Modelling and control of a system with magnetic hysteresis

Lambregts, J.A.L.M.

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University of Technology Eindhoven
Department of Electrical Engineering
Measurement and Control Section

Modelling and Control of a system with magnetic hysteresis

by J.A.L.M Lambregts

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Abstract

In this report research is done on the effects of magnetic hysteresis on a control system. A number of different hysteresis models are being investigated. To illustrate the effect of hysteresis on a control system we will consider a magnetic levitation system. A controller for such a system is designed and the effect of hysteresis on a control system is evaluated.

Contents

1) Introduction

In magnetic bearings and actuators, which makes use of electromagnets, the phenomenon hysteresis plays a role. This magnetic hysteresis has a negative influence on the performance of magnetic bearing. A magnetic hysteresis is a dynamical multi-valued non-linear process (See definition in chapter 2)

We want to study what the effect is of a magnetic hysteresis on a system. Therefore we need a model of magnetic hysteresis, so we can look at the complete process, e.g. for simulation purposes. We will consider as an illustrative example a magnetic ball levitation system. (see [9, [10], figure 1.1)

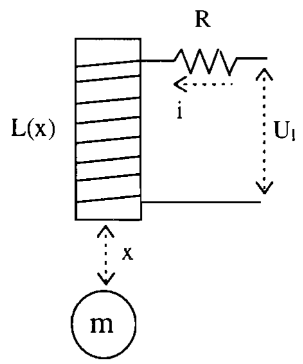


figure 1.1 a magnetic levitation system.

The system consists of an electromagnet to levitate the steel ball, an actuator to supply the voltage to the coil, a position sensor for measuring the position of the ball, and a controller. The control goal is to keep the ball levitated in one exact position. We want to study the negative influence on the performance caused by hysteresis, we also like to study possible methods to circumvent these problems. In other words how can we built a controller that counteracts on the negative influence of the magnetic hysteresis.

In chapter 2 we will give a formal definition of hysteresis. In chapter 3 we will give the magnetic properties of an electromagnet. In chapter 4 we will describe a number of models for a magnetic hysteresis. Furthermore we describe the advantage and disadvantage of the hysteresis models. In chapter 5 we will describe the effect of a hysteresis in the system. How to deal with hysteresis, and what can be done against the negative effects. In chapter 6 we will give some conclusions about the use of a system with magnetic hysteresis.

2) The definition of hysteresis

In this chapter we will formally define hysteresis. The phenomenon of hysteresis is encountered in many different areas, for example ferroelectric hysteresis, mechanical hysteresis, super conducting hysteresis, absorption hysteresis, optical hysteresis, etc. First a formal definition of hysteresis is given:

Definition: *A transducer H(t) (with an input x(t) and an output y(t) (see figure 2.1.a)) is called a hysteresis transducer if its input-output relationship is a multi valued non-linearity for which branch to branch transition occurs after input extrema (see fig 2.1.b).*

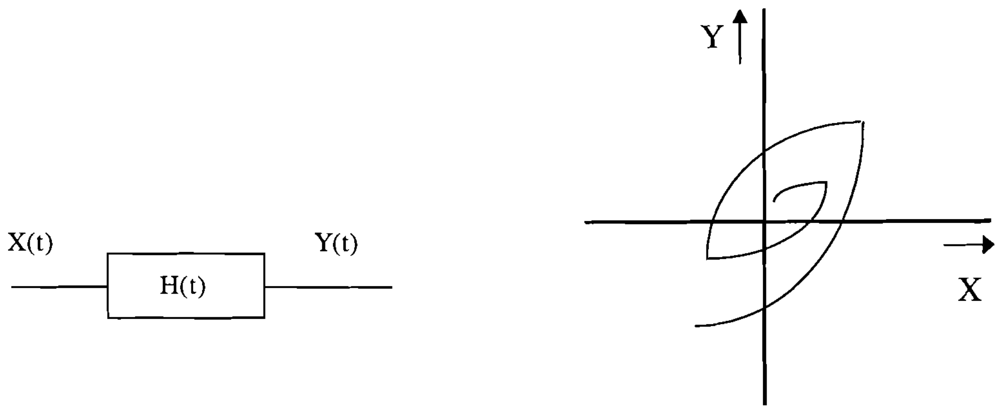


fig 2.1 :a)Hysteresis transducer b)Branch to branch transition

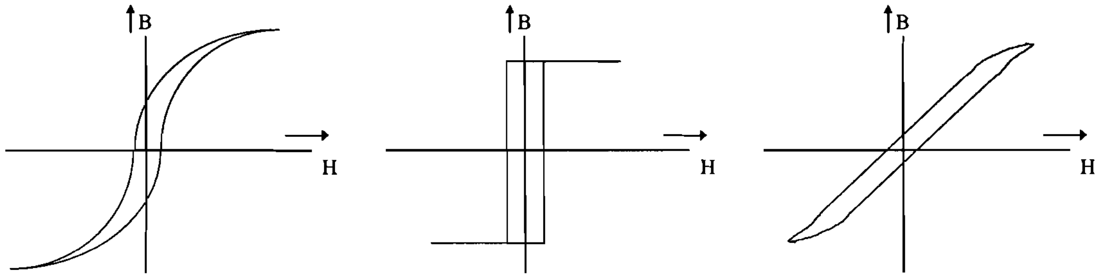
The hysteresis has the following properties:

Properties : *The future value of output y(t), (t>t0) depends on the current value of the output y(t0), the past extremal values of input x(t)) and the speed of input variations between the extremal points. In other words the past exerts in the influence upon the future through the current value of the output.*

In the existing literature the hysteresis is largely linked with the formation of hysteresis loops. This may be misleading and creates the impression that looping is the essence of hysteresis. Looping occurs when the input varies back and forth between two consecutive extremal values, while branching takes place for arbitrary input variations. The hysteresis width increases if the speed of input variations between the extremal points also increases. This dynamical effect is called loop widening.

3) Hysteresis

There are 3 common hysteresis shapes (see figure 3.1.a , b, c). The S shaped hysteresis (for example Mu metal) is the most common one. This hysteresis shape we will use in this chapter.



a) *b)* *c)*

figure3.1.a) S shape hysteresis (example Mu metal) b) square shape hysteresis (example Permenorm 5000 Z) c)flat shape hysteresis (example Ultraperm F)

3.1) The initial magnetisation curve

Assume that we have a coil with a kernel, we apply an external field with a field strength H=0 and the field density B at that moment is zero. As H increases, B rises slowly at first, passes through a steeper region and then grows more slowly with increasing H, finally remaining constant as the field strength (H) increases further. With increasing H the material saturates. (see figure 3.3.1.a) The following regions of the curve may be distinguished :

1. Slow increase of flux density from O to A
2. Steep rise of flux density between A and B
3. Falling steeples between B and C
4. Saturation region between C and D

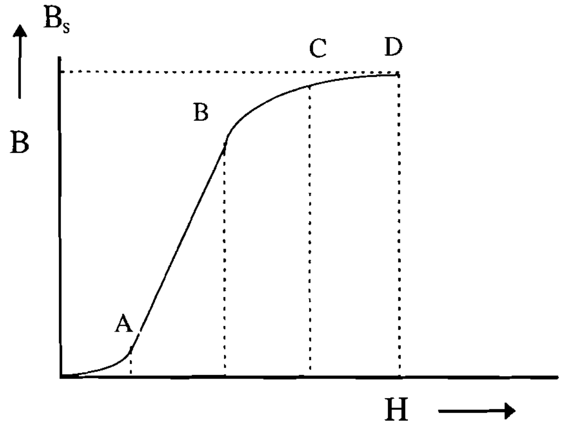


figure 3.3.1 a) Initial magnetising curve.

3.2) The limiting loop

If from the saturated state of the material one reduces the field strength (H), then the value of the field density (B) decreases not along the initial magnetisation curve but the decrease will be slower. For H=0, B still retains a positive value namely the *remanence* B_r (see figure 3.4.1). If the negative field strength is increased to saturation, the negative flux density (B) reaches saturation. The curve closes on itself if H is again brought to zero and then raised to the positive saturation value. At each further repetition of this cycle the curve always follows the same path. It is only possible to get back to the initial magnetisation curve if the material is demagnetised, as by heating above the curie temperature. The field strength necessary for making the flux density (B) zero is called *coercivity field strength* (H_c). In the limiting loop also called the mayor loop, only regions B, C and D of the magnetisation curve are found. The region A does not appear.

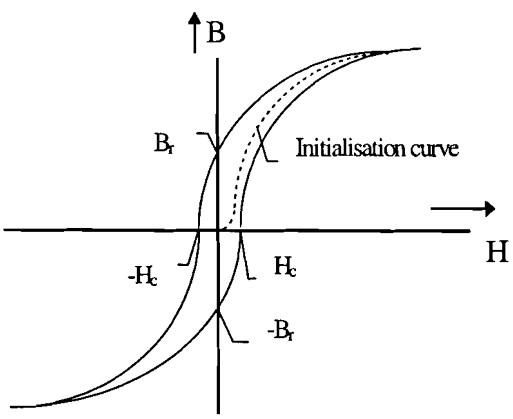


figure 3.4.) Limiting or mayor hysteresis loop

4) Modelling of hysteresis

We have searched the existing literature for magnetic hysteresis models. Our goal was to find magnetic hysteresis models that are accurate enough, and are not too complex to be simulated (like Stoner wohlfart and Preisach models) in SIMULINK. This is because these magnetic hysteresis models have to used in combination with other SIMULINK models. We have found 3 magnetic hysteresis models that have these properties, these models we will described in this chapter.

Two general approaches have been used in attempting to model hysteresis.

The first approach, is to determine the physical laws that govern the system. This approach will generally require the use of partial differential equations, for example Maxwell equations. Because the complexity of the equations, and the lack of clear understanding of the exact interaction of the physical processes involved, this approach has not provided very accurate models, even in the case of an iron core inductor where physical process are reasonable well understood.

The second approach consists of using a suitable mathematical representation which can be shown to exhibit hysteresis phenomena. It is difficult to establish the validity of a non-linear model. We do this by showing that the used model exhibits the same significant properties as the hysteresis and then verifying that the model gives realistic responses to one or more test signals. It is then assumed that since the model has the same qualitative characteristics, the model must give reasonable accurate responses to other excitations.

4.1) The Chua and Stromsmoe model

This model allows a wide control over its behaviour with varying frequency by the simple adjustment of one non-linearity. (see [4], [5]). A hysteresis loop can be exhibited with or without loop widening at higher frequencies. Thus if loop widening (with increasing frequencies) is of no concern in the problem under consideration, it can be completely eliminated. Otherwise, loop widening can be included and can be made to take affect beyond any desired frequency.

The approach of this model is to match only the external observed facets of hysteresis with appropriate mathematical counterparts. This is done partly because the physics of most hysteresis phenomena are so poorly understood. The equivalence between a model and its physical counterpart is established by observing responses to some of the more likely input signals. From these you can point out some of the model strength and weaknesses.

The Hysteresis model

$$\frac{\partial y(t)}{\partial t} = w\left(\frac{\partial x(t)}{\partial t}\right) * h(y(t)) * g(x(t) - f(y(t))) \qquad \textbf{(4.1.a)}$$

is a relation between two real valued time functions x(t) and y(t), (where x(t) is the input of the hysteresis model and y(t) is the output (see figure 2.1.a)). The functions *f*,*g*, *h* and *w* are real valued functions defined on the real axis $\Re$. $C^k(\Re)$ is the space of k times continuous differentiable functions. The three functions are assumed to satisfy the conditions,

I) g, f, h $\in C^1(\Re)$

II) g'>0, f'> 0 on $\Re$

III)f, g, h: $\Re \rightarrow \Re$ onto

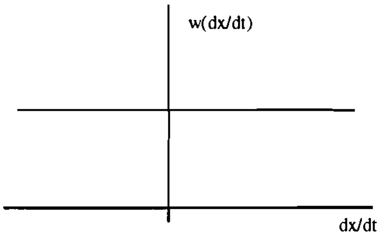
IV) 0< a <= h <= b < ∞ on $\Re$

Where the *'prime'* denotes differentiation with respect to the function's argument, and where *a* and *b* are positive constants. The function *g* will be referred to as dissipation function, the function *f* as the restoring function, the function *h* as the weighting function, and the function *w* will be called frequency dependence function. From a physical point of view the function *g* is responsible for energy dissipation, while *f* is responsible for energy storage. The mathematical properties are discussed in the literature ([4], [5]).

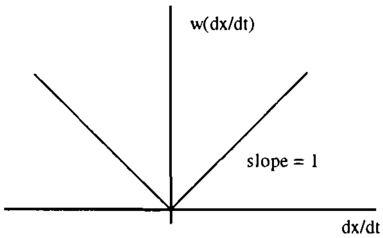
Determination of the model parameters :

The model as represented by equation (4.1.a) is able to exhibit most of the important properties observed in practice. Hence the parameters of the model (i.e. the functions w, f, g, h) can be tuned to reflect a given hysteresis. As an example we illustrate the influence of function w (.) on the hysteresis.

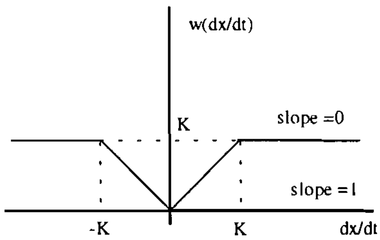
Several possible functions w are shown along with brief descriptions of their effects upon the frequency behaviour. Note: ω is the frequency of x(t)



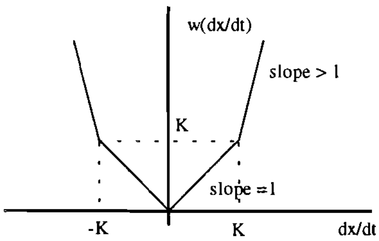
A) This function allows loop widening by increasing frequency. The loop collapse as ω = 0.



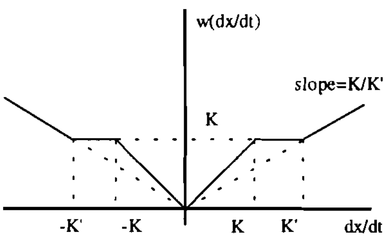
B) This function completely eliminates effect of frequency variation on the hysteresis loop. The loop at a frequency at ω = 0 is identical to that at ω = ∞. This function can be used if loop widening is not needed.



C) At low frequencies the model is frequency insensitive. The loop is unchanged as ω = 0. However as the frequency increases to the point where abs(dx/dt) begins to exceed K, loop widening with increased frequency appears. If x(t) sinusoidal, say x(t) = A.cos(ωt) then the threshold frequency of loop widening is K/A.



D) At low frequencies, the loop is insensitive to frequency variations. As frequency increases such that abs(dx/dt) >K, loop narrowing with frequency increases occurs.



E) At low frequencies, the loop is insensitive to frequency variations. At moderate frequencies, loop widening occurs with increased frequency. At still higher frequencies, loop widening becomes progressively frequency insensitive.

For the influence of the other model parameters (i.e. f, g, h) we refer to [4,5].

To show the versatility of the Chua Stromsmoe model, two physical hysteresis loops are modelled here. We consider first the hysteresis loop of a tape wound core of 50-50 percent NiFe (Orthonol). This material produces extremely square curves shown in figure 4.1.1(a) at several frequencies. Sketches of the non-linear functions used to model these loops are given in fig 4.1.1(b). The function *w* was of *type c*

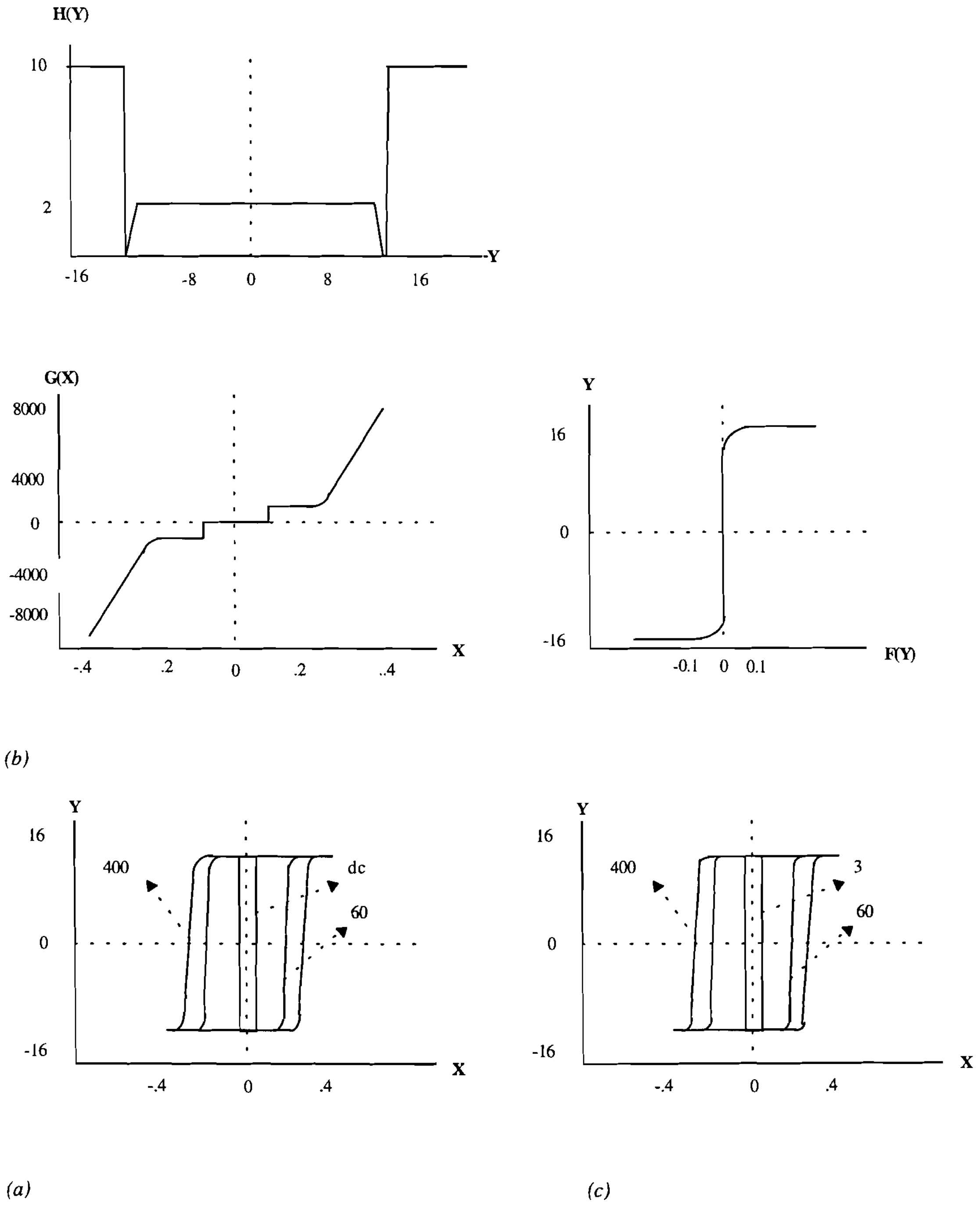
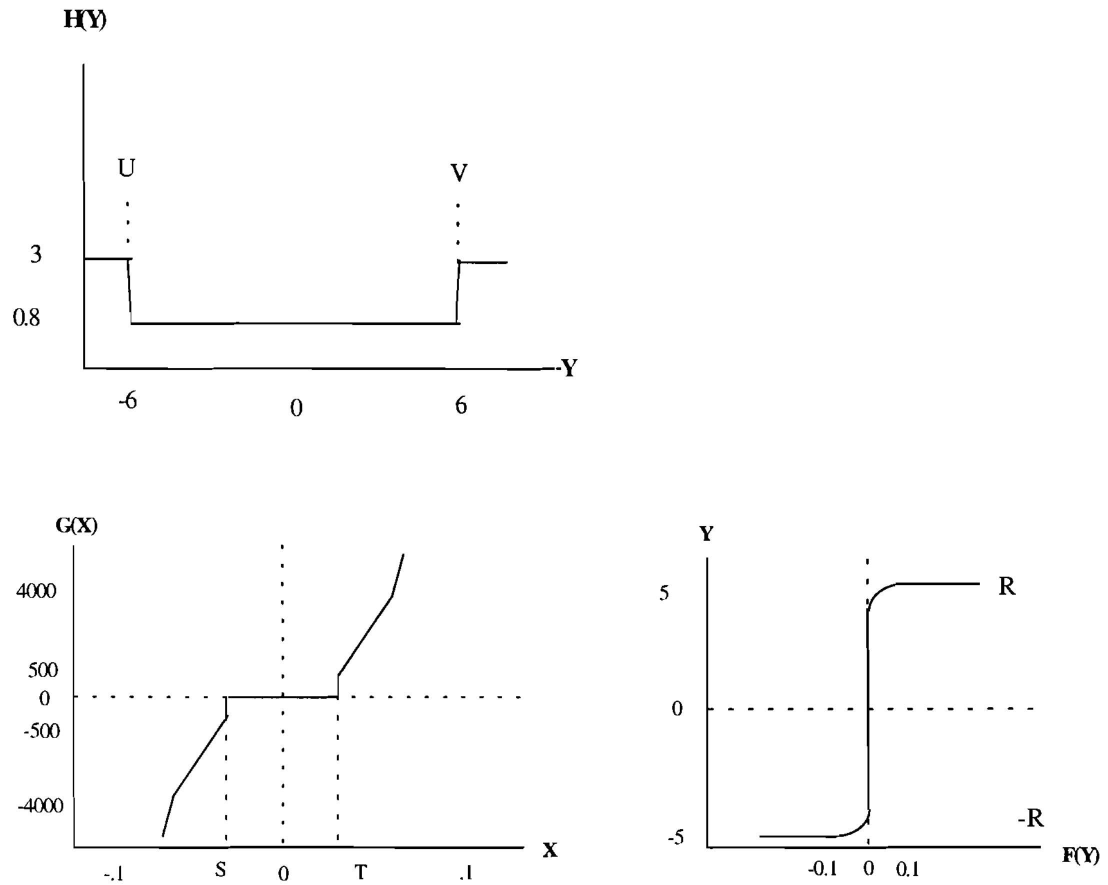
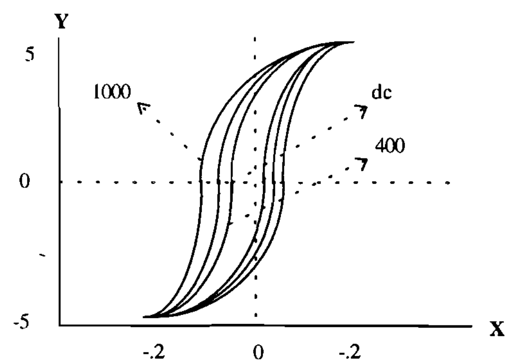


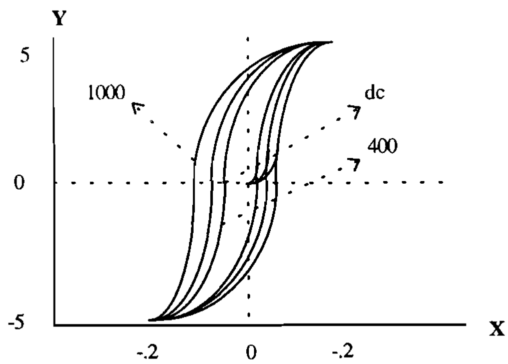
figure 4.1.1)Hysteresis loops at various frequencies for 50-50 percent NiFe. a) Manufacture's loops. b) Non-linearity's used. c) Loops provided by model.

The next example is a loop for a tape wound core of 21-79 percent Mb-Permalloy. The manufactures saturation loops are given in fig 4.1.2(a). The non-linearity's of fig 4.1.2(b) produced the model curves of fig. 4.1.2(c). The function *w* was of *type c*



(b)





(a) *(c)*

figure 4.1.2) Hysteresis loops at various frequencies for 21-79 percent Mb-Permalloy. a) Manufactures loops. b) Non-linearity's used. c) Loops provided by model.

Here equation 4.1.a is rewritten to represent the hysteresis model in B-H form:

$$\frac{\partial B(t)}{\partial t} = w(\frac{\partial H(t)}{\partial t}) * h(H(t)) * g(B(t) - f(H(t))) \qquad (4.1.b)$$

We use equation 4.1.a to show an illustrative example of a B-H hysteresis loop: (see figure 4.1.4)

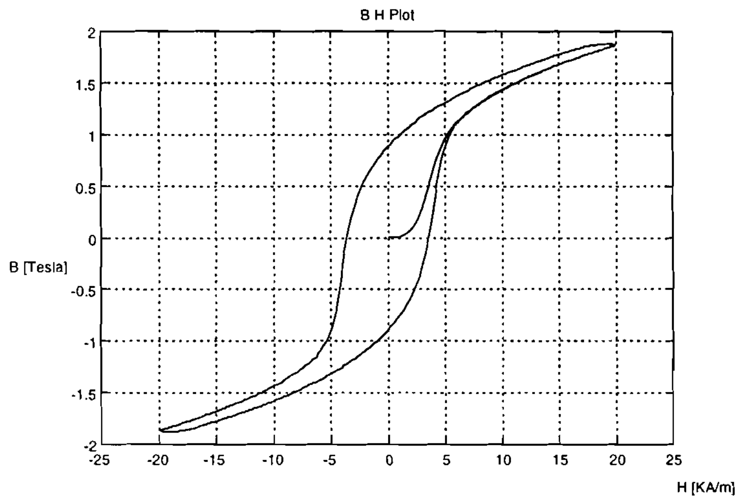


figure 4.1.4) An illustrative example of a B-H hysteresis loop

The mathematical model has been simulated using SIMULINK. The simulation model is represented in figure 4.1.5

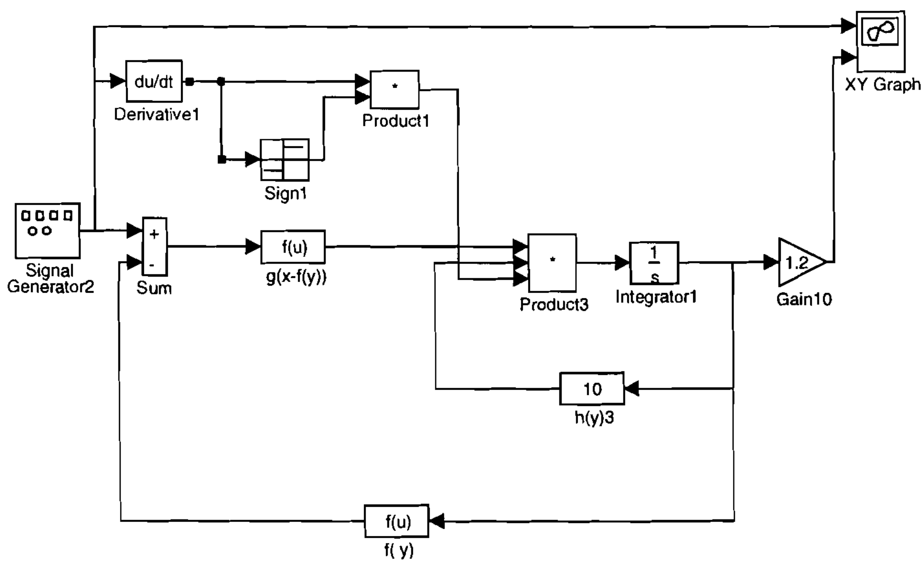


figure 4.1.5) The simulation model of Chua and Stromsmoe

4.2)The Jiles and Atherton Model

An equation describing the hysteresis loop of ferromagnetic materials has been derived theoretically based on the mean field approximation. (see [2],[3],[6])

The hysteresis is represented by the following equation :

$$\mathbf{B} = \mathbf{Bs} * \coth\left(\frac{\mathbf{B} + \lambda\mathbf{H}}{\mathbf{a}}\right) - \frac{\mathbf{B} + \lambda\mathbf{H}}{\mathbf{a}} - \frac{\mathbf{k}}{\mu 0} \qquad \textbf{(4.2.a)}$$

Examples of different values of the parameters are given in figure 4.2.1 and 4.2.2 showing the generation of families of major hysteresis loops. Further more the model can be extended to include minor loops.

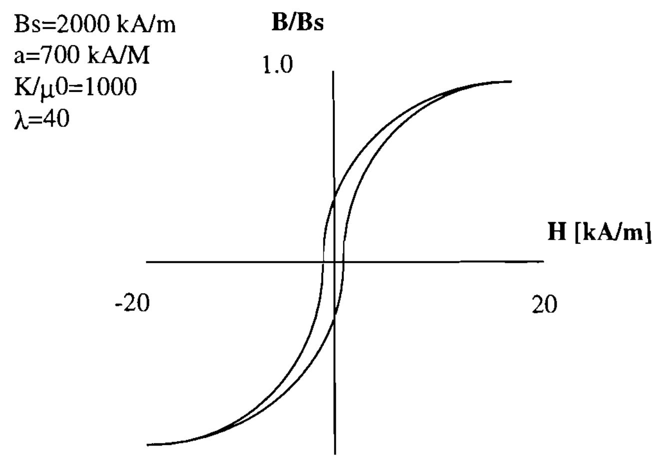


fig 4.2.1) Solutions of the model equation for the given values of the parameters

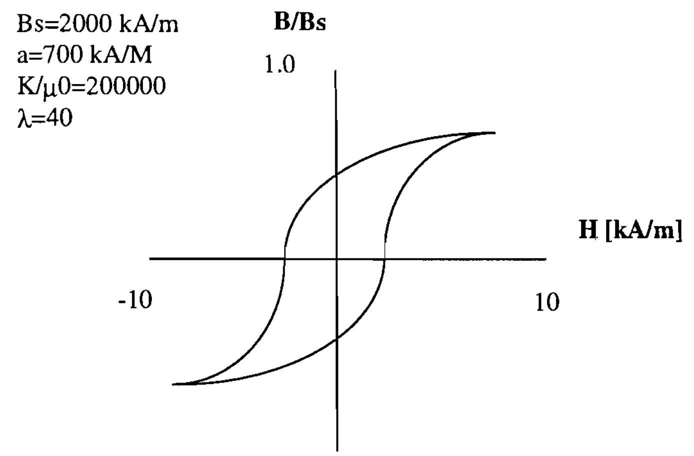


fig 4.2.2) Solutions of the model equation for the given values of the parameters

This hysteresis model has a simple structure. One can adjust the parameters Bs, a, λ, and (k/μ_0) and as a consequence of this simple structure the model is straightforward..

A hysteresis curve obtained by this model is shown in figure 4.2.3.

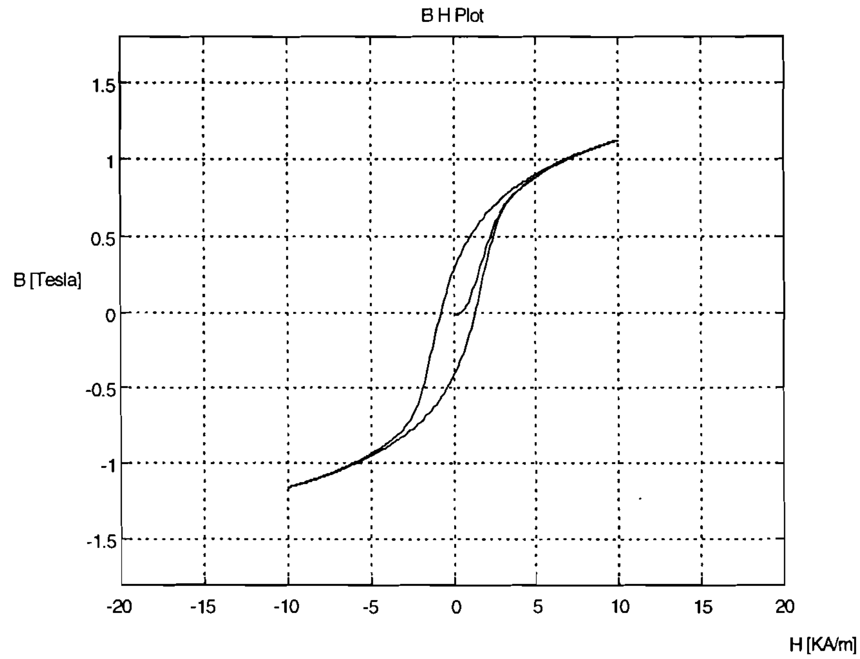


figure 4.2.3) An illustrative example of a B-H hysteresis loop

The mathematical model is simulated using SIMULINK. The simulation model is represented in figure (4.2.4)

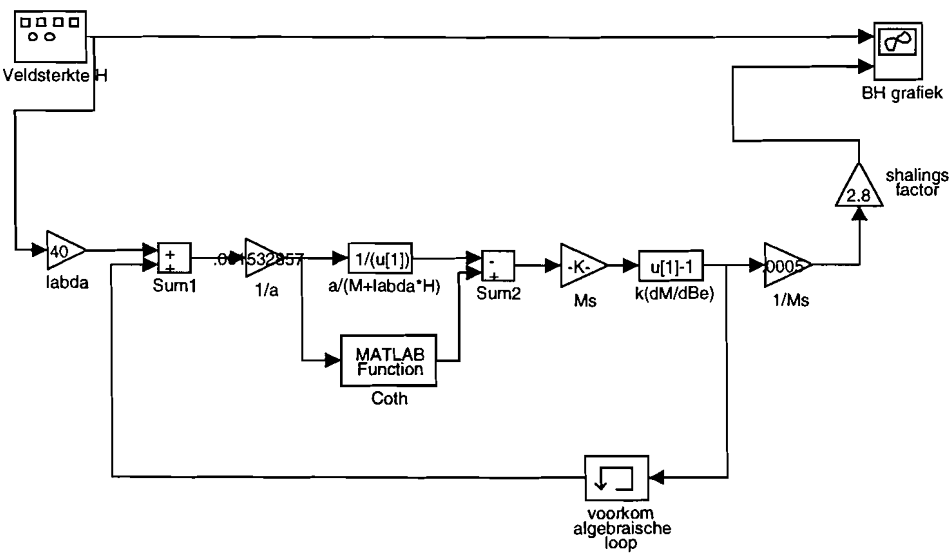


figure 4.2.4)The simulation model of Jiles and Atherton

4.3) The Karnopp Model

In the Karnopp model is a strong analogy between mechanical structural elements and magnetic circuit components (see [1]). Procedures for developing finite state models of hysteresis components are presented by using a small number of basic elements (see figure 4.3.1 and 4.3.2). Certain magnetic and mechanical variables play analogous roles, if the variables are ordered according to the scheme of table 1

	General variable	*Mechanical variable*	*Magnetic variable*
Effort	e	Force, F	magnetomotive force, M
Flow	f	velocity, v = $\dot{x}$	flux rate, $\dot{\phi}$
Displacement	q	position, x	flux, ϕ

Table 1

In [1] mechanical elements are used to build a hysteresis model. This mechanical hysteresis model can be translated into a magnetic hysteresis model with the help of table 1.

This is the simplest model that has been used to characterise a hysteresis model (proposed by Voigt and Maxwell). This model consists of two *types* of elements, a spring and a dash pot. By coupling a number of this basic elements we are able to build a hysteresis model. This model requires resistive elements because energy is dissipated and capacitive elements because extra state variables are required to record loading history.

The first element can be represented as Coulomb friction element (see figure 4.3.1).

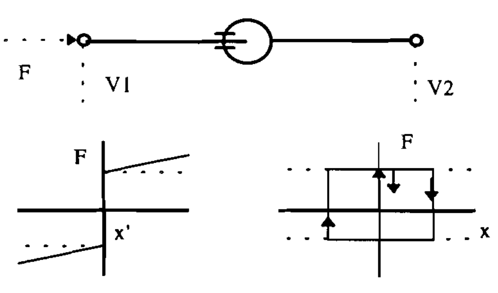


figure 4.3.1 Coulomb friction element

This element has a resistive character, any curve lying in the first and third quadrant results in a energy dissipation. The particularity of a Colomb friction is that for speed $\dot{x}$ within a band around zero no force (F) exists. The slight slope of the graph of F versus $\dot{x}$ for finite values of $\dot{x}$ is necessary to accommodate the causality. The excessively flat scopes causes numerical problems in simulation while too steep slopes will result in excessively slow reactions to dynamic changes. The slope adds some viscous friction to the hysteresis system, and in a simulation a compromise value of slope must be chosen. If the force (F) is plotted against x instead of $\dot{x}$, a square edged hysteresis is found. If x is cycled, the area in the resulting loop represents the energy lost. This simple basic element in combination with capacitive and connection elements, can represent a wide variety of mechanical and magnetic hysteresis elements.

A second special element is the limit spring (see fig 4.3.1) .

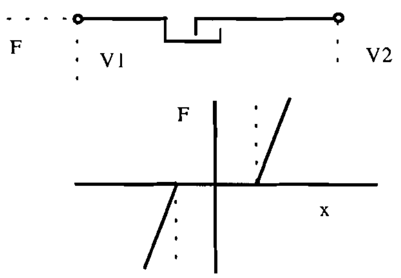


figure 4.3.2 limit spring element

Basically it is just a special form of non-linear capacitor which generates no force until a relative displacement exceeds some limits. Again the steep of the slope must be chosen as a compromise. Very steep slopes will cause numerical problems while shallow slopes will not limit the relative excursion effectively. The two slopes act together to produce a R-C time constant which may be the limiting factor in choosing the step size for integration in a digital simulation. If the slopes are chosen with care the hysteresis model will act in a nearly ideal fashion and the required time step will not be unduly small.

A hysteresis model is made up of these basic elements. (see figure 4.3.3) This model can be extended, this is done by placing a Colomb friction element parallel with a limit spring element, and place this 2 elements between B1 and B2. (see figure 4.3.3)

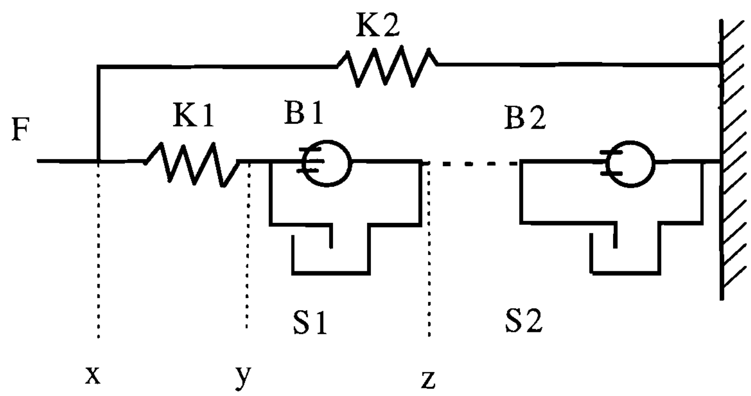


figure 4.3.3 dynamic hysteresis model

This hysteresis model can be made very accurate. The accuracy depends on the number of basic elements that are used. More basic elements makes the hysteresis model more accurate and complex. The amount of calculation time for simulation changes, with the complexity of the model. The model allows loop widening. A procedure to obtain hysteresis models using this method is described in [1].

4.4) Comparing the hysteresis models

We will shortly describe the advantages and disadvantages of the hysteresis models, and make a comparison between the models.

The Chua and Stromsmoe model

advantages:

1. We have an influence on the loop widening effect by choosing an appropriate function (w).
2. There are 4 functions (w, g, h, f) to choose, and these functions give us a lot of freedom to shape the hysteresis loop.
3. The amount of calculation time for simulating the model doesn't change, with increasing complexity of the 4 functions (w, f, g, h).

disadvantages:

1. We have a lot of freedom for choosing the specific functions (w, g, f, h), this freedom makes it complex to choose the right functions.

The Jiles Atherton model

advantages:

1. The model structure is simple, only a few parameters to adjust.
2. The amount of calculation time for simulating the model doesn't change, with different value's of the parameters.

disadvantages:

1. The model is not very accurate.
2. The model has no loop widening effect.

The Karnopp model

advantages:

1. The model can be made very accurate. The accuracy depends on the number of basic elements that are used.

disadvantages:

1. The amount of calculation time for simulating the model changes, with the complexity of the model. More basic elements makes the hysteresis model more complex.

We will now make a comparison between the models:

accuracy:

The Karnopp model and the Chua and Stromsmoe model are definitely more accurate then the Jiles Atherton model .

model structure:

The Jiles Atherton model has the simplest structure. The structure of the Karnopp model is also quite straightforward. While we have the most freedom (the freedom of choice of the functions (w, f, g, h)) in the Chua Stromsmoe model, this implies that the structure of the Chua Stromsmoe model is the most complex.

calculation time for simulating the model:

The Jiles Atherton model, has a constant calculation time for simulation. This calculation time does not depend on the specific parameter values. The Chua and Stromsmoe model, has also a constant calculation time for simulation. The calculation time for simulating the Karnopp model, depends linear on the number of basic elements that are chosen.

5) Controlling a system with hysteresis

5.1) A magnetic levitation system

We want to analyse the effect of hysteresis in a system, that is to be controlled. We will consider as an illustrative example a magnetic ball levitation system (see figure 5.1.1). A magnetic levitation system consists of an electromagnet to levitate the ball, an iron ball, a number of sensors, and an actuator. The control goal is to keep the steel ball levitated at a specific position (see [9]). Assume that the iron ball moves only in a vertical direction. For the hysteresis model we use the Chua and Stromsmoe hysteresis model. There are a number of reasons why we have chosen this model:

- We have an influence on the loop widening effect by choosing an appropriate function (w).
- There are 4 functions (w, g, h, f) to choose, and these functions give us a lot of freedom to shape the hysteresis loop.
- The amount of calculation time for simulating the model doesn't change, with increasing complexity of the 4 functions (w, f, g, h).
- For this simulation purpose the hysteresis model is accurate enough

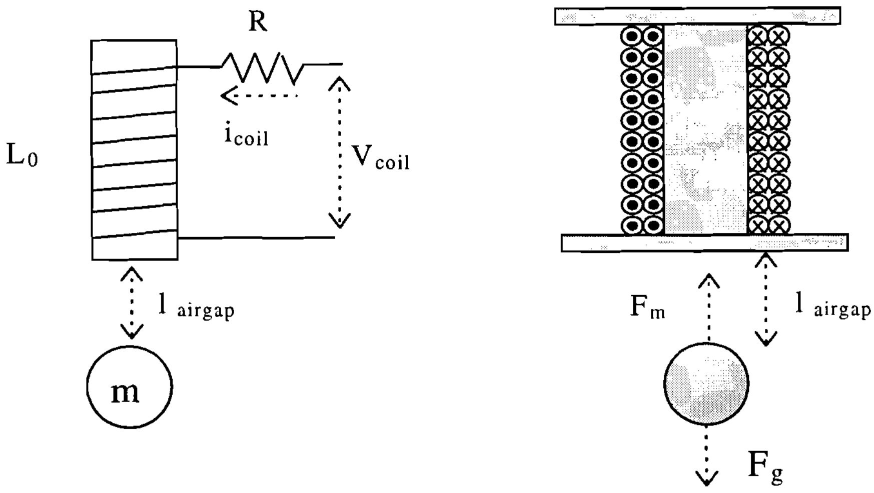


figure 5.1.1)A magnetic ball levitation system

The dynamic equations (see[9]) of the system are:

$$\mathbf{m\ddot{l}_{airgap} = F_g - F_m} \qquad \textbf{(5.1.a)}$$

$$\mathbf{V_{coil} = i_{coil} * R + \partial(i_{coil} * L)/\partial t} \qquad \textbf{(5.1.b)}$$

where

m = the mass of the levitated ball [kg]

l_{airgap} = position of the levitated ball [m]

F_m = magnetic reluctance force [N]

F_g = gravitational force [N]

R = the resistance of the coil [Ω]

i_{coil} = the current through the coil [A]

L = the inductance [H]

To calculate the reluctance force (F_m) we make the following assumptions:

1) The field in the air gap is homogeneous (see figure 5.1.2.).

2) The magnetic energy outside the air gap is not influenced by changing the position of the ball and is proportional to the acquired coil current (i^2). Although the magnetic circuit outside the air gap is poorly defined, this assumption only holds for large coils.

3) No eddy currents occur caused by changing magnetic fields, when sintered or laminated materials are used.

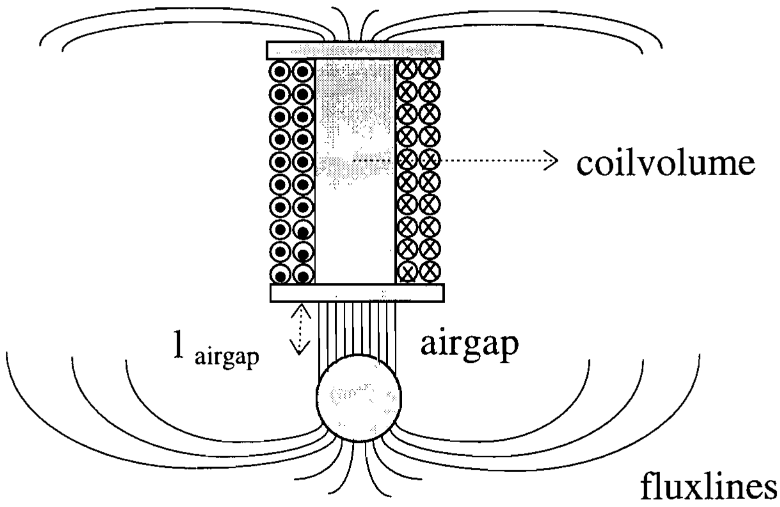


figure 5.1.2) The magnetic flux lines

Given these assumptions we can calculate the magnetic reluctance force (F_m) as follows:

$$\mathbf{F_m = \partial W/\partial l_{airgap}} \qquad \textbf{(5.1.c)}$$

Where W is the magnetic energy in the system, l_{airgap} is the distance between the coil and the ball, and L is the induction of the coil, the ball and the air gap. For the magnetic energy (W) we can write:

$$\mathbf{W=\iiint B\ H\ dV} \qquad \textbf{(5.1.d)}$$

where

B = magnetic field density [T]

H = magnetic field strength [A/m]

V= system volume [m^3]

μ = permeability of the material [H/m]

We can divide the system volume in three parts. A part inside the coil, where the magnetic field (H) is homogenous. A second part, the air gap, where we assume that the magnetic field (H) is homogenous. In the rest of the volume (this is the volume where flux lines caused by the coil, flow through, with exception of the coil and the air gap), the magnetic circuit is poorly defined and an unknown amount of magnetic energy is lost. Together with the assumptions made earlier, this implicates for the total magnetic energy (W):

$$W = \iiint_{airgap} B_{airgap} * H_{airgap} \, dV_{airgap} + \iiint_{iron} B_{iron} * H_{iron} \, dV_{coil} + W_{restvolume} \quad (5.1.e)$$

The amount of energy in the rest volume is not dependent on the position of the levitated ball and is proportional to i^2. The total energy in the air gap and coil can be derived as follows:

$$N * i_{coil} = \oint H dl \, , \quad (5.1.f)$$

which implicates:

$$N * i_{coil} = H_{airgap} * l_{airgap} + H_{coil} * l_{coil} + \int_{line} H \, dl \quad (5.1.g)$$

As the field in the coil is assumed to be homogenous, we conclude:

$$B_{coil} = B_{airgap} \quad (5.1.h)$$

Further we assume that the field strength in the ball and outside the coil is equal to:

$$\int_{line} H \, dl = \alpha * N * i_{coil} \quad (5.1.i)$$

and that the field strength in the coil is,

$$H_{coil} * l_{coil} = \beta * N * i_{coil} \quad (5.1.j)$$

where α and β can only be determined by experiment. Equations (5.1.h), (5.1.i) and (5.1.j) result in:

$$H_{airgap} = N * i_{coil} * (1-\alpha-\beta) / l_{airgap} \quad (5.1.k)$$

Now that we have found an expression for H_{airgap}, we can calculate the integral of (5.1.d) to determine the total magnetic energy (W) of the magnetic levitation system:

$$W = A_{ball} * B_{coil} * N * i_{coil} * (1-\alpha-\beta) / l_{airgap} + \gamma * (N * i_{coil})^2 \; [J] \quad (5.1.l)$$

With equation (5.1.c) we can calculate the reluctance force (F_m):

$$\mathbf{F_m = A_{ball} * B_{coil} * N * i_{coil} * (1\text{-}\alpha\text{-}\beta)/\ l_{airgap}^{\ 2}} \qquad \mathbf{[N]} \qquad \mathbf{(5.1.m)}$$

where

N = number of coil turns

A = cross sectional area of the coil [m^2]

l_{airgap} = average length of the airgap [m]

α = constant to be determined by experiment

β = constant to be determined by experiment

γ = constant to be determined by experiment [H]

Suppose we have no hysteresis, then we can substitute for B_{coil} :

$$\mathbf{B_{coil} = \mu_{coil} * H_{coil}} \qquad \mathbf{(5.1.n)}$$

$$\mathbf{B_{coil} = \mu_{coil} * i_{coil} * N/l_{coil}} \qquad \mathbf{(5.1.o)}$$

With the formula's (5.1.0 and 5.1.m) we can derive both the magnetic reluctance force (F_m) and the coil induction (L):

$$\mathbf{F_m = .5 * i_{coil}^{\ 2} * Q/\ l_{airgap}^{\ 2}} \qquad \mathbf{[N]} \qquad \mathbf{(5.1.p)}$$

$$\mathbf{L = Q/L_{airgap} + L_o} \qquad \mathbf{[H]} \qquad \mathbf{(5.1.q)}$$

where

$Q = 2 * A * N^2\ (1\text{-}\alpha\text{-}\beta)\mu_{iron}/l_{coil}$ [$kg.m^3/(A^2.s^2)$]

$L_0 = 2 * \gamma * N^2$ [H]

Equation 5.1.p and 5.1.q reflects the case that no hysteresis is present. The conclusion is that the reluctance force (F_m) is approximate proportional to the square of the current (i_{coil}) and inversely proportional to the square of the distance (l_{airgap}). (See also equation 5.2.j)

Equation 5.1.m reflects the case that hysteresis is present. As already mentioned before (see chapter 3) a magnetic hysteresis is a dynamical multi valued non-linear relationship between the magnetic field strength (H) and the field density (B). A schematic diagram of the reluctance force (F_m) is shown in figure 5.1.3:

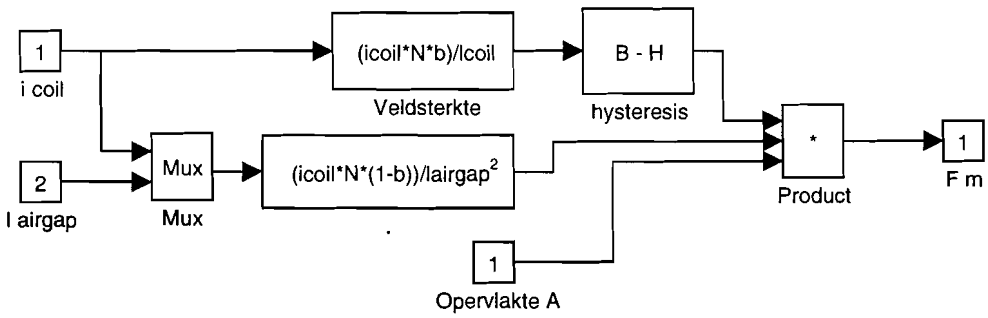


figure 5.1.3) A schematic diagram of the reluctance force (F_m).

5.2) The linearisation of the system

Now we can easily obtain the dynamic equation of the system by using (5.1.a) and (5.1.b):

$$m\ddot{l}_{airgap} = F_g - F_m$$

$$m\ddot{l}_{airgap} = mg - A_{ball} * B_{coil} * N * i_{coil} * (1-\alpha-\beta)/l^2_{airgap} \qquad (5.2.a)$$

$$V_{coil} = i_{coil} * R + \partial(i_{coil} * L) / \partial t$$

$$V_{coil} = (R + \partial L/\partial l_{airgap} * \partial l_{airgap}/\partial t) * i_{coil} + L * \partial i_{coil} / \partial t$$

$$V_{coil} = (R + Q/l_{airgap}{}^2 * \partial\, l_{airgap}/\partial t) * i_{coil} + (Q/l_{airgap} + L_0)\, \partial i_{coil} / \partial t \qquad (5.2.b)$$

The attractive force (F_m) by the activated coil is a non-linear function of both the distance (l_{airgap}) and the current (i_{coil}) through the coil (see 5.2.a). Further more we can write:

$$l_{airgap} = l_{initial} + l \qquad (5.2.c)$$

$$i_{coil} = i_{initial} + i \qquad (5.2.d)$$

$$V_{coil} = V_{initial} + V \qquad (5.2.e)$$

Linearised at a certain operating point we may write :

$$m\ddot{l} = K_i * i - K_x * l \tag{5.2.f}$$

The constants K_i and K_x are linearisation constants. Also the inductance (L) is some non-linear function of the distance (l) so that again by linearisation we can write:

$$L\dot{i} + Ri + K_L\dot{l} = V \tag{5.2.g}$$

Where R is the coil resistance, K_L some linearisation constant and V the applied voltage, which is the control input. The state space model is as follow:

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 \\ -K_x/m & 0 & K_i/m \\ 0 & -K_l/L & -R/L \end{bmatrix} * \begin{bmatrix} l \\ \dot{l} \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/L \end{bmatrix} * \begin{bmatrix} 0 & 0 & V \end{bmatrix} \tag{5.2.h}$$

The output y is formed by measuring the distance (l), the derivative of the distance ($\dot{l}$) and the current (i):

$$\dot{Y} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} l \\ \dot{l} \\ i \end{bmatrix} \tag{5.2.i}$$

The electromagnet has an inductance L_0 = 0.508 H ,a coil resistance R = 23.2 Ω, a coil length l_{coil} = 14.53 cm, and the number of windings of the coil N= 500. The mass of the ball is equal to m = 1.75 kg and the surface of the ball is equal to A = 0.03 m^2. Furthermore α=0 and β =0.999

The working point is:

- **$l_{initial}$ = 4.36 [mm]**
- **$l_{initial}'$ = 0 [ms^{-1}]**
- **$i_{initial}$ = 1.06 [A]** **(To compensate gravity)**
- **$B_{initial}$ =0.8 [T]** "

- **$H_{initial}$= 3645.8 [A/m]** "

In the existing literature [7] there are some rules that approximate give the right linearisation constants.

The magnetic force (F_m) is *approximately* proportional to the square of the current (i_{coil}) and inversely proportional to the square of the airgap (l_{airgap}):

$$F_m = K_e \left(i_{coil}^2 / l_{airgap}^2 \right) \tag{5.2.j}$$

$$1.75 * 10 = K_e \left(1.06^2 / 0.00436^2 \right)$$

$$K_e = 2.9607 * 10^{-4} \left[Nm^2 / A^2 \right]$$

Where K_e is an electromagnet constant. Linearisation around the operating point gives :

$$K_i = 2 * K_e \left(i_{coil} / l_{airgap}^2 \right) \tag{5.2.k}$$

$$K_i = 2 * 2.9607 * 10^{-4} * \left(1.06 / 0.00436^2 \right)$$

$$K_i = 33.02 \left[N / A \right]$$

$$K_x = 2 * K_e \left(i_{coil}^2 / l_{airgap}^3 \right) \tag{5.2.l}$$

$$K_x = 2 * 2.9607 * 10^{-4} * \left(1.06^2 / 0.00436^3 \right)$$

$$K_x = 8027.52 \left[N / m \right]$$

$$K_L = 2 * K_e \left(i_{coil} / l^2_{airgap} \right) \tag{5.2.m}$$

$$K_L = 2 * 2.9607 * 10^{-4} * \left(1.06 / 0.00436^2 \right)$$

$$K_L = 33.02 \left[N/A \right]$$

It is very important to choose the *correct* values of the linearisation constants. The linearisation constants depend on what for type of hysteresis (slope) and the chosen linearisation point. In our hysteresis we find for the linearisation constants the following value's:

- **K_i = 16.50 N/A**
- **K_L = 16.50 N/A**
- **Kx = 4391 N/m**

We can substitute these linearisation constants in the state space model. The poles of the open loop system are :

- **-42.6549**
- **-1.5076 + 51.8081 i**
- **-1.5076 - 51.8081 i**

We see that the open loop system is stable, but poorly damped. The pole zero map is given in figure 5.2.1.d. In the open loop system we make the initial state x_0 = 0.001 m, the response is given in figure 5.2.1.e .

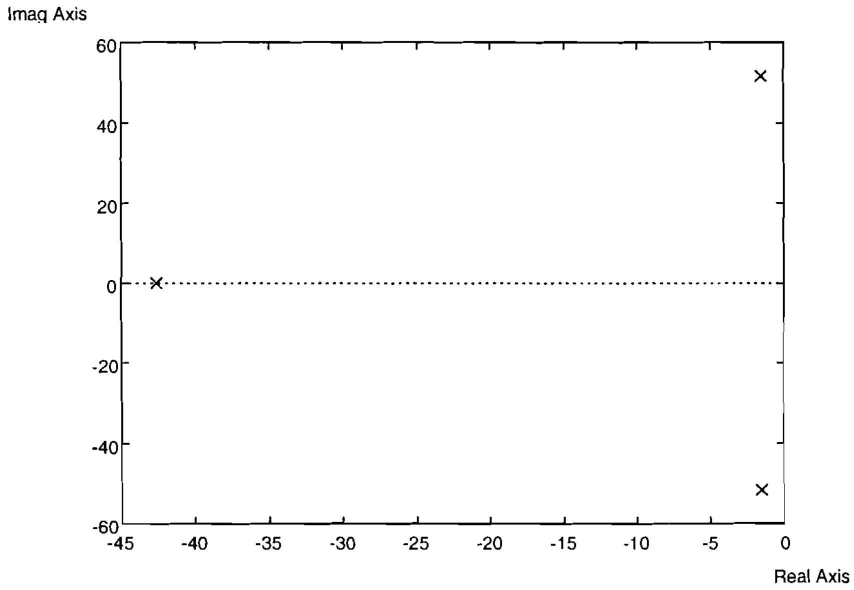


figure 5.2.1.d) The pole zero map of the open loop system

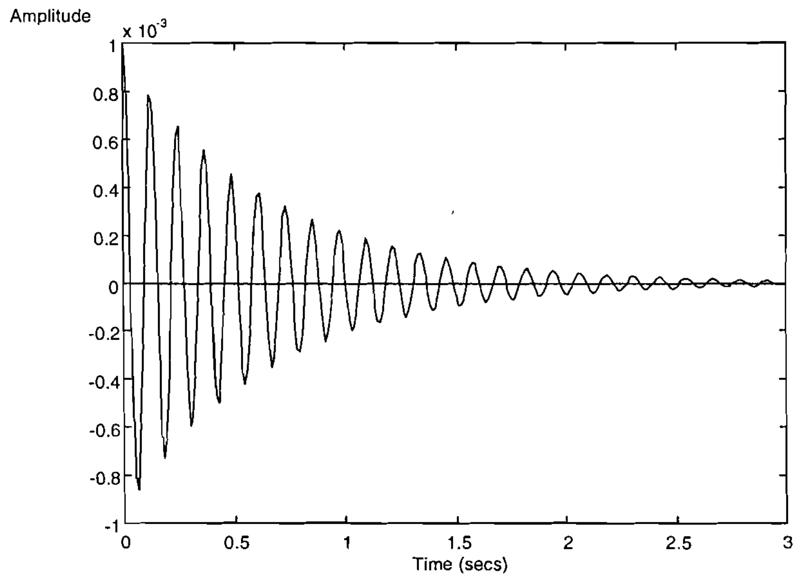


figure 5.2.1.e) The response of the open loop system if the initial state x_0 = 0.001 m

5.3 The LQ Controller

As a controller we use a LQ controller, see [11]. We shall consider the time invariant system:

$$\mathbf{x'(t)=Ax(t)+Bu(t)} \qquad \mathbf{t \geq t_0 \ \ x(t_0) = x_0}$$

where x is the process state, u is the control input and A and B are known matrices. This system is normally in rest, x(t) = 0, but due to some disturbance, the state at some time t_0 is displaced to $x(t_0) = x_0 \neq 0$. Therefore

$x(t_0) \neq x_0$, $t \geq t_0$ and the regulator problem is to apply a control u(t), such that the state x(t) is returned to zero as quickly as possible.

If the pair {A,B} is controllable we can obtain a finite energy input as u(t) = Kx(t) so that

$$x'(t) = (A - BK)\, x(t)$$

And by choosing K suitably we can make x(t) decay to zero as fast as we wish. The rate of decay depends on how negative the real parts of the eigenvalues of (A-BK) are. The more negative these are, the larger the values of K will be, and therefore the higher the required signal energy.

We should try to make a trade-off between the rate of decay of x(t) and the energy of the input. In the linear quadratic regulator problem, this is done by choosing u(t) to minimise,

$$J = \int_0^\infty (x(t)^T Q\, x(t) + u(t)^T R\, u(t)\, dt \qquad Q \geq 0 \qquad R > 0$$

$$x'(t) = Ax(t) + Bu(t) \qquad t \geq t_0 \quad x(t_0) = x_0 \qquad \{A,B\} \text{ controllable}$$

By choice of R and Q we can give different weightings to the cost of control (R) and the cost of deviations from the desired state (Q).

The optimal control is a linear feedback control,

$$u(t) = -Kx(t) \qquad t_0 \leq t \leq \infty$$

where K depends on the parameters {A, B, Q, R}.

In our example we choose for Q and R:

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1*10^4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R = 1$$

We have chosen for these matrices Q and R, because we want a small overshoot of the distance l (below 33 % of the maximum value of the disturbance on l) and a fast settling time (below the .5 second). These value's of Q and R shift the closed loop poles enough to the left, so that the controller will give the demanded performance.

The values for the controller are kl = -2935.9, kl' = 66.2 and ki = 11

The closed loop poles are:

- **- 39.0768**
- **- 14.1670 - 52.2910 i**
- **- 14.1670 + 52.2910 i**

The model of the linearised plant with controller in SIMULINK is given in figure 5.3.1

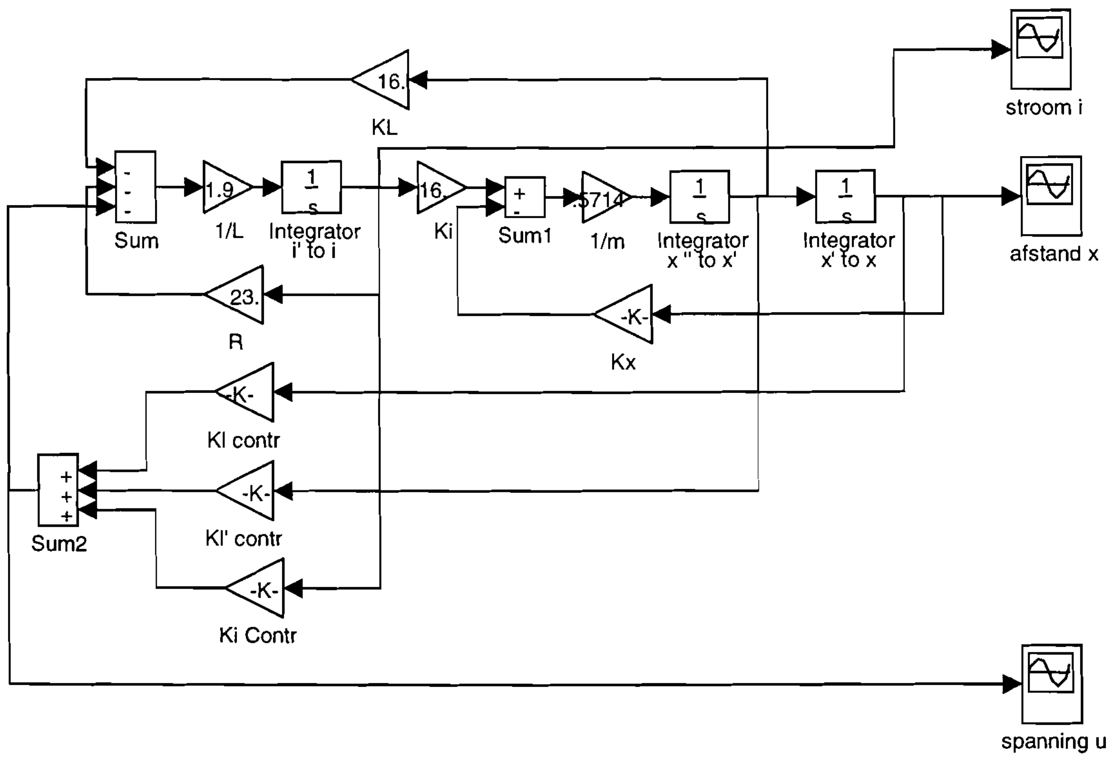


figure 5.3.1) The Linearised model in SIMULINK

5.4) The Closed loop behaviour

Let us start by evaluating the closed system without hysteresis i.e. we just consider the linearised plant controlled by the LQ controller. It is important to keep in mind that all the value's of the simulated parameters, are related to the working point. In other words, the distance $l_{airgap} = l_{inintial} + l$, the flux density $B_{total} = B_{initial} + B$, etc. (see page 28-29). Assume we want to steer the position of the ball from x = 0.001 to 0 m, we will now observe the output. (see figure 5.4.1 a, b, c)

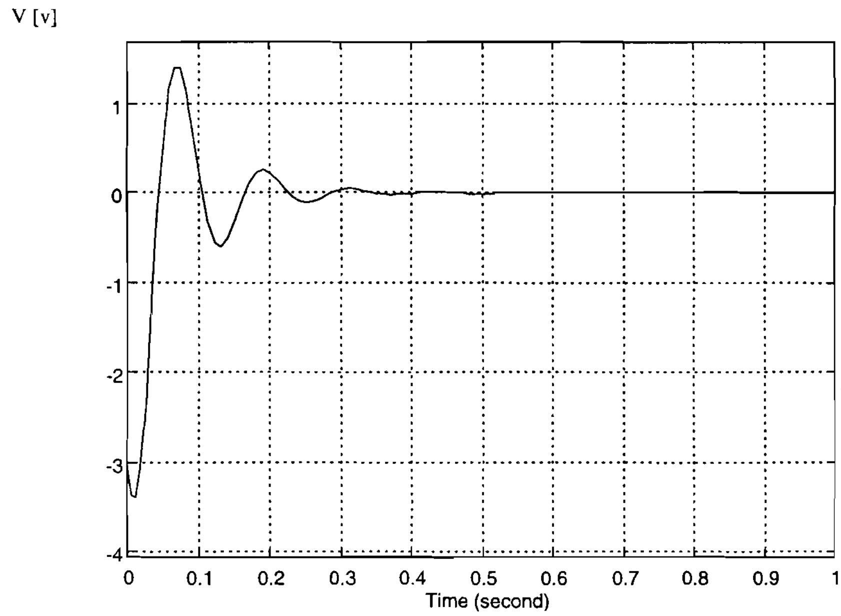


figure 5.4.1 a) The voltage V [v] over the coil

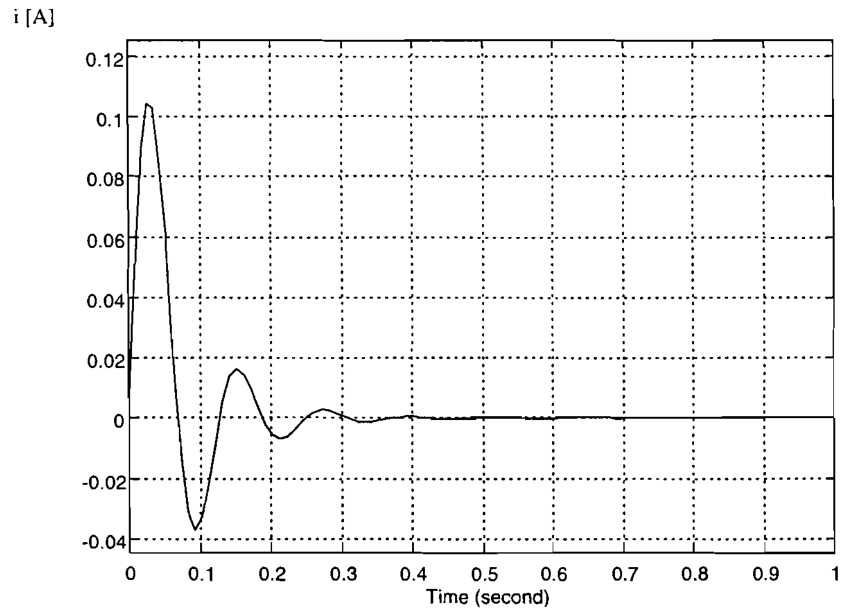


figure 5.4.1 b) The current i [A] through the coil

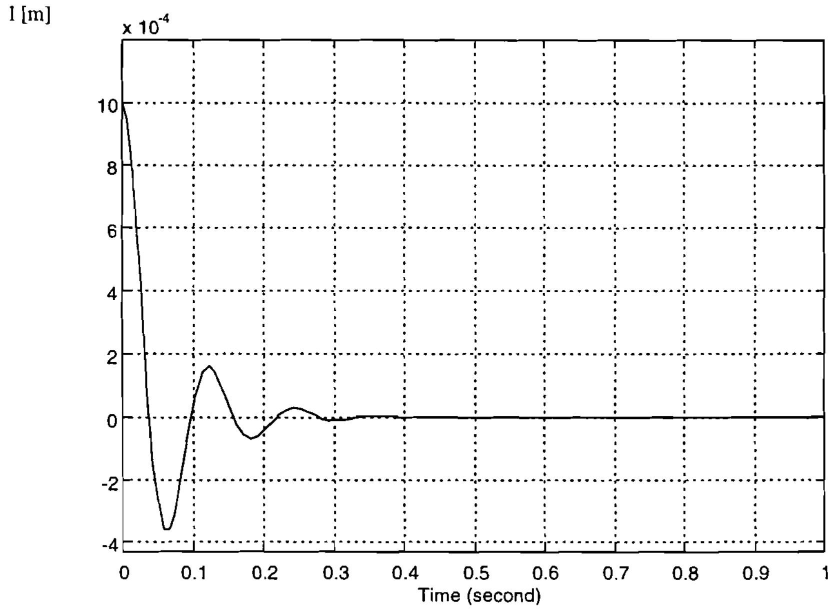


figure 5.4.1 c) The distance l [m] between the ball and the electromagnet

The controlled model has a reasonable fast response time and some overshoot. The response time can be made faster but, that means larger control gains, and thus a larger input signal (V). There are two reasons why we don't want large gains:

- We use a linearised model, we want the input signal (V) remain small so that the linearised model remains valid.
- Large control signals will drive the actuator in saturation, in practical situations.

5.5) The closed loop system with hysteresis

In the levitated ball system, magnetic hysteresis is present. We now show what the effect is of hysteresis on the controlled system. We expand the model and use the Chua Stromsmoe hysteresis model at a width zero. A detailed behaviour of the hysteresis in the plant is given in the figures 5.5.1.e,f,g. It is important to keep in mind that all the value's of the simulated parameters, are related to the working point. In other words, the distance $l_{airgap} = l_{inintial} + l$, the flux density $B_{total} = B_{initial} + B$, etc. (see page 28-29). Assume we want to steer the initial position of the ball from l = 0.001 to 0 m. We will observe what the effect is. (see figure 5.5.1)

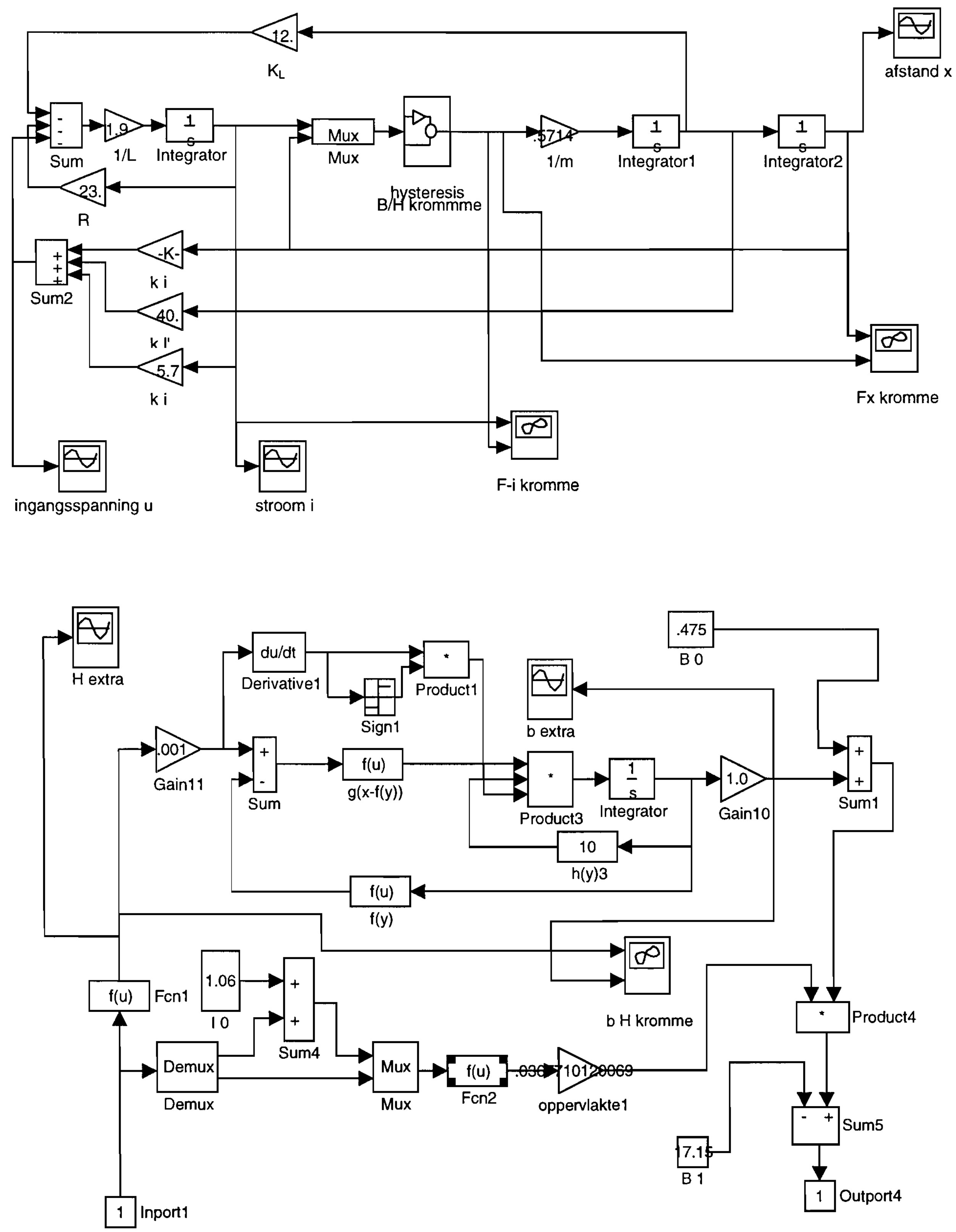


figure 5.5.1 a) The complete plant including hysteresis

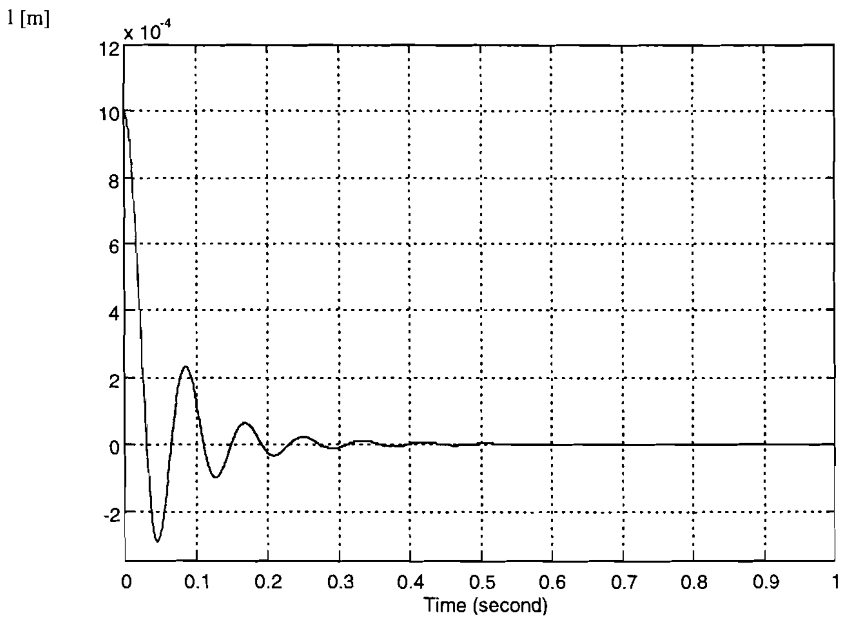


figure 5.5.1 b)The distance l [m] between the ball and the electromagnet

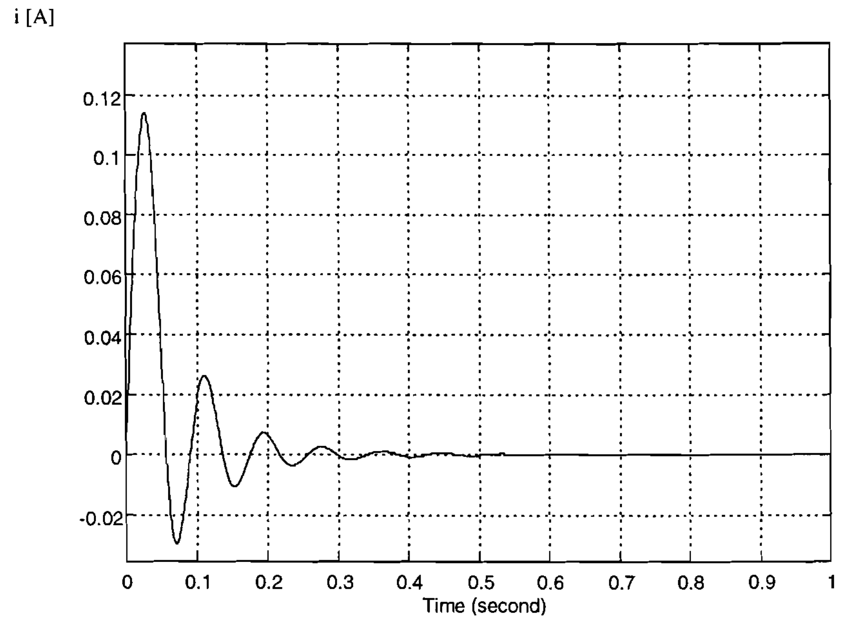


figure 5.5. c) The current i [A] through the coil

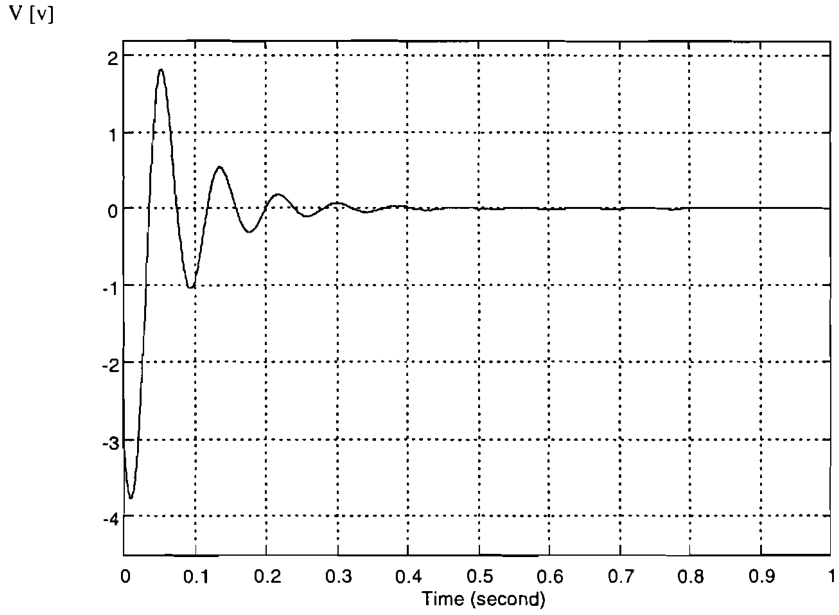


figure 5.5.1 d)The voltage V [v] over the coil.

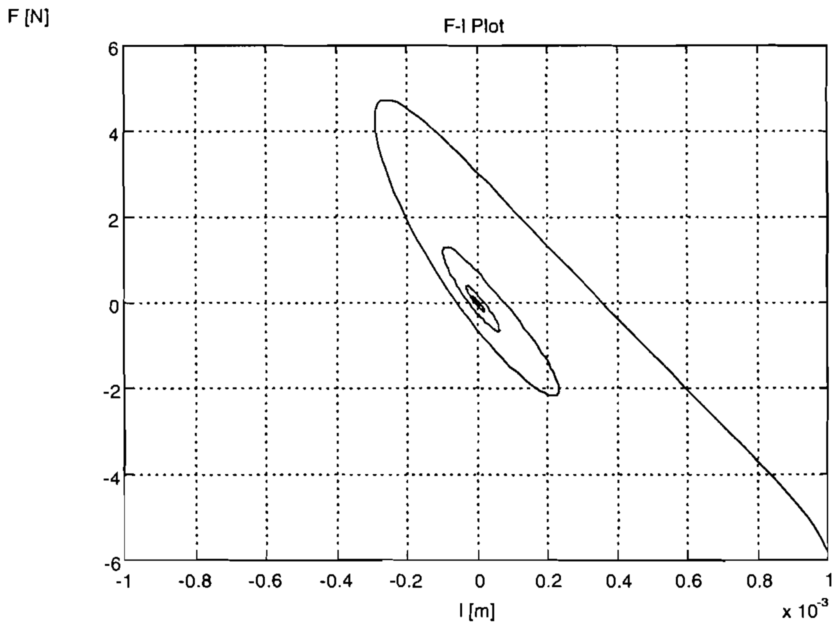


figure 5.5.1.e) The force (F) versus distance (l.)

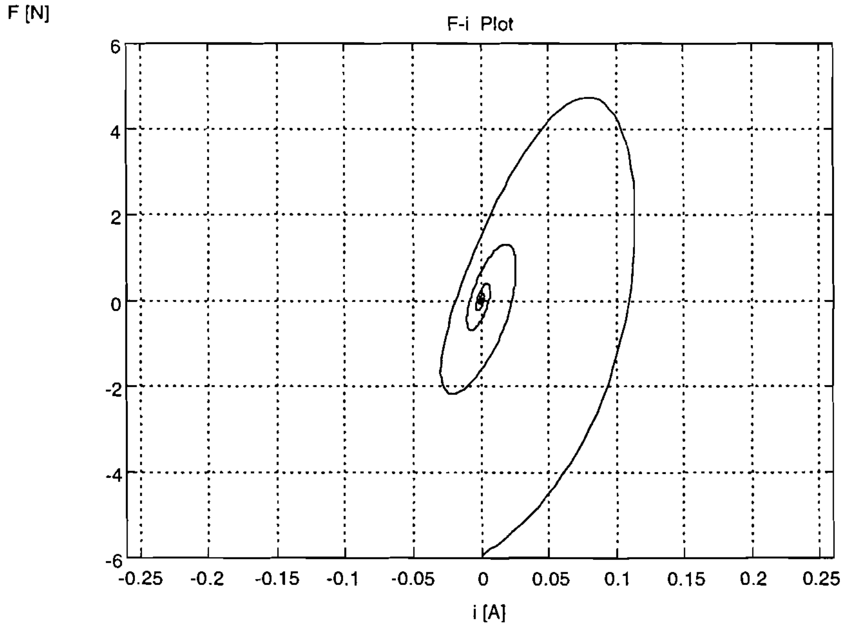


figure 5.5.1.f) The force (F) versus current (i)

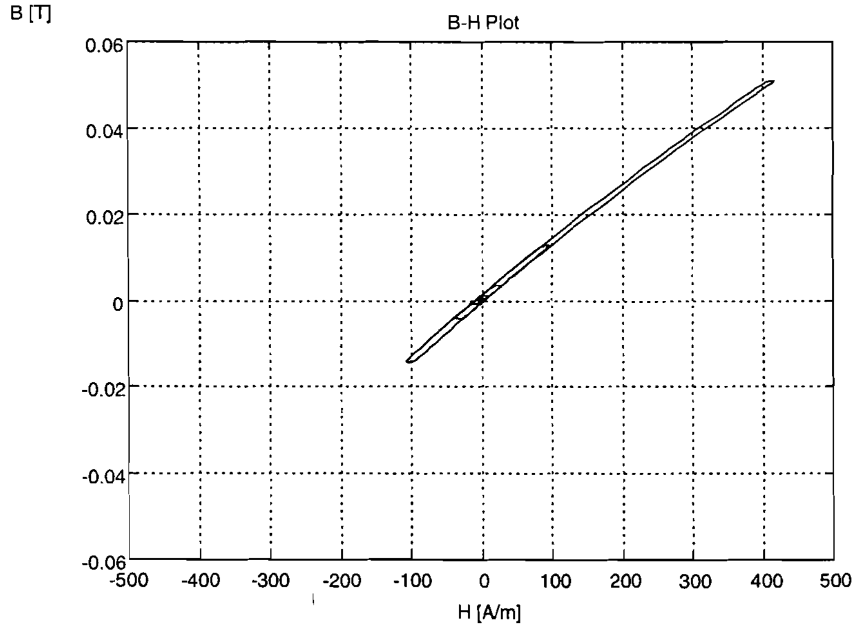


figure 5.5.1.g) The B H relationship caused by the plant.

There is a little differences in the responses of the linearised model and the model with hysteresis. This is because the non linearity of the plant in the linearisation point. Now we make the hysteresis wider to see what the effect is. (see figure 5.5.2). A detailed behaviour of the hysteresis in the plant is given in the figures 5.5.2.d,e,f.

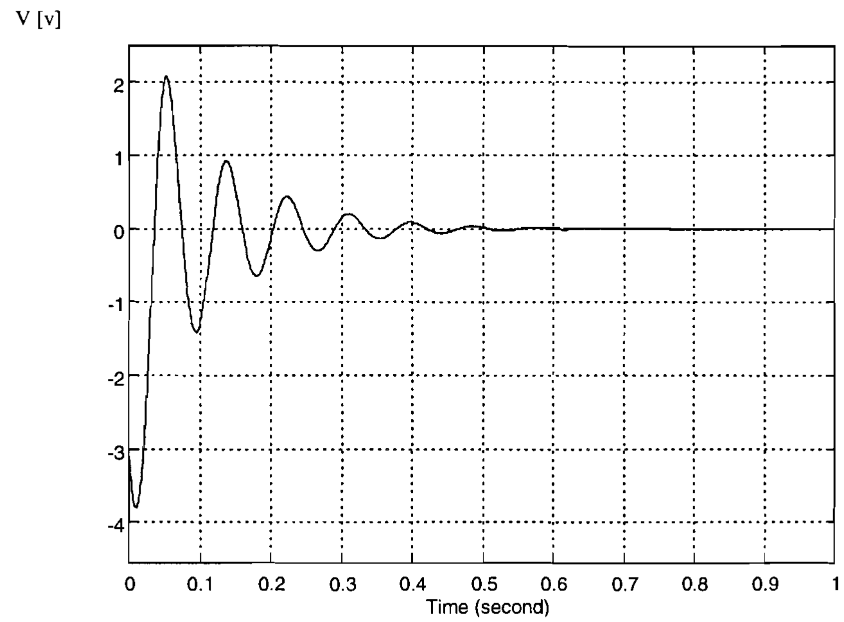


figure 5.5.2 a)The voltage V [v] over the coil

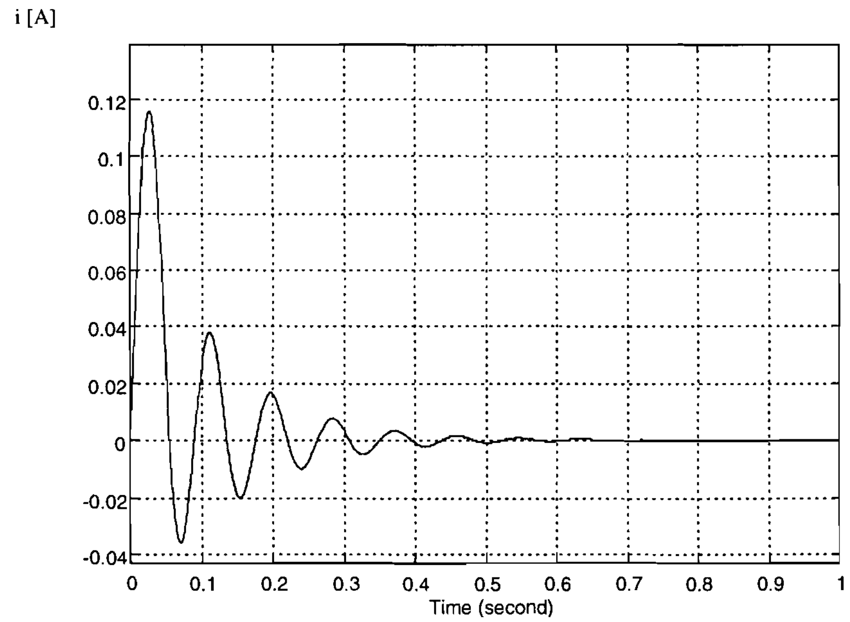


figure 5.5.2 b) The current i [A] through the coil

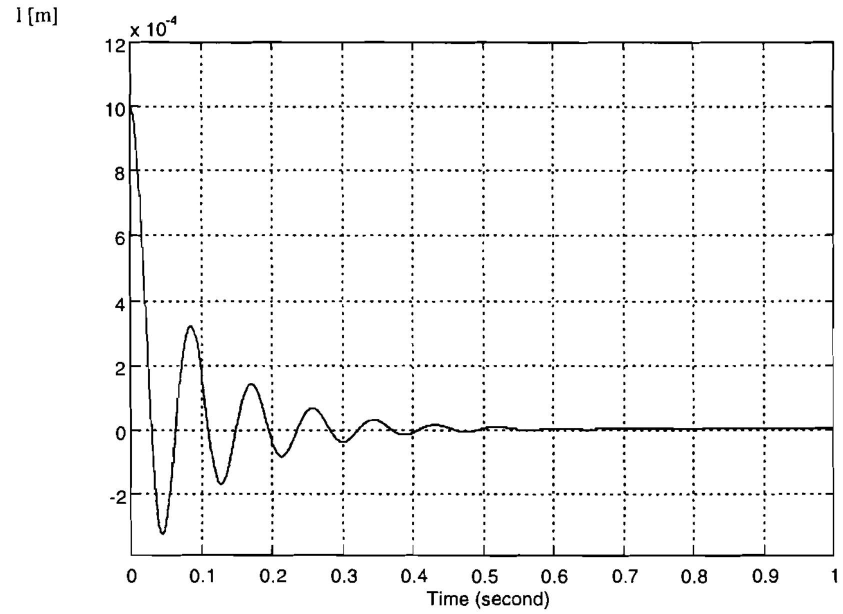


figure 5.5.2 c)The distance l [m] between the ball and the electromagnet

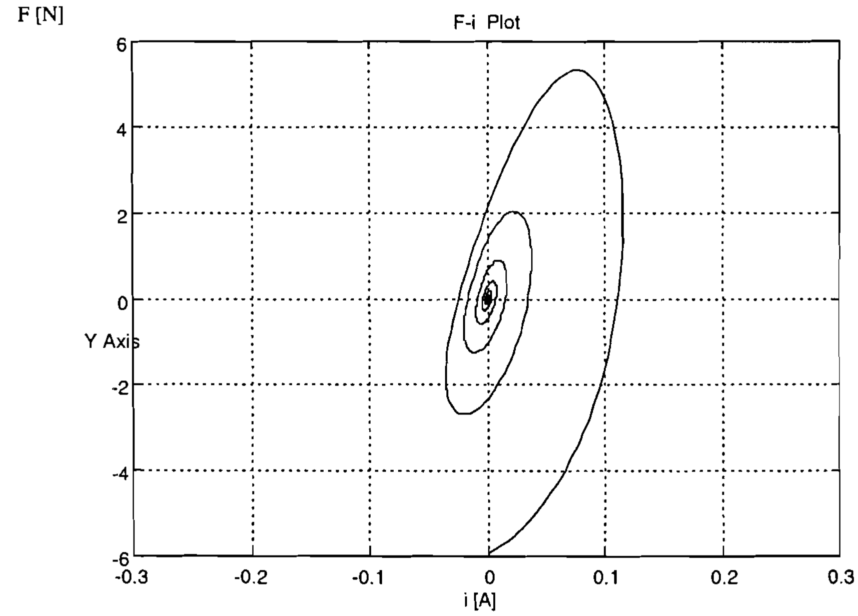


figure 5.5.2.d) The force (F) versus current (i)

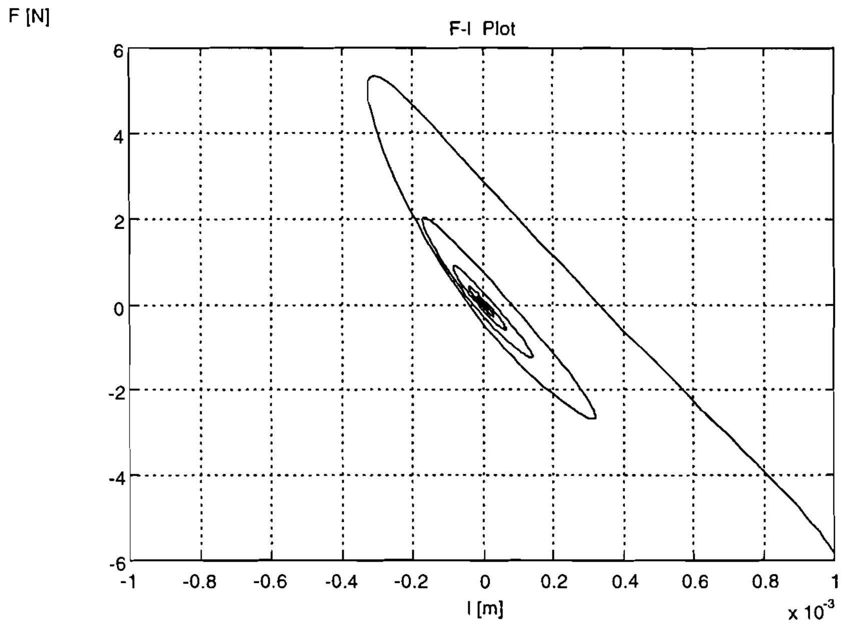


figure 5.5.2.e) The force (F) versus distance (l.)

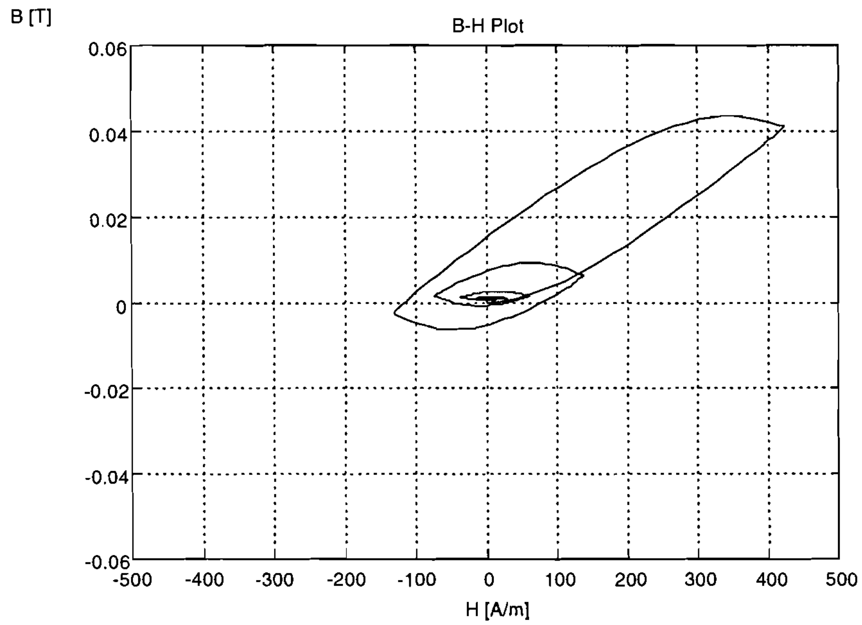


figure 5.5.2.f) The B H relationship caused by the plant.

We see that a wider hysteresis causes less damping. This is because the slope of the B-H curve changes by each extremal point of H, form a steep slope to a flat slope. The LQ controller is based on the steep slope and not on the flat slope, this "model error" will have an influence on the damping.

We want that the plant has no overshoot. We can improve the performance of the plant by choosing different values for Q and R.

We choose for Q and R:

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1*10^5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

R=1

We choose for this value of Q because we want no overshoot. The values for the controller are kl = -8312.0, kl' = 253.2 and ki = 31.2.

The closed loop poles are:

- **- 298531**
- **- 43.6453 - 62.2265 i**
- **- 43.6453 + 62.2265 i**

We now show what the effect is of hysteresis on this system. We use the Chua and Stromsmoe hysteresis model at a width zero. It is important to keep in mind that all the value's of the simulated parameters, are related to the working point. In other words, the distance $l_{airgap} = l_{inintial} + l$, the flux density $B_{total} = B_{initial} + B$, etc. (see page 28-29). Assume we want to steer the position of the ball from l = 0.001 to 0 m. We will observe what the effect is. (see figure 5.5.3)

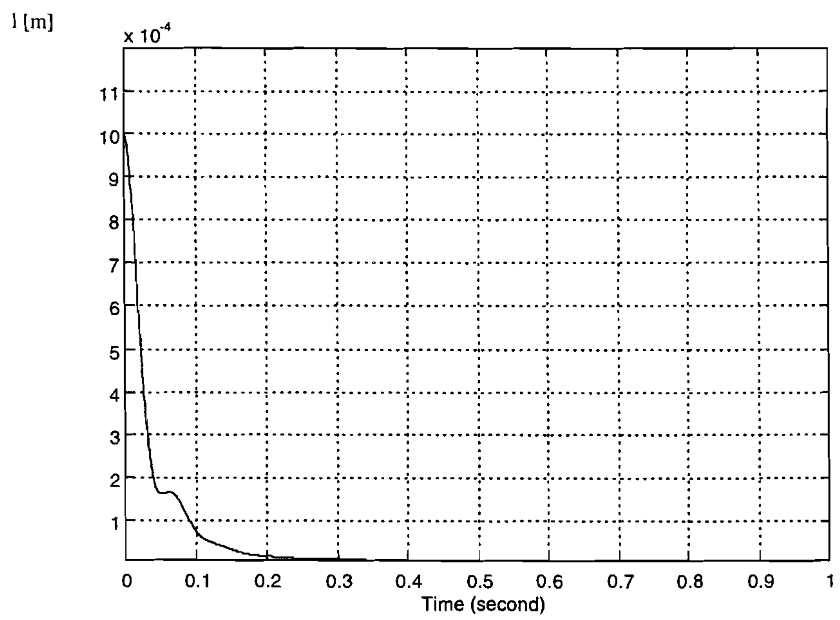


figure 5.5.3 a) The Distance l [m] between the ball and the electromagnet

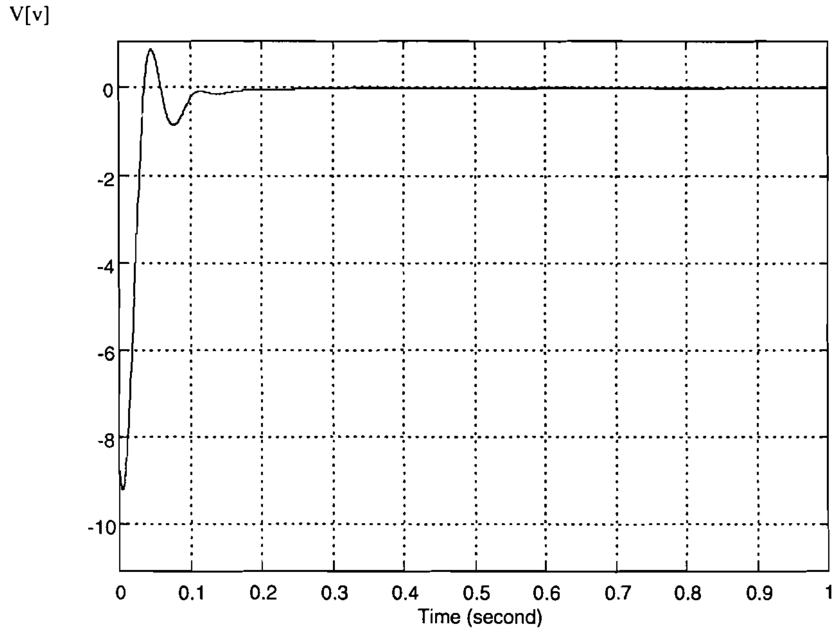


figure 5.5.3.b The Voltage V [v] over the coil

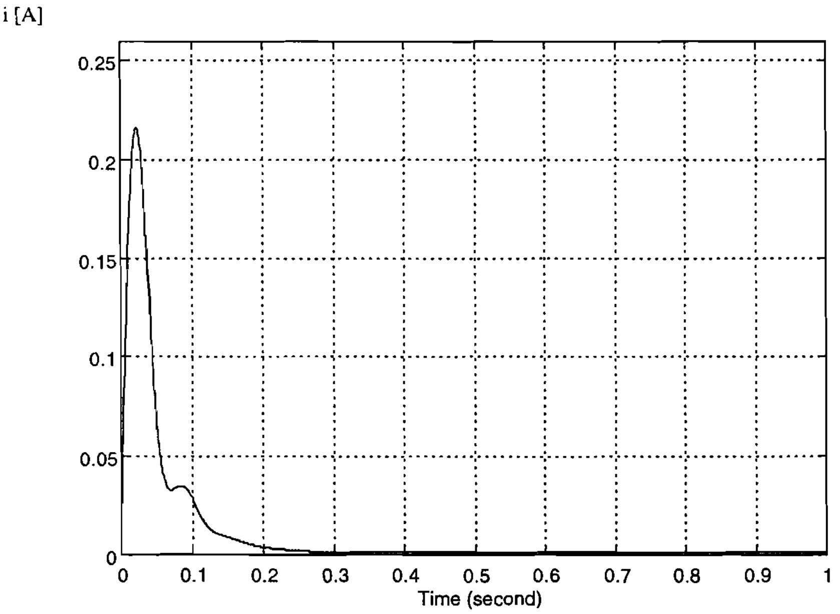


figure 5.5.3 c) The current i [A] through the coil.

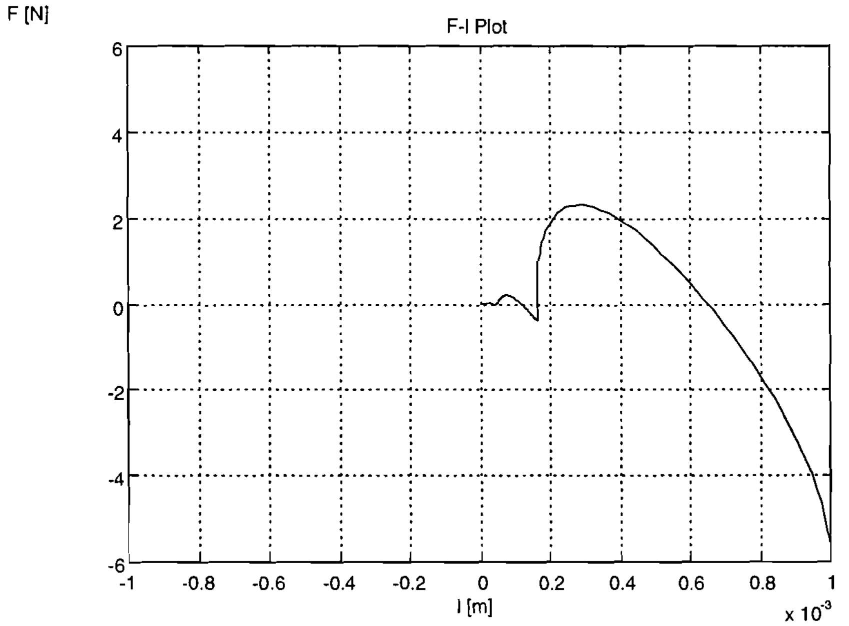


figure 5.5.3.d) The force (F) versus distance (l)

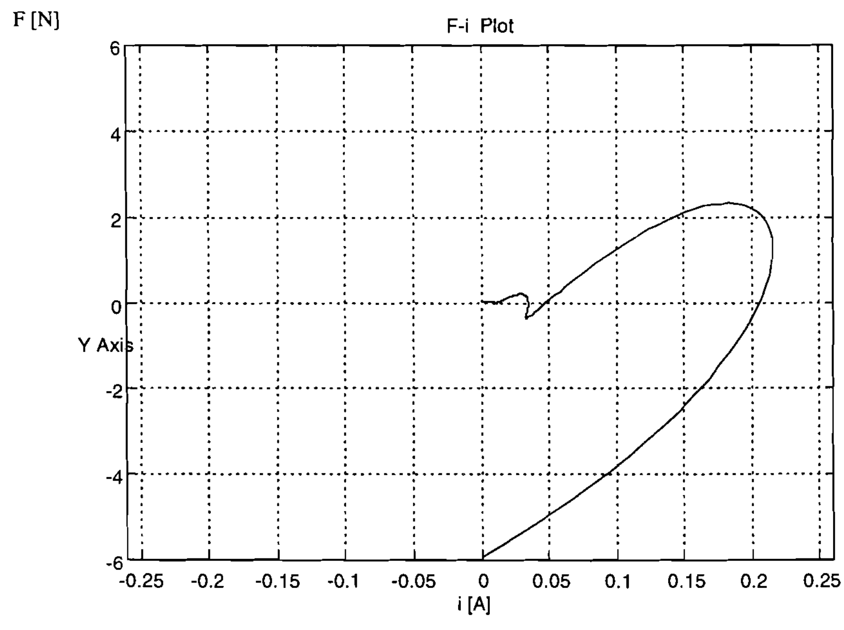


figure 5.5.3.e) The force (F) versus current (i.)

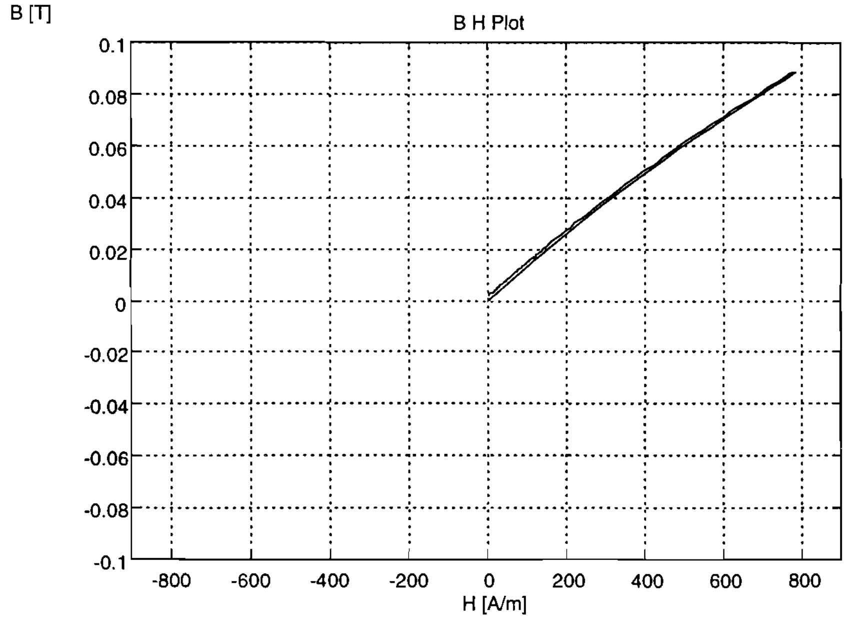


figure 5.5.3.f) The B H relationship caused by the plant.

Now we make the hysteresis wider to see what the effect is. (see figure 5.5.4)

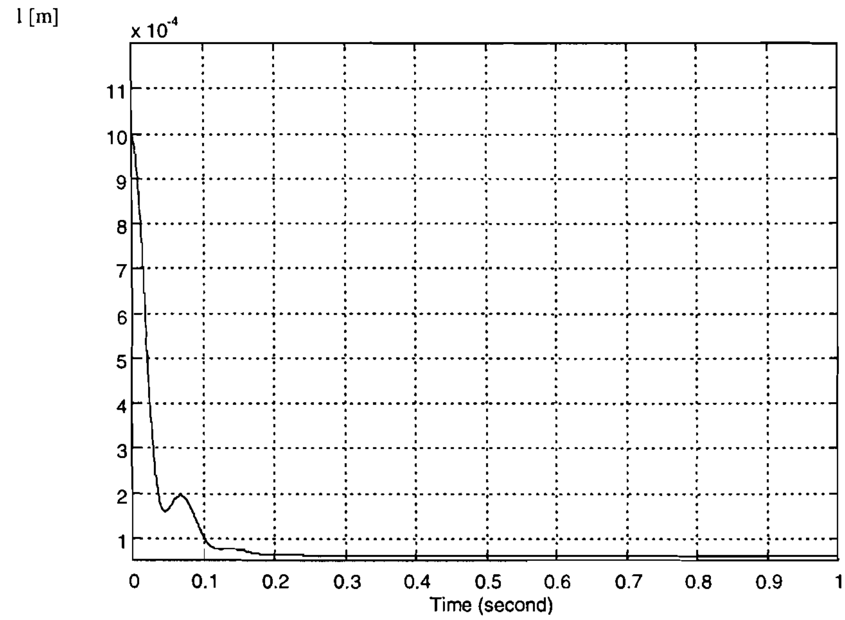


figure 5.5.4.a)The distance l [m] between de ball and the electromagnet.

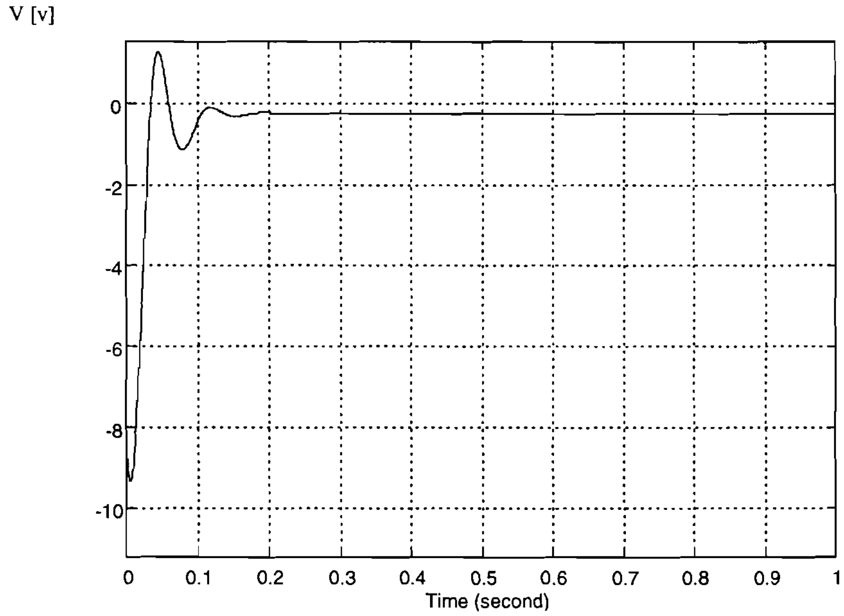


figure 5.5.4.b)The voltage V [v] over the coil.

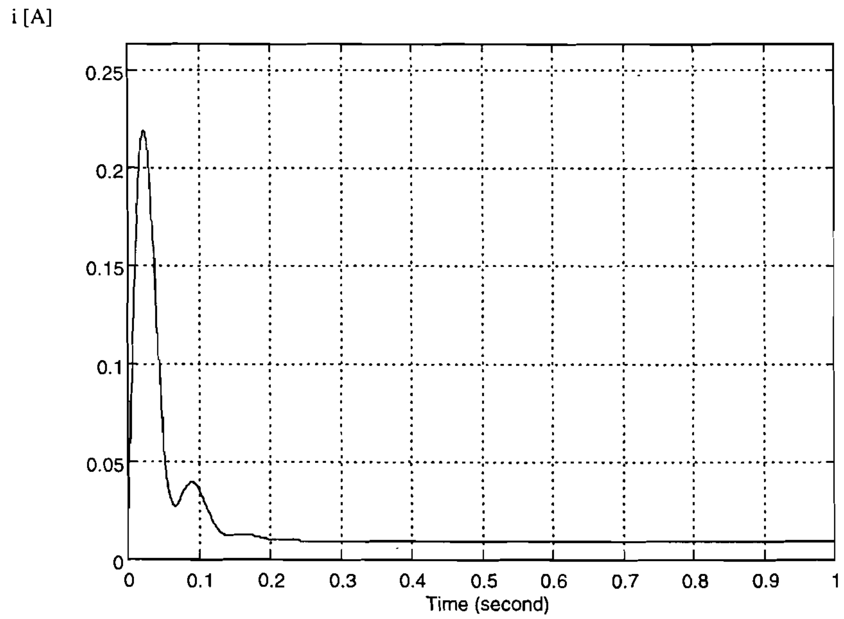


figure 5.5.4.c)The current i [A] through the coil.

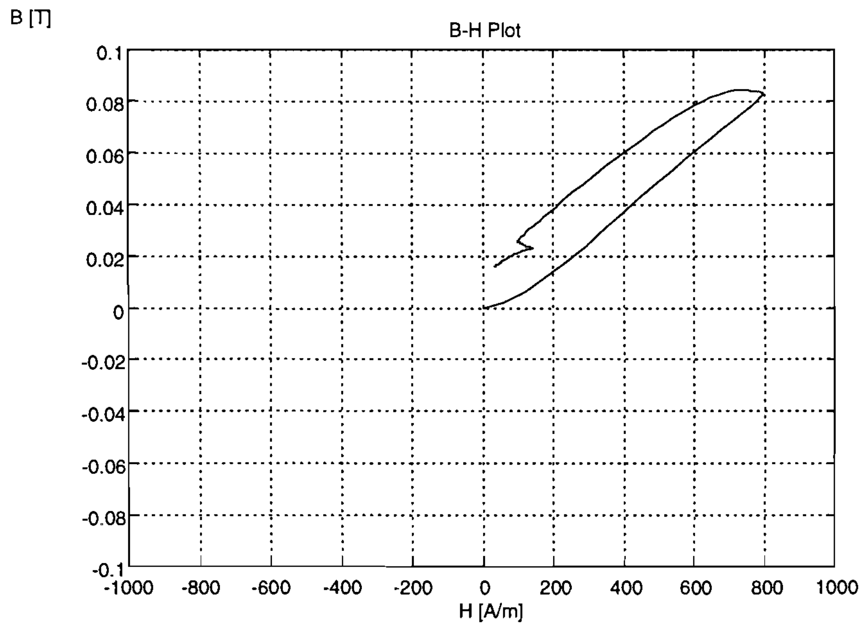


figure 5.5.4.d) The B H relationship caused by the plant.

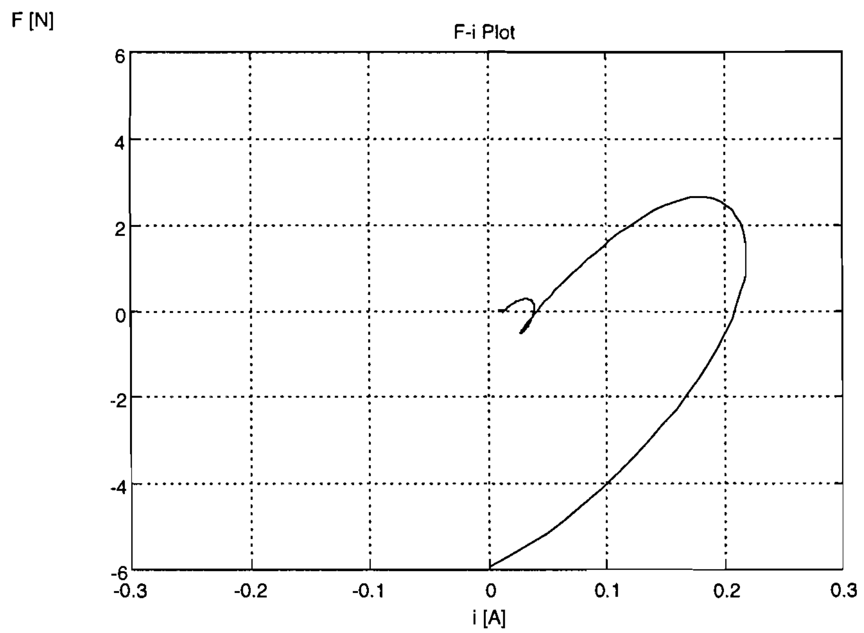


figure 5.5.4.e) The force (F) versus current (i)

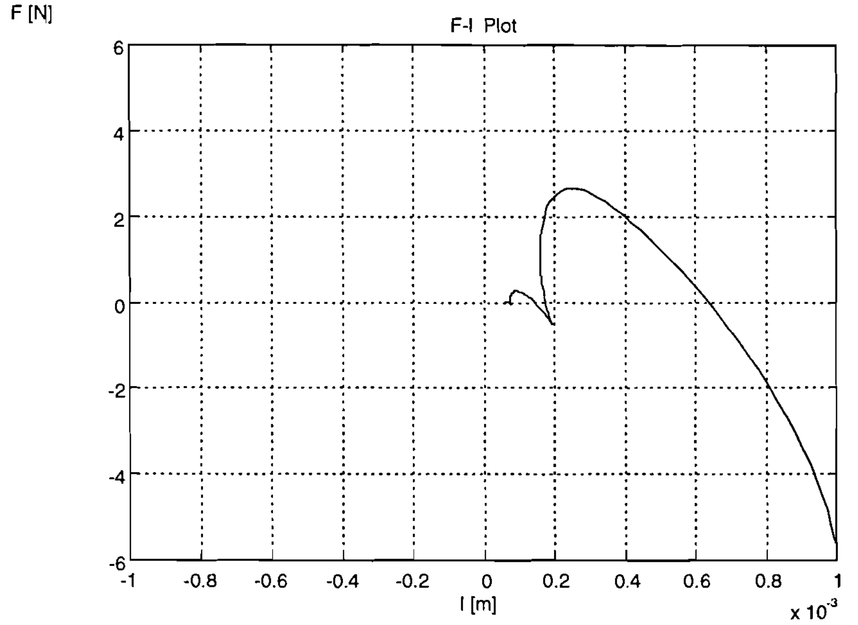


figure 5.5.4.f) The force (F) versus distance (l)

We see that the wider hysteresis causes a steady state error, and a little less damping.

If we look at results of the simulations, then we can ask ourselves a number of questions:

question 1:

Why causes a wider hysteresis less damping ?

answer:

This is because the slope of the B H curve changes by each extremal point of H, form a steep slope to a flat slope. The LQ controller is based on the steep slope and not on the flat slope, this "model error" will have an influence on the damping.

question 2:

Why causes a hysteresis with a width bigger then zero a steady state error ?

answer:

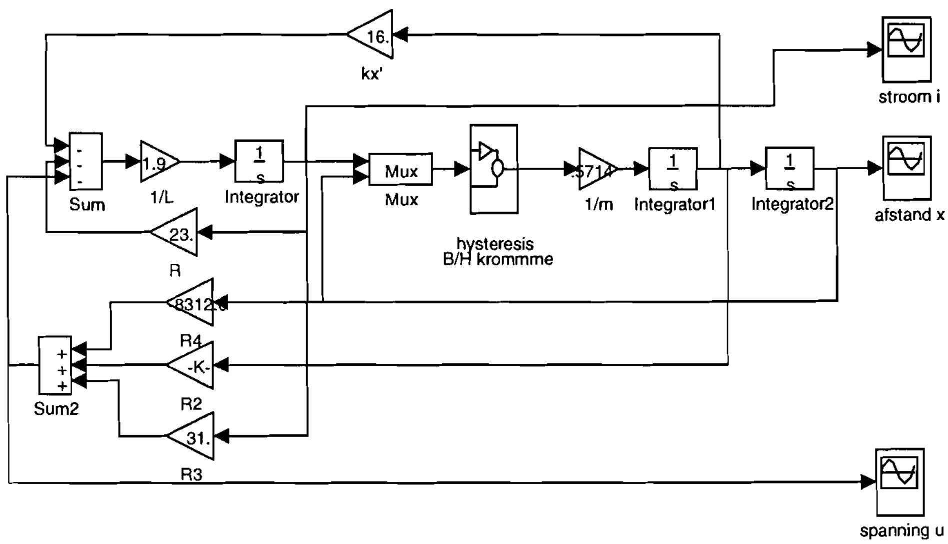
This is caused by the multi valued relationship between B and H of the hysteresis. This also causes a multi valued relationship between F and i. This implies that a control voltage V is causing a current i which result in a zero force F_m (see figure 5.5.2.c) *question 3:*

Why causes a bigger current through the coil a bigger steady state error ?

answers:

Like we discussed in question 2 the steady state error is not constant in all situations, but depends on the width of the hysteresis: The steady state error also depends on the maximum current through the coil, when the hysteresis width is kept constant. This is caused by the fact that for a bigger value of the current i we now already have a zero force (see figure 5.5.4.e).

We can ask ourselves; can we simplify the hysteresis to a simple backlash? We replace the hysteresis by a backlash and simulate the plant again to see what the result is: (see figure 5.5.5)



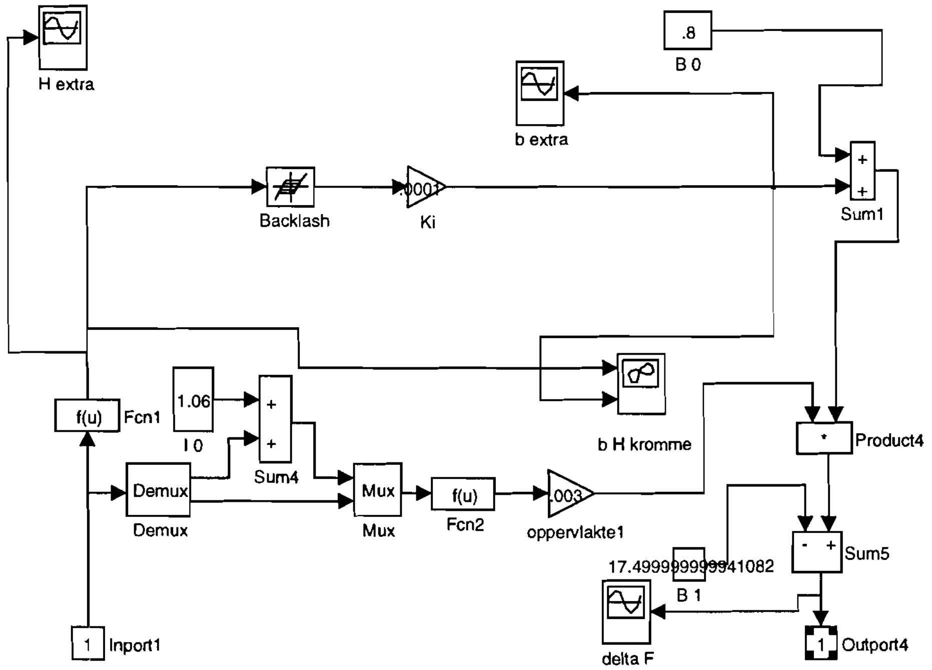


figure 5.5.5.a) Simulation model of the plant with backslash.

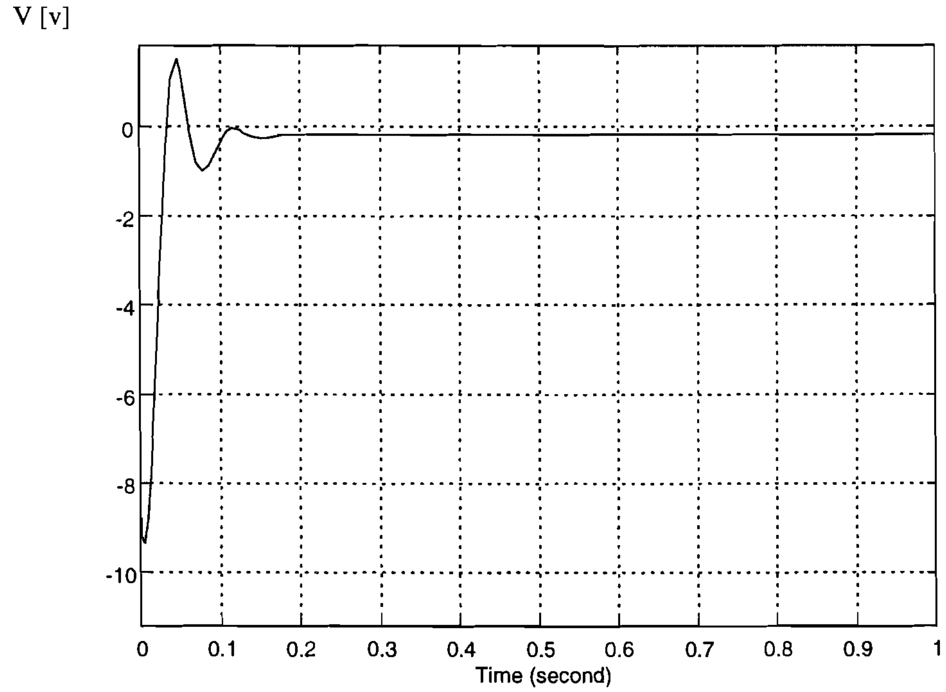


figure 5.5.5.b) The voltage V [v] over the coil.

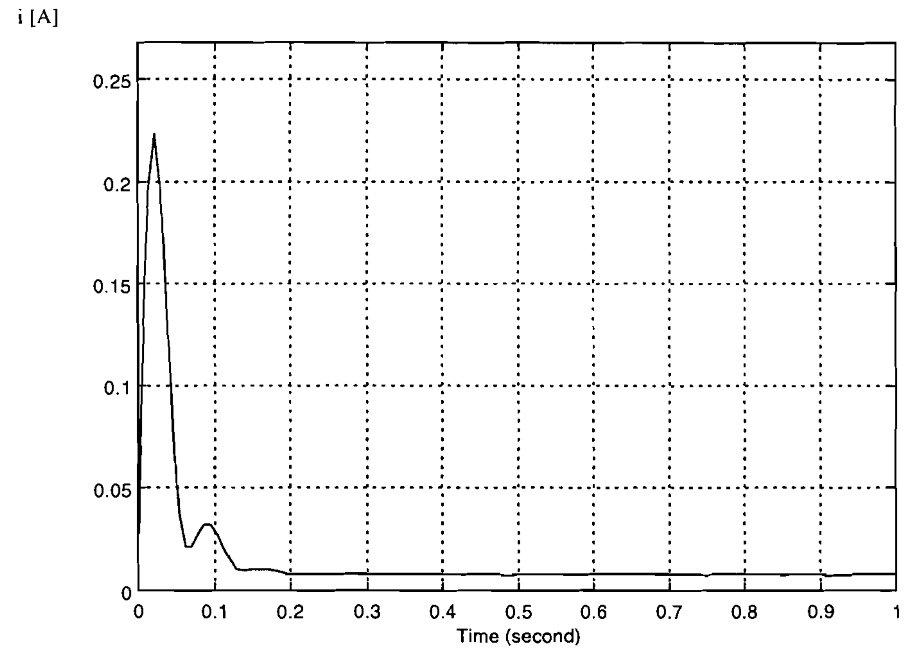


figure 5.5.5.c) The current i [A] through the coil

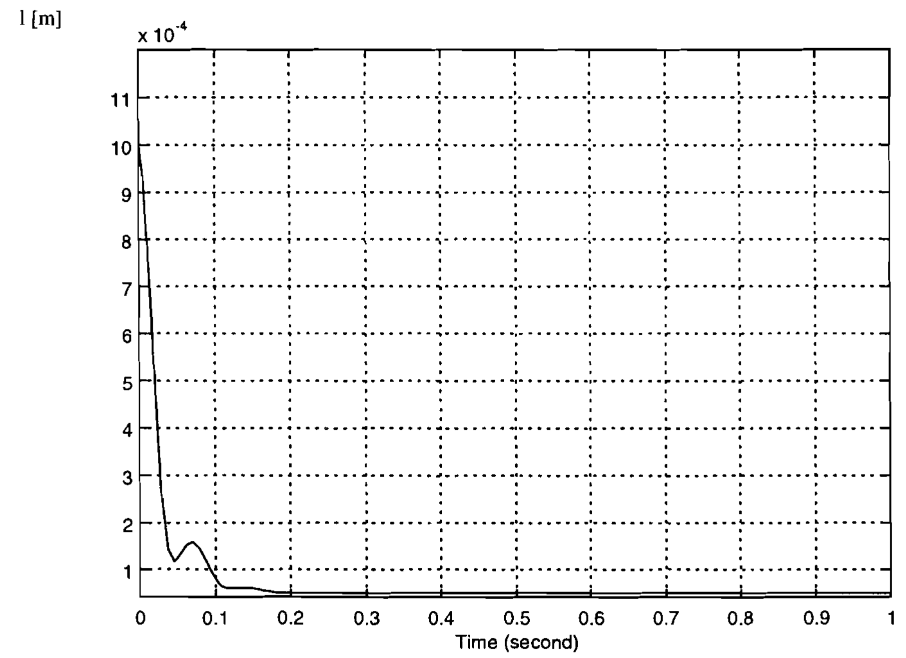


figure 5.5.5.d) The distance l [m] between the ball and the electromagnet.

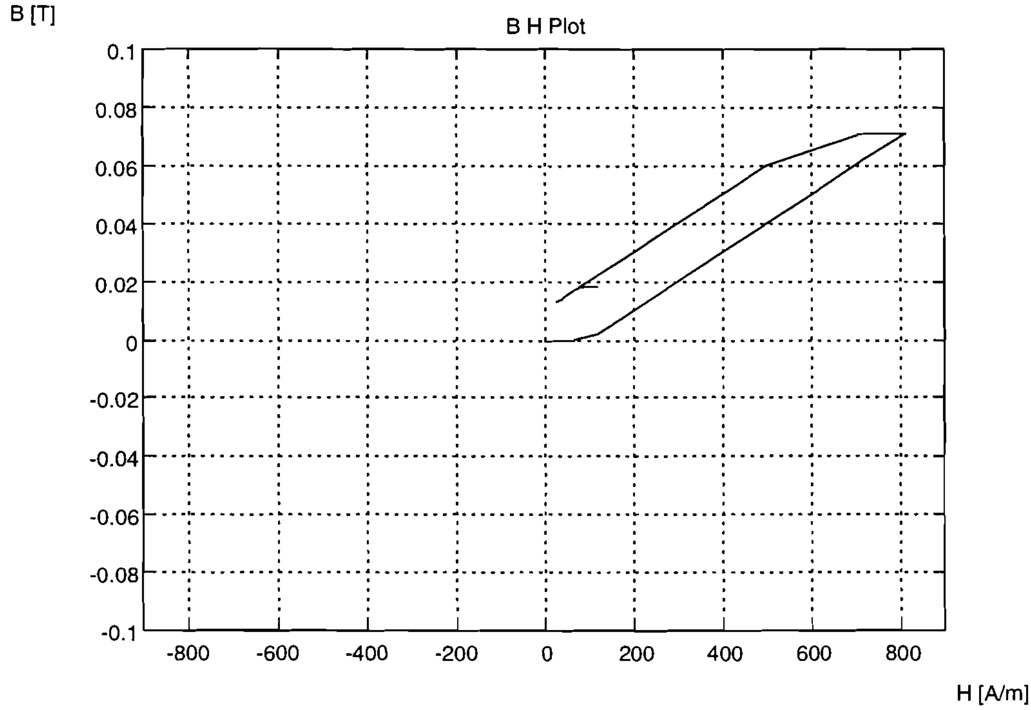


figure 5.5.5.e) The B H plot caused by the plant.

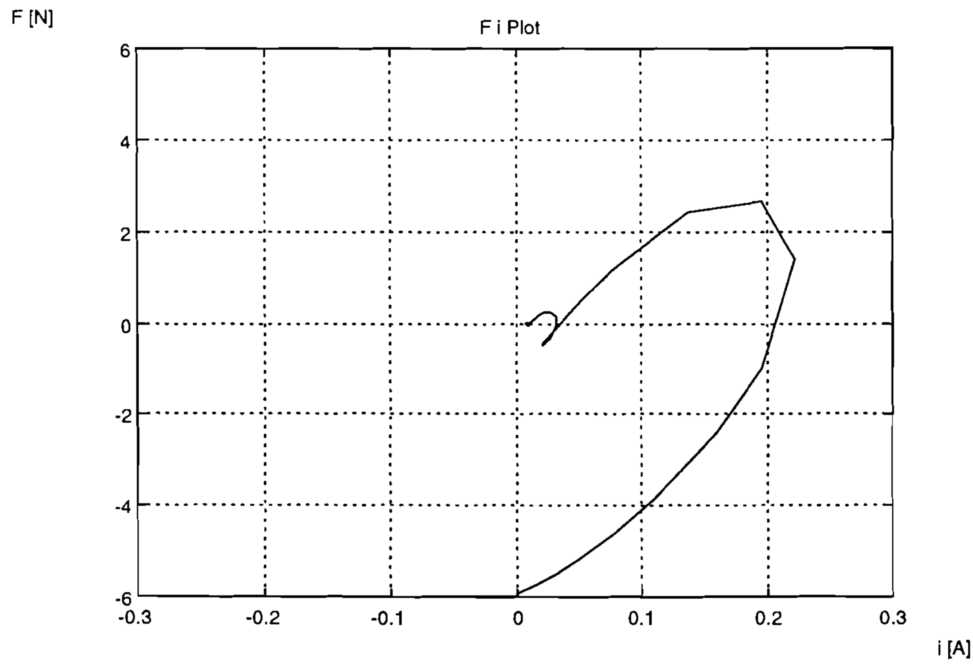


figure 5.5.5.e) The F-i plot.

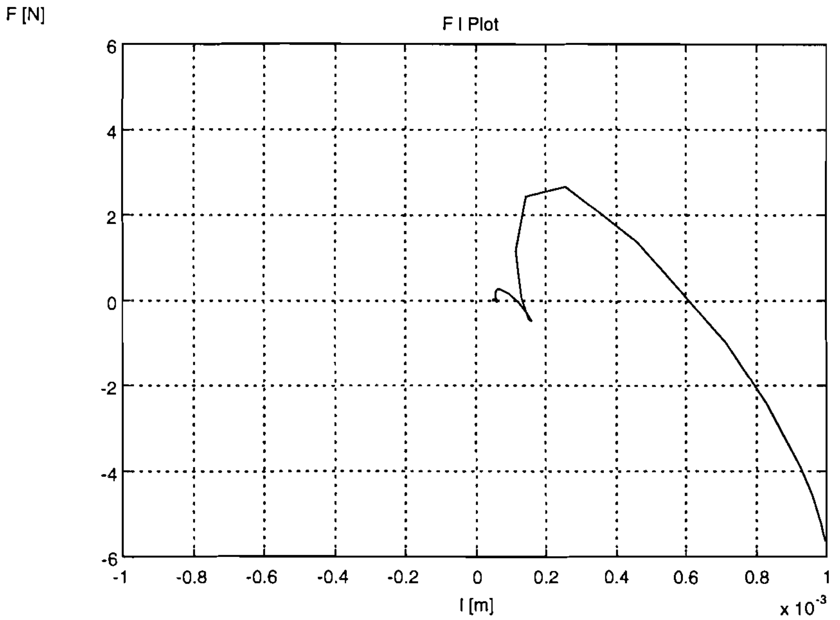


figure 5.5.5.f) The F l plot.

We see that the effect of a backlash and the Chua and Stromsmoe hysteresis model are more or less similar. If we make the dead zone bigger then also the damping will be less and the steady state error will bigger. The backlash will in most situations not represent the magnetic hysteresis accurate enough. The big difference between a backlash and a magnetic hysteresis is that the backlash has a constant dead time, and the "dead time" (the flat slope of the hysteresis by each extremal point) of the magnetic hysteresis depends on the position in the B H curve and is certainly not constant. Also the backlash has no loop widening effects.

To get a very simplistic hysteresis model, we can place after the backlash a saturation element. (see figure 5.5.5.a). This “saturation element” will take care of the saturation effect of the hysteresis. To get a S shape hysteresis (see figure 5.5.5.b) it is better to place a function (tangents hyperbolic function) behind the backlash element. To shape the hysteresis model, you can change the dead time of the backlash and the shape of the function.

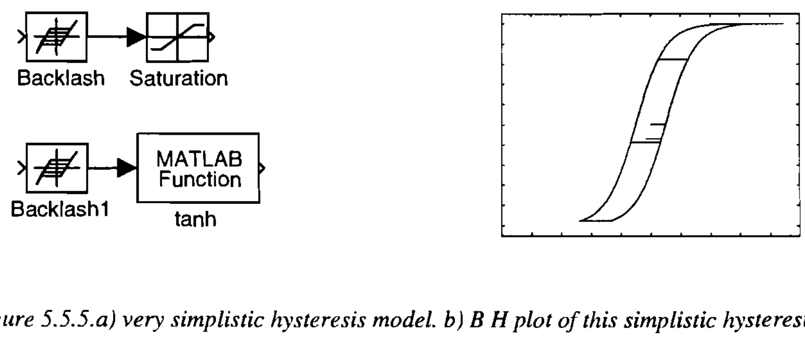


figure 5.5.5.a) very simplistic hysteresis model. b) B H plot of this simplistic hysteresis model.

6) Conclusions

We have described in this thesis 3 hysteresis models (see chapter 4); the Karnopp model, the Chua and Stromsmoe model, and the Jiles Atherton model. We will now make a brief comparison between the models:

accuracy:

The Karnopp model and the Chua and Stromsmoe model are definitely more accurate then the Jiles Atherton model .

model structure:

The Jiles Atherton model has the simplest structure. The structure of the Karnopp model is also quite straight forward. While we have the most freedom (the freedom of choice of the functions (w, f, g, h)) in the Chua Stromsmoe model, this implies that the structure of the Chua Stromsmoe model is the most complex.

calculation time for simulating the model:

The Jiles Atherton model, has a constant calculation time for simulation. This calculation time does not depend on the specific parameter values. The Chua and Stromsmoe model, has also a constant calculation time for simulation. The calculation time of the Karnopp model, depends linear on the number of basic elements that are chosen.

We have observed a number of effects of the magnetic hysteresis in the controlled system (magnetic ball levitation system, see chapter 4.1).

- A wider hysteresis causes less damping.
- The bigger the hysteresis with the bigger the steady state error is, that we get.
- A bigger current through the coil (with the same width of the hysteresis) causes a bigger steady state error.

We can take measures against the negative effects caused by the magnetic hysteresis. Possible measures are:

- To deal with the loss in damping, we can use a different LQ Controller, the controller has to shift the closed loop poles more to the left. This means larger control inputs, which can drive the actuator in saturation in practical situations, also the band width of the actuator is limited. It is better to use a different controller like a H infinite controller. This controller is less sensitive for model errors.
- To deal with the steady state error integral control can be applied.

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