model of Forecast Evolution for diminishing the forecast uncertainty. With regard to non-stationary demand, only the approach presented by Babai and Dallery (2005) was found. Althought the forecast technique utilized is not mentioned, the authors assumed that it provides an exact demand uncertainty and thus the buffer calculation only accounts for this risk over the lead time period.

So far, none of the approaches mentioned above specifically assumes non-stationary demand and makes use of a forecast technique (and which is mentioned) that enables the inventory policy to cope with the exact demand uncertainty. Recalling that literature on non-stationary times series is restricted to a limited number of approaches, the need for developing a forecast-based inventory policy that suits one of these techniques leads us to the research gap discussed in Chapter 6.

Chapter 6

Research Design

Chapter 5 introduces the research gap motivating this master thesis: the absence of literature on forecast-based inventory policies when demand is non-stationary, a lead time greater than zero is present in the production system, and the forecast technique utilized does not guarantee an exact demand uncertainty. This chapter complements Chapter 5 by describing how the research gap will be filled. In specific, this chapter presents the research assignment and the research method used to carried out the research assignment.

6.1 Research Questions

Nowadays, increasing competition has made product life cycles shorter, incrementing the randomness and unpredictability of their demand processes. As a result, managers are challenged with providing accurate forecasts when demand follows a non-stationary process.

According to Graves (1990), when demand is non-stationary, inventory managers more often rely on forecasts based on prior demands, where recent demand observations are believed to provide a better prediction for future demand. However, when demand information is limited or when the demand process or the forecast technique are not specified correctly, it is expected that the production system will incur higher inventory costs due to an increment in variability as a result of forecast bias (Kim and Ryan, 2003). Therefore, knowing that the possibility of obtaining an unbiased prediction is restricted to a limited number of non-stationary time series, the need for developing a forecast-based inventory policy where the demand process suits one of these forecasting techniques is essential for enabling the inventory policy to cope with the exact demand uncertainty.

Of the forecast-based inventory policies presented, only a few consider non-stationary demand. The majority of the forecast-based inventory policies assume stochastic stationary demand and make use of the MMFE to carry out a dynamic forecast update, where predictions are updated as new information becomes available. With regard to non-stationary demand, Babai and Dallery (2005) presented two forecast-based inventory policies. For the two of them, Babai and Dallery (2005) made use of advance demand information where demand distributions are known in advance. The authors assumed that the forecast technique utilized provides an unbiased prediction and hence the safety stock only accounts for the exact demand uncertainty.

Based on the research gap described above, the current research assignment is defined as follows:

"Design a forecast-based inventory policy able to cope with the exact demand uncertainty when demand is stochastic and follows a non-stationary process, and a lead time greater than zero is present in the production system. And, as a final goal, minimize the total holding and backordering costs."

To this aim, the following research questions are examined:

1. What forecast-based policy can be developed that specifically accounts for the exact demand uncertainty?
    (a) Which forecast technique can be utilized? How can be the demand uncertainty incorporated into the buffer calculation?
2. What is the cost difference between the proposed policy and a forecast-based policy where the forecast technique provides a biased forecast?
    (a) How does the solution proposed work under different values of demand variance, levels of non-stationary demand and service levels?

6.2 Research Method

In order to carry out the research assignment, a design science approach was followed. According to Van Aken (2005), this type of approach leads to understanding and to the development of knowledge that can be used both for implementing new ideas or design within organizations.

In order to follow this approach, the literature reviewed in Chapter 5 aimed to introduce the problem definition. From this definition, the research assignment and the research questions were developed. In the following subsections, the steps followed to carry out the research assignment and answer the research questions are presented.

The first step involved the development of two inventory policies: a forecast-based inventory policy capable of accounting for the exact demand uncertainty, and a forecast-based inventory policy that approximates this uncertainty when an increment in variability is caused by a biased prediction. In the second step, both models were simulated and results were analyzed numerically (see Figure 6.1).

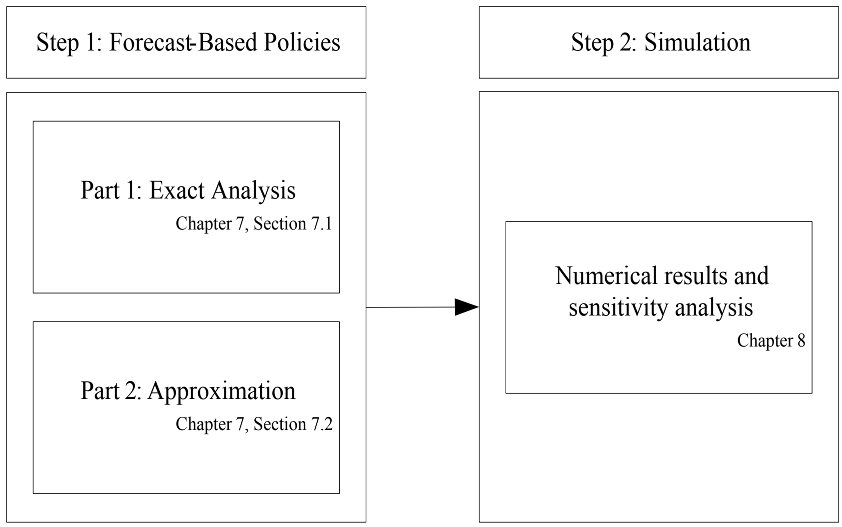

Figure 6.1: *Research Steps*

6.2.1 Forecast-Based Policies

Both models consider the scenario where a single production stage faces stochastic non-stationary demand in a multiple-period context and with a constant lead time greater than zero. Holding and backordering costs are assumed to be linear to the order quantity, where demand not satisfied is backordered. Additionally, unlimited capacity and a lot-for-lot technique are assumed.

Exact Analysis

In Section 6.1 two important model aspects are mentioned. First, the model has to consist of a dynamic base-stock level in order to satisfy the time dependent demand. Second, the forecast technique employed must provide an exact demand uncertainty based on the demand process underlying the production system.

In the model, a k–th Exponential Weighted Moving Average is utilized. Based on the demand process underlying the production system, through this forecast technique, an unbiased forecast is obtained, where the remaining demand uncertainty is equal to the random noise for time period t. In each period, the base-stock level is adjusted based on changes in the forecast and demand noise at time t, plus a constant safety stock. The safety stock is determined based on the variance of the end inventory. Although the assumption of non-stationary demand is made, in Section 7.1 it is proved that the variance of the end inventory is stationary and hence the safety stock remains constant along the periods.

Approximation

The Approximation Model also consists of a dynamic base-stock level. Different from the previous approach, the forecast technique utilized does not have to guarantee an exact forecast uncertainty and hence, an increment in the system variability is caused by predictions that are on average above or below the actual demands.

For the analysis the Exponential Smoothing and Moving Average forecasting techniques were utilized. Moreover, the scenario where partial demand information is available was assumed, where the percentages 0%, 25%, 50%, 75% and 100% of available information were considered.

Furthermore, in De la Vega (2012) a literature study regarding production systems facing random yield was carried out. This review aimed to come up with ideas for considering the bias in the forecast within the buffer calculation. For coping with yield risk, literature on this subject differs in the approach utilized. Of the approaches found in the literature a classification can be done between safety stock and yield factor. Whereas the former classification is a well-known approach for coping with quantity uncertainties, the latter approach is restricted to yield uncertainties where the order quantity is divided by the ratio between the expected number of nondefective items and the output (Ma and Murthy, 1991). Knowing that the approach of yield factor is exclusively used to cope with yield risk, the search for ideas to incorporate biases in the forecast was carried out within the safety stock approaches.

From the literature on this subject, the approach presented in Inderfurth and Gotzel (2003) is considered. Inderfurth and Gotzel (2003) presented a linear approximation for incorporating yield uncertainty into the buffer calculation. The yield uncertainty is assumed to be stationary and stochastically proportional to the order quantity. For coping with both non-stationary demand and stationary stochastic yield, the safety stock calculation is

$$SST^{a}_{t+L} = N^{-1}(\beta)\sqrt{\sum_{i=1}^{L-1}\sigma^2_{D,t+i} + \frac{\sigma^2_z}{\mu^2_z}\mu^2_{D,t+L} + \sigma^2_z\sum_{i=1}^{L-1}P^2_{t-i}}. \tag{6.1}$$

In Equation 6.1, the first term under the square root considers the demand uncertainty, and the second and third terms account for an additional uncertainty due to yield risk, derived from the current and outstanding orders respectively (the detailed derivation of the formula is given in Appendix A).

The Approximation Model takes into account biases in the forecast instead of yield uncertainty. Similar to yield risk, bias in the forecast represents a quantity uncertainty; however, whereas yield risk is proportional to the order quantities given by the master schedule, biases in the forecast are related in size to the orders derived from demand forecasts.

6.2.2 Simulation

The simulation was carried out utilizing the programming language MatLab. The analysis of the results was done in terms of total cost with the aim of identifying how the Exact Model performs compared to the Approximation Model. Moreover, a sensitivity analysis for both models was carried out under a base scenario. This analysis considered the following aspects:

1. Variation of the level of demand variance
2. Variation of the level of non-stationary demand
3. Variation of the service level

The sensitivity analysis was carried out in order to come up with remarks regarding how both models perform under different system settings and hence, be able to predict future states of the Exact Model in an unambiguous and verifiable way.

Chapter 7

Forecast-Based Inventory Policies

In this chapter, two forecast-based inventory policies are presented. Both policies make use of an adaptive base-stock level. The first model accounts for the exact forecast uncertainty and the second model makes use of an approximation for this uncertainty. For both approaches it is assumed that the demand process is stochastic and non-stationary, that the review period is equal to one, that the lead time is constant and greater than zero and that any demand not satisfied is backordered. The following subsections describe the demand process, forecast model and inventory control policy for each model.

7.1 Exact

In the development of this model, the following notation is used: d_t, μ, α, t and L, where d_t is the demand at time t; μ is the mean demand; α is the level of non-stationary demand utilized in the generation of the demand; t is the length of the demand series and L is the lead time.

7.1.1 Demand Process

In order to generate demand, the following demand process assuming normal distribution is considered

$$\begin{cases} d_1 = \mu + \varepsilon_1 & t = 1 \\ d_2 = \mu + \alpha\varepsilon_1 + \varepsilon_2 & t = 2 \\ d_t = \mu + \sum_{j=1}^{t-1} \left(\alpha^{t-j}\varepsilon_j\right) + \varepsilon_t & t = 2, 3, ... \end{cases} \tag{7.1}$$

where α $(0 \leq \alpha \leq 1)$ and μ $(\mu > 0)$ are known parameters. The variable α represents the level of stationarity (with $\alpha = 0$ when demand is stationary); ε represents the variance of the demand noise and is assumed to be an independent and identically distributed random variable, where $\varepsilon \sim N(0, \sigma_\varepsilon^2)$.

As can be observed, in each period t, the demand noise ε_i (associated with the current and the previous periods) changes the mean of the demand by adjusting its weigh (α), with $\alpha^{t-j}\varepsilon_j$ for the subsequent periods.

7.1.2 Forecast Model

In order to account for the changes in the mean of the demand, a k–th Exponential Weighted Moving Average forecast model is utilized. Through this forecasting technique, the forecast demand is updated every period after demand from the current period is observed

$$\begin{cases} F_{1,2} = \mu + \alpha\varepsilon_1 & t = 1 \\ F_{1,3} = \mu + \alpha^2\varepsilon_1 & t = 1 \\ F_{2,3} = \mu + \sum_{j=1}^{2} \alpha^{3-j}\varepsilon_j = \mu + \alpha^2\varepsilon_1 + \alpha\varepsilon_2 & t = 2 \\ F_{2,4} = \mu + \sum_{j=1}^{2} \alpha^{4-j}\varepsilon_j = \mu + \alpha^3\varepsilon_1 + \alpha^2\varepsilon_2 & t = 2 \\ F_{t,k} = \mu + \sum_{j=1}^{t} \alpha^{k-j}\varepsilon_j & t = 2, 3, \ldots \end{cases} \tag{7.2}$$

where $F_{t,k}$ denotes the forecast made at time t for period k.

Figure 7.1 presents an example of a demand series when $\sigma_\varepsilon^2 = 2$. Additionally, the following parameter values are assumed for the simulation: μ=10, α=0.2 and t=50.

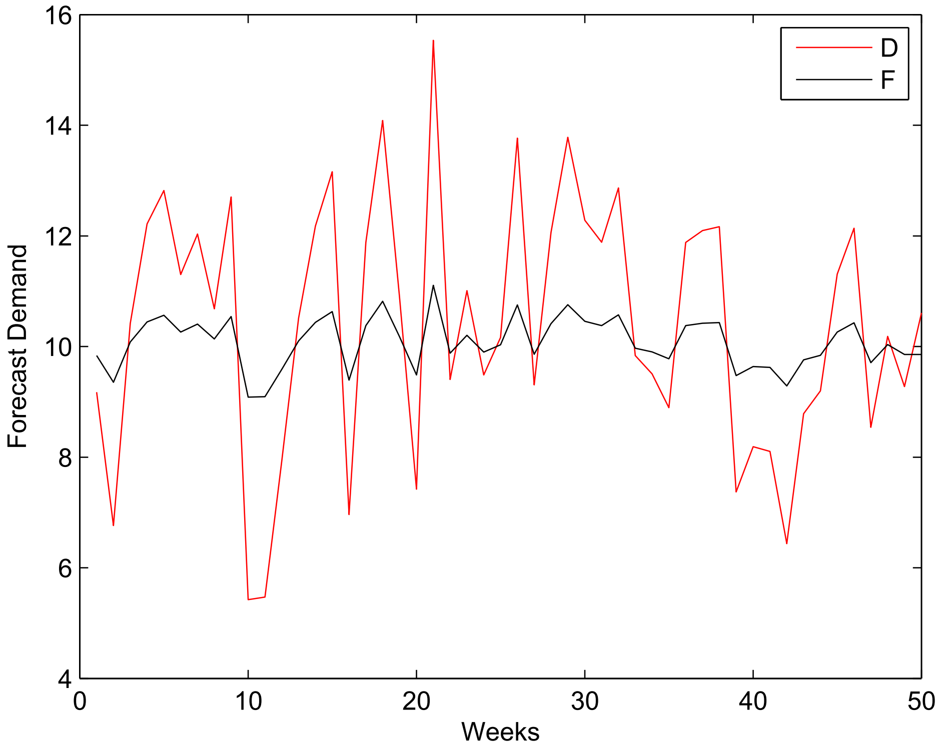

Figure 7.1: *Forecast Demand (Exact)-* σ_ε^2=*2*

7.1.3 Inventory Control Policy

It is assumed that in each period t, the end inventory from the previous period ($x_{e,t-1}$) and the order quantity (P_{t-L}), ordered L periods ago, are received

$$x_{b,t} = x_{e,t-1} + P_{t-L}. \tag{7.3}$$

Further, demand d_t is observed

$$x_{e,t} = x_{b,t} - d_t \tag{7.4}$$

where $x_{e,t}$ denotes the end on-hand inventory (or backorders if demand cannot be met) in period t. It is assumed that an initial Safety Stock can be established and that $P_t = \mu$ for $t \leq 0$.

After demand d_t is fulfilled (or partially satisfied), the Base-Stock Level is adjusted as the demand forecast changes. In order to do this, the following rule is applied for computing the order quantity

$$P_t = \left(\sum_{i=1}^{L} F_{t,t+i} - \sum_{i=1}^{L-1} F_{t-1,t+i} \right) + \varepsilon_t. \tag{7.5}$$

In Equation 7.5 the first term adjusts the Base-Stock Level as the demand forecast changes and the second term represents the demand uncertainty at time t, where $d_t - F_{t-1,t} = \varepsilon_t$.

Furthermore, in order to determine the Safety Stock, the variance of the end inventory $x_{e,t}$ is considered. It can be demonstrated that at time t the end inventory is equal to

$$x_{e,t} = SS - \sum_{j=t-L+2}^{t} \sum_{k=0}^{t-j} \alpha^k \varepsilon_j \tag{7.6}$$

when $t \geq L$. In Equation 7.6, the first and second term represent the expected mean and standard deviation in each period respectively. For proving Equation 7.6, an analysis is done assuming a constant lead time greater than zero and a pipeline filled with μs. At time 1, d_1 is observed, and the difference between the forecast demand over the lead time period and the sum of the pipeline and end-inventory is used for computing the order quantity P_1

$$
\begin{cases}
d_1 = \mu + \varepsilon_1 \\
F_{1,2} = \mu + \alpha\varepsilon_1 \\
F_{1,3} = \mu + \alpha^2\varepsilon_1 \\
. \\
. \\
F_{1,L} = \mu + \alpha^{L-1}\varepsilon_1 \\
\\
x_{b,1} = \mu + SS \\
x_{e,1} = SS - \varepsilon_1 \\
\\
P_1 = \mu + \sum_{k=0}^{L-1} \alpha^k \varepsilon_1
\end{cases}
\tag{7.7}
$$

After $L-1$ periods, right before P_1 arrives, the end inventory $x_{e,L-1}$ is

$$
x_{e,L-1} = SS - \sum_{j=1}^{L-1} \sum_{k=0}^{(L-1)-j} \alpha^k \varepsilon_j .
\tag{7.8}
$$

Finally, at time L, both $x_{e,L-1}$ and P_1 are received and demand d_L is observed

$$
\begin{cases}
d_L = \mu + \sum_{j=1}^{L-1} \alpha^{L-j}\varepsilon_j + \varepsilon_L \\
\\
x_{b,L} = SS - \sum_{j=1}^{L-1} \sum_{k=0}^{(L-1)-j} \alpha^k \varepsilon_j + \left(\mu + \sum_{k=0}^{L-1} \alpha^k \varepsilon_1 \right) \\
x_{e,L} = SS + \mu + \alpha\varepsilon_1^{L-1} - \sum_{j=2}^{L-1} \sum_{k=0}^{(L-1)-j} \alpha^k \varepsilon_j - \left(\mu + \sum_{j=1}^{L-1} \alpha^{L-j}\varepsilon_j + \varepsilon_L \right) \\
x_{e,L} = SS - \sum_{j=2}^{L} \sum_{k=0}^{L-j} \alpha^k \varepsilon_j
\end{cases}
\tag{7.9}
$$

Assuming that the process is already going on for infinite time, Equation 7.6 can be found (see Appendix B for the detailed calculation).

From the assumption that ε is a normally distributed i.i.d. random variable, the variance of the summations in Equation 7.6 can be described as the summations of the squared quantities for each epsilon

$$\sigma^2_{SUM} = \sum_{j=t-L+2}^{t} \left(\sum_{k=0}^{t-j} \alpha^k \right)^2 . \tag{7.10}$$

Finally, the Safety Stock can be derived from Equation 7.10 as

$$SS = \sigma \sqrt{\frac{1}{(1-\alpha)^2} * \left(L - 1 - 2 * \left(\frac{\alpha^L - \alpha}{\alpha - 1} \right) + \left(\frac{\alpha^{2L} - \alpha^2}{\alpha^2 - 1} \right) \right)} \tag{7.11}$$

where its value depends on the lead time, the parameter α and the level of σ (see Appendix C). To assure a specific service level, the Safety Stock given in Equation 7.11 is multiplied by the safety factor z

$$SS = z \left(\sigma \sqrt{\frac{1}{(1-\alpha)^2} * \left(L - 1 - 2 * \left(\frac{\alpha^L - \alpha}{\alpha - 1} \right) + \left(\frac{\alpha^{2L} - \alpha^2}{\alpha^2 - 1} \right) \right)} \right) . \tag{7.12}$$

When demand follows a stationary process ($\alpha = 0$), Equation 7.12 can be shortened to

$$SS = z \left(\sigma \sqrt{L-1} \right) . \tag{7.13}$$

Finally, at the end of period t, the Total Cost is calculated as

$$TC(t) = h * [x_{e,t}]^+ + b * [x_{e,t}]^- \tag{7.14}$$

where h and b represent the holding and backordering cost respectively (see Appendix D for the numerical analysis of Equation 7.14).

7.2 Approximation

For this model, the following notation is used: d_t, μ, α, α_E, T, t, L and β, where d_t is the demand at time t; μ is the mean demand; α is the level of non-stationary demand utilized in the generation of the demand; α_E is the smoothing constant for the Exponential Smoothing forecast method; T is the number of historical demands used for the Moving Average method; t is the length of the demand serie; L is the lead-time, and β is the service level.

7.2.1 Demand Process

In order to generate the demand, the same demand process as for the Exact Model is considered

$$\begin{cases} d_1 = \mu + \varepsilon_1 & t = 1 \\ d_2 = \mu + \alpha\varepsilon_1 + \varepsilon_2 & t = 2 \\ d_t = \mu + \sum_{j=1}^{t-1} \left(\alpha^{t-j}\varepsilon_j\right) + \varepsilon_t & t = 2, 3, ... \end{cases} \tag{7.15}$$

As in the previous model, α $(0 \leq \alpha \leq 1)$ and μ $(\mu > 0)$ are assumed to be known parameters. Also, ε is assumed to be an independent and identically distributed random variable, where $\varepsilon \sim N(0, \sigma_\varepsilon^2)$.

7.2.2 Forecast Model

In order to carry out the forecast, the scenarios where demand is partially known (DPK) and the Exponential Smoothing (ES) and Moving Average (MA) forecasting techniques are considered.

In the scenario where demand is partially known, the value of f represents the amount of the demand uncertainty that is known at the moment of determining the forecast

$$\begin{cases} F_1 = [\mu + f * \varepsilon_1]^+ & t = 1 \\ F_t = [d_{t-1} + f * (\varepsilon_t + \alpha\varepsilon_{t-1} - \varepsilon_{t-1})]^+ & t = 2, 3, ... \end{cases} \tag{7.16}$$

where $\varepsilon_t + \alpha\varepsilon_{t-1} - \varepsilon_{t-1}$ is the forecast uncertainty and is obtained from the subtraction $d_t - d_{t-1}$. For the different values of demand known, f can take values from 0 to 1, where 1 means that the demand uncertainty is completely known.

For the Exponential Smoothing method, the forecast is calculated as

$$\begin{cases} F_1 = \mu & t = 1 \\ F_t = \alpha_E d_{t-1} + (1 - \alpha_E) F_{t-1} & t = 2, 3, ... \end{cases} \tag{7.17}$$

where α_E is the smoothing constant of the Exponential Smoothing forecast technique.

Moreover, the Moving Average technique estimates the forecast as

$$\begin{cases} F_1 = \mu & t = 1 \\ F_t = \dfrac{1}{T}\displaystyle\sum_{i=1}^{T} d_{t-i} & t = 2, 3, \ldots \end{cases} \tag{7.18}$$

where T is the number of periods of previous demand utilized for calculating the forecast.

Figure 7.2 presents an example of a demand series when $\sigma_\varepsilon^2 = 2$. Additionally, the following parameter values are assumed for the simulation: μ=10, α=0.2, α_E=0.2, T=5, $f = [0, .25, .5, .75]$ and t=50.

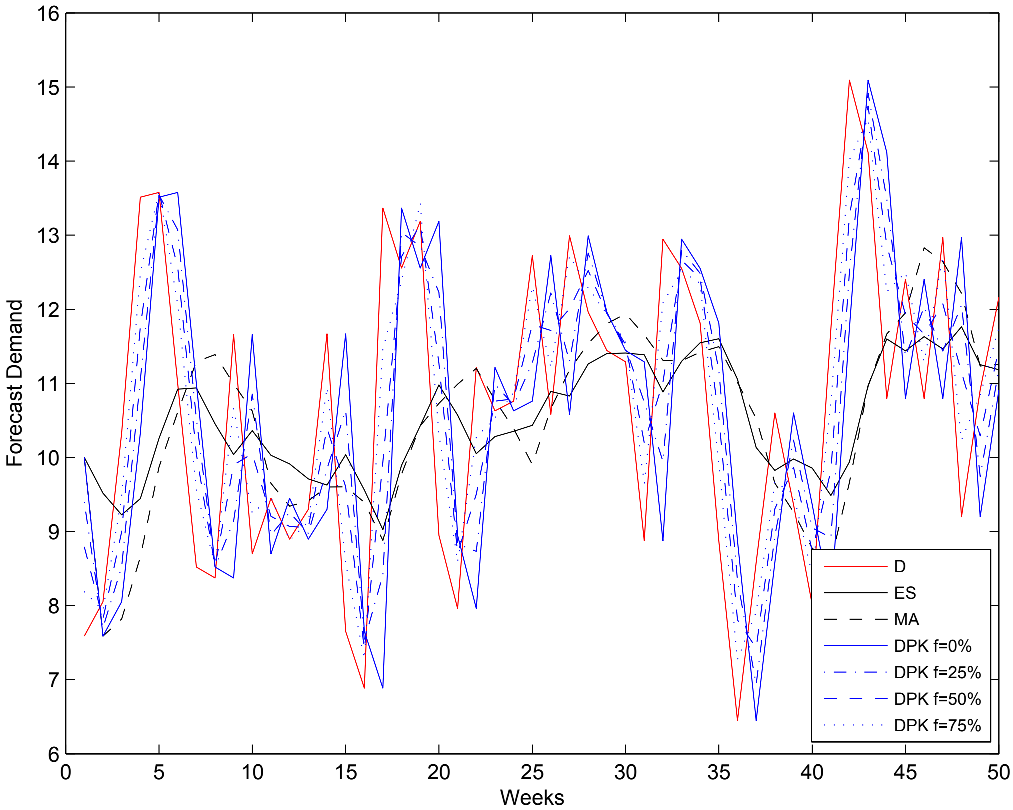

Figure 7.2: *Forecast Demand (Approximation)-* σ_ε^2*=2*

7.2.3 Inventory Control Policy

At the beginning of each review period t, the order quantity ordered $t-L$ periods ago is received and added to the end inventory from the previos period

$$x_{b,t} = P_{t-L} + x_{e,t-1}. \tag{7.19}$$

Further, demand d_t is observed

$$x_{e,t} = x_{b,t} - d_t \tag{7.20}$$

where $x_{e,t}$ is the end inventory at time t. As for the Exact Model, the inventory on hand x can take both positive and negative values; where, $[x_{e,t}]^+$ refers to the holding inventory and $[x_{e,t}]^-$ to the backordered units incurred at the end of the period t.

Further, the linear control rule for determining the order quantity is given by

$$P_t = \begin{cases} S_t - I_t & if \quad S_t > I_t \\ 0 & if \quad S_t \leq I_t \end{cases} \tag{7.21}$$

where P_t is assumed to be received at time $t+L$. The state-dependent Base-Stock Level S_t accounts for the Forecast Demand plus the Safety Stock over the period $(t, t+(L-1))$

$$S_t = \sum_{i=0}^{L-1} F_{t+i} + SS_{t,t+L}. \tag{7.22}$$

Moreover, the Safety Stock is estimated as

$$SS_{t,t+L} = N^{-1}(\beta)\sqrt{\sum_{i=0}^{L-1} \sigma^2_{d,t+i} + \sigma^2_e \sum_{i=1}^{L-1} P^2_{t-i} + \sigma^2_e \frac{\mu^2}{(\bar{e})^2}}. \tag{7.23}$$

where, $\bar{e}$ and σ^2_e denote the mean and the standard deviation of the bias in the forecast, respectively. In order to estimate this uncertainty, the ratio between the forecast demand F_t and the current demand d_t is considered

$$e_t = \frac{F_t}{d_t} \tag{7.24}$$

where e (with $0 \leq e \leq 1$) is assumed to be independent and to follow a stationary distribution; $\bar{e}$ and σ_e^2 represent the mean and variance, respectively.

The Safety Stock formula takes into account the demand uncertainty plus possible biases in the forecast. The first term under the square root in Equation 7.23 represents the variance of the demand σ_d^2 over the lead time period. The second term represents the additional uncertainty derived from a biased forecast and which is proportional to the open orders, where the term $\sum_{i=1}^{L-1} P_{t-i}$ represents the outstanding orders at time t. The last term under the square root is also the uncertainty due to bias in the forecast but proportional to the size of the current order $(\frac{\mu}{\bar{e}})$.

The Safety Stock formula is similar to the one presented in Inderfurth and Gotzel (2003), but instead of yield risk, the bias in the forecast is considered. Following the approach introduced by Bollapragada and Morton (1999), a modified demand

$$\eta_t = \sum_{i=0}^{L} (d_{t+i} - (e_{t-i} - \bar{e}) * P_{t-i}), \tag{7.25}$$

and the inventory position

$$y_t = x_{t-1} + \bar{e} * \sum_{i=0}^{L-1} P_{t-i}, \tag{7.26}$$

are considered for determining the order quantity. For calculating η_t and y_t, d and e are random variables of the demand and bias in the forecast respectively. The modified demand η has the same mean as the demand but accounts for an increased variability $(e - \bar{e})$ due to bias in the forecast derived from the current and outstanding orders. The inventory position y takes into account the inventory on hand plus a deterministic bias in the forecast $(\bar{e})$ also from the current and open orders. Furthermore, the optimal order quantity can be obtained by solving the newsvendor formula, where the probability of the modified demand η and the inventory position y are considered

$$F_{\eta_t}(y_t) = \frac{p}{p+h}. \tag{7.27}$$

Moreover, in order to come up with a linear approximation, the probability of obtaining $\eta_t \leq y_t$ is considered for calculating the order quantity

$$\{\zeta_t = \sum_{i=0}^{L} d_{t+i} - \sum_{i=1}^{L-1}(e_{t-i} * P_{t-i}) - (e_t * P_t(\hat{x})) \leq \hat{x} - \bar{e} * \sum_{i=0}^{L} P_{t-i} = \frac{b}{b+h}\}. \tag{7.28}$$

Assuming the linear approximation, where $\hat{x} = SS + L * \mu$, $S_t = SS + (L+1) * \mu$ and $P_t(\hat{x}) = \dfrac{S_t - \hat{x}}{\bar{e}}$, it is possible to obtain Equation 7.23 (see Appendix E).

Furthermore, continuing with the inventory policy, the inventory position I_t aggregates the inventory on hand from the current period t, $x_{e,t}$, plus the outstanding orders $\sum_{i=1}^{L-1} P_{t-i}$

$$I_t = x_{e,t} + \sum_{i=1}^{L-1} P_{t-i}. \tag{7.29}$$

Finally, the Total Cost at time t takes into account the total holding h and backordering b costs taking place at the end of the period, where

$$TC(t) = h * [x_{e,t}]^+ + b * [x_{e,t}]^-. \tag{7.30}$$

Chapter 8

Numerical Analysis

In this chapter, the forecast-based inventory policies of Chapter 7 are analyzed numerically. First, a base case scenario is presented. Second, a sensitivity analysis is performed for the following input parameters: demand variance (σ_ε^2), level of non-stationary demand (α) and service level (z). For each of the input parameters analyzed, the result of 5000 simulation replications is presented.

8.1 Base Case Scenario

In the base case scenario, both models were considered. The inputs of the base case scenario are:

μ	σ_ε^2	α	z	L	T	α_{ES}	h	b
10	2	0.2	95%	5	5	0.2	2	8

Table 8.1: *Input Parameters*

In addition to both models, the standard continuous review policy (s, S) is used for comparison. In the (s, S) policy, a dynamic quantity Q_t is ordered up to the level S whenever the inventory position drops below the re-order point s, where $s = \mu * L + SS$, $S = \mu * (L + 1) + SS$ and $SS = z * \left(\sqrt{L} * \sigma_d\right)$ are assumed.

Figure 8.1 shows the results of the simulation for the demand and the Exact Model. Figure 8.2 does the same for the Approximation Model, where a simulation for each of the forecasting techniques mentioned in Section 6.2 is presented. Finally, Figure 8.3 shows the results obtained through the (s, S) policy. For each graph of the policy performance, the levels of Base-Stock, pipeline, demand and inventory on hand along t are presented. Table 8.2 shows the average cost, fill rate, holding inventory and backorders for each scenario under study.

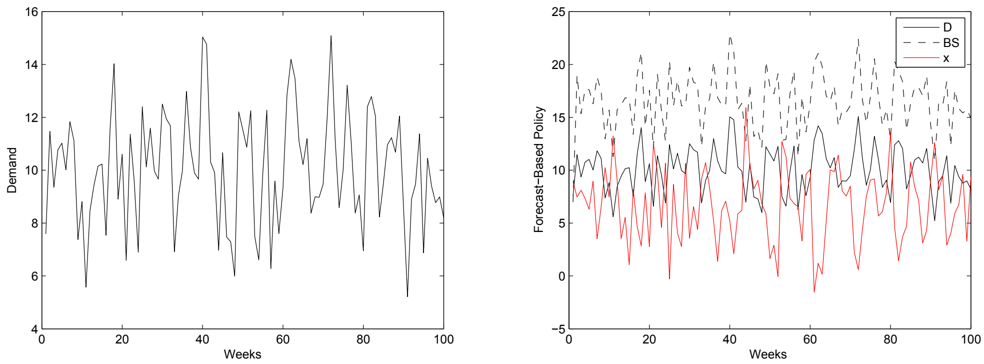

Figure 8.1: *Exact Model*

1. Exact: Under this model, an average cost equal to 13.10 was found. As can be observed in Figure 8.1, the Exact Model showed the less smooth Base-Stock levels. The perceptible change among the Base-Stock levels is because of an order quantity that accounts for the exact demand uncertainty (ε_t) and changes in the forecast, plus a constant Safety Stock. The forecast demand changes its mean as the demand noise from the current and previous periods adjusts its weight, with $\alpha^{t-j}\varepsilon_j$ for the subsequent periods.

2. Approximation: Of the different forecasting techniques, when demand is partially known, the Base-Stock levels were less smooth the higher the percentage of demand information. Unsurprisingly, the quick response to changes in demand showed a decreasing cost the higher the percentage of information. With regard to the Moving Average and Exponential Smoothing forecasting techniques, both approaches presented a slower response to changes in the demand by taking into account the mean demand from the past periods (T) and a level of correlation from the previous demand (α_E), respectively.

3. (s, S): Intuitively, one expects the highest fill rate under this policy. Results obtained confirmed this intuition where a 100% fill rate comes at the expense of high inventory levels resulting in the highest total cost. For the base case scenario, the average cost obtained was 164.09. The higher inventory levels are even greater than the order-up-to level S due to a positive lead time and thus an order quantity which does not consider the pipeline at the moment of being determined.

	(s,S)	ES	MA	0% D	25% D	50% D	75% D	100% D	Exact
Av. Cost	164.09	15.13	16.28	14.76	13.70	12.90	12.40	12.02	13.10
Fill Rate	100%	97.75%	96.60%	99.10%	99.35%	99.55%	99.67%	99.87%	99.84%
Av. Holding	82.04	6.50	6.58	6.93	6.52	6.21	6.02	5.94	6.42
Av. Backorder	0.00	0.27	0.39	0.11	0.08	0.06	0.04	0.03	0.02

Table 8.2: *Average Cost Base Case Scenario*

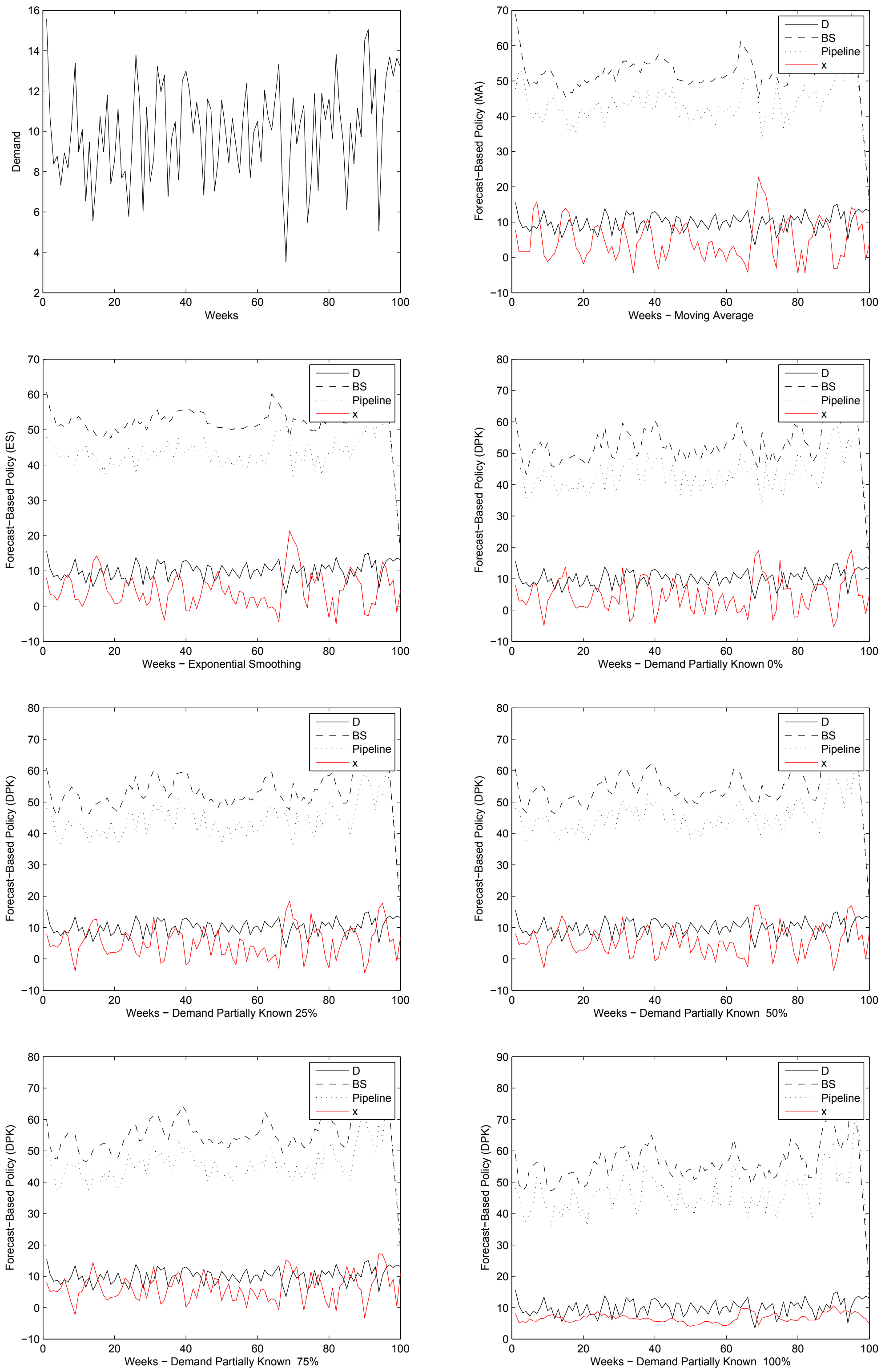

Figure 8.2: *Approximation Model*

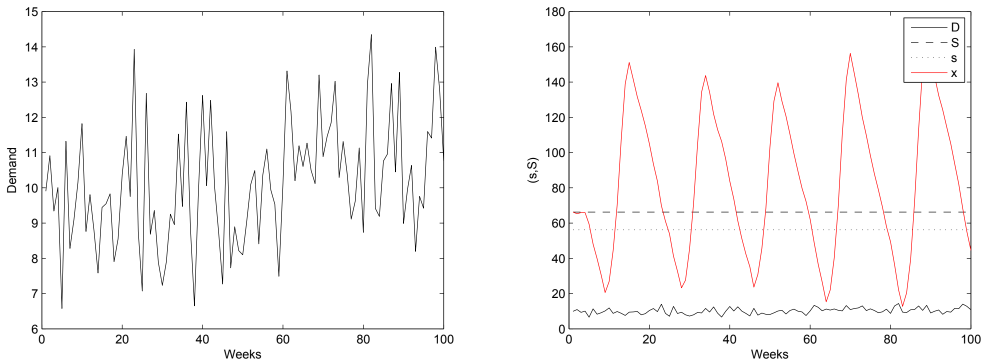

Figure 8.3: (s, S)

8.2 Sensitivity Analysis

In this section, the inputs of the base case scenario were changed one by one in order to gain insight into the impact of each input on the total cost. For this analysis, the demand variance, level of non-stationary demand and service level were considered. Appendix F presents the average cost, holding inventory and backorders for each of the input parameters under each scenario.

8.2.1 Demand Variance

For both models the presence of demand variance resulted in a substantial increase in the total cost. Figure 8.4, in which σ_ε is varied from 0 to 4, shows that after a steep increase at the beginning, the total cost continued to grow, but non-linearly. Table 8.3 shows the average cost and the percentage of increment (in bold) when σ_ε was varied from 0 to 4.

1. Exact: Under this model, the largest cost increment was found when σ_ε was varied from 0 to 1. Although the Exact model got the highest cost when demand variance is equal to 1 (in comparison with the Approximation model), this model showed the smallest cost increase as σ_ε was augmented. Moreover, contrary to what was expected, the Exact Model did not always outperform the Approximation Model. The explanation for this phenomenon can be found in the impact of the service level on the safety stock. Results obtained varying the safety factor show that, for all values of σ_ε^2, a service level greater than 95% was achieved even when a desired service level equal to 90% was specified (see Appendix G).

2. Approximation: As for the Exact Model, the largest cost increment occured when demand variance is present in the system, when σ_ε was increased from 0 to 1. Also, the increment in cost was not linear as the demand variance was augmented. With regard to the different forecasting techniques making use of demand information, the cost increment decreased at a lower rate with increasing values of demand information.

3. (s, S): Compared to the two models examined above, this policy got the highest cost along the five values of σ_ε. Different from the two models, a cost is present in the production system even if σ_ε^2 is equal to zero. This is caused by re-order level s being greater than zero even if the safety stock is equal to zero.

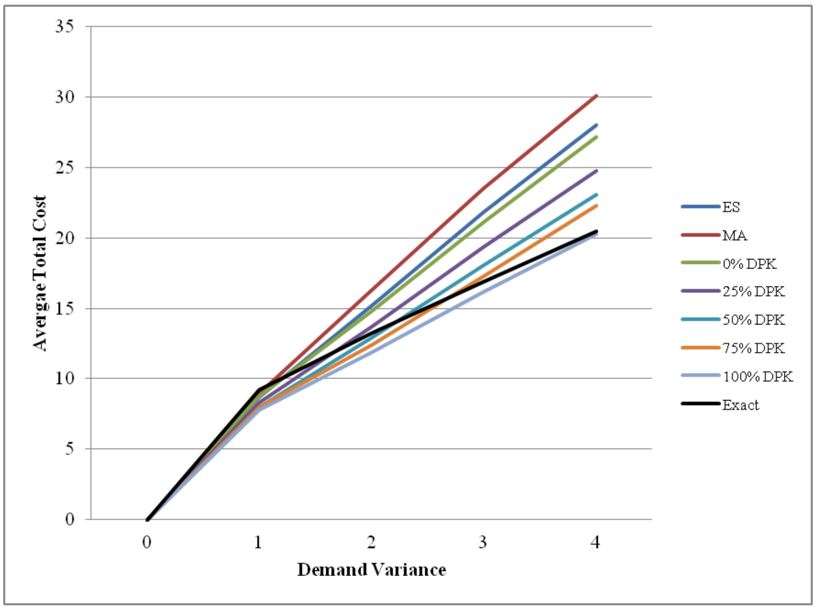

Figure 8.4: *Average Cost when varying* σ_ε^2

σ_ε	(s, S)	ES	MA	0%	25%	50%	75%	100%	Exact
0	156.8	0	0	0	0	0	0	0	0
1	159.24	8.67	8.99	8.79	8.33	8.00	7.98	7.77	9.23
1 to 2	**4.84%**	**75.05%**	**81.21%**	**68.40%**	**64.58%**	**61.26%**	**55.20%**	**52.90%**	**43.77%**
2	166.95	15.18	16.28	14.80	13.71	12.90	12.39	11.88	13.27
2 to 3	**2.79%**	**44.09%**	**44.61%**	**42.69%**	**41.19%**	**40.13%**	**39.60%**	**36.10%**	**27.43%**
3	171.61	21.87	23.55	21.12	19.36	18.08	17.30	16.16	16.91
3 to 4	**2.34%**	**28.08%**	**27.94%**	**28.78%**	**28.00%**	**27.75%**	**28.82%**	**25.30%**	**20.93%**
4	175.63	28.02	30.13	27.20	24.78	23.09	22.28	20.25	20.45

Table 8.3: *Average Cost when varying* σ_ε^2

8.2.2 Level of Non-Stationary Demand

For both models, Figure 8.5 shows an increment in the average cost as α increased.

1. Exact: Intuitively, one expects a higher inventory cost as the value of α is increased. Recalling Equation 7.11, where the safety stock depends on the value of α, it is possible to deduce that the required safety stock will increase as α gets bigger. Results obtained confirmed this belief, where the average inventory level increased while the number of backorders continued to grow at a lower rate. Moreover, as when σ_ε^2 was varied, the Exact Model did not always outperform the Approximation Model. Again, the explanation of this event was found in the impact of the service level on the safety stock. Results obtained varying the safety factor show that, for all values of α, a service level greater than 95% was achieved even when a desired service level equal to 90% was specified. Also, a service level greater than 95% was attained for low values of α when there was no desired service level (making use of Equation 7.11) (see Appendix G).

2. Approximation: Similar to the Exact Model, a higher inventory cost was obtained as the value of α was enlarged. Regarding the forecasting techniques making use of demand information, the average cost along the different values of α did not show any increment or decrement with respect to the value of demand information. Concerning the Moving Average and Exponential Smoothing techniques, both methods outperformed each other under different values of α. Whereas the performance of the Moving Average technique depends entirely on the number of periods of past demand utilized (with T=1 it is expected to perform as 0% DPK), the Exponential Smoothing depends mainly on the value of α_E (also, with α_E=1 it is expected to perform as 0% DPK), which in our case was set equal to 0.2.

3. (s, S): Under this model, the level of non-stationary did not cause substantial changes in the total cost. Whereas for low values of α the inventory cost remains relatively constant, for large values of α a small percentage of backorders started to happen.

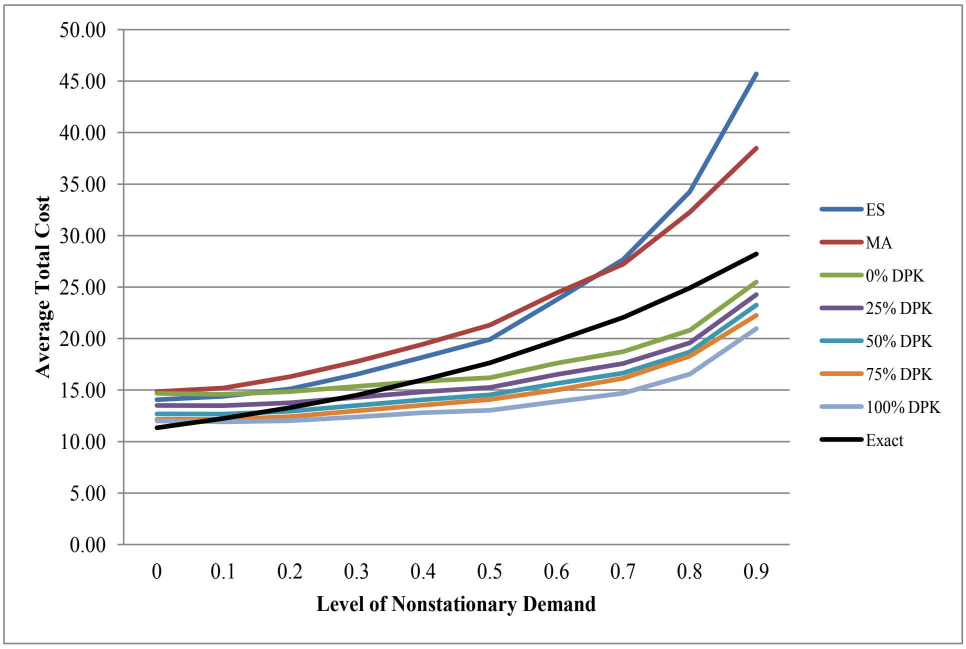

Figure 8.5: *Average Cost when varying α*

8.2.3 Variation of Service Level

Figure 8.6, in which z takes values of 90%, 95% and 99%, shows a higher inventory cost with increasing values of z.

1. Approximation: Unsurprisingly, the average cost increased as the service level was augmented. This increment is derived from higher inventory levels needed for meeting the required service level. Also, of the different forecasting techniques employed, Figure 8.6 shows that the cost difference among the methods decreases with increasing values of the service level.

2. Exact: Similar to the case of α, it is expected that the average cost will increase with higher values of service level. Although results obtained confirmed this intuition, the impact of

the service level on the average total cost is greater compared to the Approximation Model. Recalling Table 8.2, where a Fill Rate equal to 99.84% was achieved based on a service level equal to 95%, it is possible to deduce that the large increase in the total cost is caused by an increment in the holding inventory. Results obtained show real service levels equal to 99.47%, 99.84% and 100% based on desired service levels equal to 90%, 95% and 99%, respectively.

3. (s, S): As with both models, the increment of the service level resulted in higher costs. As z was varied, the cost increment was proportional to the difference between both values of z times the holding cost.

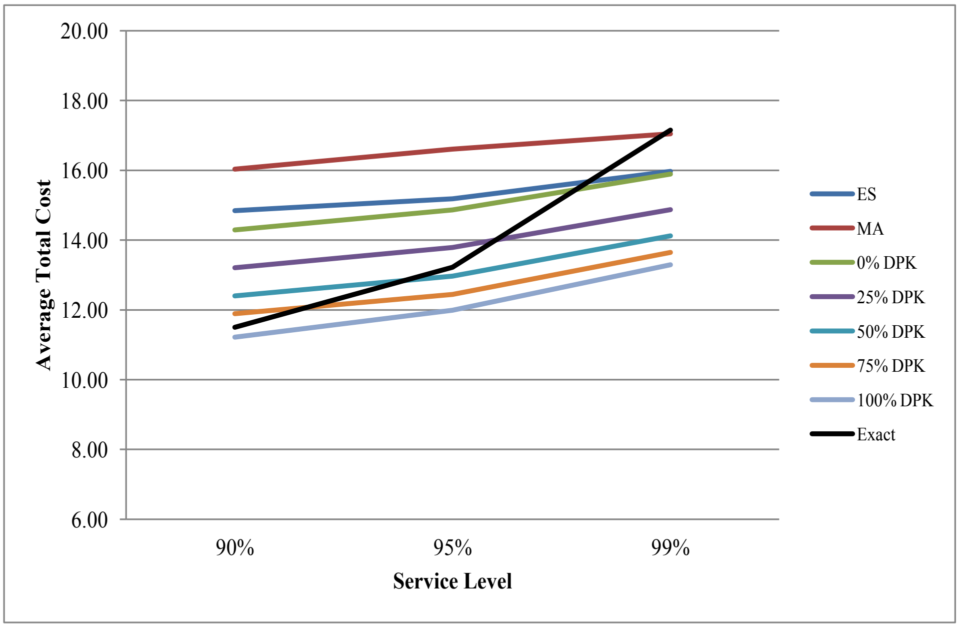

Figure 8.6: *Average Cost when varying z*

Chapter 9

Conclusion & Discussion

9.1 Conclusion

In this master thesis, two forecast-based inventory policies are presented for a single-stage and subject to a deterministic lead time and stochastic nonstationary demand. First, a forecast-based inventory model able to account for the exact demand uncertainty is proposed. This model provides an alternative option for contexts where the assuption of stationary demand is not applicable. Specifically, this model is suited for situations in which a k–th Exponential Weighted Moving Average is an appropriate forecast model. Second, a forecast-based inventory model is presented where an approximation is made to cope with an inexact demand uncertainty. In our scenario, this imprecise uncertainty is due to the misspecification of the forecast technique which results in predictions that are on average above or below demand. With the aim of understanding how to account for the imprecise uncertainty, literature on production systems facing random yield was studied. From this literature, the approach presented in Inderfurth and Gotzel (2003) is considered, but instead of yield uncertainty, this increment in variability is approximated.

Furthermore, besides formulating both inventory policies, both models are analyzed numerically and compared to each other. From this analysis, the following conclusions can be drawn:

- *When demand is stochastic and non-stationary, forecast-based inventory policies that cope with the exact demand uncertainty outperform forecast-based inventory policies making use of forecasting techniques assuming stationary demand patterns.* Evidently, the Exact Model achieves a lower total cost in comparison with the Approximation Model under the majority of the analysis presented. The sensitivity analysis shows that it is more beneficial to make use of a forecast technique that enables the Forecast-Based inventory policy to cope with the exact demand uncertainty than utilizing forecasting techniques that assume stationary demand patterns. However, with regard to the Exact Model, a remark needs to be made with respect to the impact of high service levels on the safety stock. It is expected that for high values of service level, the Approximation model will perform better than the Exact model due to an increment in holding inventory in the latter approach.

- *Recent demand observations provide a better prediction for future demand.* The numerical analysis shows that the Approximation Model making use of 0% demand information performs slightly better than the well known Moving Average (except when T=1) and Exponential Smoothing (except when α_E=1) forecasting techniques. The explanation for this phenomenon might be found in the level of uncertainty imposed by each scenario.

Whereas the 0% scenario results in a constant uncertainty equal to ε_t+$\alpha\varepsilon_{t-1}$-ε_{t-1}, the Moving Average and Exponential Smoothing techniques face a variable uncertainty derived from the demand but also from the variation of the expected demand.

- *The presence of demand variance is more important than the level of demand variance.* The numerical analysis shows that for both models the presence of demand variability caused the largest increase in cost. Although the cost kept growing as σ_ε^2 was enlarged, this increment was non-linearly. The Exact Model showed the smallest cost increase as σ_ε^2 was augmented.

- *The value of information increases with the level of demand variability.* According to the sensitivity analysis of Subsection 8.2.1, under the Approximation Model, there is a substantial cost difference along the values of σ_ε^2 across the forecasting techniques making use of demand information, where it is far more beneficial to have information when demand is more variable.

- *Forecast-based inventory policies outperform Standard policies.* Unsurprisingly, the utilization of a forecast-based inventory, regardless the forecast technique utilized, achieves a lower total cost in comparison with standard policies.

9.2 Discussion

This master thesis has three limitations with regard to the Exact Model: a model which its implementation depends upon a specific forecast technique and demand pattern, a model which operates with a periodic review equal to one where an ordering cost equal to zero is assumed and a model that assumes normal distribution and independence among the demand noise terms. These limitations, and future research to overcome them, are discussed in this section.

The model proposed in this master thesis specifically suits situations in which the k-th Exponential Weighted Moving Average forecast technique is applied. Therefore, although the Exact Model provides an option for contexts where the assumption of stationary demand is not applicable, it is expected that the best performance will be achieved when this forecast technique suits the demand process underlying the production system.

Moreover, further research should be done in contexts where the review period is greater than one. Knowing that the model proposed is an exact analysis in which each period the policy copes with the same expected mean and variance, it is expected that with a review period greater than one this won't continue to be true. In addition to adjust the base-stock level based on the forecast changes and demand uncertainty taking place over the lead time plus review period, the variance of the end inventory each period and hence the safety stock needed, are not expected to remain constant along the periods.

In Section 7.1, it was assumed that demand follows a normal distribution and demand noise terms are independent. From this assumption, the derivation of the safety stock formula is valid. However, when the error terms are correlated or when demand is not normally distributed, this derivation can not longer be applied in the analysis of the exact model. Therefore, further research concerning the inclusion of different demand distributions is fundamemtal when the assumptions of normality and independence are not applicable.

Until now, a study where the forecast-based inventory policy copes with the exact, but also total demand uncertainty is carried out. However, further research should be done considering the availability of demand information. It is expected that the utilization of demand information at the moment of determine the forecast will reduce the standard deviation faced by the policy in each period and thus the amount of safety stock required.

Moreover, with respect to the Approximation model, a remark should be made where the approximation does not account for deviations above demand. By restricting the value of e between zero and one, the buffer calculation does not account for the variance of overestimating demand of the current and outstanding orders. It is expected that an approximation of the bias in forecast, where both demand underestimation and overestimation are take into account will enable the forecast-based inventory policy to outperform the approximation presented.

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Appendix A

Safety Stock - Yield Risk

The probability of obtaining $\eta_t \leq y_t$ is

$$\{\zeta_t = \sum_{i=0}^{L} d_{t+i} - \sum_{i=1}^{L-1}(z_{t-i} * P_{t-i}) - (z_t * P_t(\hat{x})) \leq \hat{x} - \bar{z} * \sum_{i=0}^{L} P_{t-i} = \frac{b}{b+h}\}. \tag{A.1}$$

Taking into account the linear approximation $\hat{x} = SS + L * \mu$, $S_t = SS + (L+1) * \mu$ and $P_t(\hat{x}) = \dfrac{S_t - \hat{x}}{\bar{z}}$, Equation A.1 is solved as

$$\sum_{i=0}^{L} d_{t+i} - \sum_{i=1}^{L-1} (z_{t-i} * P_{t-i}) - \left(z_t * \left(\frac{SS + (L+1) * \mu - (SS + L * \mu)}{\bar{z}}\right)\right) \leq (SS + L * \mu) - \bar{z} * \sum_{i=0}^{L} P_{t-i}. \tag{A.2}$$

Rewriting Equation A.2 yields

$$SS \geq \sum_{i=0}^{L} (d_{t+i} - \mu) + \sum_{i=1}^{L-1} (z_{t-i} - \bar{z}) * P_{t-i} + \left((z_t - \bar{z}) * \frac{\mu}{\bar{z}}\right). \tag{A.3}$$

Knowing that d and z represent random values of the demand and yield risk respectively, Equation A.3 is reduced to

$$SS_{t,t+L} = N^{-1}(\beta)\sqrt{\sum_{i=0}^{L} \sigma_{d,t+i}^2 + \sigma_z^2 \sum_{i=1}^{L-1} P_{t-i}^2 + \sigma_z^2 * \frac{\mu^2}{(\bar{z})^2}}. \tag{A.4}$$

where the assumption that the yield risk is stationary is made.

Appendix B

Variance of the End Inventory

t=1

$$\begin{cases} d_1 = \mu + \varepsilon_1 \\ F_{1,2} = \mu + \alpha\varepsilon_1 \\ F_{1,3} = \mu + \alpha^2\varepsilon_1 \\ . \\ . \\ F_{1,L} = \mu + \alpha^{L-1}\varepsilon_1 \\ \\ x_{b,1} = \mu + SS \\ x_{e,1} = \mu + SS - (\mu + \varepsilon_1) \\ x_{e,1} = SS - \varepsilon_1 \\ \\ P_1 = \mu + \sum_{k=0}^{L-1} \alpha^k \varepsilon_1 \end{cases} \tag{B.1}$$

t=L-1

$$\begin{cases} d_{L-1} = \mu + \sum_{j=1}^{(L-1)-1} \alpha^{(L-1)-j}\varepsilon_j + \varepsilon_{L-1} \\ F_{L-1,L} = \mu + \sum_{j=1}^{L-1} \alpha^{L-j}\varepsilon_j \\ . \\ F_{L-1,k} = \mu + \sum_{j=1}^{L-1} \alpha^{k-j}\varepsilon_j \\ x_{b,L-1} = \mu + SS - \sum_{j=1}^{L-2} \sum_{k=0}^{(L-2)-j} \alpha^k \varepsilon_j \\ x_{e,L-1} = \mu + SS - \sum_{j=1}^{L-2} \sum_{k=0}^{(L-2)-j} \alpha^k \varepsilon_j - \left(\mu + \sum_{j=1}^{(L-1)-1} \alpha^{(L-1)-j}\varepsilon_j + \varepsilon_{L-1} \right) \\ x_{e,1} = SS - \sum_{j=1}^{L-1} \sum_{k=0}^{(L-1)-j} \alpha^k \varepsilon_j \end{cases} \tag{B.2}$$

t=L

$$\begin{cases} d_L = \mu + \sum_{j=1}^{L-1} \alpha^{L-j}\varepsilon_j + \varepsilon_L \\ F_{L,L+1} = \mu + \sum_{j=1}^{L} \alpha^{(L+1)-j}\varepsilon_j \\ . \\ F_{L,k} = \mu + \sum_{j=1}^{L} \alpha^{k-j} * \varepsilon_j \\ x_{b,L} = \left(SS - \sum_{j=1}^{L-1} \sum_{k=0}^{(L-1)-j} \alpha^k \varepsilon_j \right) + \left(\mu + \sum_{k=0}^{L-1} \alpha^k \varepsilon_1 \right) \\ x_{e,1} = SS + \alpha^{L-1}\varepsilon_1 - \sum_{j=2}^{L-1} \sum_{k=0}^{(L-1)-j} \alpha^k \varepsilon_j - \left(\mu + \sum_{j=1}^{L-1} \alpha^{L-j}\varepsilon_j + \varepsilon_L \right) \\ x_{e,1} = SS - \sum_{j=2}^{L} \sum_{k=0}^{L-j} \alpha^k \varepsilon_j \end{cases} \tag{B.3}$$

Appendix C

Safety Stock - Exact Model

The variance of the summations in Equation 7.6 is described as the summations of the squared quantities for each epsilon

$$\sigma_{SUM}^2 = \sum_{j=t-L+2}^{t} \left(\sum_{k=0}^{t-j} \alpha^k \right)^2 . \tag{C.1}$$

Rewriting Equation C.1 yields

$$\sigma_{SUM}^2 = \sum_{j=t-L+2}^{t} \left(\frac{1-\alpha^{t+1-j}}{1-\alpha} \right)^2 = \sum_{j=t-L+2}^{t} \left(\frac{1}{(1-\alpha)^2} \left(1 - 2\alpha^{t-j+1} + \alpha^{2(t+1-j)} \right) \right) . \tag{C.2}$$

Equation C.2 is solved as

$$\sigma_{SUM}^2 = \frac{1}{(1-\alpha)^2} \left((L-1) - 2 \left(\frac{\alpha^L - \alpha}{\alpha - 1} \right) + \frac{\alpha^{2L} - \alpha^2}{\alpha^2 - 1} \right) . \tag{C.3}$$

Finally, the standard deviation of demand over the lead time is

$$\sigma_{SUM} = \sqrt{\frac{1}{(1-\alpha)^2} \left((L-1) - 2 \left(\frac{\alpha^L - \alpha}{\alpha - 1} \right) + \frac{\alpha^{2L} - \alpha^2}{\alpha^2 - 1} \right)} . \tag{C.4}$$

Appendix D

Numerical Analysis Cost Function

D.1 Cost Function Numerical Analysis

In this appendix, Equation 7.14 is analyzed. The expected total cost in period t can be estimated as

$$E\left[TC_t\right] = h * E\left[x_t\right]^+ + b * E\left[x_t\right]^- \tag{D.1}$$

where, for determining $E\left[x_t\right]^+$ and $E\left[x_t\right]^-$, knowledge of the density function of the inventory x_t at the end of period t is required.

In order to analyze Equation D.1, the following notation is used: I_0, P_t, μ, σ_t^2, where I_0 stands as the initial inventory at time 0; P_t as the planned order quantity to be received at time t; μ as the mean demand and σ_t^2 as the variance of the demand at time t. Moreover, $G_t(z)$ and $g_t(z)$ represent the cumulative distribution function and the density function of the demand at time t, respectively. Finally, the assumption that demand is normally distributed is made.

The density function of the inventory at the end of period t is estimated as

$$f_1(x_1) = g_0(x_1)dx_1 \tag{D.2}$$

where x_1 can take values from $-\infty$ to $I_0 + P_0$.

For $t \geq 2$, the inventory density function of x_t is found by confining its value to the density function of the inventory from the previous period x_{t-1}. Thereafter, $f_x(x_t)$ is given by

$$f_t(x_t) = \int_{x_{t-1}=-\infty}^{I_0+\sum_{i=0}^{t-2} P_i} \int_{x_t=-\infty}^{x_{t-1}+P_{t-1}} g_{t-1}(x_{t-1} + P_{t-1} - x_t)dx_t f_{t-1}(x_{t-1})dx_{t-1}. \tag{D.3}$$

Moreover, at the end of period t, the expected inventory on hand is estimated as

$$E\left[x_t\right]^+ = \int_{x_t=-\infty}^{I_0+\sum_{i=0}^{t-1} P_i} \int_{z=0}^{x_t+P_t} (x_t + P_t - z) g_t(z) dz f_t(x_t) dx_t, \tag{D.4}$$

and the expected backorders as

$$E\left[x_t\right]^- = \int_{x_t=-\infty}^{I_0+\sum_{i=0}^{t-1} P_i} \int_{z=x_t+P_t}^{\infty} (z - x_t - P_t) g_t(z) dz f_t(x_t) dx_t. \tag{D.5}$$

D.2 Cost Function Application

In order to exemplify the application of Equation D.1, the following data is assumed for calculating the expected total cost: μ=10, I_0=5, P_0=10, P_1=10, P_2=100, σ_0^2=568, σ_1^2=4910, σ_2^2=1, h=2 and b=10. Due to the assumption that the demand is normally distributed, the error function (erf) is implemented in order to calculate the cumulative of the distribution function.

The inventory density probability at t=1 is estimated as

$$\begin{aligned} f_1(x_1) &= \frac{1}{\sqrt{\sigma_0^2 * 2\pi}} e^{-\frac{1}{2}\frac{(x_1-10)^2}{\sigma_0^2}} dx, \text{ for } -\infty \leq x_1 \leq I_0 + P_0 \\ &= \frac{1}{\sqrt{568 * 2\pi}} e^{-\frac{1}{2}\frac{(x_1-10)^2}{568}} dx, \text{ for } -\infty \leq x_1 \leq 5 + 10 \end{aligned} \tag{D.6}$$

Thereafter, the expected inventory on hand $E\left[x_1\right]^+$ and backorders $E\left[x_1\right]^-$ at the end of period 1 are

$$\begin{aligned} E\left[x_1\right]^+ &= \int_{x_1=-\infty}^{I+P_0} \int_{z=0}^{x_1+P_1} (x_1 + P_1 - z) g_1(z) dz f_1(x_1) dx_1 \\ &= \int_{x_1=-\infty}^{5+10} \int_{z=0}^{x_1+10} (x_1 + 10 - z) \frac{1}{\sqrt{4910 * 2\pi}} e^{-\frac{1}{2}\left(\frac{x-10}{\sqrt{4910}}\right)^2} dz f_1(x_1) dx_1 = 0.3345 \end{aligned} \tag{D.7}$$

$$
\begin{aligned}
E\left[x_1\right]^- &= \int_{x_1=-\infty}^{I+P_0} \int_{z=x_1+P_1}^{\infty} (z - x_1 - P_1) g_1(z) dz f_1(x_1) dx_1 \\
&= \int_{x_1=-\infty}^{5+10} \int_{z=x_1+10}^{\infty} (z - x_1 - 10) \frac{1}{\sqrt{4910 * 2\pi}} e^{-\frac{1}{2}\left(\frac{z-10}{\sqrt{4910}}\right)^2} dz f_1(x_1) dx_1 = 17.6018
\end{aligned}
\tag{D.8}
$$

Equation D.1 is recalled for calculating the expected total cost at the end of period 1

$$
E\left[TC_1\right] = 2 * 0.3345 + 10 * 17.6018 = 176.687.
\tag{D.9}
$$

Moreover, for t=2 the inventory density functions are given by

$$
f_2(x_2) = \int_{x_1=-\infty}^{I_0+\sum_{i=0}^{i=0} P_i} \int_{x_2=-\infty}^{I_0+\sum_{i=0}^{i=1} P_i} g_1(x_1 + P_1 - x_2) dx_2 f_1(x_1) dx_1
\tag{D.10}
$$

The expected holding inventory and backorders at the end of period 2 are estimated as

$$
E\left[x_2\right]^+ = \int_{x_2=-\infty}^{5+10+10} \int_{z=0}^{x_2+100} (x_2 + 100 - z) \frac{1}{\sqrt{1 * 2\pi}} e^{-\frac{1}{2}\left(\frac{x-10}{\sqrt{1}}\right)^2} dz f_2(x_2) dx_2 = 28.2709.
\tag{D.11}
$$

$$
E\left[x_2\right]^- = \int_{x_2=-\infty}^{5+10+10} \int_{z=x_2+100}^{\infty} (z - x_2 - 100) \frac{1}{\sqrt{1 * 2\pi}} e^{-\frac{1}{2}\left(\frac{z-10}{\sqrt{1}}\right)^2} dz f_2(x_2) dx_2 = 0.
\tag{D.12}
$$

Finally, the expected total cost at the end of period 2 is

$$
E\left[TC_2\right] = 2 * 28.2709 + 10 * 0 = 56.5418.
\tag{D.13}
$$

Appendix E

Safety Stock - Approximation Model

The probability of obtaining $\eta_t \leq y_t$ is

$$\{\zeta_t = \sum_{i=0}^{L} d_{t+i} - \sum_{i=1}^{L-1}(e_{t-i} * P_{t-i}) - (e_t * P_t(\hat{x})) \leq \hat{x} - \bar{e} * \sum_{i=0}^{L} P_{t-i} = \frac{b}{b+h}\}. \tag{E.1}$$

The linear approximation $\hat{x} = SS + L * \mu$, $S_t = SS + (L+1) * \mu$ and $P_t(\hat{x}) = \frac{S_t - \hat{x}}{\bar{e}}$ is considered for solving Equation E.1

$$\begin{aligned} \sum_{i=0}^{L} d_{t+i} - \sum_{i=1}^{L-1} (e_{t-i} * P_{t-i}) - e_t * \frac{SS + (L+1) * \mu - (SS + L * \mu)}{\bar{e}} \leq \\ (SS + L * \mu) - \bar{e} * \sum_{i=0}^{L} P_{t-i}. \end{aligned} \tag{E.2}$$

Rewriting Equation E.2 yields

$$SS \geq \sum_{i=0}^{L} (d_{t+i} - \mu) + \sum_{i=1}^{L-1} (e_{t-i} - \bar{e}) * P_{t-i} + (e_t - \bar{e}) * \frac{\mu}{\bar{e}}. \tag{E.3}$$

Moreover, knowing that d and e are random values of the demand and bias in the forecast respectively, the previous formula is reduced to

$$SS_{t,t+L} = N^{-1}(\beta)\sqrt{\sum_{i=0}^{L} \sigma_{d,t+i}^2 + \sigma_e^2 \sum_{i=1}^{L-1} P_{t-i}^2 + \sigma_e^2 * \frac{\mu^2}{(\bar{e})^2}}, \tag{E.4}$$

where the assumption that the variation of the expected mean of the demand distribution follows a stationary process is made.

Appendix F

Sensitivity Analysis - Results

F.1 Level of Demand Variance

σ_ε^2	0	1	2	3	4
(s, S)	156.80	159.24	166.95	171.61	175.63
ES	0	8.67	15.18	21.87	28.02
MA	0	8.99	16.28	23.55	30.13
0% DPK	0	8.79	14.80	21.12	27.20
25% DPK	0	8.33	13.71	19.36	24.78
50% DPK	0	8.00	12.90	18.08	23.09
75% DPK	0	7.98	12.39	17.30	22.28
100% DPK	0	7.77	11.88	16.16	20.25
Exact	0	9.23	13.27	16.91	20.45

Table F.1: *Average Cost when varying* σ_ε^2

σ_ε^2	0	1	2	3	4
(s, S)	75.5	79.62	83.47	85.80	87.81
ES	0	4.14	6.51	8.82	11.09
MA	0	4.18	6.60	8.98	11.42
0% DPK	0	4.35	6.97	9.68	12.52
25% DPK	0	4.14	6.54	8.99	11.48
50% DPK	0	3.98	6.22	8.51	10.84
75% DPK	0	3.87	6.01	8.22	10.60
100% DPK	0	3.81	5.94	8.08	10.13
Exact	0	4.61	6.52	7.99	9.22

Table F.2: *Average Holding Inventory when varying* σ_ε^2

σ_ε^2	0	1	2	3	4
(s, S)	0	0	0	0	0
ES	0	-0.05	-0.27	-0.53	-0.73
MA	0	-0.08	-0.39	-0.70	-0.91
0% DPK	0	-0.01	-0.11	-0.22	-0.27
25% DPK	0	-0.01	-0.08	-0.17	-0.23
50% DPK	0	0	-0.06	-0.13	-0.18
75% DPK	0	0	-0.05	-0.11	-0.13
100% DPK	0	0	0	0	0
Exact	0	0	-0.03	-0.13	-0.25

Table F.3: *Average Backorders when varying σ_ε^2*

F.2 Level of Non-stationary Demand

α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
(s, S)	164.34	164.49	164.22	165.11	165.96	165.54	168.22	168.00	172.61	177.70
ES	14.07	14.39	15.10	16.52	18.20	19.91	23.76	27.68	34.25	45.70
MA	14.84	15.21	16.30	17.77	19.46	21.31	24.43	27.22	32.27	38.49
0% DPK	14.67	14.60	14.86	15.35	15.86	16.20	17.61	18.73	20.81	25.49
25% DPK	13.51	13.49	13.77	14.30	14.83	15.25	16.51	17.57	19.58	24.26
50% DPK	12.68	12.67	12.95	13.50	14.06	14.53	15.63	16.65	18.69	23.25
75% DPK	12.16	12.16	12.42	12.99	13.54	14.07	15.00	16.14	18.30	22.25
100% DPK	12.00	11.91	12.02	12.38	12.80	13.03	13.87	14.70	16.55	20.97
Exact	11.35	12.25	13.29	14.50	16	17.65	19.82	22.04	24.92	28.21

Table F.4: *Average Cost when varying α*

α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
(s, S)	82.09	81.67	82.20	82.45	82.51	83.15	83.69	85.81	87.16	89.34
ES	6.50	6.45	6.52	6.64	6.81	7.09	7.35	7.92	9.05	11.17
MA	6.55	6.53	6.61	6.75	6.92	7.22	7.49	8.12	9.36	11.48
0% DPK	7.09	6.98	6.99	7.04	7.13	7.34	7.56	8.05	9.13	11.08
25% DPK	6.61	6.52	6.58	6.65	6.78	7.02	7.27	7.77	8.83	10.87
50% DPK	6.24	6.19	6.27	6.36	6.52	6.78	7.06	7.56	8.62	11.49
75% DPK	6.01	5.97	6.09	6.19	6.35	6.62	6.93	7.44	8.91	15.64
100% DPK	5.93	5.89	5.96	6.07	6.23	6.51	6.76	7.28	8.29	10.12
Exact	5.54	5.99	6.52	7.14	7.86	8.69	9.66	10.77	12.05	13.52

Table F.5: *Average Holding Inventory when varying α*

α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
(s,S)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02	-0.09
ES	-0.13	-0.20	-0.28	-0.34	-0.49	-0.71	-0.93	-1.37	-2.05	-2.80
MA	-0.21	-0.28	-0.39	-0.47	-0.63	-0.85	-1.03	-1.33	-1.76	-2.04
0% DPK	-0.05	-0.08	-0.12	-0.13	-0.17	-0.22	-0.25	-0.32	-0.37	-0.39
25% DPK	-0.03	-0.05	-0.08	-0.10	-0.14	-0.17	-0.20	-0.25	-0.27	-0.28
50% DPK	-0.02	-0.04	-0.06	-0.07	-0.11	-0.14	-0.15	-0.19	-0.19	-0.20
75% DPK	-0.02	-0.03	-0.04	-0.06	-0.08	-0.11	-0.12	-0.14	-0.11	-0.11
100% DPK	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Exact	-0.02	-0.02	-0.03	-0.03	-0.04	-0.05	-0.06	-0.07	-0.07	-0.12

Table F.6: *Average Backorders when varying α*

F.3 Level of Service Level

z	90%	95%	99%
(s,S)	162.67	165.08	171.36
ES	14.85	15.18	15.97
MA	16.03	16.61	17.05
0% DPK	14.29	14.86	15.89
25% DPK	13.21	13.79	14.88
50% DPK	12.40	12.97	14.12
75% DPK	11.89	12.45	13.64
100% DPK	11.22	11.99	13.29
Exact	11.50	13.23	17.15

Table F.7: *Average Cost when varying z*

z	90%	95%	99%
(s,S)	81.33	82.54	85.68
ES	6.17	6.55	7.21
MA	6.27	6.62	7.23
0% DPK	6.64	7.02	7.68
25% DPK	6.22	6.59	7.26
50% DPK	5.91	6.27	6.94
75% DPK	5.72	6.06	6.74
100% DPK	5.61	5.99	6.65
Exact	5.49	6.52	8.55

Table F.8: *Average Holding Inventory when varying z*

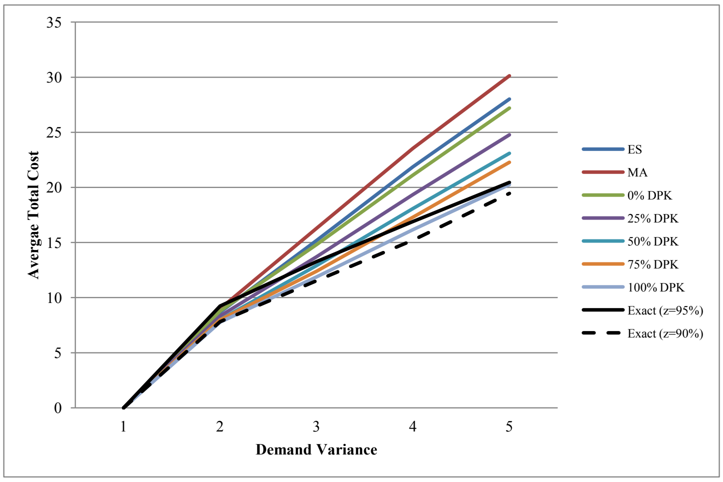
z	90%	95%	99%
(s, S)	0	0	0
ES	-0.31	-0.26	-0.19
MA	-0.44	-0.42	-0.32
0% DPK	-0.13	-0.10	-0.07
25% DPK	-0.10	-0.08	-0.05
50% DPK	-0.07	-0.05	-0.03
75% DPK	-0.06	-0.04	-0.02
100% DPK	0	0	0
Exact	-0.06	-0.02	-0.01

Table F.9: *Average Backorders when varying z*

Appendix G

Graphs of Sensitivity Analysis

Average Cost when varying σ_{ε}^{2}



Average Cost when varying α