MASTER

Analysis and control of product-process-chains

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Samenvatting

Het product-proces-keten concept wordt gebruikt in het kader van de materiële levenscyclus van een product. Het is gebaseerd op de extractie van grondstoffen, de productie naar eindproducten, de consumptie van deze producten, verwijdering of recycling. De product-proces-keten kan benaderd worden vanuit een ecologisch of een economisch gezichtspunt. Het gezichtspunt bepaalt de strategie en de instrumenten, welke gebruikt kunnen worden om een verbetering van de keten te bereiken. Om een verbetering van prestatie te bereiken het is noodzakelijk om een model te hebben van deze product-proces-ketens.

Een van de problemen is dat er nu nog geen standaard methodologie van modelleren van product-proces-ketens is ontwikkeld. In dit afstudeerrapport een 'state space representation' is gebruikt om het dynamische gedrag van de keten te beschrijven. Het product-proces-keten model blijkt niet-lineair te zijn, door een niet-lineariteit in de markt: de uitwisseling van goederen in de markt is gebonden aan het minimum van de aangeboden en gevraagde hoeveelheid goederen. In dit afstudeerrapport is een stuksgewijze lineaire set van submodellen gebruikt om deze niet-lineariteit te benaderen. De product-proces-ketens worden geanalyseerd met betrekking tot stabiliteit door gebruik van stuksgewijze kwadratische Lyapunov functies.

Het modelleren is gedaan met de opzet om een betere prestatie te krijgen van de product-proces-ketens door de gecontroleerde modellen van deze ketens te bestuderen. Met betrekking tot het voorgestelde keten model blijkt het niet mogelijk te zijn om alle toestandsvariabelen te manipuleren. Met betrekking tot sommige modules van het voorgestelde keten model is het mogelijk om deze toestandsvariabelen wel te controleren. Niettemin het gereduceerde doel om de uitwisseling van goederen op de eindproducten markt te sturen kan wel gehaald worden.

Abstract

The product-process-chain concept is used to describe the material life cycle of a product. This consists of the extraction of resources, the production to end products, the consumption of these products, the disposal or recycling. The product-process-chain can be approached from an ecological or an economic viewpoint. The viewpoint determines the strategy and the instruments which can be used to obtain an improvement of the chain behavior. To achieve an improvement of performance it is necessary to have a model of the product-process-chain.

One of the problems is that yet no standard method of modelling product-process-chains is developed. In this thesis a state space representation is used to describe the dynamic behavior of the chain. The product-process-chain model turns out to be nonlinear due to a nonlinearity in the market: the exchange of goods in the markets is bounded to the minimum of supplied and demanded goods. In this thesis a piecewise linear set of submodels is used to approximate the nonlinearity. The product-process-chains are analyzed with respect to stability using piecewise quadratic Lyapunov functions.

This modelling is done with the purpose to improve performance of the product-process-chains by analyzing the controlled models of these product-process-chains. With respect to the proposed chain model it turns out that it is not possible to control all the intermediate variables. With respect to some modules of the proposed chain model it is possible to control all these variables. Nevertheless the reduced goal of controlling the exchange of goods on the finished good market can be achieved.

Preface

After many years of being a student at the Mechanical Engineering faculty of The Eindhoven University of Technology, in the System and Control Engineering Group I finally thought 'er is een tijd van komen, en er is een tijd van gaan!'. This could only be established by starting my master's thesis. This report is the result of one year research at the Systems and Control Group of the faculty Applied Physics of the Eindhoven University of Technology.

And at the time I wrote the report, as usually, also this report was just in time. But I could not have made it, if I didnot have the support of a lot of people. In general, I want to thank everybody of the Systems and Control Group for the good advice and time they gave me.

I especially want to thank my coach Uwe Kleineidam, for helping me out with my problems, introducing new problems instead, but most of all for the discussions we had over these problems. Further I want to thank Fred Lambert for giving me information and advice on product-process-chains and environmental analysis. I also want to thank Professor Kok for the ideas and answers he gave me when discussing the preliminary results at our monthly meetings. I thank Professor van Heijningen and dr.ir. M.J.G. v.d. Molengraft for being part of the examination board. Further I want to thank drs. F.A.M. Vlemmings for the material he gave to me with respect to the steel industry.

I want to thank all the students with whom I worked. Especially Wouter and Jeroen with whom I worked at this project. Further I want to thank the students Agnes, Alex, Casper, Meike, Patrick, Remco, Reinout, and the Ph.D.-students Annelies, Georgo, Mathieu and Patrick.

Last but not least I want to thank 'de illustere bewoners van het Tachos-Huis' with whom I lived and had a lot of fun the last years.

Symbols & Abbreviations

Symbol	SI-unit	Description
$\underline{x}$	$\in \mathbb{R}^n$	state vector
$\underline{y}$	$\in \mathbb{R}^k$	measured output vector
$\underline{z}$	$\in \mathbb{R}^l$	output vector to be controlled
$\underline{u}$	$\in \mathbb{R}^m$	control input vector
$\underline{v}$	$\in \mathbb{R}^q$	measurement disturbance input vector
$\underline{w}$	$\in \mathbb{R}^p$	system disturbance input vector
$\underline{\chi}$	$\in \mathbb{R}^n$	sytem function
$\underline{\gamma}$	$\in \mathbb{R}^k$	measuremant function
$\underline{\rho}$	$\in \mathbb{R}^l$	output to be controlled function
$\underline{l}$	$\in \mathbb{R}^m$	feedback function
sp		setpoint
k		constant
e	[ktonne/year]	trade(exchange)
t	[year]	time
f	[ktonne/year]	flow
$d_{tot,s/f}$	[ktonne/year]	total demand(signal/flow)
$s_{tot,s/f}$	[ktonne/year]	total supply(signal/flow)
g_i	[ktonne]	goods in the inventory i
p	[Mf]	price
$q_{(\max)(\text{prod})}$	[ktonne/year]	(maximum) amount of (produced) goods
$c_{0(i)}$	[ktonne/year]	autonomous demand of i-th consumer
$c_{1(i)}$	[ktonne/Mfyear]	price variant demand of i-th consumer
$b_{0(i)}$	[ktonne/year]	autonomous
$b_{1(i)}$	[ktonne/Mfyear]	price variant demand of i-th supplier
v	[ktonne/year]	external demand/supply
α_0/α_1	[-]/[1/Mf]	autonomous/price variant recycling ratio
$\varepsilon\,(t)$		Heavyside step function
Abbreviations		
PPC		Product Process Chain
LCA		Life Cycle Analysis
IM		Inventory-Market
MIIM		Market-Inventory-Inventory-Market

In this list of symbols almost all the variables discussed in this report are mentioned, nevertheless in the cases where a variable is not explained in this list, it can be concluded from the context what the variable indicates and what its dimension is.

Contents

Chapter 1

Introduction

This master's thesis is entitled 'analysis and control of product-process-chains'. The research for the thesis has been done from November 1997 until October 1998. In the following chapters a description of the product process chains is given and an attempt is made to use the theory from systems and control engineering for mathematical analysis.

1.1 Outline of and goal of this research

Sustainable development and production is becoming more and more important. Since the Club of Rome's report *Limits to Growth* [DM72] more awareness is shown with respect to non-renewable resources. The limited resources will force society to search for alternatives. One of these alternatives is to recycle materials. This avoids exhausting of the limited resources and reduces the dumping of wastes. Another awareness is that the impact of production activities to the environment has to be reduced, be it only because of the population causing the demand of products to increase.

The *product-process*-chain can be described as a sequence of *processes* which are necessary for a *product* to evolve from raw material to a finished product, which is disposed after the consumption phase. Processes which are relevant in this context are the purchase of raw materials, the production of the product made from these raw materials, the purchase of the products by the consumers, the disposal of the consumed products and the recycling of materials and products. Insight in and understanding of the product-process-chains (in short PPC) will be of increasing importance, especially with respect to sustainable production.

The objective of the research-project in general has the theme 'Optimization of Product Chains' and aims at developing theory on design of sustainable product(ion) systems. In the project, the physical aspects of the product chain are taken as a starting point in the economic and environmental analysis. On the basis of material and energy flows, models are being developed for production and product chains which can be used in decision-support systems. These are of great importance for enterprises, as well as for public authorities.

This project has been set up as a cooperation project of the section Energy and Environment of the faculty of Technology Management and the System and Control Group of the faculty of Applied Physics as part of the TDO (Technology for Sustainable Development) program of the Eindhoven University of Technology.

Earlier theses in the framework of this project are: 'Een dynamisch model van een produkt-proces-keten' [Eij97], 'Een Flexibel Dynamisch Model Van Een Product-Proces-Keten' [Han98], 'Modelling and Control of Product-Chains' [Bla98] and the present thesis. The objective for this master theses was to examine whether it is possible to use the knowledge from systems and control science to analyze the (models of) product-recycling-chains with respect to aspects of sustainable development.

The work of [Eij97] can be considered as an inventory and a first model of PPC, [Han98] introduces the relationship between the producer and the consumer, [Bla98] describes a state space model and

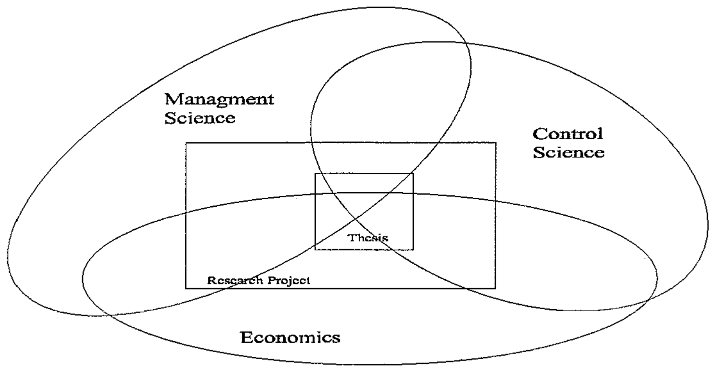


Figure 1.1: framework of the thesis

control of a paper chain. One of the problems not explicitly mentioned in the former theses is the problem of analyzing the PPCs with respect to stability. One of the objectives for this thesis is to study that problem. Therefore, the focus of the thesis cannot be on the discussion what is the solution for sustainable development.

As can be seen in the following chapters, inclusion of economic mechanisms in the model of a product-process-chain causes an essential nonlinearity due to the exchange of goods in the marketplace. This nonlinearity prevents the analysis of the PPC with the most commonly used tools for stability analysis, such has an eigenvector decomposition. Therefore other tools, such as Lyapunov functions, which are suited for this analysis are introduced and used for the analysis and model and control of PPCs.

In this paragraph the objective for this thesis will be expound. In general terms this thesis has the objective to investigate whether or not it is possible to use the knowledge from systems and control engineering to analyze the (models of) *product-process-chains.* Because this objective is very widely formulated, a more narrow description has to be given. In this thesis the PPCs are not analyzed with respect to what is the solution for sustainable development. The PPCs and its elements are analyzed with respect to the behavior of the flows of materials and the related costs in the PPC. The objective for this thesis is to model the PPCs and its elements and analyze them with respect to stability and see if it is possible to control the PPCs.

1.2 Framework of the thesis

The scope of the thesis on product-process-chains is multidisciplinary due to the properties of the PPC. Product-process-chains are in principle economic systems. But there are reasons that makes it become more important that other disciplines are used for analyzing PPC. One of these is that due to the activities in the PPC, the PPC affects the environment by its production of waste and emissions. The increased consciousness of knowledge of these effects on the environment makes it possible to reduce the burden and develop more sustainable production. Another reason is based on an economic approach and is the idea that with an efficient use of the scarce goods better profits can be obtained. These reasons make it necessary to have a multidisciplinary view to PPCs. Therefore PPCs are nowadays a research object in economics, management science, systems and control theory and a lot of other disciplines.

1.2.1 Economics and econometrics

One of the disciplines involved in the PPC is economics.

A standard definition of economics is: *economics* is the science which studies how scarce resources are employed for the satisfaction of the needs of men living in society: on the one hand, economists are interested in the essential operations of production, distribution and consumption of goods, and on the other hand, in the institutions and activities whose object it is to facilitate these operations.

Chapter 2 expounds in detail what PPCs are. At this point in time it is already clear that when the needs of people are different than the needs in which a product satisfies, it is very probable that the production of that product cannot sustain. And with the knowledge that the 'men living in society' becomes more aware of the idea of sustainable development, the environmental aspects of PPC are more part of economics as it seems to be at first sight.

Part of economics is the science called econometrics; based on models econometrists try to analyze economics. This area of economics is relatively unexplored with respect to the environmental analysis of PPC. One of the reasons is probably that its focus was primarily on the micro- and macro-economics, and as explained in chapter 2 a PPC is a meso-economic system. And another reason may be that it is just recently that more and more awareness of the environmental effects resulted in research with respect to the environmental effects of PPCs [Kan98], and the literature cited in.

With respect to economics, intertwining with the systems and control approach, analysis of PPC is not often applied. It has to be mentioned that in the early fifties a lot was done on this subject. From both the economic as the control discipline the interest in applying their knowledge on the other discipline was frequently present. Somewhere between the fifties and sixties both interests decreased, it was in the mid seventies that [AK74] [Aok76] approached the intersection again. Nowadays it is believed that applying knowledge becomes more important than only gaining knowledge.

1.2.2 Management Science

Management Science is closely related to what is known as Operational Research. *Operational Research* is defined by the community of Operational Research [OR62] as: the attack of modern science on complex problems arising in the direction and management of large systems of men, machines, materials and money in industry, business, government and defense. Its distinctive approach is to develop a scientific model of a system, incorporating measurements of factors such as change and risk, with which to predict and compare the outcomes of alternative decisions, strategies or controls. The purpose is to help management determine its policy and actions scientifically. *Management Science* is a field of study characterized by the use of mathematical and computer models for decision making.

With respect to the definition of management science it is that decision making with respect to the PPC is hard and therefore obtaining insight in the behavior of PPC can be done by using mathematical and computer models. The support of decision making is one of the reasons the management science discipline is part of the framework. Another reason the knowledge from MS with respect to how the management of a company functions is one of the essential parts in the PPC.

In the area of Management Science a lot is done with respect to PPC [Lam95] [JG93a] [FS94] .

1.2.3 Control and System theory

System theory is concerned with the description and study of input/output processes. The science of control is concerned with modifying the behavior of dynamical systems to achieve desired goals. As such, control theory adds a dimension to system theory. The engineering counterpart of control theory is usually called automatic control, emphasizing the fact that much control is done in an automatic, i.e. feedback, way.

System theory is based on describing systems and analyzing them. A product-process-chain is in fact a system with certain inputs and outputs and some dynamical behavior. For the following reasons not much attention has been payed to the PPCs. One of the problems of modelling a PPC is that it is a very complex and large system, and at this point no good descriptions on the behavior of parts of the chain are present. Another problem is that analyzing these large and complex systems is hard and until not long ago computational approaches were rare and insufficient.

On the other hand every day, mankind is subjected to the behavior of economic systems in general and of PPC in particular. From this point of view it is remarkable that literature on control of PPC is hard to find, except for some publications with respect to the control of macroeconomic systems, for instance [AY74] [Rad74a] [Rad74b] [Pin72]. Also with respect to microeconomics there are some references [DH67].

1.3 Outline of this thesis report

The report has already introduced the reader into the outline of this research and the framework of this thesis. In the following chapters the theory is explained, with this theory the models are generated and will be used for simulations. From the simulations some conclusions and recommendations are generated. In chapter two, product-process chains in general and the approaches towards the PPC management are explained. In chapter three a short introduction is given into the aspects of systems and control theory, which can be useful in the analysis of product-process-chain. With the theory the model for the PPC and its elements will be explained in chapter four. Chapter five will give some results of simulations with the PPC model of steel. The last chapter will give conclusions and recommendations which can be drawn from this research. The appendices are used to present various details.

Chapter 2

Product-Process-Chains

This chapter gives a short introduction into product-process-chains in general, and the definition for product-process-chains used in this thesis. Product-process-chains can be viewed from a lot of points, two of these viewpoints will be used for the analysis. A concise overview of strategies and instruments to influence product-process-chains is given, and in the last section of this chapter the steel product-process chain is illustrated.

The first step in explaining product-process-chains (PPCs) is to define what products, processes and chains are. *Products* in general can be defined as goods that are produced (also services, but in this thesis only 'material' products are regarded). *Processes* can be described as series of actions. For instance, production is a typical process of transformation of materials from one state into another state with the aim of augmenting the value. In contrast to the consumption process where the value is consumed to favor the needs.

More in detail a production process requires energy and information in order to transform raw and auxiliary material into finished end products, by-products and residues. Often it is not possible to make new products with only one transformation process; in these cases a series of transformations is applied to the raw material which will result in the end product. This series of processes constitute a simple process *chain*.

In industrial engineering *product*-process-chains are often used in order to describe the series of transformations from *products* to other *products* by processes. See figure 2.1 where different processes are used to transform product A into product D. Note that inventories and transportations are frequently used in product-process-chains; while no material transformation takes place here the products are transformed in time or place. Product-process-chains are used for many purposes, one

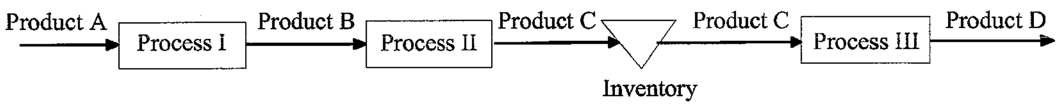


Figure 2.1: A product-process-chain for different product and different processes.

of the most important ones is the logistic purpose. The PPC gives information about the product flows from one process to another. In this thesis, the concept of PPC is used in a slightly more restricted sense. This definition is elaborated in detail in the following section.

The PPC as used in this thesis turns out to be very closely related to terms as life cycle. To prevent confusion about the definition of a lifecycle, a distinction has to be made between conceptual and material life cycle. In the case of *conceptual life cycle*, one refers to the stages in which a product is designed developed and introduced to the market. This concept is used by, e.g. designers and marketeers an describes the stage of the idea for a product, prototype to product. The other concept is the *material life cycle*. It describes the stages a product undergoes, from extracting the

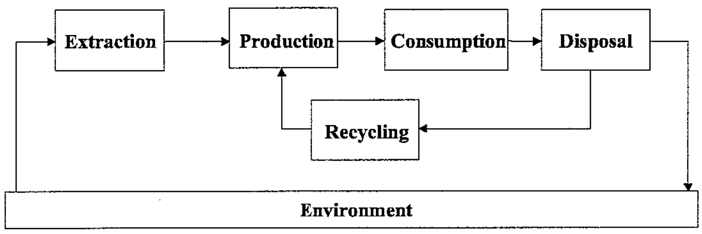


Figure 2.2: The flow of materials from the environment back to the environment, a material life cycle.

raw materials, producing these materials into products and consuming the products, and recycling or disposing of the materials.

2.1 Product-Process-Chain

In general a product-process-chain can be defined as a chain of flows of materials and products with the aim of fulfilling certain needs. In this thesis the product-process-chain is used to describe the material life cycle of a product. In this context we define the product-process-chain more specific as the set of producing, stocking and transportation activities, consuming activities, disposal activities and recycling activities with respect to a product.

In the most elementary case of a material life cycle PPC (first introduced by [Ayr89] [RF89] [Eij97]), five basic processes are present (see Fig 2.2), namely a process of resource extraction , a production process, a consumption process, a disposal process and a recycling process. The elementary processes proceed by the chain actors, so for instance the production is done by the producer. The produced and the recycled goods are stored in the inventories. The producers satisfy the demand of the consumers from the inventory. The products are consumed by the consumers, until they are obsolete and disposed of. A percentage of the disposed goods will be recycled and the rest will be dumped. The goods that are recycled and reused have to be upgraded, more specifically recycled products will be repaired and recycled materials have to be disassembled. The upgraded goods will end in the inventory of the producer in order to be put into the chain again.

The PPC can be approached in different ways, depending on the scope of research. The two approaches which will be discussed are the environmental and the economic approach. The former approach is interested in the effects on the environment due to the activities in the chain like production of goods, transportation etc. The latter approach is interested in the optimization of the PPC with respect to the economic costs and revenues of the activities in the PPC .

2.2 Environmental approach

The transformation from resource materials to finished goods by energy flows is called industrial metabolism [Lam95] [PK96]. Processes in general are not possible without the simultaneous production of waste and emission. Waste is the flow disposed of in a controlled way (mostly solid and liquid products) and emissions are uncontrolled disposals in general to surface and atmosphere. The effects of waste and emission productions on the environment are called the environmental effects of a process on the environment.

So one of the approaches toward a PPC is an environmental approach.

> A little intermezzo at this point will place this approach in a broader context. As mentioned before, two different form of life cycle are defined. With regard to the concept of a material life cycle, a widely accepted tool will be discussed in this paragraph.

> This tool for product-oriented environmental management is called life cycle assessment (LCA) and aims to quantify the effects of waste and emission production on the environment of a product during its entire life cycle. Although several definitions have been proposed, the ISO- 14000 one has been set as a world-wide standard [ISO95]: Life-cycle assessment (LCA) is a systematic set of procedures for compiling and examining the inputs and outputs of materials and energy and the associated environmental impacts directly attributable to the functioning of a product or service system throughout its life cycle. It should be stressed here that LCA is intended for comparative use, so results of LCA studies have a comparative significance rather than providing absolute values on environmental impact related to a definite product.

Essential to (material) LCA is the 'cradle to grave' approach, referring to the study of the materials flows related to production and consumption of a product from extraction to disposal. Effects of flows, arising from energy carriers and other utilities, capital goods, and by-products are all included, essentially leading to a branched product-process-chain. LCA-results usually depend on human behavior, in particular with respect to the consumption process. Therefore, integration of social and economic aspects within the analysis is frequently advocated. Although various aspects of a product (economic, social, safety) during its entire life-cycle might be studied, the concept of LCA is usually confined to quantitative and environmental LCA based on physical flows.

The environmental problems which are effects of the industrial metabolism can be divided into three classes [JG93a] [JG93b]: depletion, pollution and disturbances. Depletion is referred to as all the type of problems related to inputs from the environment (extractions for instance cause the depletion of ores). Pollution includes all the types of problems related to outputs to the environment (emissions causing ozone depletion, global warming, acidification, noise and smell), and disturbances include all problem types causing changes of structure within the environment (for instance landscape degradation). Most environmental effects are proportional to the amounts of materials flows. One of the steps in quantitative LCA proposed in [JG93a] [JG93b] is to weight the classified effects on the environmental problems (depletion, pollution and disturbances) with their specific classification factors into effect scores. The final step of classification is to normalize the effectscores, such that a total environmental effect is found.

In the determination of environmental effects two main difficulties occur compared to the measurement of economic effects (discussed in the next section). The first difficulty is that there is no standard scale, e.g. the greenhouse effect is expressed in essentially different quantities than the depletion of resources. Thus a subjectively weighted summation must be used in order to find the total environmental effects. The second difficulty is, that research on environmental problems brings constantly new insights into the severity of these problems and therefore the effectscores are not constant in time.

2.3 The Economic approach

The second approach discussed in this thesis is an economic approach. Some costs can be associated to the activities, for instance by the producer and consumer in the PPC. In [Eij97, and others] a survey of possible costs is presented which are relevant for a PPC. In general costs can be subdivided into two classes, i.e. direct and indirect costs. Direct costs are directly related to the flows of materials, and indirect costs are not directly related to these flows. The direct costs which occur in the PPC are the costs of extraction of resource costs (which have to be paid to purchase the materials which are transformed to final products), production, consumption and recycling, but also transport costs, costs for use of the products and costs made to dispose of the discarded good. Indirect costs are for instance production costs: (labor, capital, auxiliary and overhead costs), inventory costs (costs due to the maintenance of the storage), transaction costs (advertising), investments and so on.

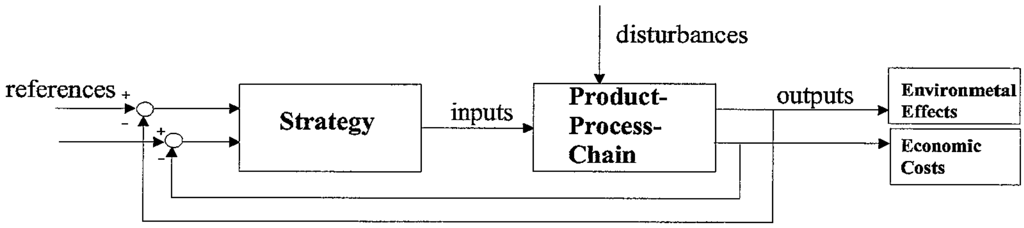


Figure 2.3: Set-up for PPC analysis

In principle all the costs can be reconstructed from the physical flow of materials. The material flow is the carrier for the variable costs, the overhead costs are not directly related to the flow. One of the goals of the actors in the PPC is to maximize benefits. To achieve this goal it becomes more and more important that the entire PPC has to be optimized instead of the individual interests of the chain actors. The form of optimization of the PPC depends on the type of product. Think for instance of a food PPC versus chemical PPC, the limited storage life versus start up and shut down problems. Somehow, the individual actors have to cooperate instead of achieving different and individual goals.

2.4 Goal definitions of PPC management

To guarantee a sustainable production in the future it becomes increasingly important for a chain actor to know what influence other actors in the PPC have on his process. If *chain management* is defined as the management of material flows that result from chains of social and economic activities [Kan98], it becomes clear that not the individual interests of the chain actors but the *integral chain* should be optimized.

In figure 2.3 a set-up for PPC analysis is presented. The outputs of the PPC are the inputs for the environmental effects and economic costs calculation modules. The outputs of the PPC are compared with the references given by an outside party. With the difference between the references and the outputs some action is initiated which will cause some effects to the PPC. The figure does not reveal what kind of strategy has to be used to achieve the desired references, nor does it reveal what the desired references should be (or on what reference the emphasis should be). With the economic costs and environmental effects as outputs it is possible to emphasize the influence of the environmental effects. In that case a reference for the environmental effects has to be chosen and a strategy which makes it possible to obtain the desired behavior of the output and the instruments which support this strategy. For instance, the emission of a toxic gas has a negative influence on the environment. The next step is to figure out some strategy for reducing the emission, and the instrument could be a restriction on certain flows in the PPC. However when the emphasis is on the economic costs, and for instance the costs made to satisfy a fluctuating demand have to be minimized, a strategy has to be chosen to diminish the fluctuations in the flows.

In [Kan98] [Bla98] the strategies and some control instruments are mentioned. Some strategies to manage the PPC are discussed. By changing the physical flows the environmental effects may be reduced. To achieve this reduction it is possible to use some strategies, for instance 1) substitution between materials, 2) recycling or reuse of materials, 3) technological change or 4) changing the pattern of consumption. The control instruments can be divided into regulatory instruments, economic instruments and social or persuasive instruments. The regulatory instruments are certain standards which the polluters have to fulfill (for instance maximum level of emissions). The economic instruments are used to directly achieve changes in economic costs which will influence decisions of the producers and consumers (for instance taxes on each product which has toxic emissions). And the social or persuasive instruments will achieve a change in behavior of the consumers and producers by changing the preferences, values etcetera by introducing education, information etc.

2.5 The Steel Product-Process-Chain

In this thesis the case of the steel PPC will be examined. The case is used to perform some modelling based on data from the steel industry, and to validate the model of the product-process-chain afterwards (see chapter 4). One of the motivations for using this branch of industry is the amount of data available since a long time. In short since the foundation of the European Coal and Steel Community (ECSC) in 1952 a lot of effort is done in collecting relevant data for the steel industry. Not only a big effort is put in collecting data, these data were also checked on correctness.

2.5.1 Historic Highs and lows of steel, a century of steel production

From [II] some facts are found over the world steel production. The facts put together are a nice introduction into the world economy and the world steel production.

> Today's global economy depends on steel. Two thirds of this century's dramatic rise in steel production has occurred since 1950. Steel is intimately linked with the world economy. Changing steel production levels reflect the major events of the last century. Pre-war steel production peaked in 1913 at 76.4 million metric tons. In 1917 it stood at 82 million tons, three times the amount as in 1900. After 1919 it fell to 45 million tons due to de-stocking. The postwar boom pushed world steel production to 121 million tons in 1929. By 1932 it had fallen to 50.7 million tons, roughly the 1906 level. Steel output reached a new peak of 136 million tons in 1937, from 1939 to 1943 it moved to 160 million. At the end of the war it slipped to 112 million. In 1974 production was 703 million tons, but fluctuation was brought on by the 1973 oil crisis, where the momentum of the 1950s and 60s was visibly slowed. After the fluctuation of the 1970s and early 1980s, production reached a new record of 786 million tons in 1989, the year when the Berlin Wall dividing Eastern and Western Europe disappeared. The collapse of the planned economies of East Europe and the former USSR removed 80 million tons from world steel production. Newly industrializing regions such as the People's Republic of China and Southeast Asia made up the shortfall. IISI (International Iron and Steel Institute) estimates world steel production will approach 820 million tons in the year 2000. Much of the growth will come from developing countries.

Since September 1998 the Asian Economic Crisis is also felt in Europe and United States of America, it is very unlikely for the world steel production to approach the estimated amount.

2.5.2 The steelmaking process

In general there are two widely used processes to make steel. The first type of process is based on the Basic Oxygen Steel making process (BOS) and the second one is based on the Electric Arc Steel making process (EAS). As a matter of fact it is not so important what the difference in chemical reaction mechanism is between these processes, but more the different amount of resource flows which are needed for the production of the same amount of steel. The BOS uses coal, ore and limestone to produce steel. And also uses an amount of steel scrap as coolant in one of its sub processes.

The EAS is a modern way of making steel and uses scrap as resource, which is then upgraded to steel. To have an idea of what the ratios are: one metric tonne [10^3 kg] of steel produced by BOS uses 1.6 metric tonne ore, 0.4 metric tonne cokes and EAS uses about 1.1 metric tonne scrap or metal of the 1.3 metric tonne of raw material needed.

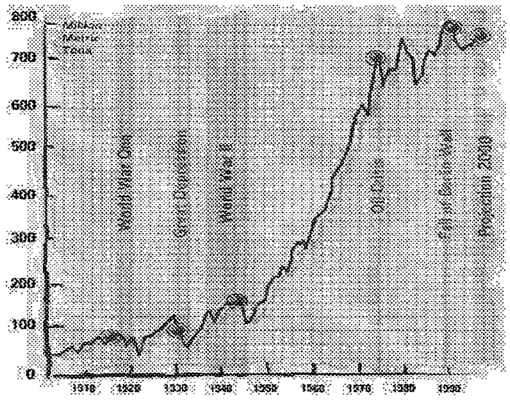


Figure 2.4: Steel production in the last century.

2.5.3 Set up of the steel PPC

In general it is important to know how a certain product is produced to get the right product-process-chain. First a rough scheme (see for instance fig. 2.1) is made where the basic processes are shown. The processes which are examined are the steel making process, the production of steel products, the steel consumption, disposal and recycling of the steel scrap.

From this rough scheme with the desired level of aggregation a more detailed PPC can be made. An assumption that all the products are homogeneous has to be made. In this thesis the assumption is made that it is possible to capture all the different types of steel product in some average steel product. It is further assumed that this average steel product is demanded by an average steel product consumer, and the products are made by some average steel product producers. This will result in a product-process-chain which looks like:

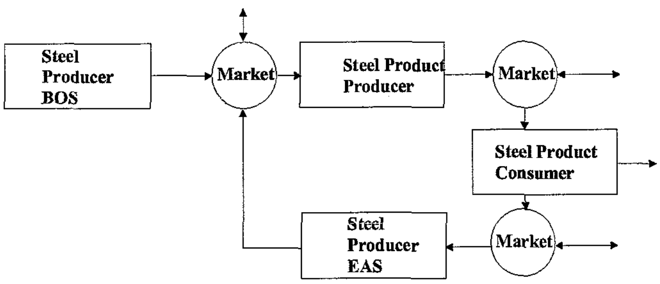


Figure 2.5: PPC set up for steel.

In this set up there are three markets, the first one is the market of resources which are needed for the steel products, the second one is the market of steel products and the last market in this model is the market where the disposed goods are purchased by the recycler. Compared to figure 2.1 the processes of consumption and disposal are put into the steel product consumer module. The consumed goods can be recycled or disposed of, depending on the choice of the consumers.

To maintain a simple representation of the PPC of steel, in this set up the recycle flow from the scrap to the steel producer BOS is not modelled.

Chapter 3

Systems, models and control

In this chapter some basic definitions regarding systems, models and control will be explained from the viewpoint of system and control theory. Some books about control theory used for this chapter are [Aok76], [PVB94], [Kok91], [Slo91], [Ste84].

3.1 System Approach

A *system approach* can be characterized by three essential parts. Talking about a system approach without defining environment, system and interactions is a waste of time. The next paragraphs will be used to give a more detailed description on the interactions between a system and its environment. Also the difference between a system and a model will be explained.

3.1.1 Environment

To define what an *environment* exactly is, is not as easy as it seems. The dictionaries describe *environment* as the circumstances, objects, or conditions by which one is surrounded or as a surrounding or associated matters that influence or modify a course of development. The definitions all have in common that an *environment* surrounds something.

The product-process-chain is primarily an economic system, because it sustains by the grace of the actors which are willing to support it financially. So before the notion of system is explained, this subsection will explain what the *environment* of the PPC is. Economics are the study of choice and decision-making in a world with limited resources [Mal72]. Economic systems basically act on three levels. These three levels are the macroeconomic, the mesoeconomic and the microeconomic level. Macroeconomics can be described as a study of economics in terms of whole systems especially with reference to general levels of output and income and to the interrelations among sectors of the economy. Microeconomics can be described as a study of economics in terms of individual areas of activity (as a firm, household). This leaves for mesoeconomics the area of sectors of industrial branches, product-process-chains etc.

So the product-process-chain will be placed in the context of mesoeconomics. The product-process-chain is a chain of actors with respect to one product and therefore cannot be described by only individual firms or individual consumers, nor can it be described by the relations among sectors.

3.1.2 System

The word *system* is a Greek word, which means *compose.* Before the product-process-chain can be studied a definition of the system *product-process-chain* has to be given. In general a definition for *system* is: with respect to its environment a separate collection, with a certain ordered structure, and can have interactions with its environment. Everything not belonging to the system is part of

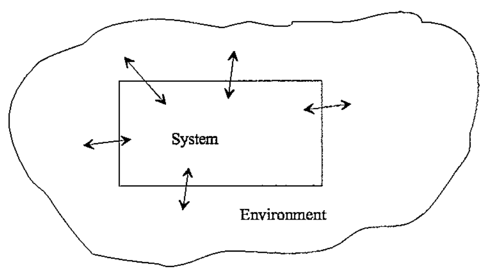


Figure 3.1: The interactions between a system and its environment

the environment. Depending on the borderline drawn between the system and its environment, one may characterize the system by input/output relations.

The product-process-chain is the collection of a extraction, a production, a stocking activity, a consuming activity, a disposal activity and a recycling activity with respect to a certain product. The producing activity is done by the producer, the consuming activity is done by the consumer, etc.. Without giving an exact description at this stage, the product-process-chain has *inputs* like raw materials (materials and semi finished products needed for the product and the production of the product) and energy. The produced goods and the recycled goods are stored in the inventory of the producer. The producers satisfy the demand of the consumers. The products are consumed by the consumers, until the products are obsolete and disposed of. A percentage of the disposed goods will be recycled and the rest will be dumped. The goods that are recycled and reused have to be upgraded and will end in the stock of the producer to be put into the chain again. One of the *outputs* of the product-process-chain is the disposed product.

3.1.3 Interactions

The product-process-chain and its environment are separated by the borderline. The *interactions* between system and environment can be divided in inputs and outputs of the system. The interactions where the environment influences the system are called *inputs* to the system. The interactions where the system influences the environment are called the *outputs* from the system. The inputs to a system can be divided into two kinds of signals. Signals which can be manipulated by an actor (part of the environment) or signals which cannot be manipulated by the actor. The former inputs are called control inputs and the latter disturbances.

Depending on the problem of interest, this will define a rough system boundary. Improving performance of the system results in a necessarily more stricter boundary, so that inputs can and have to be 'chosen'. The definition of system and system boundaries gives the designer the freedom to choose *manipulated* and *disturbance inputs.* In the PPC system the designer has the freedom to choose as control inputs e.g. levies, recycling ratios etc. The outputs can be chosen by the designer, too. Outputs for the product-process-chain can be environmental effects, flows, costs etc.

To conclude this section it has to be mentioned that considering a certain system apart from its environment depends heavily on the motivation for that particular research.

3.1.4 Classification

In general, systems can be divided into different classes. In a classification a distinction is made with respect to a certain property of a system. In this thesis a few classifications are used and will be mentioned in this paragraph. The system of the product-process-chain is a dynamic system, the relation between input and output is not static but depends on some intermediate variables. The discrimination is made on dynamic characteristics. Two more classifications can be made for the PPC with respect to the parameter characteristics and the superposition characteristics. The former

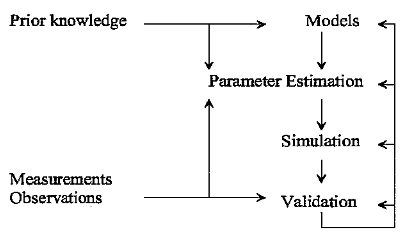


Figure 3.2: Steps in modelling [PVB94]

indicates whether the parameters are time-variant or time-invariant and the latter one indicates whether the relation between inputs and outputs is linear or non-linear. The different categories for system classification require different tools for system analysis, for instance with respect to continuous versus discrete time, the difference is to analyze a differential or a difference equation. It is obvious that with the choice of representing a system in a particular category of classification, the tools to analyze the system will result from this choice.

3.2 Models

Why models? There are many reasons to have a good insight in behavior of certain systems. The reason for the insight determines the part of interest of the system. The motivation for modelling is that a simplification of the reality gives the possibility to get quick insight into the system behavior and to be able to use powerful analysis tools. To describe a system it is not possible to use all the information of the system, a selection has to be made between relevant and irrelevant aspects of the system. This results in a particular model of the complex reality. And, for instance, simulations on the model can show how the system in reality would behave in certain situations.

In the previous chapter product-process-chains were discussed. What it is not explicitly mentioned in the previous chapter, is the fact that a PPC is a very large and complex system. In the case some improvement of performance (environmental or economic) is demanded, it is often not possible to do experiments on the real PPC to see whether this performance can be achieved. As mentioned in the previous paragraph a good alternative in analyzing the system is to make a simulation model.

In fact there is no standard recipe to obtain a model of a system. The only good advice you have is to use as much prior knowledge of the system as possible [PVB94]. In some cases it is possible to derive a model by the prior knowledge of the system's constitutive laws and parameters. This is called *white box modelling*, in the cases where there is no prior knowledge to derive the model it is called *black box modelling*. In the cases, where no knowledge about internal structures and relations is available, it is necessary to use measured inputs and outputs to obtain the model. The black box models do not give information about the underlying relations. A combination of both approaches, where prior knowledge is used in combination with the black-box model approach, is called *gray-box modelling*.

An important choice in the model making process is the selection of relevant characteristics of the system. As shown in fig 3.2 deriving a model is an iterative process. Starting with the prior knowledge about the system and an estimation of the system parameters a simulation with the model can be done. The simulated observations from the model have to be validated with the observations of the real system, when the match between these observations is not good enough the sequence of steps has to be repeated until observations from the real system validate the observations obtained from the model.

3.2.1 Graphical Models

An important issue of modelling is causality. What is the cause of the observed effect? In economic systems this is not always obvious. Do the prices cause the commodities to flow or do the commodities flows causes the prices to change? In [Bla98] one of the recommendations was to avoid this problem by applying the Bond Graph theory (See App. B) to economic systems, because bond graph modelling can deal with this causality problem. A Bond graph is composed of components that exchange

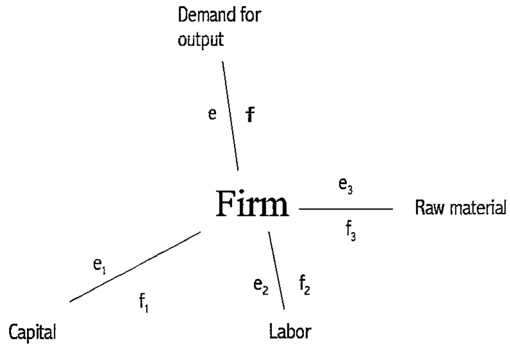


Figure 3.3: A bond graph model of a firm.

energy or power through connections (bonds) [PVB94]. Choosing energy as the exchange variable for a model leads to the use of the variables called effort (e) and flow (f). So far the assumption is made that the exchange variable is a form of energy, this limitation can be overcome by using word bond graphs. In that case for economic systems energy is interchanged by revenue flows and effort is the price and flow is the flow of goods. The big advantage of modelling in bond graphs, is that in the first steps of modelling a system, no choice has to be made between an effort input or a flow input to the modelled system, as also stated in the recommendations of [Bla98]. This property can be used to model economic systems. In economic systems it is not obvious whether the price or the flow of commodities is the actor of economic systems. The disadvantage of bondgraph modelling is that it is restricted to small systems, and systems with a known structure.

3.2.2 Mathematical Models

Another method which is frequently used, is mathematical modelling. In contrast to the bondgraphs this method of modelling requires causality. The product-process-chain will be described by dynamic models. In case of a static model the relation between input and output is of the fixed form $y = f(u)$. In the case of dynamic models intermediate variables describe the behavior of the system, this can result in different outputs for the same input.

In this thesis the representation of the system will be done in the *state space* form[1].

$$\dot{\underline{x}}(t) = \underline{\chi}(\underline{x}(t), \underline{u}(t), \underline{w}(t), \underline{\theta}(t), t) \quad \text{for } t \geq t_0; \quad \underline{x}(t_0) = \underline{x}_0 \tag{3.1}$$

$$\underline{y}(t) = \underline{\gamma}(\underline{x}(t), \underline{u}(t), \underline{v}(t), \underline{\theta}(t), t) \quad \text{for } t \geq t_0 \tag{3.2}$$

$$\underline{z}(t) = \underline{\rho}(\underline{x}(t), \underline{u}(t), \underline{\theta}(t), t) \quad \text{for } t \geq t_0 \tag{3.3}$$

With the notation $\underline{x} \in \mathbb{R}^n$, system states , $\underline{u} \in \mathbb{R}^m$ controlled inputs , $\underline{w} \in \mathbb{R}^p$ disturbances on the state, $\underline{\theta}(t)$ parameters , $\underline{y} \in \mathbb{R}^l$ system output , $\underline{v} \in \mathbb{R}^q$ disturbances on the measurements , $\underline{z} \in \mathbb{R}^k$ system output to be controlled (the notation $\underline{x} \in \mathbb{R}^n$ means $\underline{x}$ is a vector with n elements). In order to construct a model an intermediate variable is introduced called *state variable*. Without going

[1] In the equations the system explicitly depends on time. With respect to economic systems preferences are not time-invariant and therefore a system at a later time can behave differently than it did before.

to much in detail the *state variables* are used to describe the internal configuration, or state, of the model (in this case n variables are used to describe a system). Equation 3.1 describes how the system states evolve in time. In the *state* $\underline{x}(t_0)$ at time t_0 the history of the process up to t_0 is accumulated and together with the new external excitations, the behavior of the output signal is determined. Equation 3.2 describes how the measurements on the output depend on the state of the system and the inputs into the system. Equation 3.3 describes how the outputs to be controlled depend on the state and the inputs.

What are the tools? After the definition of the goal, the system is modelled. Depending on the class of system, different tools for control design are present. In the special case of linear systems it is possible to do a separate analysis and design, normally the analysis and design are intertwined.

Linearization

The equations in 3.1-3.3 are in general nonlinear differential equations. A disadvantage of nonlinear differential equations is, that no good analytical solutions exist. To get good insight into the nonlinear relations in a working point, they can be approximated by linear relations in the case that the nonlinear relations are differentiable. Transferring a nonlinear model into a linear model can be done by *linearization*. In principle this is done by approximating the nonlinear relation in a very small region around the working point $\underline{x}_0$ by linear relations. The approximation is done by introducing small deviations around point $\underline{x}_0$ and expanding the nonlinear relations in the zero and first order terms of the Taylor series of these deviation variables. This linear approximation is only valid in small regions around $\underline{x}_0$. The size of the region depends on the nonlinearity in the state equation (in case the system already was linear, the region is unbounded). The linear time-invariant system can be written as:

$$\dot{\underline{x}} = A\underline{x} + B\underline{u} \tag{3.4}$$
$$\underline{y} = C\underline{x} + D\underline{u} \tag{3.5}$$
$$\underline{z} = E\underline{x} + F\underline{u} \tag{3.6}$$

with the vectors $\underline{x} \in \mathbb{R}^n, \underline{y} \in \mathbb{R}^l, \underline{z} \in \mathbb{R}^k, \underline{u} \in \mathbb{R}^m$, and the matrices $A \in \mathbb{R}^{n*n}, B \in \mathbb{R}^{n*m}, C \in \mathbb{R}^{l*n}, D \in \mathbb{R}^{l*m}, E \in \mathbb{R}^{k*n}, F \in \mathbb{R}^{k*m}$ ($F \in \mathbb{R}^{k*m}$ is a matrix with k rows and m columns). Equation 3.4 describes for this linear system how the states will evolve in time with the inputs u. Equations 3.5 and 3.6 are the linear equivalences of eq. 3.2 and 3.3. The solution of the state $\underline{x}(t)$ with respect to time for eq. 3.4 is $\underline{x}(t_1) = e^{At_1}\underline{x}_0 + \int_{t_0}^{t_1} e^{A(t_1-s)}B\underline{u}(s)\,ds$.

Piecewise continuous functions

One of the nonlinearities arising in the product-process-chain is the exchange of goods (see chapter 4.2). This exchange of goods is bounded by the minimum of goods supplied to a market or demanded from the market. The minimum function in the form $e = \min\{s_{tot,s}, d_{tot,s}\}$ is a continuous function in its arguments $s_{tot,s}, d_{tot,s}$ .The problem of the minimum function in the exchange relation is the discontinuous derivative at the point where the two arguments of the function equal each other. To avoid this discontinuous derivative two possible approximations will be discussed here, they are: 1) the derivative can be approximated by a kind of step function, or 2) the derivative can be approximated by two functions.

The exchange $e = \min\{y, x\}$ can be written with the help of a Heavyside step function, a so called generalized function, its derivative is the Dirac δ function, as:

$$e = \varepsilon(y - x)(x - y) + y. \tag{3.7}$$

When $(y - x)$ becomes positive $\varepsilon(y - x)$ changes from zero to one. Although this function is also discontinuous in the point $x = y$, a general method of approximating the Heavyside function is to use an arctan-like function which has a continuous derivative at the point $x = y$.

$$\frac{d}{dt}e = \frac{d}{dt}(y - x)(\tfrac{1}{\pi}\arctan(x - y) + \tfrac{1}{2}) + (y) \tag{3.8}$$

In this thesis the exchange $e = \min\{y, x\}$ will be expressed by a piecewise continuous function:

$$\begin{aligned} e &= x \quad y > x \\ e &= y \quad x > y \end{aligned} \tag{3.9}$$

The derivative of the exchange function will result in:

$$\begin{aligned} \frac{d}{dt}e &= \frac{d}{dt}x \quad y > x \\ \frac{d}{dt}e &= \frac{d}{dt}y \quad x > y \end{aligned} \tag{3.10}$$

In chapter 3.3 some examples dealing with piecewise linear models are given. In state space description it is possible to use the following representation for the piecewise linear functions $\dot{\underline{x}}\,(t) = A_{i(\underline{x})}\underline{x}\,(t)$. The index of matrix $A_{i(\underline{x})}$ depends on the state and the state space can be divided into regions where the local models $(A_{i(\underline{x})})$ are valid.

3.3 Control

Why control? The basic motivation for control is to improve some performance of a system. The performance of the system itself is not sufficient for the controller, in general the three areas of improving the performance are suppressing the influence of external disturbances, ensuring the stability of processes and optimizing the performance of processes [Ste84]. Of course it is often better to change the type of process, but this is not always possible.

A few steps have to be taken in order to design a control system. There has to be a *definition* of the goal of the desired performance of the system. Further in order to analyze and design control for complex systems, these systems have to be *modelled*. To design the control structure relevant *inputs* and *outputs* of the system have to be selected. And the *control configuration* has to be selected, too. With the model, the relevant inputs and outputs of the system and the control configuration, a selection has to be made which control law will be used. The last two steps are to tune the controller and evaluate the performance of the controlled model of the system, and finally implement and test the control strategy on the real system.

A few issues which are part of the design of the control system are very important and will be explained in the following subsections. The issues are:

- if the output of the system contains enough information to capture the dynamics for the controller (*Observability*),
- if the system can be manipulated (*Controllability*) and
- if the behavior of the controlled system is stable (*Stability*).

With respect to the last issue, every controlled system is required to be stable and once designed it is indispensable to check if the overall system is stable with respect to disturbances and deviations from its initial state.

When no information of the system is used in order to assign values to the inputs, the determination of the input is called *steering*, when information from the system is used in order to manipulate the inputs it is called *control.* Equations 3.1 & 3.2 & 3.3 are used for the control law design

$$\underline{u}(t) = -\underline{l}(\underline{y}(t), \underline{r}(t), \underline{\theta}(t), t) \tag{3.11}$$

with $\underline{r}(t)$ the given references for the system.

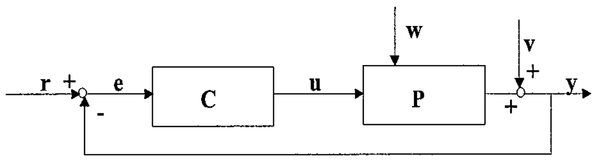


Figure 3.4: Control by feedback of the measurements.

3.3.1 Observability

One of the problems arising in designing a control law is the lack of information about the system which can be derived from the system output. To manipulate the behavior of eq. 3.1 information from the state should be reconstructed from the output of the system, therefore the system has to be observable. When the assumption is made that all information about the system is available, and no problems for reconstructing exist, equation 3.11 becomes $\underline{u}(t) = -\underline{l}(\underline{x}(t), \underline{r}(t), \underline{\theta}(t), t)$.

3.3.2 Controllability

A system is *completely controllable of state vector* if, given a desired target state $\underline{x}(t_e)$ (with $t_e > t_0$), for all initial states $\underline{x}(t_0)$ and all desired target states $\underline{x}(t_e)$ an input $\underline{u}(t)$ with $t \in [t_0, t_e]$ exists, that can bring the system from the initial state $\underline{x}(t_0)$ to the desired target state $\underline{x}(t_e)$. For the special class of linear systems a *state controllability matrix* can be computed. This *state controllability matrix* requires some explanation.

The state controllability of a linear system can be verified through some criteria, one uses the *state controllability matrix.*

$$\mathcal{C} = [\ B \quad AB \quad ... \quad A^{n-1}B\ ] \tag{3.12}$$

The proof for this test is based on the *Cayley-Hamilton Theorem* and can be found in for instance [Aok76][2].

The *state controllability matrix* $\mathcal{C}$ (eq. 3.12) is the linear subspace spanned by the column vectors $B, AB, .., A^{n-1}B$. *When the span $\mathcal{C}$ is R^n, the system with matrices A and B is completely controllable.* This span $\mathcal{C}$ is R^n, when the rank of $\mathcal{C}$ is full. When $\mathcal{C}$ has not full row rank this means that the system is not completely controllable (because according to the definition not all the coordinates in the state space can be reached). When the uncontrollable states are states with stable eigenvalues it is possible to control some parts of the system (See for instance [Kok91]). The *state controllability matrix* is only valid for small regions around the working point in the case of linearized systems.

An Example - controllability matrix

In this example the controllability matrix is computed for a system with two different input configurations. Equation 3.4 is the basis for this example. With the subscript i indicating an index ($i = 1$ or 2) two systems are given, one turns out to be completely state controllable and the other not.

$$\dot{\underline{x}} = A\underline{x} + B_i u \quad \text{with } A = \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix} \text{ and } B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \text{ and } B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{3.13}$$

The state controllability matrix is computed according to eq. 3.12 for the two cases:

$$\mathcal{C}_1 = [\ B_1 \quad AB_1\ ] = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \text{ and } \mathcal{C}_2 = [\ B_2 \quad AB_2\ ] = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \tag{3.14}$$

[2]Expanding in a polynomial in A of at most degree n-1 : $e^{At} = \alpha_0(t)I + \alpha_1(t)A + ... + \alpha_{n-1}(t)A^{n-1}$ and for the case $\int_{t_0}^{t_1} e^{A(t_1-s)}Bu(s)\,ds = B\int_{t_0}^{t_1} \alpha_0(t_1-s)u(s)\,ds + AB\int_{t_0}^{t_1} \alpha_1(t_1-s)u(s)\,ds + ...A^{n-1}B\int_{t_0}^{t_1} \alpha_{n-1}(t_1-s)u(s)\,ds$

In other words this means that the system A with Column B_1 is controllable. Here the input acts on the second component of the state, and the first component depends on itself and the second component, the system is completely controllable. In the second case with Column B_2 it is only possible to influence the first component of the state $\underline{x}$, and the second component behaves autonomously thus the system is not completely controllable.

3.3.3 Stability

In this paragraph a short explanation is given on the definition of stability. Different definitions for stability are present, for example. *Lyapunov stability* for initial conditions [Slo91], and *Bounded Input Bounded Output stability* [Ste84]. In this paragraph will be explained what these two definitions mean. And for the case of stability with respect to disturbances of the initial state some extra theory will be explained for *Linear Time Invariant* Systems and *Linear Time Varying* Systems (to be more precise Hybrid or Switched Systems). This will be used in the following chapters.

For the *Bounded Input Bounded Output Stability* a dynamic system is considered to be stable if for every bounded input it produces a bounded output, regardless of its initial state. To complete the definition, consider that 'bounded' is an input that always remains between an upper and a lower limit, unbounded outputs exists only in theory and not in practice because all physical quantities are limited, therefore unbounded means very large.

The origin of *Lyapunov's direct method of stability* is based on the idea that the total energy of an unforced dissipative mechanical system decreases as the state of the system evolves in time. The system -linear or nonlinear- will eventually settle down to its equilibrium point. Therefore the state vector approaches a constant value corresponding to the minimum energy level. A short illustration is given of the linear system state vector evolving in time.

$$\dot{\underline{x}}(t) = A(t)\,\underline{x}(t), \quad \underline{x}(t_0) = \underline{x}_0 \tag{3.15}$$

The size of a vector is measured by norms. The euclidean norm is defined as $\|\underline{s}\|_2 = \sqrt{s_1^2 + .. + s_n^2}$ (the norm expresses the power or the energy of the vector $\underline{s} \in \mathbb{R}^n$). The idea of stability is that the norm of vector $x(t)$ decreases monotonically as $t \to \infty$. The squared norm of the vector is $\|\underline{x}(t)\|_2^2 = \underline{x}^T(t)\,\underline{x}(t)$, differentiated with respect to the time this is

$$\frac{d}{dt}\|\underline{x}(t)\|^2 = \dot{\underline{x}}^T(t)\,\underline{x}(t) + \underline{x}^T(t)\,\dot{\underline{x}}(t) \tag{3.16}$$

$$= \underline{x}^T(t)\,[A^T(t) + A(t)]\underline{x}(t) \tag{3.17}$$

When the right hand side is negative definite at each t then the norm of the vector decreases and goes to 0 as $t \to \infty$.

Linear Time Invariant System

From subsection 3.1.4 it is known that systems can be discerned by its properties. From Linear Algebra it is known that a constant matrix can be described by an *eigenvalue and eigenvector decomposition* [Str88]. The solution of the differential equations 3.4-3.6 of the linear time invariant system can be found by its eigenvectors and eigenvalues. In short eigenvectors are special vectors which have the property that if the matrix A is multiplied with an eigenvector the orientation of the new vector is the same, the length is changed by a factor called the eigenvalue. So in the case of Linear Time Invariant systems it is possible to analyze the stability of the differential equation by examining the eigenvalues. A Linear Time Invariant system is stable with respect to a disturbance in the initial condition if the real arguments of all the eigenvalues of the system are negative. In other words the eigenvalues must lay in the left halfplane (negative) of the Complex space [Str88].

Time Variant, Hybrid or Switched Systems

The stability of an equilibrium can be analyzed in several ways, in the case of a linear time invariant system it is possible to search for eigenvalues which lay in left half plane. Whenever the system is not linear time invariant, but is varying with time or is a hybrid form of continuous time and discrete events (Hybrid systems) or has different types of behaviors in different regions of its state space (Switched or Switching Systems) an analysis of the eigenvalues is in most cases not appropriate or possible for stability analysis. Another way of examining the stability has to be used. This can be done by analyzing the time variation of an associated scalar 'energy-like' function, this is called the Lyapunov direct method. If it is possible to find an associated energy-like function which fulfills some conditions, it is possible to say something about the stability of one of these type of systems.

An Example - switched systems

In this example a motivation for the different types of stability analysis is given by examining two Linear Time Invariant systems and one switched system. The interest lies in the behavior of the homogeneous solution of the differential equations. In other words no inputs are present and the outputs of the system are the states of that system.

System 1:

$$\dot{\underline{x}}(t) = A_1 \underline{x}(t) \qquad \text{with } \underline{x}(0) = \underline{x}_0 \text{ and } A_1 = \begin{bmatrix} -0.1 & 1 \\ -10 & -0.1 \end{bmatrix}$$

System 2:

$$\dot{\underline{x}}(t) = A_2 \underline{x}(t) \qquad \text{with } \underline{x}(0) = \underline{x}_0 \text{ and } A_2 = \begin{bmatrix} -0.1 & 10 \\ -1 & -0.1 \end{bmatrix}$$

System 3:

$$\dot{\underline{x}}(t) = A_{i(\underline{x})} \underline{x}(t) \quad \text{with } \underline{x}(0) = \underline{x}_0$$
$$\text{for } \begin{array}{l} x_1 x_2 > 0 \ A_i = A_2 \\ x_1 x_2 < 0 \ A_i = A_1 \end{array}, \quad \begin{array}{c} \text{with } A_1 \text{ and } A_2 \text{ from} \\ \text{system 1 and 2} \end{array}$$

The eigenvalues of system 1 are: $-.10+3.16i, -.10-3.16i$ and the eigenvalues for system 2 are $-.10+3.16i, -.10-3.16i$. Both eigenvalues have a real part which is negative so the systems are exponentially stable. The eigenvalues of system 3 are the same as for system 1 and system 2. The system is not stable in contrary to what the eigenvalues suggest. In figure 3.5 the phaseportraits[3] of the three systems are given and their trajectories with initial conditions for system 1&2: $\underline{x}_0 = [-3, -3]^T$ and for system 3 $\underline{x}_0 = [-0.4, -0.4]^T$.

In this example it becomes clear that the behavior in the cells of system 3 with respect to the cell boundary determines the total behavior. The motion of system 3 is directed to infinity and so the system is unstable. (Remark that if in system 3 the conditions are changed, for instance if A_1 and A_2 are interchanged the system becomes stable).

Lyapunov functions

Lyapunov's Direct Method is based on analyzing the behavior of the systems total energy with respect to time. To analyze the energy a Lyapunov function has to be introduced. This function $V(\underline{x})$ must satisfy some conditions if the system is stable. The function $V(x)$ must be positive definite and $\dot{V}(x)$ must be negative definite.

[3] A phaseportrait or phaseplane is a graphical tool which shows in a two dimensional way the system trajectories, the vectors indicate the direction of motion.

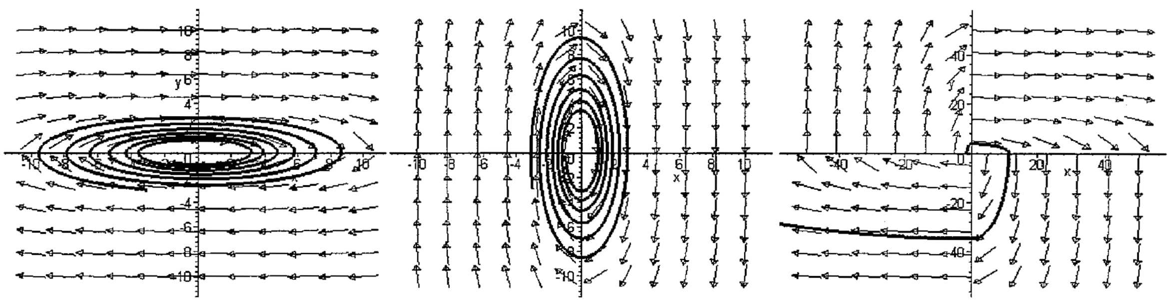


Figure 3.5: The trajectories of system 1, 2 and 3. All with the same eigenvalues.

- $V(x) > 0$
- $V(0) = 0$
- $\dot{V}(x) < 0$

When such a function exists, the system is (asymptotically) stable. Very important is that as long as no Lyapunov function is found which fulfills these conditions, nothing can be said about (in)stability. But when a function is found the system is stable.

The problem is now to find such a function, which fulfills these conditions. In [JR97] a computational approach is presented for stability analysis of nonlinear and Hybrid systems. The search for piecewise quadratic Lyapunov functions is formulated as a convex optimization problem in terms of Linear Matrix Inequalities. Because for nonlinear systems the only way to see whether a system is

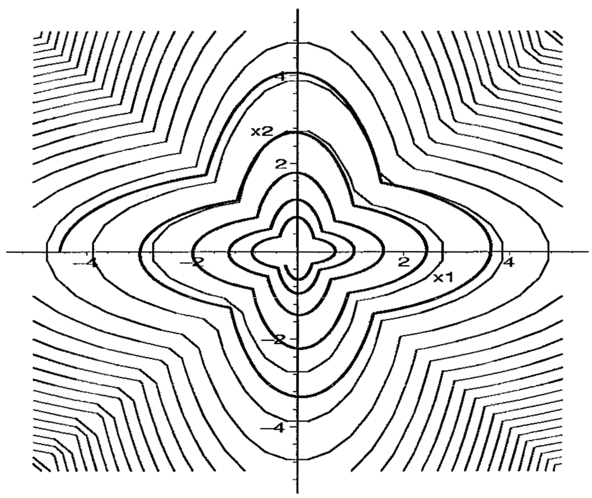


Figure 3.6: The trajectory of the flower system together with its associated Lyapunov function. The flower like curves indicate a level of energy. The energy level decreases as they move towards the origin.

stable, is to examine the Lyapunov function. An example - the flower system from [JR97] will be presented, to explain this theory. In this example a piecewise linear system is discussed.

An example -the flower system
The system is a piecewise linear system using the notation

$$\dot{\underline{x}}(t) = A_{i(\underline{x})}\underline{x}(t)$$

with the cell partition that cell 1: $x1 < x2$ and $x1 < -x2$,cell 2: $x1 < x2$ and $x1 > -x2$, cell 3: $x1 > x2$ and $x1 > -x2$, cell 4 $x1 > x2$ and $x1 < -x2$. The different matrices A_i are shown in eq. 3.18. The trajectory of a simulation moves towards the origin as is shown in the figure 3.6.

$$A_1 = A_3 = \begin{bmatrix} -0.1 & 5 \\ -1 & -0.1 \end{bmatrix}, \qquad A_2 = A_4 = \begin{bmatrix} -0.1 & 1 \\ -5 & -0.1 \end{bmatrix} \tag{3.18}$$

The system behaves like a stable system, but stability can only be guaranteed if we find a Lyapunov function which shows that the system is exponentially stable. With the computational approach for this function a piecewise quadratic Lyapunov function is found. It is $V(x) = x^T P_i x$ with

$$P_1 = P_3 = \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}, \qquad P_2 = P_4 = \begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix} \tag{3.19}$$

How can this be interpreted in terms of the energy of the system decreases in time. Along the trajectory of the simulation it can be seen that it moves towards lower energy levels. And after a long time it will end in the origin.

3.3.4 Controller design

In the special case of Linear Time Invariant systems it is possible to have a separate analysis and design, whenever controllers have to be designed for other systems, in practice the analysis and design are intertwined.

The steps which will be followed in this thesis to come to a controller design are described here and are used in chapter 5.1. With respect to a certain model, as proposed in chapter 4,

- check if the system is totally controllable for certain inputs. If the non controllable states turn out to be stable, then
- compute a controller.

There are several possible control laws, which can be used. In this thesis the emphasis is not on what controller is most suited for controlling the product-process-chains, but if it is possible to use a controller. So for instance, model predictive control could be used, but instead it is chosen to use a simple state feedback controller.

The controller in the closed loop must prevent instable behavior. In [JR97] a method is proposed to search for piecewise quadratic Lyapunov functions, as a convex optimization problem in terms of Linear Matrix Inequalities.[4] With a small extension for the Linear Matrix Inequalities it is possible search for an optimal control law. In [AR97] this is developed further and a way to compute linear quadratic controllers is shown. This type of controllers is based on linear (or affine) state feedback, in such a way that with quadratic criterium the effort to control and the result from this control input are balanced. The piecewise regulators are optimal with respect to balancing effort and result in the criterium, but it doesnot mean that total behavior of the controlled system is optimal.

3.4 Summary

With respect to gaining insight into processes (*system*), it is often found that these processes are very complex. One way to obtain insight is to select relevant information (depending on the scope

[4]In principle Linear Matrix Inequalities are of the form $A(\underline{x}) > 0$, the inequality symbol indicates that $A(\underline{x})$ is positive definite, i.e. $u^T A(\underline{x}) u > 0$ for all nonzero u.

of research), and with the selected information it is possible to make a *model* of the much more complex processes. With the relevant information put into the *model*, it is possible to analyze the behavior (with respect to *stability* for instance) of the processes. And with this model it is easier to find 'solutions' (*control design*) which yield an improvement of the processes.

With respect to the stability analysis of time-varying systems it is not possible to draw a conclusion from the eigenvalues (as can be done for linear time-invariant systems), instead the associated energy function has to be analyzed. In case of piecewise affine systems the computational approach of [JR97] can be used for analysis.

Chapter 4

Product-Process-Chain Model

4.1 Model

In this chapter some models are presented for the systems part of a product-process chain. The models will be represented in such a way that it is easy to use the representation for analyzing the system behavior and designing control laws for the system. The analysis and design can be found in chapter 5. The models are represented in such an order that the first model is a part of the second model and so on. With this sequence an easy and comprehensive way of modeling of a product-process-chain is given. At the end of the chapter a discussion about representing the models in a particular way (piecewise affine) is given. The starting point of the sections is a presentation of the models in a non linear form $\dot{\underline{x}} = \chi(\underline{x}, \underline{u})$ as in eq. 3.1. If it is possible a representation in (piecewise) linear form $\dot{\underline{x}} = A_i\underline{x} + \underline{a}_i + B_i\underline{u}$ is used. Also in the end of each section possible suggestions are given of introducing extensions to the models.

The idea of signals will be used in order to make a distinction between the kind of information used by the models. Mathematical models in general use signals to represent the real life system. The signals represent abstract information on for instance the amount of flow or velocity, represented by a symbol or a numerical value. Within the models presented in the following sections another distinction is made between the signals. In the models a difference is made between a desired signal and a signal with its actual value. Moreover, a difference will be made between signals representing material flows and signals representing a desire for certain flows. The definition used in the next sections is that a signal in these models represents the desire for an amount of goods by demand or supply. This desire can be fulfilled but this is not necessary. A flow represents the amount of goods that is actually supplied or purchased.

4.2 Market Model

One of the crucial parts of the PPC is the market mechanism. There are two approaches as to causes and effects of this mechanism: The *Walrasian* theory of price adjustment and the *Marshallian* theory of quantity adjustment [Tak85]. According to Walras' theory of dynamic markets in which the supply flow and demand flow are not equal, the unit price of a commodity changes proportionally to the difference between these flows [Bre77]. According to Marshall the rate of change of a commodity flow is proportional to the difference between the supplier's price and the demander's price [Bre77]. For further explanation on these two concepts see for instance appendix B. The Walrasian approach represents the theory of goods whereas the Marshallian approach corresponds to the theory of production adjustment [Tak85]. In this thesis the market is modelled as an exchange market. Therefore the Walrasian approach is used.

4.2.1 Price Adjustment

The price adjustment as it is modelled here, is based on the simple formula originally proposed by Walras, stating that the time derivative of the market price equals some adjustment parameter k $[US\$/ktonne]$ times the excess demand. The totals of market demand and supply signals are attained by summing the supplies and demands signals of the individual actors.

$$\dot{p} = k(d_{tot,s} - s_{tot,s}) \tag{4.1}$$

$$s_{tot,s} = \sum_{i=1}^{n_S} s_{i,s} \tag{4.2}$$

$$d_{tot,s} = \sum_{j=1}^{n_D} d_{j,s} \tag{4.3}$$

As mentioned in the previous section a distinction between the actual amount of goods exchanged and the desired amount of goods that was supposed to be exchanged, the notation which will be used is that a lower case indicates demand or supply (d or s) and the subscripts are used for denoting the demand or supply of the ith actor. The second subscript gives information whether it is the desire for a certain amount of goods or the actual amount of goods (flow is indicated with f and the signal is indicated with an s, for instance: $s_{i,f}$ $\Rightarrow$the actual supply of supplier i and $d_{tot,s}$ $\Rightarrow$the desire for an amount of goods by all the demanders. $s_{tot,s}$ $[ktonne/year]$ and $d_{tot,s}$ $[ktonne/year]$ are the totals of demand and supply signals and n_D, n_S, are the number of demanders and suppliers, respectively. Finally $s_{i,s}$ is the signal of the quantity supplied by supplier i and $d_{j,s}$ the signal of the quantity demanded by demander j.

4.2.2 Trade: The Exchange of Goods

The introduction of signals is necessary due to the following reason. An important non linearity which occurs at marketplaces follows from the fact, that there can only be traded as many goods as there are supplied or demanded for. That is, no supplier can sell more goods than his customer is willing to buy, and no demander can require his supplier to sell him more than the supplier has warehoused. The amount of goods that is traded (e) thus depends on total supply and demand to a market according to the nonlinear relation.

$$e = \min\{s_{tot,s}, d_{tot,s}\} \tag{4.4}$$

At each point in time however, one needs to allocate the supplied goods; each actor needs to know how much he has sold or purchased. Among the various actors the allocation of sales and purchases is modelled according to the ratio of their supply or demand over the total supply or demand.

$$s_{i,f} = \frac{e}{s_{tot,s}} s_{i,s} \tag{4.5}$$

$$d_{j,f} = \frac{e}{d_{tot,s}} d_{j,s} \tag{4.6}$$

Here $s_{i,f}$ are the sales of the ith supplier and $d_{j,f}$ are the purchases of the jth demander. Together with the price adjustment and this allocation of sales the market process causes the price to drop when supply exceeds demand which in turn causes demand to rise and eventually meet supply. This process develops in time as will be modelled by the proposed market model.

Possible extensions of the Market-Model

The market model is one of the elements of a product process chain model. The form of this model is relatively simple, a linear adjustment of prices, a non linearity for the minimum amount of goods exchanged and a linear rationing of allocation is suggested. For analyzing cause and effect in a market model this is sufficient. If the scope of analysis is beyond this stage, a few extensions are suggested here. With respect to the time derivative of the price, several extensions are possible, two will be discussed. The first extension is that an increase of market price will decrease the amount of goods demanded. The second extension suggests the opposite: It is possible for certain goods (prestige goods) that with an increase in market price the demand actually increases (to a certain height of course). With respect to the rationing of the exchange of goods it is possible that it is not a linear process but it is more a batch like process. So the rationing is not continuous rather discrete in the stepsize.

4.3 Production-Inventory Model

Another basic element of the product process chain is the coupling between production and inventory. The type of production-inventory control depends on the type of products. For instance in the production of food the producer aims at maintaining the time of residence of food in the inventory at a minimum level because of the limited storage life. The storage life of chemical products on the other hand is not the main problem, it is the start up and shut down of these production processes [RVH64]. The production inventory function is typical for the product process chain. Literature is abundant with respect to this topic [TH91a] [TH91b]. Several ways to determine the production coupled to inventory can be discussed, here only two of them are discussed.

Method 1: is proposed in [Bla98] in an earlier discussion of PPC. The production decision is based only on the price information on the resource and the finished-good-market. It is assumed that the producer aims at a production at maximum profit that is technical possible; so the marginal costs equal the marginal revenues (this will result in maximum profit). The decision function is a function which depends on the difference in prices. The produced goods will be warehoused in an inventory. A certain fraction of this inventory is supplied at the finished good market.

$$\dot{g} = q_{prod} - q_{pur} \tag{4.7}$$

$$q_{prod} = f\left(p_r - p_q, ...\right) \tag{4.8}$$

$$q_{pur} = \epsilon g \tag{4.9}$$

The differential equation 4.7 shows that the goods in the inventory (g) grows with the amount of produced goods (q_{prod}) and decreases with the amount of purchased goods (q_{pur}). The amount of produced goods is a function of the price at the resource market p_r and the price at the finished goods market p_q, ϵ $[1/year]$ stands for the fraction which is sold at the finished good market.

The basic idea behind this way of modeling is the following: If for instance the demand on the finished good market increases with a step, the price on this market will increase because there is an excess demand. The production function 4.8 is (directly) proportional to the price on the market, production and stock will increase and because the outflow from stock is proportional to the stock level, the supply will slowly increase, there is still an excess demand leading to an increased demand, etc. Summarizing an increase of the demand by a step will result in an inventory which will increase.

Method 2: is proposed in [TH91a]. The production decision is based on the amount of goods in the inventory. The producer holds a certain inventory norm(called setpoint in control theory) for the inventory and produces when the actual inventory lies under the norm. In this formulation the production itself does not depend on the price of resources or the finished goods.

$$\dot{g} = q_{prod} - q_{pur} \tag{4.10}$$
$$q_{prod} = f(g_{sp} - g) \tag{4.11}$$
$$q_{pur} = \epsilon g \tag{4.12}$$

The differential equation 4.10 shows that the amount of goods in the inventory (g) grows with the amount of goods produced (q_{prod}) and decreases with the amount of finished goods purchased (q_{pur}). The amount of produced goods is a function which depend on the difference between the desired inventory level (g_{sp}) and the actual inventory level (g). The amount of purchased goods is as described in section 4.2: The minimum of supplied and demanded goods. The basic idea behind this way of modeling is, that a step increase in demand will reduce the amount of products in the inventory, this can lead to a positive price adjustment because this results in a lower supply. The production takes place as described in equation 4.11, due to a larger difference between setpoint and inventory level, an amount of goods is produced so that the inventory will return to its steady state.

Both methods can be applied in a product process chain. Method two will be used in the models in the following sections.

Possible extensions of the Production-Inventory Model

As mentioned before production inventory control depends heavily on the type of product, but a combination of both methods is not unrealistic. For instance, it is possible to extend the second method with the idea of making the setpoint for the inventory a function of the price on the market. If the market price is low due to an excess of supply it can be useful to lower that setpoint, whereas if the price on the market is high it might be a good idea to increase the inventory norm in order to satisfy the excess demand.

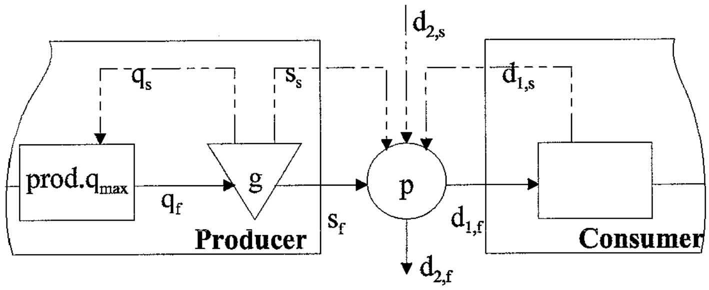


Figure 4.1: A market with supply and demand

4.4 Inventory-Market Model (*IM*)

Before a complete Product-Process-Chain is modeled, basic modules of this PPC will be modeled and analyzed. One of the essentials on the PPC are the interactions between producer and consumer. In fig. 4.1 this part of the chain is outlined. The dashed lines indicate signals and the solid lines represent flows of materials. Two degrees of freedom are present: the price on the market where demand and supply meet (p), and the amount of goods in the inventory of the firm (g). In this system one supply ($s_{1,s}$) and two demands ($d_{1,s}$, $d_{2,s}$) are present. The demand ($d_{2,s}$) is an external demand. In this system two non linearities occur: the production capacity is bounded to a certain maximum ($q_{\max}$) and the flow of goods is the minimum of the total demand or the total supply.

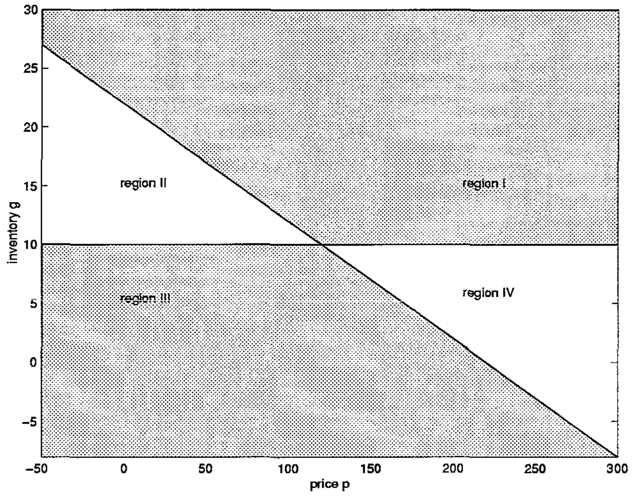


Figure 4.2: The different regions for the model with the price(p) and inventory(g) as state.

Furthermore the market, production and inventory relations are used in this model. The dynamics of this simple subsystem, fig. 4.1, is expressed by:

$$\dot{p} = k_1(d_{1,s} + d_{2,s} - s_s) \tag{4.13}$$
$$\dot{g} = q_f - s_f \tag{4.14}$$

with

$$\begin{array}{ll} q_f = \min(q_{\max}, k_2\,(g_{sp} - g)) & s_s = g \\ s_f = \min(s_s, d_{1,s} + d_{2,s}) & d_{1,s} = c_{0(1)} - c_{1(1)}p \end{array} \tag{4.15}$$

In this system k_1 , k_2 and $c_{1(1)}$ are constants typical for the market and production processes. The state space representation of this system is:

$$\dot{p} = k_1(d_{1,s} + c_{0(1)} - c_{1(1)}p - g) \tag{4.16}$$
$$\dot{g} = \min(q_{\max}, k_2(g_{sp} - g)) - \min(g, d_{1,s} + c_{0(1)} - c_{1(1)}p) \tag{4.17}$$

In the next section another representation of the market inventory system will be given. The system is represented by the equations 4.16 and 4.17, the latter equation is stating the derivative of the inventory and contains two minimum functions in its right hand side. These two minimum functions can give the inventory equation four different forms, e.g. the first argument of the first minimum function combined with the first argument of the second minimum function and so on. The four domains occur due to these two non linearities in the form of a minimum function. So actually the inventory market system can be described by four different models, the models are only valid for certain regions of the statespace. The motivation for introducing the four affine models for the four regions is that it becomes clear that the inventory market system behaves with other responses in the four different regions, as can be seen in fig. 4.2 the system has four regions. So eq. 4.17

$$\dot{g} = \min(q_{\max}, k_2(g_{sp} - g)) - \min(g, d_{1,s} + c_{0(1)} - c_{1(1)}p)$$

has the following relations for the four cases for the different regions:

$$\begin{array}{ll} \dot{g} = q_{\max} - g & \text{when } q_{\max} < k_2(g_{sp} - g) \text{ and } g < d_{1,s} + c_{0(1)} - c_{1(1)}p \\ \dot{g} = q_{\max} - \left(d_{1,s} + c_{0(1)} - c_{1(1)}p\right) & \text{when } q_{\max} < k_2(g_{sp} - g) \text{ and } g > d_{1,s} + c_{0(1)} - c_{1(1)}p \\ \dot{g} = k_2(g_{sp} - g) - g & \text{when } q_{\max} > k_2(g_{sp} - g) \text{ and } g < d_{1,s} + c_{0(1)} - c_{1(1)}p \\ \dot{g} = k_2(g_{sp} - g) - \left(d_{1,s} + c_{0(1)} - c_{1(1)}p\right) & \text{when } q_{\max} > k_2(g_{sp} - g) \text{ and } g > d_{1,s} + c_{0(1)} - c_{1(1)}p \end{array}$$

The inventory market system is summarized by a system of four affine systems. The affine representation is $\dot{\underline{x}} = A_{i(\underline{x})}\underline{x} + a_i$ with matrices A_i and vectors a_i in table 4.1. This piecewise affine notation shows that for each region of the system state space the system shows different behaviors. The piecewise affine system can only be analyzed with respect to the stability as mentioned on page 19.

matrices A_i and a_i	A_i	a_i
$q_{\max} > k_2(g_{sp} - g)$ $g > d_1 + c_{0(1)} - c_{1(1)}p$ region I	$A_1 = \begin{bmatrix} -k_1c_{1(1)} & -k_1 \\ c_{1(1)} & -k_2 \end{bmatrix}$	$a_1 = \begin{bmatrix} d_1 + c_{0(1)} \\ -d_1 - c_{0(1)} + k_2g_{sp} \end{bmatrix}$
$q_{\max} > k_2(g_{sp} - g)$ $g < d_1 + c_{0(1)} - c_{1(1)}p$ region II	$A_2 = \begin{bmatrix} -k_1c_{1(1)} & -k_1 \\ 0 & -k_2 - 1 \end{bmatrix}$	$a_2 = \begin{bmatrix} d_1 + c_{0(1)} \\ k_2g_{sp} \end{bmatrix}$
$q_{\max} < k_2(g_{sp} - g)$ $g < d_1 + c_{0(1)} - c_{1(1)}p$ region III	$A_3 = \begin{bmatrix} -k_1c_{1(1)} & -k_1 \\ 0 & -1 \end{bmatrix}$	$a_3 = \begin{bmatrix} d_1 + c_{0(1)} \\ q_{\max} \end{bmatrix}$
$q_{\max} < k_2(g_{sp} - g)$ $g > d_1 + c_{0(1)} - c_{1(1)}p$ region IV	$A_4 = \begin{bmatrix} -k_1c_{1(1)} & -k_1 \\ c_{1(1)} & 0 \end{bmatrix}$	$a_4 = \begin{bmatrix} d_1 + c_{0(1)} \\ q_{\max} - d_1 - c_{0(1)} \end{bmatrix}$

Table 4.1: matrix A and vector a for the *IM* system

Possible extensions of the Inventory-Market Model

This model is a combination of the models proposed in the previous sections. So other combinations with the suggested extensions from the previous models are also possible. Without going to much in detail a few other extensions are proposed. The Inventory Market model contains a consuming company which has no inventory. The demand can also be a function of the goods the consumer already possesses. Another suggestion is of course possible that the demand is a nonlinear function of the market price (See example [Bre91]).

An additional extension of this model is the introduction of another minimum function, the supply to the market equals at this moment the inventory. In fact this cannot always be the case. When the inventory is empty it is not possible to supply goods to the market so the supply function ($s = g$) should be extended to $s = \min(0, g)$.

4.5 Market-Inventory-Inventory-Market Model (*MIIM*)

The previous example only contained two states, i.e. one inventory and one price adjustment. In this section the system is extended to a system with two inventories and two markets. The environment of this system is the large economic world without the market where the resources needed for the production of goods and the market where the finished goods are exchanged. The production firm has two inventories and, as mentioned, the producer is connected to the two markets as can be seen in figure 4.3, so a system with four states is created. The Market-Inventory-Inventory-Market system looks like:

$$\begin{aligned} \dot{p}_1 &= k_1(d_{1,s} - (s_{1,s} + v_{1,s})) \\ \dot{g}_2 &= d_{1,f} - q_{1,f} \\ \dot{g}_3 &= q_{2,f} - s_{2,f} \\ \dot{p}_4 &= k_4(d_{2,s} + v_{2,s} - s_{2,s}) \end{aligned} \tag{4.18}$$

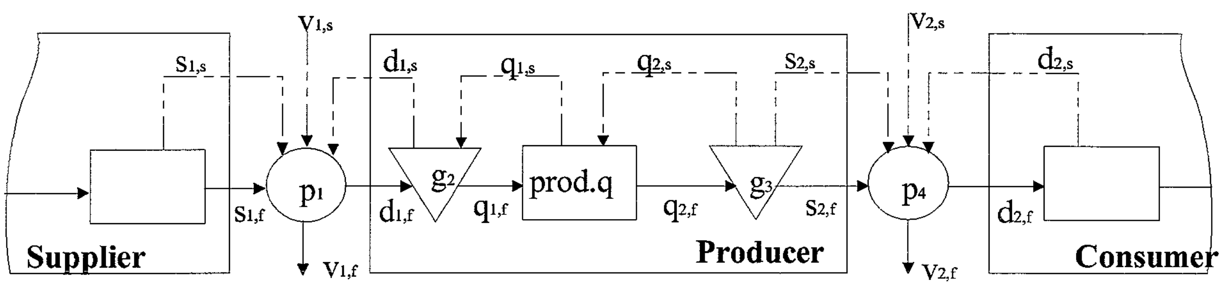


Figure 4.3: A market-inventory-inventory-market system.

With the differences between signals and 'real' material flows. As can be seen in the figure the signals are dashed and the flows are solid lines. The relations can be found in appendix A. Principally the relations are the same as described in the previous sections, with one exception. In this model a price dependent level norm is introduced.

$$g_{sp3} = f(.., +p_4, -p1, ..) \tag{4.19}$$

Which indicates that whenever the price at the market of the finished goods increases the supplier increases his inventory norm, which result in an increase of supply at the finished goods market. Whenever the prices at the resource market increases the producer will reduce his supply, because the margin between the resource and finished goods prices is decreased. The inventory norm for the resources of the producer is like described in method 2 in section 4.3. Substituting the relations from chapter A in eq. 4.18 will result in the state equations.

$$\dot{p}_1 = k_1(k_2(g_{sp2} - g_2) - (b_{0(1)} + b_{1(1)}p_1) - v_{1,s}) \tag{4.20}$$
$$\dot{g}_2 = \min(k_2(g_{sp2} - g_2), v_{1,s} + (b_{0(1)} + b_{1(1)}p_1)) - k_3(g_{sp3,0} + k_6(p_4 - p_1) - g_3) \tag{4.21}$$
$$\dot{g}_3 = k_3(g_{sp3,0} + k_6(p_4 - p_1) - g_3) - \min((d_{0(2)} - d_{1(2)}p_4) + v_{2,s}, g_3) \tag{4.22}$$
$$\dot{p}_4 = k_4((d_{0(2)} - d_{1(2)}p_4) - g_3 + v_{2,s}) \tag{4.23}$$

The nonlinear state space representation is based on the market dynamics, the production inventory control function as described earlier in section 4.3. As can seen the minimum function which represent a maximum production capacity is loosened, for simplifying the system for analysis. The description for the coefficients can also be found in chapter A. And the states are p_1, p_4 the prices on the market g_2, g_3 the amount of (resources)products in the inventories.

The system is represented by the equations 5.7 , 5.8 , 5.9 and 5.10, the equations in the middle are stating the derivative of the inventory and contain two minimum functions in their right hand side. These two minimum functions can give the inventory equations four different forms. So actually the market-inventory-inventory-market system can be described by four different models, the models are only valid for certain regions of the statespace. The motivation for introducing the four affine models for the four regions is that it becomes clear that the inventory market system behaves with other responses in the four different regions. The state space representation is of the form with $\dot{\underline{x}} = A_{i(\underline{x})}\underline{x} + a_i$ with matrices A_i and vectors a_i in table 4.2.

Possible extensions of the Market-Inventory-Inventory-Market Model

In this model it is assumed that the price on the first market itself not directly influences the production of the product which uses the good from market 1. In more realistic models, inclusion of more relations is desirable to put in to this model. In [Bla98] for instance a production function is of the form $...(pr - ps)/...$ And in for instance [Han98] a kind of model predictive control is inserted

matrices A_i and a_i	A_i	a_i
region I $k_2(g_{sp2}-g_2) > sv_1 + (b_{0(1)} + b_{1(1)}p_1)$ $(d_{20} - d_{21}p_4) + sv_2 > g_3$	$\begin{bmatrix} -k_1b_{1(1)} & -k_1k_2 & 0 & 0 \\ b_{1(1)} + k_3k_6 & 0 & k_3 & -k_3k_6 \\ -k_3k_6 & 0 & -k_3 - 1 & k_3k_6 \\ 0 & 0 & -k_4 & -k_4d_{21} \end{bmatrix}$	$\begin{bmatrix} k_1k_2g_{sp2} - k_1b_{0(1)} - k_1sv_1 \\ sv_1 + b_{0(1)} - k_3g_{sp3,0} \\ k_3g_{sp3,0} \\ k_4d_{20} + k_4sv_2 \end{bmatrix}$
region II $k_2(g_{sp2}-g_2) < sv_1 + (b_{0(1)} + b_{1(1)}p_1)$ $(d_{20} - d_{21}p_4) + sv_2 > g_3$	$\begin{bmatrix} -k_1b_{1(1)} & -k_1k_2 & 0 & 0 \\ k_3k_6 & -k_2 & k_3 & -k_3k_6 \\ -k_3k_6 & 0 & -k_3 - 1 & k_3k_6 \\ 0 & 0 & -k_4 & -k_4d_{21} \end{bmatrix}$	$\begin{bmatrix} k_1k_2g_{sp2} - k_1b_{0(1)} - k_1sv_1 \\ k_2g_{sp2} - k_3g_{sp3,0} \\ k_3g_{sp3,0} \\ k_4d_{20} + k_4sv_2 \end{bmatrix}$
region III $k_2(g_{sp2}-g_2) > sv_1 + (b_{0(1)} + b_{1(1)}p_1)$ $(d_{20} - d_{21}p_4) + sv_2 < g_3$	$\begin{bmatrix} -k_1b_{1(1)} & -k_1k_2 & 0 & 0 \\ b_{1(1)} + k_3k_6 & 0 & k_3 & -k_3k_6 \\ -k_3k_6 & 0 & -k_3 & k_3k_6 + d_{21} \\ 0 & 0 & -k_4 & -k_4d_{21} \end{bmatrix}$	$\begin{bmatrix} k_1k_2g_{sp2} - k_1b_{0(1)} - k_1sv_1 \\ sv_1 + b_{0(1)} - k_3g_{sp3,0} \\ k_3g_{sp3,0} - d_{20} - sv_2 \\ k_4d_{20} + k_4sv_2 \end{bmatrix}$
region IV $k_2(g_{sp2}-g_2) < sv_1 + (b_{0(1)} + b_{1(1)}p_1)$ $(d_{20} - d_{21}p_4) + sv_2 < g_3$	$\begin{bmatrix} -k_1s_{11} & -k_1k_2 & 0 & 0 \\ k_3k_6 & -k_2 & k_3 & -k_3k_6 \\ -k_3k_6 & 0 & -k_3 & k_3k_6 + d_{21} \\ 0 & 0 & -k_4 & -k_4d_{21} \end{bmatrix}$	$\begin{bmatrix} k_1k_2g_{sp2} - k_1b_{0(1)} - k_1sv_1 \\ k_2g_{sp2} - k_3g_{sp3,0} \\ k_3g_{sp3,0} - d_{20} - sv_2 \\ k_4d_{20} + k_4sv_2 \end{bmatrix}$

Table 4.2: matrix A and vector a for the *MIIM* system

in the production function. Here the producer tries to predict the behavior of the consumer with a simple model and uses this to anticipate on the reaction. The consumer, however reacts according to his own scope.

The production capacity can be limited to a certain maximum level dependent to an amount of Labor and Capital Goods.

4.6 Product-Process-Chain Model

The final model which will be discussed is a representation of a material life cycle product-process-chain as described in chapter 2. Because this model is the final model which contains basic modules described in earlier sections in this chapter, the assumptions made for this model will be summarized briefly. The time span is set for a few years so with this assumption it is allowed to say that technological development does not take place on the production units. And the structure of markets is not changed. Another assumption made is that there is a representative producer, which behaves like the average behavior of all individual producers of the product, consistent with this assumption there is a representative consumer which behaves like the average of all individual consumers.

The next section is about the product process chain. As can be seen in fig. 4.4 the chain exists out of an resource supplier, a producing actor, a consuming actor and a recycler. Further examining shows three markets are present, the resource market, the finished good market and the obsolete product market. The assumption which has to be made is that the three chain actors are representative members of all the firms active in the branch. Also external demands are present at these markets. In state space representation the PPC chain can be modelled with the equations of

4.25:

$$
\begin{aligned}
\dot{p}_1 &= k_1\ (d_{p_1,s} - (s_{s,s} + v_{1,s} + s_{r_2,s})) \\
\dot{g}_2 &= d_{p_1,f} - d_{p_2,f} \\
\dot{g}_3 &= s_{p_1,f} - s_{p_2,f} \\
\dot{p}_4 &= k_4\ ((d_{c,s} + v_{2,s}) - s_{p_2,s}) \\
\dot{p}_5 &= k_5\ (d_{r_1,s} + v_{3,s} - s_{c,s}) \\
\dot{g}_6 &= d_{r_1 f} - d_{r_2,f} \\
\dot{g}_7 &= s_{r_1,f} - s_{r_2,f}
\end{aligned}
\tag{4.24}
$$

The relations for the entries of eq. 4.24 can be found in appendix A. As can be seen in figure 4.4 a definite percentage of recycling takes place. The flow from the consumer to the market is a fraction from the flow that goes from the finished good market to the consumer. The fraction is modelled price dependent. So when the price on the market of recycled goods increases, less goods will be wasted. The amount of goods that can be recycled is bounded to the maximum amount of goods from the flow of goods to the consumer. For ease of analysis the restriction on maximum production capacity is relaxed.

Substituting these relations into eq. 4.24 this will results in the non linear state space representation like eq. 4.25, notice that the derivative of p_5 depends on a fraction with the size of $(\alpha_0 + \alpha_1 p_5)$ of flow of goods which is replaced by the purchased goods of the consumer $d_{c,f}$.

$$
\begin{aligned}
\dot{p}_1 &= k_1\ \left(-sv_1 - b_{0(1)} - b_{1(1)}\, p_1 - g_7 + k_2\ (g_{sp2} - g_2)\right) \\
\dot{g}_2 &= \min(k_2\ (g_{sp2} - g_2)\,, sv_1 + b_{0(1)} + b_{1(1)}\, p_1 + g_7) - k_3\ (g_{sp3} - g_3) \\
\dot{g}_3 &= k_3\ (g_{sp3} - g_3) - \min(c_{0(1)} - c_{1(1)}\, p_4 + sv_2, g_3) \\
\dot{p}_4 &= k_4\ \left(c_{0(1)} - c_{1(1)}\, p_4 + sv_2 - g_3\right) \\
\dot{p}_5 &= k_5\,(k_6\ (g_{sp6} - g_6) - \min(c_{0(1)} - c_{1(1)}\, p_4, \tfrac{\left(c_{0(1)} - c_{1(1)}\, p_4\right) g_3}{c_{0(1)} - c_{1(1)}\, p_4 + sv_2}, b) + sv_3) \\
\dot{g}_6 &= \min(\min(c_{0(1)} - c_{1(1)}\, p_4, \tfrac{\left(c_{0(1)} - c_{1(1)}\, p_4\right) g_3}{c_{0(1)} - c_{1(1)}\, p_4 + sv_2}, b) \tfrac{k_6\ (g_{sp6} - g_6)}{k_6\ (g_{sp6} - g_6) + sv_3}, k_6\ (g_{sp6} - g_6)) - k_7\ (g_{sp7} - g_7) \\
\dot{g}_7 &= k_7\ (g_{sp7} - g_7) - \min(g_7, \tfrac{k_2\ (g_{sp2} - g_2) g_7}{sv_1 + b_{0(1)} + b_{1(1)}\, p_1 + g_7})
\end{aligned}
\tag{4.25}
$$

$$
\text{with } b = (\alpha_0 + \alpha_1\, p_5) \min(c_{0(1)} - c_{1(1)}\, p_4, \tfrac{\left(c_{0(1)} - c_{1(1)}\, p_4\right) g_3}{c_{0(1)} - c_{1(1)}\, p_4 + sv_2})
$$

The parameters used in this model are also found in appendix A. The states of the Product-Process-Chain model are the prices at the markets and the inventories of the firms. With respect to this model it can be described in models valid for certain regions (like the previous sections), this modelling in regions is for the sake of completeness denoted in chapter A. Although the model in eq. 4.25 seems very complex, basically it can be represented by eight piecewise models (see A).

Possible extensions of the Product-Process-Chain Model

The product process chain system represents a minimum of the dynamics that are actually present. In reality PPCs are much more complex but one of the aims for modelling is to get a simplified structure of the reality which has just enough information to be of use for a gaining insight in specific properties of the reality. Other suggestions for this type of model that been proposed. Both the firms are at this point not directly price based active, it means that an increase in price on the finished good market itself not results in higher production.

Another extension is to model the waste flows from the production process and the recycling process.

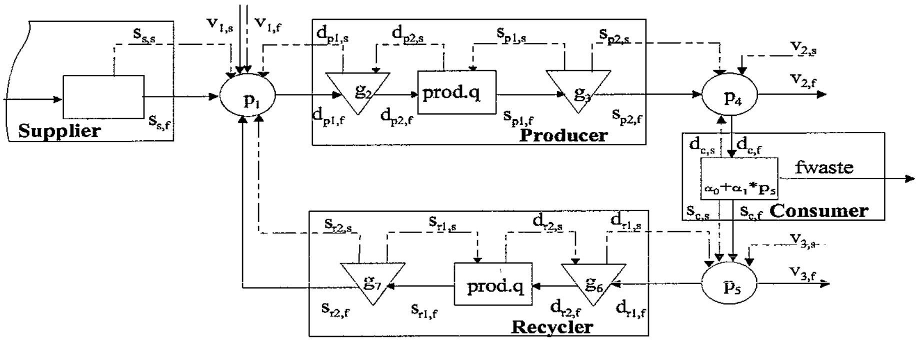


Figure 4.4: A PPC chain

4.7 Discussion

In this section a discussion is given over the piecewise affine (linear) approach of the models in a Product Process Chain. The main question is why introducing the concept of switched systems, while other approaches also seem promising. These obviously have a nonlinear character. Also the bondgraph approach has promising advantages. In this stage it is not important what choice is made at this point, it is of major importance what the consequences are of this choice. What is the use of a perfect representation in for instance a bond graph approach, if it turns out that it is impossible to analyze the models with this approach. So, with respect to the models in this chapter, in representing the problem the best choice is probably the bond graph approach and the most time consuming the piecewise affine (linear) approach. But when it comes to the property of analyzing and designing the state properties, it is the other way around. The piecewise affine (linear) approach is much more suited for this than the bondgraph approach.

Another argument is that a simple model is easier to understand than a complex model. Therefore it is more appropriate to analyze little parts of a problem than the whole problem in once. This argument puts the bondgraph approach and the piecewise affine(linear) approach as opposite to the non linear representation. Where the nonlinear representation is one formulation of the problem, the others are the sum of representations of subproblems.

At this point the piecewise affine(linear) approach seems to be the most promising way of representing and capable of analyzing the total problem of product-process-chains.

Chapter 5

Simulations, Analysis & Control

In this chapter some simulations of the systems modelled in chapter 4 can be found. The models of the sections Market and Production-Inventory will not be discussed here, because they are included in the other models. The three models discussed here, are the inventory market model, the market inventory market model and the product process chain model. Results of stability analysis of the uncontrolled and controlled systems are presented.

5.1 Way to deal with the problem

As discussed in chapter 2 the motivation for modelling product process chains is based on an economic or an environmental perspective. The different perspectives lead to different outputs for the model of PPC, for environmental aspects the output of the PPC model are the environmental effects, and for economic aspects the costs are the output (see fig. 2.3).

In principle the different types of outputs can be reconstructed from the material flows and the prices on the market. The focus of this thesis is not on the discussion how to do this. The *assumption* is made, that the states in the models are equally important in assigning values to the environmental effects and the economic costs.

The first step in control design to a system is to set the goals to be controlled. Because the assumption is made that all states are equally important this defines the control goal. In this chapter two goals are presented. The *first* control goal is to *control the complete state vector*, and for the *second* control goal the assumption is dropped that all states are equally important. It is to see if it is possible to *control a part of the state vector*, e.g. the exchange of goods in the market.

The next step in control design is to select the inputs in order to achieve the control goals. Here two approaches are chosen to determine the relevant inputs. The first one is a more theoretic approach towards the control problem and tries to find inputs which will results in controllability matrices with full rank. The motivation behind this approach is to see if it is possible to control all the components of the states, because the assumption is made that all the states are equally important. It may be possible that the input which satisfies this condition is actually not realized in the real PPC, but it may turn out to be a good idea to think of such input.

The second approach is to use economic control instruments. The input which will be used here is a price levy. The price on the market is influenced by the government through some levies, this will result in, for instance, higher prices. The influence of levies can also be seen as reducing the autonomous demand of the consumers. In the present case it is easy to model the price levies as an influence on the price derivative.

The two goals and approaches can be seen in table 5.1. From the table it becomes clear that an input selection based on complete controllability is the same for both controls described here. This leaves three cases, which are used for the analysis of the controlled systems.

For each of the models discussed in the following sections the same way to deal with the problem

	All states	**Part of the states**
Complete controllability	case I	
Price levies	case II	case III

Table 5.1: Cases described in chapter 5

is used. The first step in the analysis is to see if the autonomous system has equilibrium points and to examine if these points are stable. With the help of Lyapunov functions this is analyzed. The second step is to compute the controllability matrix for the different regions. The third step is to determine a control law which has the property to control the system to the references and to stabilize the system around these references. This control law is computed for the three cases described in table 5.1.

5.2 Inventory-Market (*IM*)

5.2.1 Stability Analysis

This part of a product process chain contains two nonlinearities: the production is limited by the production capacity and the exchange of goods is the minimum of the quantity demanded or supplied.

$$\dot{p} = k_1(d_{2,s} + c_{0(1)} - c_{1(1)}p - g) \tag{5.1}$$

$$\dot{g} = \min(q_{\max}, k_2(g_{sp} - g)) - \min(g, d_{2,s} + c_{0(1)} - c_{1(1)}p) \tag{5.2}$$

In this section two sets of parameters are used. The first parameter set is introduced for practical reasons, whereas the second set was already used to do some simulations with the Inventory-Market (*IM*) model. The values for the parameter sets are:

- set 1: $q_{\max} = 30$, $g_{sp} = 40$, $c_{0(1)} + d_{1,s} = 22$, $c_{1(1)} = 0.1$, $k_1 = k_2 = 1$
- set 2: $q_{\max} = 40$, $g_{sp} = 40$, $c_{0(1)} + d_{1,s} = 22$, $c_{1(1)} = 0.1$, $k_1 = k_2 = 1$.

The difference between the two sets lies in the production capacity. To get an idea of possible responses of this system to different initial situations see figure 5.1. Another way of representing the responses of the *IM* model is by examining the phase portrait. In fig. 5.2 the phase portrait of the system is shown (for parameter set 2). The arrows in the phase portrait indicate the evolution of states of the system. In the phaseportraits an evolution in time -from a given initial condition- is shown. The other solid lines in fig. 5.2 indicate the boundaries of the regions (see fig 4.2).

As this limited overview of the phaseportrait shows, nothing yet is said about stability of this model over the complete state space. As shown in chapter 3 it is not appropriate to examine the eigenvalues of the subsystems in order to see whether the system is stable or not. One possible way of determining the stability is to look for a Lyapunov function (see chapter 3.3.3). For the *IM*-model the computational approach (see chapter 3.3.3 as well) indeed results in a quadratic piecewise Lyapunov function. Therefore the system is globally stable. In figure 5.3 this Lyapunov function -with the level curves of equal energy- and two trajectories of the behavior of the model are shown for parameter set 2. Note that the figure shows the behavior in the deviation coordinates.

5.2.2 Control

As mentioned in chapter 5.1 three cases are used to analyze the models with respect to control.

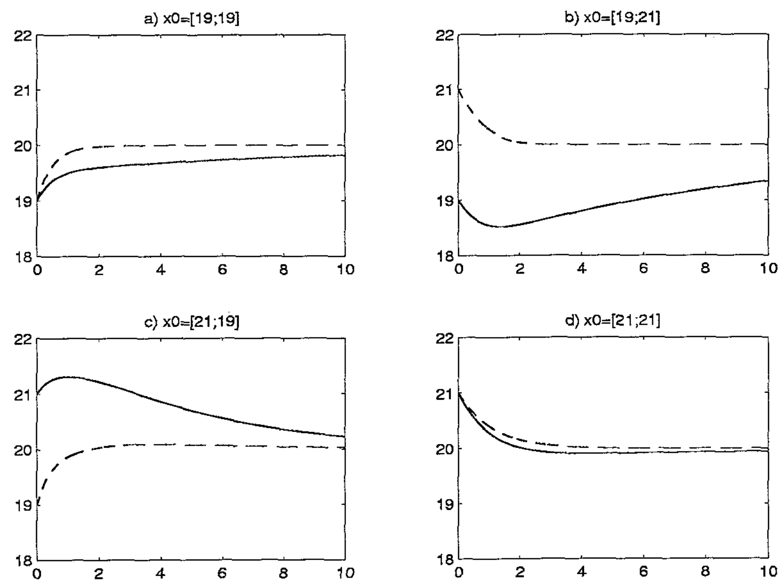


Figure 5.1: Different responses in months for the *IM* system for different initial conditions (parameter set 2). The solid line indicates the price p and the dashed line the inventory g.

Case One, complete controllability for all the states

In order to get a good insight where the problems are in the control design, first the case of a completely controllable system is analyzed. The inputs have to be determined such the controllability matrix of all of the four subsystems has full rank. The search for the input which satisfies the rank condition is done by a computer algorithm. One system which satisfies this rank condition is obtained by adding an additive input to the two state equations.[1]

$$\dot{p} = k_1(d_{2,s} + c_{0(1)} - c_{1(1)}p - g) + b_1 u_1 \tag{5.3}$$

$$\dot{g} = \min(q_{\max}, k_2(g_{sp} - g)) - \min(g, d_{2,s} + c_{0(1)} - c_{1(1)}p) + b_2 u_2 \tag{5.4}$$

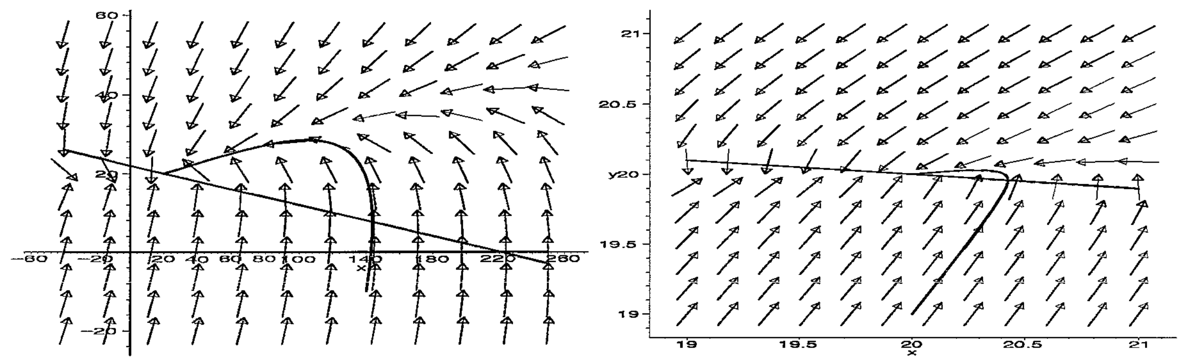
All the subsystems in the four regions satisfy the complete controllability rank condition (see chap-

controllability-matrix	$q_{\max} > k_2(g_{sp} - g)$	$q_{\max} < k_2(g_{sp} - g)$
$g > d_{2,s} + c_{0(1)} - c_{1(1)}p$	I: $\begin{bmatrix} b_1 & 0 & -k_1c_{1(1)}b_1 & -k_1b_2 \\ 0 & b_2 & c_{1(1)}b_1 & -b_2k_2 \end{bmatrix}$	IV: $\begin{bmatrix} b_1 & 0 & -k_1c_{1(1)}b_1 & -k_1b_2 \\ 0 & b_2 & c_{1(1)}b_1 & 0 \end{bmatrix}$
$g < d_{2,s} + c_{0(1)} - c_{1(1)}p$	II: $\begin{bmatrix} b_1 & 0 & -k_1c_{1(1)}b_1 & -k_1b_2 \\ 0 & b_2 & 0 & -b_2 - b_2k_2 \end{bmatrix}$	III: $\begin{bmatrix} b_1 & 0 & -k_1c_{1(1)}b_1 & -k_1b_2 \\ 0 & b_2 & 0 & -b_2 \end{bmatrix}$

Table 5.2: Controllability matrix of *IM* system with two inputs

ter 3.3.2). The controllability matrices for the regions can be found in table 5.2. This control-configuration is used to see what happens if four switched optimal controllers are computed and are used to go to a given reference state. Simulation results are given in fig. 5.4. The reference state in the simulation is the steady state of the uncontrolled system. The simulation is done with parameter set 1. The top figures indicate the behavior of the uncontrolled and the controlled system. The solid line indicates the price, the dotted line indicates the inventory. In the right figure also the 'controlled' price (dashdotted line) and the controlled inventory (dashed line) are drawn. In the bottom figure the control input is drawn (left figure) and the path of the regions through which the system goes.

[1]Note that in the case of certain 'unlucky' parameter values the rank condition is not satisfied. For simplicity we exclude these situations here as well in the following sections.



(a) overview (b) close-up

Figure 5.2: Phaseportrait of the *IM* system with set 2 with horizontally the price and vertically the inventory shown. (a) shows the boundaries and (b) a close up around the equilibrium of *IM*.

The solid line indicates the path of the uncontrolled system and the dotted line of the controlled system.

With two inputs acting on the price and inventory equation of the *IM* system it is possible to control all the four regions.

Examination of the matrices of table 5.2 shows that the system is completely controllable for the case $b_1 = 0$, too. Thus with an input on the inventory equation only the system stays complete controllable. In the case that the number of inputs is reduced to one input acting on the inventory, simulation (see fig. 5.5 based on set 1) shows that the total controller, which contains four optimal controllers for the different regions, turns out to be non optimal. The motivation for this can be seen from the right bottom of figure 5.5. The controllers in region II and region III manipulate the states towards the boundary between region II and III under the supposition that the optimal path lies in the other region. The states evolve very slowly to a negative price (sort of chattering). When the price has decreased enough, the controllers of region II and III manipulate the system in the same direction. So the states of the controlled system will move to the references. [AR97] show that, with an extension of the Linear Matrix Inequalities, it is also possible to compute a controller which satisfies some cost criterium for the entire system in the four regions. It minimizes the total costs of the input effort and the error between the reference and the state. Emphasizing the error will result in larger inputs and emphasizing on minimum input effort will result in small inputs. The controllers computed are suited for their regions, and are implemented in the closed loop.

The controllers computed according to satisfy a certain cost criterium [AR97], are implemented. In for instance fig. 5.7 it shows that it is possible to move to a given reference, in the case the system is complete controllable.

Case Two, Price levies controlling all the states

This strategy is based on economical instruments, e.g. a price levy: With this control instrument the representation of the system becomes:

$$\dot{p} = k_1(d_{2,s} + c_{0(1)} - c_{1(1)}p - g) + b_1 u \tag{5.5}$$

$$\dot{g} = \min(q_{\max}, k_2(g_{sp} - g)) - \min(g, d_{2,s} + c_{0(1)} - c_{1(1)}p) \tag{5.6}$$

The controllability matrices of the four subsystems are shown in table 5.3. The rank of the matrices in regions II and III is not full, the second row consists of zeros and therefore the inventory state cannot be controlled by the input. This fact can be logically explained: In the case that the supply to the market is smaller than the demand by the consumer, it is not possible to have any influence

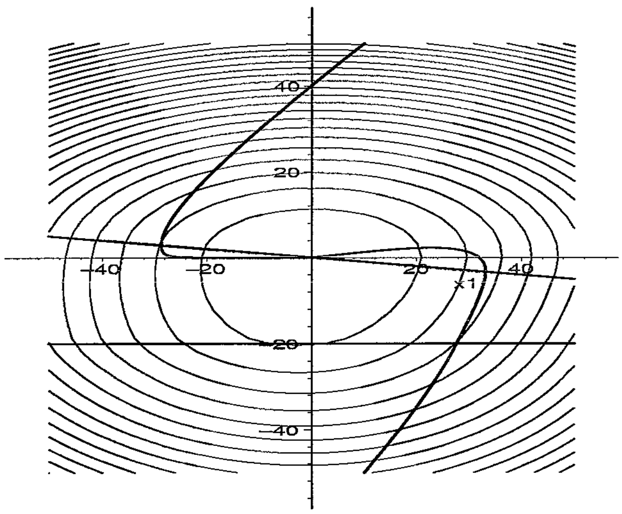


Figure 5.3: Constant energy levels of the Lyapunov function associated to the *IM* system with horizontally the price and vertically the inventory deviations. Two evolutions in time from initial conditions are represented by the thick curved lines.

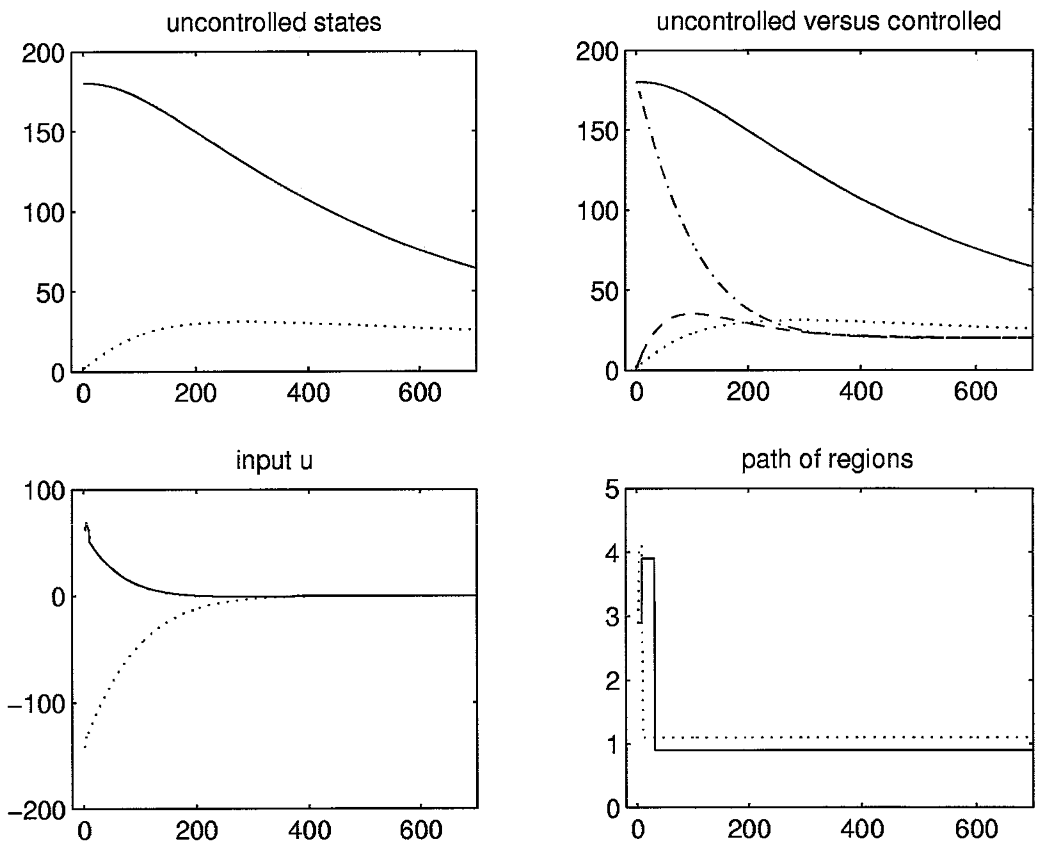


Figure 5.4: Simulation with the uncontrolled *IM* system and the controlled *IM* system by two inputs and a switched optimal controller. In the top figures the solid line indicates the uncontrolled price, the dashdotted line the controlled price. The dotted line shows the uncontrolled and the dashed line the controlled inventory.

controllability-matrix	$q_{\max} > k_2(g_{sp} - g)$	$q_{\max} < k_2(g_{sp} - g)$
$g > d_{2,s} + c_{0(1)} - c_{1(1)}p$	I: $\begin{bmatrix} b_1 & -k_1c_{1(1)}b_1 \\ 0 & c_{1(1)}b_1 \end{bmatrix}$	IV: $\begin{bmatrix} b_1 & -k_1c_{1(1)}b_1 \\ 0 & c_{1(1)}b_1 \end{bmatrix}$
$g < d_{2,s} + c_{0(1)} - c_{1(1)}p$	II: $\begin{bmatrix} b_1 & -k_1c_{1(1)}b_1 \\ 0 & 0 \end{bmatrix}$	III: $\begin{bmatrix} b_1 & -k_1c_{1(1)}b_1 \\ 0 & 0 \end{bmatrix}$

Table 5.3: Ctrbmatrix for the pricelevy on the *IM* system

on the inventory of the producer. And therefore it is not possible to have any influence on the supply of the producer.

As can be seen in fig. 5.6 this interpretation turns out to be right. The reference inventory lies in region III (indicated by *) and the starting point for this case is a point in region II (indicated by 0). In fig.5.6 the configuration with two inputs is able to move from the starting point to the reference point. The bottom figure gives the path along which the system evolves. The right top figure shows that with the price levy alone it is not possible to move to the given reference point[2]. The configuration with the price levy as control instrument is thus not totally able to move from the starting point to the reference point, only the market price can be controlled.

As can seen from fig. 5.7 the situation with two inputs is able to move from starting point to the reference point, the situation with only the price levy is not able to do so. The difference between this figure and the previous one is the situation on the market. In the former the supply limits the amount of goods exchanged, in the latter one it is the demand which determines the exchange of goods. With a price levy it is therefore not possible to control the complete statespace.

Case Three, Price levies controlling a part of the states

As it is not possible to control all the states with a price levy, a more reduced control problem is examined. It is to control the exchange of goods in the market. This turns out to be possible in some regions only. In fig. 5.6 the initial situation represents the case that the demand for the finished goods is larger than the supply. The exchange is a function of the supply to the market, e.g. in regions II and III. With the price levy it is not possible to directly reduce the flow from the marketplace to the consumers. In these regions the exchange is a function of the inventory of the producer. The control goal is to decrease inventory, from table 5.3 it is clear that this is not possible. In fig. 5.7 the demand is the limiting factor for the initial situation. The exchange in the market is a function of the price so with the price levy it is possible to reduce the flow.

It is possible to control the exchange in the market if the demand is the limiting factor.

5.2.3 Conclusion

In this very simple model, the inventory market model, three cases are analyzed. The case of complete controllability for all the states can be achieved by an input on the inventory of the producer. It turns out that, with a price levy alone, it is not possible to control the inventory state in all the four regions. In the more reduced case it is found that the levy can partly control the exchange on the market. If it is possible to have an input on both state equations it is possible to move to the given references.

In this section only a proportional state feedback law is used. Other control laws could be used, but this is not the issue here. The proportional control law is based on an criterium which weighs the states and the inputs.

[2]The problem is that the affine term cannot be cancelled by a proper input because it does not lie in the image of B.

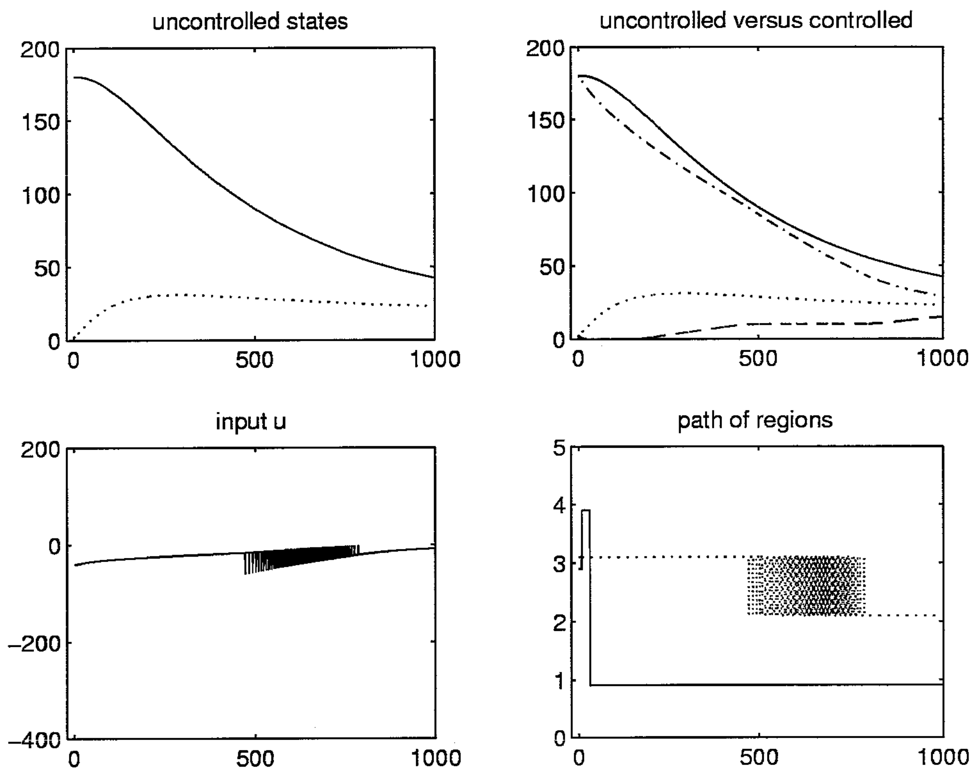


Figure 5.5: Simulation with one input and a switched optimal controller. The controlled *IM* system is not optimal due to the conflicting region controllers.

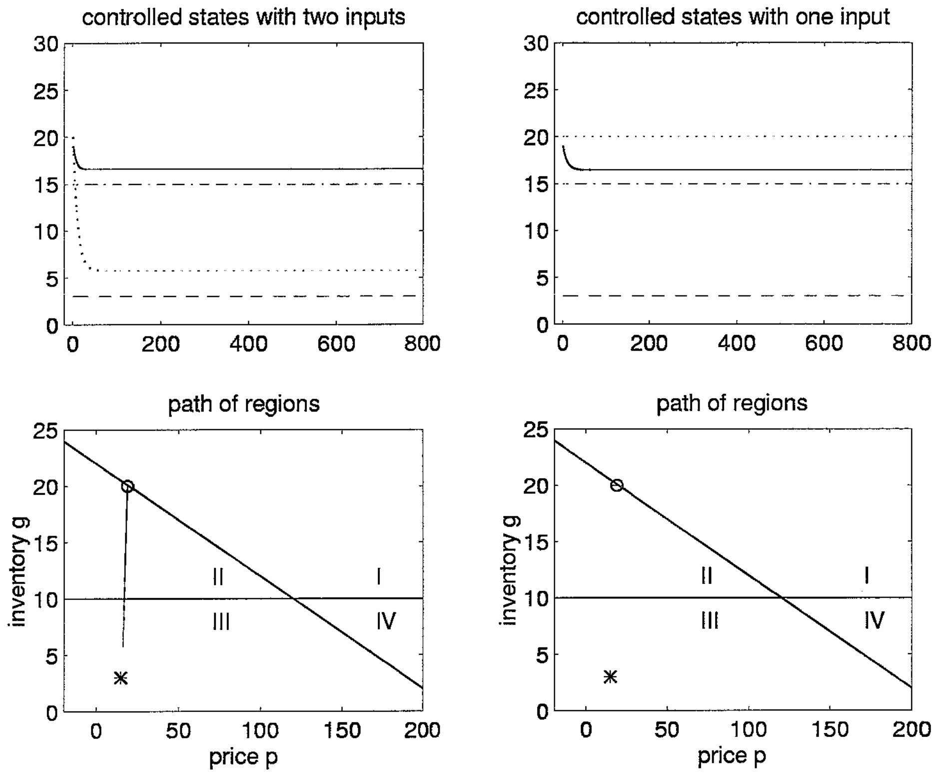


Figure 5.6: Goal to decrease the flow to the market for different strategies(for set 1). In the top figures the solid line indicates the price, the dashdot line the reference price. The dotted line shows the inventory and the dashed line the inventory reference.

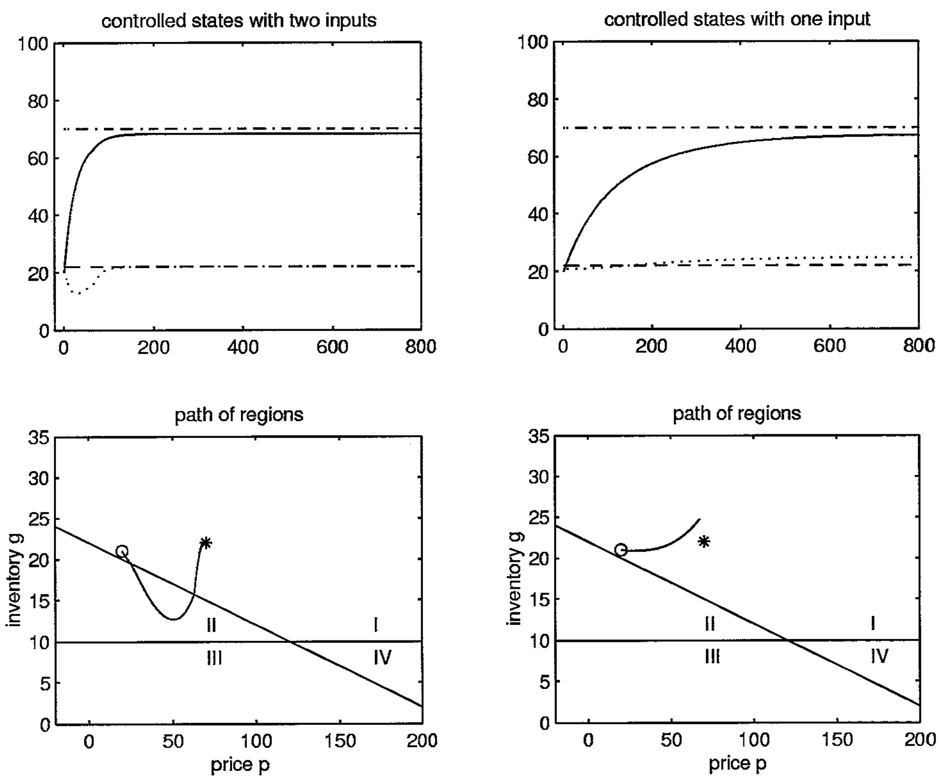


Figure 5.7: Goal to decrease the flow from the market with two different configurations(for set 1).

5.3 Market-Inventory-Inventory-Market (*MIIM*)

In this section the analysis and control of the market-inventory-inventory-market are discussed in an analogue way to the previous section. The three cases which are used for control analysis can be found in table 5.1. First we look at the uncontrolled system. The market-Inventory-inventory market model as presented in chapter 4 has four states, the two inventories and two market prices.

$$\dot{p}_1 = k_1(k_2(g_{sp2} - g_2) - (b_{0(1)} + b_{1(1)}p_1) - v_{1,s}) \tag{5.7}$$

$$\dot{g}_2 = \min(k_2(g_{sp2} - g_2), v_{1,s} + (b_{0(1)} + b_{1(1)}p_1)) - k_3(g_{sp3,0} + k_6(p_4 - p_1) - g_3) \tag{5.8}$$

$$\dot{g}_3 = k_3(g_{sp3,0} + k_6(p_4 - p_1) - g_3) - \min((d_{0(2)} - d_{1(2)}p_4) + v_{2,s}, g_3) \tag{5.9}$$

$$\dot{p}_4 = k_4((d_{0(2)} - d_{1(2)}p_4) - g_3 + v_{2,s}) \tag{5.10}$$

This model is discussed here because in contrast to the other models the second inventory setpoint depends on the market prices. It thus includes other possibilities of control by price levies. The parameters for the market-inventory-inventory-market are

- $k_1 = 2$, $k_4 = 2$, $k_2 = 1$, $k_3 = 1$, $g_{sp2,0} = 40$, $g_{sp3,0} = 40$, $b_{0(1)} = 10$, $c_{0(1)} = 16$, $b_{1(1)} = 0.1$, $c_{1(1)} = 0.2$, $sv_1 = 8$, $sv_2 = 8$.

5.3.1 Stability Analysis

A graphical way of representing the four states with for instance a phase portrait is almost impossible. Therefore in fig. 5.8 time response plot of the system behavior is shown with respect to different initial conditions. The lines indicate how the states evolve in time. The four states are represented by four different types of lines. The solid line indicates the market price (p_1), the dashed line indicates the producer's inventory level (g_2). The finished products inventory level (g_3) is indicated by the dotted line, and this leaves for the price on the market of finished goods (p_4) the dashdotted line. All are represented in deviation coordinates with respect to the steady state. As can be seen from the four different initial conditions the states will eventually settle down in the equilibrium. The figure gives an indication that the system is stable, but as long as no Lyapunov function is found, this is not certain. The computational approach (see chapter 3.3.3) results in a piecewise quadratic Lyapunov function. The Market Inventory market system turns out to be globally stable with the assumption that the parameters are kept constant.

5.3.2 Control

The model is again analyzed in three cases:

Case One, complete controllability of the states

The reason for these full ranks are the mechanisms which are present in the system. The set point for the inventory level depends on the price on the resource market and on the price of the finished good market. If for instance the price on the resource market is increased the supply of resources will increase, but the producer will reduce the inventory norm and therefore the flows of finished goods will be reduced.

It can be shown that *without* the price variant inventory norm, it is only possible to have the full rank condition satisfied in the four regions if the input acts on the finished goods inventory. One control input on the price on the second market is not sufficient in the case where the supply is less than the demand. In this case it is not possible to influence the states earlier in the chain. Due to the assigned parameters it is only possible to control all the states in all the regions by the inventory (g_3). In fig. 5.9 is shown that even with a non full rank for region II the system can be controlled to its reference point, with one price levy on the first market.

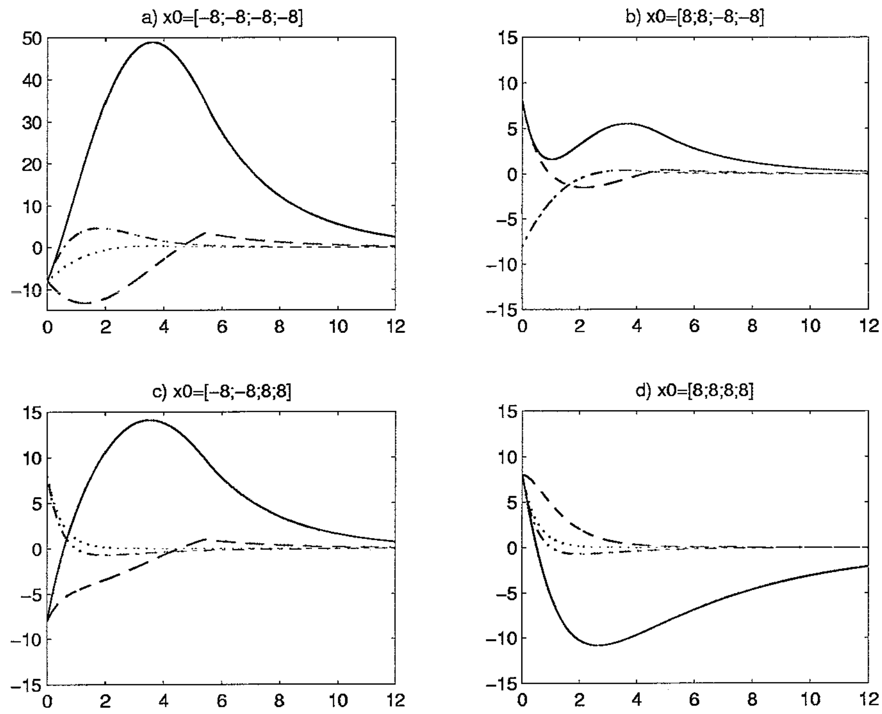


Figure 5.8: Different responses to different initial conditions for theMIIM system (p_1=solid; g_2=dashed; g_3=dotted; p_4=dashdot).

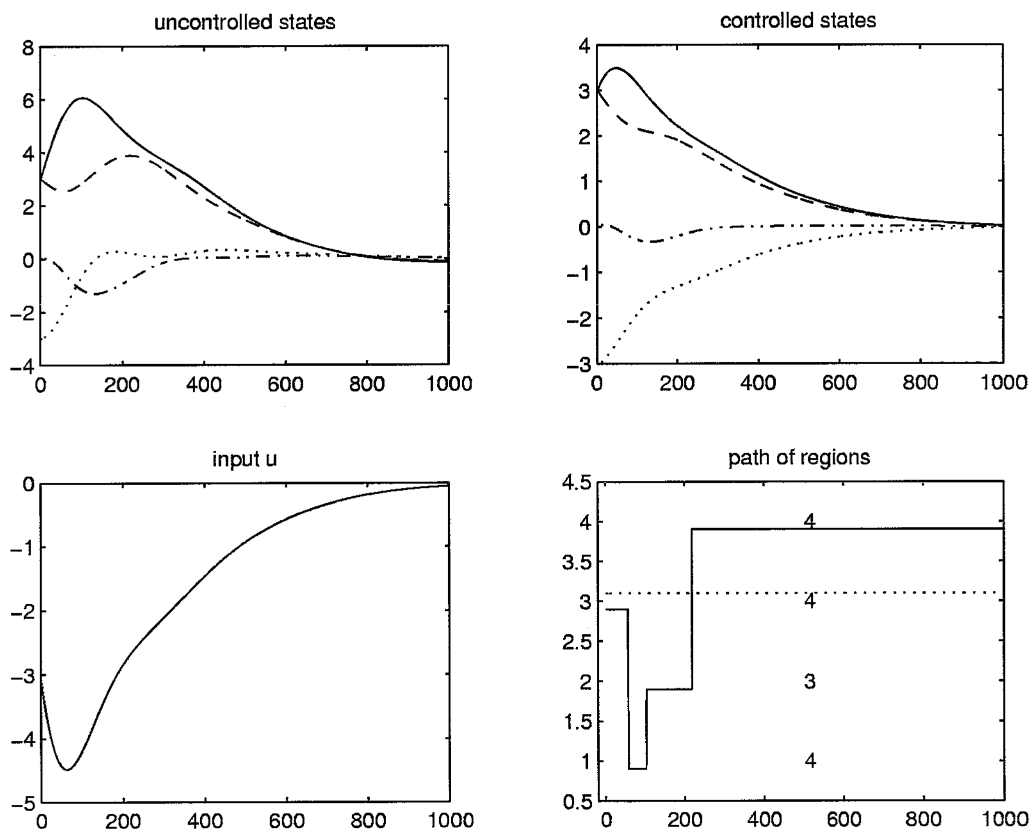


Figure 5.9: The uncontrolled and controlled *MIIM* system.

Case two, price levies controlling all the states

Case one was concerned with the question if all the states were controllable, in this case a restriction is put on the type of inputs. The used inputs are price levies for the resource market and finished goods market. It turns out that because of the price variant inventory norm - in principle all the states can be controlled. Because of the parameter constellation here this is not the case. In the region where 1) the resource supply is larger than the demand for resources and 2) the supply of finished goods is larger than the demand for the finished goods, the rank condition is not satisfied (rank = 3).

Case Three, price levies controlling a part of the states

Whereas the former cases analyze if all the states are controllable, in this case the problem is reduced to the question if it is possible to control the flow on the market of finished goods. Analyzing this question gives, that an input has to be found which acts on the producer's finished goods inventory and an input which acts on the finished goods market price. In principle an input on one of the components of the states is able to manipulate the complete state vector. Because of the parameter constellation it is not possible to manipulate all the states. In practice this is not a problem, the motivation is the (rank =3) is due to dependence between the two inventory states. If the scope is only on one of the inventories it is possible to control with two price levies the flow on the market of finished goods. In fig. 5.10 it is shown that controlling the flow in the market of finished goods is possible. This flow is a function of the state vector. In the figure the reference for the states corresponds to a reduction of the finished goods inventory and an increase of the price on the finished good market. In the left top figure the dashdotted line indicates the inventory of finished goods (g_3) and its reference, the dashed line indicates the price on the second market (p_4) and its reference. The solid line represents the price on the resource market (the reference is zero) and the dotted line the inventory of resources (g_2) (the reference is also zero). This inventory (g_2) behaves like the opposite of the finished goods inventory (g_3), as can be seen from the rank of the controllability matrix. With the inputs on the prices of the markets it is possible to control the exchange of goods in the market.

5.3.3 Conclusion

In this *MIIM* system two nonlinearities are present due to the two markets. With the price variant inventory level mechanism for the finished goods in this model all the examined input configurations turn out to be in principle completely controllable for all the regions. For certain parameter constellation the controllability matrices do not have full rank. This problem can be avoided by finding a control law which acts on the producer's inventory of finished goods.

With the controllability matrices satisfying the rank condition we know that it is possible to go from one state to another state in a finite time. In this thesis an affine state feedback is used for controlling the systems. In the cases where the constant affine vectors in the state equation can be cancelled by the control input, it is possible to move to the reference states.

In case one (complete controllability for all the states), and in case two (price levies controlling all the states), it turns out that only an input acting on the producer's finished goods inventory satisfies the conditions. In the more reduced case, of using the price levies on the resource market and the finished good market, it is possible to control the exchange on the finished good market.

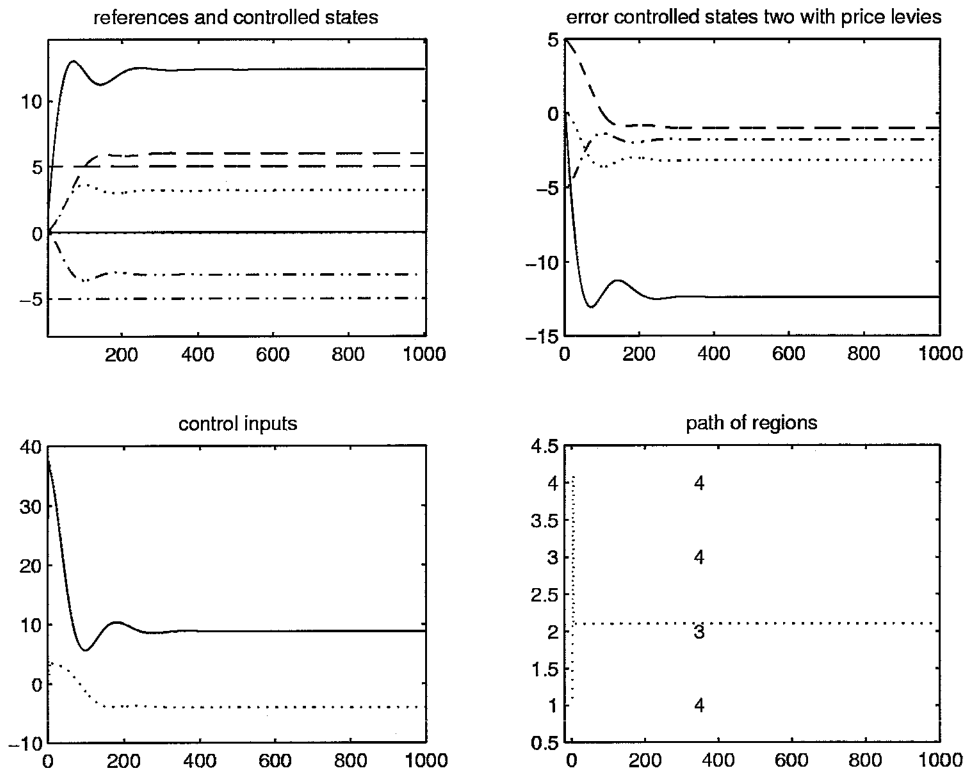


Figure 5.10: Controlling the flow of the finished goods in the *MIIM* system with two pricelevies.

5.4 Product Process Chain

In the previous sections analyses of parts of the product-process-chain are given. In this section these elements are extended to the PPC model. This section has globally the same line as the others. Before this a motivation on the choice of parameters is given. Then the uncontrolled system is analyzed with respect to stability. The next step is to see if the two control strategies can be introduced and what the results are from the simulations for these cases.

5.4.1 Identification

Before we can simulate the real PPC for Steel (see section 2.5), it is necessary to fill in the good parameters. As can be seen from fig. 3.2, for parameter estimation some measurements or observations have to be made. The simulations with these parameters are validated with the measurements or observations. When the match between simulated and measured observations is not good enough it is necessary to repeat the step of modelling and the parameter estimation.

The model proposed in section 4.6 is difficult with respect to parameter estimation. This is due to the relatively large amount of parameters in the model. The model has seven states and 20 parameters. Another problem which with respect to parameter estimation, is the limited amount of data (time-series) available. The data which was used from [II97a] [EUR97] [II97b], did unfortunately not capture all the dynamics of the PPC. The data was available at monthly, in the best case, or yearly basis.

In the simulation we do need values for the parameters, therefore the parameters are approximated as good as possible by trial and error.

- $k_1 = 0.1$, $k_2 = 1$, $k_3 = 1$, $k_4 = 1.3$, $k_5 = 2$, $k_6 = 1$, $k_7 = 1$, $g_{sp2} = 125$, $g_{sp3} = 125$, $g_{sp6} = 40$, $g_{sp7} = 40$, $b_{0(1)} = 28$, $c_{0(1)} = 57$, $b_{1(1)} = 0.04$, $c_{1(1)} = 0.01$, $sv_1 = 6.5$, $sv_2 = 9.5$, $sv_2 = 12$, $\alpha_0 = 0.1$, $\alpha_1 = 0.0033$

In the figures representing the simulations the time in years is denoted horizontally. From the data (time series) it is found that the recycler produces a yearly amount of $245 \cdot 10^3$ ktonne steel, with the Electric Arc Steelmaking process. The supplier yearly produces $433 \cdot 10^3$ ktonne with the Basic Oxygen Steelmaking process. Other steelmaking processes contribute around the $72 \cdot 10^3$ ktonne of Steel. So the world steel production in 1995 was $750 \cdot 10^3$ ktonne steel. In the chain about $642 \cdot 10^3$ ktonne steel is consumed and $276 \cdot 10^3$ ktonne is recycled to the scrap market. This should give rough indications of the amount of flows, it is further found that the dynamics of price adaption are not so sensitive for excess demand and supply, except for the scrap market. The scrap market price changed in 1995 with 30 percent due to a scrap shortage. The price of the finished goods changed with 10 percent and the resource price with 5 percent. The price level in US$ was in 1995 around 100-110 $/ktonne for scrap, for resources around $200 ktonne, and the price for finished goods around the $400.

5.4.2 Analysis

In this section the steel PPC is analyzed with respect to stability. The state equations can be found in chapter 4.6 in the eq. 4.25. The model has some nonlinearities, not only the nonlinearity due to the exchange of goods. But also through the price dependent recycling. Also the rationing introduces nonlinearities.

To examine the stability of the uncontrolled system, a similar approach is used as in the previous sections. In figure 5.11 the time response plots are shown. In the two left top figures the states of the model are drawn. The left one indicates the prices in the steel PPC, measured in US$/ktonne. The solid line indicates the steel resource price, the dashed line indicates the price of the finished steel products and the dotted line indicates the scrap price. The middle top figure indicates the inventory of the PPC actors in ktonne. the producer's resource inventory is the solid line, the producer's finished product inventory is the dashed dotted line. The recycler's resource inventory is shown with the dashed line and the recycler's product inventory is shown with the dotted line. The right top figure indicates the supply to the finished good market by the dotted line, and the demand of the steel consumers by the solid line. In the left bottom figure the total flow of steel into the steel resource market is drawn with the dotted line, and the solid line indicates the amount of recycled steel in this market. The right bottom figures are used to show the path of region which the system undergoes, and the associated Lyapunov function. This set up of the figure is also used in the other cases of the steel PPC.

From the time responses of the states it could be concluded that the system is stable, but this is not absolutely sure until a Lyapunov function is found. This function is shown in the right bottom figure, as can be seen the system is not in its equilibrium, with another simulation it is found that over a larger time period, the system eventually will come in its equilibrium.

The Steel PPC model turns out to be stable.

5.4.3 Control

Also this model is analyzed in three cases.

Case One, complete controllability of the states

This case for the steel PPC gives a negative result. By checking all the possible input configurations, as described earlier chapter 5.1, no configuration can be found which satisfied this controllability test. This is due to the parameter constellation. As also found in the *MIIM* system, it is not possible to manipulate all the states independently.

Case two, price levies controlling all the states

As shown in the previous paragraph, it is not possible to find a control configuration which can control all the states. It is therefore not strange that when restricting the control input configurations a negative result for complete controllability is obtained, too. Also this case gives negative result. In fig. 5.12 it is shown what the controlled system with price levies on the three markets can do. The same initial conditions as in fig. 5.11 are used . The controlled system turns out to evolve faster to its equilibrium than the uncontrolled steel PPC. As can be seen from the response in one year in fig. 5.11 and the response over twenty years in fig. 5.12.

As a result we maintain that it is not possible to control all the states with the use of price levies, but the control can speed up the system's dynamics.

Case Three, price levies controlling a part of the states

The last case discussed here is the case of controlling a part of the states. In this case it is tried to control the exchange in the market of finished steel products. Basically there are two things which have to be controlled. When demand is the limiting factor, the exchange flow of goods is a function of the price on the market. In the other situation that supply limits the exchange of goods, the supply of the producer has to be controlled.

The first situation is shown in fig. 5.13. The control configuration is based on three price levies. The result of controlling the exchange of steel products is that the steel product price will increase. In the second situation where supply of steel products should be controlled, the time responses of the steel PPC are shown in fig 5.14. Because the supply of steel products is decreased, the market will have an excess demand. This mechanism results in an increase of the price of the steel products. In fig. 5.13 and 5.14 two more time responses are shown. The inputs from the controller are placed in the second row, second column. The responses show that offset reduces when the time evolves. The

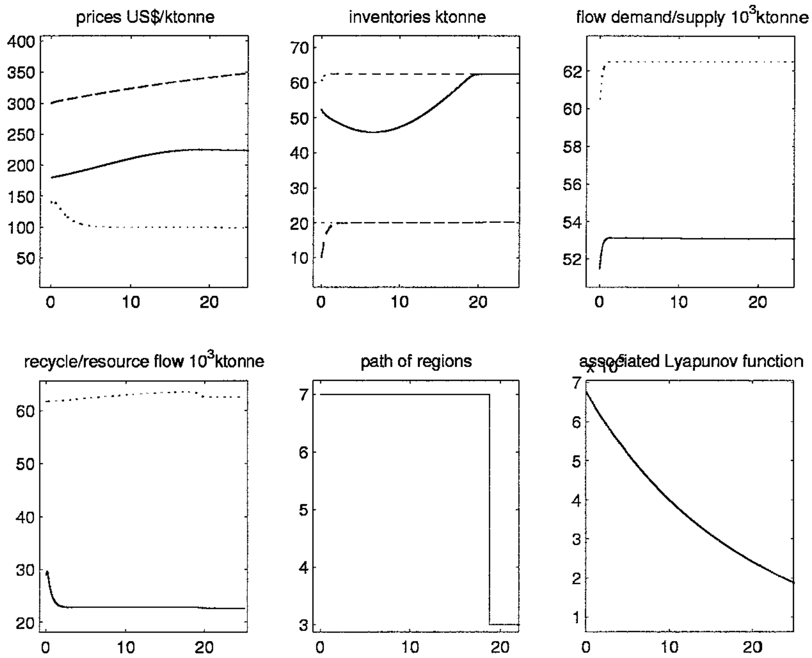


Figure 5.11: Time response plots of the Steel PPC with the time represented in years.

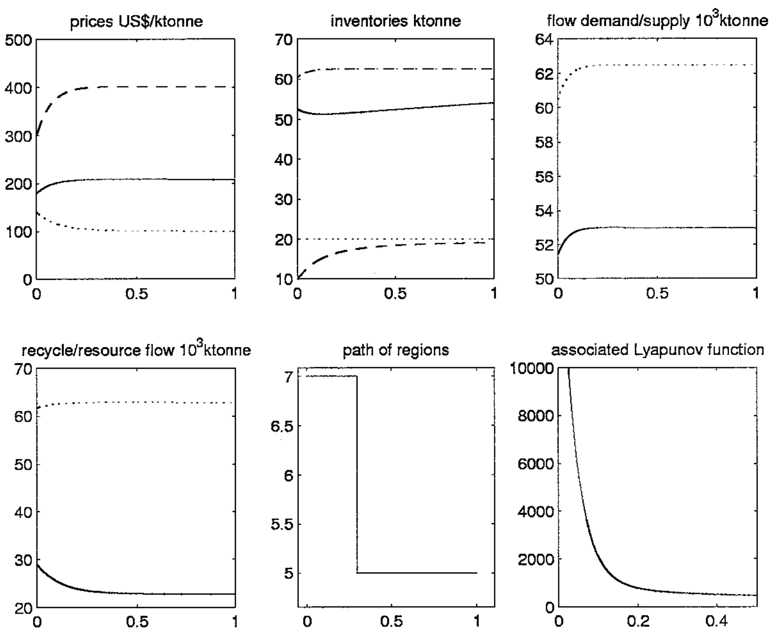


Figure 5.12: The time responses of the Steel PPC in the case of control with price levies.

other time response plot which is shown is a close up from the control goal. In 5.13 where an excess supply is, i.e. the price which should be controlled. In 5.14 where an excess demand is, the control goal is to reduce the supply from producer.

Thus, if the control goal is relaxed to controlling a part of the system's states and therefore the assumption is dropped that all states are equally important, it is possible to satisfy this goal in the case of controlling the exchange of steel products.

5.4.4 Conclusion

In this section we examine the steel product-process-chain. The three markets in this chain are the steel resource market, the finished steel products market and the scrap market. The other states are the inventories of the finished steel product producer and the (Electric Arc Steelmaking) steel producer.

In chapter 5.1, together with the PPC management set up as shown in figure 2.3, it was motivated that control of all the states was desirable. It is nevertheless only possible to control parts of the states given the proposed control configuration, for example the exchange of steel products.

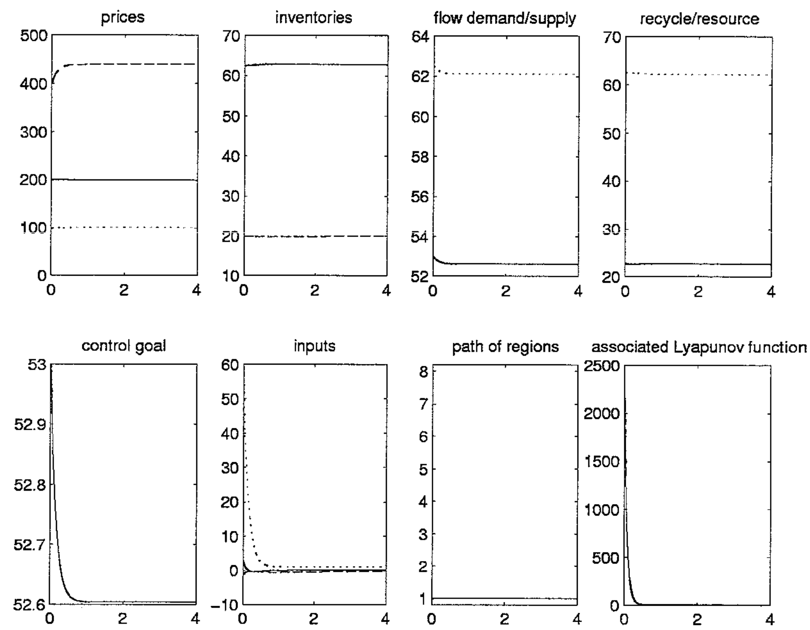


Figure 5.13: Control of the exchange of goods on the steel products market, when steel product demand is the limiting factor in the steel PPC.

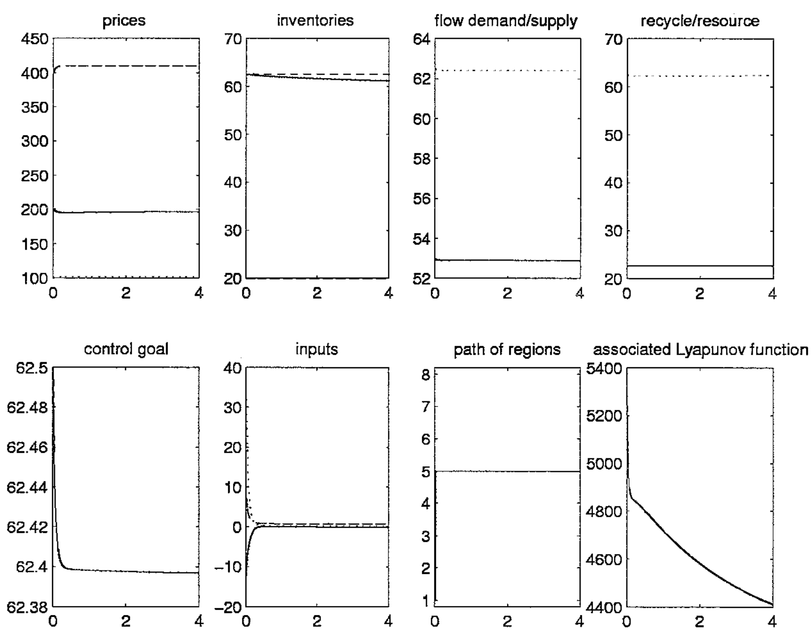


Figure 5.14: Control of the exchange of goods on the steel products market, when steel product supply is the limiting factor in the steel PPC.

Chapter 6

Conclusions and Recommendations

6.1 Conclusions

Based on the analysis and control properties of the product-process-chain and its modules some conclusions are drawn. The conclusions are based on the systems and models proposed in this report.

6.1.1 Analysis

This paragraph gives a short survey of the context of *product-process-chains.* Product-Process-Chains (PPCs) are based on a cradle-to-grave approach of the life cycle of a product. In this context two approaches to a PPC are important, an environmental approach and an economic approach.

- In modelling product-process-chains several difficulties appear, the behavior of product-process-chains depends on many parameters and many states.
- With respect to the large amount of (intermediate) variables and unknown relations in the product-process-chain, it is not possible to use Bondgraph modelling. The advantage of not assigning cause and effect in the stage of modelling cannot compensate by the fact that Bondgraph modelling cannot handle large and uncertain systems. Furthermore the tools which use bondgraph theory are limited.
- Mathematical modelling of PPCs in state space form can handle the large amount of (intermediate) variables and unknown relations. The disadvantage of assigning cause and effect, is largely compensated by the tools which are present for system analysis.
- It is possible to model a steel product process chain.
- In modelling the PPC it is found that a persistent nonlinearity occurs. This nonlinearity is introduced by the exchange of goods in the market: the exchange of goods is limited to the minimum of goods supplied to the market or demanded from the market. This nonlinearity makes it impossible to use the standard linear methods.
- The persistent nonlinearity cannot be avoided, but can be approximated. In contrast to earlier work, in this thesis the nonlinearity is treated by representing it by piecewise continuous functions.
- The piecewise continuous functions representation makes it possible to analyze the stability property of the product-process-chain systems.

6.1.2 Control

Modelling of product-process-chains in this thesis is done with the purpose of examining their activities with respect to environmental effects and economical costs. Because this objective is formulated imprecisely, a more workable definition of the control goal has to be defined. The assumption is made that the information of effects and costs can be reconstructed from the intermediate variables in the PPC model.

- With respect to the proposed PPC model in this thesis it is not possible to completely control all the intermediate variables. If all the intermediate variables are equally important, it is in theory not possible to control them with the proposed control instruments: price levies or artificial inventories inputs.
- With respect to some modules of the proposed PPC model it is possible to completely control all the intermediate variables.
- Nevertheless the reduced goal of controlling the exchange of the finished good market can be achieved.

6.2 Recommendations

This thesis aims at gaining insight into the use of systems and control theory for analyzing product process chains. Some ideas of problems which have to be analyzed and examined are given. In the recommendations a distinction is made between practical aspects to explore and ideas to think of.

6.2.1 Practical aspects

Some practical things to do, are:

With more data (time-series) it can be possible to obtain better insight in relevant mechanisms. It is important to obtain data (time-series) which capture the dynamics, which are relevant for the system's behavior.

The method described in chapter 5.1 has to be extended so an automation of analysis can be used.

6.2.2 Ideas

This thesis is based on PPC models in general, and the steel PPC in particular. From the steel PPC we cannot conclude that all other PPCs include same mechanism . The mechanisms in other PPCs have to be examined.

One of the essentials in making models is to select the relevant information and mechanisms which are present in the system of interest. In this thesis only the most simple relations are used to obtain insight in the behavior of the system. If the mechanisms are understood, it is possible to extend the relations to more realistic behavior. For instance a possible extension is the introduction of a the limited production capacity, or price variant inventory norm into the PPC model.

Other economic or physical control instruments, as for instance recycling ratios, have to be modelled.

The method used to analyze the stability property of the system, is based on Linear Matrix Inequalities. It has to be found out if it is possible to introduce extensions in the Linear Matrix Inequalities to take constraints as for example the positive direction of the flows, recycling ratios into account, or to compute other type of controllers.

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Appendix A

Model

A.1 Market-Inventory-Inventory-Market Model

in this section of the appendix the relations for the Market-Inventory-Inventory-Market model are given. The relations between the entries for eq. 4.18 are (see fig. 4.3):

$$\begin{aligned}
d_{1,s} &= k_2(g_{sp2} - g_2) \\
s_{1,s} &= b_{0(1)} + b_{1(1)}p_1 \\
d_{1,f} &= min(d_{1,s}, s_{1,s} + v_{1,s}) \\
q_{1,s} &= k_3(g_{sp3} - g_3) \\
q_{2,s} &= k_3(g_{sp3} - g_3) \\
d_{2,s} &= (d_{0(2)} - d_{1(2)}p_4) \\
s_{2,s} &= g_3 \\
s_{2,f} &= min(d_{2,s} + v_{2,s}, s_{2,s}) \\
g_{sp3} &= g_{sp3,0} + k_6(p_4 - p_1)
\end{aligned} \tag{A.1}$$

A short explanation (from eq. A.1) is given of these entries with the notation introduced in 4.2.1: $d_{1,s}$ the desired flow of goods demanded by the producer, $s_{1,s}$ the desired flow of resources supplied by the Supplier, $d_{1,f}$ the flow of resources from the market to the producer. With $q_{1,f}$ the flow from the inventory filled with resources to the production unit, $q_{2,f}$ the flow of finished goods which flow from the production unit to the fished goods inventory. And $d_{2,s}$ the desired amount of goods demanded by the Consumer, $s_{2,s}$ the amount of goods which the producer wants to purchase at the market, and $s_{2,f}$ the flow of products which are purchased to the consumer and extern demand. And a description of the parameters used in the model k_1, k_4 are the price adjustment parameters k_2, k_3 are proportional coefficients for inventory feedback g_{sp2}, g_{sp3} norms for the inventory with the norm of inventory of finished goods g_{sp3}, a function of a constant part $g_{sp3,0}$ and a price-variant part $k_6(p_4 - p_1)$ with k_6 a proportional coefficient for the feedback of the prices. And d_{20}, d_{21} autonomous and price-variant coefficients for the consumer $b_{0(1)}, b_{1(1)}$ autonomous and price-variant coefficient for the resource supplier $v_{1,s}, v_{2,s}$ an import of resources an export of finished goods and the states p_1, p_4 the prices on the market g_2, g_3 the amount of (resources) products in the inventories.

A.2 Product-Process-Chain Model

The relations substituted in eq. 4.25 can be found in the underlying table.

$$
\begin{array}{ll}
s_{s,s} = b_{0(1)} + b_{1(1)}p_1 & d_{p1,f} = min(d_{1,s}, s_{1,s} + v_{1,s}) \\
d_{p1,s} = k_2(g_{sp2} - g_2) & d_{p2,f} = k_3\ (g_{sp3} - g_3) \\
d_{p2,s} = k_3\ (g_{sp3} - g_3) & s_{p1,f} = k_3\ (g_{sp3} - g_3) \\
s_{p1,s} = k_3\ (g_{sp3} - g_3) & s_{p2,f} = \min(c_{0(1)} - c_{1(1)}\, p_4 + sv_2, g_3) \\
s_{p2,s} = g_3 & d_{c,f} = \min(c_{0(1)} - c_{1(1)}\, p_4, g_3\, \left(c_{0(1)} - c_{1(1)}\, p_4\right) / \left(c_{0(1)} - c_{1(1)}\, p_4 + sv_2\right)) \\
d_{c,s} = c_{0(1)} - c_{1(1)}\, p_4 & s_{c,f} = \min(d_{c,f}, (\alpha_0 + \alpha_1\, p_5)\, d_{c,f}) \\
s_{c,s} = \min(d_{c,f}, (\alpha_0 + \alpha_1\, p_5)\, d_{c,f}) & d_{r1,f} = \min(s_{c,f} \frac{d_{r_1,s}}{d_{r_1,s}+sv_3}, k_6\ (g_{sp6} - g_6)) \\
d_{r_1,s} = k_6\ (g_{sp6} - g_6) & d_{r2,f} = k_7\ (g_{sp7} - g_7) \\
d_{r2,s} = k_7\ (g_{sp7} - g_7) & s_{r_1,f} = k_7\ (g_{sp7} - g_7) \\
s_{r_1,s} = k_7\ (g_{sp7} - g_7) & s_{r_2,f} = \min(g_7, \frac{k_2\,(g_{sp2}-g_2)g_7}{sv_1+b_{0(1)}+b_{1(1)}\,p_1+g_7}) \\
s_{r2,s} = g_7 &
\end{array}
$$

The annotation from the previous section is used. the entries with $d_{p_1,s}$ the desired flow of resources demanded by the producer, $s_{s,s}$ the desired flow of resources supplied by the Supplier, $v_{1,s}$ the desired import of resources, and $s_{r_2,s}$ the desired amount of recycled resources. With $d_{p_1,f}$ the flow of resources from the market to the resource inventory, $d_{p_2,f}$ the flow of resources from the resource inventory to the production unit, $s_{p_1,f}$ the flow of produced goods to the fished products inventory, and $s_{p_2,f}$ the flow of finished products from the inventory to the market. With $d_{c,s}$ the desired demand for products, $v_{2,s}$ the desired amount of goods for export. With $d_{r_1,s}$ the desired amount of obsolete goods needed by the recycler, $s_{c,s}$ a fraction of the obsolete goods from the consumer which is supplied to the market, and $v_{3,s}$ the desired import of obsolete goods..And d_{r_1f} the flow of obsolete goods from the market to the inventory $d_{r_2,f}$ the flow of these goods which goes to the recycling unit, $s_{r_1,f}$ the flow of recycled goods going to the inventory, and finally $s_{r_2,f}$ the flow of recycled obsolete goods which will be purchased at the resource market.

with the following parameters k_1, k_4, k_5 price adaptation k_2, k_3, k_6, k_7 proportional coefficients for inventory feedback $g_{sp2}, g_{sp3}, g_{sp6}, g_{sp7}$ setpoints for inventory $c_{0(1)}, c_{1(1)}$ autonomous and price-variant coefficients for the consumer $b_{0(1)}, b_{1(1)}$ autonomous and price-variant coefficient for the resource supplier α_0, α_1 autonomous and price-variant ratio for recycling by the consumer sv_1, sv_2, sv_3 an import of resources an export of finished goods and an import of obsolete goods and the states p_1, p_4, p_5 the prices on the market g_2, g_3, g_6, g_7 the amount of (resources) products in the inventories.

Appendix B

Bond Graph theory

In this appendix a short explanation of the theory of the bonds will be given. In [PVB94] [DCKR90] [PG96] a more detailed explanation is given. The idea of using bond graphs for modelling economic systems comes from an article 'Structure and cause and effect relations in social system simulations '[Bre77]. In this article a simple example of an economic system with supply and demand is developed with the use of bond graphs. Other articles with respect to bond graph modelling for economic systems are [Bre82][JB82][Bre91].

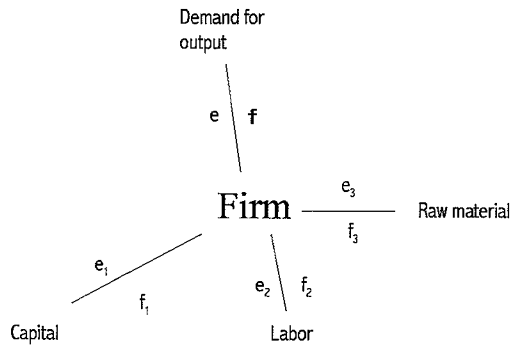

Figure B.1: a word bondgraph of a firm

B.1 Bond Graph

An attempt is made to do some system modelling on economic systems with the aid of bondgraphs. Bond graphs are graphical representations of systems. A Bond graph is composed of components that exchange energy or power through connections[PVB94]. Bond graphs can be used to model different kind of systems which manipulate energy. Choosing energy as the exchange variable for a model leads to the use of the variables called effort(e) and flow(f) and energy $E = \int e f_{dt}$ [PG96]. So far the assumption is made that the exchange variable is a form of energy(or power), this limitation can be overcome by using pseudo bond graphs or word bond graphs. The integrated product of effort and flow variables is not an energy. A lot of work is done by J.W. Brewer in the field of macroeconomic modelling using pseudo bond graphs/word bond graphs[Bre77][Bre82][Bre91].

A bond is represented by a line, the symbol(e) for effort is put above or right of the line and the symbol(f) for flow is put under or left of the line. The direction of exchange of energy(or power) is indicated by a half arrow(a full arrow indicates a signal(transport of information)). An important characteristic of a bond is the determination of causality(see fig B.2)(this is introduced for calculation of the model(physical processes do not possess causality)).

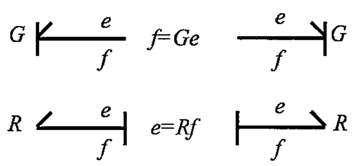

Figure B.2: causality of R and G element

The bond graphs will be primarily used for system modelling. The big advantage of bond graphs is that in the first steps of modelling a system no choice has to be made between an effort input or a flow input to the modelled system. This property will be used to model economic systems. In economic systems it is not obvious whether the price or the flow of commodities is the actor of economic systems. This can be avoided to use revenue flows, the direction of the revenue flows is easier to detect than the type of input. It has to be mentioned that bond graph modelling is not the tool for all your problems, modelled systems with many variables are hard to represent clearly and uncertainties in modelled structures or parameters can not be handled in the way differential equations handles these type of problems.

Using bondgraphs as in[Bre77][PVB94] a nice analogy is introduced between electrical, physical and economical systems. The analogy is that the idea behind this modeling is, that a flow is generated by an effort. For instance a voltage generates a current, or a hydraulic pressure generates a flow. For economical systems define revenue rate variables as

$$e = \text{unit price of commodity} \quad f = \text{commodity flow rate}$$

notice that the revenue flow rate is analogous to the power in physical systems. The revenue variables are defined as follows: *the inventory*

$$q = \int f dt$$

and the *economic impulse*

$$\lambda = \int e dt$$

notice these variables are analogous to the energy variables of physical system analysis.

According to Walras theory dynamic markets wherein the supply flow and demand flow are not equal, changes in the unit price of commodity is proportional to the difference between these flows[Bre77].

$$\frac{de}{dt} = \frac{1}{C}(f_d - f_s)$$

wherein C is a proportionality constant which is called *economic compliance.* This equation can be transformed in an equivalent form(the integral of excess supply is the inventory q which must be stored by the supplier.

$$e = \tfrac{1}{C}q \text{ or } \tfrac{d\lambda}{dt} = \tfrac{1}{C}q$$

wherein λ is the *economic impulse.*

According to Marshall the rate of change of commodity flow is proportional to the difference between the supplier's price e_s and the demander's price e_d[Bre77].

$$\frac{df}{dt} = \frac{1}{I}(e_d - e_s)$$

where I is a constant which is called the *economic inertia*. The equation can be written in equivalent forms

$$f = \frac{1}{I}(\lambda_d - \lambda_s) \text{ or } \frac{dq}{dt} = \frac{1}{I}(\lambda_d - \lambda_s)$$

Possibilities with bond graph modelling are like for instance fig. B.1[Bre77].

B.1.1 Bond Graph Modelling

Now with the basic relations an attempt can be made to model an (economic)system. The common bond graph elements can be found in table B.1. An electrical schematic can be transformed in a bond graph by following these steps[PG96]:

1. draw a '0' junction for each point in the schematic where parallel path coincide.
2. Draw a'1' junction for each component on a series path, and attach the appropriate bond graph component to that junction.
3. Draw bonds between adjacent junctions(direction of power flow).
4. Remove the '0' junction representing the reference point and remove all bonds attached to this point.
5. Remove any remaining two-port junctions and move attached nodes to the adjacent junction.

Applying these five steps to an electric schematic with two resistors and two capacitors will result in the following bond graph(see fig B.3).

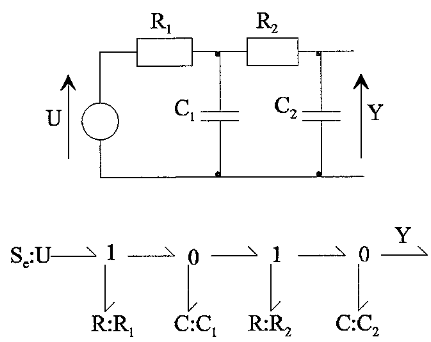

Figure B.3: bond graph equivalent of an electric schematic

B.1.2 Causality

The junctions of a bond graph constrain the possible causalities of the components of the bond graph

- One, and only one, bond connected to an 0-junction has an effort input(volgens mij klopt dit niet)
- One, and only one, bond connected to an 1-junction has a flow input

Symbol	Name	Function	Example
Sources:			
$S_e \overset{e}{\underset{f}{\text{———}}}$	e-source	e	u-source
$S_f \overset{e}{\underset{f}{\text{———}}}$	f-source	f	i-source
One-ports:			
$\overset{e}{\underset{f}{\text{———}}} R$	resistor	$e = Rf$	resistor
$\overset{e}{\underset{f}{\text{———}}} G$	conductivity	$f = Ge$	resistor
$\underset{f}{\text{———}} C$	capacity	$e = \frac{1}{C}\int f dt$	capacitor
$\overset{e}{\underset{f}{\text{———}}} I$	inertia	$f = \frac{1}{I}\int e dt$	coil
Two-ports:			
$\overset{e_1}{\underset{f_1}{\text{———}}} \mathrm{TF} \overset{e_2}{\underset{f_2}{\text{———}}}$	transformer	$e_2 = me_1$ $f_1 = mf_2$	traqnsformer
$\overset{e_1}{\underset{f_1}{\text{———}}} \mathrm{GY} \overset{e_2}{\underset{f_2}{\text{———}}}$	gyrator	$e_2 = mf_1$ $e_1 = mf_2$	DC-motor
Multiports:			
$f_3 \mid e_3$ $\overset{e_1}{\underset{f_1}{\text{———}}} 0 \overset{e_2}{\underset{f_2}{\text{———}}}$	0-junction	$e_1 = e_2 = e_3$ $f_1 + f_2 + f_3 = 0$	Parallel
$f_3 \mid e_3$ $\overset{e_1}{\underset{f_1}{\text{———}}} 1 \overset{e_2}{\underset{f_2}{\text{———}}}$	1-junction	$f_1 = f_2 = f_3$ $e_1 + e_2 + e_3 = 0$	Serial

Table B.1: Bond Graph elements

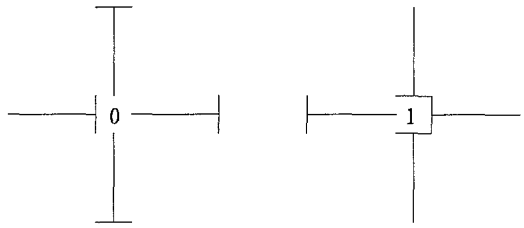

Figure B.4: junction causality

motivation is that a zero-junction represents a parallel circuit with all efforts having the same value, so there can only be one effort input. A 1-junction represents a serial circuit where all the flows have the same value, so only one flow can be the input.

An analogy to the energy conservation laws is Walras'Law, which state that the sum of value rates into a port is zero. In economic bond graphs '1' junctionsB.1 are used to describe points at which several costs are added to give the overall cost of the item, but the flow of items on each bond is identical(so *transformers* have to be used if the item exists of non identical quantities).

With the use of word(pseudo)bond graphs a firm is modeled[Bre77](see fig B.1). With this modelling of the firm two constraints have to be used, the first is the constraint called the law of good bookkeeping, namely: $ef = e_1f_1 + e_2f_2 + e_3f_3$(with $e =$ the price of the commodity and $f =$ the flow of orders of the commodity) and the second constraint is the production function $f = f(f_1, f_2, f_3)$ (a commonly used function is the Cobb-Douglas function form $f = kf_1^{\gamma_1} f_2^{\gamma_2} f_3^{\gamma_3}$).

To conclude this appendix applying bond graph theory to economic systems as suggested by [Bla98] is at this moment not useful, because it can not cope with large systems and modelled systems with many variables are hard to represent clearly and uncertainties in modelled structures or parameters can not be handled in the way differential equations handles these type of problems.