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Investigation of the electro dynamic torque of a stepper motor with a permanent magnet rotor

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Contents

Summary

In preceding investigations the electro magnetic torque of a stepper motor with a permanent magnetic rotor has been calculated as a function of the rotor-position under static conditions.
The parameters in the calculation were the magnetic potential of the stator poles and the magnetization of the rotor.
In this report the investigation is extended with a calculation of the torque under dynamic conditions, with the limitation that the rotor moves with a constant speed, while the stator coil, supplied by a DC voltage source, is commutated in the rithm of the shaft rotation.
The moment of commutation, expressed in the rotorposition, is taken as a parameter.
To verify the measurements the average torque during one revolution is calculated as a function of the switching frequency.
Comparing the measured and the calculated values the conclusion can be drawn that a further investigation has to be made concerning the magnetic reluctance of the stator circuit.

List of symbols

			dimension
A	-	surface area	m^2
A_k	-	cross-section of the stator circuit	m^2
B	-	magnetic induction	Vs/m^2
f	-	frequency of the square wave voltage	Hz
f_s	-	stepping frequency	Hz
$\underline{H}$	-	magnetic field strength	A/m
I, i	-	current in stator coil	A
i_d	-	current in damping ring	A
K	-	flux per ampere turn	Vs/A
$d\underline{l}$	-	vectorial element of length	m
$\underline{M}$	-	intensity of magnetization of the rotor material	A/m
N	-	number of windings of the stator coil	
n	-	number of revolutions per minute	
$\underline{n}$	-	nominal unit vector	
P_{diss}	-	power dissipation	Watt
P_{ec}	-	eddy current losses	Watt
P_{el}	-	supplied electric power	Watt
P_{hyst}	-	hysteresis losses	Watt
p	-	number of pole pairs	
q	-	number of stator parts	
R	-	resistance of stator coil	Ohm
R_d	-	resistance of damping ring	Ohm
$\underline{r}$	-	radius	m
$\underline{T}_e, T_e$	-	torque	Nm
U, u	-	voltage on the stator coil	V
U_p	-	magnetic potential	A
U_s, u_s	-	magnetic potential on stator poles	A
α	-	step angle, also position of the stator parts with regard to each other	
δ	-	skin depth	m
θ	-	relative position of the rotor to the stator	
θ_o	-	relative position of the rotor to the stator at time $t = o$	
μ	-	permeability	Vs/Am
μ_r	-	relative permeability	
ρ	-	specific resistance	Vm/A
σ	-	magnetic surface charge density	A/m
τ, τ_1	-	time constant	s

ϕ_m	- main flux in the stator circuit	Vs
ϕ_r	- amplitude of the rotor flux	Vs
Ω_m, ω	- rotor speed	s^{-1}
ω	- angular frequency	s^{-1}

1. Introduction

In the last few years the use of stepper motors has strongly increased in the field of control systems, especially in speedcontrol and exact positioning. This is a result of a growing application of digital techniques in measuring and controling processes, automation of production etc.
In general minimum demands of a stepper motor are made on:

- torque per unit of volume of the motor.
- fast acceleration and deceleration.
- maximum stepping rate.

To have a clear insight into the operation of the stepper motor and the limiting factors of particular constructions, it is important to give some theoretical reflections on these subjects.
In the section Electro mechanics of the Technological University Eindhoven a series of investigations is made with relation to the torque of several types of stepper motors, like: calculations of the static torque of a stepper motor with a variable reluctance rotor (L1) * and with a permanent magnet motor (L2).
The aim of this report is to give a calculation of the electro dynamic torque of a stepper motor with stator parts of solid iron and a rotor of permanent magnet material, particular at those stepping rates where the real stepping movement of the rotor has passed into a constant angular speed.
Stepper motors of this type are e.g. those of the PD-series, developed in the laboraties of the Philips' Gloeilampenfabrieken N.V. Eindhoven.
In chapter 2 the construction of this motor will be explained into more detail.
The way the electro dynamic torque of such a motor will be influenced by a number of factors is subject of this study.

* (L..) refers to list of publications.

These factors are:

a. Eddy currents in the stator iron as a result of switch over the stator field.
A fast changing stator current causes eddy currents, which retard the rise and decay of the flux in the air gap (L3). Especially at high switching rates the influence of eddy currents is clearly noticeable.

b. Hysteresis of the stator iron.
While the statorfield is switched over each half period, the whole hysteresisloop will be passed through, and the field in the air gap will be deformed.

c. The rotational voltage of the permanent magnet rotor.
This voltage in the stator coil will influence the stator current, when the supply is not an ideal current source.

d. Eddy currents in the stator poles as result of the moving rotor field.
In particular positions of the rotor, the rotor field is "short circuited" by the stator poles.
In that case the field crosses the stator poles. The moving rotor causes an alternating field, through which eddy currents arise.
The dissipated power has to be supplied by through the rotor.

When calculating the electro dynamic torque these factors have to be taken into account. Furthermore the calculation will supply information concerning the "weak points" of this stepper motor. This will be of importance for new developments of stepper motors.

2. Construction and operation of a stepper motor with a permanent magnet rotor

2.1. The Philips stepper motor of the PD-series.

2.1.1. The stator.

The stator consists of a number of q identical stator parts, each with p pole pairs. A stator part is built up by two identical stator plates and a ring of iron, which enclose a coil (fig. 1a, 1b).
When the coil is energized, inside the stator parts the stator poles are alternately north and south poles.
The q stator parts are shifted an angle $\alpha = \frac{360}{2pq}$ degrees with regard to each other.
Figure 2 shows a cross-section of a stepper motor with 4 stator parts, each with 12 pole pairs.
Each stator coil has 2 windings of the same number of turns.
These 2 windings are energized in turn, through which each pole is alternately magnetized.

2.1.2. The rotor.

The rotor consists of a cylinder of ferroxdure 100, fixed on the shaft.
The rotor is laterally magnetized and has just like the stator parts p pole pairs. Figure 3 shows the rotor of a stepper motor with 4 stator parts.

2.1.3. The operation.

The operation will be explained with the help of a model with 4 stator parts and 1 pole pair (see diagrammatical representation figure 4).
The stator parts are all energized. Table 1 relates the excitation of the windings to the position of the rotor. By switching in this sequence the rotor will rotate.

Table 1

stator part 1		stator part 2		stator part 3		stator part 4		rotor position	rotor direction
wind. 1	wind. 2	wind. 1	wind. 2	wind. 1	wind. 2	wind. 1	wind. 2		
x		x		x		x		0°	counter clockwise ↓
	x	x		x		x		45°	
	x		x	x		x		90°	
	x		x		x	x		135°	
	x		x		x		x	180°	
x			x		x		x	225°	
x		x			x		x	270°	
x		x		x			x	315°	
x		x		x		x		360°	clockwise ↑

x = winding energized.

In figure 4 mechanical switches are used. In practice an electronic switch, which energizes the windings in the right sequence, is of course a more ideal solution.

2.1.4. Movement of the rotor.

Figure 5 shows the movement of the rotor at different stepping rates.
In figure 5a the rotor moves in real steps. After the oscillation is damped out the rotor stops in its holding position, before the next step begins.
In figure 5b the stepping rate is higher. The rotor doesn't stop, but the steps are clearly observable.

At high stepping rates, or when the rotor is "damped" by an additional inertia load, the rotor speed will nearly be constant (figure 5c). In this area the investigations are made concerning the electro dynamic torque.

2.2. The test machine.

2.2.1. The construction.

The test machine is a simplification of the stepper motor. The machine consists of one stator part with 4 pole pairs (figure 6).
The stator poles are not trapezium shaped.
The coil has one winding of 1000 turns. Some stator poles carry small coils, which are suitable for measuring the flux. The induction in the air gap will be measured with a Hall probe, which is glued to the inside surface of a stator pole.
The rotor of ferroxdure is laterally magnetized with 4 pole pairs.
The length of the rotor equals the overlap of the stator poles.
On the shaft end of the rotor a disk is fixed (see figure 7).
This disk carries 8 holes on equal distances.
The position of the disk with regard to the shaft can be adjusted.
With aid of a light source, a photo diode and an additional electronic circuit an electric puls is formed each time a hole passes the photo diode.

2.2.2. The operation

The test machine is controled by an electronic switch (figure 8).
The electric pulse of the photo diode forms the input of the electronic switch. At each pulse the current in the stator coil reverses, so the test machine is used as a DC commutator machine. Figure 9 shows the quasi static torque of the machine versus the position of the rotor θ , if the coil is excited.
If the current in the stator coil reverses at a position θ between θ' and θ'', the motor delivers a positive average torque (see chapter 3).

The correspondence of the DC commutator machine to the stepper motor is the following:
The commutator machine has a fixed relative position of the rotor to the stator at the moment of switching, independent of speed and torque. The relative position can only be changed by changing the position of the disk with regard to the rotor shaft.
The stepper motor is an open loop system. The pulses to control the motor must be applied by a "strange" pulse source. The relative position of the rotor to the stator at the moment of switching will adjust itself, depending on the load and the switching rate. We have already stated that the rotor doesn't move stepwise but moves with a constant speed (figure 5c). So with a certain constant load torque , the above mentioned relative position will also be a constant.

A fixed resistance of 72 Ohm is connected in series with the statorcoil. The resistance of the statorcoil is 8 Ohm, so we have the possibility to do some of the measurements with two values of the resistance of the statorcircuit: 8 resp. 80 Ohm. (see chapter 4).
In combination with the electronic switch we use always the resistance of 80 Ohm.

3. Theoretical reflections on the electro dynamic torque

When we consider the electro dynamic torque of a stepper motor, we have to make a distinction between:

- the torque as a function of the rotorposition.
- the average torque during one step.

Figure 9 a shows a static torque of the motor versus rotor position, when the stator is energized.
When the current in the statorcoil reverses each time the rotor has moved one pole pitch, beginning with a rotor position between θ' and θ'' the average torque will be positive. In figure 9b the moment of reversing is chosen at rotor position $\theta = 0$.
Suppose the stator current is supplied by a current source and the influence of eddy currents may be neglected, then the dynamic torque will have the form as shown in the figure. When the form is a sine wave, the average torque will have the value $\frac{2}{\pi} T_{max}$.
The torque as a function of the rotor position has a basic harmonic frequency, which equals the switching frequency.
The value varies from zero to T_{max}.
The knowledge of the variation and the frequency is important with regard to the resonance and the overshoot of the motor in the application.
The average torque gives insight in e.g.:

- the power the motor can supply at different speeds.
- the frequency at which the motor can start with an additional inertia and friction load.
- the admissable change in the switching frequency under certain load conditions etc.

3.1. Torque calculation with the help of "Maxwell's stresses"

The electro magnetic torque $\underline{T}_e$ at a body can be expressed by the summation (L4):

$$\underline{T}_e = \frac{1}{\mu_0} \int_A \underline{r} \times \left\{ (\underline{B} \cdot \underline{n}) \, \underline{B} - \tfrac{1}{2} B^2 \, \underline{n} \right\} dA \qquad (3.1.)$$

with: A - surface of the body
$\underline{r}$ - radius
B - magnetic induction
$\underline{n}$ - normal unit vector on the surface

When calculating the motor torque the surface A can be both the surface around the rotor and the surface of the stator.
Is surface A situated in the air relation 3.1. becomes.

$$\underline{T}_e = \mu_0 \int_A \underline{r} \times \left\{ (\underline{H} \cdot \underline{n}) \, \underline{H} - \tfrac{1}{2} H^2 \, \underline{n} \right\} dA \qquad (3.2.)$$

with: H - magnetic field strength

In (L2) the quasi static torque of the test machine is calculated as follows.
The stator poles are assumed to have a constant magnetic potential $+U_s$ respectively $-U_s$.
The line integral for the magnetic circuit is in this case (see figure 10):

$$\oint \underline{H} \cdot d\underline{l} = \int_{iron} \underline{H} \cdot d\underline{l} + 2\,U_s = IN \qquad (3.3.)$$

with: I - current in the statorcoil
N - number of windings of the statorcoil

If we neglect the magnetic reluctance of the stator iron we learn from relation 3.3.:

$$U_s = \frac{IN}{2} \qquad (3.4.)$$

In a first approximation the magnetisation of the rotor wll be replaced by a magnetic surface charge:

$$\sigma = \underline{n}.\underline{M} \qquad (3.5.)$$

with: σ - magnetic surface charge density
$\underline{n}$ - normal unit vector on the rotor surface
$\underline{M}$ - intensity of magnetisation of the rotor material.

With a computer program the magnetic potential U_p in each point along the rotor or stator surface can be calculated and also the magnetic field strength:

$$\underline{H} = - \nabla U_p \qquad (3.6.)$$

Relation (3.2.) gives us the electro magnetic torque.

When calculating the electro dynamic torque, we use the same procedure, but in this case the relation for U_s and $\int \underline{H}.d\underline{l}$ will change.
The stator current is no longer a constant, but reverses with the consequences that eddy currents will appear. While the rotor is moving, a rotational voltage will influence the stator current.

These things are taken into account in the appendix, where a relation is found for the magnetic potential u_s:

$$u_s = \tfrac{1}{2}\left(\frac{1}{K} - \frac{1}{\mu_o \mu_r}\sum_k \frac{l_k}{A_k(f)}\right)\phi_m + \frac{\hat{\phi}_r}{2K} \cos(\omega t + \theta_o) \qquad (3.7)$$

with $$\phi_m = \phi_{(=)} \left(1 - \frac{2e^{-\frac{t}{\tau}}}{1 + e^{-\frac{\pi}{\omega\tau}}}\right) - \phi_{(\sim)} \cos(\omega t - \varphi_2 + \theta_o)$$

and $$\varphi_2 = \arctan \omega \left(\frac{K}{R_d} + \frac{KN^2}{R}\right)$$

3.2. Torque calculation with the power balance

For a machine operating in steady state publication (L5) give the relation:

$$\left[P_{el}\right]_{average} = \left[P_{diss}\right]_{average} + \left[T_e \frac{d\theta}{dt}\right]_{average} \qquad (3.8.)$$

with: P_{el} - the supplied electric power

P_{diss} - the power dissipated in copper and iron

T_e - torque supplied to the rotor

$\frac{d\theta}{dt}$ - rotor speed

When the rotor speed is a constant, like we stated in chapter 2.1.4., then:

$$\frac{d\theta}{dt} = \Omega_m$$

and the relation for the torque is now:

$$\left[T_e\right]_{average} = \frac{\left[P_{el}\right]_{average} - \left[P_{diss}\right]_{average}}{\Omega_m} \qquad (3.9.)$$

Considering our stepper motor we can make the following remarks:

- For the supplied electric power we can write:

$$P_{el} = u.i \tag{3.10}$$

with: u - the voltage over the stator coil
i - the current in the stator coil of which the form depends on: the resistance and inductance of the coil; the rotational voltage; the switching frequency.

- The dissipated power we can split up as follows:

$$P_{diss} = i^2R + P_{ec} + P_{hyst.} \tag{3.11}$$

with R - resistance of the stator coil
P_{ec} - eddy current losses, caused by changing of the flux in the iron.
These losses are inversely proportional to the specific resistivety of the iron.
$P_{hyst.}$ - hysteresis losses, which are proportional to the area of the hysteresisloop of the iron and to the switching. frequency.

- The angular speed Ω_m can be defind as follows:

$$\Omega_m = \alpha . f_s \tag{3.12}$$

with α - step angle in radians
f_s - the stepping frequency

3.3. Summary

Both ways of calculating the torque require an extensive information regarding the magnetic field in the motor.

As the configuration is very complicated, we take use of the way already mentioned in chapter 3.1. and for a great deal worked out in (L2).
Chapter 3.2. gives an idea of the losses we have to expect.

4. Measurements on the test machine

4.1. The hysteresis of the stator iron.

The hysteresisloop of the stator iron is measured as follows.
At different values of the direct current in the stator coil, the induction in the air gap is measured by means of a Hall probe. The direct current is gradually varied from - 1 amp. to + 1 amp. and again back to - 1 amp.
The position of the rotor will influence the zero of the coordinate axis. Therefore the rotor is locked so that the rotor magnetization will not cause a flux through the main circuit of the stator.
The hysteresisloop is produced in figure 11.
As a result ofthe hysteresis the induction is distorted and place shifted with regard to the magnetic field.
However the measured hysteresis is very small and the stator current will not exceed the value of 1 amp. for that reason the influence of the hysteresis on the course of the air gap induction will be neglected.

4.2. The air gap flux as a function of the time with a locked rotor.

In this measurement the electronic switch is controlled by a seperate pulse generator (see chapter 2.2), so the stator coil is supplied by a square wave voltage of which the frequency is adjustable.
The rotor is locked, so there is no rotation voltage.
The stator current is measured with the help of the voltage on a measuring resistance which is connected in series with the stator coil.
The flux in the air gap is measured with the Hall probe.
Both signals are supplied to the differential input terminals of a double beam oscilloscope.

The vertical deviation of both signals is so adjusted that on the screen the amplitudes of both signals at low frequency ($\lesssim$ 3Hz) are equal. At these frequencies both signals reach their final value (see gigure 12 a) At higher frequencies the vertical amplification is not changed. The time base (horizontal deviation) is so adjusted that a clear picture is obtained of the course of the signals as a function of the time. Figures 12 a to h show the course of the stator current and the air gap flux at different frequencies.
Especially at higher frequencies the difference in time constant is obvious. For instance at 100 Hz the top value of the current has decreased to 45% of the final value and the top value of the flux to 30% of its final value. There is also a delay of the flux, due to the eddy currents.

4.3. The rotational voltage

Figure 13 shows the R.M.S. rotational voltage as a function of the frequency. The frequency f is related to the number of revolutions as follows:

$$f = \frac{n.p}{60} \quad \text{Hz} \tag{4.1.}$$

with p - number of pole pairs
n - revolutions of the rotor per minute

This rotational voltage results in a current, if the stator coil is short circuited.Figure 14 gives this R.M.S. current versus frequency at two values of the resistance in the stator circuit.
This current strongly influences the flux in the magnetic circuit of the stator. In figure 15 we see the R.M.S. flux through the stator poles as a function of the frequency with unexcited stator and moving permanent magnet rotor. The flux is measured with the test coil shown in figure 10. The measured value is multiplied by four to get the total flux through the stator poles.

The flux drops to 50% of its value if the stator coil is short circuited.
The phase shift of the stator current with relation to the air gap flux is given in figure 16.

4.4. The "loss torque".

We call the "loss torque" the torque which is needed to rotate the rotor when the stator coil is not excited (see figure 17). This loss torque consists of:
- friction of the bearings
- iron losses (eddy current and hysteresis losses)
- copper losses, if the stator coil is short circuit.

Figure 18 shows the power dissipation with relation to these losses.

4.5. The "torque speed characteristic" of the test machine.

In chapter 2.2 we explained the working of the test machine.
The rotor of the test machine is coupled to the rotor of a cradle type dynamo meter so the average torque can be measured. The mass moment of inertia of the both rotors together is sufficient to guarantee a rotor movement as shown in figure 5 c. The final value of the stator current is 1 amp. and the supply voltage being 80 Volts DC (see figure 8).
Figure 19 shows the average torque as a function of the frequency of the stator current (and as a function of the number of revolutions per minute of the rotor).
The average torque is measured at different relative positions θ_s of the disk to the rotor.
The working at $\theta_s = 0°$ was already shown in figure 9 b.
In this case the stator current reverses at the moment the axis of the rotor poles coincide the axis of the stator poles of opposite polarity. Positive values of θ_s mean the moment of reversing is advanced. Negative values of θ_s give a delayed moment of reserving of the current.

The curve which covers all measured curves, gives the extreem average torque the motor can supply if used as a stator part of a real stepper motor.

5. Calculations

5.1. Comparison of measurement and theory.

Appendix A gives the theory to calculate the magnetic potential of the stator teeth. The final relation is mentioned in chapter 3. With the aid of the measuring results the constants of the relation (A.18) are defined.

5.1.1. The circuit flux with unexcited stator coil and moving rotor.

Defining the circuit flux ϕ_m as the N-th part of the coupled flux ϕ , we have the relation:

$$u = Ri + \frac{d\phi}{dt} \qquad (5.1.)$$

with: u - voltage of the stator circuit
i - current in the stator coil
R - resistance of the stator circuit
N - number of turns of the stator coil

Figure 13 shows the RMS voltage of u if the stator circuit is open, so i=o. If the distorsion is neglected we find the RMS value of the circuit flux from relation

$$RMS(\phi_m) = \frac{RMS(u)}{\omega N} \qquad (5.2)$$

with: ω - angular frequency

With the stator circuit short circuited, u=o in the relation (5.1). With the aid of the RMS stator current of figure 14, we find:

$$RMS(\phi_m) = \frac{R}{\omega N} RMS(i) \qquad (5.3)$$

Figure 20 shows the RMS circuit flux versus the frequency. With the aid of some values in these curves we calculate the constants of relation (A.10) of the appendix:

$$\phi_{m_1} = \frac{\hat{\phi}_r}{\sqrt{1+\omega^2 \tau^2}} \cos(\omega t - \pi - \varphi_2 + \theta_o) \qquad (5.4.)$$

$$\varphi_2 = \arctan \omega \tau$$

$$\tau = \frac{K}{R_d} + \frac{KN^2}{R}$$

The known constants in these relations are:

N = 1000 turns

R = $\sim$, 80 Ohm resp. 8 Ohm (Chapter 2.2.2.)

The calculated constants are:

$\hat{\phi}_r$ = 0,243 . 10^{-3} Vs

K = 76. 10^{-8} Vs/A

R_d = 6,2 . 10^{-4} Ohm

A number of values calculated with relations (5.4.) are indicated in figure 20. In the following calculations the resistance of the stator circuit will be 80 Ohm.

5.1.2. The air gap flux with a locked rotor and an excited stator.

In the figures 12a to 12h we find the air gap flux versus time with a locked rotor and the stator circuit controlled by the electronic switch. The course of this flux is shown once more in figure 21. This curve can approximately be expressed by the relation

$$\phi_1 = \phi_E \left(1 - 2e^{-\frac{t}{\tau_1}}\right) \qquad (5.5.)$$

with ϕ_E - the final value of the air gap flux

τ_1 - the time constant

With a time constant $\tau_1 = 15{,}3$ msec., some values of ϕ_1, are calculated and indicated in figure 21.

In the appendix we found for the time constant of the main flux (relation A.12)

$$\tau = \frac{K}{R} N \left(\frac{R}{R_d N} + N \right) \qquad (5.6.)$$

When calculating this time constant with the constants mentioned in chapter 5.1.1. we find $\tau = 10{,}7$ msec.

Considering the complicated magnetic path, the leakage flux (see chapter 6) and the influence of the permeability of the iron, it is no wonder that there is a difference between the time constant measured in the air gap and the calculated time constant of the main flux. It is difficult to predict the influence on the value of the time constant. For that reason we decided to use the time constant of relation (5.6) in relation (5.4) and the time constant $\tau_1 = 15{,}3$ msec in relation (5.7) Relation (A.13) gives the main flux if the voltage on the stator coil is a square wave voltage with an amplitude U and a period time T.

The main flux is during the interval $kT \leqslant t \leqslant (k+\frac{1}{2})\, T$:

$$\phi_{m_2} = \phi_{(=)} \left(1 - \frac{2e^{-\frac{t}{\tau_1}}}{1+e^{-\frac{\pi}{\omega\tau_1}}}\right) \qquad (5.7)$$

with $\omega = 2\pi f = \frac{2\pi}{T}$

$\phi_{(=)} = \frac{U}{R}\, KN$

U - amplitude of the square wave voltage

R - resistance of the stator circuit

f - frequency of the square wave voltage

τ_1 - time constant

The values of the constants are:

U = 80 Volt

R = 80 Ohm

τ_1 = 15,3 msec.

5.1.3. The calculation of the term $\sum\limits_k \frac{l_k}{\mu_o \mu_r A_k(f)}$

This term cannot be calculated with sufficient accuracy, while in the different cross-sections the value of the relative permeability μ_r will not be constant. Furthermore the active cross-section is depending on the frequency of switching, in consequence of the skin-effect.

The stator material is the so called Armco-iron. For low values of the magnetic field strength the permeability of the iron is:

$$\mu = 500 \text{ gauss/oersted} = 2\pi \cdot 10^{-4} \text{ Vs/Am}$$

At higher values of the magnetic field strength the value of the permeability increases to a maximum of:

4000 gauss/oersted = $16.\pi\ 10^{-4}$ Vs/Am.
As a first approximation we suppose the value of the permeability to be a constant of $2.\pi\ 10^{-4}$ Vs/Am.
Furthermore we take the active cross-section being independent of the frequency. The dimensions of the stator circuit are given in (L2). With the aid of these dimensions we find for the expression:

$$\sum_k \frac{l_k}{A_k} = 131{,}3 \quad m^{-1}$$

5.1.4. Summary of the chapters 5.1.1. to 5.1.3.

For the relation 3.7. we have found the expression:

$$u_s = \frac{1}{2}\left(\frac{1}{K} - \frac{1}{\mu}\sum_k \frac{l_k}{A_k}\right)\phi_m + \frac{\hat{\phi}_r}{2K}\cos(\omega t + \phi_o) \qquad (5.8)$$

with:

$$\phi_m = \frac{U}{R} KN \left(1 - \frac{2e^{-\frac{t}{\tau_1}}}{1 + e^{-\frac{\pi}{\omega \tau_1}}}\right) - \frac{\hat{\phi}_r}{\sqrt{1+\omega^2\tau^2}}\cos(\omega t - \varphi_2 + \phi_o)$$

$\varphi_2 =$ arctan $\omega\tau$ $\qquad$ $\omega = \frac{2\pi}{T}$

$\tau = \frac{K}{R_d} + \frac{KN^2}{R}$ $\qquad$ $kT \leq t \leq (k+\frac{1}{2})T$

The constants are:

$K = 76.10^{-8}$ Vs/A
$N = 1000$ turns
$R = 80$ Ohm
$R_d = 6{,}2.10^{-4}$ Ohm
$U = 80$ Volt
$\hat{\phi}_r = 0{,}243.10^{-3}$ Vs
$\tau_1 = 15{,}3.10^{-3}$ sec.

$$\frac{1}{\mu}\sum_k \frac{l_k}{A_k} = 0{,}208.10^{-6} \quad A/Vs$$

5.2. Computer program.

In publication (L2) we find the quasi-static torque of the test machine measured and calculated for several values of the magnetic potential on the stator poles (see figure 22). The parameter in this figure is not the magnetic potential on the stator poles, but the current in the stator coil. The relation between the magnetic potential U_s and the current in the stator coil I is in this case:

$$U_s = \frac{NI}{2} \tag{5.9.}$$

with N - number of turns

If we calculate the electronic dynamic torque, with the aid of these curves, we have to take as parameter, in stead of u_s:

$$\frac{2u_s}{N} \tag{5.10}$$

Like we have shown in figure 9b it is sufficient to calculate the electro dynamic torque during half a period of the square wave voltage on the stator coil. We have stated in chapter 2 that the rotor velocity is constant, so there is a fixed relation between the rotor position and the time. The way of calculation is the following.
With the computer the values of $\frac{2u_s}{N}$ (for u_s see relation 5.8) are calculated during the interval $kT \leq t \leq (k + \frac{1}{2})T$ with gradually increasing t. The value of θ_o in relation 5.8 is the opposite of the relative position θ_s of the rotor to the stator at the time $t = kT$. At the instantaneous rotor positions, belonging to the points of time in which the values of $\frac{2u_s}{N}$ are calculated, the electro dynamic torque is calculated with the aid of the values of the static torque shown in figure 22.

For values between the parameter values mentioned in this figure, the torque is found by interpolation. In figure 23 we give the method of determining the electro dynamic torque in a graphical way. In this figure we see the influence of the frequency and the value of θ_s on the electro dynamic torque as a function of the rotor position. It gives also an impression of the average torque.
With the computer program we also calculate the average torque during one "step" with the aid of the above mentioned electro dynamic torque versus rotor position. This average torque as a function of the frequency is plotted for several values of θ_s (see figure 24).

6. Conclusions.

6.1. Comparison of measurements and calculation results.

If we add the loss torque (figure 17) to the measured torque (figure 19), we get the average torque (figure 25) which we have to compare with the calculated torque (figure 24).
Comparing these two figures, we can make the following remarks:

a. In general the calculated torque is too high. The difference between the calculated and the measured torque is increasing as a function of the frequency (see figure 26).
In chapter 5 we stated that the magnetic reluctance is independent of the frequency. Regarding the above mentioned results, we have the indication that this approximation is too rough.

b. In the calculations at lower frequencies ($<$ 40Hz) the average torque with θ_s = 80° respectively 100° is lower than the average torque at θ_s = 60°. This is conform to the measurements. The fact that the torque will be lower at high values of θ_s, we can understand when comparing figures 23a and 23b.

c. At higher frequencies and high values of θ_s the calculated curves almost coincide. This is conform to the measurements.
The fact that these curves coincide, we can understand if we consider figure 23d. A little change in the value of θ_s will not strongly influence the average torque.

Summerizing these results we can state that the measured and the calculated curves conform rather well concerning the shape.
The size of the calculated torque differs from the measured values.

A further investigation has to be made concerning the magnetic reluctance of the stator circuit, specially with regard to the frequency.

6.2. Comments on the test results.

6.2.1. The leakage flux.

Like already stated in chapter 5 in the magnetic circuit a leakage flux excists, which influences the measuring results. If we compare the figures 15 and 20, we see that the difference between the fluxes with an open and a closed stator coil is greater in the main circuit than in the stator poles. This can be explained as follows (see figure 27).
The rotor flux coming from the north pole on the rotor surface, passing the stator and going to the south pole on the rotor surface, can take 3 ways through the stator iron.

- from the north pole into the stator pole, sideways through the air to the next stator pole and to the south pole (figure 27a).
- from the north pole of the rotor, into the stator pole, through the stator pole and the flange, via the air to the top of the next stator pole, to the south pole of the rotor (figure 27 b).
- from the north pole of the rotor through the stator pole, through the main circuit to the next stator pole and to the south pole of the rotor (see figure 27c).

If we close the stator coil the stator current will contribute to the main flux. This contribution will have a direction opposite to the exciting flux, and will equally be spread over the stator poles (see figure 27d). So the flux through a stator pole will not decrease as strongly as the flux in the main circuit if the stator coil is closed.

Regarding the leakage flux of the figures 27a and 27b it is not astonishing that the iron losses don't decrease strongly if the stator coil is closed (figure 18), in spite of the low value of the flux in the main circuit (figure 20).
Another source of iron losses is the alternating flux, caused by the moving rotor and transversing the stator poles, like shown in figure 28. This flux will not be influenced by the stator current.

6.2.2. The torque-speed characteristics.

In chapter 2.2.2. we have already described that the relative rotor position θ_s with regard to the stator, at the moment of switching, will adjust automatically, depending on the load torque. Increasing the switching frequency and increasing the load torque of a stepper motor up to the maximum average torque, the value θ_s will also increase.
The same appears with the test machine (figure 19).
To get the maximum average torque from the motor we have to increase the value of θ_s (see table 2).

Frequency	θ_s for max. torque
10 Hz	40°
25 Hz	60°
50 Hz	80°

Table 2.

If the test machine is used as a stator part of a stepper motor with q identical stator parts (see chapter 2.1.) the maximum average torque per stator part will be the curve which covers the measured curves of figure 19.

6.3. Suggestions for new stepper motor developments.

6.3.1. The investigation has proved that a stepper motor with a magnetic circuit of solid iron has a disadvantage at high frequencies, viz.:

- retardation of the flux with regard to the stator current.
- iron losses, which increase with increasing switching frequency. This results in an decreasing efficiency and an increasing temperature rise of the motor.

It's of great importance to reduce the eddy currents. The skin depth δ of the flux in the iron is proportional to:

$$\delta \;::\; \sqrt{\frac{\rho}{\mu\ \omega}}$$

with ρ - specific resistively of the iron.
ω - angular frequency
μ - permeability of the iron.

To reduce the eddy currents we have to enlarge the specific resistivety or (and) to reduce the dimensions of the solid iron parts (lamination).
If a special stator construction is chosen, in most cases the properties of the iron are fixed, so the specific resistivety too.
E.g.
- The PD-stepper motor of Philips has stator plates of iron with a high deep-draw quality. The Si-percentage is neglectable (L6) and the specific resistivety is:

$$\rho = 10.10^{-8} \quad Vm/A$$

- Laminated iron, used in electrical machines, has a Si-percentage of about 4% so the specific resistivety is:

$$\rho = 55.10^{-8} \quad Vm/A$$

Where it's possible in the construction of a stepper motor, laminated iron has to be used.
For parts of which lamination is not possible other materials have to be looked for, like:
- ferrites

- iron powder or grains with a electrically isolating filler.

In the last case the magnetic reluctance of the stator circuit will increase, so at low frequencies the torque will be reduced, but at higher frequencies the reduction of the eddy currents may compensate the increase of the magnetic reluctance concerning the influence on the torque.

6.3.2. Like already stated in chapter 6.2.1. and figure 27 the leakage flux reduces the flux in the main circuit. To reduce the leakage flux we have to enlarge the distance d (figure 27). A disadvantage is that the length of the stator poles also increases, so the reluctance of the magnetic circuit increases, especially at high frequencies.
In this case an optimum has to be found.

6.4. Suggestions for further investigations.

6.4.1. In the calculation we approximated the magnetic reluctance very roughly. It is recommendable to analyse the influence of μ_r and A_k (f), especially as a function of the frequency. Furthermore it is interesting to know, how the iron losses are spread over the stator iron. Also the influence of the dimensions of the stator poles on the iron losses (figure 28) could be a subject of further investigations.

6.4.2. In this report we have calculated and measured the average torque as a function of the frequency.
With the aid of the computer program we can also plot the electro dynamic torque versus rotor position (the instantaneous torque versus time) of the test machine.
This torque will give us an indication of the oscillations on the rotor speed we have to expect, if the inertia of the rotor and the load is known.

An extension of the calculation can be made, if we do not take the rotor speed as a constant.
With a second order differential equation calculations can be made concerning the starting and stopping characteristics, the instable areas in the stepping rate etc.

6.4.3. We have based our calculations on linear magnetic properties of the stator iron (see measurements chapter 4). A further extension of the investigation can be made, if we take the magnetic properties being non-linear. The measurements should be done at higher values of the stator current, to ensure saturation of the stator iron.

Appendix

A.1. On the basis of a simplified model, we will determine a relation for the magnetic protential of the stator-poles, if eddy currents are taken into account. (see figure 29)

We represent the influence of the eddy currents in the stator iron by a damping ring, which is fully coupled with the magnetic main circuit.

We make the following assumptions:

a. The stator coil and the damping ring are fully coupled.
b. The moving rotor causes a flux through the stator with a constant amplitude, independent of the frequency, and with a singular harmonic form.
c. The stator iron is not saturated.

A.1.1. The main flux ϕ_m in the stator cicuit is:

$$\phi_m = K\,(iN + i_d) - \hat{\phi}_r \cos(\omega t + \theta_o) \qquad (A.1.)$$

with:
- K - flux per ampere turn
- i - current in the stator coil
- i_d - current in the damping ring
- N - number of turns of the stator coil
- $\hat{\phi}_r$ - amplitude of the rotor flux
- θ_o - relative position of the rotor to the stator at time t = 0
- ω - angular velocity of the rotor.

With relation to the assumptions we made the main flux can also be expressed as:

$$\phi_m = K\,\left(iN - \frac{1}{R_d}\frac{d\phi_m}{dt}\right) - \hat{\phi}_r \cos(\omega t + \theta_o) \qquad (A.2.)$$

with: R_d - resistance of the damping ring.

Of (A.2.) we learn that the relation for the current in the stator coil is:

$$i = \frac{\phi_m}{KN} + \frac{1}{R_d N} \cdot \frac{d\,\phi_m}{dt} + \frac{\hat{\phi}_r}{KN} \cos(\omega t + \theta_o) \qquad (A.3.)$$

The voltage u on the stator coil is:

$$u = iR + N \frac{d\,\phi_m}{dt} \qquad (A.4.)$$

with: R - resistance of the stator coil.

If we combine relations (A.3.) and (A.4.), we get after some manipulations:

$$\frac{R}{KN}\phi_m + \left(\frac{R}{NR_d} + N\right)\frac{d\,\phi_m}{dt} = u - \frac{R\,\hat{\phi}_r}{KN}\cos(\omega t + \theta_o) \qquad (A.5.)$$

If u is a square wave voltage with an amplitude U, the stationary solution of this relation, during the interval $kT \leqslant t \leqslant (k+\frac{1}{2})T$, is:

$$\phi_m = \phi_{(=)}\left(1 - \frac{2e^{-\frac{t}{\tau}}}{1 + e^{-\frac{\pi}{\omega\tau}}}\right) - \phi_{(\sim)}\cos(\omega t - \varphi_2 + \theta_o) \qquad (A.6)$$

with: $\phi_{(=)} = \frac{U}{R} KN$

$$\phi_{(\sim)} = \frac{\hat{\phi}_r}{\sqrt{1 + \omega^2\tau^2}}$$

$$\varphi_2 = \arctan \omega\tau$$

$$\tau = \frac{K}{R_d} + \frac{KN^2}{R}$$

$$T = \frac{2\pi}{\omega}$$

This solution consists of two parts:

- the flux ϕ_{m_2} as a result of the excitation of the stator coil (see chapter A1.1.3)
- the flux ϕ_{m_1} as a result of the moving permanent magnet rotor (see chapters A1.1.1 and A 1.1.2)

A.1.1.1. The stator coil is open $R = \infty$

In this case relation (A.5.) becomes:

$$\frac{1}{K}\phi_m + \frac{1}{R_d}\frac{d\phi_m}{dt} = -\frac{\hat{\phi}_r}{K}\cos(\omega t + \theta_o) \quad (A.7.)$$

The solution is:

$$\phi_m = -\phi'_{(\sim)}\cos(\omega t - \varphi_1 + \theta_o) \quad (A.8.)$$

with: $\phi'_{(\sim)} = \dfrac{\hat{\phi}_r}{\sqrt{1 + \omega^2 \tau'^2}}$

$\varphi_1 = \arctan \omega \tau'$

$\tau' = \dfrac{K}{R_d}$

A.1.1.2. The stator coil is short circuited $u = o$.

Now relation (A.5.) becomes:

$$\frac{R}{KN}\phi_m + \left(\frac{R}{NR_d} + N\right)\frac{d\phi_m}{dt} = -\frac{R\hat{\phi}_r}{KN}\cos(\omega t + \theta_o) \quad (A.9.)$$

The solution of this relation is:

$$\phi_{m_1} = -\phi_{(\sim)}\cos(\omega t - \varphi_2 + \theta_o) \quad (A.10)$$

with: $\phi_{(\sim)} = \dfrac{\hat{\phi}_r}{\sqrt{1 + \omega^2\tau^2}}$

$\tau = \dfrac{K}{R_d} + \dfrac{KN^2}{R}$

$\varphi_2 = \arctan \omega \tau$

A.1.1.3. The rotor is locked in the position $\theta_o = \frac{\pi}{2}$.
u is a square wave voltage with an amplitude U.

Relation (A.5) is in this case:

$$\frac{R}{KN} \phi_m + \left(\frac{R}{NR_d} + N \right) \frac{d\phi_m}{dt} = u \qquad (A.11)$$

The solution of this relation, if u jumps from -U to +U, is:

$$\phi_m = \phi_{(=)} (1 - 2e^{-\frac{t}{\tau}}) \qquad (A.12)$$

with $\phi_{(=)} = \frac{U}{R} KN$

$$\tau = \frac{K}{R_d} + \frac{KN^2}{R}$$

If u is a square wave voltage and if the switch on phenomena are damped out, the solution during a half period is:

$$\phi_{m_2} = \phi_{(=)} \left(1 - \frac{2\, e^{-\frac{t}{\tau}}}{1 + e^{-\frac{\pi}{\omega\tau}}} \right) \qquad (A.13)$$

with $kT \leqslant t \leqslant (k+\frac{1}{2})T$

$$\omega = \frac{2\pi}{T}$$

$$\phi_{(=)} = \frac{U}{R} KN$$

$$\tau = \frac{K}{R_d} + \frac{KN^2}{R}$$

A.1.2. We call the magnetic potential on the stator poles $+u_s$ respectively $-u_s$. For the main magnetic circuit we have the following expression (figure 29):

$$\int_c \underline{H}.d\underline{l} = \int_{iron} \underline{H}.d\underline{l} + 2u_s = iN + i_d \qquad (A.14)$$

with: $\underline{H}$ - magnetic field strength
c - integration contour

If the integration contour is chosen along the H-lines, we find for the magnetic potential on the stator poles:

$$u_s = \frac{iN + i_d}{2} - \frac{1}{2}\int_{iron} H.dl \qquad (A.15)$$

We derive from relation (A.1) an expression for $iN + i_d$:

$$iN + i_d = \frac{\phi_m}{K} + \frac{\hat{\phi}_r}{K}\cos(\omega t + \theta_o) \qquad (A.16)$$

To calculate $\int H.dl$ we devide the iron circuit into k suitably chosen parts. The cross-section of part k is A_k and the length l_k.
Furthermore the active cross-section is a function of the frequency in concequence of the skin-effect, due to the eddy currents.
If we suppose that the main flux is equally spread over the active cross-section $A_k(f)$, the relation $\int H.dl$ becomes now:

$$\int_{iron} H.dl = \int_{iron} \frac{B}{\mu_o \mu_r} dl = \frac{\phi_m}{\mu_o \mu_r} \sum_k \frac{l_k}{A_k(f)} \qquad (A.17)$$

Combining relations (A.15), (A.16) and (A.17) we find for the magnetic potential:

$$u_s = \frac{1}{2}\left(\frac{1}{K} - \frac{1}{\mu_o \mu_r}\sum_k \frac{l_k}{A_k(f)}\right)\phi_m + \frac{\hat{\phi}_r}{2K}\cos(\omega t + \theta_o) \qquad (A.18)$$

with: ϕ_m - see relation (A.6)

List of publications

L1 A.J.C. Bakhuizen — De bepaling van het stationaire koppel van reluctantie stappenmotoren 1. Report EM 69-4. Technological University Eindhoven. (1969)

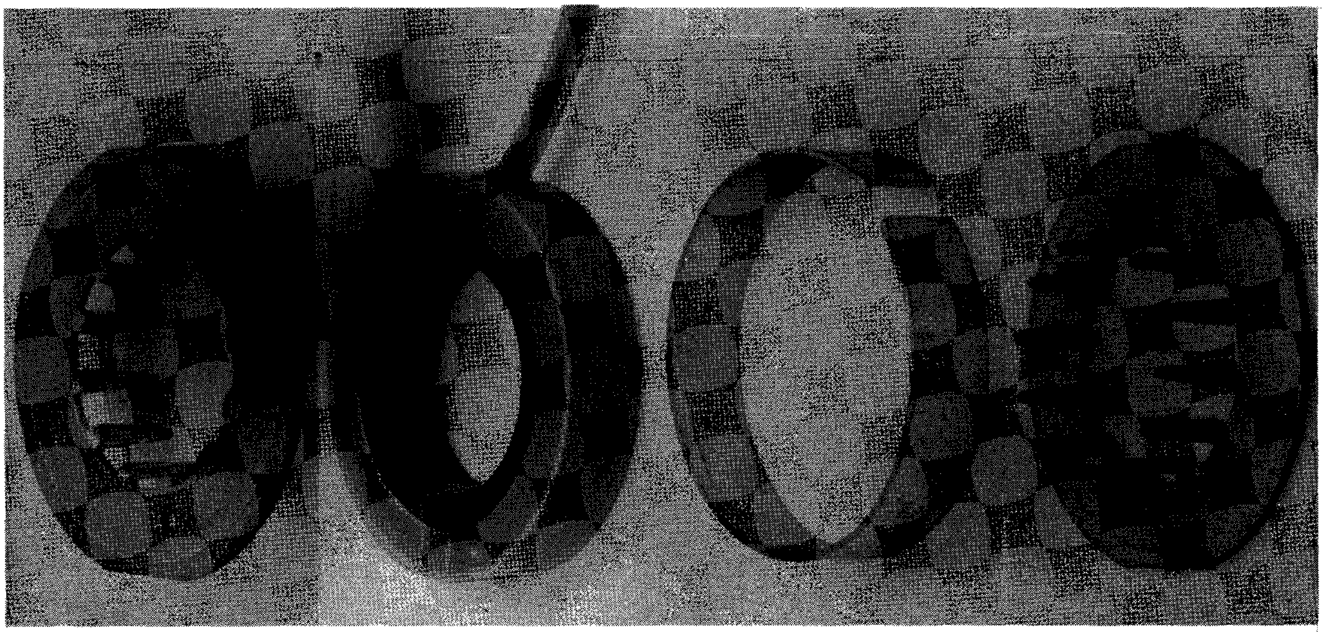
L2 B.M. Suurmeyer — A torque calculation of the multipole permanent magnet stepping motor. Report EM 69-13. Technological University Eindhoven. (1969)

L3 R. Pohl — Rise of flux due to impact excitation Retardation by eddy currents in solid parts. Proc. I.E.E. part II. (1949)

L4 J.A. Stratton — Electromagnetic Theory Mac Graw Hill Book Company Inc. New York. (1941)

L5 J.D. van Wijk — Electronic control of electromechanical energy conversion in electrical machines. Brondes Offset N.V. Rotterdam. (1969)

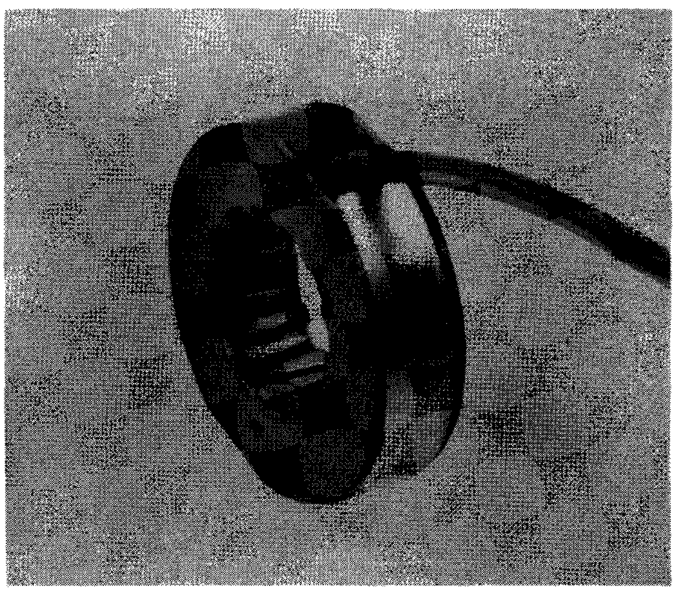
L6 C. Zwikker — Fysische Materiaalkunde deel I. Wetenschappelijke Uitgeverij N.V. (1966)



Stator plate Coil Ring Stator plate

Number of stator poles p = 12

Figure 1a.



A Stator part

Figure 1b.

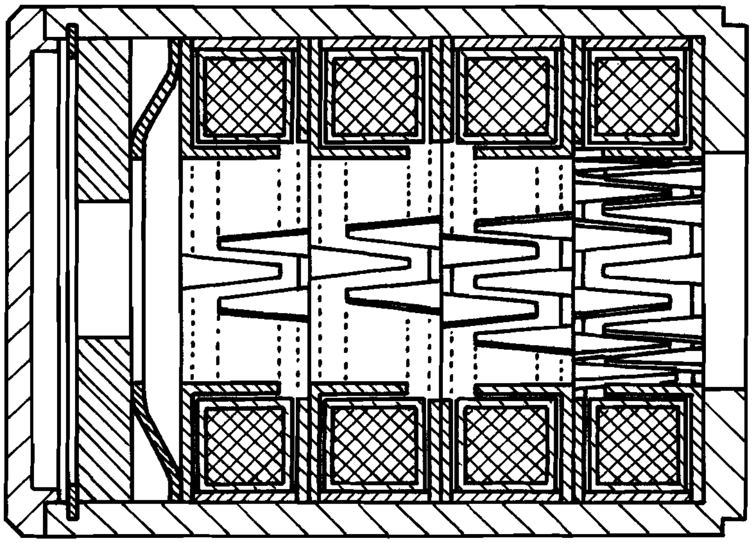


Figure 2. Stator with a number of stator parts q = 4

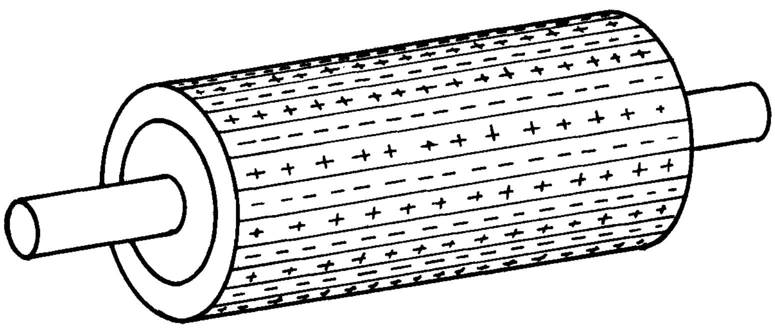


Figure 3. A laterally magnetized rotor of a motor with 4 stator parts. On the surface the north and south poles are schematically indicated.

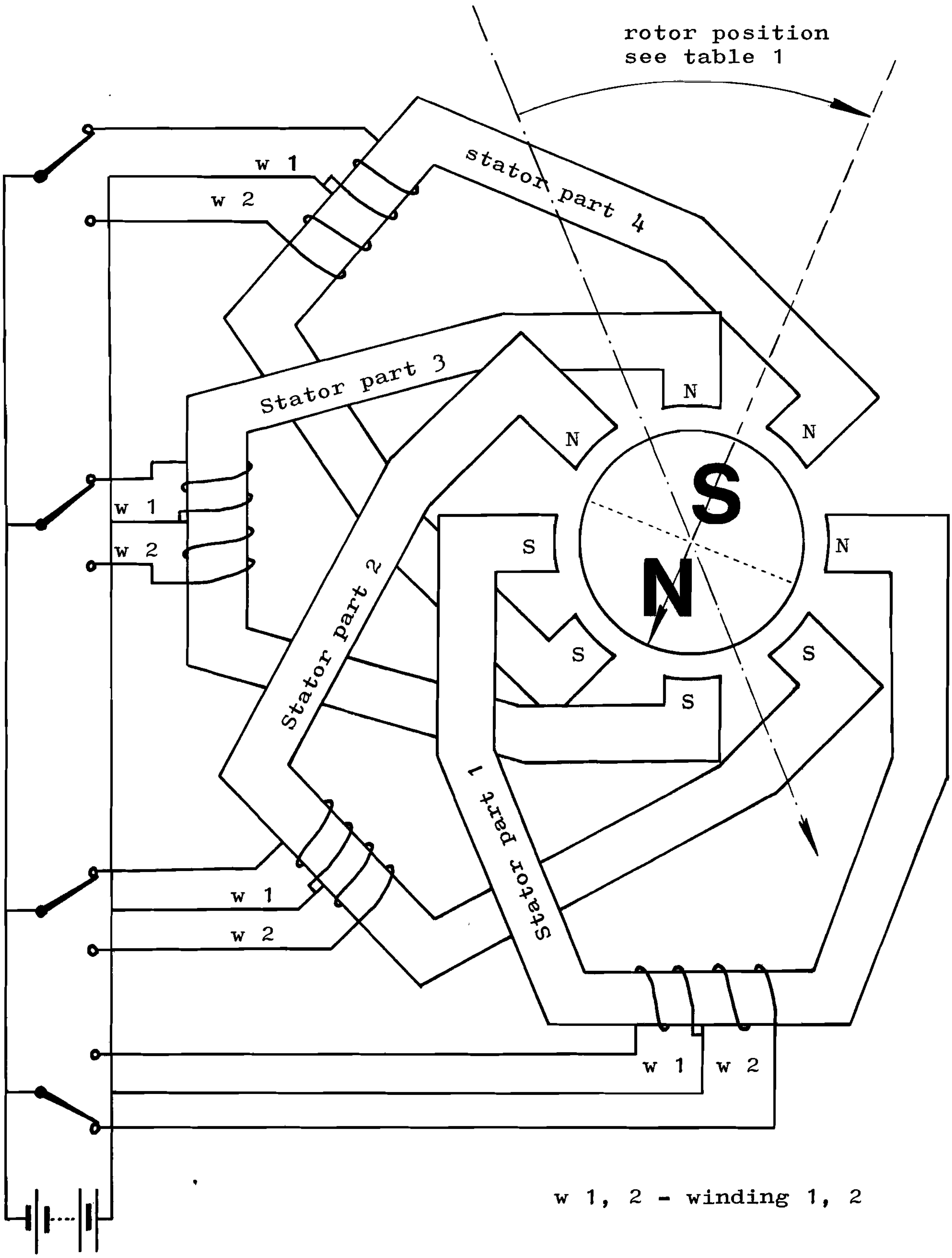

rotor position
see table 1
stator part 4
Stator part 3
Stator part 2
Stator part 1
w 1
w 2
w 1
w 2
w 1
w 2
w 1
w 2
N
N
N
N
S
S
S
S
S
N
S
w 1, 2 - winding 1, 2

Figure 4.

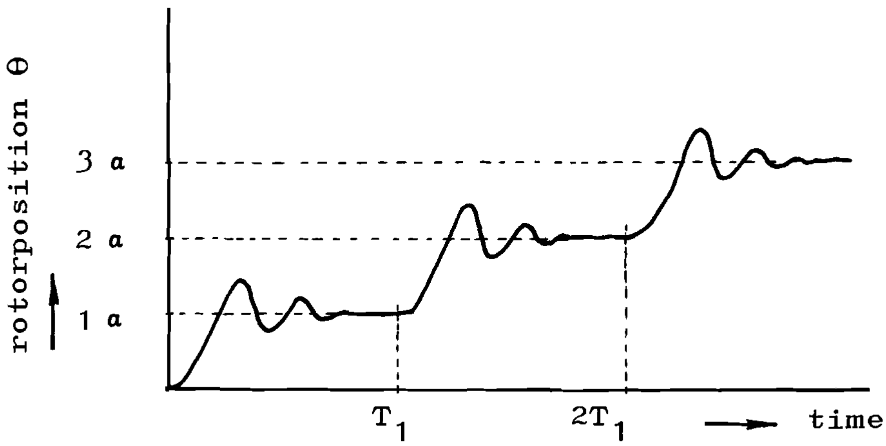


figure 5 a

Stepping rate f_1

Period time T_1

$T_1 = \frac{1}{f_1}$

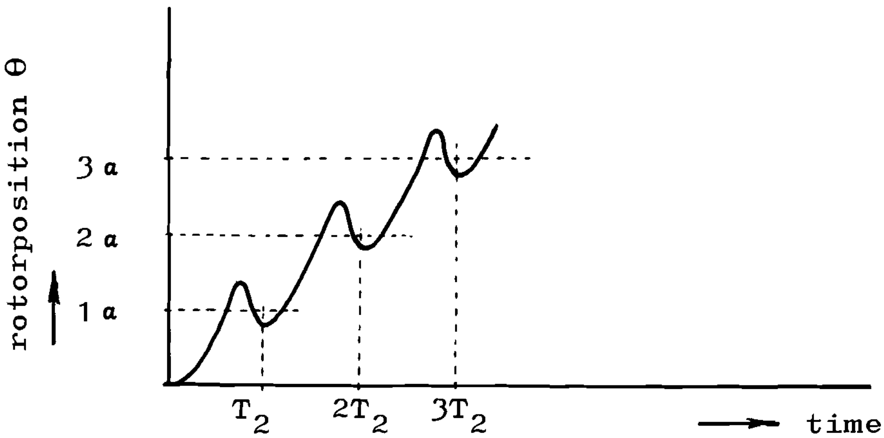


figure 5 b

Stepping rate f_2

Periode time T_2

$T_2 = \frac{1}{f_2}$

$T_2 < T_1$

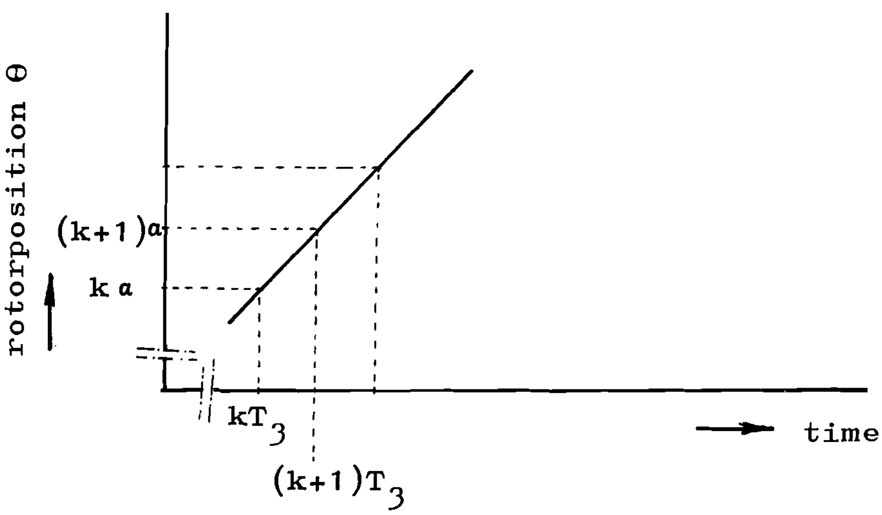


figure 5 c

Period time T_3

$T_3 \ll T_2$

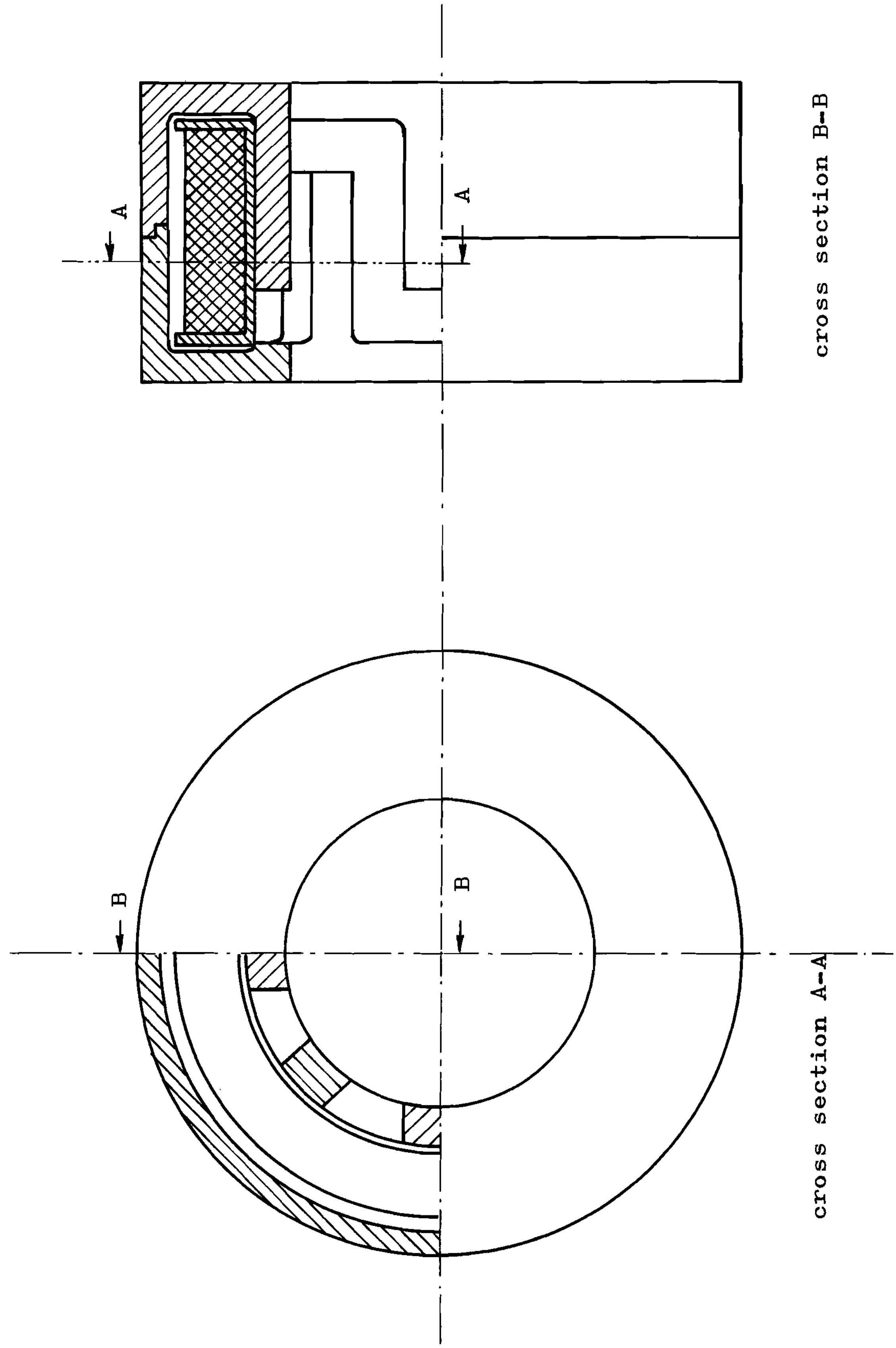


Figure 6.

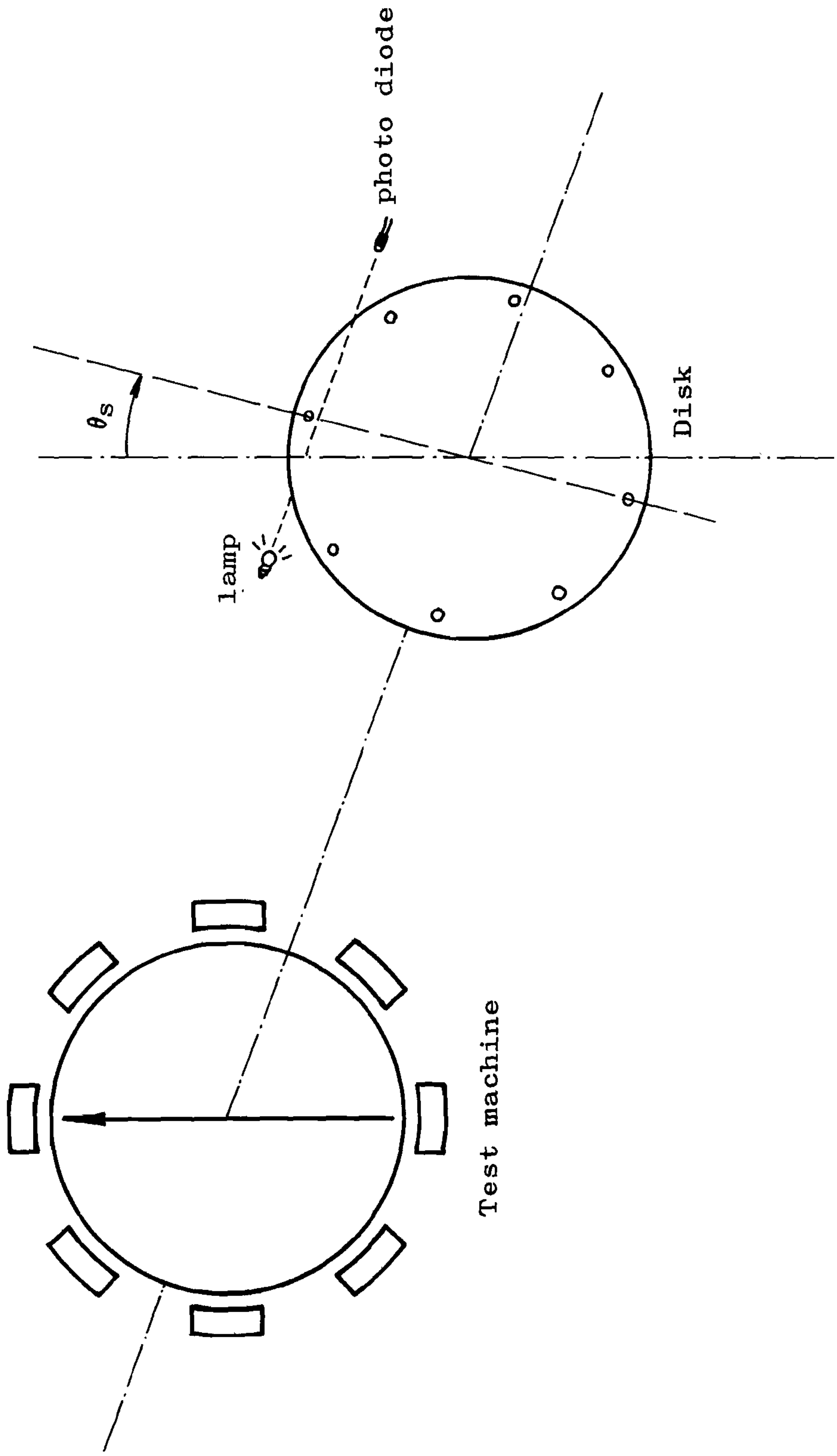


Figure 7.

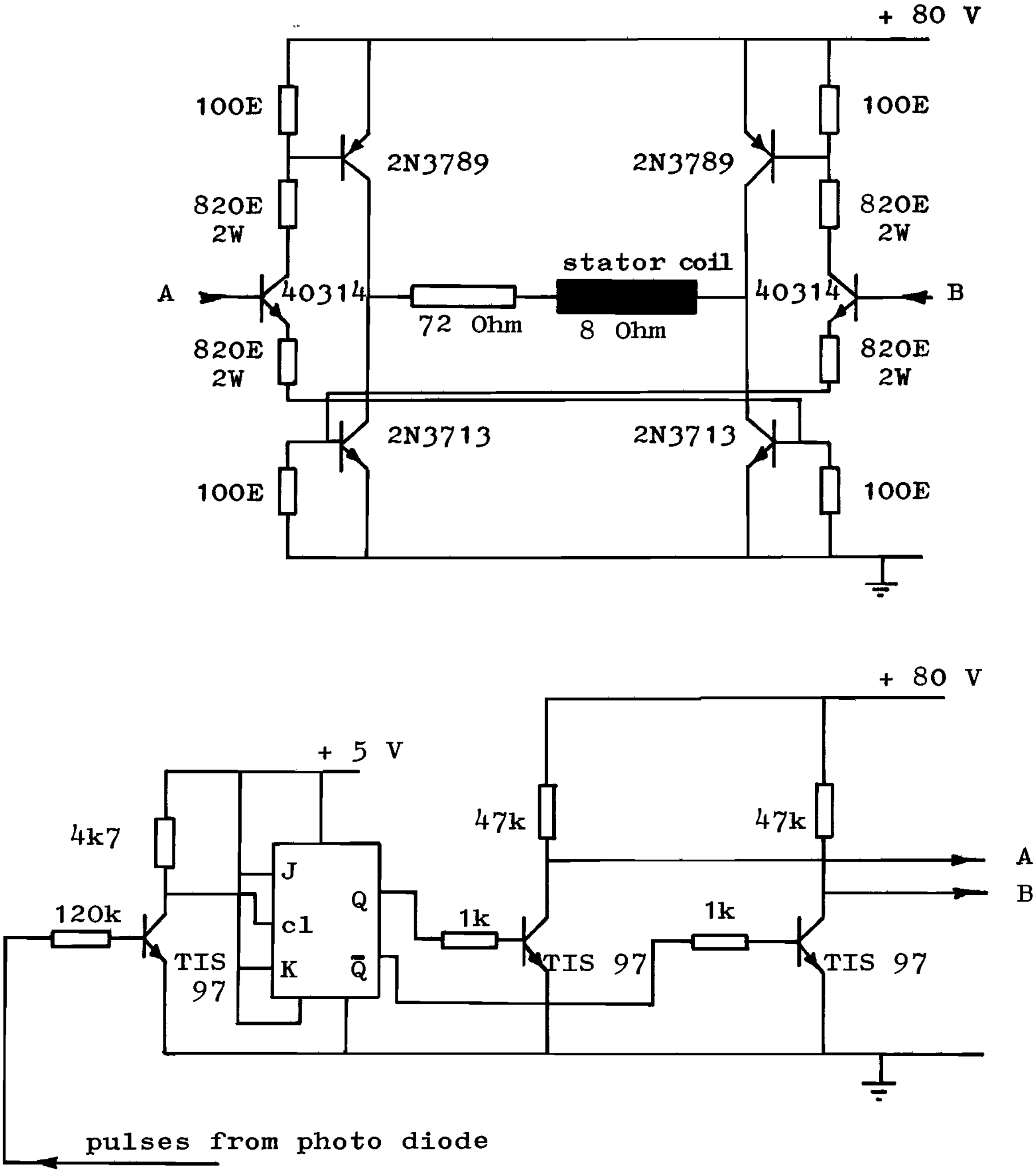

+ 80 V
100E
2N3789
2N3789
100E
820E 2W
820E 2W
stator coil
A
40314
40314
B
72 Ohm
8 Ohm
820E 2W
820E 2W
2N3713
2N3713
100E
100E
+ 80 V
+ 5 V
4k7
47k
47k
J
Q
A
B
120k
cl
1k
1k
TIS 97
K
Q̄
TIS 97
TIS 97
pulses from photo diode

Figure 8.

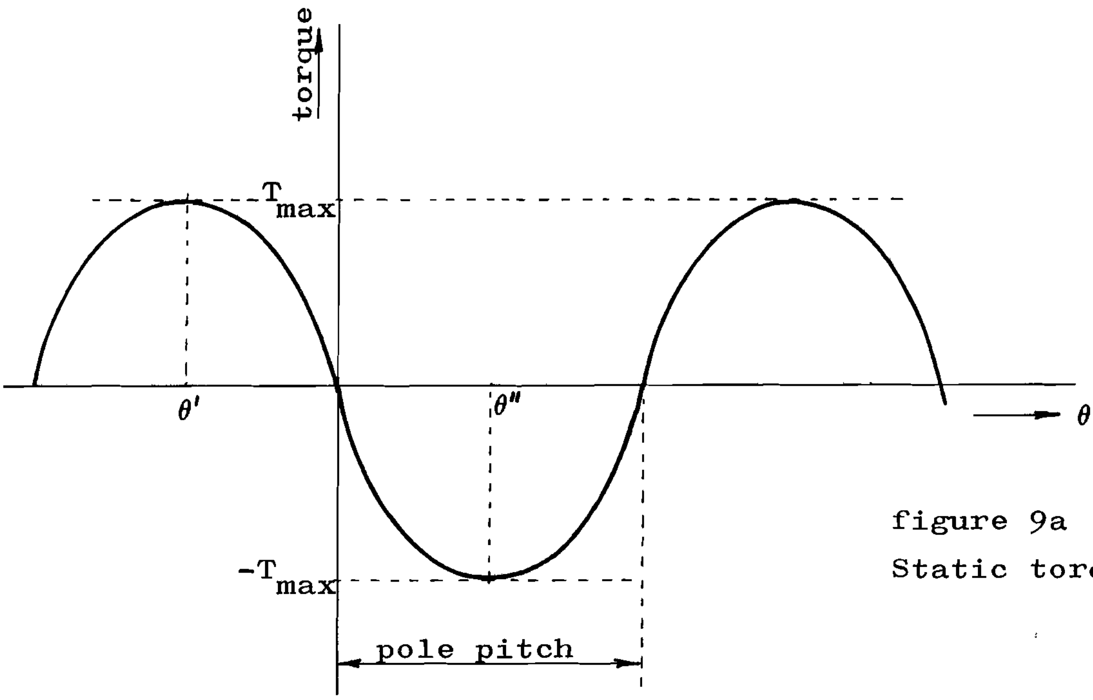


figure 9a
Static torque

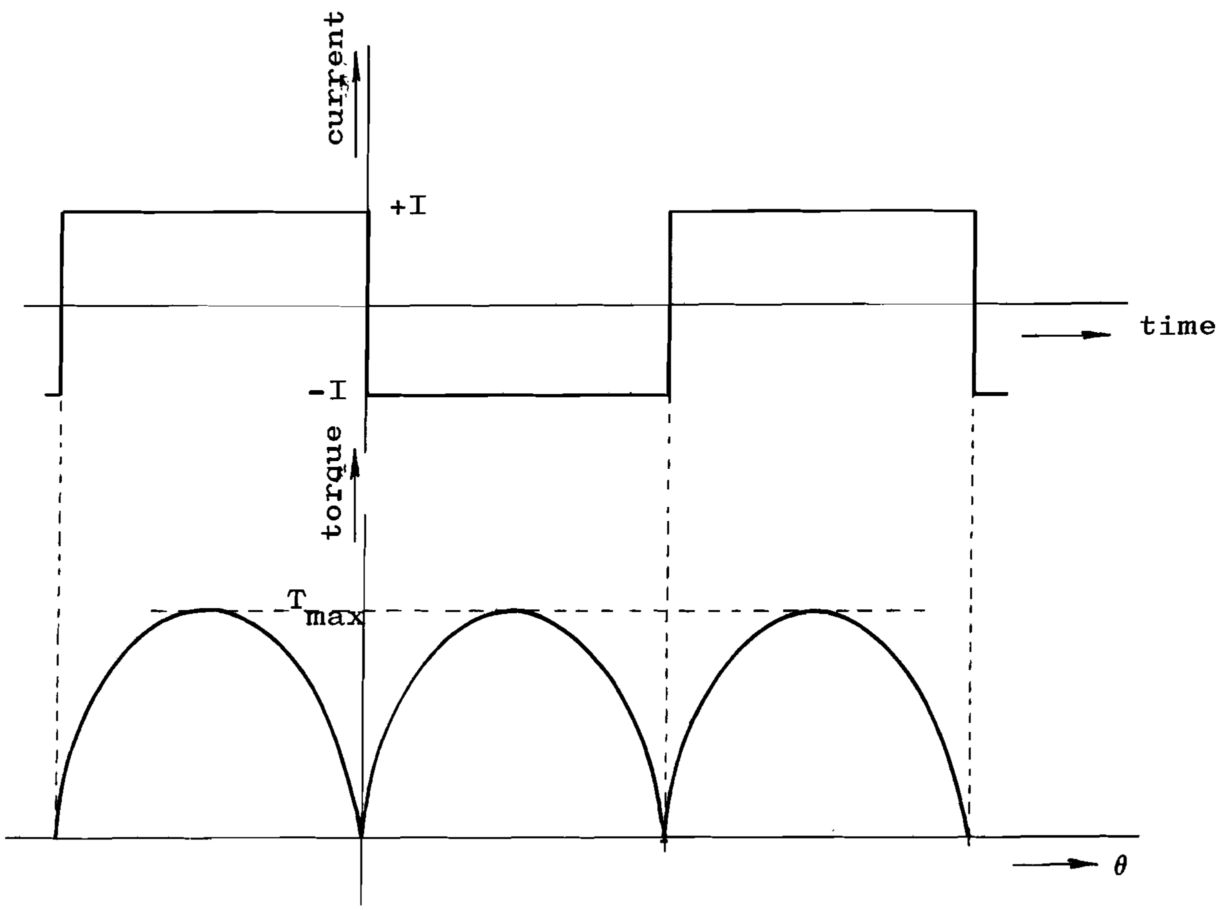


figure 9b
Torque, if the stator current is switched.

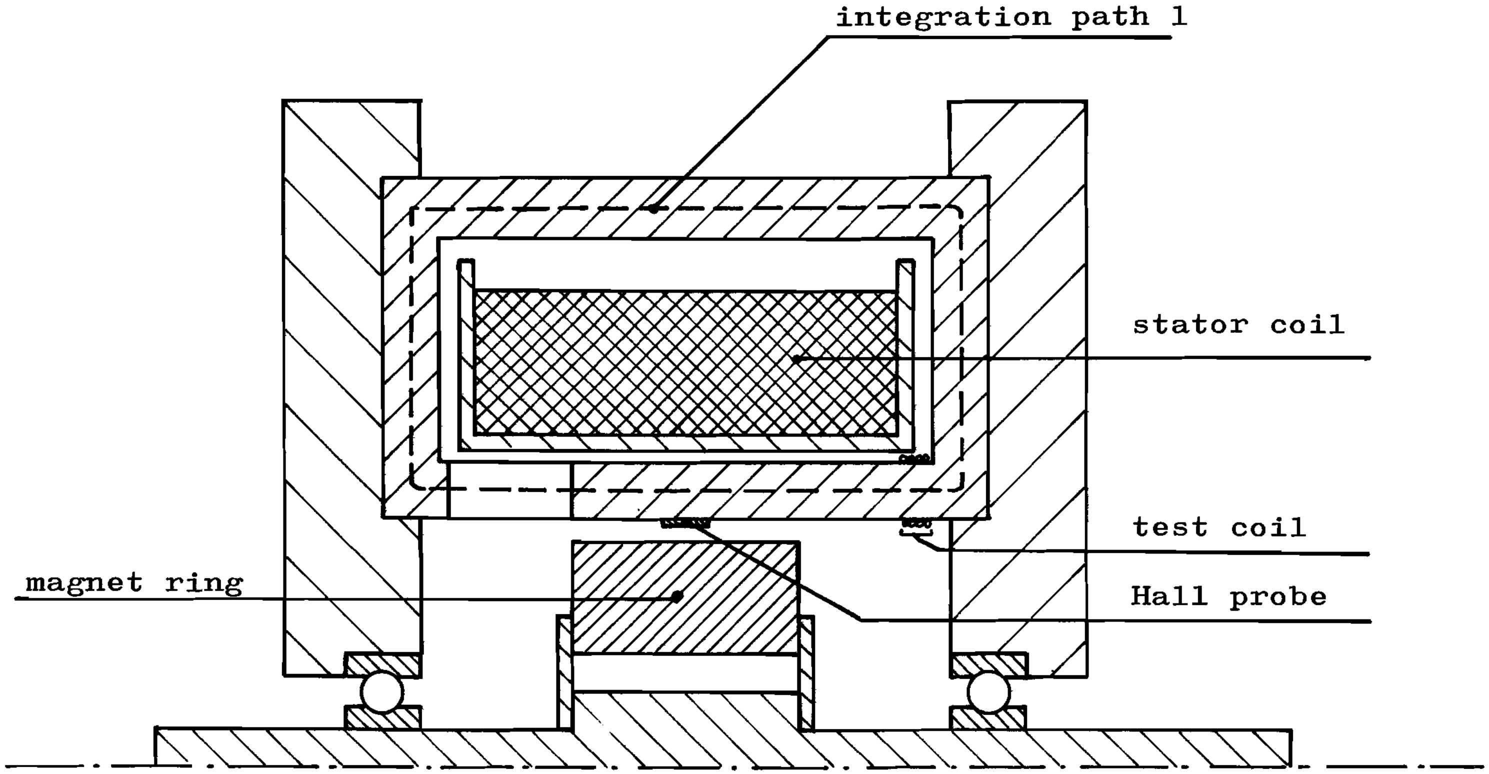


Figure 10. Cross section of the test machine.

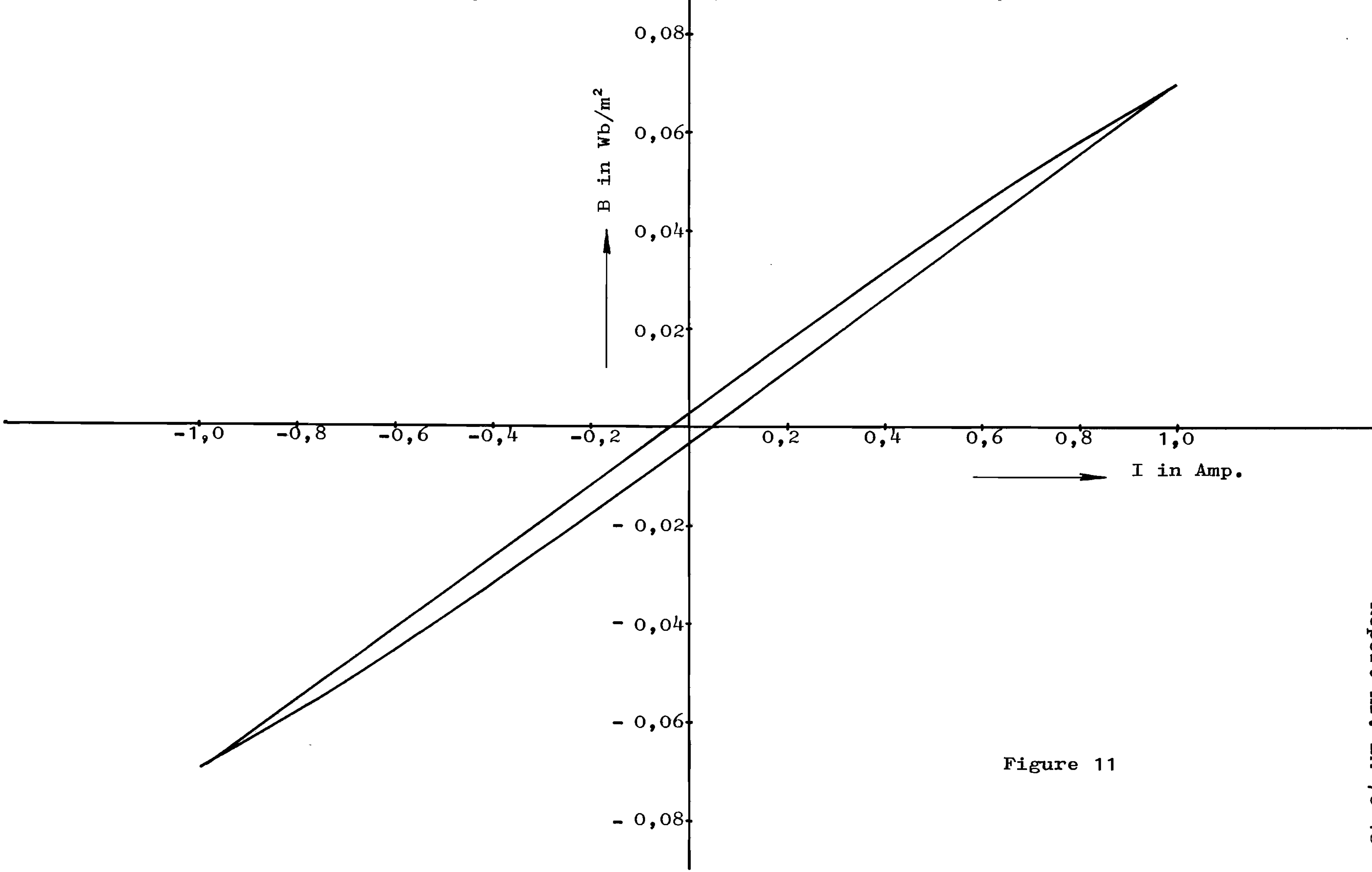
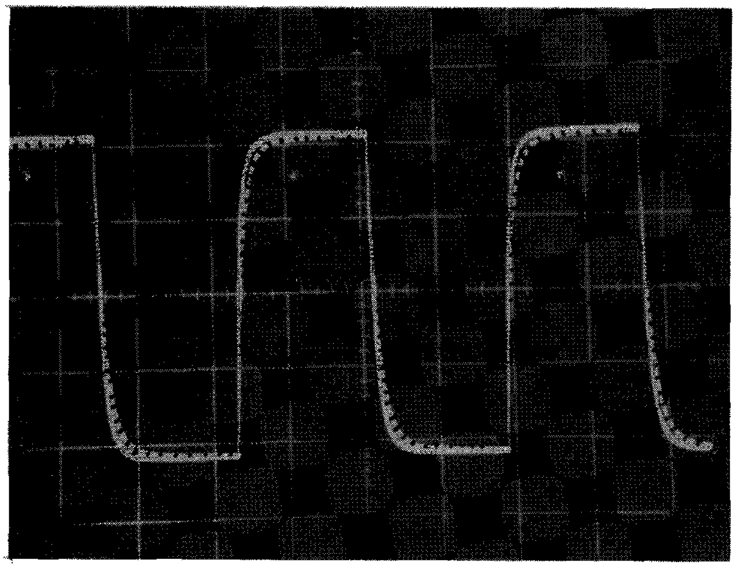

B in Wb/m²
0,08
0,06
0,04
0,02
- 0,02
- 0,04
- 0,06
- 0,08
-1,0
-0,8
-0,6
-0,4
-0,2
0,2
0,4
0,6
0,8
1,0
I in Amp.

Figure 11



 current in stator coil

 air gap flux

Figure 12a
3 Hz

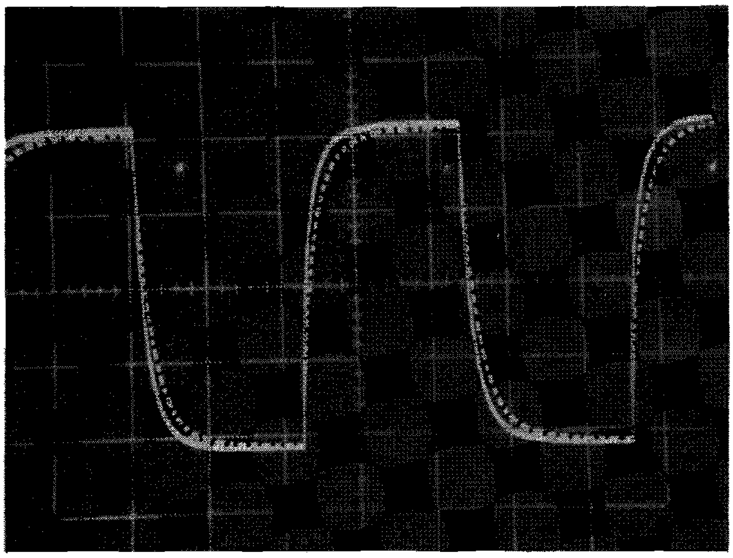


Figure 12b
5 Hz

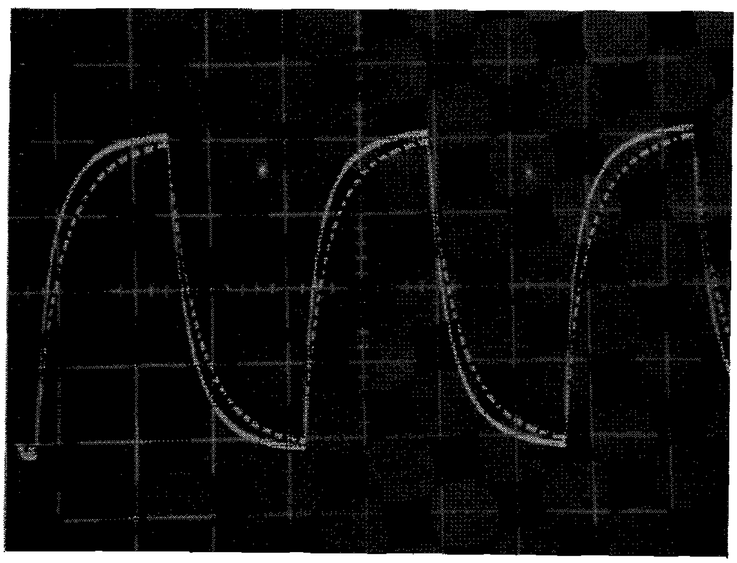


Figure 12c
15 Hz

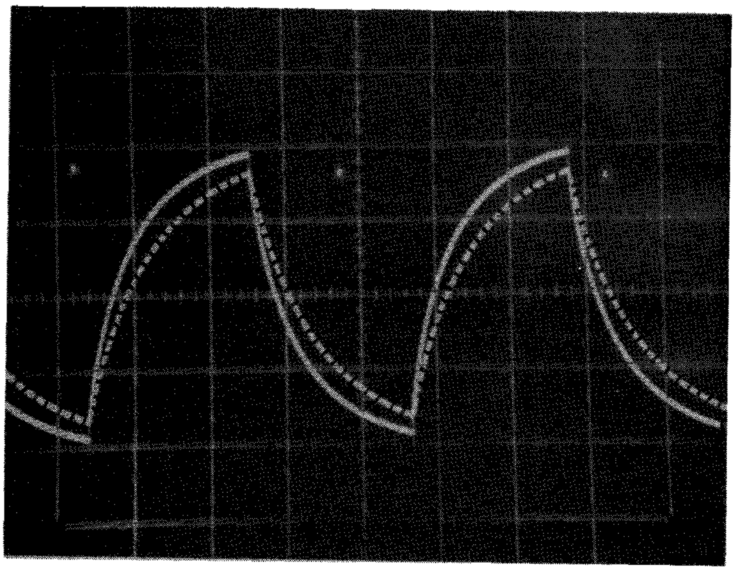


Figure 12d
25 Hz

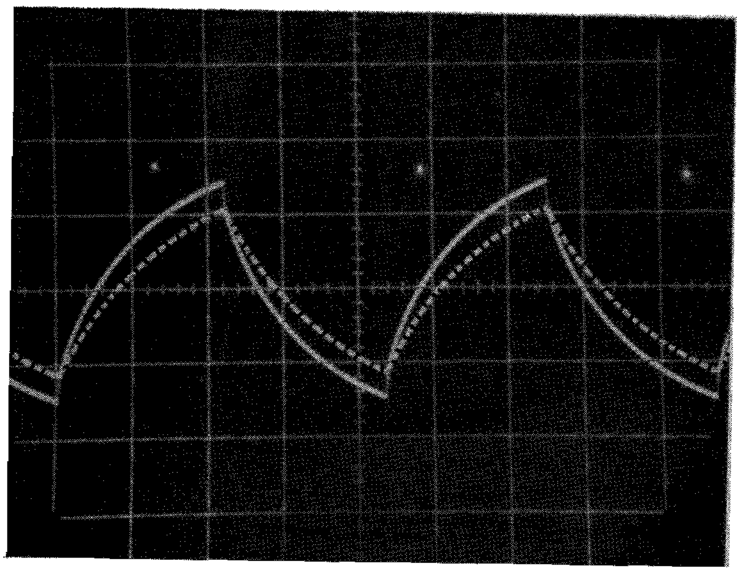


Figure 12e
50 Hz

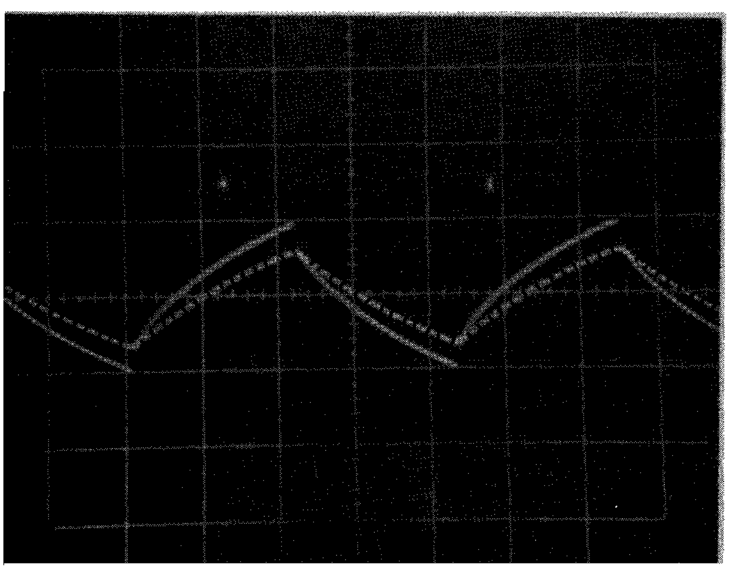


Figure 12f
100 Hz

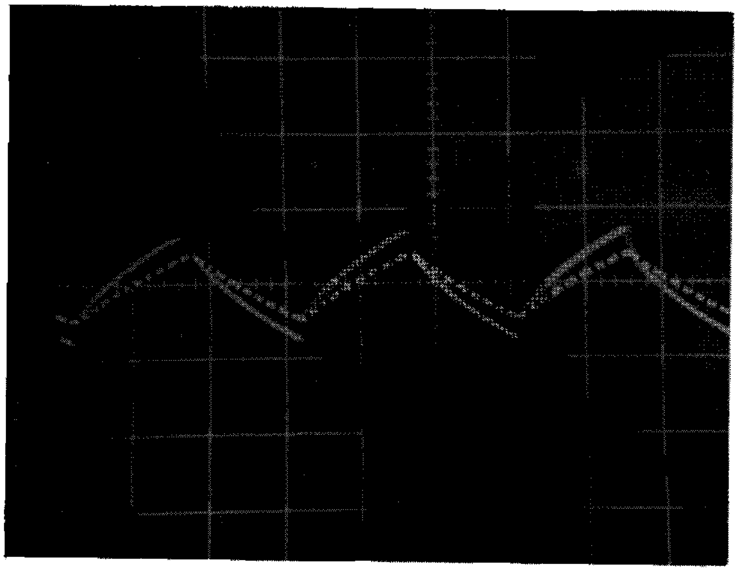


Figure 12g
150 Hz

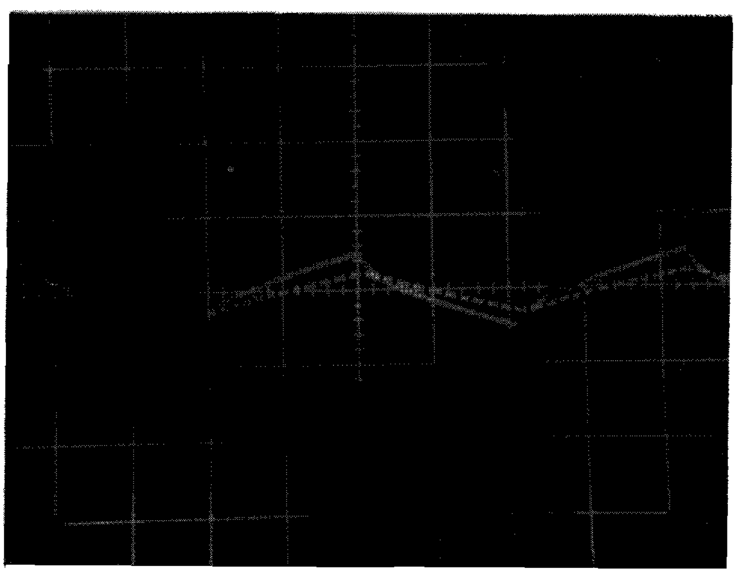
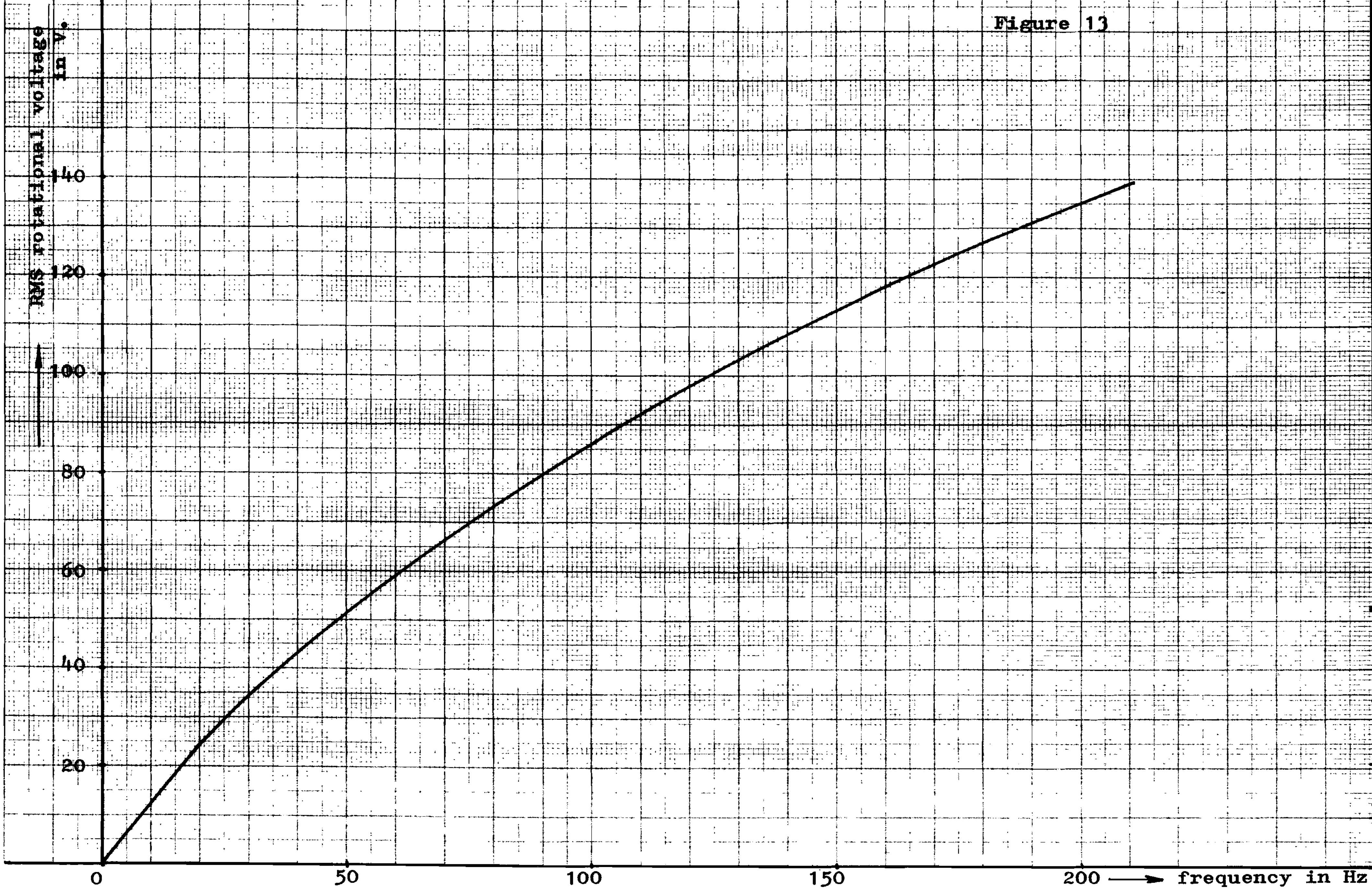


Figure 12h
250 Hz

Figure 13


RMS rotational voltage in V.
140
120
100
80
60
40
20
0
50
100
150
200
frequency in Hz

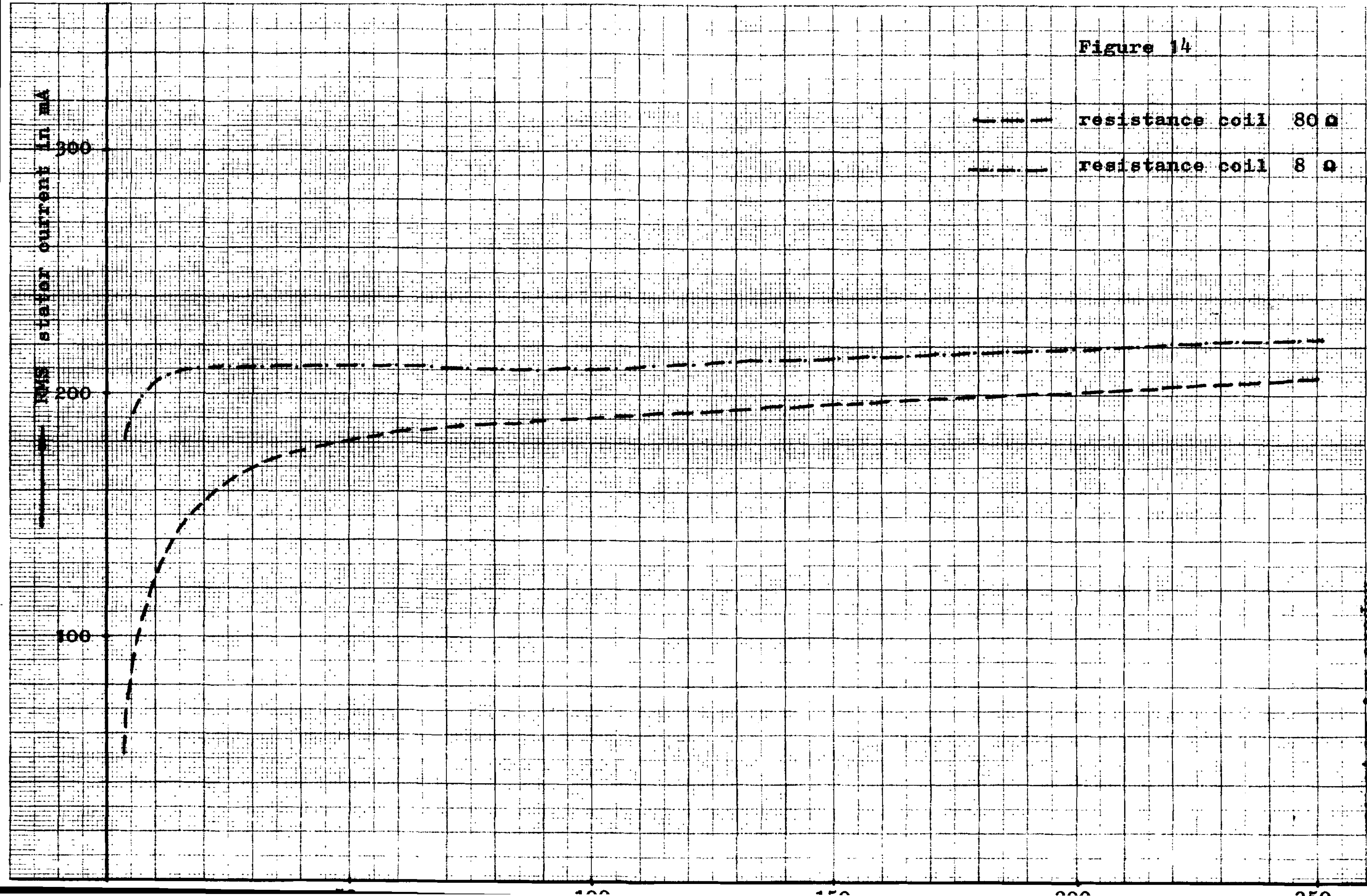
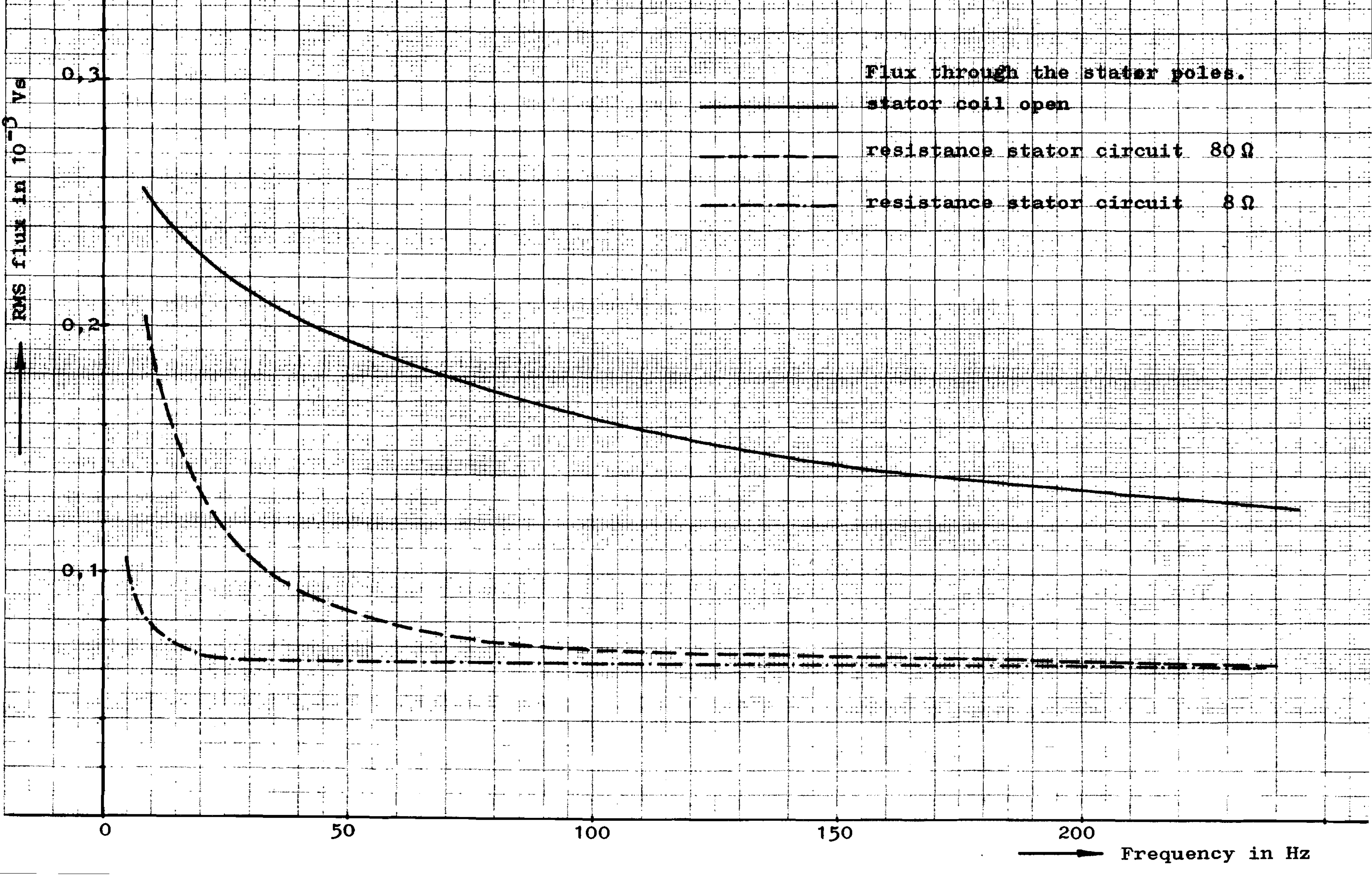


Figure 14
resistance coil 80 Ω
resistance coil 8 Ω
RMS stator current in mA
300
200
100

Figure 15


Flux through the stator poles.
stator coil open
resistance stator circuit 80 Ω
resistance stator circuit 8 Ω
RMS flux in 10^{-3} Vs
0,3
0,2
0,1
0
50
100
150
200
Frequency in Hz

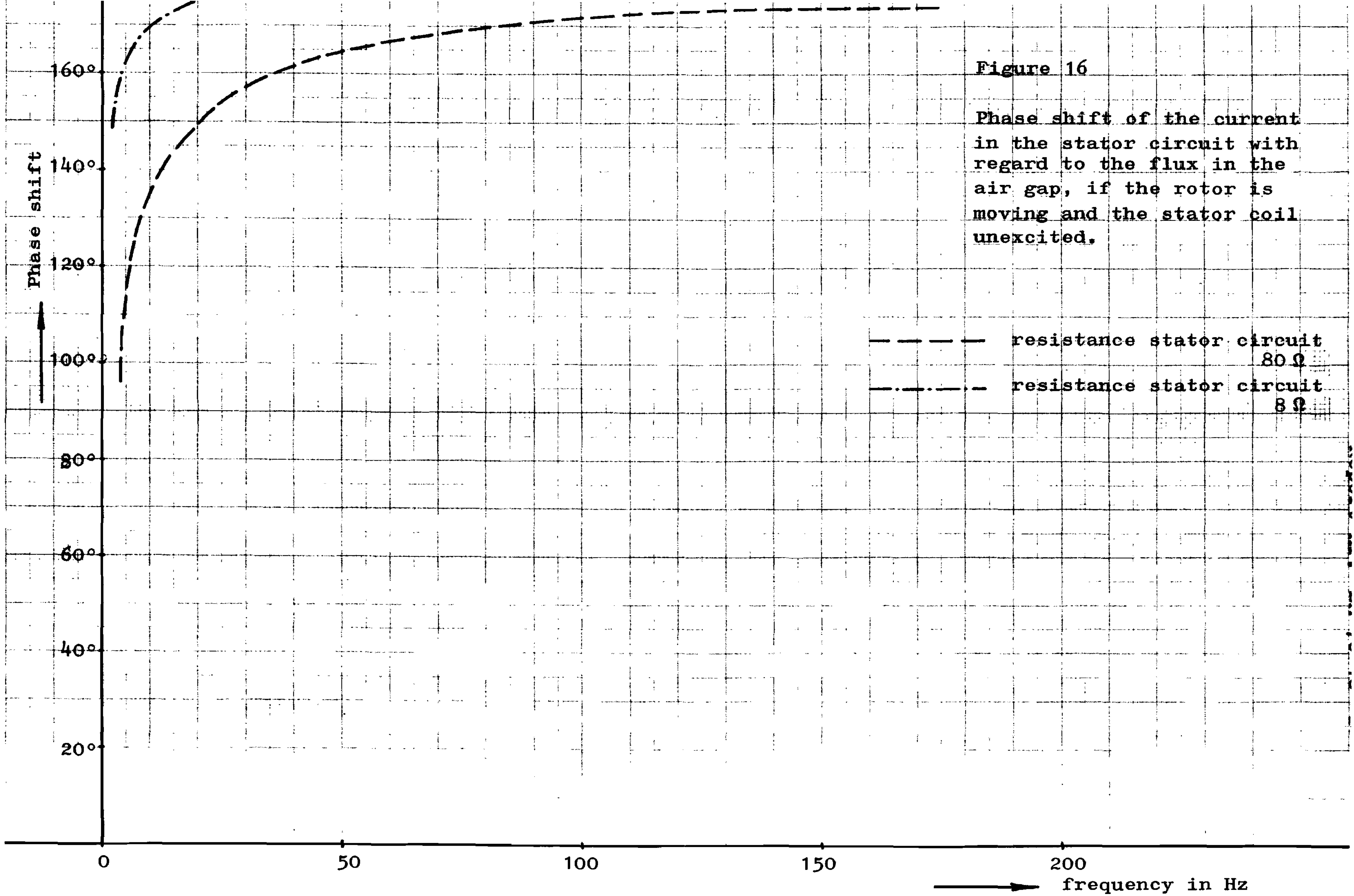
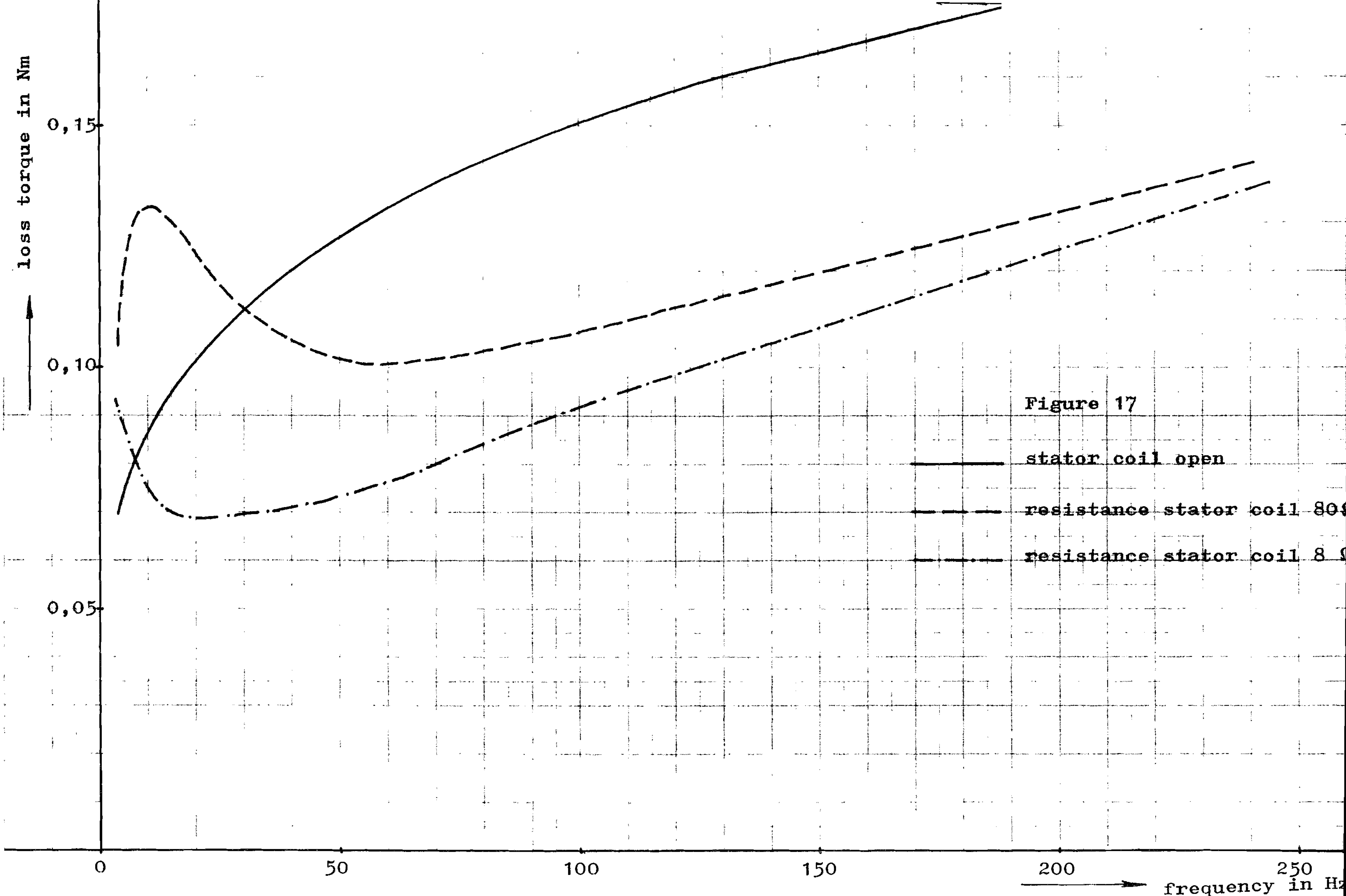


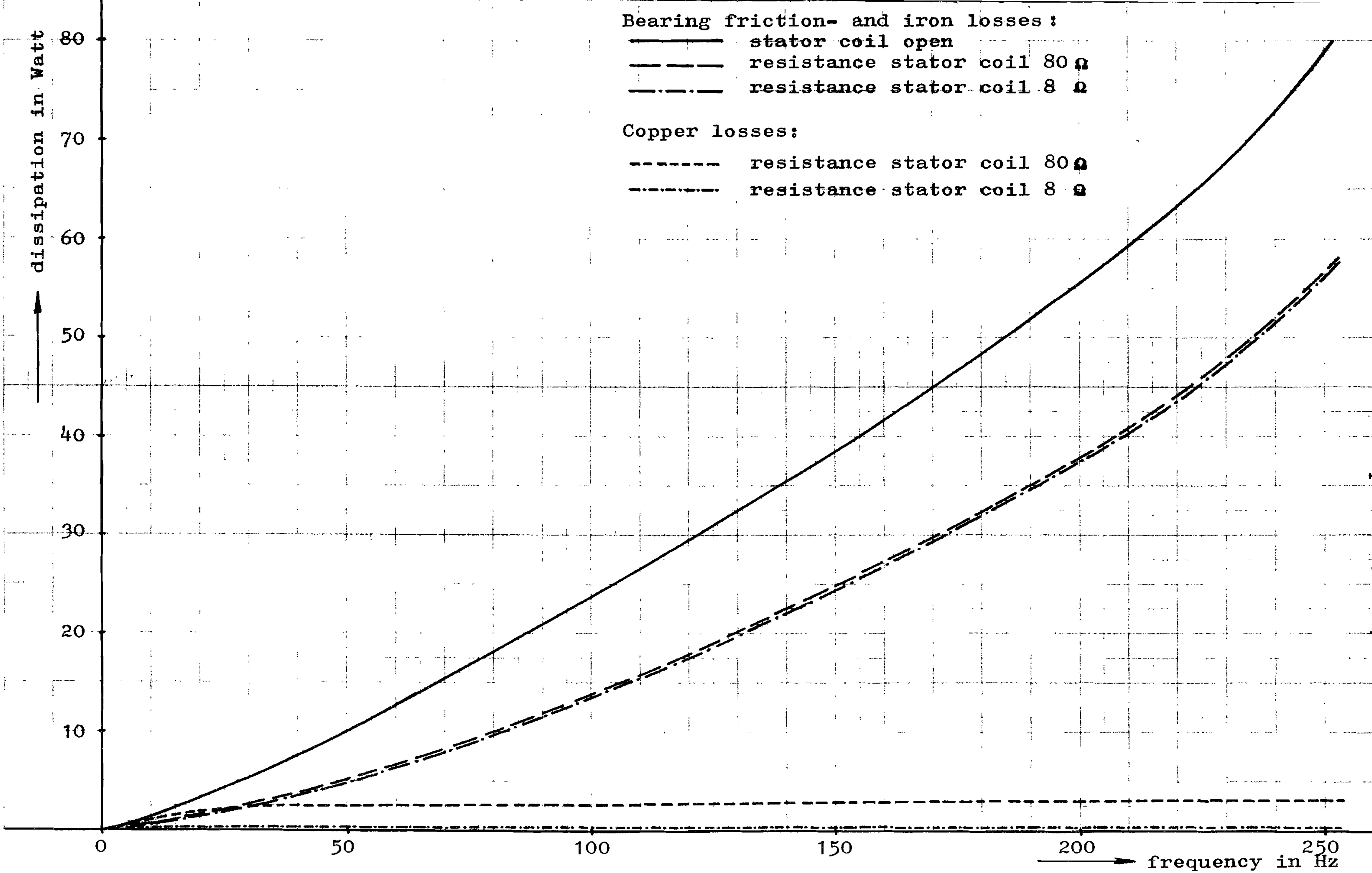
Figure 16

Phase shift of the current in the stator circuit with regard to the flux in the air gap, if the rotor is moving and the stator coil unexcited.



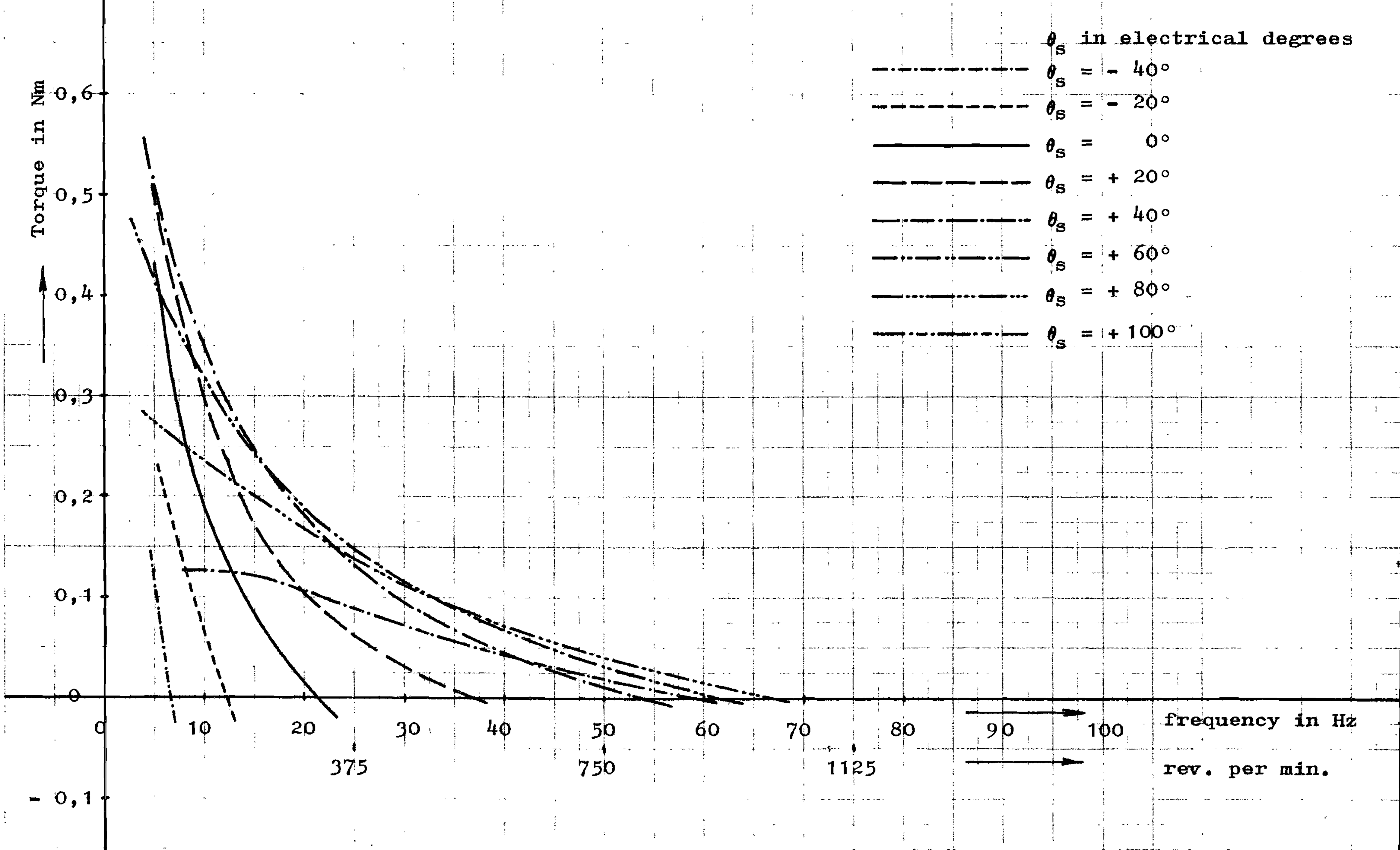
loss torque in Nm
0,15
0,10
0,05
0
50
100
150
200
250
frequency in Hz
stator coil open
resistance stator coil 80
resistance stator coil 8

Figure 17

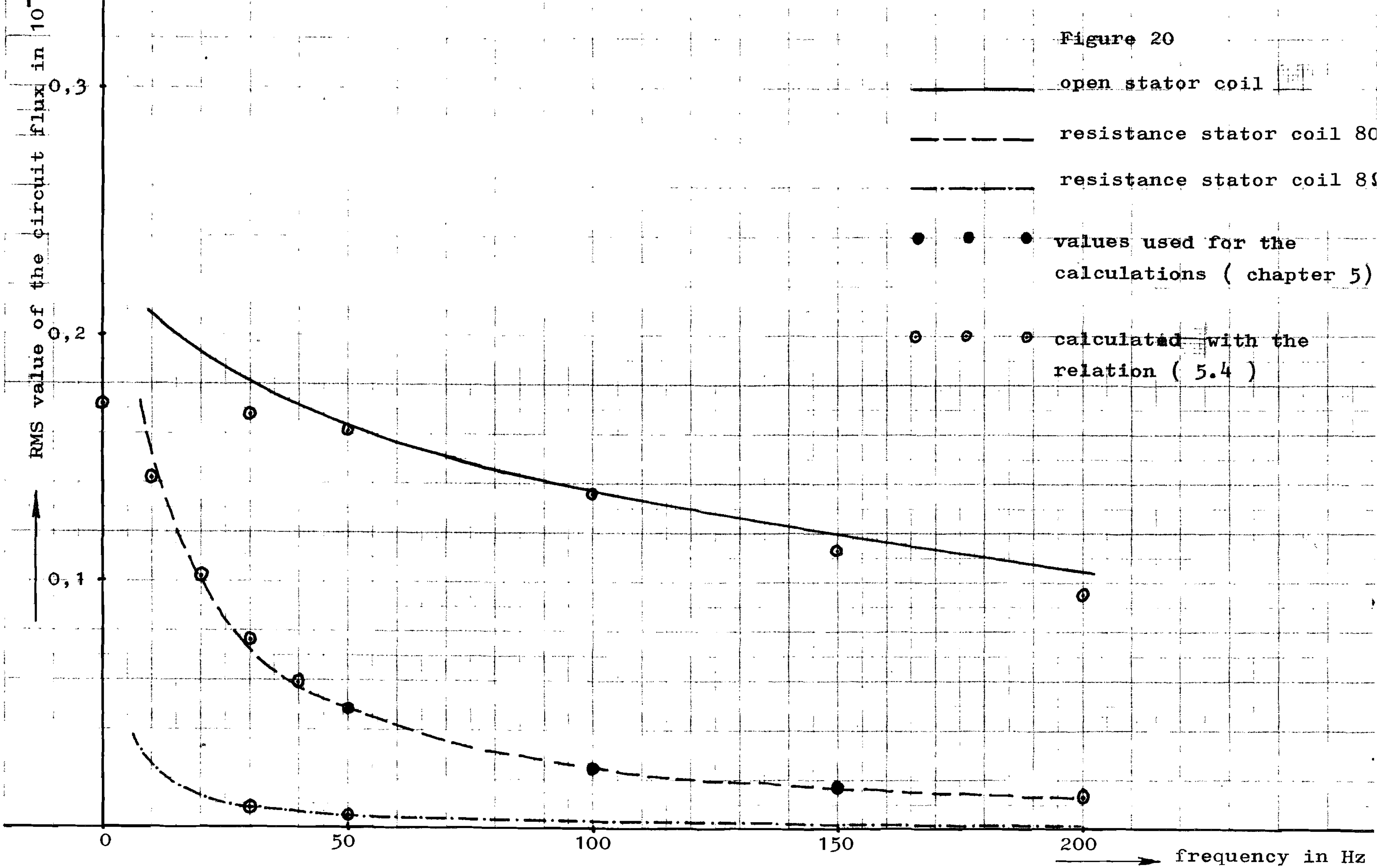


Bearing friction- and iron losses:
stator coil open
resistance stator coil 80 Ω
resistance stator coil 8 Ω
Copper losses:
resistance stator coil 80 Ω
resistance stator coil 8 Ω
dissipation in Watt
80
70
60
50
40
30
20
10
0
50
100
150
200
250
frequency in Hz

Figure 19

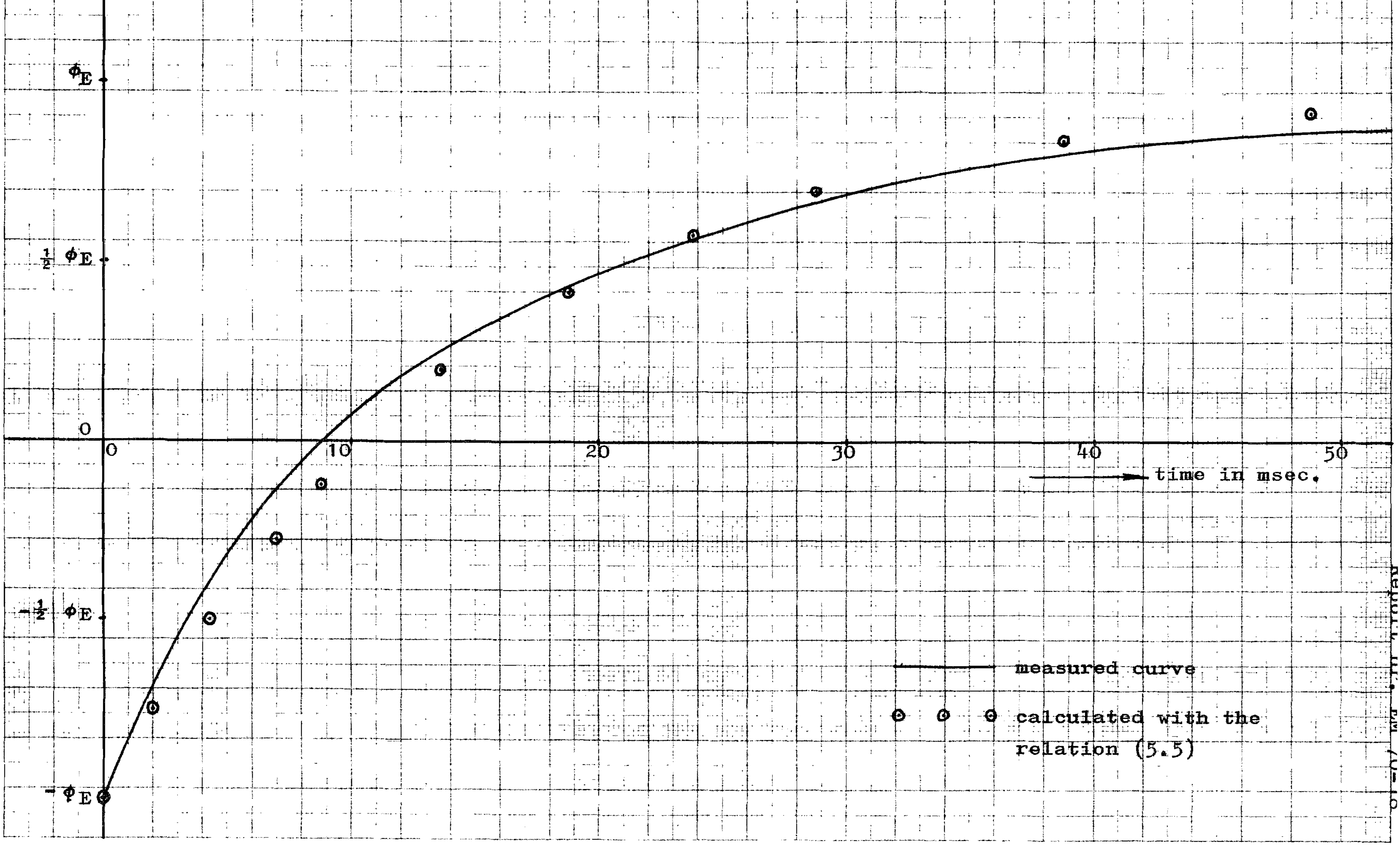


θ_s in electrical degrees
θ_s = - 40°
θ_s = - 20°
θ_s = 0°
θ_s = + 20°
θ_s = + 40°
θ_s = + 60°
θ_s = + 80°
θ_s = + 100°
Torque in Nm
0,6
0,5
0,4
0,3
0,2
0,1
0
- 0,1
0
10
20
30
40
50
60
70
80
90
100
frequency in Hz
375
750
1125
rev. per min.



RMS value of the circuit flux in 10
0,3
0,2
0,1
0
50
100
150
200
frequency in Hz
open stator coil
resistance stator coil 80
resistance stator coil 8
values used for the calculations (chapter 5)
calculated with the relation (5.4)

Figure 20



ϕ_E
$\frac{1}{2}\ \phi_E$
0
$-\frac{1}{2}\ \phi_E$
$-\phi_E$
0
10
20
30
40
50
time in msec.
measured curve
calculated with the relation (5.5)

Figure 21

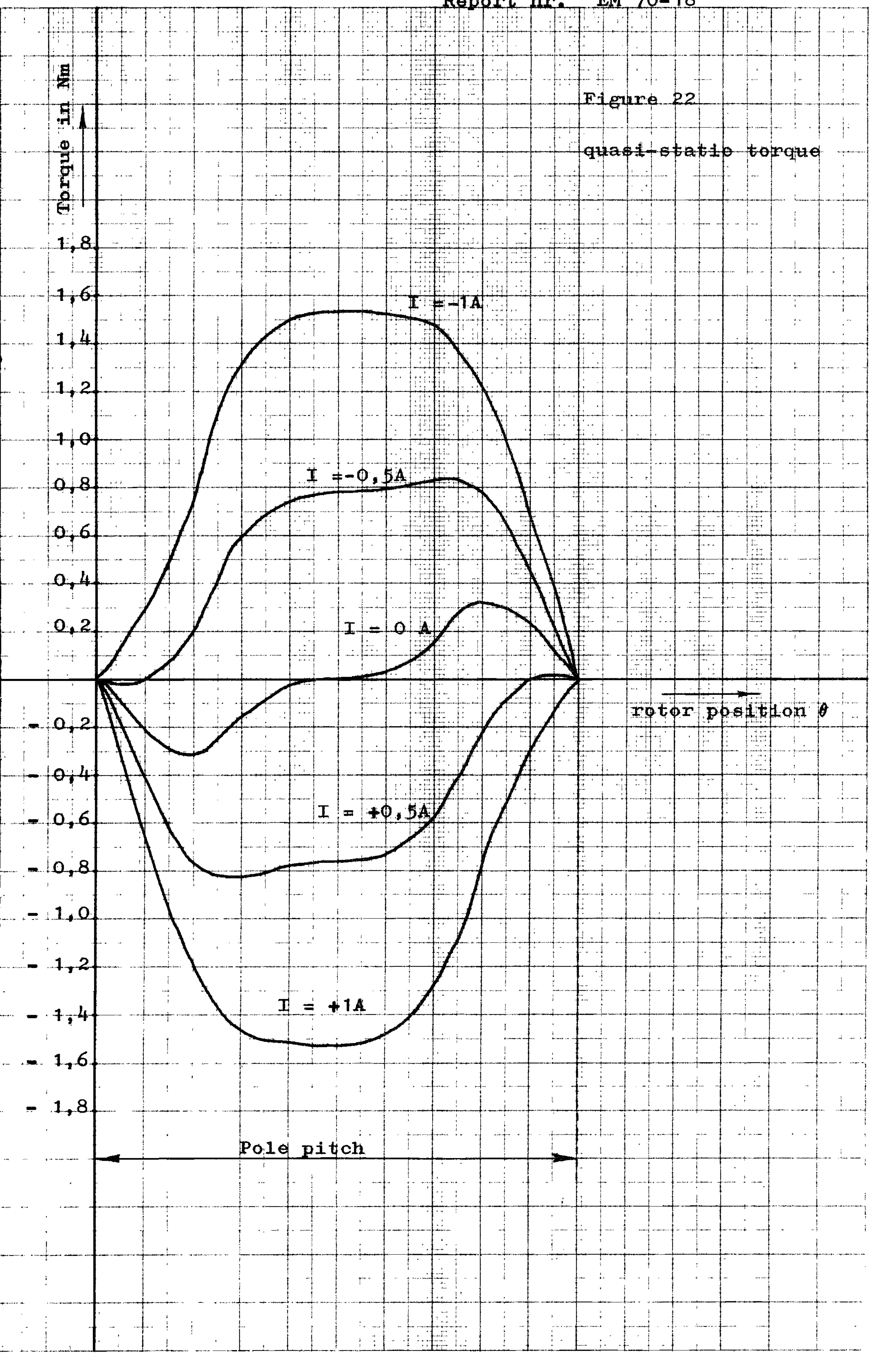


Figure 22

quasi-static torque

Figure 23 a

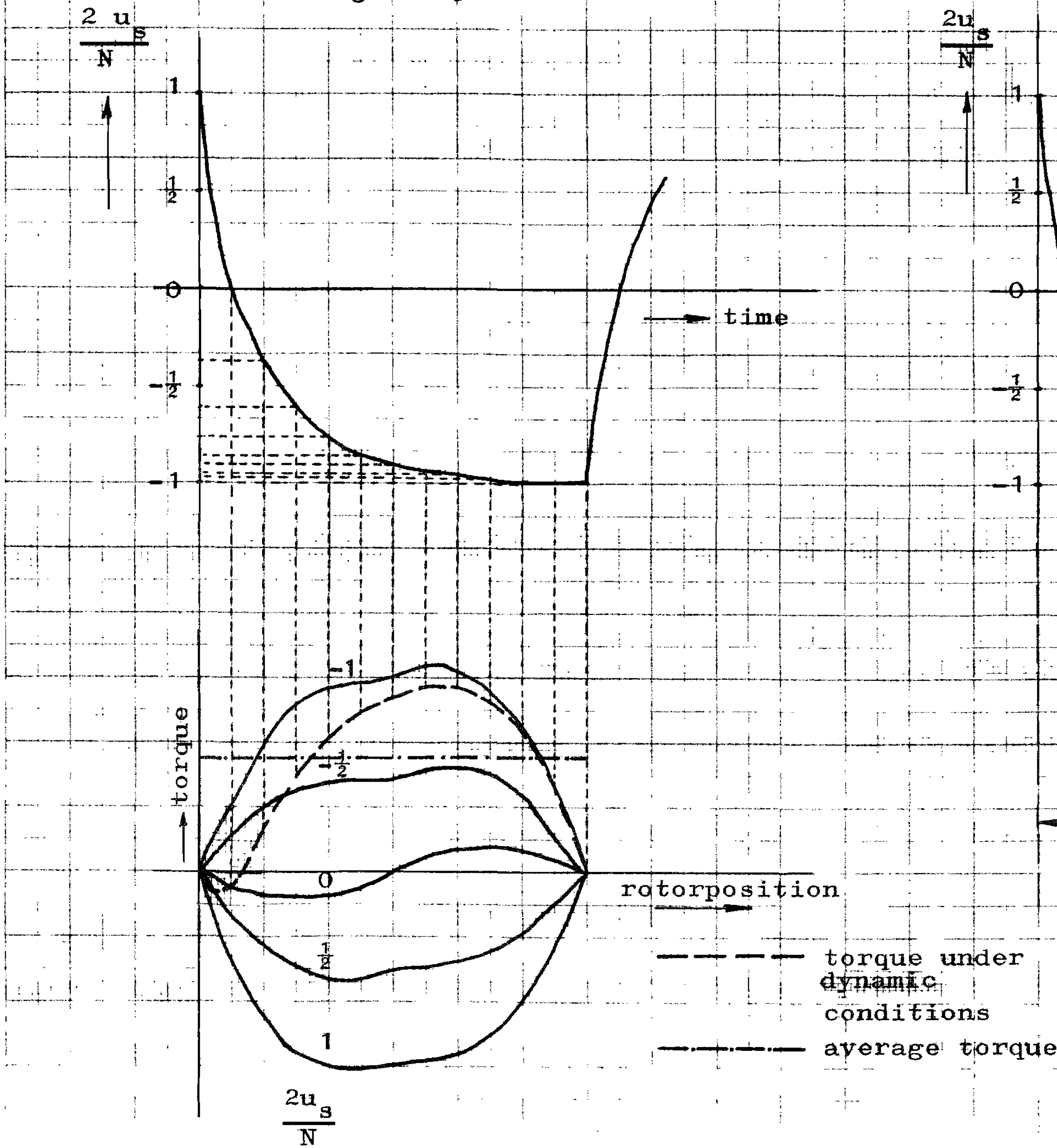


Figure 23 b

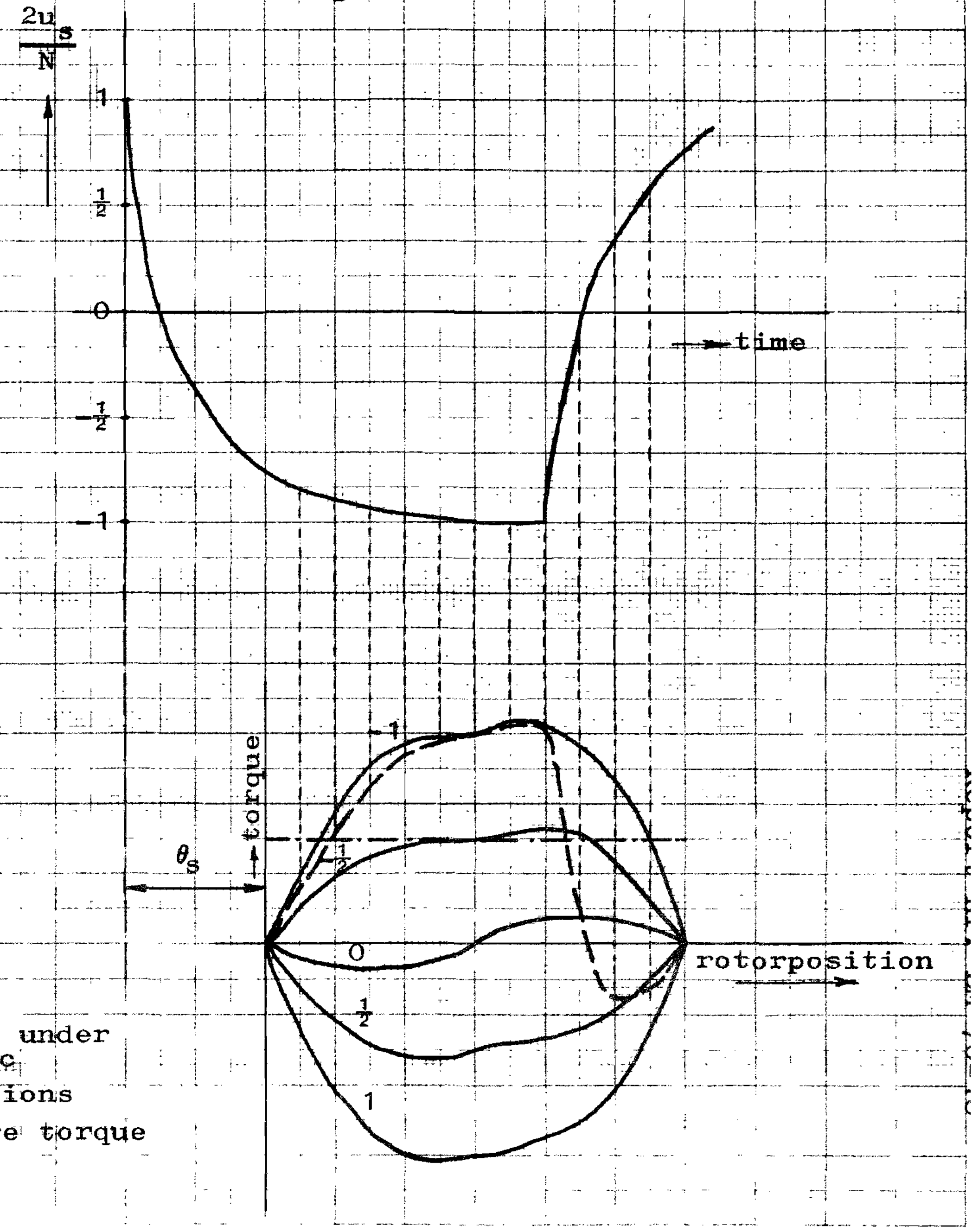


Figure 23 c

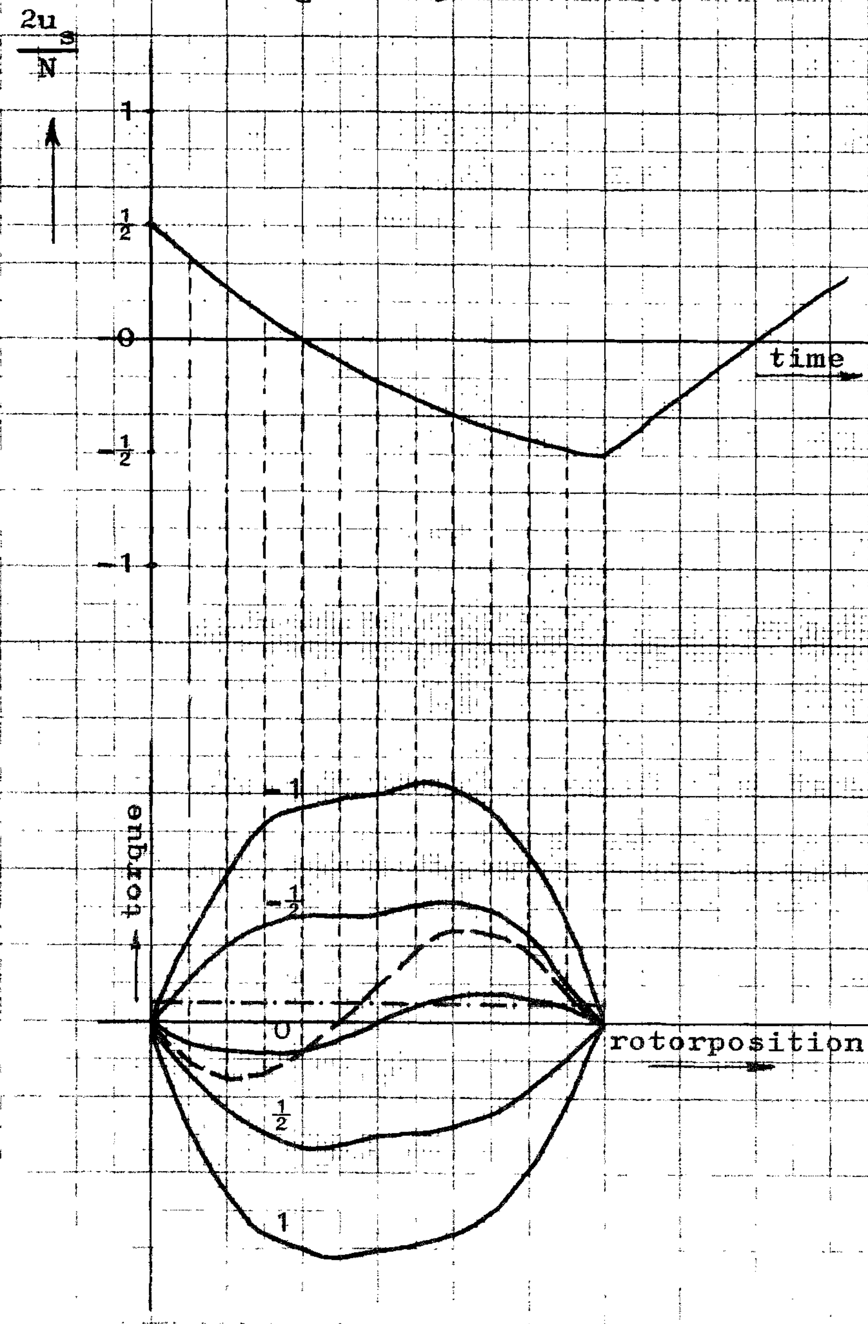
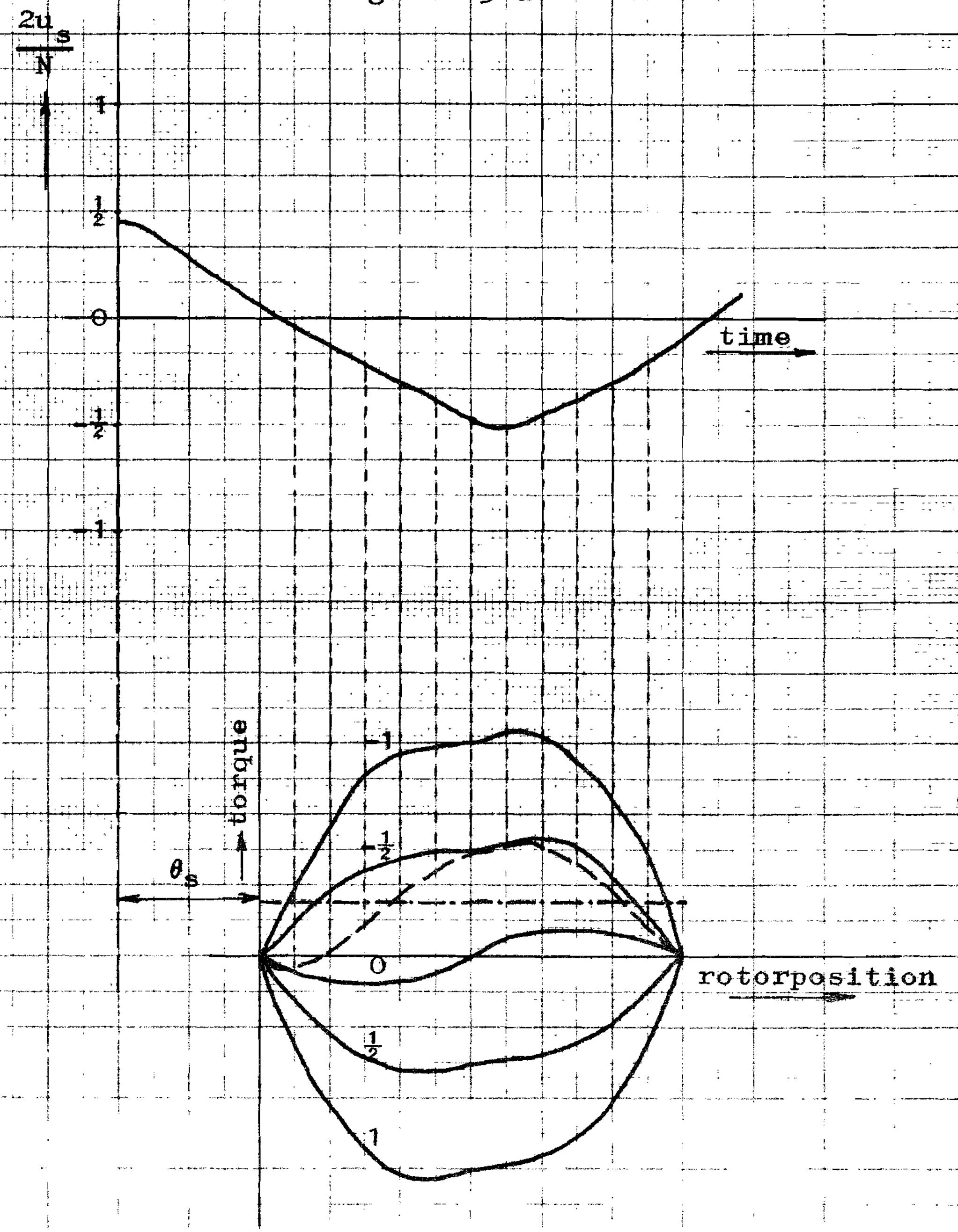


Figure 23 d



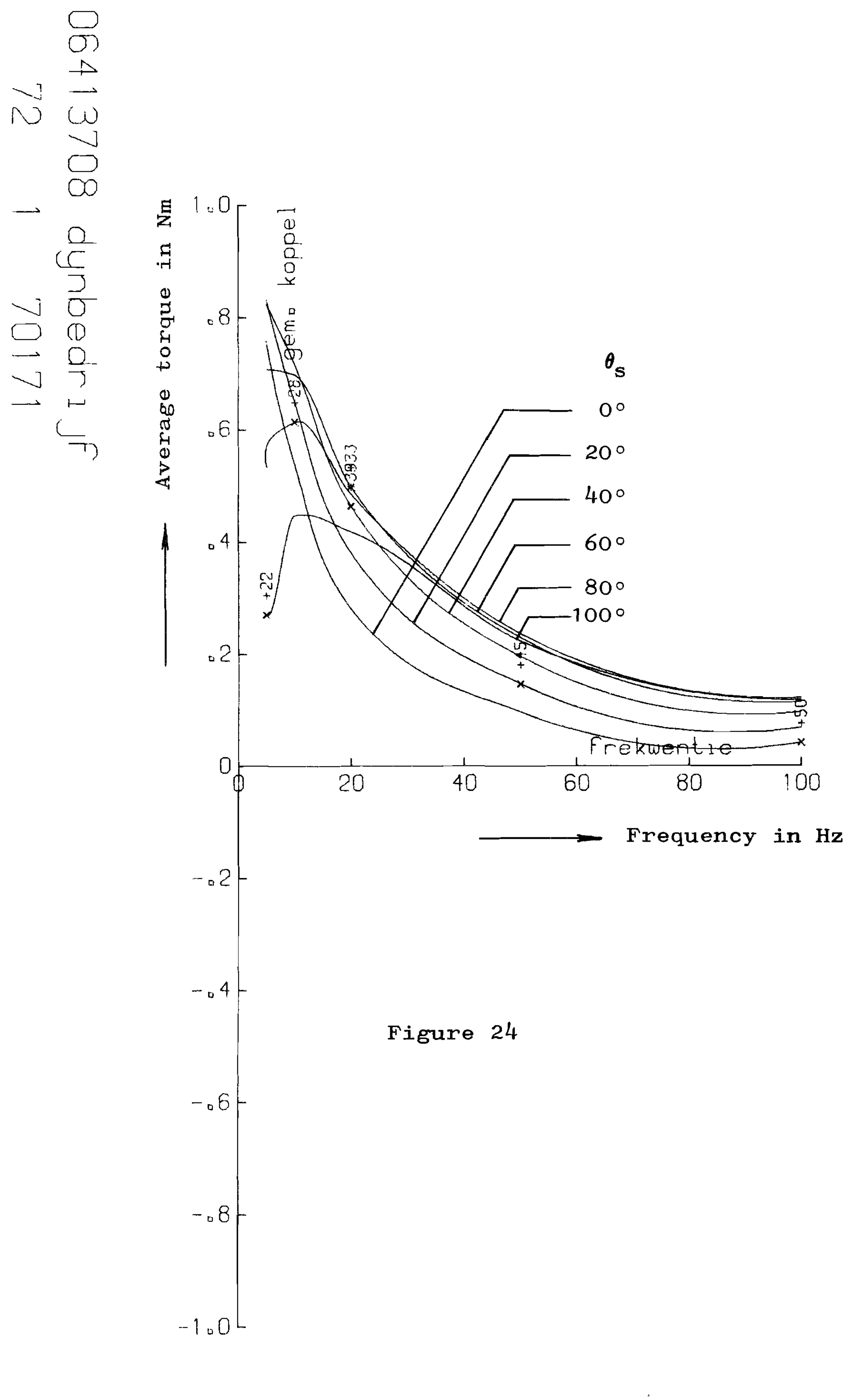

0641370 8 dynbedrijf
72 1 70171
Average torque in Nm
gem. koppel
θ_s
0°
20°
40°
60°
80°
100°
Frekwentie
Frequency in Hz
1.0
.8
.6
.4
.2
0
-.2
-.4
-.6
-.8
-1.0
0
20
40
60
80
100

Figure 24

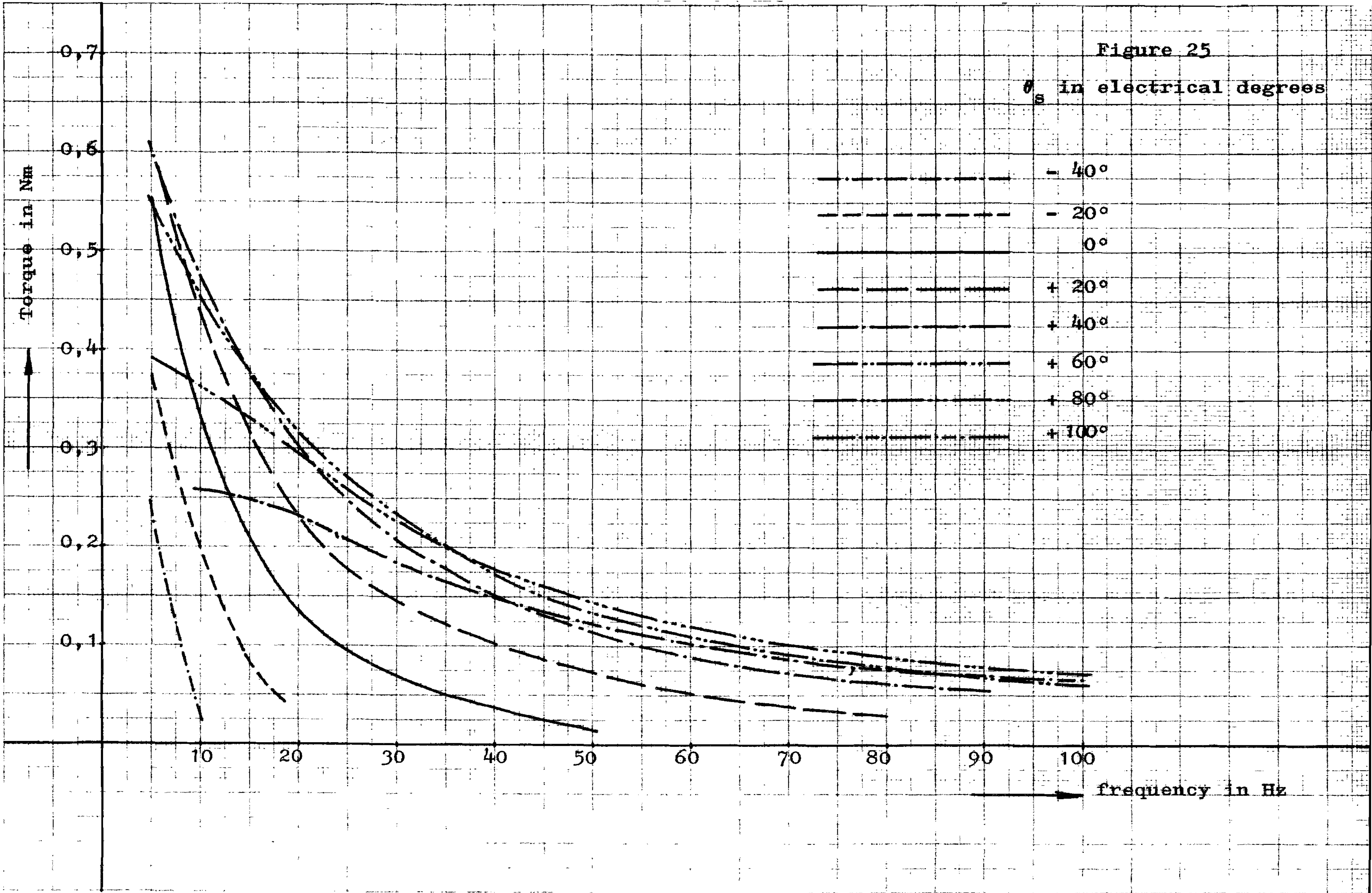


Figure 25
θ_s in electrical degrees
- 40°
- 20°
0°
+ 20°
+ 40°
+ 60°
+ 80°
+ 100°
Torque in Nm
0,7
0,6
0,5
0,4
0,3
0,2
0,1
10
20
30
40
50
60
70
80
90
100
frequency in Hz

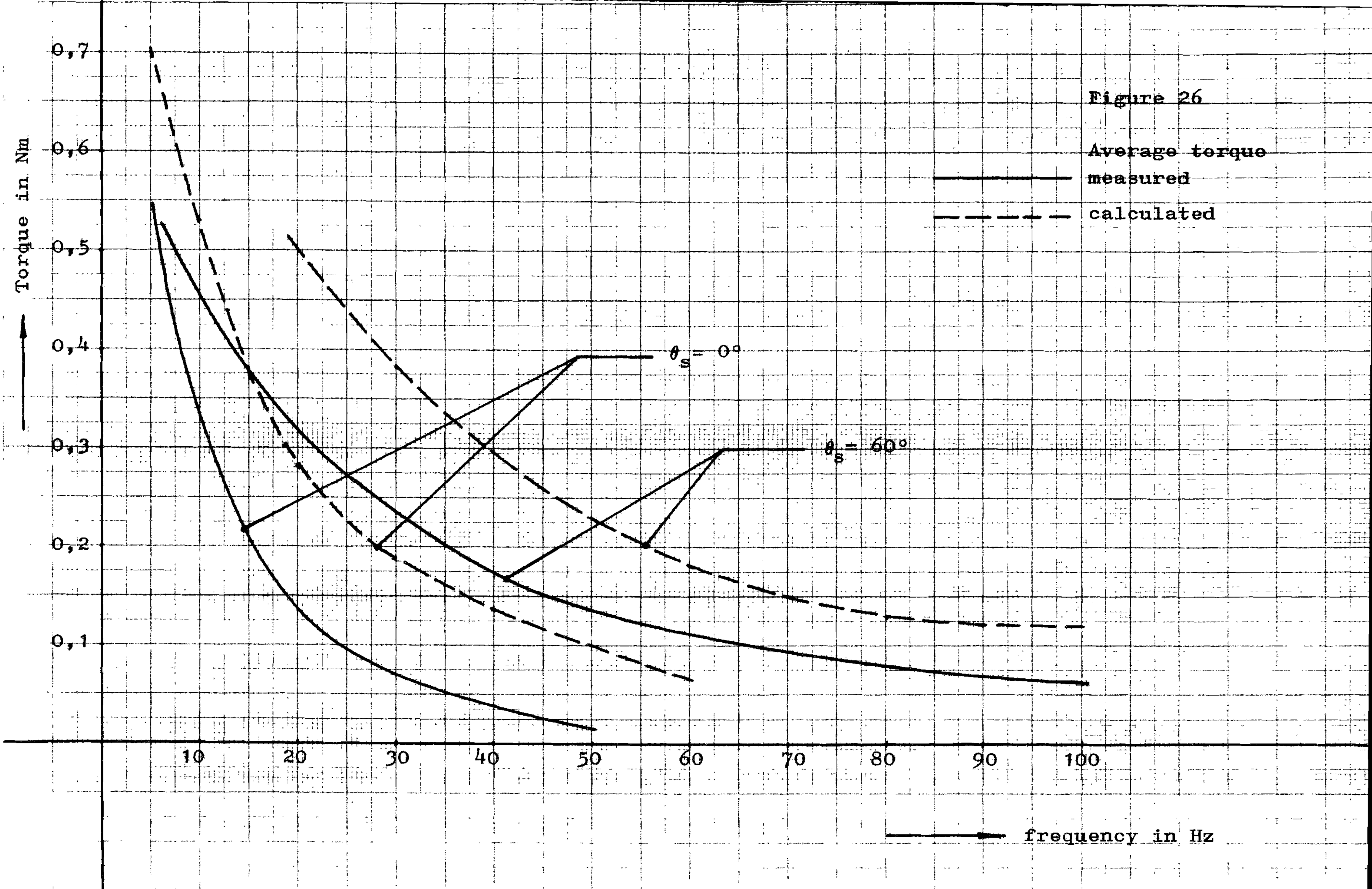


Figure 26
Average torque
measured
calculated
$\theta_s = 0°$
$\theta_s = 60°$
Torque in Nm
0,7
0,6
0,5
0,4
0,3
0,2
0,1
10
20
30
40
50
60
70
80
90
100
frequency in Hz

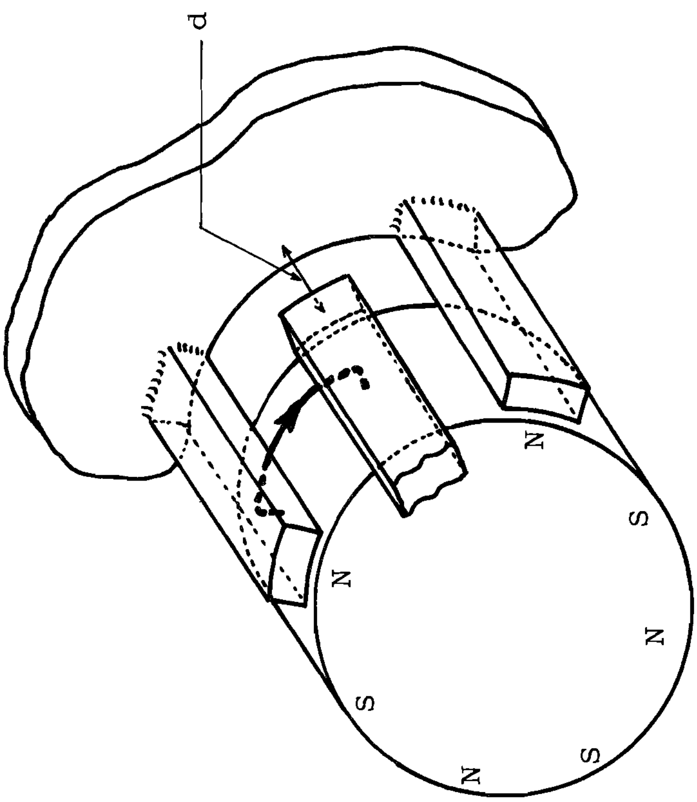


Figure 27 a

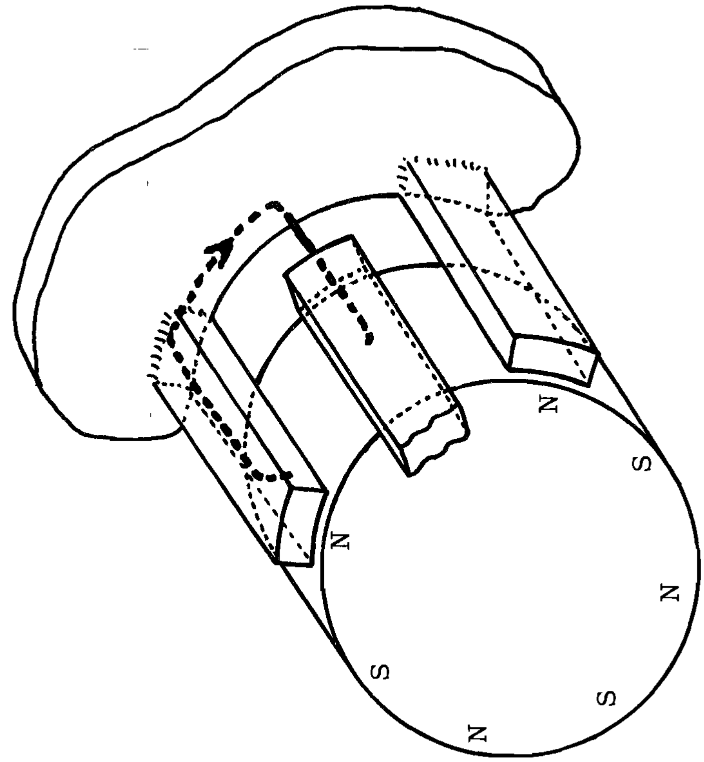


Figure 27 b

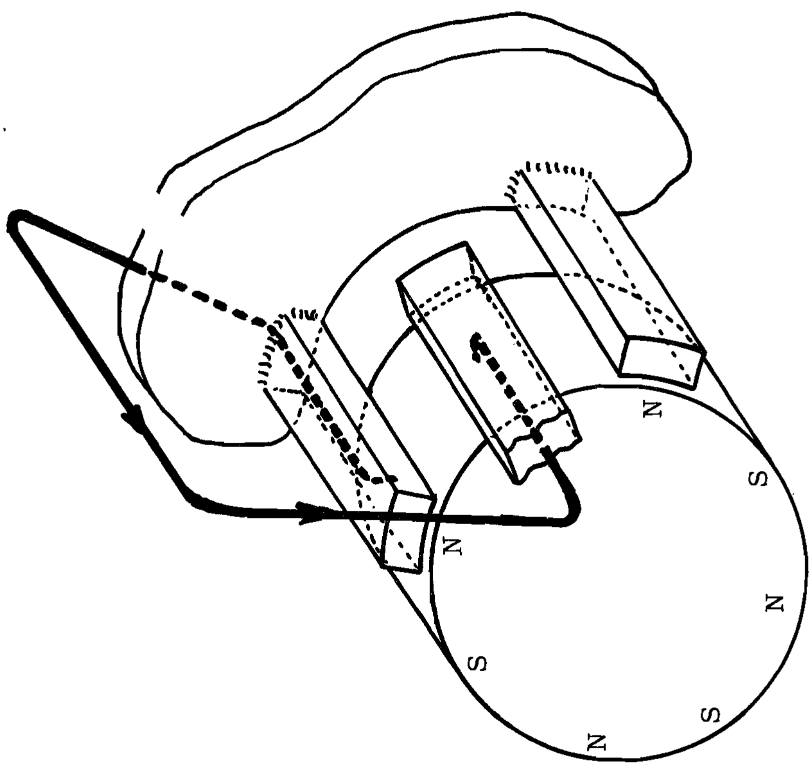


Figure 27 c

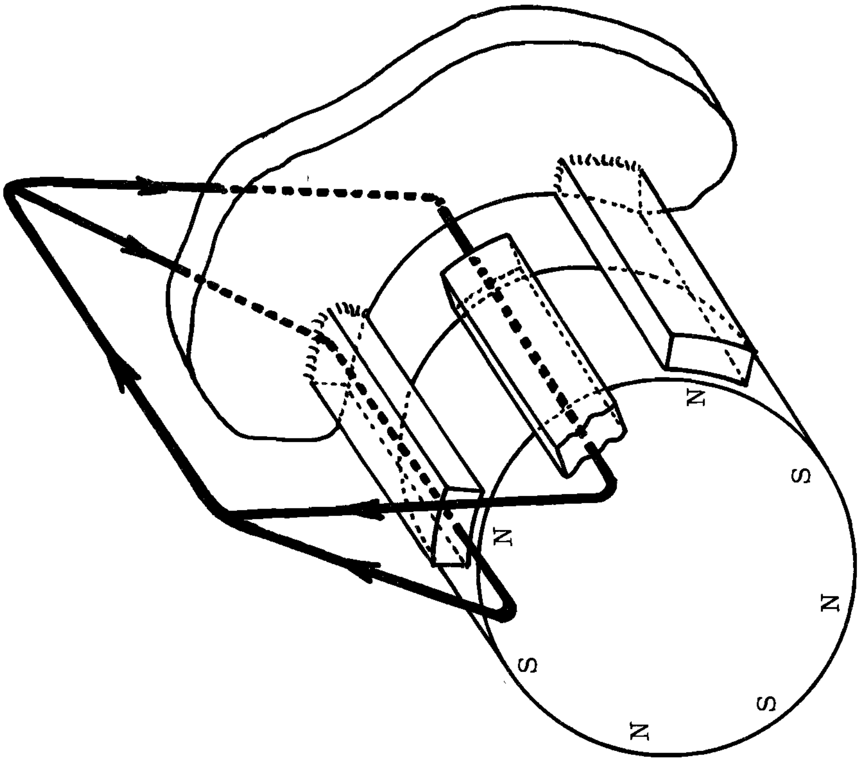


Figure 27 d

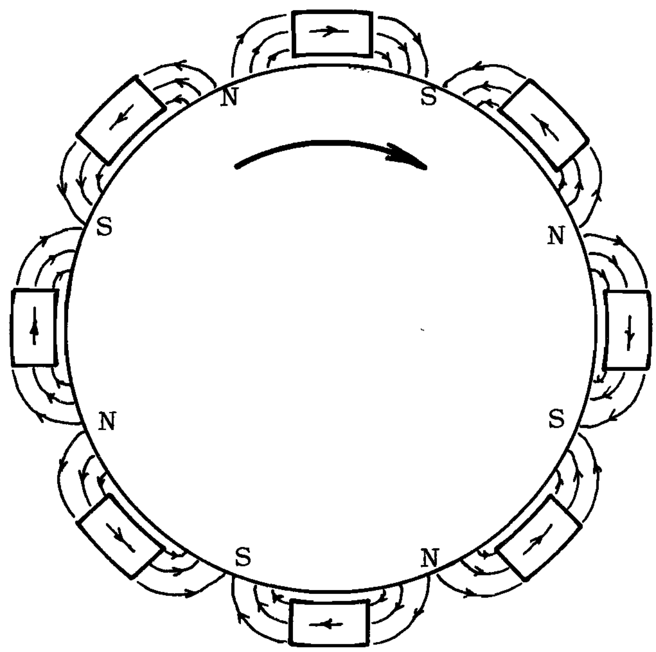


Figure 28

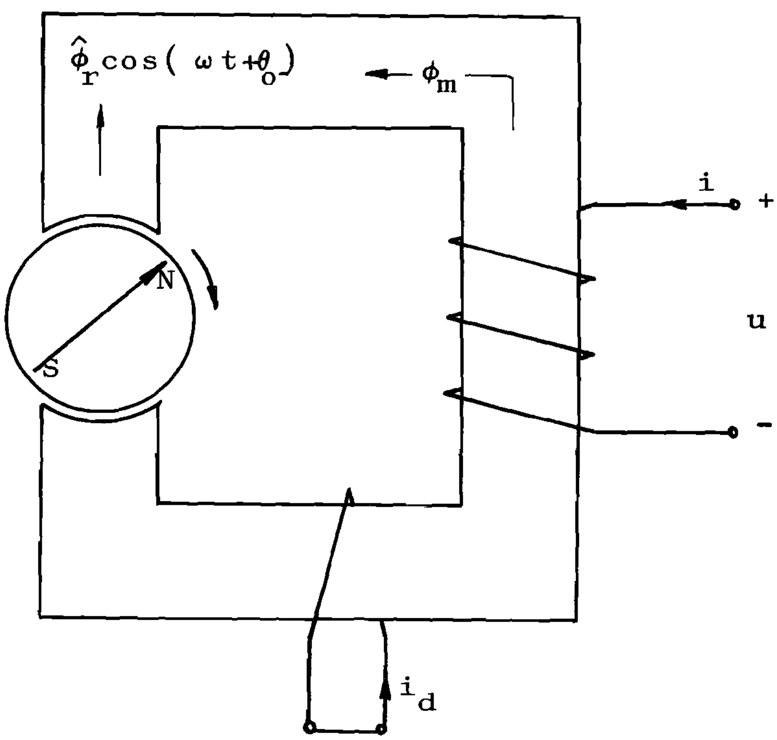


Figure 29