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Inventory optimization for perishables with the possibility of supply disruptions

Rousseau, M.

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Inventory optimization for perishables with the possibility of supply disruptions

Martine Rousseau

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Supervisors:

Dr. Z. Atan, TU/e, OPAC

Dr. T. Tan, TU/e, OPAC

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Contents

I. Executive Summary

In this study we discuss inventory optimization for perishable products with the possibility of supply disruptions.

In 2011 the northern part of Thailand was struck by a massive flooding. This part of Thailand is home for many hard drive manufacturers. The flooding caused about 25% of the global hard drive production to be completely disrupted (Anderson, 2011; Fuller, 2011). Estimations of the price increase of hard drives were between as low as 10% to high as 200% for companies using Just-In-Time policy like Dell (Anderson, 2011). This price increase is beneficial for producers with untouched facilities but is bad news for producers who use hard drives in their production, like Dell, Intel, Apple, and digital camera manufacturers (Firstpost, 2011).

Hard drives are non perishable products, so producers who use hard drives for their production could have decreased the impact of this disruption on their production by holding a significant amount of inventory of hard drives (For simplicity we ignore a short product lifecycle here). For perishable products the possibilities of increasing inventory as a buffer for supply disruptions are reduced because the inventory level is constrained by the perishability of the product.

Standard inventory theory does not take the lifetime of a product into account; in theory products can be stored for an infinite amount of time. However, in a lot of cases products have a limited lifetime, which can change the decisions on inventory levels. Examples of perishables are fresh food, medicines, blood, and photographic film.

Furthermore literature on inventory theory takes into account that an order cannot be delivered due to the non-availability of a product – however, it does not take into account that there is a possibility that the order cannot be delivered due to a supply disruption. Although there are multiple extensions in the inventory theory literature on perishables and on supply disruptions, the combination of these concepts as an extension to inventory theory is less known.

The combination of these concepts in inventory theory is an interesting field, not only because it is a quite new field, but also since good supply chain management is increasingly important for the performance of companies in general and companies which work with perishables specifically. The nature of the firm has changed in the last decades and with it, its supply chain. In the past years, supply chains became longer and more complex, because of global sourcing and vertical disintegration. The goal of inventory management, went from reducing working capital to optimizing inventory while providing a certain amount of flexibility. Next to this, perishability poses an extra difficulty on ordering and inventory decisions. As Chandler (1990) states, companies which work with perishables products, create competitive advantage over their competitors by investing more in the distribution of their products than in the production of it. So more than for other kind of products, managing the supply chain is key for perishables.

In this study we analyze the influence of several different parameters on the optimal base stock level and the corresponding cost. The main goal of the study is to obtain more insights on inventory optimization under the influence of perishability and supply disruptions. Other goals of this study are: showing the importance of taking perishability and supply disruptions into account when making decisions on the inventory level, providing support for companies to make inventory related decisions on an operational as well as a strategic level, and providing a basis for further research in this field.

To achieve the goals set, we develop a cost function for inventory related cost in which perishability and disruptions are taken into account. The development of this cost function is done in a stepwise manner. The cost function for products with a fixed lifetime of one period is similar to the cost function in the newsboy problem. In the next step we obtain the cost function for products with a fixed lifetime of three periods. Then we do the same, but now products have a fixed lifetime of four periods. Also the following step, products with a lifetime of five periods, is similar. Based on a pattern that occurs, we can develop a cost function in which the length of the fixed lifetime is a variable. The convexity of this cost function is shown in a graphical way.

After we have developed the cost function and we have good reasons to assume it is convex, we can determine the optimal base stock level by finding the minimal cost. Furthermore we can analyze the effect of different parameters on the minimal cost and with that on the optimal base stock level. For this analysis we program the cost function in Matlab and for each analysis we let one (or more) parameter(s) vary while the others remain equal. With plots we provide a graphical representation of this analysis and show the influence of each parameter on the cost function.

In some cases the influence of a parameter is quite straightforward and already proven in earlier research. This is the case for variables like unit holding cost, unit backordering cost and unit perishability cost. An increase of unit holding cost decreases the optimal base stock level, while higher unit backordering cost increases the optimal base stock level. Higher unit perishing cost, like unit holding cost, also decrease the optimal base stock level; however, here the influence stops as soon as the inventory level is below a certain level that assures that products in inventory do not perish.

Other parameters that have a straightforward influence on the optimal base stock level are the parameters which characterize the supply disruption process; the disruption probability and the recovery probability. Since inventory can act as a buffer against uncertainty, of for example order delivery, an increase of optimal base stock level occurs when the system is longer in a disrupted state. During the disruption a higher base stock level helps to fulfill demand from stock and this way prevents backorders. So in uncertain situations (high disruption probability and/or low recovery probability) the influence of perishability is not only in the cost of perishing directly, but also on the non availability of the product when a disruption takes longer than the lifetime of the products in inventory.

Another important finding is that for products with a short lifetime, the inventory related cost function is much steeper than for products with a long lifetime. With this we mean that the cost increase between the optimal base stock level and a non-optimal base stock level is higher for products with a short lifetime than for products with a longer lifetime. So for items with a short lifetime the choice of the base stock level has a bigger influence on the total cost than for products with a long lifetime. This shows the importance of taking perishability into account when determining the optimal base stock levels.

In addition to cost, we analyze the effect of perishability and disruptions on service levels. In this analysis, we find that the maximum fill rate (when the base stock level goes to infinity) is mainly influenced by the disruption distribution. On the other hand, we see that products with relative short lifetimes have an optimal base stock level that has a fill rate which is significantly lower than the maximum fill rate. However, the fill rate can be easily increased (up to the maximum fill rate) by increasing the base stock level, (and increasing the inventory related cost).

Furthermore, our model can be useful for analyzing the effects of mitigation strategies. Each strategy influences different parameters, and with that the optimal base stock level and corresponding cost change. This information can help decision makers making strategic decisions on which mitigation strategy to use in what situation.

Our results provide a basis for researchers as well as for decision makers. A drawback is however, that the results are obtained by the use of a model which was developed under certain strong assumptions. One of which is deterministic demand. Although assuming deterministic demand is a good starting point, in real life situations there might be some variance in the demand. Therefore we test the performance of our model in a stochastic environment. We do this by comparing the optimal base stock level and corresponding cost of our deterministic model with the optimal base stock level and corresponding cost as found in a Monte Carlo simulation. In this simulation, we generate normally distributed demand for a certain number of periods and determine the optimal base stock level in this specific case. We repeat this procedure a certain number of times, and then take the average of the optimal base stock levels and cost in these simulations. These averages are taken as the optimal base stock level and corresponding cost for the stochastic situation. The analysis shows that the model with the assumption of deterministic demand performs quite well in situations with stochastic demand, when variability is within a certain boundary. As is normal in cases with stochasticity, one should add a certain safety stock to the optimal base stock level from the deterministic demand model to find the optimal base stock level in the stochastic situation. The amount of this safety stock is dependent on the level of variability and on the optimal base stock determining mechanism. When this safety stock is added the inventory related costs are almost equal to the case with deterministic demand. Remarkable is that in some cases this safety stock is negative. This is for example when the optimal base stock level is relatively high and the unit perishing costs are high as well. Variability in demand might sometimes lead to low demands, yielding a high expectation of perishing cost. This in its turn might lead to a decrease in the optimal base stock level, hence a negative safety stock.

In the final analysis of this study we adapt the original model in such a way that the disruption characteristics are time dependent. We do this by making use of the idea behind Markov modulated processes. However, for our analysis we assume that the process modulating the initial process is not Markovian per se. With this analysis we show on one hand that it is fairly easy to adapt for this time dependency and on the other hand it provides some extra insights on the influence of the length of a disruption on the optimal base stock level.

1. Introduction

In this section, we define the concepts of perishability and disruptions and explain how they pose an extra difficulty on inventory decisions.

1.1 Perishability

Perishability occurs regularly and poses an extra difficulty on ordering and inventory decisions. As Chandler (1990) states, companies which work with perishables products, create competitive advantage over their competitors by investing more in the distribution of their products rather than the production of it. So more than for other kind of products, managing the supply chain is key for perishables.

Perishable products are products with limited lifetimes. Examples of perishable products are fresh food, medicines, flowers, blood, organs, photographic film, and beer. These products are different from products with a short product lifecycle, in that products with a short product life cycle are not reordered when they have become obsolete (Nahmias, 1982). The perishability of a product can be fixed or stochastic. In the latter case, decay follows a stochastic process, for example an exponential decay. While in the former case, one knows the time until the product will perish. Examples of this are products with expiry dates, like food, blood or pharmaceuticals.

When ordering new perishable products, one might be interested in the age of the products still in inventory; when inventory is relatively old, more new products might be ordered than when inventory is still relatively new. Therefore product age should be incorporated in models that analyze systems with perishability. If products have a fixed lifetime, only the age of each product in inventory should be recorded. In case of periodic review, the products will be one period older, each review period. In case of continuous review the age follows a continuous process. If products have a stochastic lifetime, not only the age should be recorded, also the probability of perishing. This can be done by the use of a (continuous/discrete-time) Markov process. Another way of modeling perishability is by queuing models with impatient customers. In which for example the inventory is represented by the customers and the perishability by the impatience of these customers.

1.2 Disruptions

Disruptions are random events that cause a supplier or other elements of the supply chain to stop functioning either completely or partially, for a (typically random) amount of time (Snyder et al., 2010). Our definition of disruptions as random events, implies that we only focus on supply disruptions that are sudden and unpredictable. Hence, disruptions that can be planned or are foreseeable in any way are not subject of this thesis. Disruptions are usually modeled by two states, the “up” and the “down” state, representing not disrupted and disrupted respectively. As in perishable inventory, we can use Markov chains to model these two states. The duration of up and down periods can have multiple distributions, but the most common assumption is that these are exponentially (continuous-time case) or geometrically (discrete-time case) distributed (Snyder et al., 2010). An implicit assumption made here is that one knows the state and/or duration distributions of a supplier or another element in the supply chain. In reality this information is often difficult to obtain accurately (Snyder et al., 2010).

Now we give some examples to shows that disruptions can have massive impacts on every element in the supply chain. These examples show that disruptions are no scarce events. Especially since we only consider two examples, and disregard many other (smaller) disruptions.

The earthquake and following tsunami that struck Japan in March 2011 is, according to HIS Isuppli, the most significant supply chain disruption that was ever recorded (Wall Street Journal, 2011). The impact of this disruption was increased by the lack of power due to the nuclear problems, and a lack of a decent transportation network. Professor Brian Tomlin explains in a video[1] that the temporary shut-down of one factory can result in the shut-down of its customer's facilities because there is not enough inventory to overlap the disruption period. This might cause other suppliers of this customer facility to shut down temporarily as well. These shortages and shut-downs might cause the prices to go up. They also cause enormous delays, causing in its turn the loss of goodwill. For example customers who bought a car from a Japanese car manufacturer had to wait much longer than usual. Toyota lost ¥ 81.8 billion of profit and two months after the earthquake still only 50% of their Japanese facilities and 40% of their facilities outside Japan were up, as they did not have enough supply from Japan (Albert, 2011). Not only the automotive industry was impacted immensely by this earthquake, also the production of silicon wafers was disrupted, causing great troubles for their customers, like the electronics and solar industry.

About half a year after the Japanese earthquake, the northern part of Thailand was struck by a massive flooding. This part of Thailand is home for many hard drive manufacturers and their suppliers. The flooding caused about 25% of the global hard drive production to be completely disrupted (Anderson, 2011; Fuller, 2011). The manufacturers expected to produce at normal rate again one year after the flooding. Analysts expected that this disruption flooding had the same effect as the Japanese earthquake. Estimations of the price increase of hard drives were between as low as 10% to high as 200% for companies using Just-In-Time policy like Dell (Anderson, 2011). This price increase is beneficial for producers with untouched facilities but is bad news for producers who use hard drives in their production, like Dell, Intel, Apple, digital camera manufacturers, but also Google and Facebook who wanted to build their own servers at that time (Firstpost, 2011). A main problem in this flooding disruption is that the manufacturers of hard drives and their suppliers were concentrated in a relatively small area. Analysts expect that in the future hard drive manufacturers build up their new facilities somewhere else, in order to have a less concentrated production. However, also the suppliers of hard drive manufacturers are concentrated in this area, so this area remains preferable. Not only hard drive manufacturers and their suppliers were directly subject to this disruption, also some semiconductor facilities and car manufacturers were disrupted by the flooding. Some of these companies are Japanese, and already had to face the Japanese earthquake and tsunami earlier that year, therefore the Japanese central bank considered setting up a Bath-lending line for these affected Japanese firms (Anderson, 2011).

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[1] Available at: http://www.tuck.dartmouth.edu/news/video/japan-supply-chain-disruptions/ [Accessed 4 February 2012]

Next we describe some of the research that already has been done in the fields of perishable inventory theory and supply disruptions.

2. Literature review

The nature of the firm has changed in the last decades and with it, its supply chain. In the past years, supply chains became longer and more complex, because of global sourcing and vertical disintegration. Furthermore markets nowadays are much more turbulent and uncertain than in the past. Also, major disruptions, like earthquakes or strikes keep occurring. Resulting from these issues, one could state that the twenty-first century supply chain has an increased level of supply chain risk.

Another topic in supply chain management is the issue of perishable products, like fresh food, medicines, blood, and photographic film. In the case of non perishable products, inventory holding cost might decrease the optimal inventory level. This is also true for perishable products, although in that case, the inventory level can also be constrained by the perishability of the product. In other words, in stable systems, one will not hold inventory for 4 weeks, if products perish within two weeks. The importance of striking this delicate balance explains why companies which work with perishables can create competitive advantage by investing in the management of the supply chain. Furthermore, we discuss mitigation strategies, that can help mitigating supply disruption risk when the possibility of holding more inventory is limited.

Also, we describe the positioning of this research into the existing literature.

2.1 Perishable inventory

Standard inventory theory does not take the lifetime of a product into account; in theory products can be stored for an infinite amount of time. However, in a lot of cases, products have a limited lifetime, which can change the decisions on inventory level. The lifetime of a product can be fixed or stochastic. The stochastic case might seem more general at first sight, but nowadays a lot of perishable products have fixed expiry dates so we focus on products with a fixed lifetime. Another restriction we make is that products are issued via a FIFO policy.

In 1964 Van Zyl was one of the first to research the topic of perishable inventory. Since that time researchers have a growing interest in the topic. Where Van Zyl (1964) assumes products perish after two time periods, Fries (1975) extends this to l time periods. For this he introduces a vector to record the age of each product in inventory. Fries (1975) also shows that when products perish after 1 time period and demand is not backlogged, it comes down to an original newsboy problem. Even when products perish after 2 time periods, the problem is still fairly simple, because one starts off with inventory of one certain age. Fries (1975) comes up with several functions that indicate whether one should order (the order-indicator function, which does not depend on the age of the inventory) and how much one should order (the target inventory function, which depends on the age of the inventory). By the use of graphs, in which each axis shows the cumulative number of inventory products of a certain age, he shows for example that when at the moment that the order decision must be made the inventory is relatively old, more should be ordered.

Nahmias (1975) comes up with a solution for more or less the same problem as Fries (1975). Nahmias (1982) also reviews the literature on perishable inventory theory, where he divides the literature not only by fixed/stochastic lifetime also by deterministic/stochastic demand. When demand is deterministic, the problem reduces to a simple EOQ problem constrained that the amount ordered will not perish before it is sold. The stochastic case is more interesting, for which we already saw the solution of Fries (1975) (and Nahmias, 1975). However both assume periodic review. Weiss (1980), assumes a fixed lifetime, but under continuous review. Furthermore he assumes that demand follows a Poisson process. He shows for both backordering and lost sales, what the optimal order policy should look like. He finds that for the case of lost sales the best strategy is a (0, S) one, while for backordering it is (s, S). The first strategy, (0, S), means that you only order up to S, at the moment the inventory is equal to 0. This strategy is called the zero inventory policy (ZIO). The latter strategy, (s, S) means that you order up to S, if inventory is below s. This difference in optimal strategy might be explained by the difference in penalty cost of not being able to fulfill demand out of inventory. In the case of backordering this is dependent of the duration, while in the case of lost sales, this is fixed. This result might be useful, but the assumptions (next to the ones already mentioned: single unit demand and zero lead time) are (together) fairly uncommon, which leads to poor performance in real life.

Around the same time as Weiss (1980), Graves (1982) develops two models for studying perishable inventory systems. Graves (1982) also assumes a deterministic life time for the products. Furthermore he assumes continuous replenishment, so in contrast with the other papers described earlier; the ordering policy is no decision variable in this article. Graves (1982) models the inventory (by assumptions like continuous replenishment, demand is a (compound) Poisson process, no backorders etc.) as a queuing problem with impatient customers. He argues that the idea of using queuing theory with impatient customers is not new. However, he uses it differently than researchers have done before. Originally the metaphor was as follows: the queue represents the inventory, the service process is the demand and, the time till customers will leave is equal to the life time of the perishable products. Although this seems very useful, the queue is usually not controllable, whereas the service process is. This is the exact opposite to what the metaphor predicts. Graves (1982) acknowledges this and swaps the two constructs (queue and service process). This leads to fairly easy functions for the expected outdates and shortages per time unit, the expected age of the oldest unit supplied for demand, and the expected inventory level. However, as with Weiss (1980) the assumptions Graves (1982) makes, are only in a few situations justified and thus the application is limited.

Goyal and Giri (2001) present a review of the more recent literature on perishable inventory. For products with a fixed lifetime, they refer to three papers which we shortly discuss in the following. Nandakumar and Morton (1993) try to improve the approximations as made by Fries (1975) and Nahmias (1975). Some assumptions are: lost sales, zero lead time, periodic review, and a fixed lifetime of K periods. They use near myopic heuristics to break the equations down into newsboy equations which provide upper and lower bounds for the inventory level. Liu and Lian (1999) also assume a fixed lifetime, but in contrast with Nandakumar and Morton (1993), they assume continuous review and try to obtain solutions for this case. They use a Markov renewal approach to obtain closed-form solutions for the steady state probability distribution of the inventory level and system performance measures. They also construct a closed-form convex cost function in both reorder point and base stock level. This shows that for the general renewal demand process an (s, S) policy is optimal and that the optimal values can be determined. Perry (1997) uses the idea of Brownian motion to model whether the inventory is too high (and perishes) or too low (leading to unsatisfied demand).

Another work, not mentioned in Goyal and Giri (2001), is the work of Chiu (2001), who approximates the optimal values for Q and r for a (Q, r) continuous review policy. Chiu (2001) provides a graphical representation of the (Q, r) policy with perishability, which might be useful for understanding calculations. A last paper we would like to mention is the one of Tekin, et al., (2001). They come up with a modified lot size-reorder control policy. This modification is based on the age of the inventory. This idea is not new, as we saw already Fries (1975) and Nahmias (1975) also take into account this age. However, what Telkin et al. (2001) do is to analyze the impact of this modification in the continuous review case. They show that for slow moving perishable inventory systems with high service levels, the modified policy is superior to the classical (Q, r) system. These more recent articles built on the ideas of the older articles mentioned but, are focused on obtaining a better solution or approximation for a more realistic situation, by using more complex methods.

2.2 Supply disruptions

Literature on general inventory theory takes into account that an order cannot be delivered due to the non-availability of a product - it often does not take into account that there is a possibility that the order cannot be delivered due to a disruption. An early contribution to the field of inventory theory with disruptions comes from Meyer, et al., (1979), although they use a different terminology. In their article, they analyze a single-stage production-storage system with deterministic demand. When the production is up, it produces more than the demand rate, until capacity is reached. Although the authors do not focus on optimization, they do find that the performance measures depend more on the average time the production is up/down, than on the specific kind of distribution of up/down times. This research implies that the inventory built up when the production rate is higher than the demand rate (i.e. when the production is up), is used as a buffer for when the demand is higher than production (i.e. production is down). Other earlier work extend this work of Meyer et al. (1979) by using stochastic demand (Posner and Berg, 1989) or by modeling disruptions as a continuous-time Markov process (Chao, 1987)

Around the same time, Bielecki and Kumar (1988) find that in some cases the zero inventory order policy (ZIO) can be optimal for the case with disruptions. At first, this result might sound counter intuitive; however, this seems reasonable when the decrease in holding cost outweigh the increase in backordering cost. This result is preferable in some cases of perishable inventory, still; backordering does not only bring along cost, one should also take into account the decrease in service level and goodwill. With the ZIO assumption in mind Parlar and Berkin (1991) analyze an EOQ approach for future supply uncertainty (EOQD (D=Disruptions)). Later, Berk and Arreola-Risa (1994) show that some of the assumptions made by Parlar and Berkin (1991) are not valid and therefore a closed form solution cannot be found anymore. Berk and Arreola-Risa (1994) do provide an approximation, as does Snyder (2008). Schmitt, et al., (2010) extend this approximation to the case in which demand is stochastic. Usually EOQ models do not take into account the stochasticity of demand. Before Schmitt et al. (2010) used stochastic demand, Bar-Lev, et al., (1993) did research on this. However, they based their work on the incorrect model of Parlar and Berkin (1991). Other drawbacks of the work of Bar-Lev et al., (1993) are the high complexity and extensive assumptions.

Returning to the case with deterministic demand brings us to the work of Weiss and Rosenthal (1992). By minimizing the expected cost, they come up with optimal inventory policies for either a supply disruption or a demand disruption. They also provide an algorithm to obtain the optimal order quantity. The research of Weiss and Rosenthal (1992) assumes that usually orders are placed at the moment inventory level reaches zero. However, as explained before, the early work of Meyer et al. (1979) shows that inventory can be used as a buffer in disrupted situations. So later in the nineties work is done with relaxing the zero inventory order assumption. For example by Parlar and Perry (1995), who leave the reorder point as a decision variable; as Weiss and Rosenthal (1992) they assume demand is deterministic. Furthermore they assume disruption periods are exponential (as also examined by Weiss and Rosenthal (1992)). Parlar and Perry (1995) also mention another issue: the cost related to determining the state of a supplier. Until now most research assumes that the state of the supplier is known at the moment of ordering. Not having this information, and incurring cost to obtain this information (for example by paying fixed ordering cost, while supplier is disrupted and cannot deliver), brings another decision variable. In another paper by Parlar and Perry (1996) they assume no cost for finding out what the state of the supplier is, and thus the time to wait until placing a next order is not a decision variable anymore. In this research they examine the situation in which there are multiple suppliers. Having multiple suppliers might be a strategy to mitigate supply disruption risk. Earlier, Anupindi and Akella (1993) worked on this topic. They do not use EOQD, but do take stochasticity of demand in account. They show, by the use of models which represent the different strategies (single or dual sourcing) that dual sourcing can be beneficial for a company. In contrast with Parlar and Perry (1996), suppliers have different cost, so there is a specific preference for a certain (cheap) supplier.

Gupta (1996) also relaxes the assumption of ZIO. With this work, we return to models with stochastic demand. When assuming lost sales, he finds that ignoring disruptions might cause high operating cost. Parlar (1997) uses more or less the same model, but he assumes random lead times and full backordering. In the end he argues that his results can be easily adapted for the lost sales case. Arreola-Risa and DeCroix (1998) take a situation in between; partial backorders, which is more general. They find that when the probability of backordering (compared to lost sales) goes up, the minimum amount ordered goes up, and if this probability increases, and at the same time, the cost for lost sales increases, s^* and S^* also go up.

2.3 Mitigation strategies

Most of the articles discussed so far, mainly analyze the system under disruption risk, and some offer an optimal inventory strategy in certain risk-situations. Except for multiple suppliers, no other ways of how to decrease the impact of a disruption on a system are given. This is done by Tang (2006). He analyzes the impact of several major disruptions on companies and how companies (both winners and losers) coped with this disruption. He comes up with a list of nine mitigation strategies, like postponement, strategic stock and a flexible supply base. He also argues that these strategies should not only be beneficial in case of disruptions, also without disruptions these strategies should add value, otherwise it is difficult to find support for implementing. Tomlin (2006) also provides a (somewhat shorter) list of mitigation strategies. After the description of the strategies, he analyzes the relative performance of each strategy in different situations.

Based on Tomlin (2006), Tomlin (2009), Snyder et al., (2010), and Tang (2006), we describe twelve general mitigation strategies and how they can be used for perishable products. The strategies are ordered in a way, such that strategies which are more on the supply side are mentioned first and strategies that focus more on the demand side follow. The first two strategies cannot be placed anywhere in the internal supply chain (from supply to marketing) process.

1. ***Financial mitigation*** is actually another word for insurance (Tomlin, 2006). Firms can insure their selves against several types of risks (also disruption risk), so that not the firm, but the bank carriers the risk. However, it might be difficult to prove the loss by the firm due to a supply disruption.
2. ***Acceptance*** is not an actual mitigation strategy. Acceptance means that a company just accepts the disruption risk it faces (Tomlin, 2006). In case the risk and its effects are fairly low, this might be a preferable 'strategy'. Still a lot of companies use this strategy, because it is difficult to come up with clear estimates of the cost of a disruption versus the cost of a mitigation strategy (Tang, 2006). That is why Tang (2006) urges for robust strategies; i.e. strategies that are not only beneficial for the company in case of a disruption.
3. ***Economic supply incentives*** means that a company (or other institution, like a government) offers financial support to its suppliers in order to set up ways to decrease the effect of disruptions (Tang, 2006). Tang (2006) discusses that this strategy might work in the flu-vaccination problem the US faced in 2004, when a lot of suppliers left the market and one of the few remaining suppliers was suspended due to contamination. One could also think of financial support to alfalfa farmers in the Central Valley. With the water problems that California faces, alfalfa (needs relative high amount of water) is not a preferred crop. However, by providing financial aid to the alfalfa farmers, supply of alfalfa can still be guaranteed in times of drought.
4. ***Flexible supply base*** is a known mitigation strategy. In this paper we already discussed it slightly (Anupindi and Akella, 1993), however, there are multiple other researchers analyzing this topic (Gürler and Parlar, 1997, Dada, et al., 2006, Federgruen and Yang, 2008/2009). Although this idea might be somewhat opposite to other current literature on supplier relations (which says to minimize number of suppliers in order to maintain good relations), Tomlin (2009) shows in his article about mitigation strategies for perishable products, that this strategy can be very useful in this setting. And as Berger, et al., (2004) show, in realistic cases, the optimal number of suppliers is still typically fairly small.

   The strategy can be brought into practice via having multiple suppliers, but also by having flexible contracts (Tang, 2006) like which for example only say that the supplier is responsible for delivering a certain amount of a product, without stating that they are responsible for producing these items their selves. This strategy (in both variances) might be useful for perishable inventory.

5. ***Contingent rerouting*** can be seen as a special case of a flexible supply base; it means that a company changes the supplier at the moment of a disruption (Tomlin, 2006). So instead of having multiple suppliers at the same time, this strategy uses only one supplier. This strategy can only be applied if the back-up supplier has some flexibility in the amount of products being produced. Sometimes the end-product also needs some flexibility, because the back-up supplier might supply a somewhat different product. An example of this strategy in perishable products is the Chiquita case; when hurricane Mitch destroyed several plants of both companies in Central America, Chiquita reacted quickly, by temporarily increasing production at other unaffected plants. Dole on the other hand was too slow and incurred a loss of 100 million dollar. (Tomlin, 2006).
6. ***Strategic stock*** is a strategy in which stock is kept at strategic places. An example of this in perishable inventory is the Centre for Disease Control (CDC) that keeps medicines in an aircraft in a strategic location in the country. As soon as a disaster happens, this aircraft can be there within 12 hours (Tang, 2006). However, this implies holding stock, which is only partly possible for perishable products. So the use of this mitigation strategy for perishable products is only minor.
7. ***Make and Buy*** implies that some products are produced in-house while other are outsourced (Tang, 2006). An example of this can be found in Chandler (1977) where he discusses that DuPont only makes a small amount of glycerin itself, while buying the remaining amount needed. Another example might be; produce specific complex medicines in-house, while outsourcing 'mature' medicines (like aspirin). If the supply of aspirins is disrupted, the in-house production could produce some aspirin in order to cope with the disruption. So this strategy can be very useful in perishable inventory theory, but as with contingent rerouting, it requires some flexibility.
8. ***Postponement*** is designing a product/process in such a way that the point of differentiation is delayed (Tang, 2006). Even without disruptions this is a good strategy and with disruptions, the postponement leads to be able to change the components quickly (without too many adaptations). Li, et al., (2006) show that a postponement strategy in EOQ-based models is indeed effective for perishable products. Next to the effectiveness, also applicability must be considered; an example the authors give for postponement in perishable items, is syrup for sodas.
9. ***Flexible transportation*** implies being able to use multiple ways of transport (different routes, but also different modes of transportation) (Tang, 2006). If for example an airport is closed (think about the trouble the eruption of an Icelandic volcano eruption caused in 2010) it would be convenient to be able to transport products via another airport, or maybe via ships. Another example is given by Tomlin (2006). New Balance rerouted ships from the West to the East coast, when the West coast dock was disrupted. Next to that they also used planes to transport their products.
10. ***Silent product roll over*** is placing new products in the market without a formal announcement (Tang, 2006). In the pharmaceutical market, often similar medicines are produced by different suppliers, and the adaption of another 'brand' can be done without a formal announcement; the

medicines are substitutable. In case a supply disruption would occur, it is easy to change to another brand, without officially informing the customer.

11. ***Revenue management via dynamic pricing and promotion*** adapts the demand of customers via changing the prices of the products (Tang, 2006). An easy example: Supply of bananas is disrupted, while supply of apples is as usual, one could lower the price of apples and increase the price of bananas. This way demand for bananas decreases and people might not even notice that there is a disruption. The field of revenue management via dynamic pricing and promotion is huge. This mitigation strategy in the perishable case, is analyzed by (among others) and Bitran and Caldentey (2003) where they give an overview of several pricing models.
12. ***Assortment planning*** is quite similar to the previous strategy. Doing this, one tries to change demand via marketing techniques like the location on the shelf, the amount of products on the shelf etc. (Tang, 2006). This is analyzed by Kök and Fisher (2007) by the use of a substitution model.

2.4 Positioning in existing literature

In Table 2.1 the positioning of this research within the existing literature is depicted. The case without perishability and without disruptions results in a simple solution; as demand is deterministic, the optimal base stock level is equal to demand. In the case that demand is not deterministic but stochastic, one could use the newsvendor model, with backordering cost as penalty cost. The newsvendor problem could also be used for the case in which the lifetime is equal to 1.

	No perishability	Perishability
No disruptions	$S^* = d$ (deterministic) Newsvendor model (stochastic)	Fries, 1975 Nahmias, 1975
Disruptions	Schmitt et al., 2010	Focus of this thesis

Table 2.1 Positioning of research questions within existing literature

Situations in which disruptions can occur and products are not perishable, are already examined by Schmitt et al., 2010. They assume demand is stochastic, while in this study we assume deterministic demand.

The other case, perishability and no disruptions, is also already examined. For fixed lifetime of products and stochastic demand, this research is done by Fries (1975) and Nahmias (1975). The case for perishable products with the possibility of disruptions is not yet studied and is examined in this thesis.

3. Goal

Disruptions can have great impacts on supply chains. More specifically, disruptions cause uncertainty regarding the delivery of orders. In the case of non-perishable products, this uncertainty can be buffered relatively easy by keeping inventory. However, in the case of perishable products, the cost of perishing might reduce the optimal inventory level. For these products, keeping inventory, offers less protection for the uncertainty caused by a disruption. Consequently, the effects of a disruption cause more complex and profound problems for managing the supply of perishables.

Further, due to the fact that buffering is only possible to a limited extent, other mitigation strategies take a more central role in the case of perishable products. These mitigation strategies focus on limiting (1) the possibility of a disruption, (2) the length of the disruption, and (3) the consequences of a disruption. Our intention is to provide a basis for analyzing the effectiveness of several mitigation strategies that all focus on one or more of these three disruption parameters.

In this study we try to obtain a robust model which supports inventory decisions and helps to analyze the effect of several parameters on the choice of the optimal inventory level under a specific policy.

After obtaining this model, we try to answer the following questions:

- How do supply disruptions influence the effect of perishability on the total inventory related cost and the optimal base stock level in a single echelon system?
- How does the disruption distribution influence the optimal base stock level of single echelon systems with perishable products?
- How do the different types of inventory related cost influence the optimal base stock level in these systems?

Through exploring these questions, we contribute to the existing inventory literature by describing the influence of the combination of supply disruptions and perishables on the base stock levels. The answers to these questions also provide implications for decision makers, on how several parameters can change the optimal base stock level. The model as obtained in this paper, helps to rationalize how the mitigation strategies work, how the optimal base stock level changes as a result of adopting a certain mitigation strategy, and resulting from that, which strategy fits best to a certain situation.

4. Model

In this section, we first explain the system and the corresponding assumptions, our assumptions on supply disruptions and the order of events. Then we derive a model that is based on minimizing the cost function to obtain an optimal base stock level. Note that the derived base stock level will only be optimal under certain conditions, which we describe below.

We consider a single-location system, carrying perishable products, fulfilling positive demand. In this single-location system, inventory levels are reviewed periodically, and new orders are placed such that after replenishment (we assume zero lead time), the inventory reaches a certain level (S). This assumption of a periodic review, base stock level control system is fairly common and is also made by other researchers (Fries, 1975; Schmitt et al., 2010). Further, we assume that linear unit inventory holding cost (denoted as h) must be paid over all products that remain in inventory at the end of each period.

Products in inventory are assumed to have a fixed deterministic lifetime (denoted in our model as x). When products remain longer in inventory than their lifetime, perishing occurs, and linear unit perishing costs (denoted as p) must be paid.

We assume a constant demand per period (d). When there are not enough products in inventory, this demand cannot be satisfied. In such cases, we assume that the unsatisfied demand is backordered and that linear unit backordering costs (denoted as b) must be paid over all backorders.

Note that we do not take into account the purchasing costs, because they do not depend on the base stock level. Also, the cost of purchasing too many products that will perish is already taken into account by the perishing cost.

The following provides an overview of our notation:

$h := inventory\ holding\ cost\ per\ unit\ per\ time\ period$

$b := backordering\ cost\ per\ item\ per\ period$

$p := perishability\ cost\ per\ item$

$d := demand\ per\ period$

$x := lifetime\ of\ products$

The decision variable, which we aim to determine, is

$S := base\ stock\ level$

We develop a model that determines the optimal base stock level by minimizing the total cost for perishable as well as non perishable products. Further, we examine situations with and without disruptions. In our model, orders are not delivered during disruptions, resulting in a decreasing stock level, which might lead to more backorders. We argue above that when products are not perishable; this effect of disruptions can be buffered by holding more inventory. However, when products are perishable, holding more inventory as a buffer for supply disruptions results in higher perishing cost, which might lead to a higher total cost than in a situation without buffering.

To describe disruptions in our cost function, we make use of two disruption characteristics; the disruption probability and the recovery probability. The disruption probability represents that probability that the system goes from an 'up' state to a 'down' state. When a system is in a 'down' state, orders cannot be delivered anymore. The recovery probability represents the probability that a system goes from a 'down' state to an 'up' state. In other words, after a recovery, the system is not disrupted anymore and orders are assumed to be delivered within lead time. The cumulative distribution functions of the disruption and recovery probability define the probability that a system goes into a disruption or recovers from it within a certain time span respectively.

The notations for the probability of disruption and recovery are:

$$\alpha := disruption\ probability$$
$$\beta := recovery\ probability$$

Knowing the disruption and recovery probability, we can determine a disruption distribution as used by Schmitt et al., (2010). These authors use the probability π_i to represent the probability of being at the end of the *i*th consecutive disrupted period.

The model that we develop shows us the change in the optimal base stock level and in the corresponding costs when disruptions occur.

Next, we describe the process by the chronologic order of events. The process is depicted in Figure 4.1. Right before the next period all perished products are disposed and the order (if any) arrives (when not disrupted). During the period demand occurs and is satisfied immediately if possible. If inventory is not sufficient to satisfy this demand, the excess demand is backordered and corresponding backordering costs occur. Then, at the end of this period all items left in inventory mature by one period (i.e. the products are marked one period older than in the prior period). When the age of a product exceeds its stipulated lifetime, the product perishes and perishing costs occur. Furthermore inventory cost need to be paid over all items left in inventory. At this time also a new order will be placed, based on the number of non perished items left.

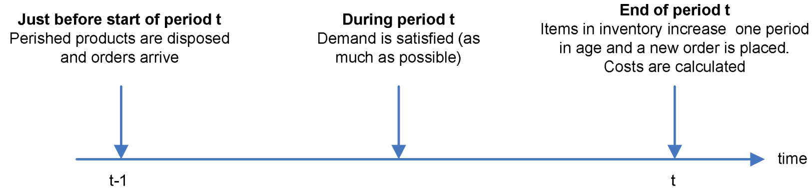

Figure 4.1 Chronologic order of actions taken

In the following sections, we obtain a general model for products with a general lifetime of x. To do so, we start with more specific situations, in which the lifetime is equal to 3, 4 and 5 periods. The situation, in which products have a lifetime of 1 period, is equal to the newsboy problem. This is not interesting for our case, since buffering against supply disruptions is not possible. When products have a lifetime of 2 periods, buffering would be possible, but only to a limited extent. We see that a general pattern occurs that enables us to go from the specific cases of lifetimes of 3, 4, and 5 periods to the general case of x periods.

4.1 Lifetime is equal to three periods

As described, we first obtain a model for the case in which the lifetime of the product, x, is equal to 3 periods. First, we make a distinction between situations with and without disruptions. In the latter situation the system is always up and delivery of orders is assumed to be within lead time. In the former situation, the system can be both 'up' and 'down', so the delivery of orders is not always assured. The cost function for situations without disruptions is used for obtaining the cost function for situations with disruptions. In the case with disruptions, we furthermore make use of the disruption distribution as used by Schmitt et al., (2010). A graphical representation of this can be found in Appendix 11.2.

A numerical example of the following steps can be found in Appendix 11.3.

4.1.1 Without disruptions

To simplify our explanation with regard to the inventory levels at each age, we assume that $S>2d$.

1. We start off with S products of age 0 in period l, after demand d, we are left with $S\text{-}d$ products of age 1. If $S\geq d$, no backorders are needed otherwise we have $(d\text{-}S)^+$ backorders (However, we assumed that $S>2d$, so in this explanation we do not have backorders, but for completeness we mention them). Furthermore zero products will perish. At the end of this first period we only need to pay inventory cost over $(S\text{-}d)^+$ products. We will order d new products. This is the warm-up period.
2. We receive the order of d units, making our starting inventory equal to S. Since we start with zero products of age 2, no products will perish. We meet demand according to a FIFO policy, resulting in $(d\text{-}S)^+$ backorders and an ending inventory of $S\text{-}d$. We will have d minus the demand that could not be fulfilled by the inventory of age 1, i.e. $(d\text{-}(d\text{-}(S\text{-}d)^+)^+)^+$ products of age 1 left. With our assumption, this equals d. Furthermore we are left with $(S\text{-}2d)^+$ products of age 2. Costs are equal to the cost in the warm-up period. We will order d products.
3. Again our starting inventory is S. If $(S\text{-}2d)^+>d$, $S\text{-}3d$ products will perish, otherwise 0. Again we have $(d\text{-}S)^+$ backorders. Due to perishing we will have $S\text{-}d\text{-}(S\text{-}3d)^+$ of ending inventory over which we have to pay inventory cost. Furthermore we have to pay cost of perishing. At the end of this period we have d products of age 1 and $(d\text{-}(d\text{-}(S\text{-}2d)^+)^+)^+$ products of age 2. So we will order $d+(S\text{-}3d)^+$ new products.
4. After delivery, we start with S products, of which none will perish and $(d\text{-}S)^+$ backorders will be needed. At the end of the period we will have $d+(S\text{-}3d)^+$ products of age 1 and $(d\text{-}(d\text{-}(d\text{-}(3d\text{-}S)^+)^+)^+)^+$ products of age 2. Therefore we will place an order of d again.
5. Equal to period n.
6. Etc

We see that, after the warm up period a pattern occurs, which re-occurs each three periods. Therefore, to find the cost function in case without disruptions, we only need to find the cost function for each of the three different periods, and divide the sum of these by three.

In Table 4.1 we provide another representation of this process.

Period	Starting inventory	Perishing products	Backorders	Ending inventory	Age 1	Age 2	Order
1. Warm up	S	0	$(d-S)^+$	$(S-d)^+$	$(S-d)^+$	0	d
2. n	S	0	$(d-S)^+$	$(S-d)^+$	d	$(S-2d)^+$	d
3. n+1	S	$(S-3d)^+$	$(d-S)^+$	$(S-d-(S-3d)^+)^+$	d	$(d-(3d-S)^+)^+$	$d+(S-3d)^+$
4. n+2	S	0	$(d-S)^+$	$(S-d)^+$	$d+(S-3d)^+$	$(d-(d-(d-(3d-S)^+)^+)^+)^+$	d
5. n+3	S	0	$(d-S)^+$	$(S-d)^+$	d	$(S-2d)^+$	d
6. n+4	S	$(S-3d)^+$	$(d-S)^+$	$(S-d-(S-3d)^+)^+$	d	$(d-(3d-S)^+)^+$	$d+(S-3d)^+$
7. n+5	S	0	$(d-S)^+$	$(S-d)^+$	$d+(S-3d)^+$	$(d-(d-(d-(3d-S)^+)^+)^+)^+$	d

Table 4.1 Ageing and ordering of inventory of products with x=3

For the first period of the pattern (f.e. period 2 in Table 4.1), the inventory over which unit holding cost need to be paid is $(S-d)^+$, the amount of backorders is $(d-S)^+$, and no products perish. For the second period of the pattern, $(S-3d)^+$ products perish and perishing cost need to be paid. Because of this perishing, the inventory over which unit holding cost need to be paid is $(S-d-(S-3d)^+)^+$. The amount that needs to be backordered remains the same compared to the first period of the pattern. Finally the third period of the pattern, is cost-wise equal to the first. The only difference is within the age distribution of the inventory.

Now we can describe the cost function as

$$C=\frac{1}{3}\left(h\left(S-d\right)^{+}+b\left(d-S\right)^{+}\right)+\frac{1}{3}\left(h\left(S-d-\left(S-3d\right)^{+}\right)^{+}+b\left(d-S\right)^{+}+p\left(S-3d\right)^{+}\right)+\frac{1}{3}\left(h\left(S-d\right)^{+}+b\left(d-S\right)^{+}\right)$$

4.1.2 With disruptions

Now we add the possibility of disruptions. We see in the previous section that there are three different situations in which the system can be. We assume that we are equally likely to be in one of the periods (one of these three situations) when a disruption occurs. Furthermore disruptions are long enough to clear out all inventory (by fulfilling demand and perishing). Now we describe the three different situations that occur after the start of a disruption.

Disruption occurs right before new cycle

Assume that a disruption occurs at the end of the warm up period as described in section 0. This disruption causes that no orders can be delivered anymore from the start of the next period until the disruption is over. Here we describe what happens with inventory, perishing and backordering.

1. Since no orders can be delivered anymore, we have a starting inventory of *S-d*. Since we had only *d* products of age 2, no products will perish. The amount of backorders is equal to $(2d\text{-}S)^+$. Ending inventory will be $(S\text{-}2d)^+$. We would like to order *2d*, however, since we assume disruptions take long enough to clear inventory and we might still have some inventory left, next period supply will also be disrupted.
2. We start with an inventory of $(S\text{-}2d)^+$, all of age 2. So every item more than demand will perish at the end of this period, so $(S\text{-}3d)^+$ products will perish. Furthermore we will have $d\text{-}(S\text{-}2d)^+$ backorders. Note that backorders only occur if no products perish and vice versa. At the end of this period, inventory will be zero, so no inventory holding cost need to be paid.
3. Assuming that the disruption still lasts, we have no starting inventory so the only costs that occur in this period are backordering cost for all demand *d*.
4. ..
5. Assume disruption has ended right before this period. Now starting inventory will be *S* again, all of age 0. This will be a warm-up period again, after which the pattern in case of no disruptions will repeat itself, until another disruption occurs.

Period	Starting inventory	Perishing products	Backorders	Ending inventory	Age 1	Age 2	Order
0. Warm up	S	0	$(d\text{-}S)^+$	$(S\text{-}d)^+$	$(S\text{-}d)^+$	0	d
1. n	$(S\text{-}d)^+$	0	$(2d\text{-}S)^+$	$(S\text{-}2d)^+$	0	$(S\text{-}2d)^+$	2d
2. n+1	$(S\text{-}2d)^+$	$(S\text{-}3d)^+$	$(d\text{-}(S\text{-}2d))^+$	0	0	0	S
3. n+2	0	0	d	0	0	0	S
4. n+3	0	0	d	0	0	0	S
5. n+4 (Warm-up)	S	0	$(d\text{-}S)^+$	$(S\text{-}d)^+$	$(S\text{-}d)^+$	0	d

Table 4.2. Ageing and ordering of inventory of products with x=3 with disruption right before period n

Now we use the probabilities π_i to represent the probability of being at the end of the *i*th consecutive disrupted period, as Schmitt et al., (2010) do in their paper.

As before, we can now determine the periodical cost, based on the values in Table 4.2:

$$C=\pi_0\left(h(S-d)^+ + b(d-S)^+\right)+\pi_1\left(h(S-2d)^+ + b\left(d-(S-d)^+\right)^+\right)+\pi_2\left(b\left(d-(S-2d)^+\right)^+ + p(S-3d)^+\right)+\sum_{i=3}^{\infty}\pi_i bd$$

Recall that $\pi_1=\pi_2$ to fulfill the assumption of an empty inventory before the disruption ends.

Disruption occurs right before second period of cycle

Now we assume the disruption occurs at the end of the first period of a new cycle. In other words at the end of step 2 in Section 4.1.1. So the *d* products that are ordered at the end of this period will not be delivered at the beginning of the next period.

1. We start off this period with *S-d* inventory. The products with age 2 will perish at the end of this period if they are not used to fulfill demand. So the amount that perishes is $(S\text{-}3d)^+$. We will have $(2d\text{-}S)^+$ backorders and the ending inventory will be equal to $(S\text{-}2d\text{-}(S\text{-}3d)^+)^+$.

2. The system is still disrupted and the starting inventory has decreased to $(S\text{-}2d\text{-}(S\text{-}3d)^+)^+$. We might have backorders of $(d\text{-}(S\text{-}2d\text{-}(S\text{-}3d)^+)^+)^+$
3. If the disruption still holds on, only backordering cost occur until the system is up again.

Period	Starting inventory	Perishing products	Backorders	Ending inventory	Age 1	Age 2	Order
0. Warm up	S	0	$(d\text{-}S)^+$	$(S\text{-}d)^+$	$(S\text{-}d)^+$	0	d
0. n	S	0	$(d\text{-}S)^+$	$(S\text{-}d)^+$	$(d\text{-}(2d\text{-}S)^+)^+$	$(S\text{-}2d)^+$	d
1. n+1	$(S\text{-}d)^+$	$(S\text{-}3d)^+$	$(2d\text{-}S)^+$	$(S\text{-}2d\text{-}(S\text{-}3d)^+)^+$	0	$(S\text{-}2d\text{-}(S\text{-}3d)^+)^+$	$2d+(S\text{-}3d)^+$
2. n+2	$(S\text{-}2d\text{-}(S\text{-}3d)^+)^+$	0	$(d\text{-}(S\text{-}2d\text{-}(S\text{-}3d)^+)^+)^+$	0	0	0	S
3. n+3	0	0	d	0	0	0	S

Table 4.3. Ageing and ordering of inventory of products with n=3 with disruption right before period n+1

In this case the costs can be described as

$$C=\pi_0\left(h(S-d)^+ + b(d-S)^+\right)+\pi_1\left(h\left(S-2d-(S-3d)\right)^+ + b\left(d-(S-d)^+\right)^+ + p(S-3d)^+\right)$$
$$+\pi_2\left(b\left(d-\left(S-2d-(S-3d)^+\right)^+\right)^+\right)+\sum_{i=3}^{\infty}\pi_i bd$$

Disruption occurs right before end of cycle

The last possibility is that the disruption starts right before the end of the cycle, say right after period n+2 (step 3) from section 0. Again we describe shortly what happens.

1. The order of $d+(S\text{-}3d)^+$ products cannot be delivered, so we have a starting inventory of $(S\text{-}d\text{-}(S\text{-}3d)^+)^+$, d of age 1 and $(d\text{-}(3d\text{-}S)^+)^+$ of age 2. If the inventory of age 2 is equal to d, it is used to fulfill demand and inventory of d remains. However, if the inventory of age 2 is equal to 0, no inventory will remain.
2. The disruption lasts and again we might use the remaining d of age 2 to fulfill demand, leaving no backorders and no inventory or we only have backorders.

Period	Starting inventory	Perishing products	Backorders	Ending inventory	Age 1	Age 2	Order
0. Warm up	S	0	$(d\text{-}S)^+$	$(S\text{-}d)^+$	$(S\text{-}d)^+$	0	d
0. n	S	0	$(d\text{-}S)^+$	$(S\text{-}d)^+$	d	$(S\text{-}2d)^+$	d
0. n+1	S	$(S\text{-}3d)^+$	$(d\text{-}S)^+$	$(S\text{-}d\text{-}(S\text{-}3d)^+)^+$	d	$(d\text{-}(3d\text{-}S)^+)^+$	$d+(S\text{-}3d)^+$
1. n+2	$(S\text{-}d\text{-}(S\text{-}3d)^+)^+$	0	$(d\text{-}(S\text{-}d\text{-}(S\text{-}3d)^+)^+)^+$	$(S\text{-}2d\text{-}(S\text{-}3d)^+)^+$	0	$(S\text{-}2d\text{-}(S\text{-}3d)^+)^+$	$2d+(S\text{-}3d)^+$
2.n+3	$(S\text{-}2d\text{-}(S\text{-}3d)^+)^+$	0	$(d\text{-}(S\text{-}2d\text{-}(S\text{-}3d)^+)^+)^+$	0	0	0	S

Table 4.4. Ageing and ordering of inventory of products with x=3 with disruption right before period n+2

$$C=\pi_0\left(h\left(S-d-(S-3d)^+\right)^+ + b(d-S)^+ + p(S-3d)^+\right)$$
$$+\pi_1\left(h\left(\left(S-2d-(S-3d)^+\right)\right)^+ + b\left(d-\left(S-d-(S-3d)^+\right)^+\right)^+\right)+\pi_2 b\left(d-\left(\left(S-2d-(S-3d)^+\right)^+\right)\right)+\sum_{i=3}^{\infty}\pi_i bd$$

4.1.3 Combination of all disruption possibilities

Now we can combine all three situations and add similar terms to find the general cost function for products with a fixed lifetime of 3 in case disruptions are taken into account. Since the three different situations that occur after a disruption are equally likely, each separate cost function in case of disruption should be multiplied by one third to determine the expected cost function.

$$C=\frac{1}{3}\left(\begin{array}{l}\pi_0\left(h\left(2(S-d)^+ +\left(S-d-(S-3d)^+\right)^+\right)+3b(d-S)^+ +p(S-3d)^+\right)\\ +\pi_1\left(h\left((S-2d)^+ +2\left(S-2d-(S-3d)^+\right)^+\right)+b\left(2\left(d-(S-d)^+\right)^+ +\left(d-\left(S-d-(S-3d)^+\right)^+\right)^+\right)+p(S-3d)^+\right)\\ +\pi_2\left(b\left(2\left(d-\left(S-2d-(S-3d)^+\right)^+\right)^+ +\left(d-(S-2d)^+\right)\right)+p(S-3d)^+\right)\end{array}\right)+\sum_{i=3}^{\infty}\pi_i bd$$

Note that we do not need to take into account the cost function for the case without disruptions separately, because these costs are already incorporated in three separate cost functions for the case with disruptions. We see that the first term (with π_0) is equal to the cost function for the situation without disruptions.

We can use this cost formula to determine the cost given a certain base stock level. The other way around, we can also determine a good base stock level by minimizing this cost function.

4.1.4 Convexity

When taking the derivative of a function, one can find the minimum by setting the derivative to zero and solve. The convexity of a function assures that the found minimum is the global minimum and the associated ‘x’ value (in this case not the lifetime, but the base stock level!) is optimal (in case one wants to minimize).

To show the convexity of the cost function found in Section 4.1.3, we investigate the convexity graphically. We do this by showing the total cost graphs for several different values of *h, p,* and *b*. We set our starting values as follows: *α=0.2, β=0.8, d=2, h=1, p=3* and *b=5*. We start by changing the value for *p*, which results in Figure 4.2. We see that for different values of *p*, convexity is maintained. The same is true for different values of *h* and *b*, as can be seen in Figure 4.3 and Figure 4.4. Also other combinations of parameter values all show convex graphs.

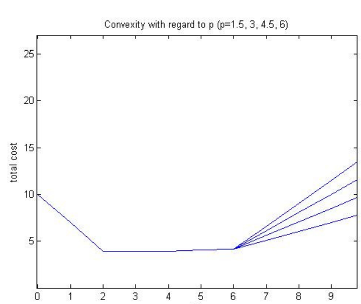

Figure 4.2 Convexity with regard to *p*

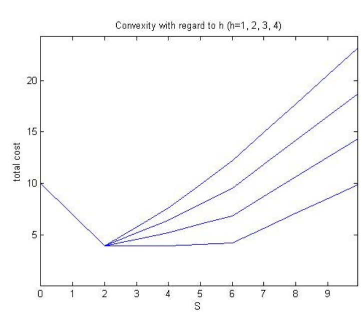

Figure 4.3 Convexity with regard to *h*

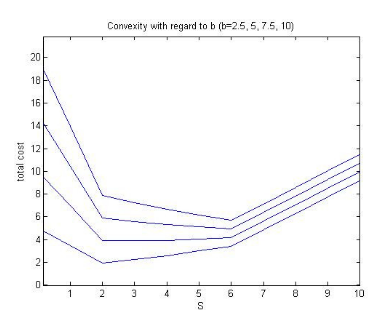

Figure 4.4 Convexity with regard to *b*

4.2 Lifetime is equal to four periods

Now we perform the same steps as for the x=3 case to obtain a cost function for products with a fixed lifetime of 4 periods.

4.2.1 Without disruptions

With a similar reasoning as in the case of $x=3$, we find that in this situation we need two warm-up periods. Furthermore we find a pattern over each four periods. The cost in case of no disruptions can be found by the following formula

$$C = \pi_0 \begin{pmatrix} \frac{1}{4}\left(h(S-d)^+ + b(d-S)^+\right) + \frac{1}{4}\left(h\left(S-d-(S-4d)^+\right)^+ + p(S-4d)^+ + b(d-S)^+\right) \\ + \frac{1}{4}\left(h(S-d)^+ + b(d-S)^+\right) + \frac{1}{4}\left(h(S-d)^+ + b(d-S)^+\right) \end{pmatrix}$$

4.2.2 With disruptions

In the $x=3$ case we had three situations in which a disruption could occur, now we have four. We shortly describe each of these situations and the cost function in case a disruption occurs in this period.

Disruption occurs right before period n

Period	Starting inventory	Perishing products	Backorders	Ending inventory	Age 1	Age 2	Age 3	Order
Warm up	S	0	$(d-S)^+$	$(S-d)^+$	$(S-d)^+$	0	0	d
Warm up	S	0	$(d-S)^+$	$(S-d)^+$	$(d-(2d-S)^+)^+$	$(S-2d)^+$	0	d
N	$(S-d)^+$	0	$(2d-S)^+$	$(S-2d)^+$	0	$(d-(3d-S)^+)^+$	$(S-3d)^+$	2d
n+1	$(S-2d)^+$	$(S-4d)^+$	$(d-(S-2d)^+)^+$	$(S-3d-(S-4d)^+)^+$	0	0	$(d-(4d-S)^+)^+$	$\mathrm{Min}(3d+(S-4d)^+,S)$
n+2	$(S-3d-(S-4d)^+)^+$	0	$(d-((S-3d-(S-4d)^+)^+)^+$	0	0	0	0	S
n+3	0	0	d	0	0	0	0	S

Table 4.5. Ageing and ordering of inventory of products with x=4 with disruption right before period n

As before we can determine the cost function with use of the values in Table 4.5. This results in the following cost function

$$C = \pi_0\left(h(S-d)^+ + b(d-S)^+\right) + \pi_1\left(h(S-2d)^+ + b(2d-S)^+\right)$$
$$+\pi_2\left(h\left(S-3d-(S-4d)^+\right)^+ + p(S-4d)^+ + b\left(d-(S-2d)^+\right)^+\right) + \pi_3 b\left(d-\left(S-3d-(S-4d)^+\right)^+\right)^+ + \sum_{i=4}^{\infty}\pi_i bd$$

Disruption occurs right before period n+1

Period	Starting inventory	Perishing products	Backorders	Ending inventory	Age 1	Age 2	Age 3	Order
N	S	0	$(d-S)^+$	$(S-d)^+$	d	$(d-(3d-S)^+)^+$	$(S-3d)^+$	d
n+1	$(S-d)^+$	$(S-4d)^+$	$(2d-S)^+$	$(S-2d-(S-4d)^+)^+$	0	d	$(d-(4d-S)^+)^+$	$\text{Min}(2d+(S-4d)^+,S)$
n+2	$(S-2d-(S-4d)^+)^+$	0	$(d-(S-2d-(S-4d)^+)^+)^+$	$(S-3d-(S-4d)^+)^+$	0	0	$(d-(4d-S)^+)^+$	$\text{Min}(3d+(S-4d)^+,S)$
n+3	$(d-(4d-S)^+)^+$	0	$(d-(S-3d-(S-4d)^+)^+)^+$	0	0	0	0	S
n+4	0	0	D	0	0	0	0	S

Table 4.6. Ageing and ordering of inventory of products with x=4 with disruption right before period n+1

Here we find the following cost function

$$C=\pi_0\left(h\left(S-d\right)^+ + b\left(d-S\right)^+\right)+\pi_1\left(h\left(S-2d-\left(S-4d\right)^+\right)^+ + p\left(S-4d\right)^+ + b\left(d-\left(S-d\right)^+\right)^+\right)$$

$$+\pi_2\left(h\left(S-3d-\left(S-4d\right)^+\right)^+ + b\left(d-\left(S-2d-\left(S-4d\right)^+\right)^+\right)^+\right)+\pi_3 b\left(d-\left(S-3d-\left(S-4d\right)^+\right)^+\right)^+ + \sum_{i=4}^{\infty}\pi_i bd$$

Disruption occurs right before period n+2

Period	Starting inventory	Perishing products	Backorders	Ending inventory	Age 1	Age 2	Age 3	Order
n+1	S	$(S-4d)^+$	$(d-S)^+$	$(S-d-(S-4d)^+)^+$	d	$(d-(3d-S)^+)^+$	$(d-(4d-S)^+)^+$	$d+(S-4d)^+$
n+2	$(S-d-(S-4d)^+)^+$	0	$(d-(S-d-(S-4d)^+)^+)^+$	$(S-2d-(S-4d)^+)^+$	0	$(d-(3d-S)^+)^+$	$(d-(4d-S)^+)^+$	$\text{Min}(2d+(S-4d)^+,S)$
n+3	$(S-2d-(S-4d)^+)^+$	0	$(d-(S-2d-(S-4d)^+)^+)^+$	$(S-3d-(S-4d)^+)^+$	0	0	$(d-(4d-S)^+)^+$	$\text{Min}(3d+(S-4d)^+,S)$
n+4	$(d-(4d-S)^+)^+$	0	$(d-(S-3d-(S-4d)^+)^+)^+$	0	0	0	0	S

Table 4.7. Ageing and ordering of inventory of products with x=4 with disruption right before period n+2

$$C=\pi_0\left(h\left(S-d-\left(S-4d\right)^+\right)^+ + p\left(S-4d\right)^+ + b\left(d-S\right)^+\right)+\pi_1\left(h\left(S-2d-\left(S-4d\right)^+\right)^+ + b\left(d-\left(S-d-\left(S-4d\right)^+\right)^+\right)^+\right)$$

$$+\pi_2\left(h\left(S-3d-\left(S-4d\right)^+\right)^+ + b\left(d-\left(S-2d-\left(S-4d\right)^+\right)^+\right)^+\right)+\pi_3\left(b\left(d-\left(S-3d-\left(S-4d\right)^+\right)^+\right)^+\right)+\sum_{i=4}^{\infty}\pi_i bd$$

Disruption occurs right before period n+3

The last possibility in which a disruption can occur is right before period n+3. Here we show how inventory and backorders change

Period	Starting inventory	Perishing products	Backorders	Ending inventory	Age 1	Age 2	Age 3	Order
n+2	S	0	$(d-S)^+$	$(S-d)^+$	d	$(d-(3d-S)^+)^+$	$(d-(4d-S)^+)^+$	d
n+3	$(S-d)^+$	0	$(2d-S)^+$	$(S-2d)^+$	0	$(d-(3d-S)^+)^+$	$(d-(4d-S)^+)^+$	Min(2d,S)
n+4	$(S-2d)^+$	0	$(d-(S-2d)^+)^+$	$(S-3d)^+$	0	0	$(S-3d)^+$	Min(3d,S)
n+5	$(S-3d)^+$	$(S-4d)^+$	$(d-(S-3d)^+)^+$	0	0	0	0	S
n+6	0	0	d	0	0	0	0	S

Table 4.8. Ageing and ordering of inventory of product with x=4 with disruption right before period n+3

$$
C=\pi_0\left(h(S-d)^+ +b(d-S)^+\right)+\pi_1\left(h(S-2d)^+ +b(2d-S)^+\right)+\pi_2\left(h(S-3d)^+ +b\left(d-(S-2d)^+\right)^+\right)
$$
$$
+\pi_3\left(b\left(d-(S-3d)^+\right)^+ +p(S-4d)^+\right)+\sum_{i=4}^{\infty}\pi_i bd
$$

4.2.3 Combination of all disruption possibilities

As for the case in which the lifetime is equal to 3, we here combine all possibilities in which a disruption can occur to obtain the general cost function for holding perishable inventory in case disruption can occur.

$$
\begin{aligned}
C=&\frac{\pi_0}{4}\Big(\left(h(S-d)^+ +b(d-S)^+\right)+\left(h(S-d)^+ +b(d-S)^+\right)+\left(h\left(S-d-(S-4d)^+\right)+p(S-4d)^+ +b(d-S)^+\right)+\left(h(S-d)^+ +b(d-S)^+\right)\Big)\\
&+\frac{1}{4}\left(\pi_1\left(h(S-2d)^+ +b(2d-S)^+\right)+\pi_2\left(h\left(S-3d-(S-4d)^+\right)^+ +p(S-4d)^+ +b(3d-S)^+\right)+\pi_3 b\left(d-(S-3d)^+\right)^+\right)\\
&+\frac{1}{4}\left(\pi_1\left(h\left(S-2d-(S-4d)^+\right)^+ +p(S-4d)^+ +b\left(d-(S-d)^+\right)^+\right)+\pi_2\left(h\left(S-3d-(S-4d)^+\right)^+ +b\left(d-\left(S-2d-(S-4d)^+\right)^+\right)^+\right)+\pi_3 b\left(d-\left(S-3d-(S-4d)^+\right)^+\right)^+\right)\\
&+\frac{1}{4}\begin{pmatrix}\pi_1\left(h\left(S-2d-(S-4d)^+\right)^+ +b\left(d-\left(S-d-(S-4d)^+\right)^+\right)^+\right)+\pi_2\left(h\left(S-3d-(S-4d)^+\right)^+ +b\left(d-\left(S-2d-(S-4d)^+\right)^+\right)^+\right)\\ +\pi_3 b\left(d-\left(S-3d-(S-4d)^+\right)^+\right)^+\end{pmatrix}\\
&+\frac{1}{4}\left(\pi_1\left(h(S-2d)^+ +b(2d-S)^+\right)+\pi_2\left(h(S-3d)^+ +b\left(d-(S-2d)^+\right)^+\right)+\pi_3\left(b\left(d-(S-3d)^+\right)^+ +p(S-4d)^+\right)\right)+\sum_{i=4}^{\infty}\pi_i bd
\end{aligned}
$$

We can rewrite this by adding all similar terms and we find

$$
C=\frac{1}{4}\begin{pmatrix}\pi_0\left(h\left(3(S-d)^+ +\left(S-d-(S-4d)^+\right)\right)+4b(d-S)^+ +p(S-4d)^+\right)\\
+\pi_1\left(2h\left((S-2d)+\left(S-2d-(S-4d)^+\right)^+\right)+b\left(3\left(d-(S-d)^+\right)^+ +\left(d-\left(S-d-(S-4d)^+\right)^+\right)^+\right)+p(S-4d)^+\right)\\
+\pi_2\left(h\left(3\left(S-3d-(S-4d)^+\right)^+ +(S-3d)^+\right)+2b\left(\left(d-(S-2d)^+\right)^+ +\left(d-\left(S-2d-(S-4d)^+\right)^+\right)^+\right)+p(S-4d)^+\right)\\
+\pi_3\left(b\left(3\left(d-(S-3d)^+\right)^+ +\left(d-\left(S-3d-(S-4d)^+\right)^+\right)^+\right)+p(S-4d)^+\right)\end{pmatrix}+\sum_{i=4}^{\infty}\pi_i bd
$$

4.3 Lifetime is equal to five periods

Now we perform the same steps as for the previous cases to obtain a cost function for products with a fixed lifetime of 5 periods. Here our assumption of *S>2d* changes into *S>3d.* We do this to simplify the inventory levels per age. This does not have any influence on the total cost function.

4.3.1 Without disruptions

We find that in this situation we need three warm-up periods. Furthermore we find a pattern over each five periods. The cost in case of no disruptions can be found by the following formula

$$C=\pi_0\left(\begin{array}{l}\frac{1}{5}\left(h(S-d)^{+}+b(d-S)^{+}\right)+\frac{1}{5}\left(h\left(S-d-(S-5d)^{+}\right)^{+}+p(S-5d)^{+}+b(d-S)^{+}\right)\\+\frac{1}{5}\left(h(S-d)^{+}+b(d-S)^{+}\right)+\frac{1}{5}\left(h(S-d)^{+}+b(d-S)^{+}\right)+\frac{1}{5}\left(h(S-d)^{+}+b(d-S)^{+}\right)\end{array}\right)$$

4.3.2 With disruptions

Now we have five situations in which disruptions can occur. We shortly describe each of these situations and the cost function in case a disruption occurs in this period.

Disruption occurs right before period n

Period	Starting inventory	Perishing products	Backorders	Ending inventory	Age 1	Age 2	Age 3	Age 4
Warm up	S	0	$(d\text{-}S)^+$	$(S\text{-}d)^+$	$(S\text{-}d)^+$	0	0	0
Warm up	S	0	$(d\text{-}S)^+$	$(S\text{-}d)^+$	d	$(S\text{-}2d)^+$	0	0
Warm up	S	0	$(d\text{-}S)^+$	$(S\text{-}d)^+$	d	d	$(S\text{-}3d)^+$	0
n	$(S\text{-}d)^+$	0	$(2d\text{-}S)^+$	$(S\text{-}2d)^+$	0	d	$(d\text{-}(4d\text{-}S)^+)^+$	$(S\text{-}4d)^+$
n+1	$(S\text{-}2d)^+$	$(S\text{-}5d)^+$	$(d\text{-}(S\text{-}2d)^+)^+$	$(S\text{-}3d\text{-}(S\text{-}5d)^+)^+$	0	0	D	$(d\text{-}(4d\text{-}S)^+)^+\text{-}(d\text{-}(S\text{-}4d)^+)^+$
n+2	$(S\text{-}3d\text{-}(S\text{-}5d)^+)^+$	0	$(d\text{-}(S\text{-}3d\text{-}(S\text{-}5d)^+)^+)^+$	$(S\text{-}4d\text{-}(S\text{-}5d)^+)^+$	0	0	0	$(d\text{-}(4d\text{-}S)^+)^+\text{-}(d\text{-}(S\text{-}4d)^+)^+$
n+3	$(S\text{-}4d\text{-}(S\text{-}5d)^+)^+$	0	$(d\text{-}(S\text{-}4d\text{-}(S\text{-}5d)^+)^+)^+$	0	0	0	0	0

Table 4.9. Ageing and ordering of inventory of products with x=5 with disruption right before period n

This results in the following cost function

$$C=\pi_0\left(h(S-d)^{+}+b(d-S)^{+}\right)+\pi_1\left(h(S-2d)^{+}+b(2d-S)^{+}\right)$$
$$+\pi_2\left(h\left(S-3d-(S-5d)^{+}\right)^{+}+p(S-5d)^{+}+b\left(d-(S-2d)^{+}\right)^{+}\right)$$
$$+\pi_3\left(h\left(S-4d-(S-5d)^{+}\right)^{+}+b\left(d-\left(S-3d-(S-5d)^{+}\right)\right)^{+}\right)$$
$$+\pi_4 b\left(d-\left(S-4d-(S-5d)^{+}\right)^{+}\right)^{+}+\sum_{i=5}^{\infty}\pi_i bd$$

Disruption occurs right before period n+1

Period	Starting inventory	Perishing products	Backorders	Ending inventory	Age 1	Age 2	Age 3	Age 4
n	S	0	$(d\text{-}S)^+$	$(S\text{-}d)^+$	d	d	$(d\text{-}(4d\text{-}S)^+)^+$	$(S\text{-}4d)^+$
n+1	$(S\text{-}d)^+$	$(S\text{-}5d)^+$	$(2d\text{-}S)^+$	$(S\text{-}2d\text{-}(S\text{-}5d)^+)^+$	0	d	$(d\text{-}(4d\text{-}S)^+)^+$	$(d\text{-}(5d\text{-}S)^+)^+$
n+2	$(S\text{-}2d\text{-}(S\text{-}5d)^+)^+$	0	$(d\text{-}(S\text{-}2d\text{-}(S\text{-}5d)^+)^+)^+$	$(S\text{-}3d\text{-}(S\text{-}5d)^+)^+$	0	0	$(d\text{-}(4d\text{-}S)^+)^+$	$(d\text{-}(5d\text{-}S)^+)^+$
n+3	$(S\text{-}3d\text{-}(S\text{-}5d)^+)^+$	0	$(d\text{-}(S\text{-}3d\text{-}(S\text{-}5d)^+)^+)^+$	$(d\text{-}(5d\text{-}S)^+)^+$	0	0	0	$(d\text{-}(5d\text{-}S)^+)^+$
n+4	$(S\text{-}4d\text{-}(S\text{-}5d)^+)^+$	0	$(d\text{-}(S\text{-}4d\text{-}(S\text{-}5d)^+)^+)^+$	0	0	0	0	0
n+5	0	0	d	0	0	0	0	0

Table 4.10. Ageing and ordering of inventory of products with x=5 with disruption right before period n+1

Here we find the following cost function

$$C=\pi_0\left(h(S-d)^+ +b(d-S)^+\right)+\pi_1\left(h\left(S-2d-(S-5d)^+\right)^+ +p(S-5d)^+ +b\left(d-(S-d)^+\right)^+\right)$$
$$+\pi_2\left(h\left(S-3d-(S-5d)^+\right)^+ +b\left(d-\left(S-2d-(S-5d)^+\right)^+\right)^+\right)+\pi_3\left(h\left(S-4d-(S-5d)^+\right)^+ +b\left(d-\left(S-3d-(S-5d)^+\right)^+\right)^+\right)$$
$$+\pi_4 b\left(d-\left(S-4d-(S-5d)^+\right)^+\right)^+ +\sum_{i=5}^{\infty}\pi_i bd$$

Disruption occurs right before period n+2

Period	Starting inventory	Perishing products	Backorders	Ending inventory	Age 1	Age 2	Age 3	Age 4
n+1	S	$(S\text{-}5d)^+$	$(d\text{-}S)^+$	$(S\text{-}d\text{-}(S\text{-}5d)^+)^+$	d	d	$(d\text{-}(4d\text{-}S)^+)^+$	$(d\text{-}(5d\text{-}S)^+)^+$
n+2	$(S\text{-}d\text{-}(S\text{-}5d)^+)^+$	0	$(d\text{-}(S\text{-}d\text{-}(S\text{-}5d)^+)^+)^+$	$(S\text{-}2d\text{-}(S\text{-}5d)^+)^+$	0	d	$(d\text{-}(4d\text{-}S)^+)^+$	$(d\text{-}(5d\text{-}S)^+)^+$
n+3	$(S\text{-}2d\text{-}(S\text{-}5d)^+)^+$	0	$(d\text{-}(S\text{-}2d\text{-}(S\text{-}5d)^+)^+)^+$	$(S\text{-}3d\text{-}(S\text{-}5d)^+)^+$	0	0	$(d\text{-}(4d\text{-}S)^+)^+$	$(d\text{-}(5d\text{-}S)^+)^+$
n+4	$(S\text{-}3d\text{-}(S\text{-}5d)^+)^+$	0	$(d\text{-}(S\text{-}3d\text{-}(S\text{-}5d)^+)^+)^+$	$(S\text{-}4d\text{-}(S\text{-}5d)^+)^+$	0	0	0	$(d\text{-}(5d\text{-}S)^+)^+$
n+5	$(S\text{-}4d\text{-}(S\text{-}5d)^+)^+$	0	$(d\text{-}(S\text{-}4d\text{-}(S\text{-}5d)^+)^+)^+$	0	0	0	0	0

Table 4.11. Ageing and ordering of inventory of products with x=5 with disruption right before period n+2

$$C=\pi_0\left(h\left(S-d-(S-5d)^+\right)^+ +p(S-5d)^+ +b(d-S)^+\right)+\pi_1\left(h\left(S-2d-(S-5d)^+\right)^+ +b\left(d-\left(S-d-(S-5d)^+\right)^+\right)^+\right)$$
$$+\pi_2\left(h\left(S-3d-(S-5d)^+\right)^+ +b\left(d-\left(S-2d-(S-5d)^+\right)^+\right)^+\right)+\pi_3\left(h\left(S-4d-(S-5d)^+\right)^+ +b\left(d-\left(S-3d-(S-5d)^+\right)^+\right)^+\right)$$
$$+\pi_4 b\left(d-\left(S-4d-(S-5d)^+\right)^+\right)^+ +\sum_{i=5}^{\infty}\pi_i bd$$

Disruption occurs right before period n+3

The next possibility in which a disruption can occur is right before period n+3. Here we show how inventory and backorders change

Period	Starting inventory	Perishing products	Backorders	Ending inventory	Age 1	Age 2	Age 3	Age 4
n+2	S	0	$(d\text{-}S)^+$	$(S\text{-}d)^+$	d	d	$(d\text{-}(4d\text{-}S)^+)^+$	$(d\text{-}(5d\text{-}S)^+)^+$
n+3	$(S\text{-}d)^+$	0	$(d\text{-}(S\text{-}d)^+)^+$	$(S\text{-}2d)^+$	0	d	$(d\text{-}(4d\text{-}S)^+)^+$	$(d\text{-}(5d\text{-}S)^+)^+$
n+4	$(S\text{-}2d)^+$	0	$(d\text{-}(S\text{-}2d)^+)$	$(S\text{-}3d)^+$	0	0	$(d\text{-}(4d\text{-}S)^+)^+$	$(d\text{-}(5d\text{-}S)^+)^+$
n+5	$(S\text{-}3d)^+$	0	$(d\text{-}(S\text{-}3d)^+)$	$(S\text{-}4d)^+$	0	0	0	$(S\text{-}4d)^+$
n+6	$(S\text{-}4d)^+$	$(S\text{-}5d)^+$	$(d\text{-}(S\text{-}4d)^+)$	0	0	0	0	0

Table 4.12. Ageing and ordering of inventory of product with x=5 with disruption right before period n+3

$$C=\pi_0\left(h\left(S-d\right)^+ + b\left(d-S\right)^+\right)+\pi_1\left(h\left(S-2d\right)^+ + b\left(2d-S\right)^+\right)+\pi_2\left(h\left(S-3d\right)^+ + b\left(d-\left(S-2d\right)^+\right)^+\right)$$
$$+\pi_3\left(h\left(S-4d\right)^+ + b\left(d-\left(S-3d\right)^+\right)^+\right)+\pi_4\left(p\left(S-5d\right)^+ + b\left(d-\left(S-4d\right)^+\right)^+\right)+\sum_{i=5}^{\infty}\pi_i bd$$

Disruption occurs right before period n+4

Period	Starting inventory	Perishing products	Backorders	Ending inventory	Age 1	Age 2	Age 3	Age 4
n+3	S	0	$(d\text{-}S)^+$	$(S\text{-}d)^+$	d	$(d\text{+}(S\text{-}5d)^+)^+$	$(d\text{-}(4d\text{-}S)^+)^+$	$(d\text{-}(5d\text{-}S)^+)^+$
n+4	$(S\text{-}d)^+$	0	$(2d\text{-}S)^+$	$(S\text{-}2d)^+$	0	d	$(d\text{+}(S\text{-}5d)^+\text{-}(4d\text{-}S)^+)^+$	$(d\text{-}(5d\text{-}S)^+)^+$
n+5	$(S\text{-}2d)^+$	0	$(d\text{-}(S\text{-}2d)^+)^+$	$(S\text{-}3d)^+$	0	0	$(d\text{-}(4d\text{-}S)^+)^+$	$(d\text{+}(S\text{-}5d)^+\text{-}(5d\text{-}S)^+)^+$
n+6	$(S\text{-}3d)^+$	$(S\text{-}5d)^+$	$(d\text{-}(S\text{-}3d)^+)^+$	$(S\text{-}4d\text{-}(S\text{-}5d)^+)^+$	0	0	0	$(S\text{-}4d\text{-}(S\text{-}5d)^+)^+$
n+7	$(S\text{-}4d\text{-}(S\text{-}5d)^+)^+$	0	$(d\text{-}(S\text{-}4d\text{-}(S\text{-}5d)^+)^+$	0	0	0	0	0

Table 4.13. Ageing and ordering of inventory of product with x=5 with disruption right before period n+4

$$C=\pi_0\left(h\left(S-d\right)^+ + b\left(d-S\right)^+\right)+\pi_1\left(h\left(S-2d\right)^+ + b\left(2d-S\right)^+\right)+\pi_2\left(h\left(S-3d\right)^+ + b\left(d-\left(S-2d\right)^+\right)^+\right)$$
$$+\pi_3\left(h\left(S-4d-\left(S-5d\right)^+\right)^+ + p\left(S-5d\right)^+ + b\left(d-\left(S-3d\right)^+\right)^+\right)+\pi_4\left(b\left(d-\left(S-4d-\left(S-5d\right)^+\right)^+\right)^+\right)+\sum_{i=5}^{\infty}\pi_i bd$$

4.3.3 Combination of all disruption possibilities

As for the other cases, we combine all possibilities in which a disruption can occur to obtain the general cost function for holding perishable inventory in case disruption can occur.

$$
\begin{aligned}
C = {} & \frac{\pi_0}{5}\Big(\big(h(S-d)^+ + b(d-S)^+\big) + \big(h(S-d-(S-5d)^+) + p(S-5d)^+ + b(d-S)^+\big) + \big(h(S-d)^+ + b(d-S)^+\big) + \big(h(S-d)^+ + b(d-S)^+\big) + \big(h(S-d)^+ + b(d-S)^+\big)\Big) \\
& + \frac{1}{5}\left(\begin{array}{l}
\pi_1\left(h(S-2d)^+ + b(2d-S)^+\right) + \pi_2\left(h\left(S-3d-(S-5d)^+\right)^+ + p(S-5d)^+ + b\left(d-(S-2d)\right)^+\right) \\
+\pi_3\left(h\left(S-4d-(S-5d)^+\right)^+ + b\left(d-\left(S-3d-(S-5d)^+\right)^+\right)^+\right) + \pi_4 b\left(d-\left(S-4d-(S-5d)^+\right)^+\right)^+
\end{array}\right) \\
& + \frac{1}{5}\left(\begin{array}{l}
\pi_1\left(h\left(S-2d-(S-5d)^+\right)^+ + p(S-5d)^+ + b\left(d-(S-d)^+\right)^+\right) + \pi_2\left(h\left(S-3d-(S-5d)^+\right)^+ + b\left(d-\left(S-2d-(S-5d)^+\right)^+\right)^+\right) \\
+\pi_3\left(h\left(S-4d-(S-5d)^+\right)^+ + b\left(d-\left(S-3d-(S-5d)^+\right)^+\right)^+\right) + \pi_4 b\left(d-\left(S-4d-(S-5d)^+\right)^+\right)^+
\end{array}\right) \\
& + \frac{1}{5}\left(\begin{array}{l}
\pi_1\left(h\left(S-2d-(S-5d)^+\right)^+ + b\left(d-\left(S-d-(S-5d)^+\right)^+\right)^+\right) + \pi_2\left(h\left(S-3d-(S-5d)^+\right)^+ + b\left(d-\left(S-2d-(S-5d)^+\right)^+\right)^+\right) \\
+\pi_3\left(h\left(S-4d-(S-5d)^+\right)^+ + b\left(d-\left(S-3d-(S-5d)^+\right)^+\right)^+\right) + \pi_4 b\left(d-\left(S-4d-(S-5d)^+\right)^+\right)^+
\end{array}\right) \\
& + \frac{1}{5}\left(\begin{array}{l}
\pi_1\left(h(S-2d)^+ + b(2d-S)^+\right) + \pi_2\left(h(S-3d)^+ + b\left(d-(S-2d)^+\right)^+\right) \\
+\pi_3\left(h(S-4d)^+ + b\left(d-(S-3d)^+\right)^+\right) + \pi_4\left(p(S-5d)^+ + b\left(d-(S-4d)^+\right)^+\right)
\end{array}\right) \\
& + \frac{1}{5}\left(\begin{array}{l}
\pi_1\left(h(S-2d)^+ + b(2d-S)^+\right) + \pi_2\left(h(S-3d)^+ + b\left(d-(S-2d)^+\right)^+\right) \\
+\pi_3\left(h\left(S-4d-(S-5d)^+\right)^+ + p(S-5d)^+ + b\left(d-(S-3d)^+\right)^+\right) + \pi_4 b\left(d-\left(S-4d-(S-5d)^+\right)^+\right)^+
\end{array}\right) \\
& + \sum_{i=4}^{\infty} \pi_i bd
\end{aligned}
$$

We can rewrite this by adding all similar terms and we find

$$
C = \frac{1}{5}\left(\begin{array}{l}
\pi_0\left(h\left(4(S-d)^+ + \left(S-d-(S-5d)^+\right)^+\right) + 5b(d-S)^+ + p(S-5d)^+\right) \\
\pi_1\left(h\left(3(S-2d)^+ + 2\left(S-2d-(S-5d)^+\right)^+\right) + p(S-5d)^+ + b\left(4\left(d-(S-d)^+\right)^+ + \left(d-\left(S-d-(S-5d)^+\right)^+\right)^+\right)\right) \\
\pi_2\left(h\left(3\left(S-3d-(S-5d)^+\right)^+ + 2(S-3d)^+\right) + p(S-5d)^+ + b\left(3\left(d-(S-2d)^+\right)^+ + 2\left(d-\left(S-2d-(S-5d)^+\right)^+\right)^+\right)\right) \\
\pi_3\left(h\left(4\left(S-4d-(S-5d)^+\right)^+ + h(S-4d)^+\right) + p(S-5d)^+ + b\left(3\left(d-\left(S-3d-(S-5d)^+\right)^+\right)^+ + 2\left(d-(S-3d)^+\right)^+\right)\right) \\
\pi_4\left(p(S-5d)^+ + b\left(4\left(d-\left(S-4d-(S-5d)^+\right)^+\right)^+\right) + \left(d-(S-4d)^+\right)^+\right)
\end{array}\right) + \sum_{i=5}^{\infty} \pi_i bd
$$

4.4 Lifetime is equal to *x* periods

Now we consider the general case in which the fixed lifetime of the product is equal to *x*.

4.4.1 Without disruptions

As we see in the other cases, when the lifetime of the products is *x* periods, we have a cycle of *x* periods after a warm up period of *(x-2)* periods. Because products perish after *x* periods, in one period of each cycle products may perish. For this period the cost consists of perishing, inventory and holding cost;

$$C_i = h\left(S - d - (S - xd)^+\right)^+ + p(S - xd)^+ + b(d - S)^+$$

For all other *(x-1)* periods the cost are the same and equal to

$$C_i = h(S - d)^+ + b(d - S)^+$$

So the total cost in case of no disruptions can be described as

$$C = h\left((x-1)(S-d)^+ + \left(S - d - (S - xd)^+\right)^+\right) + p(S - xd)^+ + xb(d - S)^+ \qquad (1)$$

4.4.2 With disruptions

Since we are equally likely to be in one of the periods when a disruption happens and inventory follows a pattern of *x* periods, there are *x* periods in which a disruption can happen. Each of these situations has a certain cost function. In this section we describe the cost functions when a disruption takes place in certain periods of the pattern/cycle. The reasoning that we follow, is similar to the specific cases of *x=3, 4, 5*.

During a disruption, costs develop in a specific way. For inventory holding cost, the amount over which unit holding cost need to be paid decreases by demand (*d* per period) and by perishing ($(S-xd)^+$, only in 1 period*)*. The timing of the period in which the inventory decreases not only by demand, but also by perishing, changes depending on starting situation (i.e. the situation when a disruption starts). Backordering costs develop in a similar way, although here the cost increase as a disruption lasts. The increase of backordering cost depends on the decrease in inventory in a delayed fashion (one period delay, since for backordering we work with the starting inventory, while for holding cost we work with ending inventory).

So, when the disruption occurs right before the first period of the cycle, we can describe the cost function as

$$
\begin{aligned}
C &= \pi_0\left(h(S-d)^+ + b(d-S)^+\right) + \pi_1\left(h(S-2d)^+ + b\left(d-(S-d)^+\right)^+\right) \\
&+\pi_2\left(h\left(S-3d-(S-xd)^+\right)^+ + p(S-xd)^+ + b\left(d-(S-2d)^+\right)^+\right) \\
&+\sum_{i=3}^{\infty}\pi_i\left(h\left(S-(i+1)d-(S-xd)^+\right)^+ + b\left(d-\left(S-id-(S-xd)^+\right)^+\right)^+\right)
\end{aligned}
\tag{2}
$$

Here we see that when the system is disrupted for two consecutive periods, products perish, resulting in perishing cost and a decrease in inventory. A period later the backordering increase due to the decrease in inventory.

The difference between the cost functions for the several different situations in which a disruption can start is due to the different relative time in which products perish. In our first cost function (2), perishing occurs after being disrupted for 2 consecutive periods, while in the second cost function (3), perishing already takes place after being disrupted for 1 period. Since the timing of perishing influences the inventory and backordering cost, each separate cost function is different, but shows similarities to the other cost functions. So again a pattern occurs, so that we can generate a formula without stating each cost function separately.

So, when the disruption occurs right before the second period of the cycle, period n+1, we find

$$
\begin{aligned}
C &= \pi_0\left(h(S-d)^+ + b(d-S)^+\right) + \pi_1\left(h\left(S-2d-(S-xd)^+\right)^+ + p(S-xd)^+ + b\left(d-(S-d)^+\right)^+\right) \\
&+\sum_{i=2}^{\infty}\pi_i\left(h\left(S-(i+1)d-(S-xd)^+\right)^+ + b\left(d-\left(S-id-(S-xd)^+\right)^+\right)^+\right)
\end{aligned}
\tag{3}
$$

When the disruption occurs right before the $(n+2)^{th}$ period of the cycle, we find

$$
\begin{aligned}
C &= \pi_0\left(h\left(S-d-(S-xd)^+\right)^+ + p(S-xd)^+ + b(d-S)^+\right) \\
&+\sum_{i=1}^{\infty}\pi_i\left(h\left(S-(i+1)d-(S-xd)^+\right)^+ + b\left(d-\left(S-id-(S-xd)^+\right)^+\right)^+\right)
\end{aligned}
\tag{4}
$$

When the disruption occurs right before the $(n+3)^{th}$ period of the cycle we find

$$C=\pi_0\left(h\left(S-d\right)^+ + b\left(d-S\right)^+\right)+\sum_{i=1}^{x-2}\left(\pi_i\left(h\left(S-\left(i+1\right)d\right)^+ + b\left(d-\left(S-id\right)^+\right)^+\right)\right)$$
$$+\pi_{x-1}\left(p\left(S-xd\right)^+ + b\left(d-\left(S-\left(x-1\right)d\right)^+\right)^+\right)+\sum_{i=x}^{\infty}\left(\pi_i bd\right) \tag{5}$$

When the disruption occurs right before the (n+4)[th] period of the cycle we find

$$C=\pi_0\left(h\left(S-d\right)^+ + b\left(d-S\right)^+\right)+\sum_{i=1}^{x-3}\pi_i\left(h\left(S-\left(i+1\right)d\right)^+ + b\left(d-\left(S-id\right)^+\right)^+\right)$$
$$+\pi_{x-2}\left(h\left(S-\left(x-1\right)d-\left(S-xd\right)^+\right)+p\left(S-xd\right)^+ + b\left(d-\left(S-\left(x-2\right)d\right)^+\right)^+\right)+\sum_{i=x-1}^{\infty}\pi_i b\left(d-\left(S-id-\left(S-xd\right)^+\right)^+\right)^+ \tag{6}$$

When the disruption occurs right before the (n+5)[th] period of the cycle we find

$$C=\pi_0\left(h\left(S-d\right)^+ + b\left(d-S\right)^+\right)+\sum_{i=1}^{x-4}\pi_i\left(h\left(S-\left(i+1\right)d\right)^+ + b\left(d-\left(S-id\right)^+\right)^+\right)$$
$$+\pi_{x-3}\left(h\left(S-\left(x-2\right)d-\left(S-xd\right)^+\right)+p\left(S-xd\right)^+ + b\left(d-\left(S-\left(x-3\right)d\right)^+\right)^+\right)+\sum_{i=x-2}^{\infty}\pi_i b\left(d-\left(S-id-\left(S-xd\right)^+\right)^+\right)^+ \tag{7}$$

Here we see that the function (5), (6), and (7) show resemblance; the cost in case the system is up, is equal in all three functions. Furthermore the terms in the first summation (second term of the function) are equal. The difference here is that the sum is taken over a different range. The third term of the function also has similar terms. The only difference is the value that is multiplied with the demand; this goes from *x* (for the holding cost term) and *x-1* (for the backordering cost term) to *x-2* and *x-3,* respectively. The terms in the last summation of these cost functions are also equal (Note that in (5), the term $(S\text{-}id\text{-}(S\text{-}xd)^+)^+$ *is always zero).*

When we recognize the similarities between the last three cost functions, we can combine (5), (6), and (7), m and expand it to the general *x* case (8). Remember that a cycle consists of *x* periods.

$$C_i=\sum_{k=3}^{x-1}\left(\begin{array}{l}\pi_0\left(h\left(S-d\right)^+ + b\left(d-S\right)^+\right)+\sum_{i=1}^{x-(k-1)}\pi_i\left(h\left(S-\left(i+1\right)d\right)^+ + b\left(d-\left(S-id\right)^+\right)^+\right)\\ +\pi_{x-(k-2)}\left(h\left(S-\left(x-\left(k-3\right)\right)d-\left(S-xd\right)^+\right)^+ + p\left(S-xd\right)^+ + b\left(d-\left(S-\left(x-\left(k-2\right)\right)d\right)^+\right)^+\right)\\ +\sum_{i=x-(k-3)}^{\infty}\pi_i b\left(d-\left(S-id-\left(S-xd\right)^+\right)^+\right)^+\end{array}\right) \tag{8}$$

As before we combine the terms (2), (3), (4), and (8) to find a general cost function (9). Since now the number of periods within one cycle is *x* we need to divide each term by *x.*

$$
\begin{aligned}
C &= \frac{1}{x}\left(\pi_0\left(h\left((x-1)(S-d)^+ + \left(S-d-(S-xd)^+\right)^+\right) + p(S-xd)^+ + xb(d-S)^+\right)\right) \\
&+ \frac{1}{x}\left(\begin{aligned} &\pi_1\left(h(S-2d)^+ + b\left(d-(S-d)^+\right)^+\right) + \pi_2\left(h\left(S-3d-(S-xd)^+\right)^+ + p(S-xd)^+ + b\left(d-(S-2d)^+\right)^+\right) \\ &+\sum_{i=3}^{\infty}\pi_i\left(h\left(S-(i+1)d-(S-xd)^+\right)^+ + b\left(d-\left(S-id-(S-xd)^+\right)^+\right)^+\right)\end{aligned}\right) \\
&+ \frac{1}{x}\left(\begin{aligned} &\pi_1\left(h\left(S-2d-(S-xd)^+\right)^+ + p(S-xd)^+ + b\left(d-(S-d)^+\right)^+\right) \\ &+\sum_{i=2}^{\infty}\pi_i\left(h\left(S-(i+1)d-(S-xd)^+\right)^+ + b\left(d-\left(S-id-(S-xd)^+\right)^+\right)^+\right)\end{aligned}\right) \\
&+ \frac{1}{x}\left(\sum_{i=1}^{\infty}\pi_i\left(h\left(S-(i+1)d-(S-xd)^+\right)^+ + b\left(d-\left(S-id-(S-xd)^+\right)\right)\right)\right) \\
&+ \frac{1}{x}\sum_{k=3}^{x-1}\left(\begin{aligned} &\sum_{i=1}^{x-(k-1)}\pi_i\left(h\left(S-(i+1)d\right)^+ + b\left(d-(S-id)^+\right)^+\right) \\ &+\pi_{x-(k-2)}\left(h\left(S-(x-(k-3))d-(S-xd)^+\right)^+ + p(S-xd)^+ + b\left(d-\left(S-(x-(k-2))d\right)^+\right)^+\right) \\ &+\sum_{i=x-(k-3)}^{\infty}\pi_i b\left(d-\left(S-id-(S-xd)^+\right)^+\right)^+\end{aligned}\right)
\end{aligned} \tag{9}
$$

From (9) we can see that the cost term of perishing is multiplied with the probability of being in 0 to *x-1* consecutive periods disrupted ($\pi_0, \pi_1, \ldots, \pi_{x-2}, \pi_{x-1}$), so we can separate this term. Furthermore, we can rewrite the second term (with π_1, π_2, and summation π_3-π_∞) such that it has the same structure as the last term, and can be incorporated in this last term. When we do this, we get the following cost function (10)

$$
\begin{aligned}
C &= \frac{1}{x}\left(\pi_0\left(h\left((x-1)(S-d)^+ + \left(S-d-(S-xd)^+\right)^+\right) + xb(d-S)^+\right)\right) \\
&+ \frac{1}{x}\left(\sum_{i=1}^{\infty}\pi_i\left(h\left(S-(i+1)d-(S-xd)^+\right)^+ + b\left(d-\left(S-id-(S-xd)^+\right)^+\right)^+\right)\right) \\
&+ \frac{1}{x}\left(\begin{aligned} &\pi_1\left(h\left(S-2d-(S-xd)^+\right)^+ + b\left(d-(S-d)^+\right)^+\right) \\ &+\sum_{i=2}^{\infty}\pi_i\left(h\left(S-(i+1)d-(S-xd)^+\right)^+ + b\left(d-\left(S-id-(S-xd)^+\right)^+\right)^+\right)\end{aligned}\right) \\
&+ \frac{1}{x}\sum_{k=3}^{x}\left(\begin{aligned} &\sum_{i=1}^{x-(k-1)}\pi_i\left(h\left(S-(i+1)d\right)^+ + b\left(d-(S-id)^+\right)^+\right) \\ &+\pi_{x-(k-2)}\left(h\left(S-(x-(k-3))d-(S-xd)^+\right)^+ + b\left(d-\left(S-(x-(k-2))d\right)^+\right)^+\right) \\ &+\sum_{i=x-(k-3)}^{\infty}\pi_i\left(h\left(S-(i+1)d-(S-xd)^+\right)^+ + b\left(d-\left(S-id-(S-xd)^+\right)^+\right)^+\right)\end{aligned}\right) \\
&+ \frac{1}{x}\sum_{i=0}^{x-1}\pi_i p(S-xd)^+
\end{aligned} \tag{10}
$$

5. Managerial implications

Now we have developed a general cost function for the inventory related cost with respect to the base stock level, we derive some implications from this model. First we take a look at the shape of the cost function with respect to the base stock level and at the influence of the cost structure of the inventory related cost. Then the influence of the disruption and recovery probabilities is be analyzed. Furthermore we analyze the influence of the lifetime on the total cost function and on the optimal base stock level. Then we study how the fill rate is influenced by perishability and supply disruptions. In the end we recapture all important implications.

5.1 Shape and cost structure

In this section we show how the total cost function is built up from holding, perishing, and backordering cost. When we increase the base stock level, holding costs should increase, while backordering cost should go down. Perishing cost should increase after the base stock level is higher than a certain threshold, and perishing occurs. Adding these cost functions, results in the total cost function, which, as we show, has a piecewise linear shape.

By inserting certain values for the parameters we can obtain a graph, like the one depicted in Figure 5.1. Here *p=6, b=5, h=1, x=3, α=0.2, β=0.1* and *d=2*. The figure shows how the different types of cost depend on the height of the base stock level. For example, we see that the purple line, which depicts holding cost, increases at different rates, a higher rate for *S<d*x* than for *S>d*x*. This can be explained by the reduction of inventory by perishing, which only occurs when *S>d*x*. Furthermore we see a blue line, which depicts perishing cost. These cost only occur when *S>d*x*, and are linear. The red line, backordering cost, has three different slopes. When we hold less inventory than the demand rate, we have fixed backorders each period, no matter what state the system is in. As we increase the base stock level, backordering only occurs in cases of disruption, so the backordering cost decrease. When *S>d*x*, the backordering cost do not depend on *S* anymore, which results in a horizontal line. The green line shows the total of these three costs with respect to S. This line shows a piecewise linear shape. Four intervals can be distinguished, *0<S<d, d<S<2d, 2d<S<3d* and *S>3d*. We indeed see that if we take the derivative of the total cost function for these four intervals, we obtain four different constant slopes, i.e.,

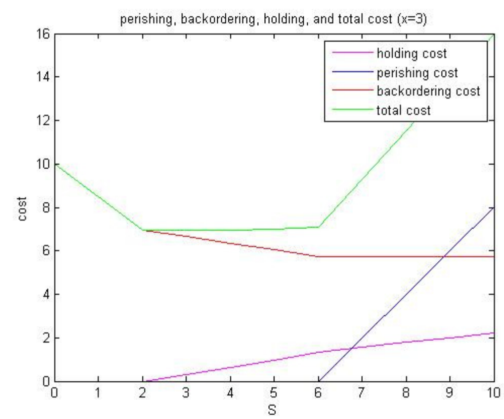

Figure 5.1 Cost functions with respect to *S*

$$\frac{\partial c}{\partial S} = \begin{cases} -\pi_0 b, \text{ for } 0 < S \le d \\ \pi_0 h - \pi_1 b, \text{ for } d < S \le 2d \\ \pi_0 h + \pi_1 h - \pi_2 b, \text{ for } 2d < S \le 3d \\ \frac{2}{3}\pi_0 h + \frac{1}{3}\pi_1 h + \frac{p}{3}, \text{ for } S > 3d \end{cases}$$

For other values of *x*, the number of intervals increases to *x+1*, with slopes as given in the following equation.

$$\frac{\partial c}{\partial S} = \begin{cases} -\pi_0 b, \text{ for } 0 < S \le d \\ \pi_0 h - \pi_1 b, \text{ for } d < S \le 2d \\ \pi_0 h + \pi_1 h - \pi_2 b, \text{ for } 2d < S \le 3d \\ \vdots \\ \left(\sum_{i=0}^{x-2} \pi_i h\right) - \pi_{x-2} b, \text{ for } (x-1)d < S \le xd \\ \left(\sum_{i=0}^{x-2} \frac{x-(i+1)}{x} \pi_i h\right) + \frac{p}{x}, \text{ for } S > xd \end{cases}$$

The total cost function enables us to find relations between parameters. To do so, we vary the value of a certain parameter, while keeping the values of the other parameters equal. This way we can see the effect of increasing/decreasing the value of a certain parameter on the optimal base stock level.

5.1.1 The sensitivity of total cost to unit holding cost

Next we show how the total cost function depends on the height of the unit holding cost. This dependency also influences the optimal base stock level. We expect that for higher unit holding cost, holding inventory is more expensive and therefore the optimal base stock level decreases. For this analysis, we keep *d* equal to two and the unit backordering and unit perishing cost fixed at 5, while we let the unit holding cost differ between 0 and 2. The π values are fixed and can be described by $\alpha=0.2$ and $\beta=0.8$ (same method as in Section 4.1.4 about convexity). This case is depicted in Figure 5.2.

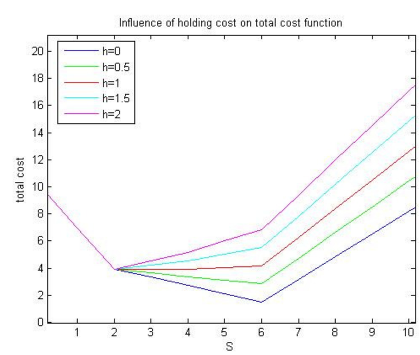

Figure 5.2 Influence of *h* on total cost

We see that for low unit holding cost, the total cost are minimal at $S = 6$ (=demand rate * lifetime). As the unit holding cost increase, the optimal base stock level changes to 2 (=demand rate). This finding is intuitive; when unit holding cost are relatively low, one can better hold some inventory to decrease other cost (backordering), while as unit holding cost increase, having more backorders might outweigh the higher holding cost.

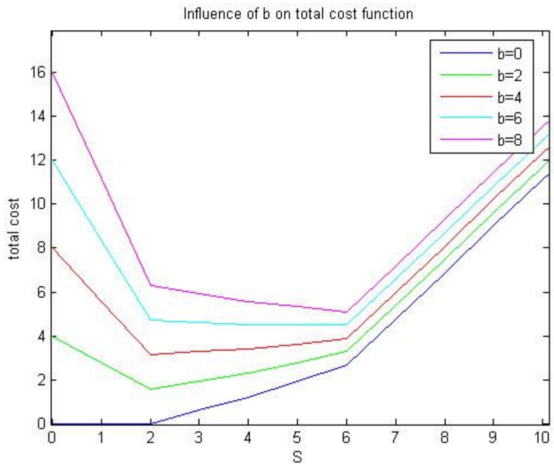

Figure 5.3 Influence of *b* on total cost

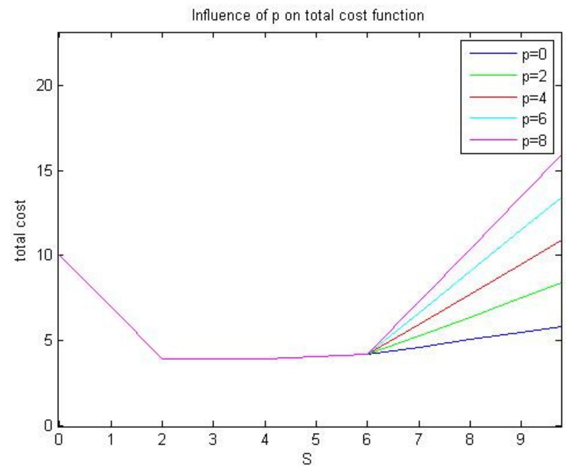

Figure 5.4 Influence of *p* on total cost

5.1.2 The sensitivity of total cost to unit backordering cost

We perform a similar analysis for the unit backordering cost. Here we expect the reverse effect, compared to the unit holding cost; with low unit backordering cost, the total cost will be minimized at a lower optimal base stock level, while high unit backordering cost yield a higher optimal base stock level. Figure 5.3 supports this reasoning. At a certain (high) level of *b,* we can observe that the total cost increase more at low base stock levels, than at high base stock levels. This can be explained by the higher rate of backorders at low base stock levels compared to higher *S*.

5.1.3 The sensitivity of total cost to unit perishing cost

The influence of unit holding and unit backordering cost on the total cost is widely known. The influence of the unit perishing cost, however, is less recognized. Since perishing occurs more often with high base stock levels, high unit perishing cost will result in lower optimal base stock levels. However since demand is deterministic, perishing cost will never occur when the base stock level remains below the demand rate*lifetime. This is also shown in Figure 5.4, where the optimal base stock level stays constant (at lifetime*demand rate) through all levels of unit perishing cost. The figure also shows that the total cost are very inflexible between base stock level 2 and 6, while the costs increase extremely beyond these levels.

An advantage of the inflexibility of the cost function with respect to the unit cost of perishing is that even if the unit cost of perishing is not exactly known, an optimal base stock level can be found without being too far away from the true optimum. This is especially beneficial since the unit costs of perishing are usually more difficult to establish than for example the unit holding cost.

5.2 Disruption distribution

So far our analysis provides some, though known insights in the effect of the cost parameters on the optimal base stock level. The next parameter we would like to analyze is the disruption distribution. Here we analyze the effect of long versus short disruptions (depending on recovery probability β), but also effect of the frequency of disruptions (depending on the disruption probability α). First we focus on α. Assume $h=1$, $b=5$, $p=3$, $d=2$, $\beta=0.5$, $x=3$. We vary the disruption probability between 0.05 and 0.8. The result of this is depicted in Figure 5.5.

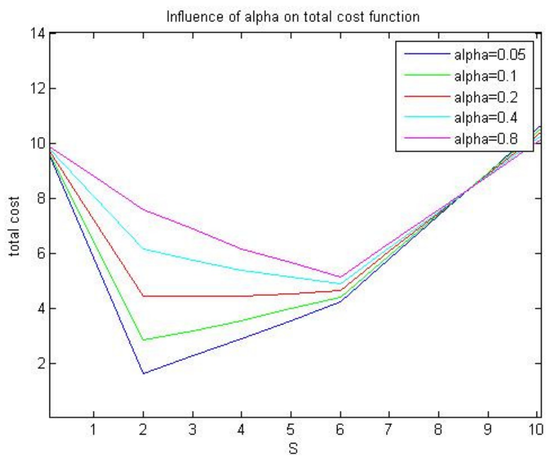

Figure 5.5 Influence of disruption probability on total cost

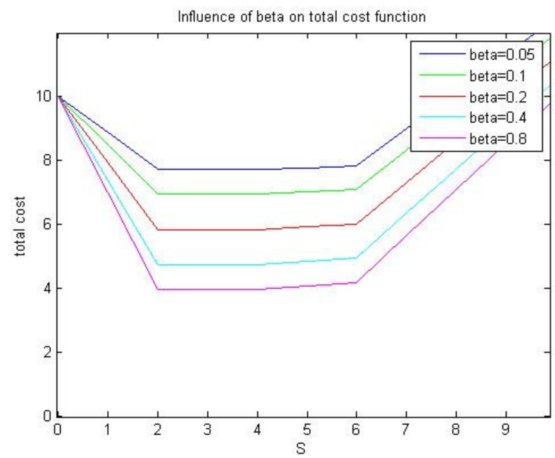

Figure 5.6 Influence of recovery probability on total cost

The higher α, the more frequent disruptions occur. During disruptions a higher base stock level helps to fulfill demand from stock. So it is reasonable that when the disruption frequency is high, the optimal base stock level is higher compared to situations with a lower disruption frequency.

In Figure 5.6 we assume the same value for the parameters as before, except now β varies and α equals 0.2. We see that for lower β, the optimal base stock level is higher. This is because in our hypothetical case disruptions are the only form of uncertainty, lower recovery probabilities imply higher uncertainty, so the optimal base stock level is likely to go up and function as a buffer for this uncertainty. This also reduces the backordering cost. Remarkable is that for high recovery probabilities, the optimal base stock level is two times the demand. This is because with high recovery probabilities, the probability of being only disrupted for one period is fairly high. When the system is disrupted for only one period, having a base stock level of two times the demand can result in the lowest total cost, because there are no backordering costs.

Another observation from these figures is that for α it is much more important to have an idea about the correct value, since a different slightly α can result in a different base stock level with significantly different cost. While for different β the cost difference between several base stock level within the range of 2 and 6 (d and x*d) is rather small. So it is less important to know the exact value of β.

5.3 Influence of perishability

Until now we discussed insights as obtained from the model with a fixed lifetime of *x*=3. Now we vary this lifetime and discuss the influence of the perishability on the total inventory related cost and optimal base stock level. For this analysis we use the model for determining the optimal base stock level in case of perishable items with a fixed lifetime of *x* and the possibility of disruptions as obtained in Section 4.4. First we show that when lifetime goes to infinity, our model is equal to a model which does not take perishability into account. Then we analyze how the minimal costs change over different lifetimes under several different situations. This analysis shows the influence of the lifetime in these different situations. Finally we analyze how the cost function and corresponding optimal base stock level change over different lifetimes.

When we would use a significant high lifetime, this model should result in the same optimal base stock level as for the case without perishability. To prove this, we show the limit of the cost function as *x* goes to infinity.

Theorem: the function obtained for perishability and disruptions converges to the function as obtained by Schmitt et al., 2010, for high values of lifetime and deterministic demand

Proof

$$\lim_{x\to\infty} C = \frac{1}{x}\pi_0\left(xh(S-d)^+ + xb(d-S)^+\right) + \frac{2}{x}\begin{pmatrix} \pi_1\left(h(S-2d)^+ + b\left(d-(S-d)^+\right)^+\right) \\ +\sum_{i=2}^{\infty}\pi_i\left(h(S-(i+1)d)^+ + b\left(d-(S-id)^+\right)^+\right)\end{pmatrix}$$

$$+\frac{x-2}{x}\begin{pmatrix} \sum_{i=1}^{x-(k-1)}\pi_i\left(h(S-(i+1)d)^+ + b\left(d-(S-id)^+\right)^+\right) \\ +\pi_{x-(k-2)}\left(h\left(S-(x-(k-3))d\right)^+ + b\left(d-\left(S-(x-(k-2))d\right)^+\right)^+\right) \\ \sum_{i=x-(k-3)}^{\infty}\pi_i\left(h(S-(i+1)d)^+ + b\left(d-(S-id)^+\right)^+\right)\end{pmatrix}$$

$$\lim_{x\to\infty} C = \sum_{i=0}^{\infty}\pi_i\left(h(S-(i+1)d)^+ + b\left(d-(S-id)^+\right)^+\right)$$

This is indeed equal to the cost function as obtained by Schmitt et al., (2010) when adapted for the deterministic demand case.

This theorem shows that under certain values of α and β, after a certain lifetime the optimal base stock level converges. A graphical representation of this can be found in Figure 5.7. The figure shows three different graphs for different levels of α and β (*h=1, b=5, p=3*). We see that under a low disruption probability (scarce disruptions) the optimal base stock level converges sooner (i.e. for shorter lifetimes) than for frequent disruptions.

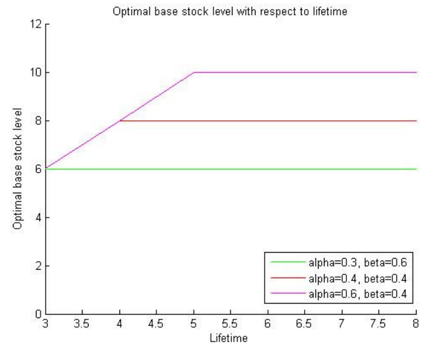

Figure 5.7 Optimal S under different lifetimes and disruption and recovery probabilities

Figure 5.8 depicts the total inventory related cost (instead of optimal base stock level) as a function of the lifetime for several α and β values. It shows that for scarce and short disruptions, the cost change (slope) is more than for long and frequent disruptions. Furthermore we see that depending on the disruption and recovery probabilities, the slope is different for short and long lifetimes. This result implies that depending on the cost structure of inventory related cost and on the height of the disruption and recovery probability, and the lifetime, investments to increase the lifetime can be beneficial, until the conversion lifetime is reached. The obtained model could be used as a basis of analyzing a decision whether or not to invest in research that could increase the lifetime.

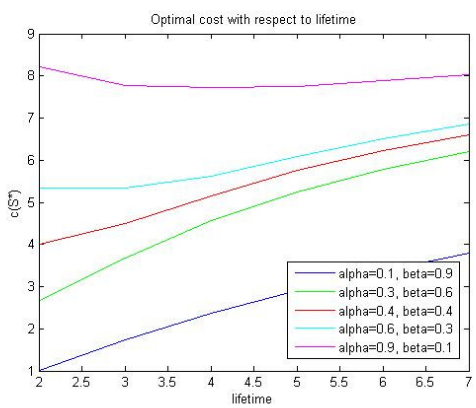

Figure 5.8 Optimal cost with respect to x for different α and β

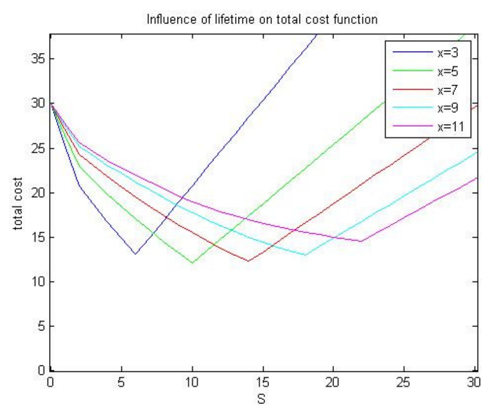

Figure 5.9 Cost functions for different lifetimes

In Figure 5.9 we see several cost functions with respect to S, for different lifetimes. In this figure $\alpha=0.5$ $\beta=0.4$, $d=2$, $b=15$, $h=1$, $p=5$. Again we see that perishability influences the optimal base stock level, but it also influences the shape of the total cost function. For products with a short lifetime the cost function is steeper around the optimal base stock level. So for these items the choice of the base stock level has a bigger influence on the total inventory related cost than for items with a longer lifetime. This emphasizes the importance of this research and the use of an appropriate cost function.

5.4 Fill rate

Often the performance of inventory policies is evaluated by certain service levels. One of these service levels is called the fill rate and represents the relative amount of demand fulfilled directly from stock on hand. In this section we analyze the fill rate in a situation with perishability and supply disruptions. We can determine the fill rate by

$$P_2 = \left(1 - \frac{E[B]}{d}\right) * 100$$

From the cost function as obtained in this study, we can find the value for *E[B]*.

$$E[B] = \pi_0 (d - S)^+ + \sum_{i=1}^{x-1} \pi_i \left(\frac{x-i}{x}\left(d - (S - id)^+\right)^+ + \frac{i}{x}\left(d - \left(S - id - (S - xd)^+\right)^+\right)^+ \right) + d\sum_{i=x}^{\infty} \pi_i$$

Inserting this in the formula for P_2 gives the formula for the fill rate.

Now we evaluate the effect perishability and disruptions have on this fill rate. When the lifetime goes to infinity, the P_2 formula is

$$\lim_{x \to \infty} P_2 = \left(1 - \frac{\sum_{i=0}^{\infty} \pi_i \left(d - (S - id)^+\right)^+}{d}\right) * 100$$

Since we focus in this study on products with a limited lifetime, we would also like to know the maximum fill rate for a product with a certain limited fill rate. We find this by taking the limit for S goes to infinity of the fill rate function. (A derivation can be found in Appendix11.5)

$$\lim_{S \to \infty} P_2 = \left(1 - \sum_{i=x}^{\infty} \pi_i\right) * 100$$

This formula shows that the maximum fill rate only depends on the disruption distribution and (slightly) on the lifetime of the products. The dependency of the lifetime decreases as the lifetime increases, since the difference between consecutive π-values decreases at higher levels of *i*.

When there are no disruptions, the expected backorders do not depend on the lifetime of the product, under our assumption of among others, deterministic demand. Therefore perishability does not influence the fill rate in absence of disruptions. However, when we assume a certain value for α which is greater than 0, perishability starts having influence on the fill rate. In Figure 5.10 we assume $\alpha=0.2$ and $\beta=0.5$. Here

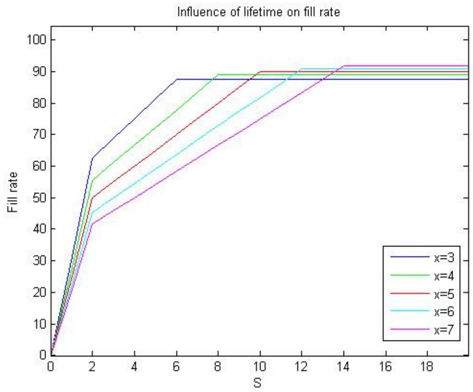

Figure 5.10 Influence of lifetime on fill rate

we plot the fill rate with respect to the base stock level for several lifetimes. We see that at a certain base stock level the fill rate converges. However, when we look at the fill rate at the optimal base stock levels (*S**=2, 2, 3, 4, 4, for *x*=3, 4, 5, 6, and 7 respectively), the fill rates are fairly low: 62.5, 55.6, 55.0, 54.6, 50 for *x*=3, …, 7 respectively. So we can imagine that in case one incorporates a preferred fill rate into the optimal base stock model, the optimal base stock level is higher than without the fill rate constraint. One can also choose to adopt the base stock level later on if necessary to achieve a certain fill rate.

Next we take a look at the influence of disruptions on the fill rate. As before we first evaluate the influence of the disruption probability under a fixed recovery probability (say 0.3) (Figure 5.11), after which we investigate the opposite case; a changing recovery probability under a fixed disruption probability (say 0.3) (Figure 5.12). For both cases we assume a lifetime of 3. In both graphs, we depict the fill rate with respect to an arbitrary value of *S*. When we determine the optimal value for the base stock level we find for, Figure 5.11 that the optimal *S** is 2, 2, 6, 6, 6 for α is 0.1, 0.2, 0.3, 0.4, and 0.5 respectively and in Figure 5.12 the optimal base stock level in each situation is equal to 6. Note that in both figures at a base stock level of 6 (=*3d*), the maximum fill rate is already reached. Also note that for the two cases in which the optimal base stock level is not 6, the fill rate relatively close to the maximum fill rate (especially in comparison to the other differences between the fill rate at *S=2* and *S=6*).

Perishability and disruptions can result in a limited converged fill rate. When limited, efforts of increasing the fill rate by holding more inventory are purposeless. One has to take this in mind when optimizing the base stock level based on the fill rate service level instead of minimizing cost.

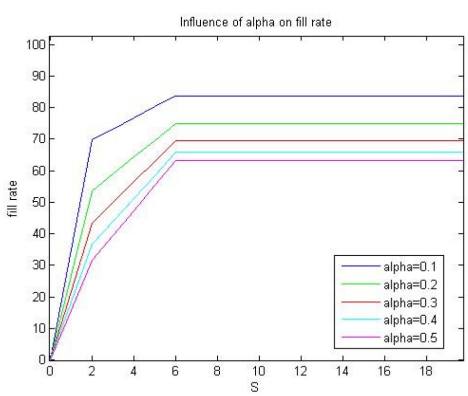

Figure 5.11 Influence of alpha on fill rate

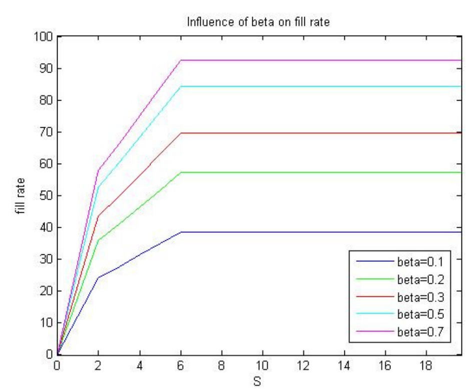

Figure 5.12 Influence of beta on fill rate

5.5 Managerial implications in short

The effects of the several cost parameters (unit holding, unit backordering, and unit perishing cost) are straightforward and already known. Something that is less known, is that the precise value of the unit perishing cost is not very important when determining the optimal base stock level. An estimate might be the purchasing price of the product. The perishability itself is quite important for the (optimal) base stock level and corresponding cost. Till a certain extent, the limited lifetime constraints the optimal base stock level. As lifetime increases, the optimal base stock level increases as well until a certain threshold is reached and the optimal base stock level converges. This finding can help in determining whether investments to increase the lifetime of a product are beneficial. Also, shorter lifetimes result in steeper cost functions, which implies that for products with short lifetimes the choice of the optimal base stock level is more important than for products with longer (or unlimited) lifetimes.

Also the effects of the disruption distribution characteristics (disruption and recovery probability) are straightforward; long and frequent disruptions lead to *higher* optimal base stock levels, while short and scarce disruptions lead to *lower* optimal base stock levels. Again, an important implication is that information regarding the precise value of the disruption probability is more important than information regarding the precise value of the recovery probability. This can play a role when determining whether to invest in information about the disruption characteristics. The disruption characteristics are also an important factor in the maximum fill rate; frequent and long disruptions lead to a lower maximum fill rate than scarce and short disruptions.

So far, the model as obtained in this thesis (1) proves that taking into account perishability for systems subject to disruptions is important, (2) provides insights about the effects of all parameters on the optimal base stock level and corresponding cost, it (3) shows that the accuracy of some parameters is more important than that of others, it (4) helps to determine whether investing in (a) information, (b) increased lifetimes, (c) lower disruption probability, or (d) higher recovery probability is beneficial.

Next we test whether the obtained model also works when certain assumptions are violated.

6. Sensitivity Analysis

In order to obtain a simple cost function, we made some strong assumptions. One of which is deterministic demand. Although this assumption is a good starting point, in real life situations there might be some variance in the demand. Therefore we would like to analyze how our model performs in a stochastic environment. To do so, we compare the optimal base stock level and cost in the stochastic case with the base stock level and cost in the deterministic demand case. In this latter case, demand is equal to 20 and the optimal S^* and $c(S^*)$ can be found by use of the formula as obtained in this study. For the stochastic case, we assume a demand which is normally distributed with mean 20 and a varying sigma. In both cases the fixed lifetime is 3. We generate a random demand for 1000 periods and determine by cost minimization the optimal base stock level for this instance. We do this 100 times and take the mean of each of these optimal base stock levels and corresponding cost.

In Table 6.1 the upper rows show the values of the parameters, the fourth row (S* model) shows the optimal base stock level and corresponding cost in the deterministic demand case respectively and the rows under row 4 show the optimal base stock level, corresponding cost and the percentage of cost difference with respect to the deterministic case for the stochastic demand case.

	h=1, p=3, b=5		h=1, p=3, b=15	h=1, p=10, b=3
α	0.2	0.3	0.6	0.4
β	0.5	0.5	0.2	0.3
S* model	20, 37.5	60,40.00	60, 167.66	40, 36.58
σ=1	24, 37.3 (0.45%)	56, 40.4 (0.95%)	62, 170.3 (1.55%)	40, 36.6 (0.06%)
σ=2	24, 37.4 (0.27%)	54, 40.7 (1.80%)	62, 171.0 (2.02%)	38, 36.6 (0.03%)
σ=3	26, 37.2 (0.85%)	54, 41.2 (2.98%)	62, 171.3 (2.15%)	38, 36.8 (0.66%)
σ=6	34, 37.5 (0.11%)	46, 42.6 (6.60%)	66, 178.1 (6.20%)	34, 37.1 (1.48%)

Table 6.1 Sensitivity analysis

In the first two columns with results, of Table 6.1 we can see how in this case *(h=1, p=3, b=5, α=0.2/0.3, and β=0.5)* the optimal base stock level in a stochastic situation is different from the optimal base stock level as obtained from the model which assumes deterministic demand. In the left hand column, the deterministic demand case has an optimal base stock level at 20, with cost of 37.5, while the right hand column has an optimal base stock level at $x*d$=3*20 with corresponding cost of 40. Although we see that for the stochastic case there is a slight difference of the heights of the optimal base stock levels, they are still quite similar to the deterministic case. This is especially true for situations in which the variance is relative small. It would suffice to adapt the optimal base stock level as obtained from the model, by adding a small safety stock to build in some safety regarding demand uncertainty. As variance grows bigger, the difference between the optimal base stock levels increases. But, more importantly, we found that in these cases the choice for the optimal base stock level is not very strict; for a large range of base stock levels the cost are more or less equal, so choosing a non-optimal base stock level from this range, will not lead to much higher costs when compared to the cost for the optimal base stock level. So when we focus on the difference between optimal cost for the deterministic and stochastic case we find that the differences mostly fall within 5%.

For the third column of the table, we simulated a situation in which, according to the deterministic demand model, the optimal base stock level should be equal to the lifetime times the demand. Again we see that in the stochastic demand case the optimal base stock levels are almost equal to the one in the deterministic case. Also in this case more variance leads to a higher optimal base stock level. But again, the percentage deviation between the optimal cost in the stochastic case and the deterministic case are almost all within 5%.

In the situation depicted in the fourth column of the table, the costs of perishing per item are relatively high. According to our model, the optimal base stock level in this case is 40. In the stochastic case we see here that more variance, leads to a lower optimal base stock level than the one as obtained in the deterministic demand case. This can be explained by the relative high perishing cost per item; when the variance of demand is relatively small, it is beneficial to keep some more inventory in case of disruptions, while not having to pay more perishing cost, since due to the low variability in demand, products will rarely perish. However, when the fluctuations in demand are relatively high, there is a higher probability of perishing at this higher base stock level. Therefore the height of the optimal base stock level decreases as variability increases. Next to this we found again that there is a broad range of base stock levels which result in more or less the same inventory related cost and the percentage cost deviation is in all cases within 2%.

All in all, this analysis shows that the model with the assumption of deterministic demand also performs quite well in situations with stochastic demand. As is normal in cases with stochasticity, one should add a certain safety stock to the optimal base stock level from the deterministic demand model to find the optimal base stock level in the stochastic situation. The height of this safety stock is dependent on the level of variability and on the optimal base stock determining mechanism. When this safety stock is added the inventory related costs are almost equal to the case with deterministic demand.

7. Time dependency of the disruption distribution

As we saw in the sensitivity analysis, the model as obtained also functions rather well in more real life situations with stochastic demand. Another assumption we made to obtain a simple cost function is that the probabilities of disruption and recovery are time independent. So no matter the state of the 'world', these probabilities are always the same. We can imagine that in real life, these probabilities are dependent on the state of the 'world'. For example, when we know that in the coming period farmers will have a short term strike; the disruption probability might increase, but due to the short period of the expected disruption, the recovery probability might be higher. To make our model more realistic, we would like to extend our current model so that it takes this into account. To do this we make use of the idea of Markov modulated processes (MMP). In MMP a Markov process determines in which state the world is, and dependent on this state of the world a certain process occurs (Yechiali and Naor, 1971). We adapt this idea of MMP in such a way that not a Markov process determines the state in which the world is, but an arbitrarily chosen probability distribution. Assume that the 'world' has two states, state 0 and state 1. Disruptions characteristics in state 0 are different from the ones in state 1. With a certain probability pr_0 one is in state 0 and with probability pr_1 the state is 1. The sum of these two probabilities is 1. Of course one can extend this to a world in which more states are possible, but for our analysis two suffices.

The total cost function changes to

$$C = pr_0 C\left(state = 0\right) + pr_1 C\left(state = 1\right)$$

In this function the only difference between the cost in state 0 and state 1 is the disruption distribution. With this 'two state world-model' we do not only want to show a more realistic model, we can also analyze more precise the influence of the length of a disruption on the optimal base stock level. To do so, we fixed the length of disruptions in each state at a certain value. When for example in a certain state the length of a disruption is six periods, the transition probability matrix is given as follows

$$A_{state} = \begin{matrix} \alpha & 1-\alpha & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}$$

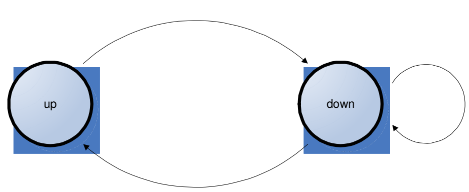

Figure 7.1 Transition diagram

Another way of depicting this can be found in Figure 7.1, in which the probability from up to down is the disruption probability, the loop at the down state represents the fixed number of periods a disruption lasts, and the probability from down to up is 1 divided by the length of the disruption (*6* in this case).

We analyze this model to find more precise insights regarding the length of disruptions. For this analysis we distinguish between three situations: 1) in both states disruptions last longer than the lifetime of the product minus one (assumption set earlier to empty all inventory), 2) in state 1 the disruption is longer than *x-1* and in state 0 the disruption lasts *x-2* periods, which is shorter than *x-1*, 3) in both states disruptions last *x-2* periods.

7.1 Situation 1: Disruptions in both states last longer than *x-1*

When both disruptions last longer than *x-1*, the cost function is similar to the one obtained before, except now we know in each case exactly how long the disruption lasts and with that we have a different disruption distribution.

$$C=\frac{1}{x}\sum_{t=0,1} pr_t \left(\begin{array}{l} xp\left(S-xd\right)^+ + \pi_{0,t}\left(h\left(\left(x-1\right)\left(S-d\right)^+ + \left(S-d-\left(S-xd\right)^+\right)^+ \right) + xb\left(d-S\right)^+ \right) \\ +\pi_{1,t}\left(\begin{array}{l} h\sum_{i=1}^{x-2}\left(\left(x-1-i\right)\left(S-\left(i+1\right)d\right)^+ + \left(i+1\right)\left(S-\left(i+1\right)d-\left(S-xd\right)^+\right)^+ \right) \\ +b\sum_{i=1}^{x-1}\left(\left(x-i\right)\left(d-\left(S-id\right)^+\right)^+ + i\left(d-\left(S-id-\left(S-xd\right)^+\right)^+\right)^+ \right) \end{array} \right) \end{array} \right)$$

Note that we only use π_0 and π_1, this is because $\pi_1=\pi_2=\ldots=\pi_t$, since the length of disruptions are fixed and therefore long term probability of being *z* consecutive periods disrupted is equal to being *z+1* consecutive periods disrupted (as long as *z+1* is smaller than the length of a disruption).

When we assume $\alpha=0.5$, $h=2$, $b=5$, $p=3$, $d=2$, $x=3$, $t_0=12$, $t_1=3$, in which *t* stands for the duration of a disruption in a certain state. We find optimal base stock levels and corresponding cost for several arbitrarily chosen long term probabilities of being in either state 0 or state 1, as depicted in Table 7.1. This table shows that for each probability distribution chosen (pr_0 and pr_1) the optimal base stock level is equal to 4.

pr_0	0.9	0.7	0.5	0.3	0.1
pr_1	0.1	0.3	0.5	0.7	0.9
S*	4	4	4	4	4
c(S*)	1.52	1.98	2.44	2.91	3.37

Table 7.1 Analysis of influence of length on S*

7.2 Situation 2: Disruptions in state 0 last *x-2* and in state 1 longer than *x-1*

To obtain the model from Section 4 we assumed that disruptions take as long as needed to clear out all inventory. By that assumption the starting situation after a disruption is standardized. Now we do not need this assumption anymore, since we exactly know how long disruptions take, and thus what the starting situation is and what kind of cost terms occur in each situation in which a disruption can occur. When we assume that in state 0 the disruption lasts *x-2* periods, we can determine the cost function as follows.

$$
\begin{aligned}
C = \frac{pr_0}{x} \Bigg(& \pi_{0,0}\Big(h\Big((x-1)(S-d)^+ + \big(S-d-(S-xd)^+\big)^+ \Big) + xb(d-S)^+ + xp(S-xd)^+ \Big) \\
& + \pi_{1,0} \left(\begin{array}{l} h\left(\sum_{i=1}^{x-2} (x-(i+1))(S-(i+1)d)^+ + (i+1)\big(S-(i+1)d-(S-xd)^+\big)^+ \right) \\ + b\left(\sum_{i=1}^{x-2} (x-i)\big(d-(S-id)^+\big)^+ + i\Big(d-\big(S-id-(S-xd)^+\big)^+\Big)^+ \right) \\ + p(x-2)(S-xd)^+ \end{array} \right) \Bigg) \\
+ \frac{pr_1}{x} \Bigg(& \pi_{0,1}\Big(h\Big((x-1)(S-d)^+ + \big(S-d-(S-xd)^+\big)^+ \Big) + xb(d-S)^+ \Big) \\
& + \pi_{1,1} \left(\begin{array}{l} h\sum_{i=1}^{x-2} \Big((x-(i+1))(S-(i+1)d)^+ + (i+1)\big(S-(i+1)d-(S-xd)^+\big)^+ \Big) \\ + b\sum_{i=1}^{x-1} \Big((x-i)\big(d-(S-id)^+\big)^+ + i\Big(d-\big(S-id-(S-xd)^+\big)^+\Big)^+ \Big) \end{array} \right) \\
& + xp(S-xd)^+ \Bigg)
\end{aligned}
$$

The cost function consists of two parts; the first depicts the cost in state 0, and the second of state 1 (similar to the cost function in Section 4.4.) Note that the cost functions for state 0 and 1, apart from a different disruption distribution, only differ in the cost of perishing; in state 0 these cost depend on the disruption distribution, while in state 1 this is not the case. Because this difference is significantly small, there will be no difference between the optimal base stock levels for different values of pr_0 and pr_1. This can be seen in Table 7.2 where, $\alpha=0.5$, $h=3$, $b=15$, $p=7$, $d=2$, $x=3$, $t_0=1$, $t_1=12$.

pr_0	1	0.9	0.7	0.5	0.3	0.1	0
pr_1	0	0.1	0.3	0.5	0.7	0.9	1
S*	6	6	6	6	6	6	6
c(S*)	12	9.21	7.64	6.07	4.50	2.93	2.57

Table 7.2 Analysis of influence of length on S*

7.3 Situation 3: Disruptions in both states last *x-2* periods

Now, we can easily find the influence of the length of disruptions on the optimal base stock level when the disruptions last $x-2$ periods. We just set pr_0 equal to 1 and thus pr_1 will be 0 (Table 7.2)

Summarizing we can say that for these cases the length of a disruption does not influence the optimal base stock level, but it does influence the associated cost. The direction of this influence depends on the values for the parameters.

7.4 Higher unit backordering cost for the first backorder

We can imagine that there are cases in which there is a difference between optimal base stock levels of situations with different probability distributions (more/less disruptions of a certain kind). This might be the case when for example the cost of a first backorder is more expensive than the other backorders. We also can imagine that this is a situation which can occur in real life. For example, when the losing some customers as a result to not be able to fulfill demand from stock on hand, results in the loss of other customers by loss of goodwill. A very extreme example of this is when a person needs transplantation blood every period. When this person cannot receive blood for one period, he might not survive. The first backorder would then lead to the disappearance of future deliveries, which would make this first backorder more costly.

To take the possibility of preventing any backorders into account, we assume that disruptions might last not long enough to clear out all inventory. This way, there is the possibility to still have inventory left after the disruption is ended and the system is up again. When this is the case it might be beneficial to keep some more inventory so that the number of backorders can be reduced to zero and therefore not have the relative high cost for the first backorder. Assume $\alpha=0.1$, $h=3$, $b=5$, $b1=100$, $p=7$, $d=2$, $x=3$, $t_0=1$, $t_1=12$. In which $b1$ is the cost for the first backorder. When optimizing this situation for several values of $pr0$ and $pr1$, we find optimal base stock levels and corresponding cost as depicted in Table 7.3.

pr_0	1	0.9	0.7	0.5	0.3	0.1	0
pr_1	0	0.1	0.3	0.5	0.7	0.9	1
S*	4	4	4	4	4	2	2
c(S*)	5.46	16.14	37.5	58.86	80.23	100.82	110.00

Table 7.3 Analysis of high unit backordering cost for first backorder

Here we indeed see that when the unit backordering costs for the first backorder are significantly higher than for the remaining backorders the optimal base stock level goes up for those cases in which the relative amount of short disruptions is high. The optimal base stock level goes up to 4, since this amount is needed to keep fulfilling demand from inventory when the disruption lasts only one period.

In conclusion to this analysis we can state that for equal unit backordering cost the length of a disruption only influences the height of the optimal base stock level slightly (as seen in Section 5.2). When unit backordering cost are linear, holding more inventory to keep fulfilling demand during a short disruption does not make sense since the cost function only changes with regard to the perishability cost, and not the backordering cost. So when in a long lasting disruption the optimal base stock level is equal to the demand, this will also be the case for short disruptions. However, when it is costly to have a first backorder, the cost structure changes and the possibility of keeping inventory to secure demand fulfillment during a disruption offers cost advantages. Therefore, in this case, the length of disruptions has more influence on the optimal base stock level.

8. Mitigation strategies

Disruptions lead to a decreasing inventory level and with that an increasing amount of backorders. So far, we mainly researched the possibility of decreasing the amount of backorders by increasing the base stock level. However, because of cost and perishability, this is not always a good strategy to reduce backorders. However, as we saw in Section 2.3, there are more strategies that companies use to mitigate supply disruption risk and with that decrease the number of backorders. In this section we describe how these mitigation strategies affect the parameters in the model to reduce the amount of backorders. Changing a certain parameter might also change other corresponding results, which we show as well. Furthermore, we discuss the advantages and disadvantages of each strategy.

8.1 No change of parameters

When companies accept risk or use insurance to mitigate disruption risk for perishable products, they actually do not change any parameter in our model. Therefore they could determine the optimal base stock level with use of the cost function as obtained by Schmitt et al., (2010). An advantage of these strategies is that no information about the disruption distribution is needed. When disruptions (almost) never occur and last only very short, acceptance might be a good strategy (financial mitigation might be unnecessarily costly). Even when taking perishability and disruptions into account, acceptance turns out to be the best strategy (i.e. when S^* is equal to d). However, when disruptions occur more often and/or last longer, acceptance can become very costly and insurance companies on the other hand will not finance the losses any longer.

8.2 Reducing disruption probability

When economic supply incentives are correctly used to reduce the disruption risk, this mitigation strategy changes the cost function by decreasing the disruption probability. When the disruption probability decreases, the optimal base stock level and the corresponding cost might decrease. Furthermore, the maximum fill rate increases. Decreasing the disruption probability also leads to an earlier convergence of the optimal base stock level. This means that for products with shorter limited lifetimes the optimal base stock level is similar as for products with unlimited lifetimes. A drawback of this strategy is that it is difficult to establish; significant investments are needed, and the results of these investments (decreasing disruption probability) is difficult to estimate upfront. Furthermore, when a supplier needs financial support for securing supply, this might be a sign that the business is not profitable. Therefore this strategy should only be adopted if the investment is profitable, either in costs or in quality of service.

8.3 Increasing recovery probability

Flexible supply base, contingent rerouting, make and buy, and flexible transportation can increase the recovery probability; when a disruption still lasts at a certain supplier, the other supplier or route can take over the supply, ending the disruption for the company, while the real disruption might still last. It might even be that the suppliers, routes, or contracts are so flexible that they can immediately take over the disrupted supply flow such that even the disruption probability decreases. When one or both parameters change, a decreased optimal base stock level might result. More importantly, the inventory related costs are likely to go down, while the maximum fill rate increases. An advantage of decreasing backorders by increasing the recovery probability is that decrease of cost is not mainly caused by a decrease in base stock level. This means that, as we saw before, the choice of the base stock level is not very strict. This is beneficial, since it might be difficult to establish what the exact increase of recovery probability is. A disadvantage of flexible supply base and flexible transportation is that they require attention during times of no disruption. For make and buy and contingent rerouting this is only true to a limited extent. Therefore, when these strategies are possible, they are profitable to adopt in many cases.

8.4 Decreasing demand

Silent product roll-over, revenue management via dynamic pricing and promotion and assortment planning let the demand go down during times of disruption. This results in lower temporary lower demand that cannot be fulfilled and thus less backorders. With silent product roll-over demand of the disrupted items might vanish fully. When the system is mainly a push system, and demand depends on inventory, these strategies are very useful. A drawback however, is that when demand decreases the number of items in inventory that perish might increase.

8.5 Increasing lifetime

In this section, we discuss a mitigation strategy that, to the best of our knowledge, is not addressed by previous research. In other words, this mitigation strategy is a result of our findings. As we saw in Section 5.3, it might sometimes be beneficial to invest to increase the lifetime of the products. With a longer lifetime the optimal base stock level might increase in its turn, leading to less backorders. Furthermore, when lifetimes are longer, the choice of the optimal base stock level is less important. This strategy can only be successfully adopted when the convergence lifetime is not reached yet, and when the optimal base stock level is determined by x^*d, otherwise the optimal inventory level does not increase.

So far, we discuss when strategies are adoptable. The effectiveness of each strategy, however, depends on the specific parameter values of the situations, and therefore cannot be assessed. When the parameter values are known, the model can be used more precisely for deciding which strategy to adopt.

9. Conclusion

In this study we analyzed the influence of several parameters on the inventory related cost function and the associated optimal base stock level for perishables with the possibility of disruptions. In our model, the inventory related cost function is a piecewise linear function in which the several cost parameters influence the total cost as expected; high unit holding and high unit perishing cost might decrease the optimal base stock level, while high unit backordering cost might increase the optimal base stock level. Also the disruption distribution parameters influence the optimal base stock level in a way that is intuitively expected; higher probability of being disrupted and a lower probability of recovering once disrupted yield in higher optimal base stock levels, while the opposite might decrease the optimal base stock level.

Furthermore we showed in this study that perishability influences the optimal base stock level in such a way that it puts a limit on it. Next to this, the cost function that we obtain, proofs that for products with short lifetimes the choice for the optimal base stock level is more important than for products with long lifetimes, since in the former case the cost increase much more when choosing the non-optimal base stock level. The cost function also might be used as a supporting tool for decisions on investments in lifetime increasing innovations.

After we developed the model and discussed the managerial implications of it, we tested the model in other cases as assumed before. We first performed a sensitivity analysis which showed that even when demand is stochastic, our model (based on a deterministic demand assumption) performs rather well. This shows that the model as obtained is fairly robust. Next we adapted the model to a case in which there can be several different disruption distributions for different states of the world. This adapted model enabled us to find more precise insights about the influence of the duration of a disruption on the optimal base stock level for perishables. We find for example that for equal unit backordering cost the possibility of having no backorders at all in short disruptions is not used, while this possibility is used when the cost of the first backorder are higher than the cost of the remaining backorders.

In this study we showed the importance of this research, furthermore we obtained a model that provided insights that can be used by future researchers and decision makers. In a later stage the model itself might even be used as a direct supporting tool for decision makers. Since our model is general and robust, it is applicable in a wide range of situations. However, when obtaining our model, we assumed that demand and lifetime are deterministic and known. And although we show that our model performs quite well in a stochastic demand case, it might be even more useful to obtain a similar model with stochastic demand. Another recommendation for further research might be the evaluation of stochastic perishability. In our model the optimal base stock level is always smaller than x^*d, this might change when perishability is stochastic and having backorders is relatively more expensive then perishing products.

From another point of view, it might also be interesting to perform research in the cost of obtaining reliable information about the disruption distribution. In this study we assumed that the disruption distribution is known, however, oftentimes this is not the case and it might only be obtained at a certain cost and at low reliability. We did argue that in our specific case it is more useful to know the correct value for α, rather than for β.

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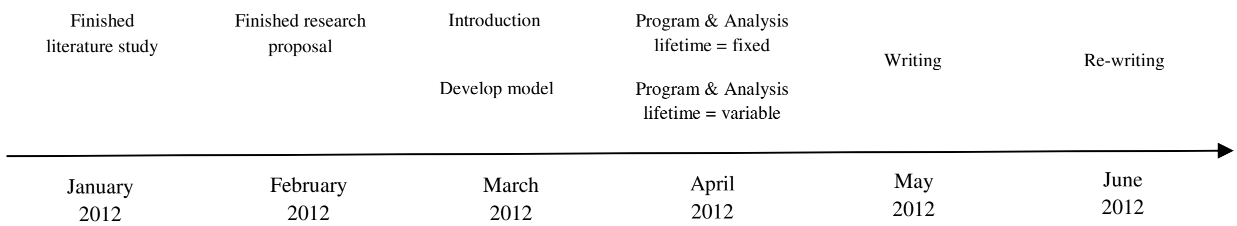
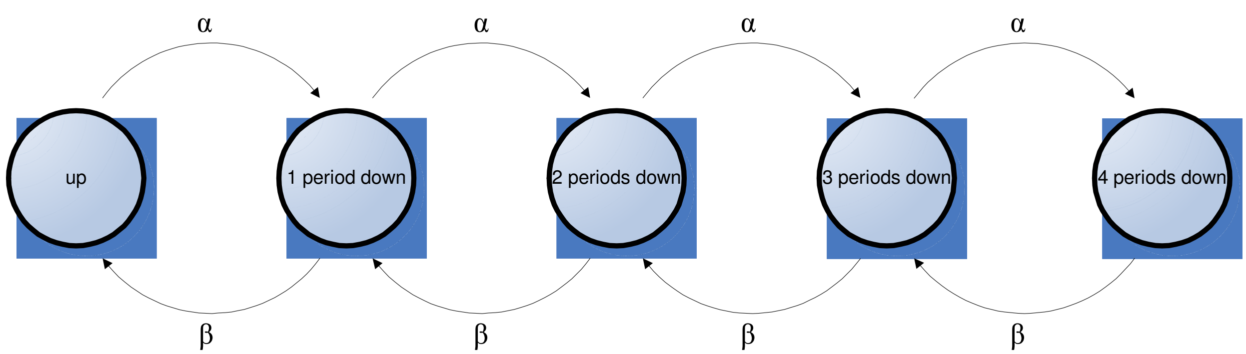
11. Appendix

11.1 Timeline



11.2 Disruption distribution as used in Schmitt et al. (2010)



11.3 Numerical example

For inventory of products with a fixed lifetime of 3, we assume that the parameters have the following values.

$S = 10$

$d = 2$

$b = 3$

$h = 1$

$p = 5$

Without disruptions

1. We start off with S=10 products of age 0 in period l, after demand d=2, we are left with S-d=8 products of age 1. S is larger than d, so we will have no backorders. Furthermore zero products

will perish. At the end of this first period we only need to pay inventory cost over 8 products. We will order 2 new products. This is the warm-up period.

2. We receive the order of 2 units, making our starting inventory equal to 10 (=S). Since we start with zero products of age 2, no products will perish. We meet demand according to a FIFO policy, resulting in an ending inventory of 8 again. We will have 2 products of age 1 (the order that arrived this period) and 6 of age 2. We will order again 2 products.
3. Again our starting inventory is S. The products of age 2 that are left after fulfilling demand will perish, so S-3d = 4 products will perish. Due to this perishing we will have $S-d-(S-3d)^{+} = 4$ ending inventory over which we have to pay inventory cost. Furthermore we have to pay cost of perishing. At the end of this period we have 2 products of age 1 (the order that arrived at the beginning of this period) and 2 products of age 2 (the order that arrived in the previous period was not needed yet to fulfill demand). So we will order 6 new products.
4. After delivery, we start with S products, of which none will perish. At the end of the period we will have 6 products of age 1 and 2 products of age 2. Therefore we will place an order of 2 again.
5. Equal to period *n*.
6. Etc

We see that after the warm-up period, a pattern occurs every three periods.

Period	Starting inventory	Perishing products	Backorders	Ending inventory	Age 1	Age 2	Order
Warm up	10	0	0	8	8	0	2
n	10	0	0	8	2	6	2
n+1	10	4	0	4	2	2	6
n+2	10	0	0	8	6	2	2
n+3	10	0	0	8	2	6	2
n+4	10	4	0	4	2	2	6
n+5	10	0	0	8	6	2	2

Table 11.1 Ageing and ordering of inventory of products with x=3

Now we can describe the cost by determining the cost in each of these three periods and take the mean.

$$C=\frac{1}{3}\left(h\left(S-d\right)^{+}\right)+\frac{1}{3}\left(h\left(S-d-\left(S-3d\right)^{+}\right)^{+}+p\left(S-3d\right)^{+}\right)+\frac{1}{3}\left(h\left(S-d\right)^{+}\right)$$

With disruptions

Now we add the possibility that disruptions can occur. We assume that we are equally likely to be in one of the periods when disruptions happen. Furthermore disruptions are long enough to clear out all inventory (by fulfilling demand and perishing). Now we describe the three different situations that occur after the start of a disruption.

Disruption occurs right before new cycle

Assume that a disruption occurs at the end of step 4 as described in section 0. This disruption causes that no orders can be delivered anymore from the start of period 5 until the disruption is over. Here we describe what happens with inventory, perishing and backordering costs.

1. Since no orders can be delivered anymore, we have a starting inventory of S-d=8. Since we had only d products of age 2, no products will perish. Ending inventory will be S-2d=6. We would like to order 4, however, since we assume disruptions take long enough to clear inventory and we might still have some inventory left, next period supply will also be disrupted, so although we want to order, no orders can be delivered.
2. We start with an inventory of 6, all of age 2. So every item more than demand will perish at the end of this period, so 4 products will perish. At the end of this period inventory will be zero.
3. Assuming that the disruption still lasts, we have no starting inventory so the only costs that occur in this period are backordering cost for all demand d.
4. ..
5. Assume disruption has ended right before this period. Now starting inventory will be 10 again, all of age 0. This will be a warm-up period again, after which the pattern in case of no disruptions will repeat itself. Until another disruption occurs.

Period	Starting inventory	Perishing products	Backorders	Ending inventory	Age 1	Age 2	Order
n+2	10	0	0	8	6	2	2
n	8	0	0	6	0	6	4
n+1	6	4	0	0	0	0	10
n+2	0	0	2	0	0	0	10
n+3	0	0	2	0	0	0	10
n+4 (Warm-up)	10	0	0	8	8	0	2

Table 11.2. Ageing and ordering of inventory of products with x=3 with disruption right before period n

Now we use the probabilities π_i to represent the probability of being at the end of the *i*th consecutive disrupted period, as Schmitt et al., (2010) do in their paper.

The periodical costs are then equal to

$$C = 8\pi_0 + 6\pi_1 + 4*5*\pi_2 + \sum_{i=3}^{\infty} 3*2*\pi_i$$

Disruption occurs right before second and third period of cycle

It is left to the reader himself to do the same exercise as before, however, now with the disruption occurring in different periods of the cycle

11.4 Matlab code for model with variable lifetime

```
%Lifetime is l periods

alpha=0.8;
beta=0.2;

pimatrix=[1-alpha alpha 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0;
    0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0;
    0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0;
    0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0;
```

```
    beta 0 0 0 0 1-beta 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0;
    beta 0 0 0 0 0 1-beta 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0;
    beta 0 0 0 0 0 0 1-beta 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0;
    beta 0 0 0 0 0 0 0 1-beta 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0;
    beta 0 0 0 0 0 0 0 0 1-beta 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0;
    beta 0 0 0 0 0 0 0 0 0 1-beta 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0;
    beta 0 0 0 0 0 0 0 0 0 0 1-beta 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0;
    beta 0 0 0 0 0 0 0 0 0 0 0 1-beta 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0;
    beta 0 0 0 0 0 0 0 0 0 0 0 0 1-beta 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0;
    beta 0 0 0 0 0 0 0 0 0 0 0 0 0 1-beta 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0;
    beta 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1-beta 0 0 0 0 0 0 0 0 0 0 0 0 0 0;
    beta 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1-beta 0 0 0 0 0 0 0 0 0 0 0 0 0;
    beta 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1-beta 0 0 0 0 0 0 0 0 0 0 0 0;
    beta 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1-beta 0 0 0 0 0 0 0 0 0 0 0;
    beta 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1-beta 0 0 0 0 0 0 0 0 0 0;
    beta 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1-beta 0 0 0 0 0 0 0 0 0;
    beta 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1-beta 0 0 0 0 0 0 0 0;
    beta 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1-beta 0 0 0 0 0 0 0;
    beta 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1-beta 0 0 0 0 0 0;
    beta 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1-beta 0 0 0 0 0;
    beta 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1-beta 0 0 0 0;
    beta 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1-beta 0 0 0;
    beta 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1-beta 0 0;
    beta 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1-beta 0;
    beta 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1-beta;
    1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]^100
p=pimatrix(1,:)
sum(p)


h=1;
b=5;
per=3;
d=2;
l=5;

cost=zeros(1,61);
    for S=0:60

        line1=p(1)*((h*(l-1)*max(0,S-d)+max(0,S-d-max(0,S-l*d)))+l*b*max(0,d-
S));
        line2=p(2)*(2*h*max(0,S-2*d-max(0,S-l*d))+b*(max(0,d-max(0,S-
d))+max(0,d-max(0,S-d-max(0,S-l*d)))));
        line3=0;
        for g1=3:30

        line3=line3+p(g1)*2*(h*max(0,S-g1*d-max(0,S-l*d))+b*(max(0,d-max(0,S-
(g1-1)*d-max(0,S-l*d)))));
        end
        line4=0;
        for k=3:l
        for g2=2:l-k+2
        line4=line4+p(g2)*(h*max(S-g2*d,0)+b.*max(0,d-max(0,S-(g2-1)*d)));
        end
```

```
        line4=line4+p(l-k+3)*(h*max(0,S-((l-(k-3))*d)-max(0,S-
l*d))+b*max(0,d-max(0,S-(l-(k-2))*d)));
        for g3=(l-k+4):A
        line4=line4+(p(g3)*(h*max(0,S-g3*d-max(0,S-l*d))+b*max(0,d-max(0,S-
((g3-1)*d)-max(0,S-l*d)))));
        end
        end
        line5=0;

        for g5=1:5
            line5=line5+p(g5)*per*max(0,S-l*d);
        end

        cost(S+1)=(line1+line2+line3+line4+line5)/l;

    end

    cost;

    opt_S=1000000;
    min_cost=10000000000;

    for S=0:30
        if cost(S+1)<min_cost
            min_cost=cost(S+1);
            opt_S=S;
        end
    end

    opt_S;
    min_cost;

x=0:60;
plot(x,cost,'red');
```

11.5 Derivation of limit P_2 as S goes to infinity

$$\lim_{S\to\infty} P_2 \left(1 - \frac{\pi_0 (d-S)^+ + \sum_{i=1}^{x-1} \pi_i \left(\frac{x-1}{x} \left(d - (S-id)^+ \right)^+ + \frac{i}{x} \left(d - \left(S - id - (S-xd)^+ \right)^+ \right)^+ \right) + d \sum_{i=x}^{\infty} \pi_i}{d} \right) * 100$$

$$\lim_{S\to\infty} P_2 \left(1 - \frac{\sum_{i=1}^{x-1} \pi_i \left(\frac{i}{x} \left(d - \left(S - id - (S-xd) \right) \right)^+ \right)^+}{d} + \sum_{i=x}^{\infty} \pi_i \right) * 100$$

$$\lim_{S\to\infty} P_2 \left(1 - \frac{\sum_{i=1}^{x-1} \pi_i \left(\frac{i}{x} \left(d - (x-i) d \right)^+ \right)}{d} + \sum_{i=x}^{\infty} \pi_i \right) * 100$$

$$\lim_{S\to\infty} P_2 \left(1 - \sum_{i=x}^{\infty} \pi_i \right) * 100$$

11.6 Matlab code for Monte Carlo Simulation

```
t=1000;
d=zeros(1,t);
mud=20;
sigma=2;
disruption=zeros(1,t);
dsr=zeros(2,t); %will later be a vector of random number between 0 and 1, if
this number is < alpha, a disruption occurs.
alpha=0.6;
beta=0.2;
per=3;
b=15;
h=1;
k=80;
average_total_cost=zeros(1,k);
upprob=zeros(1,k);
sample_average=zeros(1,31);

for S=20:2:70;


for g=1:k

for i=1:t
```

```
    dsr(1,i)=rand();
    dsr(2,i)=rand();
end


for i=1:t
    d(i)=round(norminv(rand(),mud,sigma));
end

d;

for i=3:t
    if disruption(1,i-1)==1
        if disruption(1,i-2)==1
            if dsr(2,i)<beta
                    disruption(1,i)=0;
            else disruption (1,i)=1;
            end
        else disruption(1,i)=1;
        end
    elseif dsr(1,i)<alpha
        disruption(1,i)=1;
    else disruption(1,i)=0;
    end
end

disruption();
 upprob(1,g)=1-(sum(disruption)/t);

inv=zeros(3,t);
inv(1,1)=max(0,S-d(1));
inv(2,1)=0;
inv(3,1)=0;
order=zeros(2,t);
order(1,1)=d(1,1);
order(2,1)=0;

for i=2:t
    if (disruption(i)==1)
        inv(1,i)=0;
        order(2,i)=max(0,d(1,i)-inv(1,i-1)-inv(2,i-1));
    else
        inv(1,i)=max(0,order(1,i-1)-max(0,d(i)-inv(1,i-1)-inv(2,i-1)));
        order(2,i)=max(0,d(1,i)-inv(1,i-1)-inv(2,i-1)-order(1,i-1));
    end

    inv(2,i)=max(0,inv(1,i-1)-max(0,d(1,i)-inv(2,i-1)));
    inv(3,i)=max(0,inv(2,i-1)-d(1,i));
    order(1,i)=S-inv(1,i)-inv(2,i);

end
inv=inv();
order=order();
```

```
totalcost=[sum(inv())*h;order(2,1:t)*b;inv(3,1:t)*per];

average_total_cost(1,g)=sum(sum(totalcost()))/t;

end

sample_up_prob=mean(upprob());
sample_average(1,S)=mean(average_total_cost);
end
sample_average
```

11.7 Matlab code for Modulated Process

```
alpha=0.5;
h=3;
b=15;
per=7;
d=2;
l=3;
dur0= 1;
dur1= 12;
pr0= 0;
pr1= 1-pr0;

A=[1-alpha alpha;1 0]^1000;
B=[1-alpha alpha 0 0 0 0 0 0 0 0 0 0 0; 0 0 1 0 0 0 0 0 0 0 0 0 0; 0 0 0 1 0
0 0 0 0 0 0 0 0; 0 0 0 0 1 0 0 0 0 0 0 0 0; 0 0 0 0 0 1 0 0 0 0 0 0 0; 0 0 0
0 0 0 1 0 0 0 0 0 0; 0 0 0 0 0 0 0 1 0 0 0 0 0; 0 0 0 0 0 0 0 0 1 0 0 0 0;0 0
0 0 0 0 0 0 0 1 0 0 0; 0 0 0 0 0 0 0 0 0 0 1 0 0;0 0 0 0 0 0 0 0 0 0 0 1 0;0
0 0 0 0 0 0 0 0 0 0 0 1;1 0 0 0 0 0 0 0 0 0 0 0 0 ]^5000;
pi0=A(1,1:2)
pi1=B(1,1:13)

C=zeros(1,9);
for S=0:8

    line1=(pi0(1,1)*h*((l-1)*max(S-d,0)+(max(S-d-max(S-l*d,0),0)))+l*b*max(d-
S,0)+l*per*max(S-l*d,0));
    line2=(pi0(1,2)*h*(max(S-2*d,0)+2*max(S-2*d-max(S-l*d,0),0)));
    line3=0;
    for k=1:(l-1)
        line3=line3+(pi0(1,2)*b*((l-k)*max(d-max(S-k*d,0),0)+k*max(d-max(S-
k*d-max(S-l*d,0),0),0)));
    end
    line3a=pi0(1,2)*per*(l-2)*max(S-l*d,0);
    line4=(pi1(1,1)*h*((l-1)*max(S-d,0)+(max(S-d-max(S-l*d,0),0)))+l*b*max(d-
S,0))+l*per*max(S-l*d,0);
    line5=pi1(1,2)*h*(max(S-2*d,0)+2*max(S-2*d-max(S-l*d,0),0));
    line6=0;
    for k1=1:(l-1)
```

```
        line6=line6+(pi1(1,2)*b*((l-k1)*max(d-max(S-k1*d,0),0)+k1*max(d-
max(S-k1*d-max(S-l*d,0),0),0)));
    end
C(1,S+1)=(pr0*(line1+line2+line3+line3a)+pr1*(line4+line5+line6))/l;
end

C

x=0:8;
plot(x,C,'magenta')
```