Capacitated production and inventory control for MTS perishable items which are subject to lumpy demand

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in partial fulfilment of the requirements for the degree of

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“In der Beschränkung zeigt sich erst der Meister”
Johann Wolfgang von Goethe, 1749 – 1832

I. PREFACE

This report presents the results of my research project in order to receive the Master's degree in Operations Management and Logistics at Eindhoven University of Technology, The Netherlands. This project was carried out from September 2007 until February 2008 at the production planning and purchasing department of XXXX in XXXX.

In these last six months of my study I have experienced a lot of challenges and opportunities. I found it very interesting to work at a company as XXXX and during my time in XXXX I have been able to develop myself both theoretically and personally. I would like to thank all people who made this project possible and who gave me help and support.

First, I would like to thank Dr. Ir. Flapper, my first supervisor of Eindhoven University of Technology. His enthusiast feedback and critical remarks encouraged me to continuously improve my work. Next, I would like to thank Prof. Dr. Fransoo, my second supervisor of Eindhoven University of Technology, for the clear insights he gave me.

I would like to thank XXXX, my supervisor of XXXX, for always being available for helping me and making me feel comfortable at the production planning and purchasing department. Besides my supervisor, I would like to thank all my colleagues at XXXX for giving me information and explanation.

In my personal life, I would like to thank all my friends for their interest and support. Especially, I would like to thank Niek and Vincent for giving me critical feedback on my work and for motivating me to keep striving for good results. I would like to thank Gaby for her great support in all times.

Last, but not least, I would like to thank my parents for their infinite support during my studies.

II. ABSTRACT

This Master's thesis project describes the development of a heuristic solution procedure that determines the order-up-to levels for products with a fixed shelf life which are subject to lumpy demand and produced to stock on a production resource with limited capacity. In order to anticipate on demand that has very pronounced peaks that can be several times greater than average demand, the order-up-to level is adjusted in the course of time by expressing the order-up-to level in terms of customer demand forecasts. Simulation of the heuristic solution procedure for products produced by a case company shows that the new production and inventory control policy reduces the disposal costs and increases the service level. Simulation also gives insight in the impact of different characteristics of the products and the production system on the profit, the service level, and the disposal costs.

III. MANAGEMENT SUMMARY

The problem
A large portion of make-to-stock producers face lumpy demand. Lumpy demand is defined as demand that has very pronounced peaks that can be several times greater than average demand. Lumpy demand can create significantly more complex production problems than stationary demand. This is because firms that face lumpy demand typically do not have sufficient capacity to meet demand during peak periods and must build inventory in the low demand periods in anticipation of high demand later. In practice, the lumpy demand environment with limited production capacity can be difficult to deal with and can lead to both large inventories of some products and large shortages of others. When products have a fixed finite lifetime (perishable products), the situation gets much more complex, since excess inventory has to be disposed, which is often much more expensive than keeping large inventories. For these products, a practical production and inventory control policy has to be found that makes a trade-off between spill and shortages in a lumpy demand environment with production capacity limitations.

Academic literature
Our approach in finding well performing production and inventory control policies for perishable products which are produced on a capacitated production resource draws from both perishable and non-perishable production and inventory control theory. For production and inventory control of perishables which have a non-zero replenishment lead time, near-to-optimal policies have been found in the class of simple order-up-to rules. For production and inventory control of non-perishables which are produced on a production resource with limited capacity, a single order-up-to point is of interest rather than production quantities or reorder points.

Capacitated production and inventory control for perishables
A periodic review order-up-to production and inventory control policy is proposed that yields near-to-optimal results for both production and inventory control problems of perishable products with a non-zero replenishment lead time and production and inventory control problems of products that are produced on a production resource with limited capacity.

Since in a lumpy demand environment the average demand rate changes in the course of time, for every product i the order-up-to level in week w, S_{iw}, is expressed in terms of number of weeks of expected demand, based on the demand forecast of the next $R + L + c$ weeks ($0 < c \leq \text{ISL}$), in which R is the review period, L is the replenishment lead time, and c is a variable that allows for building up inventory during multiple weeks. The review period R is set equal to one time unit. For perishable products it is important to be able to replenish fresh inventory regularly. By setting R equal to one, every period there is a possibility to produce. Limited production capacity is allocated among products by not producing the least critical product in a particular week until the capacity constraint is satisfied.

Heuristic
Due to a lack of an optimal solution of simple form, a heuristic is developed for determining Ψ_i for every perishable product. The heuristic calculates the profit for different settings of Ψ_i and sets Ψ_i to a value that yields the highest profit for every product. In order to calculate the profit for a particular setting of Ψ_i, in every week a production decision is made for a production batch that will be received a replenishment lead time later. The production decision in a particular week is made such that the inventory level is raised up to the order-up-to level S_{iw}. A production decision is made by choosing one of the alternative production batch sizes that are pre-specified for every product. Production decisions in every week are

restricted by a net production capacity constraint. Due to complexity reasons, the detailed scheduling of production batches on the production resource is not incorporated into the model. When the net production capacity constraint in a particular week is not met, planned production for the least critical product (based on the inventory level compared to the expected demand) is set to 0 in this week. This means that this product becomes more critical in the next week. When the net production capacity constraint in a particular week is satisfied, demand occurs (no backordering is allowed), stock-outs are observed, the production batch about which was decided a replenishment lead time earlier becomes available, inventory becomes one week older, and disposals are observed.

Case study

The performance of the periodic review order-up-to production and inventory control policy is evaluated by a case study at a case company. The case company produces ready-to-serve fruit juices and fruit beverages in cartons for the Dutch and Belgian market. In 2007, the target service level of X% for Y products was not achieved. The service level is defined by the case company as the percentage of the customer demanded products that are delivered within the time that is required. Additionally, 50% of the total disposal costs in 2007, namely € X,-, were due to Y products. Characteristics of the Y products (i.e. limited shelf life, long replenishment lead time) and of the production system with which the Y products are produced (i.e. limited production capacity) are such that the case company has difficulties in dealing with the lumpiness in the customer demand.

Experimental design

The new production and inventory control policy is tested by first determining Ψ_i for every Y product using the heuristic that was developed. Then the performance of these settings of Ψ_i is tested on a different dataset using simulation. Different experiments are executed in order to evaluate the new production and inventory control policy in the current situation and to determine the impact of the different characteristics of the products and the production system on the profit.

Conclusions

Based on the case study general conclusions have been drawn on production and inventory control for products with a fixed shelf life which are subject to lumpy demand and produced to stock on a production resource with limited capacity.:

- Our periodic review order-up-to production and inventory control policy yields good results. Our policy yields good results due to the following reasons:

    1. The order-up-to level is adjusted in the course of time by expressing the order-up-to level in terms of customer demand forecasts in the next $R + L + c$ weeks. In this way inventory can be build up in periods with lower demand for periods with peak demand.
    2. Production capacity is allocated among products by not producing the least critical product until the production capacity constraint is satisfied.
    3. The order-up-to parameters are determined using empirical customer demand and customer demand forecast data, such that all true characteristics are incorporated.
    4. The heuristic compensates for structural forecast errors.
    5. The heuristic compensates for the sizes of the alternative batches.

- Increasing the internal shelf life reduces the disposal costs but does not necessarily increase the service level

- Reducing the replenishment lead time reduces the disposal costs and increases the service level when better customer demand forecasts can be used for production decisions

- Increasing the production capacity does not reduce the disposal costs and does not necessarily increase the service level

- The internal shelf life and the replenishment lead time interact

- Reducing the forecast error by 25% reduces the disposal costs and increases the service level

Further, conclusions can be drawn on the working of our heuristic:

- Inventory holding costs have an impact on the Ψ_i's that are chosen

- The heuristic compensates for a structural forecast error

- For products with big alternative batch sizes relative to the average customer demand, Ψ_i will be relatively high. As a result, disposal costs for products with relatively big alternative batch sizes are relatively high.

Recommendations
Based on the general results that are generated in this Master's thesis project, the following things were recommended to the case company:

- Implement the periodic review order-up-to production and inventory control policy that was researched in this Master's thesis project in order to increase the service level and to reduce the disposal costs.

- As a next step, first determine the Ψ_i's without taking into account inventory holding costs, since the inventory holding costs are not important for the case company.

- As a second next step, reduce the forecast error in order to reduce the disposal costs and to increase the service level by using forecast methods for lumpy demand.

- Research the impact of the production capacity when the customer demand for Y products will increase by 40%: when there are three week periods in which the total customer demand is greater than the available production capacity, the impact of increasing the internal shelf life and of increasing the production capacity will be different.

- Reduce the alternative batch sizes for products with big alternative batch sizes relative to the average customer demand.

TABLE OF CONTENTS

1. INTRODUCTION

A large portion of make-to-stock producers face lumpy demand. Lumpy demand is defined as demand that has very pronounced peaks that can be several times greater than average demand. Lumpy demand is for example caused by consumers buying quantities that not reflect their immediate needs during consumer promotions. Lumpy demand must be planned for in the same way one would plan for seasonal demand peaks (Schuster and Finch, 1990) and can create significantly more complex production problems than stationary demand (Metters, 1998b). This is because firms that face lumpy demand typically do not have sufficient production capacity to meet demand during peak periods and must build inventory in the low demand periods in anticipation of high demand later (Krane and Braun, 1991; Fair, 1989).

In practice, the lumpy demand environment with limited production capacity can be difficult to deal with. Bush and Cooper (1988) and Buxey (1993) indicate that firms facing these conditions tend to have no formal planning mechanism. Bush and Cooper (1988) indicate that firms tend to build far too much inventory in general. Fisher et al. (1994) say that this environment leads to both large inventories of some products and large shortages of others.

When products have a fixed finite lifetime (perishable products), the situation gets much more complex, since excess inventory has to be disposed, which is often much more expensive than keeping large inventories. Examples of perishable products include blood, certain pharmaceuticals, and food. Perishable products that have not been sold to the customer before the shelf life has expired have to be disposed.

For the production and inventory control problem for perishable products in a lumpy demand environment with capacity constraints there has been no simple, closed-form optimal solution presented in literature. In this Master's thesis project the production and inventory control is researched for multiple products which:

- are produced-to-stock,
- are subject to lumpy demand,
- are produced on a production resource with limited capacity, and
- have a fixed finite lifetime

We only consider production and inventory control for end products: products which have undergone all processing steps and are ready for selling to the customer. Our aim is not to provide a comprehensive heuristic or advanced theoretical statements on optimal policies, but to provide a policy that yields good results in practice and to provide insights in the field of capacitated production and inventory control problems for perishables. For this aim we propose a heuristic that determines the parameters of a production and inventory control policy for perishable products that are subject to lumpy demand and are produced on a production resource with limited capacity.

The outline of this Master's thesis project is as follows: first, in chapter 2 a literature review is given on both production and inventory control for perishable products and production and inventory control for non-perishable products that are produced on a production resource with limited capacity. In chapter 3, a production and inventory control policy is proposed for perishable products that are produced on a production resource with limited capacity. In chapter 4, the heuristic that determines the parameters for the production and inventory control policy is described and in chapter 5 the case company at which the production and inventory control policy was tested is introduced. In chapter 6, experiments for testing the production and inventory control policy are given and the results are discussed. Finally, in chapter 7 conclusions with managerial insights and with general insights for academic research are discussed.

2. ACADEMIC LITERATURE

Our approach in finding well performing production and inventory control policies for perishable products which are produced on a capacitated production resource draws from both perishable and non-perishable production and inventory control theory. In this chapter, academic literature in both fields is discussed.

Production and inventory control for perishables
For production and inventory control of perishables which have a non-zero replenishment lead time, near-to-optimal policies have been found in the class of simple order-up-to rules.

For perishable products with a non-zero replenishment lead time, optimal ordering policies cannot be found. Therefore, for these problems authors try to find the best policy from a pre-specified class. Inventory control problems for perishable products have been studied extensively in literature since the early 1960s (e.g. Van Zyl, 1964; Nahmias, 1972, 1975; Fries, 1975). A literature review on research until the 1980s is given by Nahmias (1982). The 'simplest' problem for perishable products that has been studied is the single period, single item model, commonly called as the newsvendor problem (Hadley and Whitin, 1963). It is observed that most of the models proposed in the perishable inventory literature (e.g. Fries, 1975) do not consider the case of positive lead time for manufacturing or procurement. According to Schmidt and Nahmias (1985), for cases of positive replenishment lead time, the solution is analytically intractable and no closed form expression exists. The optimal inventory reorder policy is probably a complex nonlinear function of a state variable that includes all orders that were placed and the elapsed time since their placement (Schmidt and Nahmias, 1985). Schmidt and Nahmias (1985) say that it is unlikely that anyone can find the optimal ordering policy when there is a positive lead time. A reasonable alternative to optimal policies which is commonly employed is to find the best policy for a pre-specified class (Schmidt and Nahmias, 1985). Two types of basic inventory policies (so-called pre-specified classes) can be identified: a continuous and a periodic review policy. A continuous review policy indicates that the inventory is continuously reviewed, whereas in the case of a periodic policy, the inventory is only observed at certain moments in time. Further, inventory control policies can be classified as either order-up-to policies or fixed-order-quantity policies (Silver et al., 1998). In an order-up-to policy, every time at which it is decided that inventory should be replenished, the inventory is raised-up-to an order-up-to level S. In a fixed-order-quantity policy, every time at which it is decided that inventory should be replenished, a fixed order quantity Q, or a multiple of Q (nQ), is reordered.

Due to its simple structure and wide applicability, the (s, S) policy has been used often as a pre-specified class from which the best policy is searched (e.g. Schmidt and Nahmias, 1985; Pal, 1989; Kalpakam and Sapna, 1994). The (s, S) policy requires continuous review of inventory. However, in many situations this is not possible or practical. In these situations a periodic review policy is more realistic. Chiu (2000) uses the pre-specified (R, S) policy in his study, which is a periodic review order-up-to policy. This policy in the presence of perishable products had not yet been adequately discussed due to the complexity of the problem. Further, this policy was chosen because in practice, the use of periodic review procedures in perishable inventory systems is necessary when a continuous review procedure is costly and when replenishment orders should be placed periodically. Other reasons to adopt this policy is the low unit price of the perishable item (which does not need to consider the number of units in stock which have perished at all times but only at the review points of time), and the short review interval (which is less than the lifetime of the item).

Near-to-optimal ordering policies for perishable products have been found in the class of simple order-up-to rules. Cohen (1976) showed the convexity of expected costs in the order-up-to level S. For a production and inventory control problem of perishable products with highly variable and uncertain customer demand and a positive lead time, Haijema et al. (2007) found near-to-optimal policies in the class of simple order-up-to rules.

Capacitated production and inventory control for non-perishables

For production and inventory control of non-perishables which are produced on a production resource with limited capacity, a single order-up-to point is of interest rather than production quantities or reorder points. A significant theoretical difference between the uncapacitated and capacitated production and inventory control problem for multiple products is the dependence of the optimal solution on the initial inventory vector (Metters, 1998a). A single produce-up-to point has been shown to be an optimal policy in the stationary demand capacitated production problem for a single product (Federgruen and Zipkin, 1986a, 1986b). In the multi-product stationary demand problem, a single produce-up-to value is also an important aspect of the solution (Evans, 1967), however, no solution procedure is given to determine this vector. Furthermore, for the multi-product problem a produce-up-to vector must be specified for each initial inventory vector for each time period. Consequently, it is not practical to specify optimal solutions for moderately sized problems. In his paper, Metters (1998a) uses heuristics that yield optimal results for an uncapacitated, non-seasonal environment (produce-up-to policies) to test in a capacitated, seasonal environment. He achieved near-to-optimal results.

3. CAPACITATED PRODUCTION AND INVENTORY CONTROL FOR PERISHABLES

For the capacitated production and inventory control of perishable products we propose a periodic review order-up-to policy that yields near-to-optimal results for both problems of perishable products with a non-zero replenishment lead time and problems of products that are produced on a production resource with limited capacity. In section 3.1 we discuss the parameters of the policy and in section 3.2 we discuss the capacity allocation method.

3.1 Parameters

The order-up-to level is adjusted in the course of time by expressing the order-up-to level in terms of customer demand forecasts. In a lumpy demand environment the average demand rate changes in the course of time. Therefore, the values of the production and inventory control policy should change in the course of time. Silver et al. (1998) say that changes in the review period R should be avoided. Silver et al. (1998) say that one appealing approach to adjusting S in the course of time is to compute the value of S in a particular period t using customer demand (forecast) information over the immediately following interval of duration $R + L$, where L is the replenishment lead time. This is equivalent to using a rolling horizon of length $R + L$. Tests by Kaufman (1977) have revealed that this simple approach performs well. For the situation with limited production capacity, in order to be able to anticipate on peaks in the customer demand by building up inventory in multiple periods, we add a period c to $R + L$. For every product i the order-up-to level in week w, S_{iw}, is expressed in a number of weeks of demand, based on the demand forecast of the next $R + L + c$ weeks:

$$S_{iw} = \Psi_i * \left(\frac{\sum_{r=w}^{w+R+L+c-1} F_{i,w-1,r}}{R+L+c} \right) \tag{1}$$

where

Ψ_i is the number of weeks of expected demand in which the order-up-to level S_{iw} is expressed (this decision variable should be determined for every product)
F_{ivw} is the demand forecast for product i made in week v for demand in week w.

The period c should be chosen such that production can be spread over multiple weeks in order to build up inventory for a period with peak demand. The period c depends on the production capacity and should always be smaller than or equal to the internal shelf life of the products.

The review period R is set equal to one time unit. For perishable products it is important to be able to replenish fresh inventory regularly. By setting R equal to one, every period there is a possibility to produce.

3.2 Capacity allocation

Limited production capacity is allocated among products by not producing the least critical product in a particular week until the capacity constraint is satisfied. A lot of research has been executed on how to allocate limited production capacity among multiple products (e.g. Glasserman, 1996; Korpela et al., 2002). One option is to allocate production capacity such that the maximum rate of decrease of the stock out probability is maximized (Glasserman, 1996). We allocate production capacity among products by not producing the least critical product in a particular week w until the capacity constraint is satisfied. The least critical product in a particular week w is the product with the highest inventory level (i.e. the total inventory that is on-hand plus on order) relative to expected demand. The replenishment lead time is defined as the time that elapses from the moment at which it is decided to place a production order, until the related products are physically on the shelf ready to satisfy customer demand. When no production batch is planned for the least critical product in week w, this product becomes more critical in week w + 1. The least critical product is calculated as follows:

$$\arg\max_i \left(\frac{\left(I_{iw} - \sum_{r=w}^{w+R+L+c-1} F_{i,w-1,r} + \sum_{r=w}^{w+L-1} RB_{ir} \right)}{\frac{1}{R+L+c} * \sum_{r=w}^{w+R+L+c-1} F_{i,w-1,r}} \right) \qquad (2)$$

where

I_{iw} = The total inventory (on hand) for product i at the beginning of week w.

F_{ivw} = The demand forecast for product i made in week v for demand in week w

RB_{iw} = The production batch size for product i that will be received in week w: $RB_{iw} = PB_{iw-L}$

$\arg\max_i (x)$ = The product i for which the value of x attains its maximum value

4. HEURISTIC

In this chapter a heuristic is presented that determines Ψ_i for every product. The heuristic calculates the profit for different settings of Ψ_i and sets Ψ_i to a value that yields the highest profit. In order to calculate the profit for a particular setting of Ψ_i, every week w a production decision is made for a production batch that will be received in week $w + L$ by choosing one of the K alternative pre-specified production batch sizes. The production decision in week w is made such that the inventory level at the end of week $w + L$ is raised up to S_{iw}. Due to complexity reasons, detailed scheduling of production batches is not incorporated into the heuristic. Set-up times are incorporated by subtracting the setup time in a production cycle from the production capacity (Glasserman, 1996). Limited production capacity (C_w) is allocated to products using formula (2). When the net production capacity constraint in a particular week is satisfied, demand occurs (no backordering is allowed since lost sales are more appropriate for lumpy demand environments (Metters, 1998b)), stock-outs are observed, the production batch about which was decided in week w - L becomes available, inventory becomes one week older, and disposals are observed.

The parameter Ψ_i is determined for every perishable product i with a given internal shelf life (ISL). The internal shelf life is defined as the maximum period that can elapse between production of the product and delivery of the product to the customer. When the ISL has expired, products have to be disposed. In the heuristic, inventory with different ages is considered. The inventory state for product i at the beginning of week w is denoted by $x_{iw} = (x_{iw0}, \ldots, x_{iwISL})$, where x_{iwr} is the number of products i with an age of r weeks at the beginning of week w. Every week a transition is made in the inventory state of product i. Transitions are determined by two elements: the production decision and the customer demand that occurs. Products are issued FIFO, i.e. oldest products are sold first. Pierskalla and Roach (1972) have shown this to be the optimal issuing policy for a perishable item when the objective is to minimize total inventory costs or the quantity of stock reaching the final age category. When discussing the transition from one state to the next, the following elements are distinguished:

- *No demand:* First assume that there is no demand. As the age of all products increases by one week, with part of the inventory expiring, the transition from week w to week w+1 then becomes

  $$(x_{iw0}, \ldots, x_{iwISL-1}, x_{iwISL}) \rightarrow (RB_{iw}, x_{iw0} \ldots x_{iwISL-1})$$

  Where RB_{iw} is the received production batch size for product i in week w
  In this situation x_{iwISL} items would have been disposed since no demand occurred.

- *Demand:* First the oldest products are sold. When demand exceeds total inventory, lost sales occur. An example is given for a product with an ISL of three weeks for which in week w 10 products are produced on Friday and for which demand during week w equals 20

  (10, 12, 21, 33) → (10, 10, 12, 21)

  In this situation 13 products must be disposed. When demand would have been 40, the situation would have been as follows

  (10, 12, 21, 33) → (10, 10, 12, 14)

  In this situation no products must be disposed. The heuristic is given in figure 1.

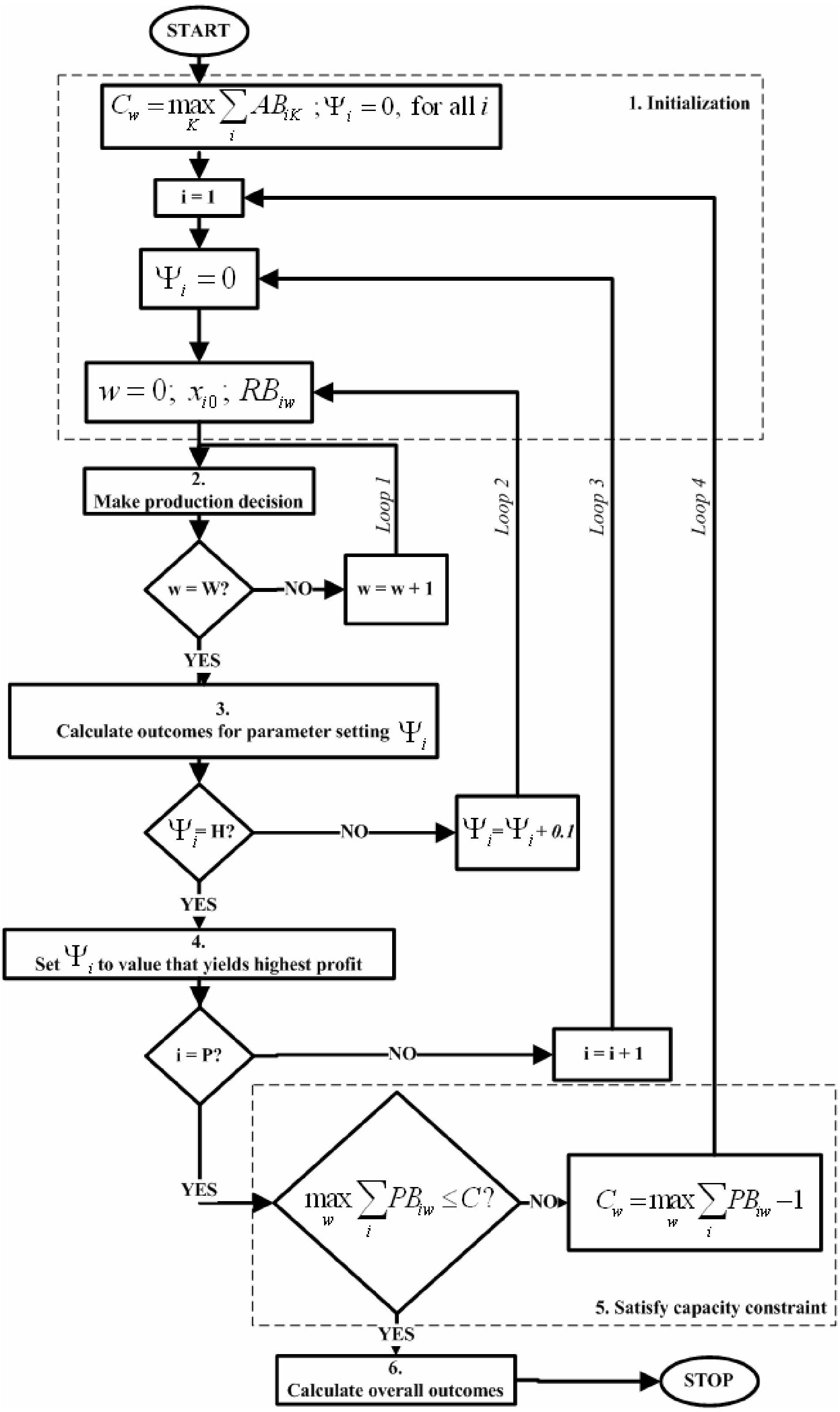

Figure 1: Heuristic model to determine values of Ψ_i that yield the highest profit

The different steps of the heuristic that is given in figure 1 are described in the remainder of this section.

1. Initialization

In order to satisfy the net production capacity constraint C, the heuristic is executed iteratively. Before an iteration starts, initialization is needed. Four different loops exist in this heuristic (see figure 1), namely:

Loop 1: For a single product i and a single value for Ψ_i, a period of W weeks w is evaluated.

Loop 2: For a single product i, H values of Ψ_i are evaluated, from $\Psi_i = 0$ to $\Psi_i = H$.

Loop 3: For all products i the Ψ_i that yields the highest profit is determined.

Loop 4: The production capacity in the heuristic in every week w (C_w) is reduced step-by-step until the net production capacity constraint (C) is met.

Initializations are:

$C_w = \max_n \sum_i AB_{in}$:	the production capacity in every week w (C_w) is set equal to the sum of the maximum alternative production batch sizes n of all products i (AB_{in})
$\Psi_i = 0$, for all i:	for all products evaluation starts at $\Psi_i = 0$
$i = 1$:	the heuristic starts determining the best Ψ_i for product 1
$w = 0$:	the period starts at week 0
x_{i0} and $RB_{i0} \dots RB_{iL}$:	are chosen such that, in the first weeks the total demand during the replenishment lead time can be covered,

where

x_{i0} is the inventory state for product i at the beginning of week 0), and $RB_{i0} \dots RB_{iL}$ is the received production of product i in the weeks before the first planned production can be received.

2. Make production decision

At the beginning of every week w (current week is called week 0) for every product i a production decision is made about production that will be received in week L. This production decision is called the planned production batch size: the planned production batch size for product i in week w is denoted by PB_{iw}. For this production decision alternative production batch sizes can be chosen. Every week the inventory level is raised-up-to S_{iw} by planning a production batch that is equal to one of the alternative production batch sizes:

IF $0 < \left(S_{iw} + I_{iw} - \sum_{r=w}^{w+L} F_{i,w-1,r} + \sum_{r=w}^{w+L-1} RB_{ir} \right) \leq \mathrm{AB}_{i1}$ THEN AB_{i1}

ELSE

IF $\mathrm{AB}_{i1} < \left(S_{iw} + I_{iw} - \sum_{r=w}^{w+L} F_{i,w-1,r} + \sum_{r=w}^{w+L-1} RB_{ir} \right) \leq \mathrm{AB}_{i2}$ THEN AB_{i2}

ELSE

IF

$$\vdots$$

ELSE

IF $\mathrm{AB}_{iK-1} < \left(S_{iw} + I_{iw} - \sum_{r=w}^{w+L} F_{i,w-1,r} + \sum_{r=w}^{w+L-1} RB_{ir} \right) \leq \mathrm{AB}_{iK}$ THEN AB_{iK}

ELSE 0

where

I_{iw} = The total inventory for product *i* at the beginning of week *w*.

F_{ivw} = The demand forecast for product *i* made in week *v* for demand in week *w*

AB_{in} = Alternative production batch size *n* for product *i*, with *n* = 1, .., K and $\mathrm{AB}_{i1} < \mathrm{AB}_{i2} < \ldots < \mathrm{AB}_{iK}$

RB_{iw} = The received production batch size for product *i* in week *w*: $\mathrm{RB}_{iw} = \mathrm{PB}_{iw-L}$

Limited production capacity is allocated among products using formula (2). When the planned production satisfies the capacity constraint, demand occurs and lost sales are observed. Due to complexity reasons, no detailed scheduling of the production is incorporated into this heuristic. Since no detailed scheduling is incorporated into this heuristic, the time at which a production batch becomes available for delivery to the customer is not modeled. In this heuristic in a particular week *w* first demand is issued after which a planned production batch is received. When production batches are received, the inventory becomes one week older. When inventory has become one week older, the disposals for that week are observed and the next week is considered (*loop 1*).

3. Calculate outcomes for parameter setting Ψ_i

When for a particular product *i* *W* weeks are planned for a particular value of Ψ_i, the profit and the service level are calculated. The average profit per week for product *i* (AWP_i) is defined by:

$$\begin{aligned} AWP_i = (\sum_w \left(gsp_i - cp_i\right)\left(D_{iw} - \left(D_{iw} - I_{iw}\right)^+\right) - \\ s_i(D_{iw} \text{ - } I_{iw})^+ + d_i x_{i(w+1)(ISL+1)} + (h_i/52) * \left(I_{i(w+1)} + I_{iw}\right)/2 + \\ v_i * \frac{\left(I_{i(w+1)} + I_{iw}\right)/2}{p_i} + \gamma_w * rc\left(\sum_i \frac{I_{iw}}{p_i} - D\right) + q_i \beta_{iw})/52 \end{aligned} \qquad (3)$$

where $x^+ = \max(0, x)$

The first term is related to the revenues minus costs-to-make, where gsp$_i$ is the gross sales price for product i, cp$_i$ is the cost price for product i, D$_{iw}$ is the customer demand for product i in week w, and I$_{iw}$ is the total inventory for product i at the beginning of week w. The second term is related to lost sales, where s$_i$ is the unit lost sales cost for product i. This unit lost sales costs are costs on top of the lost revenues due to for example a loss of goodwill of the customer and are expressed in a percentage of the difference between gsp$_i$ and cp$_i$. The third term is related to disposals, where d$_i$ is the unit disposal cost for product i (which is equal to cp$_i$). Since x$_{\text{iwr}}$, the number of products i with an age of r weeks at the beginning of week w, is only observed at the beginning of the week, after inventory has become one week older, products at the beginning of next week that have an age of ISL+1 have to be disposed. The fourth term is related to inventory holding. Inventory holding costs consist of an interest rate (opportunity cost) that is paid for the value of products kept in inventory and a rate that is paid per pallet for keeping pallets in inventory. In the fourth term h$_i$ is the annual unit inventory holding cost for product i and v$_i$ is the rate for keeping one pallet in the warehouse for one week. p_i is the factor that converts a unit into a pallet for product i. The fifth term is used to punish when the storage capacity restriction is not satisfied. γ_w is a dummy that activates the formula when the storage capacity restriction is not satisfied: $\gamma_w = \begin{cases} 1 \text{ } if \text{ } \sum_i roundup\left(\frac{I_{iw}}{p_i}\right) > D \\ 0 \text{ else} \end{cases}$. The conversion of the inventory in products to pallets should be rounded up to the nearest integer since pallets that are not totally utilized require the space for a full pallet. rc is the cost to store one pallet in a different warehouse per week. D is the storage capacity of the warehouse for the products under consideration. The sixth term is related to the startup of production, where q$_i$ is the startup cost for product i and β_{iw} is a production dummy for product i in week w,

$$\beta_{iw} = \begin{cases} 1 \text{ } if \text{ } PB_{iw} > 0 \\ 0 \text{ } else \end{cases} \qquad (4)$$

The service level that is achieved for product i (SL$_i$) is calculated as follows:

$$SL_i = \left(\sum_w \frac{D_{iw} - \left(D_{iw} - I_{iw}\right)^+}{D_{iw}}\right) * 100\% \qquad (5)$$

4. Set Ψ_i to value that yields highest profit

When for a particular product *i* the profit and the service level are calculated for every setting of Ψ_i (*loop 2*), the Ψ_i for product *i* is set equal to the value that yields the highest profit. Then, the same steps are executed for the following product (*loop 3*).

5. Satisfy production capacity constraint

When for every product *i* the optimal value for Ψ_i is chosen under the production capacity restriction C_w, the production capacity constraint (*C*) is evaluated:

$$\left(\max_{w} \sum_{i} PB_{iw} \right) \leq C \,? \quad (6)$$

When the production capacity constraint is satisfied, a feasible solution is generated. When the production capacity restriction is not satisfied by current solution, the production capacity restriction C_w has to be reduced and the calculations have to be executed again (*loop 4*). Production capacity is reduced by setting C_w equal to the maximum production capacity that was needed in the current solution and reducing this production capacity by one:

$$C_w = \left(\max_{w} \sum_{i} PB_{iw} \right) - 1 \qquad (7)$$

6. Calculate overall outcomes

When a feasible solution has been found, the overall performance of this solution must be calculated such that the overall solution can be compared to solutions generated by runs with different parameter settings. The total average profit per week (AWP) can be calculated by:

$$AWP = \sum_{i} AWP_i \qquad (8)$$

The overall service level (SL) can be calculated by:

$$SL = \left(\sum_{i} \sum_{w} \frac{D_{iw} - \left(D_{iw} - I_{iw} \right)^{+}}{D_{iw}} \right) * 100\% \qquad (9)$$

Non-negativity constraints

The following non-negativity constraints have to be taken into account:

I_{iw}, PB_{iw}, RB_{iw}, D_{iw}, S_{iw}, Ψ_i, x_{iwr}, AB_{in}, C_w, F_{ivw}, L_i, $ISL_i \geq 0$

5. CASE STUDY

The production and inventory control policy that was proposed in chapter 3 will be evaluated by a case study at a case company. The case company produces ready-to-serve fruit juices and fruit beverages in cartons for the Dutch and Belgian market.

In 2007, the target service level of X% for Y products was not achieved (see table 1). The service level is defined by the case company as the percentage of the customer demanded products that are delivered within the time that is required.

Table 1: Achieved service level in 2007
per product group

< *Deleted due to confidentiality* >

Additionally, 50% of the total disposal costs in 2007, namely € X,-, were due to Y products.

Characteristics of the Y products (i.e. limited shelf life, long replenishment lead time) and of the production system with which the Y products are produced (i.e. limited production capacity) are such that the case company has difficulties in dealing with the lumpiness in the customer demand. In the current situation, the case company uses a production and inventory control policy that is based on minimum and maximum inventory levels. Minimum and maximum inventory is expressed in terms of weeks of expected customer demand. The minimum level for Y products equals 1 and the maximum equals 2.5. For different Y products production can be planned according to pre-specified technically restricted alternative batch sizes.

Lumpy demand

Y products are promoted almost every week of the year. Promotion*s* are defined by communicating to the public in an attempt to influence them toward buying products. The case company does not initiate promotions. Customer demand during a promotion period can be several times greater than customer demand in a week without promotions. Additionally, there is a "post-promotion lag": in weeks after weeks in which a peak in the customer demand volume is observed the customer demand volume drops below the average level. This customer demand is called lumpy demand.

Limited shelf life

The shelf life of Y products is short. Y products have an internal shelf life of three weeks. The internal shelf life is defined as the maximum period that can elapse between production of the product and delivery of the product to the customer. An internal shelf life of three weeks means that all products which cannot be sold to the customer within three weeks after production, have to be disposed.

Limited production capacity

Y products are produced on a production resource with limited capacity. Production capacity is defined as the maximum amount of products that can be produced per time unit utilizing current resources In table 2, average, minimum, and maximum customer demand per week is given for Y products in 2007. Table 3 shows the net production capacity that can be used with and without using planned overtime production. Overtime production capacity is the amount of time that can be used for production beyond the normal production capacity that is available when workers work the working hours that are specified in their contracts. Planned overtime production capacity is used in periods in which peak customer demands are expected. The net production capacity is defined as the production capacity taking into account planned and routine stoppages.

Table 2: Customer demand for Y products in 2007

Customer demand	Units/week
Average	644,134
Minimum	303,000
Maximum	1,188,972

Table 3: Net production capacity for Y products in 2007

Production capacity	Units/week
Average	1,032,083
Maximum overtime	1,330,706

In tables 2 and 3 it can be seen that, on average, customer demand is lower than the available production capacity. However, in weeks with peak demands the average production capacity is lower than the customer demand. Additionally, due to technically restricted pre-specified batch sizes, planned production in a particular period does not represent customer demand in that period. When maximum overtime production capacity is used, in weeks with peak customer demand the production capacity is greater than the customer demand. However, it takes at least two weeks to arrange planned overtime production capacity. Further, the marketing department of the case company targets a 40% sales increase for Y products in 2008. In this situation, maximum overtime production capacity is lower than the maximum customer demand. In 2007, the average production capacity utilization for Y products was 89.5%. Production capacity utilization is defined as the total time that is used divided by the total time that is available on a production resource.

Long and variable replenishment lead time

The replenishment lead time for Y products is long and variable. The replenishment lead time is defined as the time that elapses from the moment at which it is decided to place a production order, until the related products are physically on the shelf ready to satisfy customer demand (Silver et al., 1998). For Y products, when inventory is observed at the beginning of week w, the related production batch is received in week $w + 2$. Unplanned overtime production capacity is used in order to finish production batches that are delayed due to variability in the replenishment lead time. On average, in a week the production capacity is extended by 3% unplanned overtime production capacity. Since the replenishment lead time is longer than the time that customers are willing to wait for their products when they are ordered, all products are produced according to a make-to-stock strategy.

6. EXPERIMENTAL DESIGN AND RESULTS

The new production and inventory control policy is tested by first determining the parameter Ψ_i for every Y product using the heuristic that was given in chapter 4. Then the performance of these parameter settings is tested on a different dataset using simulation. In section 6.1 information on Y products, that is relevant for determining the parameters of the production and inventory control policy and for evaluating the performance of this policy, is given. In section 6.2 it is described how the simulation model was validated and verified. In section 6.3 the different experiments are given and in section 6.4 the results of these experiments are discussed.

6.1 Background

The heuristic is simulated for Y products that were produced on the packaging lines 19 and 20 in the period week 33 2006 until week 6 2008. Y products that were outsourced in this period are not taken into account. The products under analysis are given in table 4. The packaging lines 19 and 20 were dedicated to the products that are given in table 4.

Table 4: Products under analysis

Product	
Y_1	Y_6
Y_2	Y_7
Y_3	Y_8
Y_4	Y_9
Y_5	Y_{10}

Profit is calculated for different values of the variable Ψ_i, from 0 to H, where H equals five. Five is chosen since trial runs showed that for values that are bigger than five the disposal costs are such that a solution that yields the highest profit cannot be generated. Alternative production batch sizes for the different Y products are given in table 5. In order to incorporate quality losses into the model, alternative production batch sizes are multiplied by the average production yield (which equals 99.7%).

Table 5: Alternative production batch sizes for the products under analysis

	Alternative batch size				
Product *i*	**1**	**2**	**3**	**4**	**5**
Y_1	95165	134606	2*95165	95165+134606	2*134606
Y_2	26914	37292	53827	76412	102865
Y_3	33600	59684	89911	127639	
Y_4	87996	108641	124833	166625	
Y_5	45394	94899			
Y_6	52374	81918	101611	122289	
Y_7	48262	86035	105974	130106	
Y_8	46873	46873	46873		
Y_9	36932	36932	36932		
Y_{10}	82292	96656	114434	140680	184941

Empirical customer demand per week and customer demand forecast per week for the period week 33 2006 until week 6 2008 are used as an input for the simulation. The first half of these data is used for determining the order-up-to parameter for every Y product and the second half of these data is used to evaluate the production and inventory control policy. The advantage of using empirical data for simulation is that the model's true characteristics are maintained and that no probability distribution function is used to approximate the system's real characteristics.

Delays in production are incorporated by issuing less customer demand in a particular week from the production batch that is received in that week. On average, every week the production capacity is extended by 3% unplanned overtime production capacity in order to finish production batches that are delayed due to variability in the replenishment lead time. Due to the delay, customer demand can be fulfilled from the inventory, but not from the production batch that is delayed. The percentage of customer demand that can be issued from a particular production batch depends on the fact whether the product is a startup product or not and is given in table 6 (delays in production are taken into account). Startup products are given in table 7. The way in which demand is issued form the inventory available and the way in which production batches are added to the inventory was explained in chapter 4.

Table 6: Percentage of customer demand that can be satisfied from the production batch in a particular week

Product	% of customer demand
Startup	70%
Non-startup	30%

Table 7: Y start-up products

Product
Y_1
Y_2
Y_3

In order to incorporate sequence dependent cleaning times, following Glasserman (1996), sequence dependent cleaning times are subtracted from the available capacity. In 2007, 6.0% of the total time that was available for Y products production was used for planned stoppages, and 13.4% was used for routine stoppages. When no planned overtime production capacity is used, three shifts of 36 hours are available for production. This time is available on both packaging lines. On packaging line 19, the first 10 hours cannot be used for production due to startup of the equipment at the beginning of the week; on packaging line 20 the first hour cannot be used for production due to startup of the equipment. At the end of a week cleaning is needed on both packaging lines which takes 4 hours. The production speed of each packaging line equals 6,500 products/hour. This means that the total production capacity in a week, without planned overtime production equals 1,280,500 products. 19.4% of this time is used for planned and routine production stoppages, so the net capacity that is available in a week without planned overtime production equals 1,032,083 products. In this simulation, the production capacity that is available in a week without planned overtime production is called *setting 1.*
When planned overtime production capacity is used, both packaging lines can be used for production from Monday 2 am until Saturday 6.30 pm. Using maximum planned overtime production capacity, the production capacity in a particular week equals 1,330,706 products. In this simulation the production capacity that is available in a week with maximum planned overtime capacity is called *setting 2*.

The variable c that was introduced in formula (1) is set equal to 1. This means that production can be spread over two weeks in order to anticipate on peak demand periods. When looking at tables 2 and 3, this seems to be an appropriate choice for our data. Due to time constraints, no different settings for c could be evaluated.

Different input parameters for the simulation are given in tables 8 and 9.

Table 8: Input parameters for the products under analysis

Product *i*	gsp_i	cp_i	s_i	h_i	v_i	p_i	q_i
Y_1	€ X	€ X	10%	0.1	€ X	960	€ X,-
Y_2	€ X	€ X	10%	0.1	€ X	960	€ X,-
Y_3	€ X	€ X	10%	0.1	€ X	960	€ X,-
Y_4	€ X	€ X	10%	0.1	€ X	960	€ X,-
Y_5	€ X	€ X	10%	0.1	€ X	960	€ X,-
Y_6	€ X	€ X	10%	0.1	€ X	960	€ X,-
Y_7	€ X	€ X	10%	0.1	€ X	960	€ X,-
Y_8	€ X	€ X	10%	0.1	€ X	960	€ X,-
Y_9	€ X	€ X	10%	0.1	€ X	960	€ X,-
Y_{10}	€ X	€ X	10%	0.1	€ X	960	€ X

Table 9: Input parameters for the system under analysis

Parameter	Value
D	2,650
rc	€ X

6.2 Verification and validation

In order to make sure that our simulation model is free of any programming bugs and performs in the way it is expected, it is verified and validated.

The simulation model in this Master's thesis project was first verified by verifying the corresponding heuristic model. The heuristic model was verified by discussing the formulas with different persons. Once coded, the simulation model was verified using standard functions of Visual Basic and by observing the behavior of specific parts of the system individually.

The simulation model in this Master's thesis project was validated by comparing the output obtained from the simulation model to numbers in practice. Since empirical customer demand data and customer demand forecast data were used for simulation, numbers from practice from the same period as the empirical customer demand data and customer demand forecast data were compared to the output of the simulation model. Since a more advanced production and inventory control method was used in the simulation model, simulation results were expected to be better than results that were achieved in practice in the same period. Further, the simulation model was validated by subjecting it to extreme conditions such as unlimited capacity, unlimited shelf life, and a zero replenishment lead time. The outcomes of the system under these extreme conditions were observed. If the model is an accurate representation of the system, then the results for the extreme cases will be reflected in the form of unacceptable outputs of the simulation. The last method that was used to validate the simulation model was to plot inventory against time and detect any abrupt behavior of the system. All outcomes of the validation were as expected.

6.3 Experiments

Different experiments are executed in order to evaluate the performance of the new production and inventory control policy in the current situation and to determine the impact of the different characteristics of the products and the production system on the profit. The different experiments that are executed are given in table 10.

Table 10: Experiments

Experiment	C	ISL	L
1	1	3	2
2	1	3	1
3	1	3	0
4	1	4	2
5	1	4	1
6	1	4	0
7	1	5	2
8	2	3	2
9	2	3	1
10	2	3	0
11	2	4	2
12	2	4	1
13	2	4	0
14	2	5	2

In the first experiment, the new periodic review order-up-to production and inventory control system is evaluated in the current circumstances by determining the parameter settings that yield the highest profit. *We expect that the disposal costs will be reduced and the service level will be increased since the periodic review order-up-to policy yields near-to-optimal results for both production and inventory control problems of perishable products with a non-zero replenishment lead time and production and inventory control problems of products that are produced on a production resource with limited capacity.*

In the following experiments, characteristics of the products and of the production system are changed. First, in experiments 2 and 3, the replenishment lead time is reduced by one and two weeks. *We expect that reducing the replenishment lead time will reduce the disposal costs and will increase the service level since we can make a production decision later with better customer demand forecasts.* The second law of forecasting applies: the farther in the future we must forecast, the worse the forecast.

In experiment 4, the internal shelf life is increased by one week in the current situation. In experiment 7, the internal shelf life is increased by two weeks. *We expect that increasing the internal shelf life will reduce the disposal costs and will increase the service level since products have to be disposed later and can be used longer to issue customer demand from.*

In experiments 5 and 6, both the internal shelf life is increased and the replenishment lead time is reduced. *We expect that increasing the internal shelf life and reducing the replenishment lead time simultaneously will both reduce the disposal costs and increase the service level due to both the second law of forecasting and the fact that products have to be disposed later and can be used longer to issue customer demand from.*

In experiments 8 until 14, the same settings are evaluated for the production capacity setting 2, that is, the production capacity in every week is such that maximum planned overtime production capacity can be used. *We expect that increasing the production capacity will increase the service level since production is less restricted.*

All experiments as given in table 10 are also executed for the situation in which the forecast error is reduced by 25%. The forecast error is defined by the difference between the customer demand forecast and the actual customer demand. *We expect that reducing the customer demand forecast by 25% will increase the service level and will reduce the disposal costs since better forecasts can be used to make production decisions.*

6.4 Results

The outcomes of the experiments are given in table 11. In the different experiments outcomes were generated for the current situation, for a decrease in the replenishment lead time, for an increase in the internal shelf life, and for an increase in the production capacity.

Table 11: Experiment outcomes

Experiment	**C**	**ISL**	**L**	**Profit** per year	**Disposal costs** per year	**Service level**
1	1	3	3	€ X,-	€ X,-	X %
2	1	3	2	€ X,-	€ X,-	X %
3	1	3	1	€ X,-	€ X,-	X %
4	1	4	3	€ X,-	€ X,-	X %
5	1	4	2	€ X,-	€ X,-	X %
6	1	4	1	€ X,-	€ X,-	X %
7	1	5	3	€ X,-	€ X,-	X %
8	2	3	3	€ X,-	€ X,-	X %
9	2	3	2	€ X,-	€ X,-	X %
10	2	3	1	€ X,-	€ X,-	X %
11	2	4	3	€ X,-	€ X,-	X %
12	2	4	2	€ X,-	€ X,-	X %
13	2	4	1	€ X,-	€ X,-	X %
14	2	5	3	€ X,-	€ X,-	X %

Current situation

In line with our expectations, the new production and inventory control policy increases the service level and decreases the total disposal costs (experiment 1). In table 11 it can be seen that with the new periodic review order-up-to production and inventory control policy a service level of X% is achieved. Compared to practice, the disposal costs are reduced by approximately 50%.

Increased internal shelf life

In line with our expectations, increasing the internal shelf life reduces the disposal costs. However, *in contrast with our expectations*, increasing the internal shelf life does not increase the service level. In table 11 it can be seen that increasing the internal shelf life by one week reduces the disposal costs by approximately € X,- (experiments 1 and 4). However, a lower service level is achieved. When the internal shelf life is increased by two weeks in the current situation, the disposal costs are reduced by approximately € X,- (experiments 1 and 7). Also in this situation a lower service level is achieved. These results are in line with Myers (1997) who found that increasing the internal shelf life of products does not necessarily increase the service level. Myers (1997) concluded that increasing the shelf life does increase the service level when demand in a given period exceeds the combined capacity for that period and the M prior periods, in which M equals the shelf life of the product. In 2006 and 2007 we never found a three week period in which the total customer demand was greater than the available production capacity for Y products.

Reduced replenishment lead time

In line with our expectations, reducing the replenishment lead time reduces the disposal costs. However, *in contrast with our expectations*, reducing the replenishment lead time does not (necessarily) increase the service level. In table 11 it can be seen that reducing the replenishment lead time by one week reduces the disposal costs by approximately € X,- (experiments 1 and 2). However, a lower service level is achieved. When the replenishment lead time is reduced by two weeks the disposal costs are reduced by approximately € X,- (experiments 1 and 3). Also the service level is increased by approximately 1%. Much better results (i.e. disposal costs reductions, service level increases) are achieved when the replenishment lead time is reduced by two weeks than when the replenishment lead time is reduced by one week. When the replenishment lead time is reduced, production decisions can be made later, such that a better customer demand forecast can be used. The law of forecasting applies: the farther in the future we must forecast, the worse the forecast.

In the data of the case company we see that often the customer demand forecast that is made two weeks in advance is not better than the forecast that is made three weeks in advance. The customer demand forecast that is made one week in advance is often better than the forecast that is made two weeks in advance. This explains that better results are achieved when the replenishment lead time is reduced by two weeks than when the replenishment lead time is reduced by one week.

Williams and Patuwo (1999) found for a periodic review policy for a single product with a useful lifetime of two periods, subject to a known positive order lead time and a lost sales policy that the optimal order quantity is a function of the order lead time and the quantity of goods on-hand and on order. When the replenishment lead time is reduced, the optimal order quantity is also different (according to Williams and Patuwo (1999)). However, in our experiments the pre-specified alternative batch sizes were not changed. Therefore, the pre-specified batch sizes are expected to affect our results. Further investigation is needed on how the pre-specified alternative batch sizes restrict the improvements that can be achieved by reducing the replenishment lead time.

Increased production capacity

In contrast with our expectations, increasing the production capacity does not increase the service level. In table 11 it can be seen that increasing the production capacity reduces the service level by approximately 0.7% (experiments 1 and 8). Since in 2006 and 2007 we never found a three week period (i.e. a period equal to the internal shelf life) in which the total customer demand was greater than the available production capacity in our dataset, peaks in customer demand could always be dealt with by producing excess inventory during periods with lower customer demand. Our production and inventory control strategy plans for this. When the production capacity becomes more restrictive (e.g. when the customer demand increases) such that there are three week periods in which the total customer demand is greater than the available production capacity, the impact of increasing production capacity is expected to be different.

Besides the counterintuitive result that increasing the production capacity does not increase the service level, in table 11 we can also see that the disposal costs are increased by approximately € X,- (experiments 1 and 8). Due to the limited production capacity, production of the least critical product is shifted to the next week. In this way the limited production capacity reduces inventory. Lower inventory reduces disposal costs.

A combination of changes in the characteristics of the products and of the production system

In line with our expectations, increasing the internal shelf life and reducing the replenishment lead time simultaneously reduces the disposal costs and increases the service level. In table 11 it can be seen that both increasing the internal shelf life by one week and reducing the replenishment lead time by one week reduces the disposal costs by approximately € X,- (experiments 1 and 5). Also the service level is increased. The effect of reducing the replenishment lead time and increasing the internal shelf life simultaneously is greater than the sum of the effects of increasing the internal shelf life (experiments 1 and 4) and reducing the replenishment lead time (experiments 1 and 2) individually. These results indicate that the internal shelf life and the replenishment lead time interact.

Reducing the forecast error

In line with our expectations, reducing the forecast error reduces the disposal costs and increases the service level. In table 12 the results of the experiments with a 25% lower forecast error are given.

Table 12: Experiment outcomes with 25% reduced forecast error

Experiment	**C**	**ISL**	**L**	**Profit** per year	**Disposal costs** per year	**Service level**
1	1	3	2	€ X,-	€ X,-	X %
2	1	3	1	€ X,-	€ X,-	X %
3	1	3	0	€ X,-	€ X,-	X %
4	1	4	2	€ X,-	€ X,-	X %
5	1	4	1	€ X,-	€ X,-	X %
6	1	4	0	€ X,-	€ X,-	X %
7	1	5	2	€ X,-	€ X,-	X %
8	2	3	2	€ X,-	€ X,-	X %
9	2	3	1	€ X,-	€ X,-	X %
10	2	3	0	€ X,-	€ X,-	X %
11	2	4	2	€ X,-	€ X,-	X %
12	2	4	1	€ X,-	€ X,-	X %
13	2	4	0	€ X,-	€ X,-	X %
14	2	5	2	€ X,-	€ X,-	X %

In table 12 it can be seen that in the current situation, reducing the forecast error by 25% reduces the disposal costs by approximately € X,- (experiment 1, table 11 and experiment 1, table 12). The service level is increased by approximately 0.4% (experiment 1, table 11 and experiment 1, table 12). Further, increasing the internal shelf life, reducing the replenishment lead time and increasing the production capacity have the same effects when the forecast error is reduced by 25% (table 11, 12).

In order to get more insight in the working of our heuristic, in table 13 the Ψ_i's are given for the different products in the current situation.

Table 13: Parameter settings for the current situation

Product	Ψ_i
Y_1	1.6
Y_2	2.0
Y_3	0
Y_4	0.9
Y_5	0.4
Y_6	2.0
Y_7	0.8
Y_8	2.8
Y_9	1.6
Y_{10}	1.6

Structural forecast error

Our production and inventory control policy compensates for structural forecast errors. In table 13 it can be seen that Ψ_i for the different products varies between 0 and 2.8 and does not correlate with the cost price and the gross sales price as given in table 8. Although the cost price for product Y_3 is relatively low, Ψ_i for this product is set equal to 0. Further investigation shows that for this product the customer demand forecast is structurally higher than the actual customer demand (see figure 2). Our heuristic deals with this structurally higher customer demand forecast by setting a low Ψ_i. Since it is unknown if the customer demand forecast will remain structurally higher than the actual customer demand, the calculated Ψ_i's should be updated frequently.

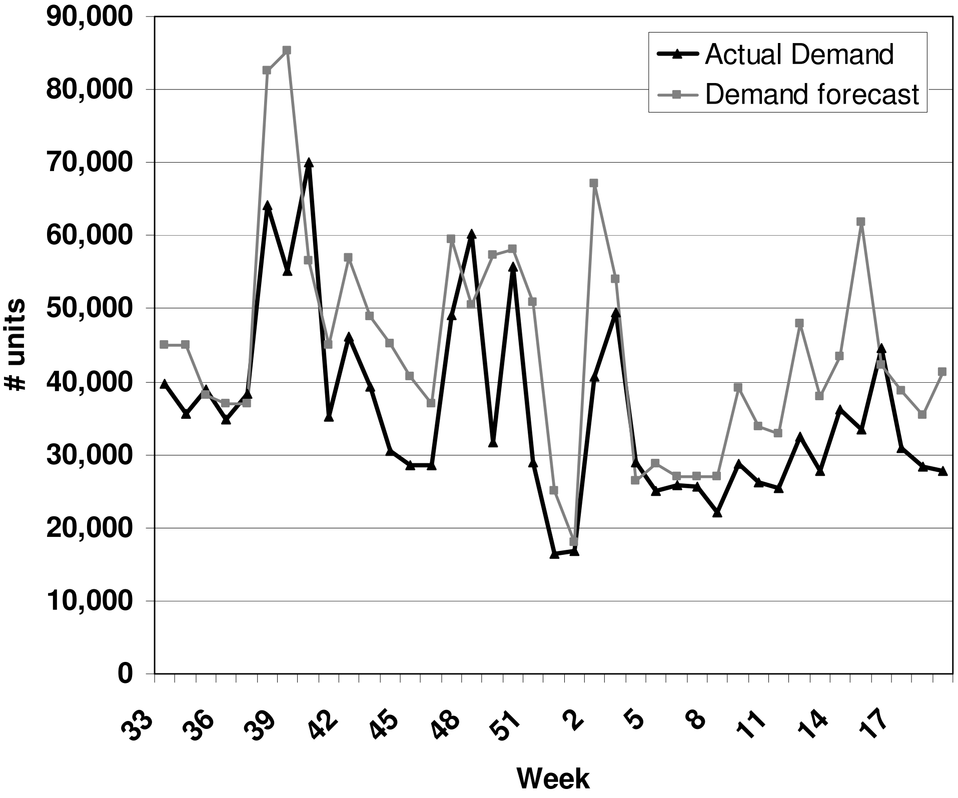

Figure 2: Customer demand forecast and actual customer demand for product Y_3 in week 33 2006 until week 19 2007

For product Y_2 Ψ_i is relatively high. For this product periods can be seen in which the customer demand forecast is structurally lower than the actual customer demand (see figure 3). Our heuristic compensates for this by choosing Ψ_i relatively high.

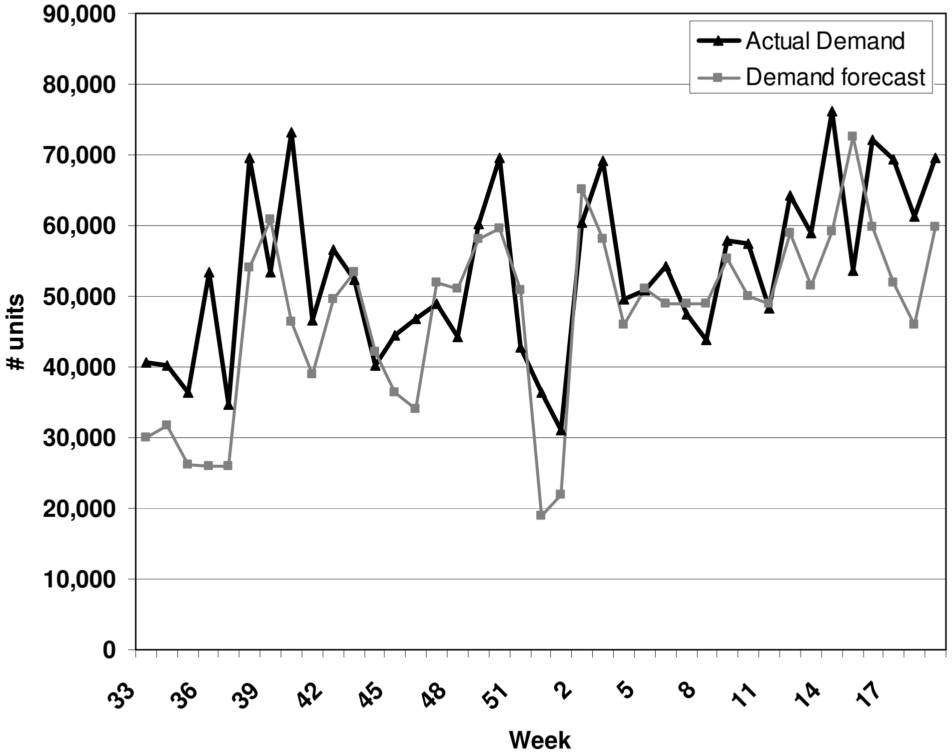

Figure 3: Customer demand forecast and actual customer demand for product Y_2 in week 33 2006 until week 19 2007

Big pre-specified alternative batch sizes

For products with big alternative batch sizes relative to the average customer demand the disposal costs are relatively high. In table 13 it can be seen that for product Y_8 Ψ_i is relatively high while for this product the unit disposal cost is relatively high. For this product the customer demand forecast is not structurally higher or lower than the actual customer demand. The smallest alternative batch size for this product is 1.26 times the average customer demand. As a consequence, in periods with a low average customer demand, big production batches relative to the average customer demand should be planned. The effect of big alternative batch sizes relative to the average customer demand is illustrated using figure 4.

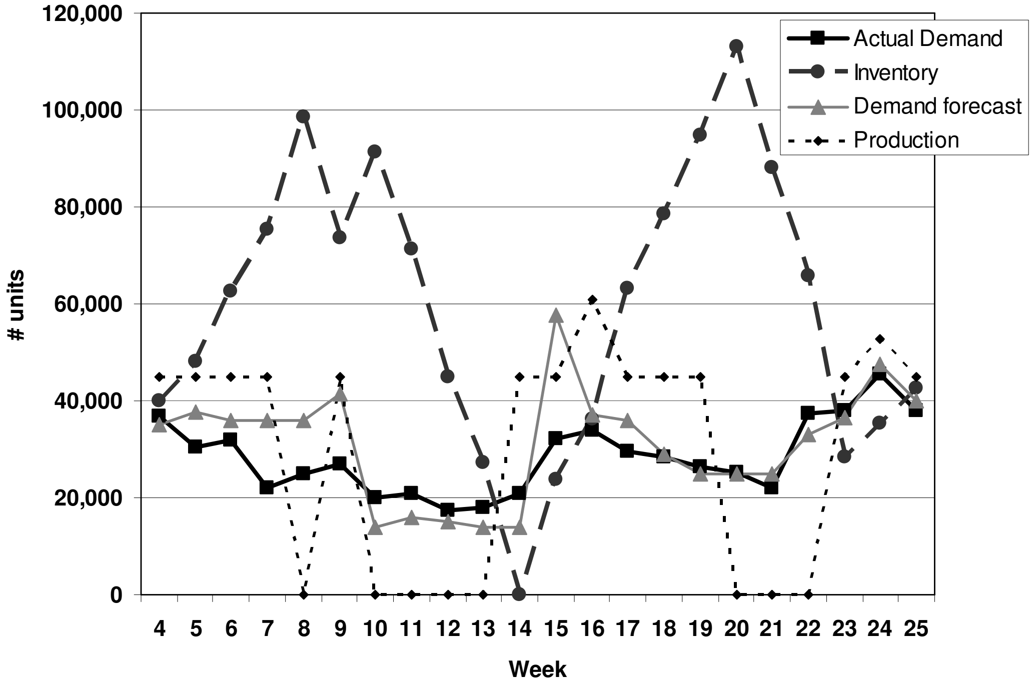

Figure 4: Customer demand forecast, actual customer demand, production, and inventory for product Y_8

In figure 4 we can see that a stock-out occurs in week 15. In this figure Ψ_i is set equal to 1. We see that the production batches that are planned are always greater than the customer demand levels. In week 10 until week 13 no production batches are planned since aggregate inventory is enough to satisfy customer demand from. However, inventory does not only decrease when customer demand is issued, but also when products are disposed. Although inventory is enough to satisfy customer demand from, due to disposal of inventory, in week 14 stock-outs occur. Our method prevents these stock-outs by choosing Ψ_i relatively high for products that have big alternative batch sizes in comparison to the average customer demand level. A higher Ψ_i will prevent that no production is planned four weeks in a row when customer demand is much lower than average. More disposal costs are incorporated with a higher Ψ_i.

Inventory holding costs

For the case company under analysis, disposal costs and lost sales costs are more important than inventory holding costs. However, inventory holding costs have an impact on the Ψ_i's that are chosen. In every experiment, for every product the Ψ_i that yields the highest profit is chosen. The choice of Ψ_i is illustrated in figure 5.

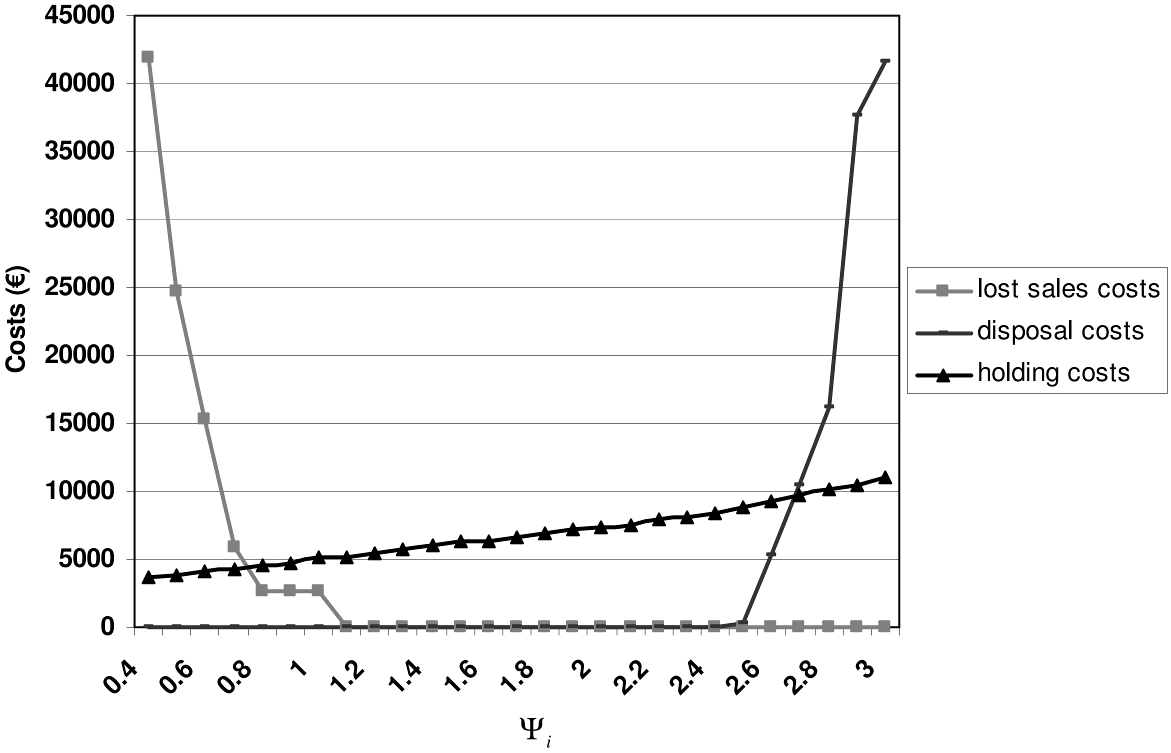

Figure 5: Illustration of the heuristic solution

In figure 5 it can be seen that for low values of Ψ_i the lost sales costs are relatively high and the disposal costs are relatively low. On the other hand, when Ψ_i is high, the disposal costs are relatively high and the lost sales costs are relatively low. There is a range in Ψ_i for which both the lost sales costs and the disposal costs are equal to zero (i.e. 1.2 – 2.6). For the product that is given in figure 5 the heuristic that was developed in chapter 3 will set Ψ_i equal to 1.2 since at this setting for Ψ_i the lost sales costs will be equal to zero and the inventory holding costs will be low. However, the difference in profit between $\Psi_i = 1.2$ and $\Psi_i = 2.6$ equals approximately 0.02% due to an increase in inventory holding costs. In theory, the heuristic sets Ψ_i equal to 1.2 since this value of Ψ_i yields the highest profit; for the situation of the case company the unit lost sales cost, the unit disposal cost, and the unit inventory holding cost are such that the unit inventory holding cost is practical irrelevant to the solution. Therefore, for the case company Ψ_i can be chosen such that Ψ_i lies between 1.2 and 2.6 based on the unit lost sales costs and the unit disposal cost in order to achieve a higher service level.

7. CONCLUSIONS AND RECOMMENDATIONS

In this chapter, the main conclusions of this project are summarized (section 6.1) and recommendations are given for the case company (section 6.2).

7.1 Conclusions

The aim of this Master's thesis project was to find a practical production and inventory control policy that makes a trade-off between spill and shortages in a lumpy demand environment with production capacity limitations and perishability of products. A periodic review order-up-to policy was introduced and a heuristic was developed to determine the order-up-to parameter of this policy for every product. A case study was used to evaluate the new production and inventory control policy. Based on this case study general conclusions can be drawn on production and inventory control for products with a fixed shelf life which are subject to lumpy demand and produced to stock on a production resource with limited capacity.:

- Our periodic review order-up-to production and inventory control policy yields good results. Our policy yields good results due to the following reasons:

    1. The order-up-to level is adjusted in the course of time by expressing the order-up-to level in terms of customer demand forecasts in the next $R + L + c$ weeks. In this way inventory can be build up in periods with lower demand for periods with peak demand.
    2. Production capacity is allocated among products by not producing the least critical product until the production capacity constraint is satisfied.
    3. The order-up-to parameters are determined using empirical customer demand and customer demand forecast data, such that true customer demand characteristics are incorporated.
    4. The heuristic compensates for structural forecast errors.
    5. The heuristic compensates for the sizes of the alternative batches.

- Increasing the internal shelf life reduces the disposal costs but does not necessarily increase the service level

- Reducing the replenishment lead time reduces the disposal costs and increases the service level when better customer demand forecasts can be used for production decisions

- Increasing the production capacity does not reduce the disposal costs and does not necessarily increase the service level

- The internal shelf life and the replenishment lead time interact

- Reducing the forecast error by 25% reduces the disposal costs and increases the service level

Further, conclusions can be drawn on the working of our heuristic:

- Inventory holding costs have an impact on the parameters that are chosen

- The heuristic compensates for structural forecast errors

- For products with big alternative batch sizes relative to the average customer demand, the parameter that is chosen will be relatively high. As a result, disposal costs for products with relatively big alternative batch sizes are relatively high.

7.2 Recommendations

Based on the general conclusions that were drawn in the previous section, we recommend to the case company to do the following:

- Implement the periodic review order-up-to production and inventory control policy that was researched in this Master's thesis project in order to increase the service level and to reduce the disposal costs.

- As a next step, first determine the parameters without taking into account inventory holding costs, since the inventory holding costs are not important for the case company. First increase Ψ_i for products with a relatively low unit disposal cost (Metters, 1998b)

- As a second next step, reduce the forecast error in order to reduce the disposal costs and to increase the service level by using forecast methods for lumpy demand.

- Research the impact of the production capacity when the customer demand for Y products will increase by 40%: when there are three week periods in which the total customer demand is greater than the available production capacity, the impact of increasing the internal shelf life and of increasing the production capacity will be different.

- Reduce the alternative batch sizes for products with big alternative batch sizes relative to the average customer demand.

7.3 Future research

More sophisticated and complex heuristics could, no doubt, yield better results. The heuristic in this Master's thesis project is chosen for its simplicity and use in practice. The heuristic that was developed in this Master's thesis project can be improved on the following points:

- Execute the heuristic faster by making use of the Newton–Raphson method. A description of the Newton–Raphson procedure can be found in Williams and Patuwo (1999) and Williams (1995).

- Investigate the impact of the pre-specified alternative production batch sizes on the performance of the order-up-to strategy. Due to the pre-specified alternative production batch sizes, the order-up-to level can never be exactly reached. Production decisions are taken such that an alternative production batch is planned that raises the inventory level up to a level that is greater than or equal to the order-up-to level. The impact of the pre-specified alternative production batch sizes can be investigated by executing the same experiments without pre-specified production batch sizes.

- Investigate what the impact is on the service level and on the disposal costs of increasing the internal shelf life and of increasing the production capacity when there are periods of the length of the internal shelf life of the products in which the total customer demand exceeds the total available production capacity.

List of definitions

Baseline demand = Demand that would occur without promotion activities initiated by customers.
Best before date = The date after which a good taste cannot be guaranteed.
Bottleneck = One process in a chain of processes, such that its limited capacity reduces the capacity of the whole chain.
Degrees Brix (˚Bx) = A measurement of the mass ratio of dissolved sucrose to water in a liquid.
Disposal costs = the amount of money incurred for the action of getting rid of products that are not sold within the ISL period
End product = A product, which has undergone all processing steps and is ready for selling to the customer. In this report an end products is denoted by product
Forecast error = The difference between the customer demand forecast and the actual customer demand.
Incremental demand = demand that occurs as a result of promotion activities initiated by customers.
Internal shelf life = The maximum period that can elapse between production of the end product and delivery of the end product to the customer
Inventory holding costs = The cost of carrying items in inventory (includes opportunity costs of the money invested and the expenses incurred in running a warehouse)
Inventory level = The total inventory that is on-hand plus on order.
Lumpy demand = Demand that has very pronounced peaks that can be several times greater than average demand
Net production capacity = The production capacity taking into account planned and routine stoppages.
Non-deliveries = Products that can not be delivered to the customer within the time that is required.
Order time = the time that elapses between observing the on-hand stock level and the ordering of raw materials
Overtime production capacity = The amount of time that can be used for production beyond the normal production capacity that is available when workers work the working hours that are specified in their contracts
Perishable product = A product with a fixed finite lifetime
Product group = A group of end products that are similar based on one or more characteristics
Production and inventory control policy = The set of policies that monitor levels of inventory and determine what levels should be maintained, when stock should be

replenished, and how large production batches should be.

Production capacity = The maximum amount of products that can be produced per time unit utilizing current resources.

Production capacity utilization = The total time that is used divided by the total time that is available on a production resource.

Promotion = Communicating to the public in an attempt to influence them toward buying products.

Raw materials lead time = the time that elapses between the point at which the first raw material is ordered and the point at which production can start.

Replenishment lead time = The time that elapses from the moment at which it is decided to place a production order, until the related products are physically on the shelf ready to satisfy customer demand

Review period = The time that elapses between two subsequent observations of inventory

Rush order costs = Costs that are made for reducing the replenishment lead time

Safety stock = The amount of inventory in the DC to allow for the uncertainty of demand and the uncertainty of supply in the short run

Service level = the percentage of the customer demanded products that are delivered within the time that is required.

Shelf life = The time that elapses between production and the best before date

Simulation = The imitation of reality for studying the effect of changing parameters in a model as a means of preparing a decision.

Stock-out = A stock-out occurs when FF is not able to deliver the product directly from stock.

Time until obsolescence = The time between the point that the first raw material is pumped into the receiver tank until the time when the production batch is pasteurized.

Total demand = Baseline demand plus incremental demand

List of variables

AB_{in} = Alternative production batch size n for product i
AWP = Total average profit per week
AWP_i = Average profit per week for product i
β_{iw} = Dummy that indicates if product i is produced in week w or not
cp_i = Cost price for product i
C_w = Net production capacity in week w
C = Production capacity constraint that should be satisfied
d_i = Unit disposal cost for product i
D = Storage capacity of the warehouse
D_{iw} = Customer demand for product i in week w
F_{ivw} = Customer demand forecast for product i made in week v for demand in week w
γ_w = Dummy that indicates whether the storage capacity restriction in week w is satisfied or not
Ψ_i = The number of weeks of expected demand in which the order-up-to level
S_{iw} is expressed
gsp_i = Gross sales price for product i
h_i = Annual unit inventory holding cost for product i
I_{iw} = Total inventory for product i at the beginning of week w
ISL_i = Internal shelf life of product i
L = Replenishment lead time
ND_{iw} = The non-deliveries (in CU) for product i in week w.
p_i = Pallet conversion factor for product i
PB_{iw} = Planned production batch size for product i in week w
q_i = Production start up cost for product i
rc = Cost to store one pallet in a different warehouse for one week
R = Review period
RB_{iw} = Received production batch size for product i in week w
s_i = Unit lost sales cost for product i
S_{iw} = Order-up-to level for product i in week w
SL = Overall service level
SL_i = The service level that is achieved for product i
v_i = Rate for keeping one pallet in the warehouse for one week
x_{iw} = inventory state for product i at the beginning of week w
$\mathrm{x}_{\mathrm{iwr}}$ = the number of CUs of product i with an age of r weeks at the beginning of week w

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