

MASTER

Comparison of three eye movement recording techniques for the clinical vestibular practice electrooculography, video oculography and scleral search coils

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Comparison of three eye movement recording techniques for the clinical vestibular practice: Electrooculography, video oculography and scleral search coils.

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Summary

In this report a technique is presented to improve the temporal resolution of low-frequency online video oculography. Since saccades are the fastest eye movements one can make, an accurate recording of these movements requires the highest sampling frequency. It is shown that saccades can in good approximation be considered to be bandwidth limited with an upper frequency limit of 25-30 Hz; small amplitude saccades having a higher bandwidth than large amplitude saccades. This means that the 50 Hz sampling frequency of on-line video oculography is in theory sufficient to prevent aliasing. The properties of three eye movement recording techniques are investigated: electrooculography (EOG), video oculography (VOG) and the scleral search coil (SSC) technique. For this purpose, horizontal and vertical saccades of varying amplitude are simultaneously recorded with all three techniques. Both the static and dynamic properties are examined. Fixation position are well-correlated for the VOG and SSC system, whereas for the EOG system the correlation with the VOG and SSC is significantly less due to changes in skin impedance. The measured peak velocities of the saccades are determined for all three recording techniques. For the EOG signal it is shown that the asymmetric placement of the temporal and nasal electrode results in erroneous values for the peak velocity. Moreover, it is shown that the low-frequency video evetracker yields reliable values for the saccadic peak velocity, which are equal to or even somewhat higher than those of the scleral search coil technique. This discrepancy is assumed to be a result of lens slippage of the coil.

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List of Abbreviations

AC	anterior canal
CMOS	complementary metal oxide semiconductor
DFT	discrete Fourier transform
EOG	electrooculography
IDFT	inverse discrete Fourier transform
IR	infra-red
IROG	infra-red oculography
LC	lateral canal
OKN	optokinetic nystagmus
Р	Purkinje image
PC	posterior canal
rms	root mean square
SSC	scleral search coil
VET	video eyetracker
VOG	video oculography
VOR	vestibulo-ocular reflex

Chapter 1 Introduction

Measurement of eye movements plays an important role in the diagnosis of vestibular disorders. Looking at a subject's eyes without optical aids can only result in very crude qualitative observations. In order to observe and quantify eye movements in detail, several techniques to measure eye movements have been developed. With the existence of these different techniques, the question arises which method is suited best for the diagnosis of vestibular disorders.

When comparing the different methods, a distinction has to be made between clinical and technical aspects; the method which yields the highest sensitivity and accuracy in theory may not be applicable in clinical practice¹. Important clinical issues are the patient's comfort, the suitability for routine evaluations, the degree of restriction of the visual field, the possibility to measure with and without visual stimuli, the question whether the method is appropriate for patients with specific disorders, the possibility of on-line recording and analysis and the working domain of the method. Technical aspects can be divided into static and dynamic aspects. Some important static aspects are the calibration, linearity, reproducibility and spatial resolution of the method, whereas temporal resolution and bandwidth, sometimes restricted by the limited sampling frequency, are dynamic aspects.

Despite the fact that during the last decades several investigations have been performed in which the different systems for measuring eye movements were compared, still confusion and disagreement exists among physicians about the question which method is suited best for the diagnosis of disorders. The aim of this study is to investigate the properties and suitability for clinical vestibular practice of three eye movement recording methods: electrooculography, video oculography and scleral search coils.

First, the relevance of the investigation of eye movements for the diagnosis of vestibular disorders is presented in chapter 2. In chapter 3, the most frequently used eye movement recording techniques are presented. For these techniques the principles of measurement are described together with their properties. Then, in chapter 4 a technique is presented to increase the temporal resolution of the video system. In this chapter also the differentiation of the eye position signal with respect to time is discussed. Chapter 5 deals with the measurement protocols and the data processing of each technique. The results are shown in chapter 6 and are discussed in chapter 7. In chapter 8 the conclusions are presented. Finally, chapter 9 deals with the recommendations for future studies.

 $^{^{1}}$ Carpenter [Carpenter, 1988] defined the sensitivity of a measuring system as the smallest eye movement that can be measured and accuracy as the nearness of the calculated, absolute eye position to the actual absolute eye position.

Chapter 2

Vestibular system

In man, the two vestibular organs are located in the bony labyrinth of the inner ear, one in each inner ear. The membranous labyrinth is surrounded by perilymph and situated in the bony labyrinth. The membranous labyrinth is filled with endolymph which has a density slightly greater than that of water. Each vestibular labyrinth can be divided in two functionally and anatomically distinct parts: the three semicircular canals and the otolith organs, the utricle and the saccule. The three semicircular canals detect angular accelerations of the head, the utricle and the saccule linear accelerations. The vestibular apparatus is the primary sensor for the detection of the orientation and movement of the head in space. This information is of vital importance for the ability to maintain balance during both locomotion and rest, the orientation in space and the stabilization of images on the retina during head movements. Image stabilization is controlled by the so-called vestibulo-ocular reflex (VOR).

Although the vestibular system is the primary sensor for head movements, information about these movements is also provided by the visual system, the proprioceptive system and the auditory system. Information from these systems is integrated in the cerebellum and the vestibular nuclei [Kingma et al., 2003]. A proper integration of this information results in an adequate interpretation of head movement and stabilization of gaze.

2.1 Statolith organs

The two otolith organs, the utricle and the saccule, are covered with sensory epithelium, the maculae. The sensory epithelium of the utricle forms an angle with that of the saccule of about 90° [Kingma et al., 2003]. The hair cells are located in the maculae and are in good approximation arranged in a plane. Each hair cell contains about 30-200 stereocilia which are oriented in rows of ascending height, the longest one is called the kinocilium. The stereocilia and kinocilia of the hair cells in the maculae are situated in a gelatinous mass. On top of this mass, relatively heavy statoconia are located $\rho_m = 2.7$ g/ml. When the head experiences a linear acceleration, the statoconia lag behind the bottom side of the maculae due to inertia which causes a deflection of the stereocilia. This deflection of the stereocilia results in an opening or closing of the transduction channels of the hair cells, which changes the membrane potential of the hair cells and, as a result, its firing rate. This is shown in figure 2.1.

Deflection of the stereocilia towards the kinocilium leads to excitation (depolarization) of the hair cell, whereas deflection of the stereocilia in the opposite direction leads to inhibition (hyperpolarization). A healthy vestibular system has a resting firing rate of about 70 to 100 spikes per second and a resting membrane potential of -50 to -70 mV [Vrijling, 2004]. Hair cells are more sensitive to excitation than to inhibition. Besides linear accelerations, the statolith system also detects the position of our head with respect to gravity. On physical grounds, no distinction can be made by the statolith system between linear accelerations and tilted head positions.



Figure 2.1: Schematic representation of the function of a hair cell. Deflection of the stereocilia results in a changing firing rate in the nervus vestibularis.

2.2 Semicircular canals

The three semicircular canals, the lateral (horizontal), posterior (inferior) and anterior (superior) canals, detect angular accelerations of the head. The three canals are aligned in such a way that their corresponding planes are approximately orthogonal as shown in figure 2.2.

In upright position, the lateral canals form an angle of about 30° with the horizontal (axial) plane. The two vertical canals, posterior and anterior, form an angle of approximately 45° with the sagittal plane. Each semicircular canal consists of about two-third of a planar torus and has an enlargement of one end that is called the ampulla [Vrijling, 2004]. Inside the ampulla, the cupula is located. The cupula is a gelatinous barrier that encloses the connection of the semicircular canal to the utricle on one end. The other end of the canal is in open connection with the utricle. Within the cupula, the hair cells are situated. The kinocilia and stereocilia of the hair cells are seated in the crista ampullaris.

When the head is rotated, the endolymph lags behind the rotation of the head due to its inertia. This causes a deflection of the cupula and thus a deflection of the stereocilia, resulting in a change of its firing rate. When excitation occurs in one labyrinth caused by a movement of the head, inhibition occurs in the other due to the mirror geometry of the two labyrinths. For example, if excitation occurs in the left anterior semicircular canal, inhibition occurs in the right posterior canal. The signals from the left and right semicircular canal afferents are combined in the vestibular nuclei in a push-pull configuration to decrease the amount of asymmetry and to increase the sensitivity of the signal [Vrijling, 2004].

2.3 The vestibulo-ocular reflex

In order to prevent vision from blurring during head movements, it is important to stabilize images on the retina. During head movement, this is accomplished by rotating the eyes at a velocity that has the same magnitude, but the opposite direction with respect to the velocity of the head. These eye movements are caused by reflex mechanisms.

When the head movement is prolonged, the eyes move conjugately in the opposite direction and then quickly reset in the direction of the head movement. After this reset, a new region of gaze will be stabilized on the retina. This rhythmic alternation of conjugate (slow compensatory) and fast-phase eye movements is called nystagmus.

Stabilization of gaze is achieved by eye movement reflexes caused by both visual (optokinetic nystagmus: OKN) and vestibular (Vestibulo-ocular reflex: VOR) information. The visual system



Figure 2.2: Orientation of the three semicircular canals. A: Sagittal cross-section, B: Axial cross-section. The lateral canals (LC) are situated in a plane at an angle of about 30° with the axial plane and the posterior (PC) and superior (AC) canals form an angle of approximately 45° with the sagittal plane.

has a latency time of about 70 ms and is therefore not capable of image stabilization during fast head movements. Since the vestibular signal is sent to the oculomotor nuclei via a direct pathway, the so-called three-neuron arc, the system has a latency time of approximately 10 ms [Huygen and Kingma, 1993]. Therefore, gaze stabilization during fast head movements is mainly achieved by the vestibulo-ocular reflex.

Chapter 3 Measurement of eye movements

When measuring eye movements, an important distinction has to be made between the two- and three-dimensional eye position (2D and 3D, respectively). In this report, the 2D eye position corresponds with the direction of the line of sight. In order to uniquely characterize the 3D eye position, also a rotation around the line of sight, the so-called torsion, is necessary. For the measurement of the 3D trajectory of the eye, the time-dependent eye position needs to be determined with respect to a chosen reference position. Usually, this position is defined as the direction in which the eye is positioned when the subject is looking straight ahead with the head in upright position.

To describe the 3D position of the eye in space, it is first important to discuss the differences between a head-fixed and an eye-fixed coordinate system [Haslwanter, 1995]. Let $\{\vec{h}_1, \vec{h}_2, \vec{h}_3\}$ be a right-handed head-fixed coordinate system such that \vec{h}_1 coincides with the line of sight when the eye is situated in the reference position. Let $\{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$ be an eye-fixed coordinate system, meaning that the coordinate system moves with the eye, and let $\{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$ coincide with $\{\vec{h}_1, \vec{h}_2, \vec{h}_3\}$ in the reference position as shown in figure 3.1^1 .



Figure 3.1: The position of the eye can be described in both a head-fixed coordinate system $\{\vec{h}_1, \vec{h}_2, \vec{h}_3\}$ and an eye-fixed coordinate system $\{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$.

 $^{^{1} \}rm Adapted\ from\ http://web.mit.edu/bcs/schillerlab/research/B-EyeMovements/B1-3.html.$

A rotation of the eye-fixed coordinate system from the reference position can be described by

$$\vec{e_i} = \mathbb{R} \cdot \vec{h_i}, \qquad i = 1, 2, 3, \tag{3.1}$$

where \mathbb{R} is the so-called rotation matrix. The components of the vectors $\vec{e_i}$ are expressed relative to the head-fixed coordinate system. From figure 3.1 it can be seen that the orientation of $\vec{e_1}$ specifies the direction of gaze. Haslwanter [Haslwanter, 1995] demonstrated that rotations about head-fixed axes and rotations about eye-fixed axes in the reverse sequence lead to the same final eye position.

3.1 The Fick and Helmholtz system

An important distinction between a head-fixed and an eye-fixed coordinate system is the orientation of the axes. In a head-fixed coordinate system, the orientation of the axes of rotation is independent of the current eye position. In an eye-fixed coordinate system, the axes of rotation depend on the eye position, as can be seen from figure 3.1. Each rotation about an eye-fixed axis changes the orientation of the eye-fixed coordinate system with respect to the head-fixed coordinate system. Rotations about head-fixed axes are called active rotations. Rotations about eye-fixed axes are called passive rotations. Since for the human eye the orientation of the axes of rotation depends on the current eye position, generally eye-fixed coordinate systems are used. When using an eye-fixed coordinate system, the rotation of the eye is in fact described by three independent rotations: a horizontal (around \vec{e}_3), vertical (around \vec{e}_2) and a rotation around the line of sight \vec{e}_1 . In appendix A it is shown that when the eye position is described by consecutive rotations about eye-fixed axes, the final eye position depends on the order in which these rotations are performed. As a result, it is important to specify the sequence of the consecutive rotations; the coordinate system is non-commutative.

In the Fick coordinate system, the eye position is characterized by a horizontal rotation, followed by a vertical and then by a torsional rotation. The horizontal, vertical and torsional rotations in this system are denoted by θ_F , ϕ_F and ψ_F respectively. The rotation matrix corresponding to the Fick-sequence of rotations is

$$\mathbb{R}_{Fick} = \mathbb{R}_3(\theta_F) \mathbb{R}_2(\phi_F) \mathbb{R}_1(\psi_F). \tag{3.2}$$

As remarked in the previous section, for an eye-fixed coordinate system the reversed sequence of rotations lead to the final eye position. As a result, the left rotation matrix on the right-hand side of equation (3.2) describes the first rotation, the second matrix the second rotation and the matrix on the right the last rotation.

The Fick-sequence of rotations is arbitrary. An alternative eye-fixed coordinate system is the so-called Helmholtz system. In this system the eye position is characterized by a vertical rotation, followed by a horizontal and then a torsional rotation. The rotation matrix corresponding to the Helmholtz-sequence is

$$\mathbb{R}_{Helmholtz} = \mathbb{R}_2(\phi_H) \mathbb{R}_3(\theta_H) \mathbb{R}_1(\psi_H).$$
(3.3)

It is important to keep in mind that the eye position is characterized by the value of the rotation matrix \mathbb{R} and that \mathbb{R}_{Fick} and $\mathbb{R}_{Helmholtz}$ only give different parametrizations for the same rotation matrix \mathbb{R} .

An interpretation of the elements of \mathbb{R} can be found by looking at equation (3.1). The columns of \mathbb{R} represent the vectors of the eye-fixed coordinate system $\{\vec{e_1}, \vec{e_2}, \vec{e_3}\}$ expressed in the headfixed coordinate system $\{\vec{h_1}, \vec{h_2}, \vec{h_3}\}$. This means that different values in the rotation matrix \mathbb{R} indicate a different orientation of the eye.

In appendix B is shown that by using a coordinate system of so-called rotation vectors, the problem of non-commutativity can be solved.

3.2 Methods of measuring eye movements

3.2.1 Electrooculography (EOG)

The principle of electrooculography is based on the fact that the cornea is positively charged with respect to the retina. This results in a potential difference of about 1 mV between the cornea and the retina. This means that the eye acts like an electric dipole with dipole moment p [Feynman et al., 1989]

$$p = qd, \tag{3.4}$$

with q the electric charge and d the separation of the charges. An electrode placed at a position \vec{r} in space will measure a potential V given by

$$V(\vec{r}) = \frac{1}{4\pi\epsilon} \frac{\vec{p} \cdot \vec{r}}{r^3},\tag{3.5}$$

with ϵ the so-called permittivity and r the distance between the dipole and the electrode.

Vertical eye movements are recorded from electrodes placed above and below the eyes. Horizontal eye movements can be measured from two bitemporal electrodes, placed on each temple near the outer canthus of the eye, which provides a summation of movements of both eyes together. A binocular recording can be preformed by placing two sets of electrodes nasally and temporally if horizontal eye movement recordings of each separate eye are desired [Barber and Stockwell, 1980].

The electrodes typically record a potential difference of about 20 μ Vdeg⁻¹ [Carpenter, 1988]. The smallest eye movement that can be detected varies per subject but is typically in the order of 0.5° [Baloh and Honrubia, 1990]. The advantages of this method are that it is non-invasive, the signal can be sampled at a relatively high frequency of 1 kHz, a wide range of eye movements can be recorded (±80° for horizontal and ±50° for vertical eye movements with a linearity up to ±30° horizontal and ±20° vertical [Young and Sheena, 1975, Kumar and Krol, 1992]), recordings can be made when the eyes are closed, it is unaffected by movements of the head, and relatively cheap.

However, the technique is far from ideal due to a number of disadvantages. First, only twodimensional eye movements can be recorded, as rotation of the eye around the optical axis does not induce observable changes in the EOG signal due to symmetry of the dipole. Furthermore, it is known that the EOG signal is not constant in magnitude but depends on the metabolic state of the eye (for instance temperature, pCO_2 , etc.) and visual stimulation. North [North, 1965] investigated the accuracy and precision of electrooculography and the effect of variations in illumination. He found a damped oscillation (with a period of approximately half an hour) evoked by an increase of illumination, which can persist for more than one hour, even when the illumination is kept constant at the new level. When the system is frequently calibrated, the accuracy of EOG is typically $1 - 2^{\circ}$ [Young and Sheena, 1975, Kumar and Krol, 1992, Baloh and Honrubia, 1990].

Another disadvantage is the presence of noise caused by muscle activity, eyelid closure and changes of skin impedance. A slow change in skin impedance can be seen as baseline drift. In order to remove the high frequency noise from the signal, low-pass filtering is required, which results in a limited bandwidth² of about 60 Hz [Reulen et al., 1988].

Finally, the placement of the electrodes plays an important role. When the electrodes are not properly aligned, the line of interconnection between the two electrodes will have a component in the direction orthogonal to the direction of interest, resulting in cross talk between the horizontal and vertical channels. Moreover, when each eye is recorded separately, asymmetric signals for rightward and leftward eye movements are often found. This phenomenon is sometimes ascribed to the fact that the distance from the nasally placed electrode to the centre of the eyeball is greater than that of the temporally placed electrode [Kumar and Krol, 1992]. However, in appendix C it is shown that the difference in distance from the eye to the both electrodes will not result in an asymmetric EOG signal. A possible explanation for the asymmetric EOG signal that is suggested

 $^{^{2}}$ According to the definition, the bandwidth of a filter is that frequency at which the frequency response of the filter is attenuated by 3 dB. However, the maximum bandwidth of a recorded signal is always equal to or less than half the sampling rate at which the signal was recorded, as will be shown in section 4.2.1.



Figure 3.2: Schematic representation of a monocular EOG recording of the left eye under the assumption that the reference position θ_0 of the eye makes an angle θ with the eye position θ' at which the potential difference between the electrodes E_t and E_n is zero.

in this report can be obtained by looking at figure 3.2. In this figure a representation is shown of a monocular EOG recording of the left eye. Since the temporal electrode E_t is situated somewhat behind the eye whereas the nasal electrode E_n is located effectively in front of the eye, the reference position θ_0 of the eye makes an angle θ with the eye position θ' at which the potential difference between the electrodes is zero. From equation (3.5) it follows that the potentials measured by the temporal and nasal electrodes, V_t and V_n respectively, are given by

$$V_t(\alpha) = -\frac{p}{4\pi\epsilon r^2}\sin(\theta + \alpha), \qquad (3.6)$$

$$V_n(\alpha) = \frac{p}{4\pi\epsilon r^2}\sin(\theta + \alpha).$$
(3.7)

For the potential difference between the temporal and nasal electrodes of the left eye ΔV_l it follows that

$$\Delta V_l(\alpha) = V_n(\alpha) - V_t(\alpha) = \frac{p}{2\pi\epsilon r^2}\sin(\theta + \alpha).$$
(3.8)

Since from the geometry of the head it can be estimated that θ is in the order of $20^{\circ} - 30^{\circ}$ it follows that

$$\Delta V_l(-\alpha) \neq -\Delta V_l(\alpha), \tag{3.9}$$

which results in an asymmetric EOG signal. For the right eye it follows that the measured potential difference V_r is given by

$$\Delta V_r(\alpha) = V_t(\alpha) - V_n(\alpha) = \frac{p}{2\pi\epsilon r^2}\sin(-\theta + \alpha).$$
(3.10)

Note that in order to have an eye rotation to the right correspond with an increasing potential difference between the electrodes, for the right eye the potential of the temporal electrode is subtracted from that of the nasal electrode.

3.2.2 Oculography by scattering

Scattering oculography uses a spot of (preferably infra-red to avoid visual perception and stimulation) light that is projected on the corneal-scleral junction, called the limbus, and a nearby photo detector that can pick up the scattered light, as shown in figure 3.3. This technique is based on



Figure 3.3: Principle of oculography by scattering. Taken from [Carpenter, 1988].

the fact that the white sclera reflects light to a greater extent than the darker iris. Under the assumption that the beam of light that is directly reflected on the surface of the eye is not received by the photo detector, the amount of light that is detected will be more or less proportional to the area of sclera lying under the spot of light, which in turn is proportional to the angular deviation of the eye for small eye movements. The time resolution is better than 1 ms [Carpenter, 1988], which means a sampling frequency of more than 1 kHz.

With the use of two narrow lines of light at right angles and two photo detectors, it is possible to record both horizontal and vertical eye movement differentially at the same time. It is claimed that this improves both the linearity and reduces noise, caused by fluctuations in the illumination of the eye. The sensitivity can further be increased by using chopped illumination and phase locked amplification to make the detection of the eye movements independent of noise and disturbances from surrounding light sources [Frietman et al., 1984]. This setup gives a sensitivity of 3' of arc for horizontal movements, and 10' of arc for vertical movements [Carpenter, 1988] and has a field of view of about $\pm 18^{\circ}$ in horizontal and $\pm 8^{\circ}$ in vertical direction.

Reulen et al. [Reulen et al., 1988] and later Kumar and Krol [Kumar and Krol, 1992] developed a system that uses a horizontal array of infra-red LEDs in combination with an array of phototransistors placed parallel to and below the LED array to record horizontal movements. Furthermore a pair of phototransistors is mounted above the LED array to record vertical movements. The system uses a sampling frequency of 500 Hz. In order to suppress high frequency noise, a low-pass filter is set at 100 Hz. The sensitivity of the system is 1' of arc for an artificial eye, with an accuracy of 6'. For a real human eye the sensitivity is 6' of arc (accuracy not quantified). The horizontal and vertical range are $\pm 30^{\circ}$ and $\pm 20^{\circ}$, respectively. Important disadvantages of this technique are that the surrounding illumination must be quite dim to obtain a good signal-to-noise ratio and that head movement relative to the photo detector will be interpreted as eye rotation, so the system must be tightly fixed to the head.

3.2.3 Reflection oculography

Reflection oculography makes use of the fact that an incident ray of light, impinging on the outer surface of the eye, is reflected on various surfaces of different refractive index when passing through the different eye components. The incident beam is first reflected at the outer corneal surface, which results in the first Purkinje image. Further reflections occur at the cornea-anterior chamber, the anterior chamber-lens and the lens-vitreous transition surfaces. Together, these reflections form the four Purkinje images as shown in figure 3.4 [Carpenter, 1988]. When the eye rotates, the position of these Purkinje images will move as well. By recording the position of the first Purkinje image, which has the highest light intensity, the rotation of the eye can be measured.

Advantages of this method are that the brightness of the image and the contrast between the image and the surroundings can be made very large, the method is non-invasive, and both



Figure 3.4: Principle of oculography by reflection. The incident beam I is reflected on various surfaces of different refractive index forming the four Purkinje images P1, P2, P3 and P4. Taken from [Carpenter, 1988].

horizontal and vertical eye movements can be recorded. Disadvantages are that this technique is extremely sensitive to movements of the head relative to the measuring device, and depends on the geometry of the eye, which differs for different subjects. The influence of head movement can be reduced by measuring both the first Purkinje image (P1) and the fourth Purkinje image (P4), the reflection that occurs at the posterior surface of the lens. This is the so-called double Purkinje reflection technique. Since P1 is formed by a convex surface and P4 by a concave surface, P1 and P4 move by equal amounts under translation. Under rotation they move relative to one another. The accuracy of the system is 2' [Young and Sheena, 1975], but this accuracy is doubted during fast eye movements like saccades, since there exists evidence that saccadic accelerations result in significant deviations of the lens from the optical axis, caused by the inertia of the lens [Deubel and Bridgeman, 1995]. Using this technique a sensitivity of 10 - 20'' of arc can be achieved over a range of $10 - 20^\circ$ at sampling frequencies up to 1 kHz. The field of view is $\pm 15^\circ$ [Young and Sheena, 1975]. For the detection of the less intense fourth Purkinje image it is important that the light source is sufficiently bright.

3.2.4 Scleral search coil (SSC)

The scleral search coil technique was introduced by Robinson [Robinson, 1963] and is considered to be the gold standard for the detection of eye movements because of its high spatial accuracy (< 1' of arc) [Collewijn et al., 1975] and temporal resolution (< 1 ms) [Van der Geest and Frens, 2002]. A small induction coil with N turns is embedded in a silicon annulus, which is placed onto the eye after applying anesthetic eye drops. The annulus is held to the eye by suction as shown in figure 3.5^3 . The subject is then placed in an alternating magnetic field \vec{B} . As a consequence, an alternating voltage V will be generated in the coil according to Faraday's law

$$V = -\frac{\mathrm{d}\Psi}{\mathrm{d}t},\tag{3.11}$$

where Ψ is the total magnetic flux through the coil. For the case that all N turns contain the same amount of flux, this equation can be written as

$$V = -N\frac{\mathrm{d}\phi}{\mathrm{d}t},\tag{3.12}$$

³Adapted from Instruction Manual Scleral Search Coil System S3020, Skalar Medical.



Figure 3.5: Placement of the scleral search coil on the eye.

where ϕ is the flux through a single turn given by

$$\phi = \int_{A} \vec{B} \cdot d\vec{A}.$$
 (3.13)

In case of a homogeneous magnetic field, this equation becomes

$$\phi = \vec{B} \cdot \vec{A}. \tag{3.14}$$

If the orientation of the coil is characterized by a coil vector \vec{c} , which is perpendicular to the surface of the coil and has a length proportional to the surface area surrounded by the coil, the induced voltage is proportional to the cosine of the angle between \vec{B} and \vec{c} . Let \vec{B} consist of three orthogonal magnetic fields which are oriented parallel to the axes of the head-fixed coordinate system $\{\vec{h}_1, \vec{h}_2, \vec{h}_3\}$ as shown in figure 3.6

$$\vec{B}_i = \vec{h}_i |b_i| \sin(\omega_i t + \psi_i), \qquad i = 1, 2, 3,$$
(3.15)

with $|\vec{b}_i|$ the amplitude, ω_i the frequency and ψ_i the phase shift of the magnetic field in the direction of \vec{h}_i . Further, let $\{\vec{c}_1, \vec{c}_2, \vec{c}_3\}$ denote three orthogonal coils which are attached parallel to the eye-fixed coordinate system $\{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$. With the use of equation (3.1), these eye-fixed coordinates can be expressed in head-fixed coordinates $\{\vec{c}_1', \vec{c}_2, \vec{c}_3'\}$. For coil *j* it follows that

$$\vec{c}_j = |c_j| \mathbb{R} \vec{h}_j = c_j \begin{pmatrix} \mathbb{R}_{1j} \\ \mathbb{R}_{2j} \\ \mathbb{R}_{3j} \end{pmatrix}.$$
(3.16)

By using equations (3.11), (3.14), (3.15) and (3.16) it follows that the voltage induced by the magnetic field B_i in coil c_j , V_{ij} , is given by [Haslwanter, 1995]

$$V_{ij} = \mathbb{R}_{ij} |b_i| |c_i| \omega_i \cos(\omega_i t + \psi_i).$$
(3.17)

This means that the voltage induced by the magnetic field B_i in coil c_j is proportional to the element \mathbb{R}_{ij} of the rotation matrix \mathbb{R} .

In many laboratories only two orthogonal magnetic fields are available; usually one is parallel to \vec{h}_3 and the other is parallel to \vec{h}_2 . With the use of dual search coils, which have one coil parallel to the axis \vec{e}_1 and a second coil wound in such a way that it has an effective area in parallel with \vec{e}_2 , the following elements of the rotation matrix are measured

$$\mathbb{R} = \begin{pmatrix} - & - & - \\ H & T_2 & - \\ V & T & - \end{pmatrix}.$$
(3.18)



Figure 3.6: Principle of the scleral search coil with three orthogonally magnetic fields and three orthogonal mounted search coils. The coil vectors $\{\vec{c}_1, \vec{c}_2, \vec{c}_3\}$ are parallel to the eye-fixed coordinate system, whereas the magnetic fields $\{\vec{B}_1, \vec{B}_2, \vec{B}_3\}$ are parallel to the head-fixed coordinate system. Taken from [Haslwanter, 1995].

Note that in each coil an alternating voltage will be measured which is equal to the sum of both magnetic fields. From equation (3.17) it follows that the eye rotation can be obtained by frequency or phase decoding of the magnetic fields. Once the elements of \mathbb{R} are known, the Fick and Helmholtz angles can be obtained from the Fick and Helmholtz rotation matrices given in appendix A.

The possibility to record with a high sampling frequency results in a wide bandwidth of 500 Hz [DiScenna et al., 1995]. The system noise has been estimated to be in the order of 0.004° , which results in a theoretical sensitivity in the order of 0.01° [Clarke et al., 2002]. A major drawback of this technique is the need for a subject to wear a relatively large contact lens, held to the eye by suction, which causes irritation to the eye and limits the examination time to a maximum time of about 30 minutes. Another disadvantage is the possibility that slippage of the contact lens occurs during fast eye movements.

3.2.5 Video oculography (VOG)

The development of small video cameras made it possible to build small head-mounted video systems to record images of the eye. These systems make use of infra-red video cameras since the eye is illuminated with infra-red light with a wavelength invisible to the human eye, to prevent the subject from being distracted by this light. An example of such an image made with the Maastricht video eyetracker (VET) is shown in figure 3.7 A. As can be seen from this figure, the pupil is much darker than the rest of the eye. This means that the location of the pupil can be obtained by considering only those pixels which have a grey value below a certain threshold. In figure 3.7 B it is shown how the value of this threshold can be determined by looking at the grey values of the pixels located on a line (the dotted line in figure 3.7 A) which goes through the pupil.

Since the size of the pupil varies in time, it is necessary to determine one point in the pupil that is always located at the same position, independent of the pupil size; the centre of the pupil. This centre can be determined by various algorithms. In practice however, the algorithm must be accurate while using as little computation time as possible to be able to perform real-time



Figure 3.7: A: Image of an eye made with the Maastricht video eyetracker. B: Grey value pattern of the pixels located on the dotted line in A.

measurements. Moreover, since sometimes part of the pupil disappears behind the eyelid, the determination of the centre should also be possible when only a part of the pupil is visible. A method that meets these requirements is the method of Teiwes as described by Van der Glas [Van der Glas, 1999].

In short, the method determines circles through points on the edge of the pupil. The left and right border of the pupil are determined on two horizontal lines, as shown in figure 3.8. The



Figure 3.8: Determination of the centre of the pupil. Adapted from [Van der Glas, 1999].

x-value of the point between the left (x_L) and right (x_R) border on each line is determined

$$x_i = \frac{x_{iL} + x_{iR}}{2}, \qquad x_j = \frac{x_{jL} + x_{jR}}{2}.$$
 (3.19)

The x position of the centre of the pupil x_p is now obtained by averaging x_i and x_j . The y position can now be determined by using the fact that the distance from the centre of a circle to a point

on the border of this circle is equal to the radius of the circle

$$r^{2} = l_{i}^{2} + (y_{i} - y_{p})^{2}, \qquad r^{2} = l_{j}^{2} + (y_{j} - y_{p})^{2},$$
 (3.20)

with

$$l_i = \frac{x_{iR} - x_{iL}}{2}, \qquad l_j = \frac{x_{jR} - x_{jL}}{2}.$$
 (3.21)

From equation (3.20) it follows that the y position of the centre of the pupil y_p can be determined by

$$y_p = \frac{l_i^2 - l_j^2 + y_i^2 - y_j^2}{2(y_i - y_j)}.$$
(3.22)

This procedure can be repeated for all relevant combinations of horizontal lines through the pupil. In order to eliminate erroneous x- and y values, the centre of the pupil is then taken as the median of all determined values.

For the calculation of torsional eye movements an algorithm is used that determines the correlation of an iris signature of the current image and an iris signature of a reference image.

Commercially available VOG systems capable of on-line analysis sample at a low frequency of approximately 50 Hz [Schmid-Priscoveanu and Allum, 1999] to 60 Hz [Allum et al., 1998]. Besides these systems also off-line high speed sampling systems are developed. Using high frame rate CMOS sensors, Clarke et al. [Clarke et al., 2002] developed a system capable of off-line recording of three-dimensional eye movements with a sampling frequency up to 400 Hz, a noise level of 0.01° (for an artificial eye) and an accuracy of 0.05° .

However, off-line recording is not suited for the diagnosis of vestibular disorders in clinical practice, since direct feedback to the patient during the examination is essential. A disadvantage of a video eye-tracking system is the difficulty in tracking the eye position in the dark⁴ due to the large pupil diameter, which results in an increased occlusion of the pupils by the eyelids. This is mainly a problem for on-line recording techniques, since complex off-line analysis methods are capable of calculating the eye position even when only a small part of the pupil is visible.

In table 3.1 an overview of the properties of the different recording techniques, gathered from the literature as described in the previous sections, is given.

 $^{^{4}}$ It is important that some examinations are performed in the dark in order to prevent that the patient fixates on visual targets

DG	off-line	400 Hz		200 Hz	0.05°	0.02°			$\pm 40^{\circ}$	$\pm 25^{\circ}$			$\pm 20^{\circ}$	$\pm 20^{\circ}$	$40^{\circ} \ge 25^{\circ}$		Yes
NC	on-line	50-60 Hz		25-30 Hz	0.05°	0.04°			$\pm 40^{\circ}$	$\pm 25^{\circ}$			$\pm 20^{\circ}$	$\pm 20^{\circ}$	$40^{\circ} \ge 25^{\circ}$		Yes
SSC		1 kHz		500 Hz	0.01°	0.01°			$\pm 40^{\circ}$	$\pm 45^{\circ}$			$\pm 25^{\circ}$	$\pm 25^{\circ}$	unlimited		Yes
Reflection	Double Purkinje	1 kHz		500 Hz	0.03°	$0.003 - 0.006^{\circ}$			$\pm 15^{\circ}$				$\pm 15^{\circ}$		$\pm 15^{\circ}$		No
ering	array	500 Hz		100 Hz	0.1°	$0.02-0.1^{\circ}$			$\pm 30^{\circ}$	$\pm 20^{\circ}$			$\pm 20^{\circ}$		unlimited		No
Scatte	chopped	$3 \mathrm{~kHz}$		$1.5 \mathrm{~kHz}$	0.1°	$0.05-0.15^{\circ}$			$\pm 18^{\circ}$	$\pm 8^{\circ}$					$18^{\circ} \ge 8^{\circ}$		No
EOG		1 kHz		60 Hz	$1-2^{\circ}$	0.5°			$\pm 80^{\circ}$	$\pm 50^{\circ}$			$\pm 30^{\circ}$	$\pm 20^{\circ}$	unlimited		No
Method		Sampling	frequency	Bandwidth	Accuracy	Sensitivity	Dynamic	range	Horizontal	Vertical	Linear	range	Horizontal	Vertical	Field of	view $h \ge v$	Torsion

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3.3 Comparison of eye movement recording techniques

During the last decades, several investigators performed experiments in which they compared the different measurement techniques outlined in section 3.2.

3.3.1 SSC versus VOG

[Houben et al., 2006] used an infrared (IR) video-based eye-tracking system Houben et al. (Chronos Vision) capable of recording eye movements in three dimensions. The three-dimensional movements of both eyes were recorded simultaneously with the Chronos system and scleral search coils under static and dynamic conditions. The Chronos system has laterally mounted digital infrared cameras and uses a sampling frequency of 200 Hz. The images are stored on a hard disc for off-line calculation of eye positions. The calculation of the horizontal and vertical eye position is based on a circle approximation technique which fits a circle through the pupil perimeter. Torsional eye position is calculated by correlation of an iris signature of the current frame with a reference signature. The coil signal was low-pass filtered with a cut-off frequency of 500 Hz and was sampled on-line at a frequency of 200 Hz. In addition the coil signal was sampled at 1 kHz and stored on a hard disc for off-line analysis. The A/D converter had a precision of 12 bits. Velocity signals were calculated using a five-point central difference algorithm. For the fixation positions, the mean square differences between the calculated and actual positions, averaged over all subjects were 0.56° (horizontal) and 0.62° (vertical) for the video system and 0.096° (horizontal) and 0.080° (vertical) for the coil system. The mean square differences between coil and video for torsional eye positions was 1.9° . For saccadic eye movements, there was a tendency of higher peak velocities measured with the video system than with the coil system, which was presumed to be a result of the differences in signal noise level (0.02°) for the coil system and 0.2° for the video system). During testing of the vestibular system by means of whole body translation, differences between the coil and video signal were statistically significant at the 0.05 level⁵. These differences found between the coil and video system may be a result of possible slippage of the headband of the video system. Despite the good agreement of the two methods, Houben et al. concluded that for measuring eye movements with high precision or at high frequency head motion, the scleral search coil technique is superior to the video-based technique, due to the poorer time resolution of the video device, possible movement of the head device, and instabilities of the pupil tracking algorithm of the video device.

Van der Geest and Frens [Van der Geest and Frens, 2002] compared a video-based eye-tracking system (EyeLink system) with the scleral search coil technique. The EyeLink system records eye movements in two dimensions, using two miniature infrared cameras. A comparison was made between the properties of saccadic eve movements and fixation points, recorded simultaneously with both methods. The video system tracked the centre of the pupil by an algorithm with a theoretical noise-limited resolution of 0.01° and a velocity noise below 3° s⁻¹. The sampling rate was 250 Hz for the video system and 500 Hz for the coil system. Both signals were off-line recorded on separate computers. The instantaneous velocity was approximated by dividing the difference of two consecutive sampling points by the inter-sample interval⁶. The video and coil signals were both low-pass filtered using a Butterworth filter with a cut-off frequency of 120 Hz. For the fixation positions it was found that the standard deviation in fixation error between the coil and video system was 1°. Furthermore, it was shown that there was no systematic drift in the output of the video system over a recording time of several minutes. For the saccadic eye movements, the video system found systematically higher values for the amplitude (8% horizontal and 4% vertical), saccadic peak velocities (7%) and duration times (3%). Van der Geest and Frens supposed that possible explanations for these discrepancies were the visco-elastic coupling between the lens and the cornea and the high frequency noise of the video signal.

DiScenna et al. [DiScenna et al., 1995] used a video-based system (EL-MAR system) for measuring horizontal and vertical eye movements synchronously with the scleral search coil technique

 $^{^{5}}$ This means that the possibility that the observed differences are coincidental differences is smaller than 5%.

⁶This differentiation method introduces a phase shift of $\pi f/f_s$ between the eye position and velocity signal.

(during one session also a monocular EOG was recorded). The EL-MAR system consists of two small video cameras which record images of each eye and tracks simultaneously two or three corneal reflections. From the differences between the centre of the pupil, calculated by the video images, and the corneal reflection, the rotation of the eye is calculated. By using this technique, the method becomes insensitive for transversal motion of the video eyetracker, since these calculated differences remain constant during such motion. The video signal was sampled at a frequency of 120 Hz⁷. Signals from the SSC and video system were passed through Butterworth filters prior to digitization at 120 Hz to avoid aliasing⁸. The velocity signal was obtained by applying a two-point central difference algorithm. DiScenna et al. found an overall good agreement between the video-based system and the coil technique, without quantifying this agreement. The video signal showed a velocity noise of $4 - 5^{\circ} \text{ s}^{-1}$, which is 2 to 3 times higher than for the coil technique. The peak velocity of saccades recorded by the video system tended to be greater than that of the coil system. This difference was attributed to the algorithm for the calculation of the eye position used by the video system. In the experiments, the EL-MAR system was found to be superior to EOG.

3.3.2 EOG, VOG and oculography by scattering

Allum et al. [Allum et al., 1998] investigated the problem of automated analysis of ocular nystagmus recorded with different techniques: EOG, VOG and infrared (scattering) oculography (IROG). In their investigation, only a qualitative comparison was made between the different recording techniques. The analysis algorithms were developed to operate on eye position recordings sampled at 100 or 120 Hz. However, since an on-line analysis was required and the VOG system had a sampling limitation of 60 Hz, the relatively high sampling frequency, needed for the analysis, represented a problem. To overcome this problem, the input signal was resampled using a simple linear interpolation algorithm. For the EOG and IROG signals, a second order low-pass Bessel filter with a corner frequency of 30 Hz was used. The eye velocity signal was calculated by differentiating the sampled eye position using a finite impulse response differentiation filter. The mean and variance of the slow phase of the eye velocities were estimated using the last 100 samples of the current eye velocity, which were exponentially weighted in favor of the most recent. It was concluded that the exact analysis of small, slow saccades requires sampling rates higher than 100 Hz, an A/D resolution higher than 10 bits, and low measurement noise. Allum et al. found that since the sampling frequency of VOG is too low and in EOG recordings the noise is often too high, only IROG seems to satisfy the given conditions for the accurate measurements of eye movements. Furthermore, it was concluded that VOG (50 Hz, 10 bit) and IROG (100 Hz, 12 bit) were equally superior to EOG for recording horizontal vestibulo-ocular reflex responses. For optokinetic responses, however, no differences between the techniques were observed.

Schmid-Priscoveanu and Allum [Schmid-Priscoveanu and Allum, 1999] compared different kinds of recording techniques: EOG, VOG and infrared (scattering) oculography (IROG) by measuring the mean slow phase velocity during different kinds of examinations. Eye movements were recorded from the right eye simultaneously with EOG and VOG or EOG and IROG. The EOG was performed using a 30 Hz low-pass filter, a sampling frequency of 100 Hz and a 12 bit A/D converter. For measuring the EOG signal, a right-sided monocular electrode configuration was used. The video system recorded the eye position in two dimensions at a sampling frequency of 50 Hz. Eye movements were recorded by a CCD miniature camera. The infrared system used a sampling frequency of 100 Hz and a 12 bit A/D converter. When measuring the mean slow-phase velocity during horizontal optokinetic nystagmus and the vestibulo-ocular reflex, the three recording techniques did not differ significantly (p > 0.05). However, the EOG-signal was less stable and contained more noise. For the vertical OKN, the mean slow-phase velocity measured with EOG was significantly higher than for both IROG (p < 0.001) and VOG (p < 0.05). These differences where ascribed to the limited dynamic range of the latter two systems.

⁷The sampling frequency of the SSC signal was not mentioned.

 $^{^{8}}$ Only the SSC signal and the corneal reflection signal of the video evetracker can be filtered this way.

3.3.3 EOG versus scattering oculography

Hess, Müri and Meienberg [Hess et al., 1986] investigated possible methodical differences between EOG and scattering oculography by measuring horizontal saccadic eye movements in normals. In their investigation three control groups were used. In group 1, EOG and scattering oculography signals were simultaneously recorded from the right eye, using various cut-off frequency limits (30 Hz and 70 Hz) for both methods. In group 2, the EOG signal was recorded from the left eve with an upper frequency limit of 30 Hz. The EOG signal was frequently recalibrated to minimize electrode drift. In group 3, the scattering system recorded eye movements from each eye separately with an upper frequency limit of 70 Hz. The data from control groups 2 and 3 were compared. For each experiment horizontal midline crossing saccades of 20° and 30° in amplitude were recorded. Each eye was illuminated with infrared light. A pair of photosensors was aimed at the iris-sclera border, using the differential electrical current of the two photosensors as a measure for the eye position. In order to obtain saccade velocities, the position signals of the EOG and the scattering system were differentiated by identical active differentiators with a cut-off frequency of 120 Hz. In the EOG signal, artifacts from extra-ocular muscle activity were considerable when using the 70 Hz low-pass filtering, causing difficulties in the measurements. When using the 30 Hz filtering settings, these artifacts were considerably reduced, but the peak velocities were reduced by 3% and their duration was prolonged by 6%. The most important differences between the two techniques were encountered in the abducting (horizontal movement from the nose) saccades. For this case it was found that the EOG method resulted in a statistically significant underestimation of peak velocities and an overestimation of saccade durations. The oculography scattering system measured up to 20% faster velocities. For adducting (horizontal movement towards the nose) saccades no systematic deviation was found.

3.3.4 EOG, SSC and scattering oculography

Yee et al. [Yee et al., 1985] investigated the velocities of voluntary, vertical saccadic eye movements separately recorded with EOG, SSC and infrared limbus tracking (IR) and compared the three techniques. The noise level of the position signal for monocular EOG recordings was $0.2-0.4^{\circ}$. An analog low-pass filter with a cut-off frequency of 42 Hz was used to remove high frequency noise. Then the signal was sampled at a sampling rate of 200 Hz. The velocity signal was calculated from the position signal by using the two-point central difference method. It followed that the analog and digital filtering (which is a result of the two-point central difference method, as will be described in section 4.3.2) led to an underestimation of the peak velocity (2 to 6% for saccades of 10° and 0.6 to 1.2% for 35°). The IR system used photocells to detect the density of infrared light and was mounted in such a way that both horizontal and vertical eye movements could be recorded. The signal-to-noise ratio was in the order of $0.1 \, (rms)^9$. An analog low-pass filter with a cut-off frequency of 100 Hz was used and the signal was sampled at a sampling frequency of 200 Hz. The two-point central difference algorithm was used to obtain the velocity signal. For the magnetic scleral search coil measurements, the signal was filtered using an analog low-pass filter of 1 kHz and sampled at a frequency of 200 Hz¹⁰. The two-point central difference algorithm was used to calculate the velocity. The linear range of the system was $\pm 20^{\circ}$. With the search coil, a difference up to 20% in velocities between up and down saccades was found in a normal subject. However, there was no significant pattern of differences in the velocities of up and down saccades among subjects. In the EOG signal, artifacts from eyelid movements, resulting in overshooting trajectories, had major effects on the calculated peak velocities of saccadic eye movements. Peak velocities of up saccades were consistently larger than those of down saccades (e.g. peak velocities of 30° up saccades were up to 40% larger than those of equally sized down saccades). Moreover, the velocities of up and down saccades were significantly larger for the EOG than for the search coil

 $^{^{9}}$ The stated value for the signal to noise ratio is doubted, since it implies that the level of the noise was higher than the signal.

 $^{^{10}}$ The choice of the 1 kHz low-pass filter is doubted, since in order to prevent aliasing the signal should be low-pass filtered at a frequency of 100 Hz.

(p < 0.005). With the IR system, only vertical saccades up to 20° could be recorded accurately. The peak velocity of up saccades was underestimated and differences in velocity of up and down saccades of up to 56% were found. The differences in velocity between up saccades with IR and with the search coil were significant (p < 0.01), whereas for the down saccades no significant differences were found.

3.3.5 EOG versus infrared reflection oculography

Boghen et al. [Boghen et al., 1974] performed a study to establish normative data which are required to accurately define pathologic slowness of saccades. In their study, simultaneous recordings of horizontal saccades of 5° , 10° , 20° and 30° of only the right eye were made using EOG and infrared reflection. The full system bandwidth of the EOG recording technique was 25 Hz with a response time for the differentiator of 15 ms. The full system bandwidth of the infrared reflection recording technique was 100 Hz with a differentiator response time of 4 ms. The rate at which the signals were sampled was not mentioned. From the recordings it was found that the infrared reflection method gives higher values for the peak velocity than the low bandwidth EOG technique. In order to find the relation between the infrared reflection and the EOG technique, they determined the ratio between the peak velocity calculated from the infrared reflection and the EOG signal for the various amplitude saccades. For large amplitude saccades (30°) this ratio is 1.14 and increases for smaller amplitude saccades in previous studies and their experimental study, Boghen et al. concluded that studies using higher bandwidths yielded higher and more accurate peak velocities.

3.3.6 EOG versus SSC in animals

Schlag et al. [Schlag et al., 1983] made a qualitative comparison of EOG and SSC in an alert monkey and cat. Simultaneous recordings of eye movements were obtained by EOG and SSC. The used electrodes were a mixture of silver-silver chloride-bentonite and were surgically implanted in holes drilled in the orbital bone. A search coil was inserted around the left eyeball. The sampling frequency was 1 kHz¹¹. In order to obtain the velocity, the two-point differentiation algorithm was used (using consecutive sampling points). Except for discrete events (mainly blink episodes, only occurring in the EOG signal) the recorded traces were very similar and hard to distinguish. Other events that caused discrepancies between the tracings were drift in the EOG signal and crosstalk between channels. Moreover, it was found that some vertical saccades recorded by EOG displayed an overshoot. Regarding the accuracy and stability it was found that the SSC was superior to EOG. One should note that in this study an invasive placements of the electrodes and the search coil was used. The conventional clinical configurations for measuring EOG and SSC signals as described in section 3.2 are likely to yield poorer similarity.

3.4 Discussion

Although a lot of studies have been performed in order to determine which method is suited best for measuring eye movements for the diagnosis of vestibular or oculomotor disorders, still confusion and disagreement exists among clinical practitioners. This is caused by several aspects. First, the type of eye movement which is to be recorded plays an important role; for the accurate recording of very fast eye movements like saccades, it is stated that it is important to use a relatively high sampling frequency of at least 300 Hz [Juhola et al., 1985], whereas for the measurement of the mean slow phase velocity during vestibular testing a lower sampling frequency of 50 Hz seems to be sufficient [Schmid-Priscoveanu and Allum, 1999]. Also the direction of the eye movement is important; since for the VOG and scattering method no signal can be obtained when the pupil

 $^{^{11}\}mathrm{From}$ the article it is not clear whether this sampling frequency was the same for both methods.

disappears behind the eyelid, these techniques are not suited for measuring vertical eye movements of large amplitude [Schmid-Priscoveanu and Allum, 1999].

In order to examine whether a measuring technique yields reliable results, sometimes a (direct) comparison is made between the eve position and velocity signals obtained from this technique and the (simultaneously) recorded SSC signals. Significant differences in these signals are often ascribed to the inaccuracy or limitations of the other measuring technique. Although the SSC technique is considered to be the gold reference for the accurate measurement of eye movements because of its high spatial and temporal resolution, the method is not ideal for clinical practice and care has to be taken when analyzing fast eye movements like saccades. It was shown by Frens and Van der Geest [Frens and van der Geest, 2002] that the placement of scleral search coils onto the eyes influences the kinematics of saccades. In their investigation saccadic eye movements of human subjects were recorded with a video system sampling at 250 Hz while scleral search coils were placed onto the subject's eyes. The results were compared with recordings in which no coils were placed onto the eyes. It was found that saccades last longer (8%) and become slower (5%)when a scleral search coil is mounted on the eye. Frens and Van der Geest concluded that the coils appear to change the oculomotor command signals that drive the saccadic eye movements. Another important aspect when considering fast eye movements is the risk of slippage of the search coil, since the search coil is held to the eye by suction. In their experiments, Houben et al., Van der Geest and Frens, and DiScenna et al. found that there was a tendency of higher velocities measured with the video system than for the coil system. Since these signals were simultaneously recorded with the video and coil system, this difference can not be attributed to the phenomenon described by Frens and Van der Geest. A possible explanation for this observed difference in velocity can be slippage of the contact lens, which will result in lower apparent eve velocities¹².

Although electrooculography is frequently used in clinical practice for the measurement of peak velocities of saccadic eye movements, it is questioned whether EOG is suited for the accurate measurement of saccades. Especially for the analysis of vertical saccades, EOG is limited because of considerable overshoot that occurs for both upward and downward saccades due to eyelid movement, resulting in an increase of the peak velocity. This effect might contribute to the higher values of the vertical eye velocities obtained by the EOG technique observed by Yee et al. [Yee et al., 1985] and Schmid-Priscoveanu and Allum [Schmid-Priscoveanu and Allum, 1999]. Chioran and Yee [Chioran and Yee, 1991] investigated the origin of these artifacts by comparing voluntary vertical saccades recorded by EOG and the SSC technique. They found that the amplitude of these artifacts was decreased but not completely eliminated by eyelid fixation. As a consequence they observed decreased peak velocities. Nevertheless, EOG measurements with eyelid fixations were still less accurate than those obtained by the search coil. Chioran and Yee concluded that there is evidence that the observed artifacts may be caused by a combination of eyelid electrical activity and resistance effects.

However, the accurateness of the EOG technique is not only questioned for vertical saccades but also for horizontal saccades. Carpenter [Carpenter, 1988] remarked that EOG may lead to underestimations of the peak velocity. This remark was mainly based on an article of Boghen et al. [Boghen et al., 1974] described in section 3.3.5. Although Boghen et al. observed that the peak velocities derived from the infrared reflection signal were higher than those derived from the EOG signal, they implicitly ascribed this phenomenon to the lower system bandwidth of electrooculography. Nevertheless, also in other studies it was found that the EOG technique led to lower saccadic peak velocities [Hess et al., 1986, Juhola, 1986]. It is questioned whether this phenomenon is a result of the different electronics and filter settings that were used or that the EOG technique is not suited for measuring such high eye velocities.

 $^{^{12}}$ Lens slippage of the scleral search coil was detected with the VOG system by Clarke and shown in a poster presentation in Freiburg.

Chapter 4

Theory

In order to make an objective comparison between the three different recording techniques, it is necessary to determine to which degree the recording techniques yield accurate results. For this determination, however, it is not only important to have information about a number of relevant properties and limitations of the methods, but also the characteristics of the measured quantity have to be known, in order to estimate what values of these properties are required to reach a reliable conclusion. In this section, first some important static properties of the recording techniques will be described, followed by a section dealing with the dynamic characteristics of the techniques and the quantity of interest. Finally, attention is paid to the differentiation of the eye position signal with respect to time in order to obtain the eye velocity signal.

4.1 Accuracy, precision and resolution

To many people accuracy and precision mean the same thing. In a quantitative measurement, however, accuracy and precision have very different meanings. Accuracy describes the closeness of the measured value to the reference value of the measured quantity, while precision is a measure for the degree of reproducibility. Precision is defined as the spread that occurs in the measured value when the measurement is repeated. This is illustrated in figure 4.1^1 . Precision is usually



Figure 4.1: Definition of accuracy and precision.

¹Taken from http://en.wikipedia.org/wiki/Accuracy.

defined in terms of the standard deviation of a measurement. Therefore, random noise affects the precision of a measurement, whereas drift affects the accuracy of a measurement. Another important property of a recording technique is its resolution or sensitivity, which is defined as the smallest, detectable change of the measured quantity.

4.2 Dynamic characteristics

4.2.1 Sampling theorem and aliasing

When recording a signal which varies in time, the used sampling frequency f_s is of high importance, since a sampling frequency that is too low introduces a loss of information. It is evident that for a rapidly varying signal the sampling frequency must be higher than for a slowly varying signal. For any sampling frequency f_s there exists a special frequency f_c , called the Nyquist critical frequency, given by [Press et al., 1995]

$$f_c \equiv f_s/2. \tag{4.1}$$

The Nyquist critical frequency is important for two related reasons. The first reason is known as the sampling theorem; if a continuous function x(t), sampled at a sampling interval $T_s = 1/f_s$, is bandwidth limited with a maximum frequency component, f_{max} , equal to or smaller than f_c , then the function x(t) is completely determined by its samples x[n] and is given explicitly by the equation

$$x(t) = \sum_{n=-\infty}^{+\infty} x[n] \frac{\sin[2\pi f_c(t-nT_s)]}{\pi(t-nT_s)},$$
(4.2)

with x[n] given by

$$x[n] = x(nT_s), \qquad n = 0, \pm 1, \pm 2, \pm 3, \dots$$
 (4.3)

Second, the Nyquist theorem states that in order to prevent losing information, the (continuous) signal has to be bandwidth limited to frequencies equal to or smaller than f_c . By using equation (4.1) it follows that the minimal required sampling frequency is given by

$$f_s = 2f_{max},\tag{4.4}$$

with f_{max} the bandwidth limit of the signal. For the situation that x(t) is not bandwidth limited to less than the Nyquist critical frequency, it turns out that any frequency component outside the frequency range $(-f_c, f_c)$ is falsely translated into that range. This phenomenon is called aliasing and is caused by the discrete sampling of the signal. If the signal is not bandwidth limited or this limit is not known, aliasing can be eliminated by analog filtering of the continuous signal before the signal is sampled. Alternatively, if one wants to verify that a signal is sampled at a high enough sampling rate, the spectrum of the sampled signal can be calculated by applying a discrete Fourier transformation as described in section 4.2.2. When this Fourier transformation is already approaching zero as the frequency approaches f_c , it can be assumed that the used sampling frequency was sufficiently high to prevent aliasing.

4.2.2 Frequency analysis

A dynamic signal which is described in the time domain (x = x(t)) can also be represented in the frequency domain (X = X(f)). According to the theory of Fourier, any periodic signal $x_p(t)$ with a period T_0 can be described by a linear combination of harmonic signals, called a Fourier series, with a fundamental harmonic of $f_0 = 1/T_0$

$$x_p(t) = \sum_{k=-\infty}^{\infty} c_k e^{2\pi i k f_0 t},$$
(4.5)

where c_k is called the Fourier coefficient which corresponds to a frequency kf_0 and is given by

$$c_k = \frac{1}{T_0} \int_{t_0}^{t_0 + T_0} x_p(t) e^{-2\pi i k f_0 t} dt, \qquad k = 0, \pm 1, \pm 2, \pm 3, \dots$$
(4.6)

The frequency spectrum of x_p can be found by calculating $|c_k|$ and $\arg(c_k)$ and plotting them versus their corresponding frequencies. Since the signal considered has a finite period T_0 , the spectrum will only exist for the frequencies kf_0 ($k = 0, \pm 1, \pm 2, \pm 3, ...$), resulting in a line spectrum. The distance between two lines is given by $1/T_0$ and is called the resolution of the spectrum. When the signal of interest is known only on a certain time interval ($t_0, t_0 + T$) but not periodic, equations (4.5) and (4.6) can still be used by applying a periodic extension of the signal outside the known interval.

When the signal of interest is not periodic and a periodic extension of the signal is not possible or desired, it is still possible to obtain the frequency spectrum by considering the signal to be periodic with infinite period: $T_0 \to \infty$. Since the resolution of the spectrum is given by $1/T_0$, the resolution becomes infinitely small for $T_0 \to \infty$, which results in a continuous frequency spectrum. Another consequence of this approach is that the discrete Fourier series in equation (4.5) is modified to an integral

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{2\pi i f t} df, \qquad (4.7)$$

with the Fourier transform X(f) given by

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-2\pi i f t} dt.$$
(4.8)

Sampling a signal at a sampling frequency $f_s = 1/T_s$ on the interval $(t_0, t_0 + T)$ results in N+1 sampling points with $N = T/T_s$. For this situation the (continuous) signal is only known at the sampling points and the Fourier coefficients c_k cannot be calculated by the use of equation (4.6). This equation can be approximated by the discrete sum

$$c_k = \frac{1}{N} e^{-2\pi i k t_0/T} \sum_{n=0}^{N} x(t_0 + nT_b) e^{-2\pi i k n/N}, \qquad k = -N/2, ..., 0, ..., N/2.$$
(4.9)

For this situation, equation (4.5) can be written as

$$x(t_0 + nT_s) = \sum_{k=0}^{N} c_k e^{2\pi i k n/N}, \qquad n = 0, 1, ..., N.$$
(4.10)

Equations (4.9) and (4.10) form the basis for the discrete Fourier transformation (DFT). Let X[k] be the discrete Fourier transform of the sampled signal x[n], which is defined as

$$X[k] = \sum_{n=0}^{N} x[n]e^{-2\pi i k n/N}, \qquad k = -N/2, ..., 0, ..., N/2.$$
(4.11)

The frequency corresponding to each value for k is given by k/T. The inverse discrete Fourier transform (IDFT) is defined as

$$x[n] = \frac{1}{N} \sum_{k=0}^{N} X[k] e^{2\pi i n/N}, \qquad n = 0, 1, ..., N.$$
(4.12)

Note that equation (4.11) differs from equation (4.9) in two aspects. First, in equation(4.11) it is assumed that $t_0 = 0$ since the sampled values are written as a series x[n]. As a consequence, a phase shift is introduced in the spectrum of X[k]. Since one is often only interested in the amplitude

spectrum and the relative change in phase, this assumption has generally no consequences. The second difference is a multiplication by a factor N. By multiplying by this factor, X[k] becomes in fact an amplitude density which makes it easier to compare with the continuous Fourier transform.

When calculating the Fourier transform of a signal on a time interval $(t_0, t_0 + T)$, care has to be taken that this algorithm implicitly uses the assumption that the signal is periodic outside this interval by using a periodic extension. For the situation that $x(t_0) \neq x(t_0 + T)$, this results in a discontinuous periodic function. This leads to so-called leakage in the spectrum from frequencies of high amplitude to frequencies of low amplitude. To solve this problem, the original function is multiplied by a window weighting function which will force the end points of the function to zero before the Fourier transform is performed. A frequently used weighting function is the Hanning window

$$w[n] = 0.5\{1 - \cos(2\pi n/N)\}, \qquad 0 \le n \le N.$$
(4.13)

4.2.3 Fourier analysis of saccades

As described in section 4.2.1, for the determination of the minimal sampling frequency required to accurately reconstruct the continuous eye position signal from the sampled signal, it is important to know the bandwidth of the type of eye movement of interest. This minimal sampling frequency is given by equation (4.4). Since saccadic eye movements are the fastest eye movements one can make, they have the most extended bandwidth of all movements. This means that a sampling frequency capable of the accurate recording of saccadic eye movements is also capable of recording all other types of eye movements of interest. Harris et al. [Harris et al., 1990] performed a Fourier analysis of saccades in monkeys and humans. In their analysis they used recordings of saccadic eye movements, measured with the scleral search coil technique. These recordings were processed with a high resolution, infinite Fourier transform algorithm. To avoid leakage due to truncation with finite Fourier transforms, they assumed that for a saccade, starting at point A and ending at point B after the completion of the saccade. In figure 4.2 the power spectrum of a human horizontal saccade of 10.3° is shown. The inset shows the original saccade in the time domain. In this figure it can be seen that at low frequencies there is a constant roll-off of 20 dB/decade,



Figure 4.2: Power spectrum of a 10.3° horizontal saccade. Adapted from [Harris et al., 1990]. Inset: original saccade in the time domain.

which is typical for saccadic eye movements. This roll-off can be explained by the examination of a simplified model in which the saccadic eye movement is considered to be instantaneous

$$s(t) = AH(t) - A/2, (4.14)$$

with s(t) the eye position at time t, A the amplitude of the saccade and H(t) the Heaviside function which has value zero for t < 0 and one for t > 0. The Fourier transform of s(t) is given by

$$S(f) = -\frac{iA}{2\pi f}.\tag{4.15}$$

The power spectrum of S(f) is represented by the dashed line in figure 4.2. As can be seen in the figure, the fact that saccades do not occur instantaneously results in a downwards departure from the theoretical 20 dB/decade roll-off at a frequency of 20 - 30 Hz, indicating a lack of energy at higher frequencies. Since larger amplitude saccades have longer durations, this departure begins at lower frequencies as the amplitude of the saccade increases [Harris et al., 1990]. The results of the analysis performed by Harris et al., together with the theoretical 20 dB/decade roll-off, indicate that for the accurate description of saccadic eye movements, the high frequency components are negligible compared to the low frequency components. This means that saccadic eye movements are in good approximation bandwidth limited with an upper frequency of 20 - 30 Hz. As a result, the Nyquist theorem states that the original time continuous signal can be completely reconstructed from the sampled signal if the condition $f_s \geq 2f_{max}$ is satisfied.

Hypothesis: Saccadic eye movements are in good approximation bandwidth limited with an upper frequency of 20 - 30 Hz and in order to accurately record saccadic eye movements, the sampling frequency must be at least two times higher than the bandwidth limit.

4.3 Differentiation method

The output of all eye measurement systems described in section 3.2 is the eye position. However, for clinical practice not only the eye position but especially the eye velocity is an important parameter for the diagnosis of disorders. To accurately calculate the eye velocity signal from the eye position signal, an appropriate algorithm which differentiates the eye position with respect to time is essential. For the choice of this algorithm, several aspects have to be taken into consideration. First, the time between two subsequent (sampling) points t_s plays an important role. In order to get a good approximation of the instantaneous eye velocity, it is important that t_s is small enough. However, when t_s becomes very small, care has to be taken that round off errors, caused by the A/D conversion, and signal noise do not introduce erroneous values for the instantaneous eye velocity. These effects are shown in appendix D. The influence of noise can be reduced by filtering the eye position signal or by choosing a differentiation algorithm with an appropriate bandwidth, meaning that the algorithm yields accurate results for all relevant frequency components but suppresses the frequency components which contain predominantly noise. Furthermore, the algorithm should not introduce a phase shift in the eye velocity signal.

4.3.1 Design of differentiation algorithm

Differentiation of a signal in the time domain corresponds to a multiplication with $2\pi i f$ in the frequency domain

$$\frac{\mathrm{d}^n g(t)}{\mathrm{d}t^n} \Longleftrightarrow (2\pi i f)^n G(f),\tag{4.16}$$

in which g(t) and G(f) represent the signal in the time and frequency domain, respectively. From this equation it follows that the frequency response characteristics for a first order differentiation in the frequency domain can be written as

$$H(f) = 2\pi i f. \tag{4.17}$$

For an optimal design of a differentiation filter, its frequency response F(f) is the best approximation of H(f). Since H(f) is an anti-symmetric function, the frequency response of the approximating filter F(f) can be written as a linear combination of anti-symmetric functions [Usui and Amidror, 1982]

$$F(f) = i \sum_{k=1}^{N} C_k \sin(2\pi k f/f_s), \qquad (4.18)$$

where f_s is the sampling frequency at which the signal was recorded and N is the number of anti-symmetric functions for the approximating filter F(f). This equation can be written as

$$F(f) = \frac{1}{2} \sum_{k=1}^{N} C_k \{ (e^{2\pi i k f/f_s}) - (e^{-2\pi i k f/f_s}) \}.$$
(4.19)

Via an inverse z-transformation, using $z = e^{2\pi i f/f_s}$, this equation can be written in the discrete time domain

$$f[n] = \frac{1}{2} \sum_{k=1}^{N} C_k \{ \delta[n+k] - \delta[n-k] \}.$$
(4.20)

For an input signal y[n], the output of the filter $\dot{y}[n]$ is given by

$$\dot{y}[n] = \frac{1}{2} \sum_{k=1}^{N} C_k \{ y[n+k] - y[n-k] \}.$$
(4.21)

Since both H(f) and F(f) are pure imaginary functions, the phase of the approximation filter F(f) is the same as that of the ideal H(f)

$$\varphi = \begin{cases} \frac{\pi}{2} & \text{if } f > 0, \\ -\frac{\pi}{2} & \text{if } f < 0. \end{cases}$$
(4.22)

In order to quantify the agreement of the filter with the ideal frequency response, the square-error measure E can be calculated

$$E = \int_{-f_s/2}^{f_s/2} |H(f) - F(f)|^2 df = \int_{-f_s/2}^{f_s/2} |H(f) - i \sum_{k=1}^N C_k \sin\left(2\pi k f/f_s\right)|^2 df,$$
(4.23)

where the limits of integration, $-f_s/2$ and $f_s/2$, are determined by the Nyquist criterium. Minimizing the square-error for the coefficients C_k results in N equations from which these coefficients can be determined. However, for bandwidth limited signals the square error measure E only needs to be minimized for frequencies up to the highest relevant frequency f_{max} present in the signal

$$E = \int_{-f_{max}}^{f_{max}} |H(f) - F(f)|^2 df = \int_{-f_{max}}^{f_{max}} |H(f) - i\sum_{k=1}^{N} C_k \sin\left(2\pi kf/f_s\right)|^2 df.$$
(4.24)

4.3.2 Two-point central difference differentiation algorithm

In the introduction of this section it was mentioned that the influence of noise can be reduced by choosing a differentiation algorithm with an appropriate bandwidth. In order to illustrate this idea, the simple case will be discussed in which the approximating filter F(f) consists of a single anti-symmetric function

$$F(f) = iC_k \sin(2\pi f/f_s).$$
 (4.25)

Under the assumption that the low frequency components are most important, which is often the case in the presence of high frequency noise, the low frequency error has to be minimized. This can be done by considering the slope of the function for f = 0

$$\left. \frac{\mathrm{d}H(f)}{\mathrm{d}f} \right|_{f=0} = \left. \frac{\mathrm{d}F(f)}{\mathrm{d}f} \right|_{f=0} \Longrightarrow C_k = f_s = 1/T_s,\tag{4.26}$$

with T_s the sampling time. From equation (4.21) it follows that the output of the filter $\dot{y}[n]$ is given by

$$\dot{y}[n] = \frac{1}{2T_s} \{ y[n+1] - y[n-1] \}.$$
(4.27)

This equation is the well-known two-point central difference differentiation algorithm with a spread of two, where the spread is defined as the distance d between the two sampling points used to calculate the derivative. In the early days of computer usage for the analysis of biological or biomechanical data, this algorithm has been studied extensively, since the use of micro- or minicomputers implied the need for simple and fast differentiation methods. Usui and Amidror [Usui and Amidror, 1982] showed this method to be not only very practical for use, but even almost optimum. However, this algorithm does not only differentiate the signal, but also filters it. Bahill et al. [Bahill et al., 1982] demonstrated that this algorithm in fact can be modeled as an ideal differentiator in series with a low-pass filter. This phenomenon can be shown by considering the frequency response of the corrresponding approximating filter F(f) given by

$$F(f) = \frac{i\sin(2\pi f/f_s)}{T_s}.$$
(4.28)

In figure 4.3 a this response is shown for $T_s = 1$ ms together with the response of an ideal differentiation filter. Equations (4.17) and (4.28) can be used to calculate the ratio of the absolute value of the frequency responses

$$\left|\frac{F(\omega)}{H(\omega)}\right| = \frac{\sin \omega T}{\omega T}.$$
(4.29)

This ratio is plotted in figure 4.3 b. As can both be seen from this figure and calculated by using



Figure 4.3: a: Absolute value of the gain for an ideal derivative and the two-point central difference algorithm for a sampling time T_s of 1 ms. b: Bode diagram of the ratio of the absolute value of the frequency responses of the two-point central difference method and the ideal derivative [Bahill et al., 1982].

equation (4.29) the ratio is within 1% of unity for frequencies up till about 40 Hz ($f/f_s = 1/25$). It is within 10% of unity for frequencies up till 125 Hz ($f/f_s = 1/8$). For the sampling time of 1 ms, the algorithm acts as a low-pass filter with a bandwidth (attenuation of the signal by 3 dB) of 221 Hz. Bahill et al. [Bahill et al., 1982] found that the general formula for the bandwidth f_{max} of the two-point central difference algorithm is given by

$$f_{max} = \frac{0.443f_s}{d}.$$
 (4.30)

When the bandwidth of the signal and the spread of the used differentiation algorithm are known, the minimum sampling frequency necessary to obtain accurate values for the velocity can be determined using this equation.

Juhola et al. [Juhola et al., 1985] studied the influence of the sampling frequency f_s on the calculated peak velocity of saccadic eye movements, determined with the two-point central difference differentiation algorithm. In this study, an artificial saccade of 20° with a peak velocity of 656° s⁻¹ was used. They found that in order to obtain accurate values for the maximum eye velocity, a sampling frequency of at least 300 Hz is required, as shown in figure 4.4. When comparing these results with equation (4.30), the erroneous conclusion could be drawn that a saccadic eye movement of 20° contains frequency components up to about 74 Hz. However, in figure 4.2 it can be seen that the high frequency components (higher than 30 Hz) appear to be negligible compared to the low frequency components. This apparent discrepancy can be explained by noticing that using an appropriate differentiation algorithm is necessary but not sufficient to obtain accurate values for saccadic peak velocities. When a too low sampling frequency is used, the possibility exists that there will be no sampling point near the time where the peak velocity is reached. In this case, the differentiation algorithm will yield accurate values for the eye velocity only for those times where the signal was sampled. This will also be shown in section 5.4.3.



Figure 4.4: Peak velocities of a simulated 20° saccade determined at varying sampling frequencies. Adapted from [Juhola et al., 1985].

Chapter 5

Methods

5.1 Subjects

Voluntary horizontal and vertical saccades were recorded in three normal, young adults. Their ages ranged from 23 to 28 years. For one of the subjects, the SSC signal could be only recorded for the left eye due to a broken connection cable of the right lens. None of the subjects used medication or alcohol prior to time of testing. Furthermore, no evidence of ophthalmologic or neurologic disorders was known. Each of the subjects was familiar with wearing contact lenses. During the measurements, the subjects were seated in a chair. The illumination was kept at a constant dim level. The subjects were asked to focus on a dot, projected on a television screen, which moved abruptly from side to side. The distance from the rotatory chair to the screen was 1 m. The diameter of the dot was in the order of 0.5 cm, corresponding with an angle of approximately 0.5° . Movements of the head were minimized using a head-rest attached to the chair. This method of head fixation was favored over the use of a mouth bar because of the subject's comfort and to minimize the possibility of electromyographic artifacts in the EOG signal [Iancono and Lykken, 1981].

5.2 Instrumentation

5.2.1 Visual stimulator

For the projection of the dots on the screen, a television was connected to the visual stimulator, $IDEE/MI^1$. The visual stimulator is an autonomous unit with which various visual stimuli can be projected on a television screen or a beamer. It consists of an industrial personal computer with VGA-card and a hard disk. An application on the computer generates images on a television screen via the VGA-card. The visual stimulator communicates with a host computer via a serial connection. Via an analog output (8 bits A/D converter, Range: 0 - 5V), the position of the dot was recorded with the program Balancelab 1.10, IDEE/MI.

5.2.2 SSC system

The SSC signal was recorded with the Skalar scleral search coil system S3020 (Skalar Medical). This system used two orthogonal a.c. magnetic fields (carrier frequency 20 kHz); one parallel to \vec{h}_3 , the other to \vec{h}_2 in the head-fixed coordinate system. The two magnetic fields were in temporal quadrature (meaning 90° out of phase). The SSC signal was amplified by an analog amplifier with a bandwidth of 200 Hz. For the scleral search coils, the Skalar Medical combination annulus was used. These coils consisted of a silicone rubber suction ring in which two induction coils were wound. The coils were wound in such a way that their surface vectors were parallel to \vec{e}_1 and \vec{e}_2

¹Instrument Development Engineering and Evaluation/Maastricht Instruments bv.

in the eye-fixed coordinate system, respectively. Since the a.c. magnetic fields were in temporal quadrature, the induction voltages were phase-sensitive demodulated. The signal was recorded on a separate personal computer at a sampling rate of 1 kHz.

5.2.3 EOG system

The EOG signal was recorded with Ag-AgCl skin electrodes. Before the placement of the electrodes, the subject's skin was intensively cleaned with aether petroleum, after which it was scrubbed at the electrode placement sites. For the recording of horizontal eye movements of each eye separately, the electrodes are placed in the binocular configuration as described in section 3.2.1. For the measurement of vertical eye movements, one electrode was placed directly above the eyebrow and one below the inferior rim of the orbit for each eye. Finally, a ground electrode was attached to the forehead. In order to minimize the effect of high frequency induction voltages in the electrodes caused by the a.c. magnetic fields, all electrode wires were twisted carefully to minimize the magnetic flux through their loops.

The EOG signal was amplified and analog low-passed filtered at 100 Hz, after which the signal was digitized by a 12 bit A/D-converter (Keithley KPCI-3101) and sampled at a frequency of 1 kHz. The recordings were made with Balancelab 1.10.

5.2.4 VOG system

The Maastricht video eyetracker, IDEE/MI, was used as VOG system. Each eye was illuminated by four infrared LEDs (type: Siemens LD271), which had a wavelength of 950 nm. Via an infrared reflecting mirror, the images of each eye were recorded by a 1/3" black and white video CCD chip (type: Videology 21D379). The images were sampled interlaced (the odd and even lines of the image were sampled separately) at a sampling rate of 50 Hz, after which they were recorded on tape by digital video cameras (Sony DVcam). In order to obtain the eye position signal, each image was analysed with the method as described in section 3.2.5. Although in this study an off-line processing of the data was performed, it is mentioned that theoretically this processing can be performed on-line.

In order to prevent the video eyetracker from slipping, it was firmly attached to the head.

5.3 Measurement protocols

Before each measurement, the scleral search coils were calibrated. For this purpose, a calibration device, on which the search coils were attached, was placed in the a.c. magnetic fields of the SSC system. Using this device, the search coils could be rotated about the eye-fixed axes by well-known angles. This way, the gain and offset of the analog amplifier could be adjusted.

For each subject, the same measurement procedure was performed twice: first while the eye position was simultaneously recorded with the EOG and VOG system, during the second procedure the eye position was simultaneously recorded with the EOG, VOG and SSC method. This was done for two reasons. First, this way the effect of the scleral search coil on the dynamics of the eye could be determined by a comparison of both procedures. Second, by measuring the EOG signal both in the absence and presence of the a.c. magnetic fields, artifacts in the EOG signal caused by the magnetic fields could be identified.

Each measurement procedure consisted of the following parts:

- Determination of the reference position of the eye. During this measurement the subjects were asked to focus on a dot straight in front of them projected on the television.
- Horizontal calibration. The subjects were asked to fixate on a spot projected on the screen which instantly moved 20° to the left or right with the reference position located in the middle.
- Vertical calibration. The same procedure was used as for the horizontal calibration, with the difference that the spot moved in the vertical direction.
- Measurement of horizontal saccades of varying amplitude. During this measurement, the subjects were again asked to fixate on the spot projected on the screen. As for the calibration procedures, the spot instantly moved over different angles to the left and right. In order to follow the spot, voluntary horizontal midline-crossing saccades had to be made by the subjects. The angle over which the spot moved started at a small value (5°) and increased during the experiment in steps of 5° up to a rotation angle of 25°. Also the maximum rotation angle of 28^{circ}, limited by the dimensions of the television screen, was measured. For each rotation angle, three saccades to the left and right were made before the angle of rotation was increased.
- Measurement of vertical saccades of varying amplitude. For this measurement the same procedure was followed as for the recording of the horizontal saccades with the difference that the spot moved in the vertical direction. Furthermore, the maximum rotation angle was 22°.

The calibrations were used for calibration of the EOG and VOG signals and for the SSC signal as well even though the system was already calibrated with the calibration device. The main reason for this, is that when the scleral search coil is placed on the eye it adheres to the limbus of the eye. As a result, deformation of the lens occurs which changes the surface area of the coils and thus the magnetic flux through this area.

Finally, it is mentioned that since saccades are ballistic eye movements², sometimes small over- or underestimations for the saccadic amplitude occur. As a consequence, the amplitude of the measured saccades can differ from the step size of the dot on the screen.

5.4 Data processing

Before an accurate comparison between the different systems can be made, it is first important to analyse the raw data to determine how the signals must be processed in order to obtain accurate and reliable results. In the following sections, this analysis is described for each system.

5.4.1 SSC system

The raw eye position data obtained with the SSC system for two horizontal saccades with amplitudes of 5° to the right and 28° to the left are shown in figure 5.1 **A** and **B**, *respectively*. As can be seen from the figure, the SSC signal has a very high signal-to-noise ratio. The noise level of the eye signal obtained from the SSC system is in the order of 0.05° .

In order to test the hypothesis stated in section 4.2.3 that saccades can in good approximation be considered to be bandwidth limited with an upper frequency of 20-30 Hz, the power spectra of a number of saccades of varying magnitude recorded with the SSC system were determined. Because the system had a bandwidth of 200 Hz, the used sampling frequency of 1 kHz was sufficiently high to avoid aliasing.

Since even the duration of a relative large amplitude saccade of 40° does not exceed about 100 ms [Carpenter, 1988], a discrete Fourier transform using data only containing the pure saccadic eye movement will result in a very poor resolution of the frequency spectrum. To solve this problem, the signal was extrapolated under the assumption that before the onset of the saccade, starting at point A and ending at point B, the eye had always been at point A and would always be at point B after the completion of the saccade. The problem of leakage, due to the fact that the starting point A did not coincide with endpoint B, was solved by forcing the begin and end points of the signal to zero using a Hanning window as described in section 4.2.2. In figure 5.2 the calculated

 $^{^{2}}$ Since saccades are very fast eye movements, the trajectory cannot be changed once the saccade has started.



Figure 5.1: Recording of two saccades with the SSC system. **A**: Eye position signal of a 5° saccade to the right. **B**: Eye position signal of a 28° saccade to the left. **C**: Velocity signal of the saccade shown in figure **A** obtained with the two-point differentiation algorithm with a spread of two (grey) and a spread of six (black). **D**: Velocity signal of the saccade shown in figure **B** obtained with the two-point differentiation algorithm in figure **B** obtained with the two-point differentiation algorithm.

power spectra are shown for saccades of varying magnitude; 5° , 10° , 20° and 28° . The resolution of the power spectra was chosen to be 0.1 Hz by extrapolating the signal to 10 s.

It can be seen from the figure that in the low frequency domain the characteristics correspond to those predicted by the instantaneous step model described in section 4.2.3: a constant rolloff of 20 dB/decade, whereas larger amplitude saccades have larger absolute values in the low frequency domain. In the higher frequency domain, a downward departure of this constant rolloff can be seen, indicating a lack of energy at high frequencies, as observed by Harris et al. [Harris et al., 1990].

From the figure it can be seen that this downward departure occurs at approximately 25-30 Hz for the small amplitude saccades and at lower frequencies for the larger amplitude saccades. This indicates that for the accurate description of eye movements, the higher frequency components are relatively more important for the small than for the large amplitude saccades. As a consequence, in order to accurately record small amplitude saccades. This phenomenon is also evident when the signal is considered in the time domain. According to Carpenter [Carpenter, 1988], for healthy subjects there exists a linear relation between the duration and the amplitude of a saccade. The duration of saccadic eye movements larger than 5° in amplitude is roughly 20-30 ms plus about 2 ms for every degree of amplitude. In order to, be able to reconstruct the original signal from the sampled signal,



Figure 5.2: Power spectra of saccades of varying amplitude recorded with the SSC system. Top to bottom: 28° , 20° , 10° and 5° . The dotted lines represent the power spectra of the idealized, instantaneous saccades with corresponding amplitudes.

it is evident that the time between two sample points may not exceed the duration of the saccade. Otherwise, even with beforehand information about the exact shape of saccadic eye movements, it would be impossible to determine the time of the saccadic onset, making a determination of the latency time, as described in section 2.3, impossible. To satisfy this condition, a sampling frequency of at least 50 Hz is required.

Another feature of the power spectra in figure 5.2 are the local minima. When an incorrectly limited time window is used, a similar pattern of minima can occur. However, since Harris et al. [Harris et al., 1990] found these same minima using an infinite Fourier transform algorithm, these minima can not be ascribed to this phenomenon. Using both computer simulations and electrophysiological recordings, Harris et al. found that these minima are caused by a pulse-step signal that drives the saccadic eye movement. Moreover, the first of these minima occurs at a frequency that is the reciprocal of the duration of the pulse.

In order to obtain the eye velocity, the eye position is differentiated with respect to time. From equation (4.30) it follows that the two-point central difference differentiation algorithm with a spread of two has a bandwidth of approximately 220 Hz. However, since from the power spectra it can be concluded that all relevant information of saccadic eye movements is contained in the low frequency components, the bandwidth of the differentiation algorithm should not be chosen too large in order to suppress noise in the velocity signal introduced by the differentiation of high frequency noise. In practice, the use of the differentiation method with a spread of six (bandwidth = 74 Hz) results in reliable velocity signals, as shown in figure 5.1 C and D. The noisy grey velocity signal is obtained using the two-point differentiation algorithm with a spread of two, whereas the thick black velocity signal is obtained using a spread of six.

In order to obtain the velocity signal of the SSC technique, in this study the two-point central difference differentiation algorithm is used with a spread of six.

5.4.2 EOG system

In figure 5.3 A, the raw data of the eye position of a 6° saccade, recorded with the EOG method at a sampling frequency of 1 kHz, is shown. Comparison of this figure with figure 5.2 shows that



Figure 5.3: Data of a 6° saccade recorded with the EOG system. A: Position signal of original data (grey) together with the filtered data containing only frequency components up to 25 Hz (black). B: Data containing the frequency components higher than 25 Hz. C: Velocity signal calculated from A using the two-point central difference differentiation algorithm. D: Velocity signal calculated from B.

the EOG signal contains significantly more noise than the SSC signal. This can also be concluded from the calculated power spectra that are shown in figure 5.4. For the calculation of these power spectra the same routine was used as described in the previous section. When comparing this figure with figure 5.2, it can be seen that for frequencies higher than 30 Hz, the spectra recorded with the SSC method decay more rapidly and to a lower noise level than the spectra recorded with the EOG system. This indicates that these frequency components contain considerable amounts of noise for the EOG system and can lead to undesired disturbances in the calculated eye position and velocity.

In order to show that even for the small amplitude saccades all relevant information is contained in the frequency components up to 25 Hz, the eye position data of a 6° saccade was transformed to the frequency domain using the discrete Fourier transform. Before this transform was applied, a Hanning window was used in order to avoid errors caused by the truncation of the signal. Next, the



Figure 5.4: Power spectra of saccades of varying amplitude recorded with the EOG system. Top to bottom: 30° , 20° , 10° and 5° . The dotted lines represent the power spectra of the idealized, instantaneous saccades with corresponding amplitudes.

frequency components corresponding to frequencies higher than 25 Hz were given the value zero, to eliminate these components and to maintain the same resolution in the time domain. Although such a steep window in the frequency domain can cause ringing in the time domain when an inverse Fourier transform is performed, this approach was chosen since this way no information was lost for the frequency components below 25 Hz. Thereafter, an inverse discrete Fourier transform was used to transform the filtered signal back into the time domain. Finally, a correction was made for the applied Hanning window by using an inverse Hanning window in the time interval of interest, to be able to compare the original data with the filtered data. The original eye position data is shown in figure 5.3 A together with the filtered data. The data corresponding to frequencies higher than 25 Hz are shown in figure 5.3 B³. In the bottom figures C and D, the velocity profiles calculated from the upper figures using the two-point central difference differentiation algorithm are shown. Note that since the high frequency noise is already filtered out of the signal, is it not necessary to use the six-point differentiation algorithm as for the SSC technique.

As can be seen in this figure, filtering the signal this way leads to a significant increase of the signal-to-noise ratio. Furthermore, by comparing the original data with the filtered data, it appears that all relevant information of the eye position signal is completely contained in the frequency components up to 25 Hz. This observation is confirmed by regarding the signal composed of the higher frequency components (figure 5.3 **B**), which appears to contain no relevant information and is dominated by noise. From this figure the noise level can be determined to be in the order of 0.3° .

³In order to obtain this data, the eye position data was transformed to the frequency domain using the same procedure as for the filtering of the EOG signal, which is described in this section. For this case, however, the frequency components corresponding to frequencies of 0 - 25 Hz were given the value zero.

Also for the eye velocity signal, obtained from the filtered eye position signal, the signal-to-noise ratio is highly increased and the filtered eye velocity signal appears to be a good approximation of the true eye velocity.

In conclusion, in order to increase the signal-to-noise ratio, in this study the EOG signal is filtered as described above. The velocity signal is then obtained by the two-point central difference differentiation algorithm with a spread of two.

5.4.3 VOG system

An important disadvantage of the VOG system is its low sampling frequency of 50 Hz. Before showing the raw and processed VOG data, it will first be shown that even with the use of higher order and thus more accurate differentiation algorithms, no reliable value for the peak velocity can be obtained. Next, it will be shown that a saccade with an amplitude larger than 5° which is sampled at a frequency of 50 Hz can be quite accurately resampled at higher sampling frequencies using equation (4.2). In order to illustrate these two phenomena, saccadic eye movements recorded with the SSC system are used, since this system provides accurate position and velocity signals and thus a good comparison can be made.

From equation (4.30) it follows that the bandwidth of the two-point central difference differentiation algorithm is 11 Hz for a sampling frequency of 50 Hz. Since the frequency response of the first order derivative H(f) is better approximated when more anti-symmetric functions are used, as described in section 4.3.1, an algorithm using higher order anti-symmetric functions for the calculation of the derivative will yield better results. In equation (4.21) it can be seen that when using higher order anti-symmetric function, more data points are used to differentiate the signal.

In figure 5.5 the velocity signal of a 15° saccade recorded with the SSC system (sampling rate 1 kHz) is shown. The grey continuous line corresponds to the eye velocity obtained as described in section 5.4.1. Then, the position signal was decimated at a sampling rate of 50 Hz. The velocity signals obtained from the 50 Hz signal by using a two-, four- and six-point central difference differentiation algorithm are included in figure 5.5. The coefficients C_k in equations (4.21) are determined by minimizing equation (4.24) for the coefficients C_k with $f_{max} = 25.^4$

In figure 5.5 it can be seen that the higher order differentiation algorithms yield better approximations of the instantaneous velocity. However, as already stated in section 4.3.2, not only the used differentiation algorithm is an important aspect, but also the location of the sampling points plays an important role, which can be seen by comparing the left and right figures in figure 5.5. In the right figure, there is no sampling point near the peak velocity, resulting in a poor estimation of the peak velocity. In the left figure, the same original velocity signal is shown, but in this figure there is a sampling point at the time where the eye reaches the peak velocity. The presence of this sampling point results in a better estimation of the peak velocity. This means that in order to find an accurate estimation of the peak velocity of saccades it is not only necessary to use a differentiation algorithm with an appropriate bandwidth, but also a sampling frequency of at least 300 Hz is required as is shown in section 4.3.2.

As shown in the previous sections, saccadic eye movements can in good approximation be considered to be bandwidth limited with an upper frequency limit of about 25 - 30 Hz. As a consequence, for a signal sampled at a frequency of 50 Hz, the eye position for times in-between two consecutive sampling points can be obtained fairly accurately using the Nyquist sampling theorem (equation (4.2)). In order to show this, saccadic eye movements were recorded with the SSC signal at a sampling frequency of 1 kHz. Next, all frequency components corresponding with frequencies higher than 25 Hz were removed from the signal using the same procedure as for the filtering of the EOG signal described in section 5.4.2. The resulting signal can be considered to be obtained directly by using a 25 Hz low-pass filter before sampling at a rate of 1 kHz. Next, the filtered signal was resampled at a frequency of 50 Hz, after which the original signal was

⁴For the two-point algorithm $C_1 = 50s^{-1}$ for the four-point algorithm, $C_1 = 89.5s^{-1}$ and $C_2 = -19.7s^{-1}$, and $C_1 = 91.9s^{-1}$, $C_2 = -35.0s^{-1}$ and $C_3 = 9.3s^{-1}$ for the six-point algorithm.



Figure 5.5: Influence of the differentiation algorithm. The original velocity signal derived from the position signal of a 15° saccadic eye movement sampled at 1 kHz (grey) is shown together with the eye velocity signals derived from the eye position data sampled at 50 Hz using different differentiation algorithms. The presence of a sampling point near the peak velocity (left) leads to a better approximation of the estimated peak velocity than the situation where the sampling point is not in the vicinity of the peak velocity (right).

reconstructed (interpolated to the original sampling frequency of 1 kHz) by using equation (4.2). The original eye position and eye velocity data of two saccades of 5° and 25° are shown in figure 5.6, together with the resampled (50 Hz) and reconstructed signals. The eye velocity signals were obtained by differentiating the corresponding eye position signals with respect to time, using the two-point central difference differentiation algorithm. From this figure it can be seen that a significantly better approximation for the peak velocity can be obtained by using the Nyquist sampling theorem to reconstruct the original signal. Moreover, better approximations are obtained for relatively large amplitude saccades. It can be seen that for the 25° saccade a reliable value for the peak velocity is obtained, whereas for the small amplitude saccade of 5° an underestimation of approximately 10% is found. Furthermore, it follows from the figure that for both the position and velocity signal of the 5° amplitude saccade ringing occurs at the start and end of the saccade. Also for the large 25° amplitude saccade this effect can be seen in the velocity signal. This is an indication that in the original signal the frequency components higher than 25 Hz still contained a small amount of relevant information.

From these results it can be concluded that in order to improve the time resolution of the raw VOG data, the signal can fairly accurately be reconstructed at higher frequencies using the method described above for the SSC signal. Thereafter, the eye velocity signal can be obtained by using the two-point central difference differentiation algorithm. In figure 5.7 the raw 50 Hz eye position and velocity signal, and the corresponding 1 kHz reconstructed signals are shown for a

 5° and a 25° degrees saccade recorded with the VOG system.

Finally, it is remarked that during eye blinks it was impossible for the VOG system to obtain reliable estimates of the position of the pupil centre due to eyelid closure. These events were marked and afterwards the position was estimated by using linear interpolation. This approach was chosen in order to be able to reconstruct the signal using equation (4.2) without introducing large errors in the eye position directly before or after the eye blink.

In conclusion, it was shown in this section that in order to find accurate estimations of saccadic peak velocities, the saccadic position signals must have a time resolution in the order of 3 ms. Furthermore, it was shown that by using the Nyquist sampling theorem, it is theoretically possible to obtain any desired time resolution for a saccadic position signal sampled at 50 Hz. In this study, this technique is used to obtain saccadic eye position signals with a time resolution of 1 ms. Thereafter, the velocity signals are calculated from the position signals using the two-point central difference differentiation algorithm.



saccade followed by a small correction saccade. C: Eye velocity signal of the position signal shown in A. D: Eye velocity signal of the eye position the signal is reconstructed at a frequency of 1 kHz by using equation (4.2). A: Eye position signal of a 5° saccade. B: Eye position signal of a 25° Figure 5.6: Position and velocity signals obtained with the SSC system sampled at 1 kHz. The position signal is resampled at a rate of 50 Hz. Next, signal shown in **B**.



Figure 5.7: Position and velocity signals obtained with the VOG system. The raw 50 Hz VOG data (solid line) is shown together with the reconstructed 1 kHz data (dashed line). A: Eye position signal of a 5° saccade. B: Eye position signal of a 25° saccade. C: Eye velocity signal of saccade shown in A. D: Eye velocity signal of saccade shown in B:.

Chapter 6

Results

In order to make a quantitative comparison between the three methods (EOG, VOG and SSC), the signals are analysed in two ways. First, the static properties are investigated by examining the absolute eye position during the time intervals of fixation. Second, the dynamic properties are considered by determining the begin- and end position of each saccadic eye movement of 5° or larger. Besides the amplitude of the saccades, also the peak velocity is determined.

6.1 Comparison of the signals

Each signal was processed as described in chapter 5. Then, the signals were calibrated using the horizontal and vertical calibration data. In figure 6.1, part of the position signal of the left eye of subject 3, recorded simultaneously with the EOG, VOG and SSC method during horizontal midline-crossing saccades, is shown together with the position of the spot on the screen. In the figure the VOG and the SSC signals appear to be almost identical and are hard to distinguish, whereas the EOG system is less stable, due to baseline drift and eye blink artifacts present in the signal. Moreover, a change in skin impedance taking place during the measurement, resulted in an erroneous amplification of the EOG signal.

In figure 6.2, the position and velocity signal of two horizontal saccades with amplitudes of 4° to the right and 25° to the left, shown in figure 6.1, are presented in more detail. As can be seen in the figure, all three recording methods yield the same peak velocity for the 4° amplitude saccade, despite the observed ringing in the position and velocity signals of the EOG and VOG systems. However, for other small saccades with an amplitude in the order of 5° sometimes an underestimation up to 10% of the peak velocity is obtained with the EOG and VOG system ,as shown in figure 5.6. For the large amplitude saccade of 25° , the VOG and SSC signals yield similar position and velocity signals. However, the peak velocity determined by the EOG system is higher due to the erroneous amplification of the EOG signal.

For completeness, in figure 6.3 also two vertical saccadic eye movements of 10° up and 22° down are shown. The signals are simultaneously recorded with the EOG, VOG and SSC method. As can be seen in this figure, the EOG signal displays a severe overshoot for the 10° upwards saccade, which results in an overestimation of the peak velocity. This phenomenon was also observed by Yee et al. [Yee et al., 1985]. They concluded that this artifact was probably caused by eyelid movement. Furthermore, Yee et al. found that immobilization of the eyelids decreased the effects of the artifact, but still no reliable EOG signal was obtained to calculate an accurate value for the peak velocity.



Figure 6.1: Position signal of the left eye of subject 3 for horizontal midline-crossing saccades recorded simultaneously with the EOG, VOG and SSC system. Also the position of the spot projected on the screen is shown.



Figure 6.2: Position signal of the left eye of subject 3 for two horizontal saccadic eye movements simultaneously recorded with the EOG, VOG and SSC method. A: Eye position signal of a 4° saccade to the right. B: Eye position signal of a 25° saccade to the left. C: Eye velocity data of the saccade shown in A. D: Eye velocity data of the saccade shown in B.

Figure 6.3: Position signal of the left eye of subject 3 for two vertical saccadic eye movements simultaneously recorded with the EOG, VOG and SSC method. A: Eye position signal of a 10° upwards saccade. B: Eye position signal of a 22° downwards saccade. C: Eye velocity data of the saccade shown in A. D: Eye velocity data of the saccade shown in B.

6.2 Accuracy and sensitivity

In this section, the static properties of the methods are considered for both horizontal and vertical saccades. First, the absolute eye position obtained from each method is determined during every time interval of fixation. The difference between these positions and the position of the dot on the screen is calculated. This difference is a measure of the accuracy of the recording technique.

Next, the sensitivity of each method is determined. In order to estimate the sensitivity, the noise level of the signals during the periods of fixation is determined. This is done by averaging the standard deviation calculated for each fixation period. As a rule of thumb, the sensitivity of the system is taken to be equal to twice this noise level, to be able to distinguish an eye movement from the noise.

6.2.1 Horizontal fixation

In figure 6.4 the difference between the measured eye position and the position of the spot on the screen is shown as a function of time for one subject. In the upper figure, this difference is shown for the EOG method for the left and right eye of subject 3, together with the difference for both the VOG and SSC method for the left eye only. In the lower figure, the VOG and SSC signals are displayed in more detail. As can be seen from the upper figure, the accuracy of the VOG and



Figure 6.4: Difference between the recorded absolute eye position and the position of the spot on the screen during fixation as a function of time. In the upper figure this difference is shown for the EOG system for the left and right eye of subject 3, together with the left eye for the VOG and SSC methods. In the lower figure this difference is shown in more detail for the VOG and SSC methods.

SSC systems is much better than that of the EOG signal, since the latter is considerably affected by baseline drift. For the right eye, this baseline drift is about $0.25^{\circ}s^{-1}$. For all subjects, baseline drift was observed during the experiment.

In the lower figure it can be seen that the difference of the screen position and the eye position measured with the VOG and SSC methods is in the order of $\pm 1^{\circ}$. However, it can also be seen that the differences of the VOG and SSC methods are highly correlated, which indicates that the accuracy of both systems is better than $\pm 1^{\circ}$. This correlation can be explained by two phenomena. First, since the spot on the screen had a diameter corresponding to an angle of 0.5° , it is possible that the subject not always fixated on the same part of the spot. Second, the accuracy of the position of the spot recorded with Balancelab was in the order of 0.1° because the digital output signal of the visual stimulator recorded with Balancelab was converted with an 8 bit A/D converter. As a consequence, a better estimation of the accuracy of the systems is obtained by comparing the eye positions recorded with the VOG method to the positions recorded with the SSC method. In figure 6.4 it can be seen that the maximum difference in eye position recorded by these both methods is in the order of 0.5° . As a result, the accuracy of both the VOG and SSC systems can be assumed to be better than 0.5° .

The calculated accuracy and precision for each subject are shown in table 6.1. From this table, a number of properties of each signal can be concluded.

Measurement	Sub	ject 1	Sub	ject 2	Sub	ject 3			
EOG, VOG	Accuracy	Sensitivity	Accuracy	Sensitivity	Accuracy	Sensitivity			
EOG left	4°	0.6°	4°	0.3°	8°	1°			
EOG right	3°	0.7°	4°	0.3°	4°	0.6°			
EOG comp	2°	0.4°	3°	0.2°	4°	0.2°			
VOG left	1°	0.2°	1.5°	0.1°	0.5°	0.1°			
VOG right	1°	0.2°	2.5°	0.2°	0.5°	0.1°			
EOG, VOG,SSC									
EOG left	7°	0.5°	8°	0.4°	5°	0.6°			
EOG right	3°	0.6°	23°	0.7°	15°	0.9°			
EOG comp	4°	0.5°	10°	0.4°	5°	0.5°			
VOG left	0.7°	0.2°	5°	0.3°	0.5°	0.2°			
VOG right	0.7°	0.2°	2°	0.1°	0.5°	0.2°			
SSC left	0.7°	0.2°	1°	0.1°	0.5°	0.2°			
SSC right	0.7°	0.2°	1°	0.1°	-	-			

Table 6.1: Accuracy and sensitivity for the measurement of horizontal eye movements of each recording technique for all three subjects.

First, it is not possible to determine a well-defined value of the accuracy for the EOG system since the EOG signal is sometimes subject to severe baseline drift. In general, this baseline drift results in increased erroneous values for the absolute eye position as time elapses. In order to increase the accuracy of the EOG signal, the system must frequently be recalibrated. Even doing so, the EOG method is not suited for the accurate measurement of absolute horizontal eye positions.

Second, in the table it can be seen that, except for subject 2, the accuracy of the VOG signal is better than 1° and comparable to that of the SSC method. Moreover, the placement of the scleral search coil during the second series of experiments did not seem to represent a problem for the video eyetracker, as can be seen from the determined accuracies. For subject 2 the accuracy of the VOG system was significantly worse than for the other two subjects. The main reason for this was the fact that for subject 2, part of the pupil was constantly covered by the upper eye lid. Another complication was that subject 2 had deep brown eyes, making it hard to distinguish the pupil from the iris. This combination made an accurate determination of the position of the centre of the pupil difficult.

Finally, in table 6.1 it can be seen that the sensitivity found for the EOG method varies

per subject and has a range of $0.3^{\circ} - 1^{\circ}$ for the monocular configuration. The sensitivity of the bitemporal configuration, however, is more stable and has a value smaller than 0.5° for all subjects. The reason for this is that the bitemporal signal can be considered to be the average of the both monocular signals. As a consequence, noise artifacts occurring in one of the two monocular signals are averaged. For both the VOG and SSC signals the sensitivity has a stable value in the order of 0.2° .

6.2.2 Vertical fixation

For the determination of the accuracy and sensitivity of the vertical saccades, the same procedure is followed as for the horizontal saccades described in the previous section. Unfortunately, this could only be done for subject 3, since for the other two subjects, the vertical EOG signals were highly disturbed by interference with the a.c. magnetic fields of the SSC system. The calculated accuracy and sensitivity for this subject are shown in table 6.2.

Table 6.2: Accuracy and sensitivity of the measurement for vertical eye movements of each recording technique for subject 3.

Measurement	Subject 3							
EOG, VOG	Accuracy	Sensitivity						
EOG left	5°	1°						
EOG right	4°	0.9°						
VOG left	1°	0.2°						
VOG right	1°	0.2°						
EOG, VOG,SSC								
EOG left	3°	0.7°						
EOG right	4°	0.7°						
VOG left	1°	0.2°						
VOG right	1°	0.2°						
SSC left	0.8°	0.1°						

For the vertical saccades the same features can be observed with respect to the accuracy and sensitivity as for the horizontal saccades. Note that for the vertical eye movements no composed EOG signal of both eyes can be recorded.

For the EOG signal it is again not possible to obtain a well-defined value for the accuracy due to baseline drift. As a consequence, the EOG method is not suited for the measurement of absolute vertical eye positions. For the VOG and SSC signal, the accuracy is better than or equal to 1° .

Further, the sensitivity of the EOG system is in the order of $0.7^{\circ} - 1^{\circ}$, whereas for the VOG and SSC methods this is in the order of 0.2° .

Since the vertical saccades could only be measured for one subject it is difficult to draw general conclusions about the results.

6.3 Comparison of saccadic amplitude

In the previous section, the accuracy and precision of the recording techniques were determined. It was shown that the accuracy of the EOG technique was highly affected by baseline drift. However, in clinical vestibular practice one is interested in the saccadic amplitude instead of the absolute eye position. Since the duration of a saccadic eye movement is in the order of 30 - 100 ms, the influence of baseline drift on the saccadic amplitude is negligible. In this section, the saccadic amplitudes recorded by each method are compared.

6.3.1 Horizontal saccades

In order to compare the measurement techniques that were used, the amplitude of each recorded saccadic eye movement was determined. In figure 6.5 the horizontal saccadic amplitudes determined with the EOG and VOG methods are shown versus the amplitudes determined with the SSC method. In this figure, the amplitude of the right (left) eye measured with the EOG and VOG methods is plotted against the amplitude of the right (left) eye recorded with the SSC method. Note that with the SSC method no composed signal could be measured. In order to be able to make a comparison for the composed EOG signal as well, it was plotted against the average value of the left and right eye of the SSC method in the left graph.



Figure 6.5: Correlation between the different recording techniques for the measured amplitudes of horizontal saccades. Left: EOG versus SSC. Right: VOG versus SSC.

From the figure it is evident that there is a linear relation between both the saccadic amplitudes measured with the EOG and SSC method, and the amplitudes measured with the VOG and SSC method. This indicates that neither the EOG nor the VOG tends to saturate at larger saccadic amplitudes measured. The values of the slopes found for each subject are displayed in table 6.3.

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Measurement	Subject 1	Subject 2	Subject 3							
EOG, VOG	EOG plotted vs VOG									
EOG left	1.018 ± 0.005	1.132 ± 0.008	1.055 ± 0.009							
EOG right	0.974 ± 0.005	1.118 ± 0.009	1.026 ± 0.006							
EOG comp	0.995 ± 0.003	1.126 ± 0.005	1.040 ± 0.007							
EOG, VOG,SSC	EOG and VOG plotted vs SSC									
EOG left	0.902 ± 0.006	0.92 ± 0.01	1.14 ± 0.01							
EOG right	0.916 ± 0.005	0.97 ± 0.03	-							
EOG comp	0.910 ± 0.003	0.93 ± 0.02	-							
VOG left	1.041 ± 0.003	0.905 ± 0.005	0.988 ± 0.003							
VOG right	1.000 ± 0.003	0.989 ± 0.005	-							

Table 6.3: Slopes of the relations between the EOG, VOG and SSC signals for the measured amplitudes of horizontal saccades.

From the values of the slopes displayed in this table it can be concluded that for the saccadic amplitude the VOG technique shows a better agreement with the SSC method than the EOG technique; the values for the slopes found for the VOG technique show a significantly smaller deviation from 1 than the values for the slopes of the EOG technique. With the exception of the left eye of subject 2, the slope of the video eyetracker differs at most 4% from the value 1. The reason for the relatively large deviation of 10% from 1 for the left eye of subject 2 is again the fact that part of the pupil was covered by the eyelid. For the EOG system, the deviations were caused by a change in the amplification factor that took place in the time interval from the time of calibration until the time of measurement. This resulted in an erroneous amplification factor of the signal up to 14%.

6.3.2 Vertical saccades

For the comparison of the amplitudes of the vertical saccades recorded with the different techniques, the same procedure is followed as described in the previous section. In figure 6.6 the vertical saccadic amplitudes determined with the EOG and VOG methods are shown versus the amplitudes determined with the SSC method. Note that in this figure only the left eye is shown since no signal of the right eye could be recorded with the SSC system for subject 3. As can be seen



Figure 6.6: Correlation between the different recording techniques for the measured amplitudes of vertical saccades. Left: EOG versus SSC. Right: VOG versus SSC.

in this figure, the linear relation between the saccadic amplitudes measured with the VOG and SSC method is better than that of the EOG and SSC method. Moreover, from a comparison of figure 6.6 with figure 6.5, it appears that the linear relation between the EOG and SSC technique is better for horizontal than for vertical saccadic eye movements. However, since the vertical eye movements could only be measured for one subject with the EOG technique and, moreover, only the left eye could be compared with the SSC method, no general conclusion can be drawn. The values of the slopes found for subject 3 for vertical saccadic eye movements are shown in table 6.4.

Table 6.4: Slopes of the relations between the EOG, VOG and SSC signals for the measured amplitudes of vertical saccades.

	-
Measurement	Subject 3
EOG, VOG	EOG plotted vs VOG
EOG left	1.06 ± 0.02
EOG right	1.05 ± 0.01
EOG, VOG,SSC	EOG and VOG plotted vs SSC
EOG left	1.13 ± 0.02
VOG left	1.016 ± 0.006

6.4 Comparison of the main sequence

In clinical practice, not only the static properties of a recording method, such as accuracy and sensitivity, are important, but also its dynamic properties. In order to come to a reliable diagnosis, it is also important to have an accurate velocity signal. In this section, the relation between the saccadic amplitude and peak velocity, the so-called main sequence [Bahill et al., 1975], is investigated.

6.4.1 Horizontal saccades

In figure 6.7 the main sequence of horizontal midline-crossing saccades of subject 2, recorded with the EOG, VOG and SSC system, is shown for the left eye. Positive amplitudes correspond to saccades to the right whereas negative negative amplitudes correspond to saccades to the left.



Figure 6.7: Main sequence of horizontal midline-crossing saccades of the left eye of subject 2 simultaneously recorded with the EOG, VOG and SSC method. Positive amplitudes correspond to saccades to the right. Negative amplitudes correspond to saccades to the left. Also the fits of an exponential curve through the data points are shown.

This figure shows that for this subject all three recording techniques yield similar main sequence relations.

In order to be able to determine whether the main sequence relations differ significantly for the three subjects, it is necessary to make a quantitative comparison. The main sequence data were



Figure 6.8: Main sequence of horizontal midline-crossing saccades of the both eyes of subject 1 simultaneously recorded with the VOG and SSC method. Positive amplitudes correspond to saccades to the right. Negative amplitudes correspond to saccades to the left. Also the fits of an exponential curve through the data points are shown.

fitted by a theoretical curve in such a way that the mean square error between the theoretical line and the data points was minimized. In the absence of a definite model to explain the data, two types of curves were tried, in line with the generally accepted approach reported in the literature.

The first was an exponential curve of the form

$$v_{peak} = C(1 - \exp[-b(A_{sac})]),$$
 (6.1)

with v_{peak} the peak velocity, A_{sac} the saccadic amplitude, and C and b constants with dimensions degrees s⁻¹ and degrees⁻¹, respectively. This curve for the main sequence was suggested by several investigators [Van der Geest and Frens, 2002, Houben et al., 2006, Baloh et al., 1975]. For this curve, the peak velocity saturates to a maximum peak velocity for large saccades, which is also found in literature [Carpenter, 1988]. The value of constant C corresponds to this maximum saturated peak velocity. The value of constant b corresponds to the rate of saturation for increasing amplitudes.

The second curve that was tried was a power-law curve of the form

$$v_{peak} = CA_{sac}^{\ b},\tag{6.2}$$

suggested by Baloh et al. [Baloh et al., 1975]. When using this function for the description of the main sequence, two problems arise that are not taken into consideration by Baloh et al.. First, the dimension of the constant C is not well-defined, but depends on the value of constant b ($[C] = m/(sdegrees^b)$). Second, according to this curve no saturation of the peak velocity occurs for large amplitude saccades.

When these two curves were fitted to the main sequence plots, two observations suggested that the exponential curve yields the best agreement with the data. First, the sum of squares¹ SS was significantly less (for most cases less than half) for the exponential curve than for the power-law. Moreover, the exponential function showed a better correlation coefficient R^2 than the power-law curve function. Second, by visual inspection of the data it appeared that the main sequence experienced an asymptotic behavior. This was in agreement with the observations of Baloh et al..

¹The sum of squares is defined as the sum of the squared differences between the data points and the curve.

																														_
		R^2	0.90	0.95	0.97	0.89	0.90		0.96	0.93	0.96	0.96	0.93	0.98	ı		R^2	0.97	0.91	0.97	0.93	0.91		0.97	0.95	0.98	0.95	0.97	0.96	ľ
		SS	930	587	301	630	927		1365	1709	963	635	1143	194	I		SS	477	950	390	821	906		1151	1380	486	952	552	586	
Subject 3	5	$b [\deg^{-1}]$	$\boldsymbol{0.12\pm0.01}$	0.076 ± 0.008	0.097 ± 0.006	0.15 ± 0.01	0.12 ± 0.01		0.09 ± 0.01	0.07 ± 0.01	0.08 ± 0.01	0.13 ± 0.01	0.17 ± 0.02	0.16 ± 0.01	I		$b [\mathrm{deg}^{-1}]$	0.077 ± 0.006	$\boldsymbol{0.13\pm0.01}$	0.104 ± 0.006	0.14 ± 0.01	0.13 ± 0.01		0.07 ± 0.01	0.08 ± 0.01	0.09 ± 0.01	0.13 ± 0.01	0.12 ± 0.01	0.16 ± 0.01	
bject.		$C \left[\deg s^{-1} \right]$	510 ± 17	568 ± 28	526 ± 13	456 ± 12	533 ± 19		560 ± 29	567 ± 39	544 ± 25	427 ± 13	431 ± 14	383 ± 6	I		$C \left[\deg s^{-1} \right]$	555 ± 19	492 ± 16	504 ± 12	531 ± 14	460 ± 15		599 ± 32	560 ± 28	505 ± 14	460 ± 16	445 ± 13	424 ± 10	
sach su		R^2	0.98	0.96	0.98	0.96	0.95		0.98	0.87	0.93	0.95	0.94	0.94	0.91		R^2	0.99	0.98	0.99	0.96	0.93		0.95	I	I	0.97	0.97	0.96	0.06
ed for e	ight	SS	557	895	457	689	937		456	1793	938	918	907	858	696	eft	SS	346	370	332	667	1244		833	I	I	411	543	389	418
<i>l curves determin</i> Subject 2	Saccades to the ri	$b [\deg^{-1}]$	0.049 ± 0.005	0.052 ± 0.006	0.051 ± 0.004	0.07 ± 0.01	0.07 ± 0.01		0.048 ± 0.005	0.06 ± 0.01	0.06 ± 0.01	0.07 ± 0.01	0.08 ± 0.01	0.08 ± 0.01	$\underline{0.10\pm0.01}$	Saccades to the l	$b [\deg^{-1}]$	0.049 ± 0.003	0.067 ± 0.004	0.059 ± 0.003	0.086 ± 0.006	0.09 ± 0.01		0.048 ± 0.005	ı	ı	0.10 ± 0.01	0.083 ± 0.008	0.11 ± 0.01	0.11 ± 0.01
he exponentia	01	$C \left[\deg s^{-1} \right]$	749 ± 47	736 ± 55	736 ± 41	636 ± 43	$\overline{632\pm54}$		726 ± 51	608 ± 73	576 ± 47	638 ± 47	566 ± 34	553 ± 29	496 ± 24		$C \left[\deg s^{-1} \right]$	804 ± 32	686 ± 21	728 ± 25	590 ± 24	589 ± 30		681 ± 73	ı	ı	549 ± 22	636 ± 30	544 ± 15	501 + 16
rs for t		R^2	0.90	0.91	0.94	0.96	0.96		0.98	0.96	0.98	0.97	0.92	0.99	0.98		R^2	0.93	0.90	0.93	0.93	0.95		0.95	0.93	0.96	0.92	0.92	0.93	0.03
vramete		SS	329	491	218	397	418		171	231	172	203	733	83	97		SS	454	537	317	640	610		183	448	204	515	408	300	385
Table 6.5: Fit po Subject 1	5	$b \; [\deg^{-1}]$	0.12 ± 0.01	$\boldsymbol{0.08 \pm 0.01}$	0.103 ± 0.007	0.125 ± 0.008	0.113 ± 0.008		0.11 ± 0.01	0.13 ± 0.01	0.11 ± 0.01	0.11 ± 0.01	0.10 ± 0.01	0.112 ± 0.004	0.15 ± 0.01		$b [\mathrm{deg}^{-1}]$	0.07 ± 0.01	0.12 ± 0.01	0.10 ± 0.01	0.11 ± 0.01	0.10 ± 0.01		0.12 ± 0.01	0.11 ± 0.01	0.12 ± 0.01	0.11 ± 0.01	0.13 ± 0.01	0.12 ± 0.01	0.11 ± 0.01
		$C \left[\deg s^{-1} \right]$	511 ± 11	470 ± 15	475 ± 11	473 ± 10	487 ± 12		438 ± 9	348 ± 9	391 ± 9	441 ± 9	461 ± 11	393 ± 6	369 ± 4		$C \left[\deg s^{-1} \right]$	507 ± 20	502 ± 15	470 ± 14	495 ± 15	534 ± 18		311 ± 10	397 ± 14	343 ± 10	409 ± 15	383 ± 11	350 ± 11	370 + 19
Measurement		EOG, VOG	EOG left	EOG right	EOG comp	VOG left	VOG right	EOG, VOG, SSC	EOG left	EOG right	$EOG \ comp$	VOG left	VOG right	SSC left	SSC right		EOG, VOG	EOG left	EOG right	EOG comp	VOG left	VOG right	EOG, VOG, SSC	EOG left	EOG right	$EOG \ comp$	VOG left	VOG right	SSC left	SSC right

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In table 6.5 the parameters for the exponential curves that were fitted to the main sequence are shown. Interesting significant differences between the signals are highlighted in gray. Three different features can be observed. In order to distinguish the data corresponding to each feature, the values are underlined, bold and italic.

First, for subject 1 and 3 an asymmetry can be seen in the EOG signal (shown in bold). For the experiment without the scleral search coil, it can be seen that the constant b has a higher value for adducting saccades than for abducting saccades². Moreover, for the experiment with the scleral search coil also an asymmetric value of the constant C can be observed for subject 1. In section 6.5 this phenomenon will be described in more detail.

Second, for all three subjects, table 6.5 shows that the values found for C for the VOG method are always higher than or equal to the corresponding values found for the SSC method (shown in italic). This can also be concluded from figure 6.8 by visual inspection.

Finally, for subject 2 the values of C and b for the EOG technique differ systematically from those for the VOG technique (underlined); the EOG method yields higher values for the constant C and lower values for b. However, it is remarked that for subject 2 the main sequences saturated for larger amplitude saccades than for the other subjects. This is also indicated by the small values found for constant b. As a consequence, larger errors are found for the constants C and b for this subject than for the other subjects.

6.4.2 Vertical saccades

In this section, the main sequence of vertical saccades is considered. In figure 6.9 the main sequence of vertical midline-crossing saccades of the left eye, simultaneously recorded with all three recording technique, is shown. From this figure, a number of phenomena can be seen. First, it is evident



Figure 6.9: Main sequence of vertical midline-crossing saccades of the left eye of subject 3 simultaneously recorded with the EOG, VOG and SSC method. Positive amplitudes correspond to upward saccades. Negative amplitudes correspond to downward saccades.

that for each measurement technique the variability of the mean sequence of vertical saccades is much larger than for the horizontal saccades (cf figure 6.7 and 6.8). Therefore, there was no added value in fitting a theoretical curve to the main sequence as was done for the horizontal

 $^{^2\}mathrm{Adducting}$ saccades are horizontal saccades towards the nose, abducting saccades are horizontal saccades away from the nose.

saccades. This indicates that the movement of horizontal saccades is determined better than that of vertical saccades. Second, the figure shows that with the EOG method considerably higher peak velocities are found for upward saccades, which is presumably caused by eyelid movement. This phenomenon was also observed by Yee et al. [Yee et al., 1985].

6.5 Asymmetry of the EOG signal

The observed asymmetry in the values of C and b for adducting and abducting saccades for subjects 1 and 3 for the EOG technique can also visually be seen from the main sequence relations. In



Figure 6.10: Main sequence of horizontal midline-crossing saccades for both eyes of subject 1 simultaneously recorded with the EOG (upper) and VOG (lower) technique. In the EOG signals an asymmetry can be seen between adducting and abducting saccades that is absent in the VOG signals.

figure 6.10, the main sequences of subject 1 simultaneously measured with the EOG (upper) and VOG (lower) technique are shown. In the figure, also the best fits of the exponential function (equation (6.1)) are shown. The values of the constants C and b of these curves, fitted to the main

sequences, are displayed in table 6.5.

From the upper figure and table 6.5 it follows that for the EOG technique significantly higher values for the peak velocity are found for adducting than for abducting saccades. In the lower figure it can be seen that for the VOG technique no significant differences are observed for the peak velocities between adducting and abducting saccades. Since the eye movements were simultaneously recorded with both the EOG and VOG technique, the observed difference in the main sequence relations must be due to an artifact of either the EOG or VOG technique. There are several important factors indicating that the differences between the main sequence relations are due to an artifact of the EOG technique.

First, also for the horizontal saccadic eye movements simultaneously recorded with the EOG, VOG and SSC technique, the EOG technique recorded higher values for the peak velocity for adducting than for abducting saccades. Although the differences in peak velocity were considerably smaller for the measurements with the scleral search coil than for those without it, they were still significant. The VOG and SSC technique showed no significant difference in peak velocities between adducting and abducting saccades.

Second, in section 3.2.1 it was shown that the asymmetric placement of the electrodes results in asymmetric EOG signals. From equation (3.8) it follows that the time-dependent potential difference between the nasal and temporal electrodes for the left eye is given by

$$\Delta V_l[\alpha(t)] = \frac{p}{2\pi\epsilon r^2} \sin[\theta + \alpha(t)].$$
(6.3)

The eye velocity v measured with the EOG technique is proportional to the time derivative of this potential difference

$$v_l \sim \frac{\mathrm{d}}{\mathrm{d}t} \Delta V_l[\alpha(t)] = \frac{p}{2\pi\epsilon r^2} \frac{\partial \alpha(t)}{\partial t} \cos[\theta + \alpha(t)], \qquad (6.4)$$

with v in degrees per second. From this equation, it follows that if the eye velocity signal were perfectly symmetric, meaning that the maximum velocity is reached when the eye is exactly in the middle of the saccadic movement, no difference in peak velocities between left- and rightwards saccades would be observed.

For small amplitude saccades, the eye velocity signals are in good approximation symmetric. For these saccades the eye is accelerating during the first half of the saccade. Then, the eye starts to decelerate before the maximum eye velocity is reached in order to avoid overshooting the target. As a consequence, the maximum velocity is in good approximation reached in the middle of the saccade.

For the large amplitude saccades, however, the eye reaches it maximum velocity before it reaches the position in the middle of the saccade. This results in an asymmetric velocity signal. As a consequence, a difference in calculated peak velocity between left- and rightward saccades of large amplitudes is expected, according to equation (6.4). Since $\cos(x) \approx 1$ for small values of x and $\cos(x) < 1$ for larger values of x, it follows that this effect becomes larger for larger values of θ .

Since the horizontal saccades were simultaneously measured with the EOG and VOG technique, the position signal of the eye $\alpha(t)$ is known from the VOG technique. This signal is used to show the effect of asymmetric placement of the electrodes on the calculated peak velocity measured with the EOG technique. In order to do so, the eye position $\alpha(t)$ is substituted in equation (6.3). Next, the derivative of this equation, is calculated as a function of the offset angle θ . Since the eye position $\alpha(t)$ is only known at the sampling times, this derivative is approximated by

$$v \sim \frac{\mathrm{d}}{\mathrm{d}t} \left[\Delta V_l \alpha(t) \right] \approx \frac{p}{4\pi T_s \epsilon r^2} \sin(\theta + \alpha[n+1]) - \sin(\theta + \alpha[n-1]), \tag{6.5}$$

with T_s the sampling time in seconds³. From the obtained signal, the maximum velocity is determined. This is done for various values of θ up to 45°. The maximum velocity found for each offset angle θ is expressed in a percentage of the maximum velocity recorded for $\theta = 0$. In

 $^{^{3}}$ Since the 50 Hz sampling rate of the video system was too low, the reconstructed 1 kHz signal was used.

figure 6.11 this is shown for both small saccades with amplitudes in the order of 10° (left) and large amplitude saccades of 25° (right). For each figure, four different saccades, approximately equal in amplitude, were used: one rightward and one leftward recorded for each eye. The solid lines correspond to rightward saccades, the dashed lines to leftward saccades. Note that positive values of θ correspond to the left eye, whereas negative values of θ correspond to the right eye due to the mirror geometry between the eyes. From this figure it can be seen that for small amplitude saccades no significant difference for the values of the peak velocity between adducting and abducting saccades is found, even for large values of θ . For the large amplitude saccades however, it appears that the peak velocities of adducting saccades become increasingly larger than those of abducting saccades for increasing values of θ . For an offset angle θ 20°, a difference of 20% is observed. From figure 6.11 it can be concluded that the asymmetric placement of electrodes can lead to erroneous peak velocities of large saccadic eye movements.



Figure 6.11: Theoretical peak velocity measured with the EOG technique for different values of θ , calculated from the eye position signal recorded with the VOG method. Left: Small amplitude saccade of 10°. Right: Large amplitude saccade of 25°.

Chapter 7 Discussion

When the values for the accuracy and sensitivity found for all three subjects are compared with those found in literature and shown in table 6.1, it can be concluded that for all recording techniques equal or better values for these properties are published than observed in this study. For the EOG method, the reported sensitivity of 0.5° is in the same order as found for all three subjects, i.e. $0.3^{\circ} - 0.7^{\circ}$. The accuracy, however, was observed to be ill-defined due to baseline drift. In this study, the accuracy was observed to be 3° and larger. Therefore, the reported accuracy of $1 - 2^{\circ}$ is doubtful, especially for the clinical practice, due to the relatively long time that is needed for the EOG signal to stabilize at the illumination level. For the VOG and SSC system, the observed values for the accuracy and sensitivity are about a factor 10 larger than those reported in the literature. A possible explanation for this difference is that often the values for the accuracy and sensitivity be reached with the techniques are given. In practice, however, during fixations such small values for these properties are not found due to small eye movements. Moreover, in the literature sometimes accuracy is confused with sensitivity.

In this chapter, first all relevant phenomena observed in this study for each recording technique are discussed. After doing so, the requirements of an eye movement recording technique for vestibular clinical practice are considered, in order to conclude which method is best suited for clinical practice.

SSC technique

The SSC signal was sampled at a frequency of 1 kHz. In order to obtain the eye velocity signal, the two-point central difference algorithm with a spread of six was used, as a spread of two resulted in a noisy velocity signal.

In the literature, several investigators discussed the influence of the placement of scleral search coils on the kinematics of eye movements [Frens and van der Geest, 2002, Robinson, 1964]. Frens and Van der Geest recorded saccadic eye movements of seven subjects with an infrared video system (250 Hz) while the subjects had scleral search coils attached to the eyes and compared the main sequence properties with recordings in which no coils were inserted. They found that with the insertion of the coils, the saccades became slower than without. Moreover, they concluded that this observed difference in eye velocity cannot be attributed to purely mechanical factors such as inertial load on the eyeball or increased friction, since the influence of coils in both eyes was also observed when one coil was mounted in one eye only. As a consequence, Frens and Van der Geest concluded that the tactile feedback of the eye most likely changes the oculomotor command signals that drive the saccadic eye movements. This interpretation agrees with the knowledge that contact lenses are sometimes prescribed to patients with congenital nystagmus to reduce these spontaneous eye movements by tactile feedback (Kingma, p.c.). This conclusion is supported by investigations performed by Robinson. In his investigation, Robinson used a mechanical description of the muscle-eyeball system. Furthermore, the assumption was made that

the eye was driven by a pulse-step signal¹. In his study, Robinson showed that the mass of the eye ball, and hence inertial load of the search coil, plays an negligible role on the kinematics of saccades.

In our study, only a significant difference in the main sequence properties for saccades, recorded with the video eyetracker, with or without the insertion of coils, was observed for subject 3. For this subject, the insertion of coils resulted in decreased values for the peak velocity, which can be concluded from the values of the constant C shown in table 6.5. This is in agreement with the observations of Frens and Van der Geest. A possible explanation for the fact that this difference in the main sequence properties is only observed for subject 3 could be that subjects 1 and 2 wear contact lenses in daily life, whereas subject 3 wears glasses. This might be an indication that the kinematics of the eye are affected by the daily use of contact lenses, caused by a persistent change of the oculomotor signals.

From both visual inspection (see figure 6.8) and the calculated values of constant C it can be concluded that a systematic difference exists also in the main sequence properties for saccades recorded with the VOG and SSC technique; the video eyetracker systematically yields higher values for the peak velocity than the SSC technique. Since the main sequences were obtained from saccades simultaneously recorded with both VOG and SSC, the observed difference is caused by an artifact of either the VOG or SSC technique². Since this phenomenon was also observed in other studies in which a comparison was made between the values of the peak velocity of saccades recorded with both VOG and SSC [Houben et al., 2006, DiScenna et al., 1995, Van der Geest and Frens, 2002], in which several different video eyetrackers were used, this phenomenon could be a result of lens slippage of the search coil and suggests that the video eyetrackers yield the correct values. Observation of our video recordings indeed show that the search coil is not really fixed on the eye, in agreement with reports by others (non-published data Clarke et al.). We therefore conclude that the SSC technique, that is often mentioned to be the golden standard of eye movement recordings, suffers from at least two limitations.

EOG technique

As described in section 5.4.2, the EOG signal was first electrically filtered using a low-pass filter with a bandwidth of 100 Hz. Thereafter, it was sampled at a frequency of 1 kHz. Next, the frequency components corresponding to frequencies higher than 25 Hz were eliminated using a discrete Fourier transform. The velocity signal was obtained by using the two-point central difference differentiation algorithm with a spread of two.

As already mentioned, the accuracy of the EOG method is ill-defined due to baseline drift. Especially for long periods of measurements, this can result in errors in absolute eye position up to 20° . The accuracy can be improved considerably by frequent calibration of the signal. However, even doing so, no good accuracy can be guaranteed. The sensitivity is in the order of $0.5 - 1^{\circ}$, due to the presence of high frequency noise.

It was shown in chapter 6 that for two subjects asymmetric values for the saccadic peak velocity were observed for abducting and adducting saccades, due to the asymmetric placement of the temporal and nasal electrode. In section 6.4 it was mentioned that these asymmetric peak velocities are only observed for the situation that the eye velocity signal is not perfectly symmetric. For this case, the eye position where the maximum velocity is reached α_{max} is not exactly in the middle of the saccade. This means that the maximum eye velocity is reached at different eye positions for left- and rightward saccades; $\alpha_{max,left} \neq \alpha_{max,right}$, which results in different values for the calculated saccadic peak velocity, as can be seen from equation (6.4).

For subject 2, however, no significant asymmetry in saccadic peak velocity for abducting or adducting saccades could be observed. One explanation for this phenomenon is that for subject

¹This assumption is in agreement with the feature of the local minima observed in the power spectra.

 $^{^{2}}$ Note that this observed difference in the main sequence relationships can not be ascribed to the effect of the scleral search coil on the kinematics of the eye since the saccades were simultaneously measured with both the VOG and SSC method.

2 the saccadic velocity signals were still highly symmetric for large amplitude saccades. Another possible explanation could be a smaller offset angle θ because of a different geometry of the head.

Using equation (3.8), it is theoretically possible to determine the offset angle θ and correct for this angle³. Since the only unknown parameters in this equation are the dipole moment p and the offset angle θ when the eye is situated at a known position α , in theory θ can be determined by having the subject fixate on two different positions. In practice, however, a third fixation on the reference position $\alpha = 0$ is necessary to correct for a possible offset of the head position.

This far, only the monocular configuration for the measurement of horizontal eye movements has been discussed. By using the composed configuration, a better accuracy and sensitivity can be achieved. Since the composed configuration can be considered as the average signal for both eyes, artifacts occurring in the signal of one eye are averaged over both eyes. As a consequence, the accuracy and sensitivity are significantly improved. The sensitivity for the composed configuration is in the order of $0.3 - 0.5^{\circ}$. Although improved, the accuracy for the recording of horizontal eye movements is still very sensitive to baseline drift. Moreover, also for the measurement of saccadic peak velocities, the composed configuration yields better results than the monocular configuration.

VOG technique

The video eye tracker recorded (interlaced) images of the eye with a frequency of 50 Hz. In order to improve the temporal resolution, the Nyquist sampling theorem was used to resample the signal at a rate of 1 kHz.

The performance of the video eyetracker depends on the geometry of the subject's eyes; for the situation that the subject has slant-eyes, the upper part of the pupil is constantly covered by the upper eyelid, making the detection of the centre of the pupil difficult and sensitive to errors. Moreover, a very dark iris results in little contrast between the grey values of the pupil and the iris in the video image. As a consequence, it is hard to distinguish the pupil from the iris. These two phenomena were clearly observed for subject 2.

Furthermore, in section 5.4.3 it was shown that even with the use of the Nyquist sampling theorem to reconstruct the signal at times in-between two sampling points, the low sampling rate of 50 Hz is insufficient for the accurate determination of the saccadic peak velocity for saccades having an amplitude of 5° or smaller. As a consequence, the 50 Hz sampling frequency can lead to an underestimation of the peak velocity of about 10% for 5° saccades. However, for saccadic eye movements larger than 5° a reliable estimation of the peak velocity is obtained.

Vertical saccades

Due to interference of the a.c. magnetic field of the search coil system in the vertical EOG signals, only for subject 3 a direct comparison of vertical saccades could be made for all three recording techniques. Nevertheless, from the main sequence of this subject shown in figure 6.9 two interesting features can be observed. First, it can be seen that from the EOG signal incorrectly high values for the saccadic peak velocities are obtained, which results from saccadic overshoot caused by eyelid movement. Yee et al. [Yee et al., 1985] showed that fixation of the eyelids considerably, but not completely, reduced this effect. Second, when comparing figure 6.9 with figure 6.7, it is evident that for all three recording methods a large spread in peak velocity is observed. Since figure 6.6 shows that the VOG and SSC signals are highly correlated, it follows that the observed spread is due to the natural variability of vertical saccadic peak velocities. This indicates that the movement of horizontal saccades is better determined than that of vertical saccades This observation is in agreement with the literature [Collewijn et al., 1988].

Clinical practice

In vestibular clinical practice, several examinations exist to test patients on vestibular disorders. In order to come to the right diagnosis, the recording technique that is used must satisfy the demands

 $^{^{3}}$ In practice, due to baseline drift and artifacts this may not be possible

needed to obtain reliable signals. With the exception of the saccadic test, the vestibular physician is interested in the slow phase velocity of the vestibular ocular reflex. Since the dominant frequency of this slow phase velocity is in the low frequency domain (0.025 Hz for the caloric test), usually a sampling frequency of 50 Hz suffices. Furthermore, an accuracy and sensitivity of approximately 1° are desired [Kingma, 2007]. From this study it follows that both the VOG and SSC technique meet these requirements. It is remarked that when the EOG signal is stabilized and frequently calibrated, the accuracy of the EOG technique is in general acceptable.

For the saccadic test however, the recording techniques must satisfy different demands. In clinical practice, saccades are recorded at a sampling rate of 200 Hz. For saccades of 5° and larger, the amplitude and velocity need to be measured with an accuracy of 0.5° and $5^{\circ}s^{-1}$, respectively (Kingma p.c.). From these requirements it follows that the EOG technique is not suited for the measurement of saccadic amplitude and velocity. Using the Nyquist sampling theorem, it was shown that for small (5°) amplitude saccades the saccadic peak velocity measured with the VOG technique can be underestimated by about 10%, whereas reliable results are obtained for larger amplitude saccades. For the SSC technique it is assumed that the measured peak velocity is underestimated as a result of lens slippage and that the coil reduces the eye movements.

So far only the technical aspects have been discussed. However, also the clinical aspects play an important role. Although the SSC technique theoretically yields the highest sensitivity and accuracy (with the exception of saccades), it is not suited for vestibular clinical practice. Important issues are the discomfort of the patient, a restricted measuring time of about 30 minutes and the unsuitability for routine evaluations. Moreover, since the coils are connected to the recording device by copper wires, it can only be used for experiments in which the head of the patient is at rest.

Another important aspect is the measurement of 3D eye movements. Using electrooculography, only the 2D eye position can be measured. This means that this method cannot be used for recording of eye torsion. In contrast, with the video eyetracker it is possible to measure eye torsion.

Even though the VOG technique yields more reliable signals than the EOG technique, the EOG method is still favored by many vestibular physicians. A very important aspect is the fact that some patients are unable to maintain their eyes open during the investigation. In such cases, no eye position signal can be obtained from the video eyetracker, whereas the EOG method can still record eye movements when the eyes are closed.

Chapter 8 Conclusions

In this study, it is concluded from the power spectra of saccadic eye movements that saccades can be considered to be bandwidth limited with an upper frequency limit of about 25 - 30 Hz; small (5°) amplitude saccades having a higher frequency limit (30 Hz) than large (25°) amplitude saccades (20 Hz). As a result, the 50 Hz video signal can be considerably improved by using the Nyquist sampling theorem to reconstruct the signal at times in-between two sampling points. By doing so, reliable values for the saccadic peak velocity can be obtained for saccades larger than 5° .

Furthermore, from the main sequence relations of the three recording methods, EOG, VOG and SSC, it can be concluded that

- the EOG technique yields erroneous asymmetric values for the saccadic peak velocity, caused by the asymmetric placement of electrodes.
- the insertion of the scleral search coil can lead to a significant decrease in saccadic peak velocity for some subjects.
- the peak velocities of saccadic eye movements obtained from the SSC method appear to be slightly underestimated due to lens slippage.
- the movement of horizontal saccades appears to be better determined by the oculomotor system than that of vertical saccades.

Finally, from a technical point of view, the VOG and SSC technique are best suited for the diagnosis of vestibular disorders. From a clinical point of view, however, the SSC method is not applicable for routine clinical examinations. Another aspect is the recording of eye torsion. With electrooculography only 2D eye movements can be recorded, and therefore eye torsion cannot be measured using this technique. In contrast, it can be measured using video oculography. On the other hand, for subjects with slant-eyes or examinations in which the pupil makes large vertical excursions, no reliable signal can be detected with the VOG technique, which favors EOG over VOG.

Chapter 9

Recommendations

- In this study, the temporal resolution of the 50 Hz VOG signals was improved using the Nyquist sampling theorem. This was only done for saccadic eye movements of 3 healthy subjects. In order to prove that this technique can also be used for subjects with vestibular disorders, a patient study should be performed.
- In order to obtain the eye position signal at a time t using this resampling technique, also the eye position at future times is necessary. However, for clinical practice online recordings are necessary. One possible solution to this problem is to show the 50 Hz data during the measurement and use the resampling technique before performing analyses. Another possible solution is to use only a part of the signal in a window of 1 or 2 s. This way, the resampled signal can be shown with a delay of about 1 s.
- In this study only saccadic eye movements were taken into consideration. In order to verify that this technique also yields reliable results for other clinical tests, a comparison could be made by measuring simultaneously the EOG and VOG signals in clinical practice.
- The asymmetric placement of the temporal and nasal electrodes leads to erroneous asymmetric values for the peak velocity of saccades. By determining the offset angle θ shown in figure 3.2, it is in theory possible to compensate for this effect. The effect is also canceled by using the bitemporal (composed) configuration.
- From the main sequence relations recorded with the VOG technique, it was shown that for subject 3 higher values of the saccadic peak velocity were observed without than with the insertion of the scleral search coil. This phenomenon was not observed for subjects 1 and 2. Since these two subjects wear contact lenses in daily life, whereas subject 3 usually wears glasses, it appears that the daily use of contact lenses causes a persistent change in the oculomotor command signals. In order to confirm this observation, a larger subject study should be performed.
- From a comparison of the main sequence relationships of the VOG and SSC technique it is observed that with the VOG technique systematically higher values for the peak velocity are found. A possible explanation for this effect could be lens slippage of the search coil. In order to prove this assumption, the position of the coil on the eye during saccades could be determined by looking at the images obtained from the video eyetracker.
- A comparison between the main sequence relations of horizontal and vertical saccades of subject 3 indicates that the dynamic character of horizontal saccades is better defined than that of vertical saccades. Since this comparison could only be made for subject 3, a larger subject study is necessary to verify this observation.

Appendix A

Rotation matrix of the Fick and Helmholtz system

Once the rotation matrix $\mathbb R$ is know, the Fick rotation angles can be calculated by using [Haslwanter, 1995]

$$\mathbb{R}_{Fick} = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix},$$
(A.1)

with

$$\begin{aligned} A_{11} &= \cos(\theta_F) \cos(\phi_F), \\ A_{12} &= \cos(\theta_F) \sin(\phi_F) \sin(\psi_F) - \sin(\theta_F) \cos(\psi_F), \\ A_{13} &= \cos(\theta_F) \sin(\phi_F) \cos(\psi_F) + \sin(\theta_F) \sin(\psi_F), \\ A_{21} &= \sin(\theta_F) \cos(\phi_F), \\ A_{22} &= \sin(\theta_F) \sin(\phi_F) \sin(\psi_F) + \cos(\theta_F) \cos(\psi_F), \\ A_{23} &= \sin(\theta_F) \sin(\phi_F) \cos(\psi_F) - \cos(\theta_F) \sin(\psi_F), \\ A_{31} &= -\sin(\phi_F), \\ A_{32} &= \cos(\phi_F) \sin(\psi_F), \\ A_{33} &= \cos(\phi_F) \cos(\psi_F). \end{aligned}$$
(A.2)

The Helmholtz rotation angles can be found by using

$$\mathbb{R}_{Helmholtz} = \begin{pmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{pmatrix},$$
(A.3)

with

$$B_{11} = \cos(\theta_H) \cos(\phi_H),$$

$$B_{12} = -\sin(\theta_H) \cos(\phi_H) \cos(\psi_H) + \sin(\phi_H) \sin(\psi_H),$$

$$B_{13} = \sin(\theta_H) \cos(\phi_H) \sin(\psi_H) + \sin(\phi_H) \cos(\psi_H),$$

$$B_{21} = \sin(\theta_H),$$

$$B_{22} = \cos(\theta_H) \cos(\psi_H),$$

$$B_{23} = -\cos(\theta_H) \sin(\psi_H),$$

$$B_{31} = -\cos(\theta_H) \sin(\phi_H),$$

$$B_{32} = \sin(\theta_H) \sin(\phi_H) \cos(\psi_H) + \cos(\phi_H) \sin(\psi_H),$$

$$B_{33} = -\sin(\theta_H) \sin(\phi_H) \sin(\psi_H) + \cos(\phi_H) \cos(\psi_H).$$
(A.4)

The need to specify the sequence of consecutive rotations is shown by the following example. First, consider a rotation of the eye 15° to the left (about the axis \vec{e}_3), followed by a rotation of 25° down (about the rotated axis \vec{e}_2), i.e. $(\theta_F, \phi_F, \psi_F) = (15, 25, 0)$. The orientation of the eye will then be given by the rotation matrix

$$\mathbb{R}_{Fick} = \begin{pmatrix} 0.88 & -0.26 & 0.41 \\ 0.23 & 0.97 & 0.11 \\ -0.42 & 0 & 0.91 \end{pmatrix}.$$
 (A.5)

Second, consider a rotation of the eye 25° down (about the axis \vec{e}_2), followed by a rotation of 15° to the left (about the rotated axis \vec{e}_3), i.e. $(\theta_H, \phi_H, \psi_H)$. The orientation of the eye is now given by

$$\mathbb{R}_{Helmholtz} = \begin{pmatrix} 0.88 & -0.23 & 0.42\\ 0.26 & 0.97 & 0\\ -0.41 & 0.11 & 0.91 \end{pmatrix}.$$
 (A.6)

As can be seen from equations (A.5) and (A.6), a change in the order of the consecutive rotations yields different rotation matrices, and thus different eye positions.

Appendix B Quaternions and rotation vectors

As shown in the previous section, the use of rotation matrices to describe 3D eye position have the disadvantage that the three axes of rotation, as well as the sequence of the rotations about these axes are defined arbitrarily, with different sequences leading to different eye rotations. Another disadvantage is that rotation matrices are not the most efficient way to describe a rotation since they contain nine elements where only three are actually needed to uniquely characterize a rotation. A more efficient way to characterize eye rotations is to use a vector with the direction of the vector given by the axis of the rotation and the length given by the size of the rotation. Two kinds of such descriptions of rotations are: quaternions and rotation vectors [Haslwanter, 1995].

B.1 Quaternions

A quaternion q which uniquely characterizes a rotation over an angle θ about an axis \vec{n} is a four-component vector given by

$$q = q_0 + (q_1^* i + q_2^* j + q_3^* k) = q_0 + \vec{q} \cdot \vec{I},$$
(B.1)

with

$$\vec{q} = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} \text{ and } \vec{I} = \begin{pmatrix} i \\ j \\ k \end{pmatrix},$$
 (B.2)

where $\{i, j, k\}$ are defined by

The element q_0 is called the scalar component whereas \vec{q} is referred to as the vector component of the quaternion q. Note the q is a four-component vector and q_1 , q_2 and q_3 can be seen as the projections of the vector component q on I. The elements have the following properties

$$\begin{aligned} |\vec{q}| &= \cos(\theta/2), \\ |\vec{q}| &= \sqrt{q_1^2 + q_2^2 + q_3^2} = \sin(\theta/2), \\ \vec{q} \text{is parallel to } \vec{n}. \end{aligned}$$
(B.4)

From these equations it follows that quaternions describing rotations have length 1, i.e. $\sqrt{q_0^2 + q_1^2 + q_2^2 + q_3^2} = 1$. The connection between the quaternion q and the rotation matrix \mathbb{R} , both describing the rotation of a vector \vec{x} about an axis \vec{n} by an angle θ is given by [Haslwanter, 1995]

$$q \circ (\vec{x} \cdot \vec{I}) \circ q^{-1} = (\mathbb{R} \cdot \vec{x}) \cdot \vec{I}, \tag{B.5}$$

$$q^{-1} = q_0 - \vec{q} \cdot \vec{I}. \tag{B.6}$$

The equation for the combination of two quaternions q and p is given by

$$q \circ p = \sum_{i=0}^{3} q_i I_i * \sum_{j=0}^{3} p_j I_j.$$
 (B.7)

B.2 Rotation vectors

The scalar component q_0 of a unit quaternion does not contain any relevant information that is not already given by the vector component \vec{q} . Therefore, it can be eliminated using so-called rotation vectors. The rotation vector \vec{r} corresponding to the quaternion q describing a rotation over an angle θ about the axis \vec{n} , is given by

$$\vec{r} = \frac{\vec{q}}{q_0} = \tan(\theta/2) * \frac{\vec{q}}{|\vec{q}|} = \tan(\theta/2) * \vec{n}.$$
 (B.8)

The relation between the rotation vector and the rotation matrix \mathbb{R} which describes a rotation by an angle θ about an axis \vec{n} is given by [Haslwanter, 1995]

$$\vec{r} = \frac{1}{1 + (\mathbb{R}_{11} + \mathbb{R}_{22} + \mathbb{R}_{33})} * \begin{pmatrix} \mathbb{R}_{32} - \mathbb{R}_{23} \\ \mathbb{R}_{13} - \mathbb{R}_{31} \\ \mathbb{R}_{21} - \mathbb{R}_{12} \end{pmatrix}.$$
 (B.9)

The equation for the combination of two rotation vectors $\vec{r_q}$ and $\vec{r_p}$ is given by

$$\vec{r}_q \circ \vec{r}_p = \frac{\vec{r}_q + \vec{r}_p + \vec{r}_q \times \vec{r}_p}{1 - \vec{r}_q \cdot \vec{r}_p},$$
 (B.10)

with $\vec{r_p}$ the first and $\vec{r_q}$ the second rotation.

If sufficient elements of rotation matrix \mathbb{R} are known, the eye position can be described by a rotation vector by using equation (B.9).

where

Appendix C

EOG signal obtained with different distances of electrodes

In this section it is shown that the difference in distance from the temporal and nasal electrode to dipole moment of the eye does not result in an asymmetric EOG signal. In figure C.1 a schematic representation of this situation is shown. From equation (3.5) it follows that the potential measured



Figure C.1: Schematic representation of a monocular EOG recording of the left eye under the assumption that the distance from the dipole \vec{p} to the temporal placed electrode E_t is less than that to the nasal placed electrode E_n .

by the left and right electrodes, V_t and V_n respectively, are given by

$$V_t(\alpha) = -\frac{p}{4\pi\epsilon} \frac{\sin(\alpha)}{r_t^2},\tag{C.1}$$

$$V_n(\alpha) = \frac{p}{4\pi\epsilon} \frac{\sin(\alpha)}{r_n^2}.$$
 (C.2)

From these equations it follows that when the eye is rotated by an angle α (to the right), the measured potential difference ΔV is given by

$$\Delta V(\alpha) = V_n(\alpha) - V_t(\alpha) = \left(\frac{1}{r_n^2} + \frac{1}{r_t^2}\right) \frac{p}{4\pi\epsilon} \sin(\alpha).$$
(C.3)
When the eye is rotated by an angle $-\alpha$ (to the left), this potential difference is

$$\Delta V(-\alpha) = V_n(-\alpha) - V_t(-\alpha) = \left(\frac{1}{r_n^2} + \frac{1}{r_t^2}\right) \frac{p}{4\pi\epsilon} \sin(-\alpha) = -\Delta V(\alpha).$$
(C.4)

From this equation it follows that a rotation of the eye by $-\alpha$ degrees results in a measured potential that has the same magnitude but opposite sign as a rotation by $+\alpha$ degrees. This means that the asymmetric behavior of the EOG signal is not caused by a difference in r_t and r_n .

Appendix D Effect of A/D conversion

In order to record or process a signal, in general the analog signal x(t) is converted to a digital signal x[n] at a sampling frequency f_s . Suppose this conversion is preformed using N bits. The analog signal is then converted into a digital number in the interval $[0, 2^N - 1]$. Suppose that α_{max} is the largest angle that can be converted for both positive as negative angles. For this situation $-\alpha_{max}$ and α_{max} correspond to the digitized values 0 and $2^N - 1$, respectively. The smallest detectable angle is given by

$$\Delta \alpha_{min} = \frac{2\alpha_{max}}{2^N - 1}.\tag{D.1}$$

From this equation the largest round-off error made by A/D conversion is

$$|E_{max}| = \frac{\alpha_{max}}{2^N - 1}.\tag{D.2}$$

The consequence of A/D conversion on the spatial resolution of the signal is thus given by equation (D.2).

An inadequate A/D conversion has not only consequences for the spatial resolution of a signal but also on the temporal resolution. This can be shown as follows. Shortening of the sampling time T_s in equation (4.27) results in a better approximation of the instantaneous derivative. However, a too small sampling time can result in large errors for the velocity due to round-off errors made by the A/D conversion and/or the presence of noise. The relation between the measured value of the signal y[n] and the actual value y'[n] in the absence of noise and round-off errors is given by

$$y[n] = y'[n] + E + E',$$
 (D.3)

with E' the error caused by the presence of noise and E the round-off error. Substituting equation (D.3) into equation (4.27) yields

$$\dot{y}[n] = \frac{y'[n+1] - y'[n-1]}{2T} + \frac{E_1' - E_2'}{2T_s} + \frac{E_1 - E_2}{2T_s}.$$
(D.4)

Using equation (D.2), the maximum error in the velocity $|S_{max}|$ caused by A/D conversion is given by

$$|S_{max}| = \frac{|E_{max}|}{2T} = \frac{\alpha_{max}}{2T(2^N - 1)},$$
(D.5)

and the maximum error in the velocity $|S'_{max}|$ caused by noise

$$|S'_{max}| = \frac{|E'_{max}|}{2T}.$$
 (D.6)



Figure D.1: Maximum velocities of a simulated 20° saccade for different A/D conversion resolutions and different sampling frequencies, taken from [Juhola et al., 1985].

As can be seen from these equations, the errors caused by noise and A/D conversion both increase with increasing sampling frequency. The error caused by the A/D conversion can be reduced by increasing the number of bits used for the A/D conversion. In a study of Juhola et al. [Juhola et al., 1985] the effects of the A/D conversion and the sampling frequency on the computation of the maximum velocity of saccadic eye movements was investigated. For this simulation an artificial saccade of 20° with an average maximum velocity of 656° s⁻¹ was used. The velocity of the saccade was calculated using the two-point central difference method. In figure D.1 the maximum velocities of a simulated 20° saccade for different resolutions of the A/D converter at different sampling frequencies are shown.

In this figure it can be seen that the calculated maximum velocity becomes incorrectly high for low resolutions of the A/D converter at high sampling frequencies. This figure also reveals that not only a sampling frequency that is too high leads to an incorrect maximum velocity, but also a sampling frequency that is too low.

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