MASTER

Parameter estimation on a aortalike tube with a distributed parameter model

Hopmans, A.C.M.

Award date:
1974

Group Measurement and Control
DEPARTMENT OF ELECTRICAL ENGINEERING
UNIVERSITY OF TECHNOLOGY
EINDHOVEN, THE NETHERLANDS

PARAMETER ESTIMATION ON A AORTALIKE TUBE WITH A DISTRIBUTED PARAMETER MODEL

by A.C.M.Hopmans

Submitted in partial fulfillment of the requirements for the degree of ir. (M.Sc.) at the Eindhoven University of Technology. The work was carried out from January 1973 until January 1974, in the Measurement and Control Group under directorship of Prof.dr.ir.P.Eykhoff.
Advisor: ir.C.A.M.van den Brekel.

SUMMARY

This report deals with a distributed linear model of the aorta. It is a mathematical description of pressure-flow-relationship in the vessel, represented in the frequency domain.
Some criteria are given for limiting the frequency range of the model.

The model has been used to estimate the parameters of a tubelike aorta, that are of physiological importance. The adjustments have been performed using hill-climbing techniques, viz.: Gauss-Newton and Marquardt, in the frequency domain. It is shown that the parameters, within a physiologically important region, may be estimated well with Gauss-Newton; beyond this region the method of Marquardt is a necessary and powerful tool.
Some suggestions for further work are given.

CONTENTS

1. INTRODUCTION

There have been made several attempts to develop models for vessels, subsystems of the bloodcirculation and the entire cardiovascular system.
In medical science they may be used as models for the patient; for education purposes, to predict operative effects etc.
Among measurement and control engineers, however, modelbuilding is a well known technique to estimate unmeasurable parameters from measurable signals.
So modelling of the bloodcirculation may also be used to find physiological relevant, in vivo unmeasurable, parameters from measurable signals as pressure and flow.

Mostly the models of (parts of) the bloodcirculation are based on linear differential equations, describing pressure and flow relations in the system. Moreover they are in general rather fine or coarse lumped e.g. one vessel is represented by many sections or one section is the representation of several vessels, parallel or in series.

In the case of one vessel, lumping means, that pressure and flow are considered to be constant over a certain length along the vessel. So, this is an approximation of the partial differential equations describing that vessel.

This report deals with a mathematical model for the main vessel in our body, the aorta. It is distributed in the parameters and essentially linear, therefor a description in the frequency domain is possible.
The advantage of this model over a lumped linear one is,that it is an exact representation of the already simplified description of the pressure-flow relations in the vessel, while the lumped model is an approximation of that description. Moreover this model needs no hybrid computer but only a digital one.
After some refinements this model may be used to estimate the physiological important parameters of the aorta from pressure and flow curves of a patient.

For the present, the aorta is considered as a straight tube with constant diameter and wall thickness, and we assume the aorta has no branches. Because it is difficult to obtain good patientsignals at our university and moreover because it is impossible to find such an idealized aorta, the model will be tested on a mechanical analogue of the aorta, namely a cylindrical rubber tube, length 40 cm, driven by a mechanical pump. Using the pressure and flow signals of this analogue, its parameters will be estimated.

Chapter 2 of this report deals with the model building: the description of the model in the frequency domain, the search for a numerically stable model equation and the optimal number of harmonics, that is admitted in the model.
In Chapter 3 some theoretical aspects of parameter estimation in the frequency domain are discussed.
Chapter 4 gives some results of parameter choice and sensitivity, while Chapter 5 givesinformation about the estimation programs, as well as estimation results.
These results are obtained from parameter estimations with the distributed model on other models of the aorta viz. a 8-sections lumped parameter model implemented on an analogue computer driven by a function generator and the rubber tube already mentioned.

2. THE MODEL BUILDING

This Chapter describes briefly the equations of motion of rather idealized blood in an equally idealized aorta, and the pressure-flow relations which can be derived from these equations. Furthermore it shows why the model description is in the frequency domain, it makes a choice from different model equations, and it gives criteria for choosing the optimal number of harmonics that will be admitted in the model.

2.1 The basic equations for the model

For deriving the general equations of motion of the blood in a vessel, the fluid is mostly considered as an incompressible Newtonian fluid. This property and the assumption that the blood flow is laminar yields the Navier-Stokes equations:

$$\rho \frac{\partial \underline{v}}{\partial t} + \rho(\underline{v}\cdot\text{grad } p)\underline{v} = \underline{K} - \text{grad}\, p + \eta \Delta \underline{v} \tag{2.1}$$

Here is $\underline{v}$ the velocity vector, $\underline{K}$ the force vector per unit volume, p the pressure, Δ the Laplace operator, ρ the mass-density and η the viscosity ~~of~~ the fluid.

These equations may be linearized if the system (vessel + blood) satisfies some conditions, e.g.: the maximal axial material velocity is much smaller than the wave velocity, the radial (material) velocity is much smaller than the axial velocity, no influence of gravity (so $\underline{K} = \underline{0}$).

Using these linearized equations, the equation of continuity and by several restrictions (ref.1) one may find the pressure flow relations in this rather idealized vessel.
These equations are:

$$-\frac{\partial p}{\partial z} = Ml\,\frac{\partial i}{\partial t} + Wsl\cdot i \tag{2.2a}$$

$$-\frac{\partial i}{\partial z} = Cl\,\frac{\partial p}{\partial t} + Cl\cdot Wpl\cdot\frac{\partial^2 i}{\partial z\,\partial t} \tag{2.2b}$$

Here p(z,t) is the pressure in the vessel on the spot z, and at time t; i(z,t) the flow with same conditions, z the axial direction, t the time, Ml the inertance and Wsl the resistance to flow. Cl, the compliance, and Wpl represent the viscoelastic effects of the vessel wall. All coefficients are per unit length. (2.2a) en (2.2b) are the basis for both distributed and lumped models of vessels. The lumped version approximates (2.2), a distributed model provides an exact solution of (2.2).
There is no general analytic time-solution of (2.2) but there is an harmonic solution. On this base and because (2.2) is linear it is possible to make a model in the frequency domain.

One may also find a numerical solution in the time domain using an iterating procedure on a digital computer, but this is a very time-consuming task.

Using Fast Fourier Transform procedures one has a very fast algorithm for the transformation of the pressure and flow signals measured in the human aorta, the tube etc, from time to frequency domain, and, if wanted, the model output back to the time domain.

So a distributed model described in the frequency domain will be much faster than one described in the time domain.It has the disadvantage however, that implementation of non-linearities is inherently quite complicated.
The harmonic solution of (2.2) can be found by inserting in the basic equations the following expressions for p(z,t) and i(z,t):

$$p(z,t) = P(z)\, e^{j\omega t} \qquad (2.3a)$$

$$i(z,t) = I(z)\, e^{j\omega t} \qquad (2.3b)$$

This yields:

$$-\frac{dP(z)}{dz} = (Wsl + j\omega Ml)\, I(z) \qquad (2.4a)$$

$$-\frac{dI(z)}{dz} = Wpl.j\omega Cl.\frac{dI(z)}{dz} + j\omega Cl.P(z) \qquad (2.4b)$$

Or with:

$$Z1 = Ws1 + j\omega M1 \quad (2.5)$$

$$Z2 = Wp1 + 1/j\omega C1 \quad (2.6)$$

One has to solve:

$$-\frac{dP(z)}{dz} = Z1.I(z) \quad (2.7a)$$

$$-\frac{dI(z)}{dz} = \frac{1}{Z2}.P(z) \quad (2.7b)$$

The general solution of (2.7) is:

$$I(z) = A\exp(-\gamma z) + B\exp(\gamma z) \quad (2.8a)$$

$$P(z) = Z0.A.\exp(-\gamma z) - Z0.B.\exp(\gamma z) \quad (2.8b)$$

Here A and B are integration constants, z is the distance along the vessel, Z0 the characteristic impedance and γ the propagation exponent.

So: $$Z0 = \sqrt{Z1.Z2} \quad (2.9)$$

$$\gamma = \sqrt{\frac{Z1}{Z2}} \quad (2.10)$$

With $I0 = A + B$

$$P0 = Z0(A - B)$$

One may find expressions for P(z) and I(z)in terms of P0 and I0 (2.11)

$$I(z) = I0.\cosh(\gamma z) - \frac{1}{Z0}.P0.\sinh(\gamma z) \quad (2.11a)$$

$$P(z) = -Z0.I0.\sinh(\gamma z) + P0.\cosh(\gamma z) \quad (2.11b)$$

Interpretation of equation (2.11):
I(z) and P(z) are complex amplitudes of flow and pressure on the spot z of the tube, which can be expressed by (2.11) in terms of the amplitudes at the beginning of the vessel.
These equations are the basic equations for a variety of possible model descriptions in the frequency domain.

2.2 The 8-sections model

Before giving a further description of the model in the frequency domain, it is necessary to tell something about a lumped model.
We used such a model to obtain some physiologically looking signals with a low noise level. For, if one is starting with a new model, for example (2.11) may be such a mathematical description, it is very useful to test it for numerical stability and necessary frequency band with "process-signals" (PO and IO in (2.11)) that are alike the physiological ones, but with a low noise level.
The 8-sections model of Tomesen (ref.2) may serve as a "low noise signal generator", because it is a good approximation of (2.2) and hence of a vessel.
A short derivation and set up of this model follows.
For easy reference we repeat (2.2).

$$-\frac{\partial p(z,t)}{\partial z} = Ml\,\frac{\partial i}{\partial t} + Wsl.i \qquad (2.2a)$$

$$-\frac{\partial i(z,t)}{\partial z} = Cl\,\frac{\partial p}{\partial t} + Cl\cdot Wpl\,\frac{\partial^2 i}{\partial z \partial t} \qquad (2.2b)$$

Actually, pressure p and flow i in the tube are continuously dependent of distance z along it (see fig 2.1a). When lumping however, these signals are considered to be independent of the distance along the vessel for a certain length.
The same situation is occuring in lumping a transmission line.
Fig 2.1a gives a segment of the vessel while fig 2.1b gives its electrical lumped representation [1].

---

[1] Note that this section is not equal to the L-section of a real transmission line, $\frac{R_{pl}}{\Delta z}$ and $C_l\,\Delta z$ are in series instead of parallel.

This can easily be verified:

The electrical section in fig 2.1b is described by (2.12):

$$p_z - p_{z+\Delta z} = Rs1.\Delta z.i_z + L1.\Delta z \frac{\partial i_z}{\partial t} \qquad (2.12a)$$

$$\frac{\partial p_{z+\Delta z}}{\partial t} = \frac{\partial}{\partial t}(i_z - i_{z+\Delta z}) \frac{Rp1}{\Delta z} + \frac{1}{C_1 \Delta z}(i_z - i_{z+\Delta z}) \qquad (2.12b)$$

Let $\Delta z \rightarrow 0$ then:

$$-\frac{\partial p}{\partial z} = Rs1.i + L1 \frac{\partial i}{\partial t} \qquad (2.13a)$$

$$-\frac{\partial i}{\partial z} = C1 \frac{\partial p}{\partial t} + \frac{\partial^2 i}{\partial z \partial t}.Rp1.C1 \qquad (2.13b)$$

Let Rs1 $\hat{=}$ Ws1, L1 $\hat{=}$ M1, Rp1 $\hat{=}$ Wp1 and C1 $\hat{=}$ C1 (2.14)

Then (2.13) is exactly analogous to (2.2), and the harmonic solution of (2.13) is (2.11).

From now on we shall use the electrical representation of the vessel described by (2.13). Then Z1 and Z2 of (2.5) and (2.6) become:

$$Z1 = Rs1 + j\omega L1 \qquad (2.15)$$

$$Z2 = Rp1 + 1/j\omega C1 \qquad (2.16)$$

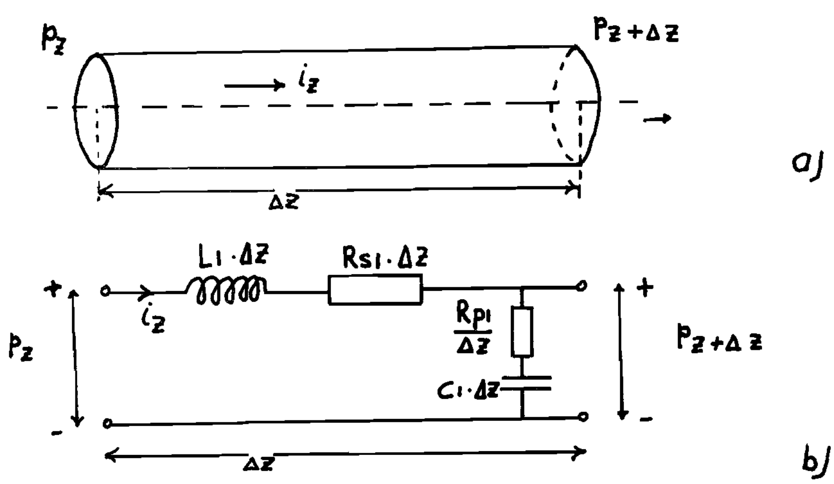


fig 2.1 One segment of a blood vessel (a)
and its electrical lumped representation (b).

Tomesen (ref. 2) has shown that a aorta segment of 40 cm can be well approximated by 8 sections of fig 2.1b. Each section has a length of 5 cm. The termination exists of two resistances and one capacitance (fig 2.2).

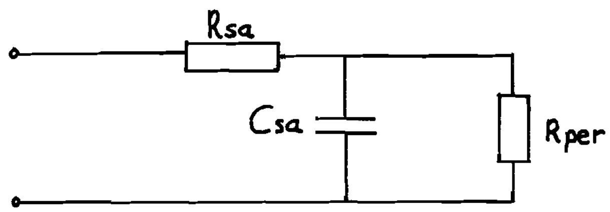


fig 2.2 The termination of the 8-sections model of Tomesen.

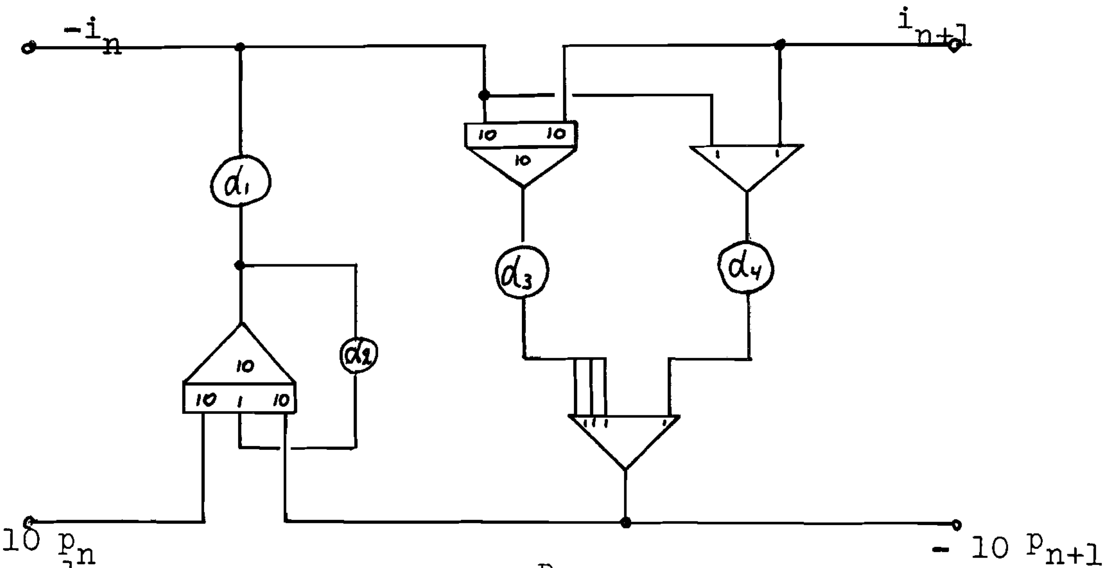
Rsa = resistance caused by systemic arteries, Rper = resistance caused by periferal vessels, Csa = capacitance of the systemic arteries.
The model is driven by a function generator which produces a pressure curve, similar to physiological pressure curves measured behind the arcus aorta.

The parameter values of model and termination are those used by Timmer (ref.3) and Leliveld (ref.4). They seem to be reasonable physiological values (see table 2.1).

parameter per unit length	values in medical units	parameter per 5 cm	values in medical units
Rsl	1.67×10^{-4}	Rs	8.35×10^{-4}
Ll	2.74×10^{-4}	L	1.37×10^{-3}
Rpl	1.37×10^{-1}	Rp	2.74×10^{-2}
Cl	1.468×10^{-2}	C	7.34×10^{-2}
Rsa	8×10^{-2}	medical units: resistance:1 mmHg.s/ml inertia :1 mmHg.s^2/ml compliance:1 ml/mmHg	
Csa	1		
Rper	1.58		

table 2.1 Parameter values of the 8-sections model.

The 8-sections and the termination has been implemented on an analog computer (Hitachi 350). Fig 2.3 gives the representation of one 5 cm section on the computer.

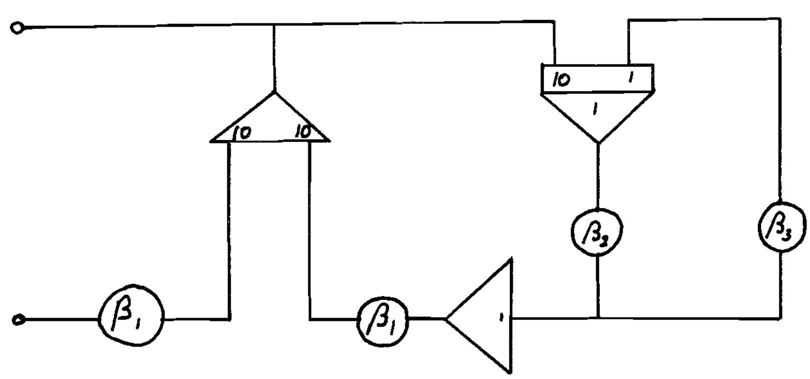


$\alpha_1 = \frac{1}{1000\ L} = .73$ $\alpha_2 = \frac{Rs}{10\ L} = .061$

$\alpha_3 = \frac{1}{30\ C} = .45$ $\alpha_4 = 10\ Rp = .274$

fig 2.3 The representation of one section (5 cm) on the analog computer.

The termination is represented in fig 2.4



$\beta_1 = \frac{1}{100\ Rsa} = .12$

$\beta_2 = \frac{1}{Csa} = 1$

$\beta_3 = \frac{1}{Rper} = .633$

fig 2.4 Termination of the 8-sections model.

Appendix 1 gives the complete analog representation of model and termination.

2.3 Three formulas for the output pressure of the model

Before making a choice from possible output pressure formulas, derived from (2.11), it is better to see after wishes, possibilitie and the intention of the model.

The intention is to use the model for estimating the parameters of the process, viz. tube, aorta. Then a good agreement of model and process is necessary. It is clear that this depends greatly on the restrictions and approximations used by De Pater (ref.1) for deriving equation (2.2). For, these equations are also the basis for this distributed model. We shall assume that this is a good description of the pressure flow relations in the aorta and rubber tube.

There is a practical wish to use a minimal number of signals in the estimating process, on the other hand we can only obtain three signals from the rubber tube (ref.4), because there are only two pressure transducers and one flow transducer. So the model description may use input and output pressure and input or output flow.

Because there is very little known about the termination of the aorta it is preferable to make no use of an approximation of the structure of it.

We have chosen for model parameter adjustment by comparing model and process output pressure, where the model is the distributed one based on (2.11).

$$I(z) = I_0 \cosh(\gamma z) - \frac{1}{Z_0} P_0 \sinh(\gamma z) \quad (2.11a)$$

$$P(z) = -Z_0 I_0 \sinh(\gamma z) + P_0 \cosh(\gamma z) \quad (2.11b)$$

The aorta and rubber tube,we consider, have a length of 40 cm, so $z = d = 40$ cm. From now on, we will write for input pressure and flow of the process: Pin and Iin instead of P_0 and I_0 respectively, and for these quantities at the end of the tube: Pout and Iout instead of $P(z)$ and $I(z)$.

So (2.11) becomes:

$$Iout_m[k]=Iin[k]\cdot\cosh(\gamma[k]\cdot d) - \frac{1}{Z_0[k]}\cdot Pin[k].\sinh(\gamma[k]\cdot d) \quad (2.17a)$$

$$Pout_m[k]=-Z_0[k]Iin[k]\sinh(\gamma[k]\cdot d) + Pin[k].\cosh(\gamma[k]\cdot d) \quad (2.17b)$$

m denotes model output, k the k-th harmonic.
Note that (2.17) gives an expression, in which the output signals are considered to be unknown, while the input signals are known (measured from the process). Of course other combinations of two signals can be measured and then modified expressions relate unknown and known signals. With the requirement that the measured

signals must be a pressure and a flow (because of the available transducers) and because we want to adjust on the output pressure, follows that there is just one another model description, namely if Pin and Iout are known:

$$Pout_m[k]=\left\{Pin[k] - Z_0[k].\sinh(\gamma[k]\cdot d).Iout[k]\right\}/\cosh(\gamma[k]\cdot d) \quad (2.18a)$$

$$Iin_m[k] =\left\{\frac{Pin[k]}{Z_0[k]}.\sinh(\gamma[k]\cdot d) + Iout[k]\right\}/\cosh(\gamma[k]\cdot d) \quad (2.18b)$$

So (2.17b) and (2.18a) are two formulas to compute model ouput pressure.

<u>a</u> Formula (2.17b) appears to be numerically unstable, especially for higher k or frequency.
Because the model has a low pass character $Pout_m[k]$ must be smaller than $Pin[k]$ for higher k (the tube is passive), however on the same moment k becomes greater (see (2.10)).
Hence, for $k\rightarrow\infty$, (2.17b) goes to:

$$Pout_m[k]= -Z_0[k].Iin[k].e^{\gamma[k]\cdot d} + Pin[k].e^{\gamma[k]d}$$

and $e^{\gamma[k]\cdot d}$ becomes very great (with respect to Pin and Iin).
So for higher k, $Pin[k]$ and $Z_0[k].Iin[k]$ must be almost the same, to make $Pout_m[k]$ small, and when these terms are subtracted great loss of digits happens.
As stated $Z_0[k].Iin[k] \rightarrow Pin[k]$ for $k\rightarrow\infty$ because

$$Zin[k] = \frac{Pin[k]}{Iin[k]} \longrightarrow Zo[k] \text{ for } k\rightarrow\infty :$$

$$\mathrm{Zin}[k] = Z_0[k]\frac{ZA.\cosh(\gamma d) + Z0.\sinh(\gamma d)}{ZA.\sinh(\gamma d) + Z0.\cosh(\gamma d)} \longrightarrow$$

$$\frac{ZA.e^{\gamma d} + Z0.e^{\gamma d}}{ZA.e^{\gamma d} + Z_0.e^{\gamma d}}.Z_0 = Z_0[k].$$

Here ZA is the terminal impedance.
Fig. 2.5 gives the output pressure in the time domain of the 8-sections model (flat-curve) and of the model with frequency band 0 - 13 Hz based on (2.17) (rough curve). The numerical unstabillity has the greatest effect on the flat part of the output pressure, as can be seen on the oscillations there.
Fig.2.6 shows the same pressure curves, but now the frequency range of the model is 30 Hz. Moreover the output pressures in fig 2.5 have been shifted over a half period with respect to fig 2.6. This has no further meaning. It is clear that in fig 2.6 the effect of the numerical unstabillity is greater, the oscillations are heavier.
The 8-sections model has served as process for the distributed model, so it delivers the necessary signals Iin and Pin.

b Formula (2.18a) appears also to be unstable, but now especially for low k. Then,namely Pin[k] and $Z_0[k].\sinh(\gamma d).\mathrm{Iout}[k]$ are nearly the same and when subtracted, loss of digits happens. We have checked this for k = 3 and we found that a disturbance of 5 % in the input signals of the model output equation: Pin and Iout delivers a deviation in Pout_m of 100 %.
Fig 2.7 gives again the output pressure of the 8-sections model (x-curve) and of the model with frequency band 0 - 15 Hz based on (2.18a). This figure makes it visible that the lower harmonics have been determined badly.

Another expression for the model output pressure is possible, if we use the terminal admittance:

$$YA[k] = \frac{\mathrm{Iout}[k]}{\mathrm{Pout}[k]}$$

With (2.18a) follows:

$$\mathrm{Pout} = (\mathrm{Pin} - Z_0.\sinh(\gamma d).\mathrm{Pout}.YA)/\cosh(\gamma d)$$

Bringing Pout to the left gives:

o see page 19

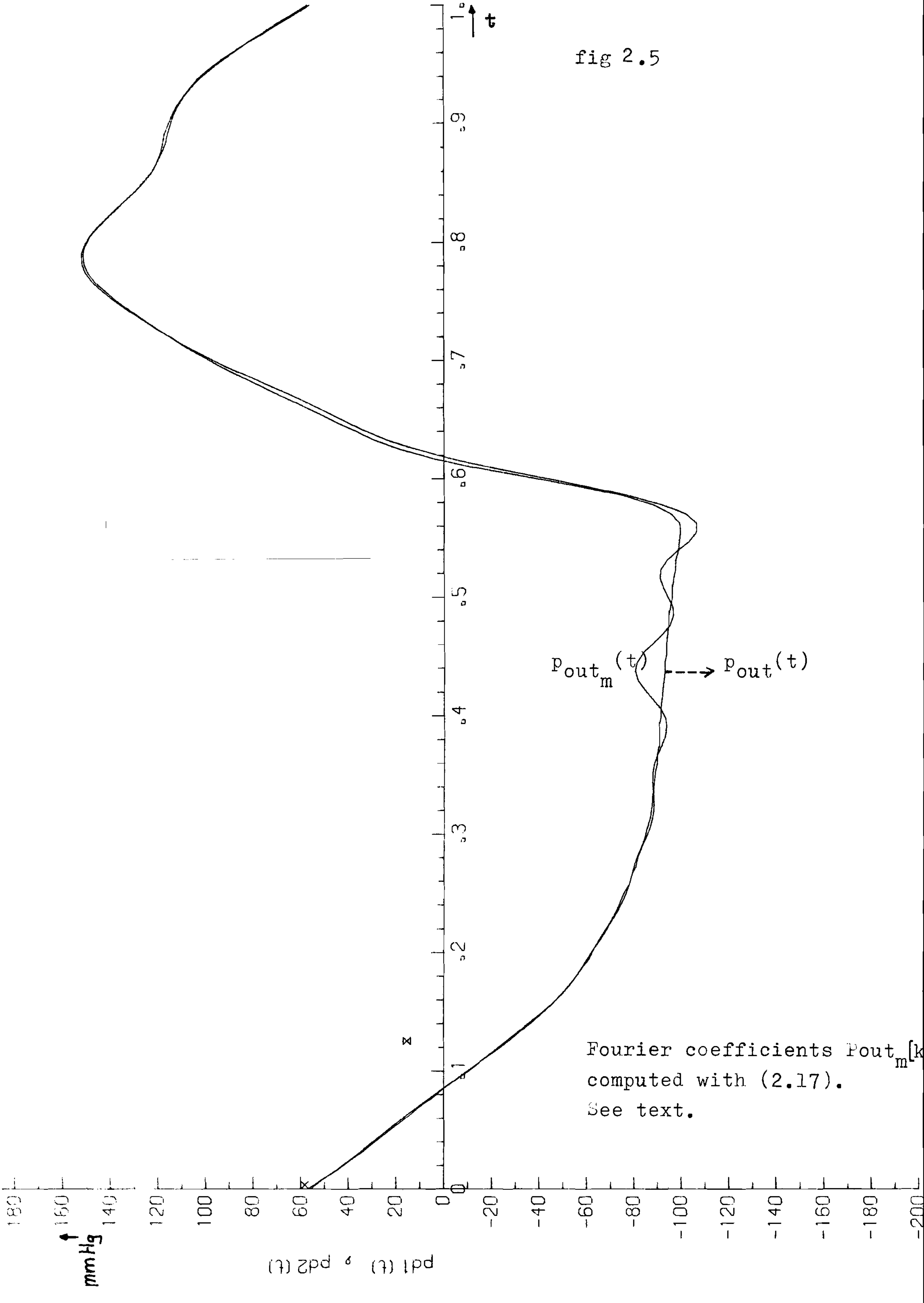
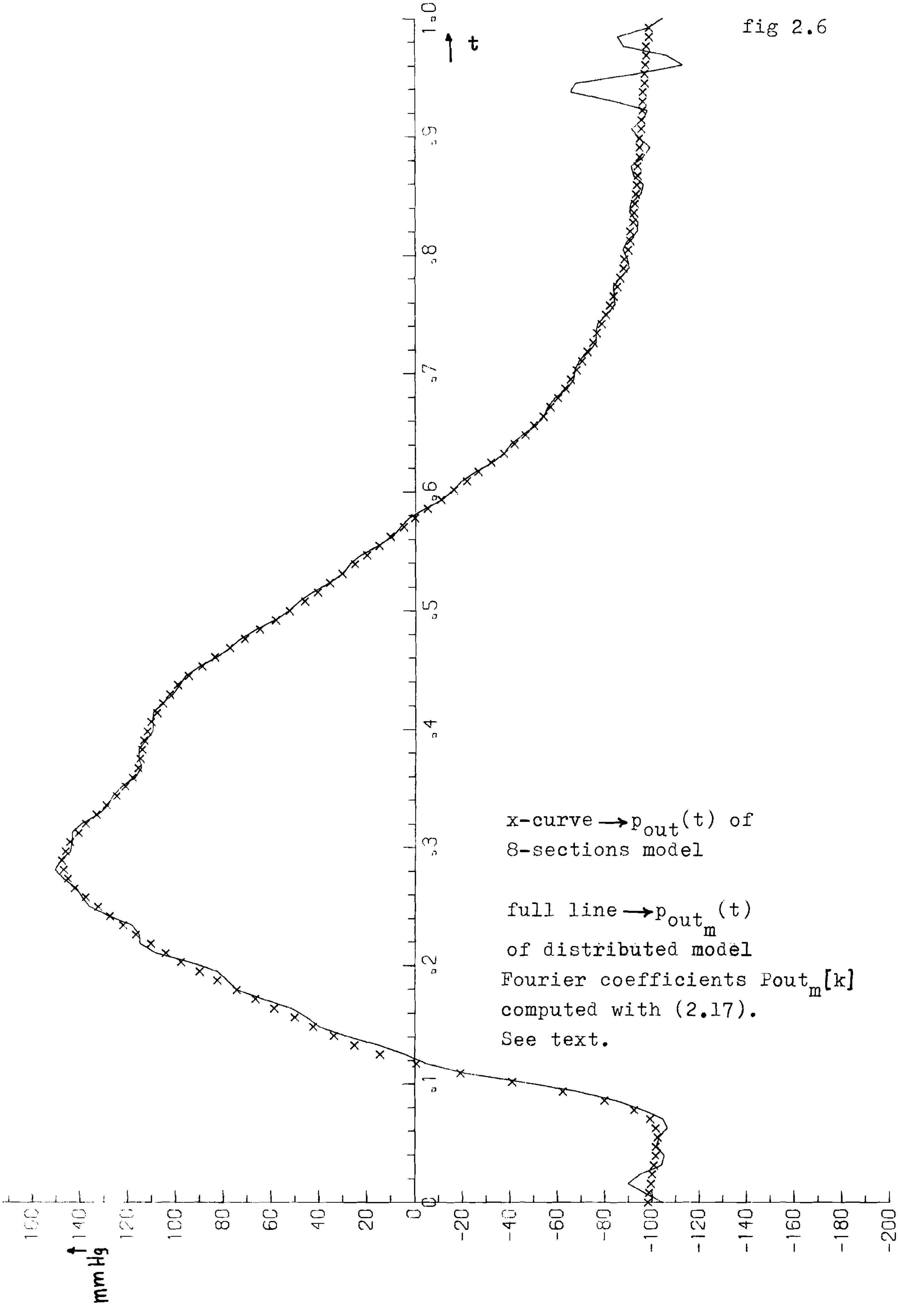


fig 2.5

Fourier coefficients $Pout_m[k$
computed with (2.17).
See text.

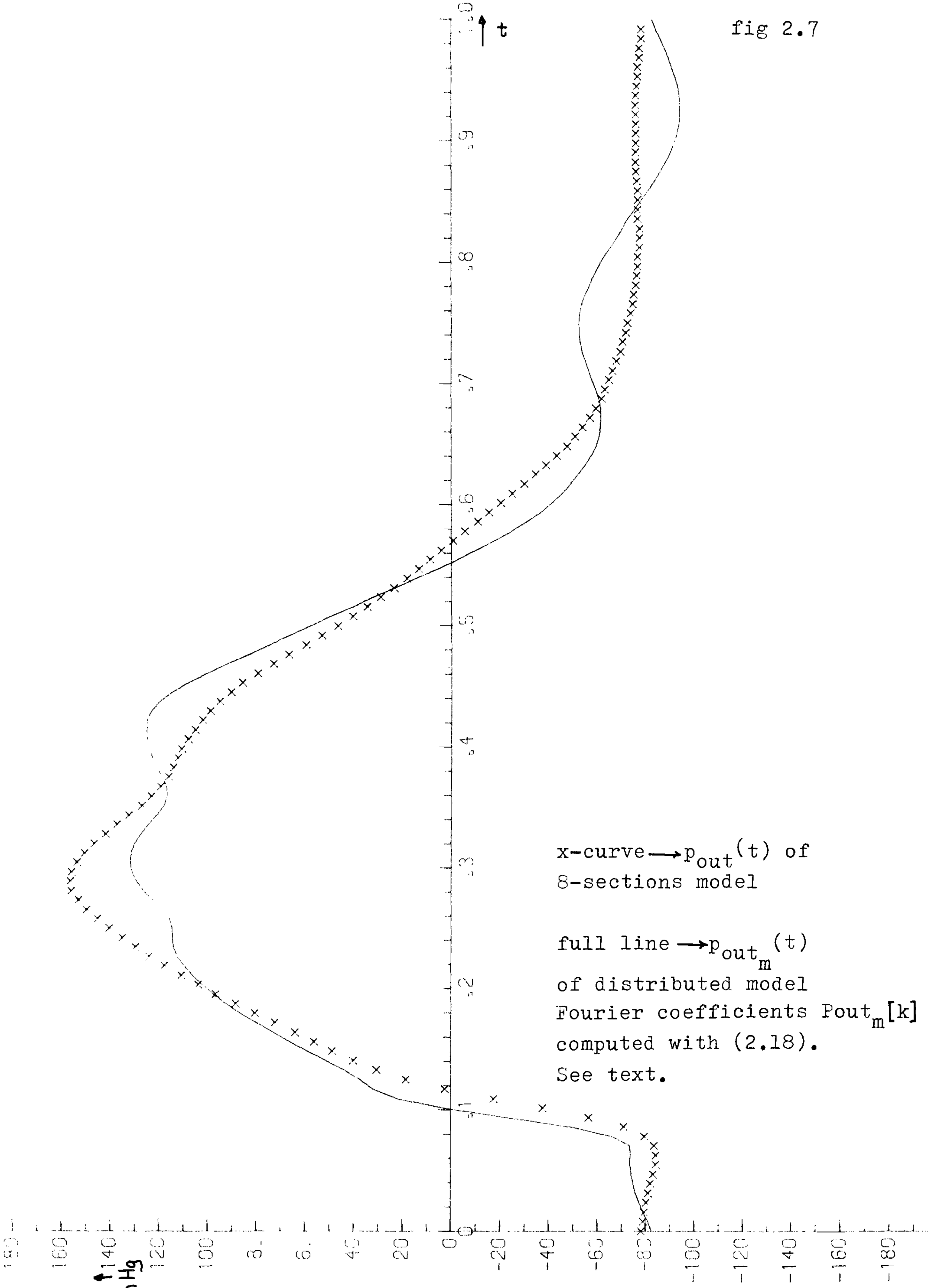
fig 2.6



x-curve → $p_{out}(t)$ of 8-sections model

full line → $p_{out_m}(t)$ of distributed model Fourier coefficients $Pout_m[k]$ computed with (2.17). See text.

fig 2.7



x-curve → $p_{out}(t)$ of 8-sections model

full line → $p_{out_m}(t)$ of distributed model Fourier coefficients $Pout_m[k]$ computed with (2.18). See text.

$$\underline{c}\ Pout_m[k] = \frac{Pin[k]}{\cosh(\gamma[k]\cdot d) + Z_0[k].\sinh(\gamma[k]\cdot d).YA[k]} \qquad (2.19)$$

Formula (2.19) appears to be stable for relevant k. It is necessary to emphasize that it is very difficult to prove if a particular complex formula is numerically stable or unstable, because not only additions and subtractions may cause unstability, but also complex multiplications and divisions. Actually,these complex operators are additions, subtractions, multiplications and divisions of real numbers.
For instance the multiplication of

$$\sinh(\gamma[k].d).YA[k] = (a + jb).(c + je) = ac - be + j(ae + bc)$$

may give rise to loss of digits if ac and be are nearly the same, but a = re(sinh(γd)) depends on the frequency k, the setting of Rsl,Ll,Rpl and Cl which determine γ, the length of the tube d;

and c depends on the kind of termination and the frequency
So, it will be clear that it is hardly to say if a formula stays stable for different settings, terminations, frequencies etc.
We have checked (2.19) for values of Rsl, Ll, Rpl and Cl as printed in table 2.1, for the termination of the 8-sections model and for a frequency range 0 - 64 Hz. It appears that formula (2.19) is the most stable one.
Fig 2.8 shows again output pressure curves of the 8-sections model and the model based on (2.19), with frequency band 0 - 13 Hz and the terminal admittance computed from the structure of fig 2.2 and element values of table 2.1.
If we determine Ya[k] from the measured process output flow and pressure, then the structure of the termination has not to be known and (2.19) can be replaced by (2.20):

$$Pout_m[k] = \frac{Pin[k]}{\cosh(\gamma[k]\cdot d)+Z_0[k].\sinh(\gamma[k]\cdot d).\frac{Iout[k]}{Pout[k]}} \qquad (2.20)$$

Note that the ratio Iout/Pout is independent of the process, so there is no coupling between process and model. This is very important with regard to bias in estimated parameters, for in the case there is coupling it will presumably cause bias.

Fig 2.9 gives again the output pressure curves, but now the computation of model output pressure is done with (2.20), while the frequency band of the model is 0 - 13 Hz.

This figure shows a very good agreement between the two curves.

From now on we shall use (2.20) as model output pressure formula.

Remark:

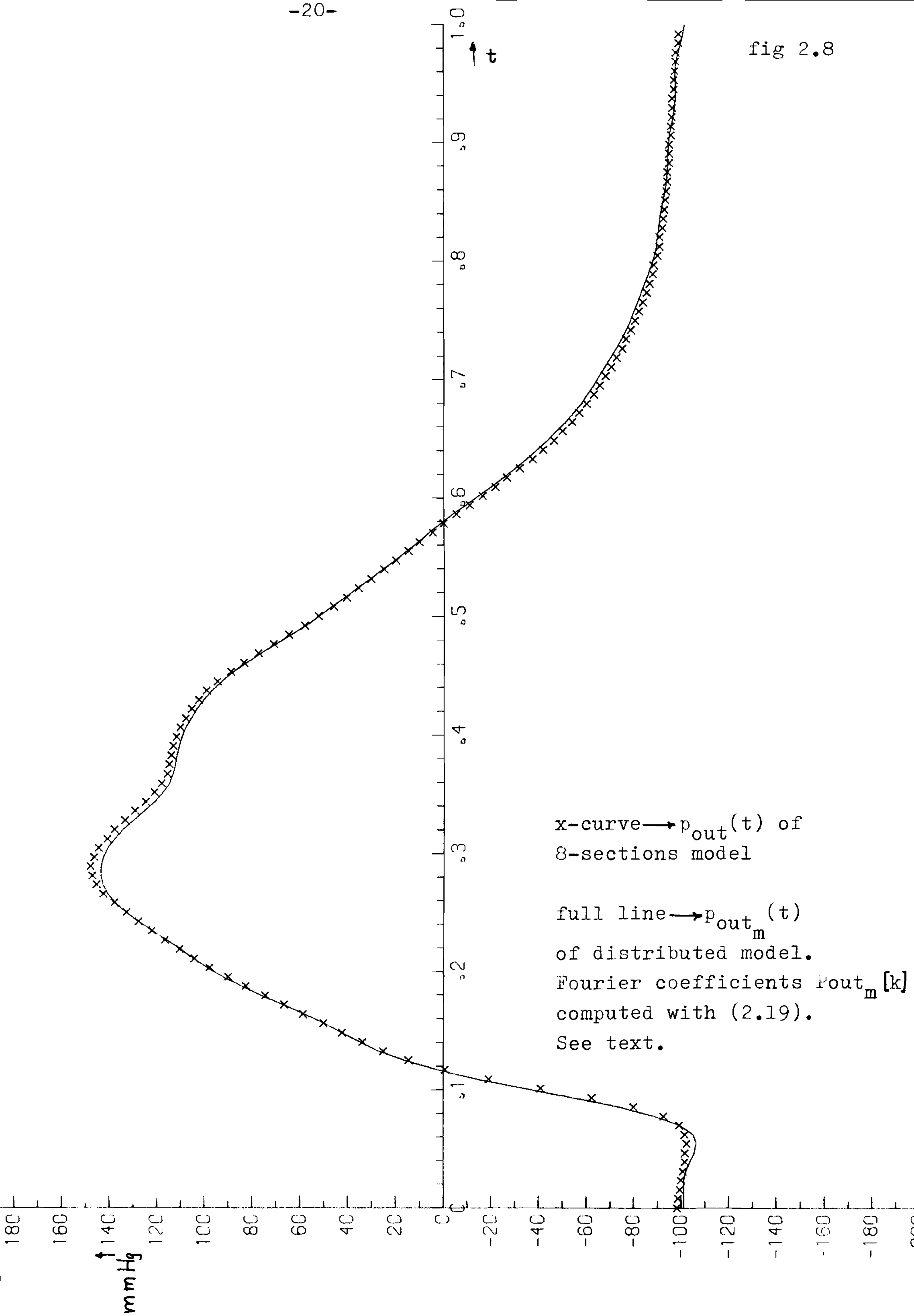
It must be emphasized here that we have not strived after completeness with respect to all possible descriptions of a particular output pressure formula ((2.17b) and (2.18a)), to see if there is a description which will be numerically stable. However, more extended information about the way we checked some formulas on stability can be found in the logbook of this work, which is at the Measurement and Control Group of the Eindhoven University of Technology.

For instance we have tried:

$$Pout_m = (Pin - Z_0.Iin).e^{\gamma d} + (Pin + Z_0.Iin).e^{-\gamma d}$$

instead of (2.17b). This description gave no further improvement, as will be clear from page 14. For k = 500, for instance, $(Pin - Z_0.Iin).e^{\gamma d}$ is of the order of magnitude of 10^{+22}, while the second term is about 10^{-19} and Pout[500] about 10^{-4}.

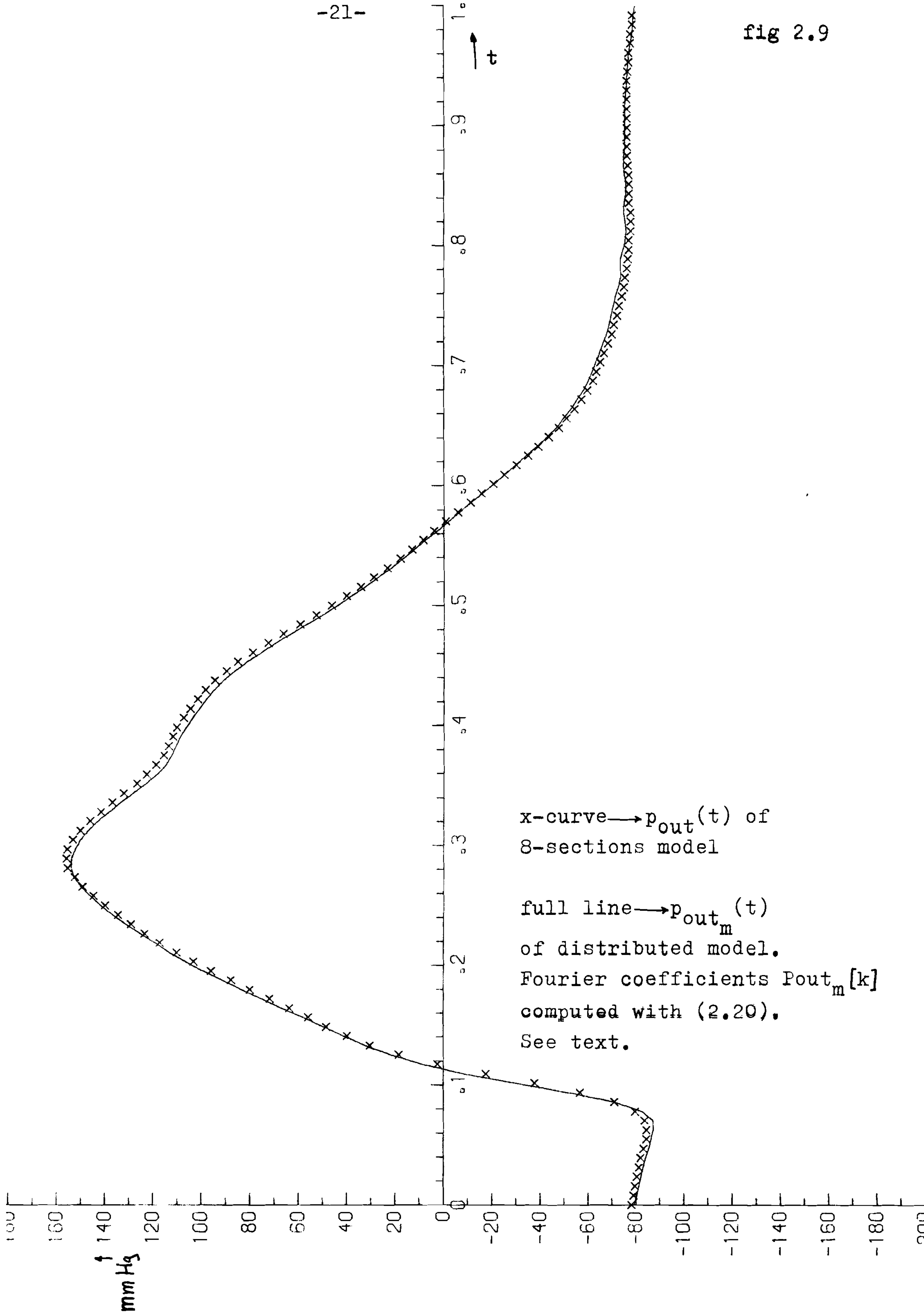
fig 2.8



x-curve ⟶ $p_{out}(t)$ of 8-sections model

full line ⟶ $p_{out_m}(t)$ of distributed model. Fourier coefficients $Pout_m[k]$ computed with (2.19). See text.

fig 2.9


t
mm Hg
x-curve → $p_{out}(t)$ of 8-sections model
full line → $p_{out_m}(t)$ of distributed model.
Fourier coefficients $Pout_m[k]$ computed with (2.20).
See text.

The diagram of fig 2.10 shows the set up for comparing the output pressures $Pout_m$ and Pout. The process is excited by a function generator (basic frequency 1 Hz), it is terminated by impedance ZA.
Pressure and flow of input and output: $p_{in}(t)$, $p_{out}(t)$ and $i_{out}(t)$ respectively are sampled with a PDP-8 computer with sampling frequency f_s=1024 Hz (This is very high for physiological signals). After the sampling, one period (1024 samples) of each signal is sent into the Fast Fourier Transform procedure. Then we obtain $f_s/2$=512 frequency components of each signal, viz: Pin[k], Pout[k] and Iout[k]. The terminal admittance is by definition: YA[k]=Iout[k]/Pout[k]. Now, Pin[k] and YA[k] are the arrays of input for the model. The model equation computes $Pout_m[k]$, where $0 < k_{max} \leq 512$. We have chosen for k_{max}=12,13 and 30 respectively. This gives 12,13 and 30 model pressure components, these can be compared with the corresponding process components: Pout[k]. By adding zero's to the $Pout_m$-spectrum, namely for $k>k_{max}$ until 64, one is always able to transform $Pout_m[k]$ to 128 time samples. Then one may compare these samples with the corresponding process ouput samples. The differences between the comparison of frequency components and time samples will be discussed in Chapter 3 the first paragraph.

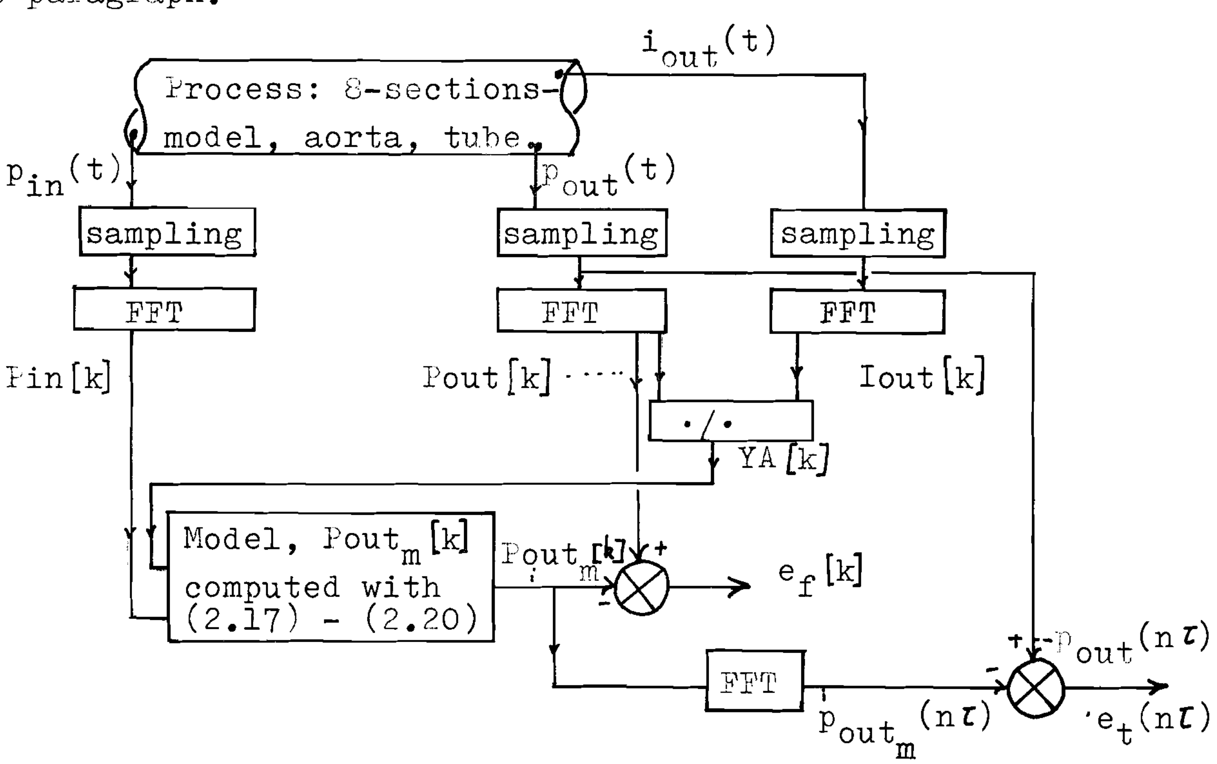


fig 2.10 Block-diagram of set up for comparing process and model output pressure.

2.4 The frequency range of the distributed model

Summarizing the previous sections: there is a distributed model, described by (2.11) in the frequency domain; it computes h frequency components of the model output pressure $Pout_m$ with (2.20). The Inverse Fourier Transformation yields the model output in the time domain.
Here h is the highest relevant harmonic.
There are various criteria to determine the limit of the optimal frequency band. These criteria are:

1 That harmonic who's amplitude is less than 1% of the maximum harmonic sets the cut-off frequency.

2 That frequency where the noise is equally big as the signal determines the cut-off frequency.

3 The model computes components using (2.20). When, from a certain k, the error propagation is such,or when the model and process Fourier coefficients, from a certain k, are so different, that no further improvement with regard to Least-Squares agreement of process and model output can be expected, then the cut-off frequency is reached.

4 It appears that the number of harmonics has influence on the place of the optimal parameter set; in other words, it is possible that minimal difference, in Least Squares sense, between model and process output pressure is reached for other parameter values by a model with a smaller frequency band than by a model with a greater frequency band. So the model frequency range has to be such, that with more frequencies included no other "optimal" parameter set will be found.

We have chosen for the first criterion; this choice, however, has relations with criteria 3 and 4.
Analysis of the Fourier spectrum of the output pressure of the 8-sections model makes it clear that the 13th harmonic is the first one that is smaller than 1% of the greatest (the first one). See appendix 2.

With respect to the second criterion, it may be shown that the noise level (sampling noise) in 22th harmonic of Pout is about 50%. So according to the second criterion the cut-off frequency would be 21Hz (when the basic frequency is 1Hz).

Model equation (2.20) uses YA[k]=Iout[k]/Pout[k] and Pin[k], so error propagation may be expected. Table 2.2 gives the absolute value of the admittance of the termination, in column 2 computed with the known structure and element values as implemented on the analog computer (fig 2.2); in column 3 computed from the 8-sections ouput; in column 4 is the relative error $\frac{|YAm| - |YA|}{|YA|} x100\%$.

frequency	|YA[k]| from structure	|YA[k]| from Iout and Pout	relative error
1	5.42205	5.43563	.26
2	8.65284	8.70618	.61
3	10.2619	10.3323	.69
4	11.0822	11.1550	.68
5	11.5352	11.7204	1.6
6	11.8059	11.9660	1.4
7	11.9787	11.9369	.35
8	12.0950	11.9344	1.3
9	12.1767	12.0619	.94
10	12.2362	12.1372	.81
11	12.2808	12.1253	1.3
12	12.3150	12.2251	.73
13	12.3419	12.7157	3.0
14	12.3633	12.7084	2.8
15	12.3807	12.2207	1.3
16	12.3949	12.4912	.78
17	12.4608	12.9671	4.5
18	12.4168	11.6350	6.3
19	12.4252	12.3570	.55
20	12.4324	15.7687	27.
21	12.4387	12.6596	1.8
22	12.4441	9.60721	23
23	12.4488	28.0420	125
24	12.4530	48.0254	286

Table 2.2 Admittance of termination of 8-sections model computed from structure and from output signals.

Above 22 Hz the errors are greater than 100% and they are less than 10% for frequencies under 20 Hz.
Furthermore the error in Pin[k] will be less than 10% of the signal amplitude for frequencies under 18 Hz.
So to get a small error propagation, we admit in accordance of criterion 1, only 13 harmonics.

Remark: The basis of criterion 4 is a strange experience we had, studying the parameter sensitivity of the 8-sections model. It appears that the minimal PI-value 1) is found for other values of Rpl and Cl when the model frequency range is 7 instead of 13 or 18 harmonics (in this case the variable parameters are Rpl and Cl; Ll and Rsl are constant). See table 2.3

number of harmonics	minimal PI reached for Rpl in med.un.:	minimal PI reached for Cl in med.un.:
7	.205	.0156
13	.151	.0156
18	.151	.0156

In the 3 situations were $Rsl = 1.67 \times 10^{-4} \frac{mm\ Hg}{ml/sec}$

$Ll = 2.74 \times 10^{-4} \frac{mmHg}{ml/sec^2}$

Table 2.3 Dependance of optimal parameter set on the number of harmonics in the model.

This phenomenon may have some relation with the following effect:
Say the distributed model includes p harmonics:
Then:

$$PI_p = \underbrace{\sum_{k=-p}^{+p} \left|Pout[k] - Pout_m[k]\right|^2}_{I} + \underbrace{\sum_{k=-\infty}^{-(p+1)} \left|Pout[k]\right|^2}_{II} + \underbrace{\sum_{k=p+1}^{\infty} \left|Pout[k]\right|^2}_{III} \quad (2.21)$$

-----
1) PI is the Performance Index, a measure for the disagreement of process output and model output. See Chapter 3 and 4.

II and III of (2.21) are independent of the model parameters.
Hence:

$$II + III = c_1 \tag{2.22}$$

If the model is an exact description of the process it is possible to find one optimal parameter set (this appears from the PI-surfaces see Chapter 4), where $Pout_m[k] \equiv Pout[k]$ for $-p \leqslant k \leqslant p$. Then the PI-minimum will be c_1 for this optimal set.

A model with q harmonics (q>p) leads to a minimal

$$PIq = c_2 = \sum_{k=-\infty}^{-(q+1)} \left|Pout[k]\right|^2 + \sum_{k=q+1}^{\infty} \left|Pout[k]\right|^2 < c_1,$$

when $Pout_m[k] \equiv Pout[k]$ for $-q \leqslant k \leqslant q$.
Because q>p and there is only one parameter set, where $Pout_m[k] \equiv Pout[k]$ for all k between -p and p, this set will also be the optimal one in the case of q harmonics.

However if the model is no exact description of the process it will never be possible to find a set where $Pout_m[k] \equiv Pout[k]$ for $-p \leqslant k \leqslant p$. Then holds for the optimal set:

$$\min PI_p = \underbrace{\sum_{k=-p}^{+p} \left|Pout[k] - Pout_m[k]\right|^2}_{I} + c_1 \tag{2.23}$$

with I is minimal.
But with q harmonics:(q>p)

$$\begin{aligned}\min PI_q &= \sum_{k=-q}^{+q} \left|Pout[k] - Pout_m[k]\right|^2 + c_2 = \\ &= \underbrace{\sum_{k=-p}^{+p} \left|Pout[k] - Pout_m[k]\right|^2}_{I} + \underbrace{\sum_{k=-q}^{-(p+1)} \left|Pout[k] - Pout_m[k]\right|^2}_{II} + \\ &+ \underbrace{\sum_{k=p+1}^{q} \left|Pout[k] - Pout_m[k]\right|^2}_{III} + c_2\end{aligned} \tag{2.24}$$

It is possible that for the parameter set where I is minimal (and so (2.23)), sum II and III together with c_2 is larger than c_1, namely when $Pout[k]$ and $Pout_m[k]$ have for some

$p + 1 \leqslant k \leqslant q$ and $-q \leqslant k \leqslant -(p + 1)$, a phase difference $90^0 < \alpha < 270^0$. So one could imagine there will be a set of parameters, where I is not exactly minimal but where the total sum I + II + III is minimal.

It is clear that this effect only happens when the model is no exact description of the process.

As may be seen from table 2.3, 13 harmonics leads to the same optimal set as the model with 18 harmonics does.

3. PARAMETER ESTIMATION IN THE FREQUENCY DOMAIN

As said before, the model will be used to estimate physiological relevant process parameters, e.g. Rsl, Ll, Rpl and Cl or combinations from these; see (2.11), (2.15) and (2.16). However, the process is non-linear in these parameters.
So the minimum of a error function PI (Performance Index), which is a measure for the disagreement between model and process output pressure, will be searched with hill-climbing techniques.
Because there is lack of knowledge concerning statistical parameters of noise, we have chosen for Least-Squares estimators, so the error function will in general have the form:

$$PI = \underline{e}^T\underline{e} = \sum_{i=m}^{n} e_i^2 \qquad (3.1)$$

where $\underline{e} = \underline{y}-\underline{x}$ = process output vector minus model output vector (fig 3.1)

This holds if $\underline{e}$ is a vector with only real elements, e.g.: a vector of time samples of a real function. In the case that $\underline{e}$ is a complex vector, e.g.: a vector of Fourier coefficients, the next expression has to be used:

$$PI = \underline{e}^H\underline{e} = \sum_{i=m}^{n} |e_i|^2 \; {}^{1)} \qquad (3.2)$$

Because the model is a description of pressure flow relations in the frequency domain we will use equation (3.2). Then there is no need for transforming the frequency components $Pout_m$ of the model to the time domain, each time when a new parameter updating $\underline{\beta}^{N-1} \rightarrow \underline{\beta}^N$ has been performed.[2] So, with this model, frequency domain estimators have the profit of using less computer time in comparison with estimators in the time domain (at least when we consider only the computing time of the estimation part).

-----

1) $\underline{e}^H$ means the Hermitian of vector $\underline{e}$.

2) In general we mean with $\underline{\beta}$ the set of model parameters, N indicates the number of iterations, see paragraph 3.2.

The Least-Squares estimators in time and frequency domain gave rise to same estimates as can be seen in the first following paragraph. The second paragraph gives a derivation of the Gauss-Newton hill-climbing algorithm in the frequency domain. Marquardt's method is subject in the third paragraph.

3.1 The equivalence between Least-Squares estimates in time and frequency domain

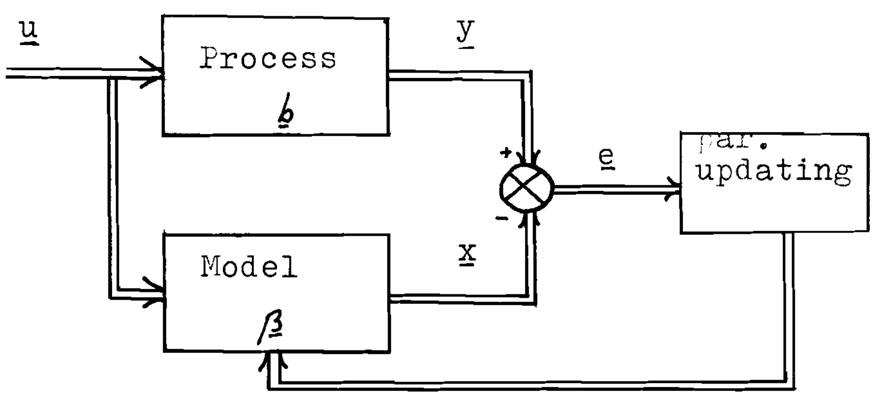


fig 3.1 Block diagram of model adjustment.

Time domain approach:

$\underline{u}$, $\underline{y}$, $\underline{x}$ and $\underline{e}$ are vectors consisting of the n samples of one period of the corresponding signals.

So:

$$\underline{y} = \{y(0), y(\tau), \ldots\ldots, y((n-1)\tau)\}^T \tag{3.3}$$

$$\underline{x} = \{x(0), x(\tau), \ldots\ldots, x((n-1)\tau)\}^T \tag{3.4}$$

$$\underline{e}_t = \{e(0), e(\tau), \ldots\ldots, e((n-1)\tau)\}^T \tag{3.5}$$

Here is τ the time between two succeeding sample moments, and $T = n\tau$ is one period.

The Least-Squares error function PI has the form:

$$PI = \int_{t_0}^{t_0+T} e_t^2 \, dt \quad ^{1)} \tag{3.6a}$$

where $e_t(t)$ is a continuous function of time, indicating the difference $y(t)-x(t)$ (not sampled output).

1) (3.6a) may be approximated by (3.6b):

$$PI \approx \underline{e}_t^T \underline{e}_t = \sum_{i=0}^{n-1} e_t(i\tau)^2 = \sum_{i=0}^{n-1} \{y(i\tau) - x(i\tau)\}^2 \tag{3.6b}$$

Frequency domain approach:

$\underline{U}$, $\underline{Y}$, $\underline{X}$ and $\underline{e}_f$ are, in this case, vectors consisting of Fourier coefficients of the corresponding periodic signals. In the case that the basic frequency is 1Hz the kth harmonic agrees with k Hz.

Then:

$$\underline{Y} = \left\{ Y(-h),\ldots\ldots,Y(0),\ldots\ldots,Y(h)\right\}^T$$
$$\underline{X} = \left\{ X(-h),\ldots\ldots,X(0),\ldots\ldots,X(h)\right\}^T \qquad (3.7)$$
$$\underline{e}_f = \left\{ e_f(-h),\ldots\ldots,e_f(0),\ldots\ldots,e_f(h)\right\}^T$$

Here is h the highest relevant frequency component in the signals.

Least-Squares means here:

$$PI = \underline{e}_f^H \underline{e}_f = \sum_{i=-h}^{+h} e_f^*(i).e_f(i) = \sum_{i=-h}^{+h} |e_f(i)|^2$$
$$= \sum_{i=-h}^{+h} |Y(i) - X(i)|^2 \qquad (3.8)$$

The theorem of Parseval gives the relation between (3.6a) and (3.8). In the discrete version it has the form:

$$\frac{1}{T}\int_{t_0}^{t_0+T} e_t^2 dt = \sum_{i=-h}^{+h} |e_f(i)|^2 \qquad (3.9)$$

Because of (3.9), it is clear that the minimum of $\int_{t_0}^{t_0+T} e_t^2 dt$ will be reached for the same parameterset $\hat{\beta}$ as the minimum in $\underline{e}_f^H \underline{e}_f(\underline{\beta})$.

3.2 The Gauss-Newton method

This method is based on the linear approximation of the model-output X[k].A short derivation follows:

The criterion function: $PI = \underline{e}_f^H \underline{e}_f = (\underline{Y}-\underline{X})^H(\underline{Y}-\underline{X})$ (3.10)
has to be minimized. The minimum is reached when $\frac{\partial PI}{\partial \underline{\beta}} = \underline{0}$.
Here:

$$\underline{\beta} = (\beta_1, \cdots, \beta_p)^T$$

is the model parameter vector.

$$\underline{Y} = (Y_{-h}, \ldots, Y_0, \ldots, Y_h)^T$$

is the vector of frequency components of the process output.

$$\underline{X} = (X_{-h}, \ldots, X_0, \ldots, X_h)^T$$

is the vector of frequency components of the model output.
So:

$$\frac{\partial PI}{\partial \underline{\beta}} = \frac{\partial(\underline{Y}-\underline{X})^H}{\partial \underline{\beta}}(\underline{Y}-\underline{X}) + \frac{\partial(\underline{Y}-\underline{X})^T}{\partial \underline{\beta}}(\underline{Y}-\underline{X})^* = \underline{0} \tag{3.11}$$

H means Hermitian, T =transposed, * = complex conjungate.

We may approximate (3.11) by using a linear approximation for $\underline{X}$:

$$\underline{X}(\underline{\beta}^{N+1}) = \underline{X}^{N+1} \simeq \underline{X}^N + \left.\frac{\partial \underline{X}}{\partial \underline{\beta}^T}\right|_{\underline{\beta}^N} (\underline{\beta}^{N+1} - \underline{\beta}^N) \tag{3.12}$$

$\underline{X}^N$ means model output obtained in the N-th iteration.

When inserting (3.12) into (3.11) we obtain:

$$-\left.\frac{\partial \underline{X}^H}{\partial \underline{\beta}}\right|_{\underline{\beta}^N} \left\{\underline{Y}-\underline{X}^N - \left.\frac{\partial \underline{X}}{\partial \underline{\beta}^T}\right|_{\underline{\beta}^N} (\underline{\beta}^{N+1} - \underline{\beta}^N)\right\} +$$

$$-\left.\frac{\partial \underline{X}^T}{\partial \underline{\beta}}\right|_{\underline{\beta}^N} \left\{\underline{Y}^* - \underline{X}^{N*} - \left.\frac{\partial \underline{X}^*}{\partial \underline{\beta}^T}\right|_{\underline{\beta}^N} (\underline{\beta}^{N+1} - \underline{\beta}^N)\right\} = \underline{0} \tag{3.13}$$

Rearranging (3.13) yields:

$$\left.\frac{\partial \underline{X}^H}{\partial \underline{\beta}}\right|_{\underline{\beta}^N} (\underline{Y} - \underline{X}^N) + \left.\frac{\partial \underline{X}^T}{\partial \underline{\beta}}\right|_{\underline{\beta}^N} (\underline{Y} - \underline{X}^N)^* =$$

$$= \left\{ \left.\left(\frac{\partial \underline{X}^H}{\partial \underline{\beta}} \cdot \frac{\partial \underline{X}}{\partial \underline{\beta}^T}\right)\right|_{\underline{\beta}^N} + \left.\left(\frac{\partial \underline{X}^T}{\partial \underline{\beta}} \cdot \frac{\underline{X}^*}{\partial \underline{\beta}^T}\right)\right|_{\underline{\beta}^N} \right\} \cdot (\underline{\beta}^{N+1} - \underline{\beta}^N) \qquad (3.14)$$

So the updated parameter set $\underline{\beta}^{N+1}$ can be obtained by adding a correction term to $\underline{\beta}^N$:

$$\underline{\beta}^{N+1} = \underline{\beta}^N + \left\{ \left.\frac{\partial \underline{X}^H}{\partial \underline{\beta}} \cdot \frac{\partial \underline{X}}{\partial \underline{\beta}^T}\right|_{\underline{\beta}^N} + \left.\frac{\partial \underline{X}^T}{\partial \underline{\beta}} \cdot \frac{\partial \underline{X}^*}{\partial \underline{\beta}^T}\right|_{\underline{\beta}^N} \right\}^{-1} \cdot \left\{ \left.\frac{\partial \underline{X}^H}{\partial \underline{\beta}}\right|_{\underline{\beta}^N} (\underline{Y} - \underline{X}^N) + \frac{\partial \underline{X}^T}{\partial \underline{\beta}} (\underline{Y} - \underline{X}^N)^* \right\} \qquad (3.15)$$

Because the sum of a matrix and its complex conjungate is two times the real part of this matrix, (3.15) may be rearranged to:

$$\underline{\beta}^{N+1} = \underline{\beta}^N + \left[2\mathrm{re} \left\{ \left.\frac{\partial \underline{X}^H}{\partial \underline{\beta}} \cdot \frac{\partial \underline{X}}{\partial \underline{\beta}^T}\right|_{\underline{\beta}^N} \right\} \right]^{-1} \cdot \left[2\mathrm{re} \left\{ \left.\frac{\partial \underline{X}^H}{\partial \underline{\beta}}\right|_{\underline{\beta}^N} (\underline{Y} - \underline{X}^N) \right\} \right] \qquad (3.16)$$

This is the correction obtained with the Gauss -Newton method, the physical interpretation of this method can be found in ref.5.

3.3 The method of Marquardt

Starting from a point $\underline{\beta}^N$, this method tries to minimize PI on a circle, (hyper)-sphere respectively.

The idea behind this method is that, as the convergence of the Gauss-Newton method is not assured (certainly not when the PI surface is highly non-quadratic), while the Steepest-Descent method does converge, one better can combine these methods, to have the speed of convergence of Gauss-Newton (quadratic convergent in the neighbourhood of the minimum) and the assured convergence of the Steepest-Descent.

As will be shown in the next chapter , the PI surfaces have local maxima at the left and right hand side of the absolute minimum. In the method of Gauss-Newton, the parameters may not vary beyond the region bounded by the parameter values corres-ponding with the inflection points of the PI-curve, while in the case of Steepest-Descent the parameter region is bounded by the local maxima themselves. (see fig 3.2). This means a considerable enlargement of the admissable parameter region. In the next chapter this will be related with physiological parameter bounds.

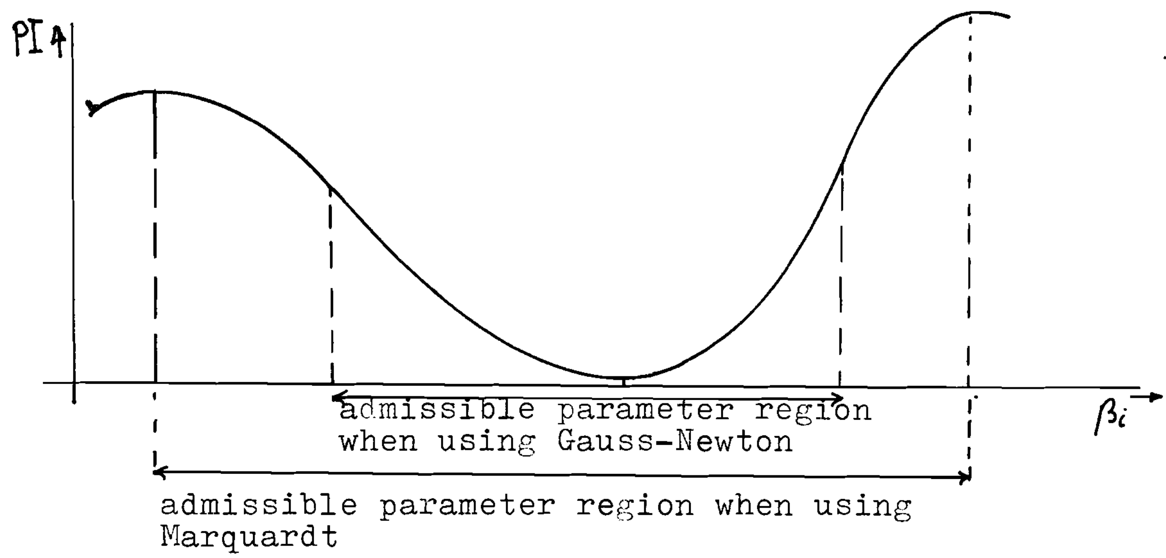


fig 3.2 Performance Index as function of parameter β_i.

Remark.

The convergence of the Gauss-Newton method towards the minimum is restricted because it actually solves (3.11), viz.: $\frac{\partial I I}{\partial \underline{\beta}}=\underline{0}$, with a tangent method (fig 3.3).

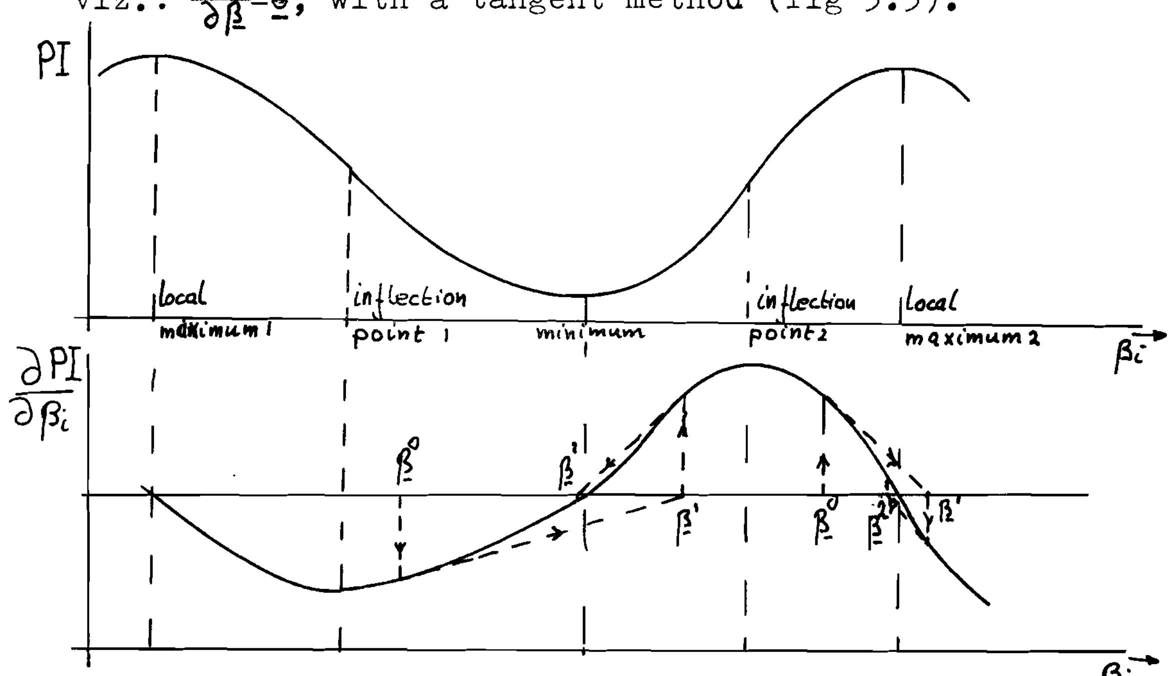


fig 3.3 PI and $\frac{\partial PI}{\partial \beta_i}$ as function of β_i.

Starting at the right of inflection point 2 or at the left of inflection point 1 means that the Gauss-Newton method finds

A short derivation of Marquardt's method in the frequency domain is given.
In order to find a correction step $\Delta\underline{\beta} = \underline{\beta}^{N+1} - \underline{\beta}^{N}$ which will minimize $PI(\underline{\beta}^{N+1})$ under the constraint $\Delta\underline{\beta}^T\Delta\underline{\beta} = r^2$, (r is the radius of the (hyper-) sphere) we have to optimize, in accordance to the method of Lagrange, an extended function $\Phi(\underline{\beta}, r)$:

$$\Phi(\underline{\beta}, r) = (\underline{Y}-\underline{X})^H(\underline{Y}-\underline{X}) + \tfrac{1}{2}\lambda(\Delta\underline{\beta}^T\Delta\underline{\beta} - r^2) =$$

$$= PI + \tfrac{1}{2}\lambda(\Delta\underline{\beta}^T\Delta\underline{\beta} - r^2) \tag{3.17}$$

For the optimization with respect to $\Delta\underline{\beta}$ holds:

$$\frac{\partial \Phi}{\partial \Delta\underline{\beta}} = \frac{\partial PI}{\partial \Delta\underline{\beta}} + \lambda\Delta\underline{\beta} = \underline{0} \tag{3.18}$$

With the linearization of $\underline{X}$:

$$\underline{X}^{N+1} = \underline{X}^{N} + \left.\frac{\partial \underline{X}}{\partial \underline{\beta}^T}\right|_{\underline{\beta}^N} \cdot \Delta\underline{\beta} \tag{3.19}$$

where $\Delta\underline{\beta} = \underline{\beta}^{N+1} - \underline{\beta}^{N}$ (3.20)

and because of:

$$\frac{\partial \underline{X}^{N+1}}{\partial \Delta\underline{\beta}^T} = \left.\frac{\partial \underline{X}}{\partial \underline{\beta}^T}\right|_N \quad \text{(follows from (3.19))} \tag{3.21}$$

it follows that:

$$-\left.\frac{\partial \underline{X}^H}{\partial \underline{\beta}}\right|_N \left\{\underline{Y}-\underline{X}^N - \left.\frac{\partial \underline{X}}{\partial \underline{\beta}^T}\right|_N \Delta\underline{\beta}\right\} - \left.\frac{\partial \underline{X}^T}{\partial \underline{\beta}}\right|_N \left\{\underline{Y}-\underline{X}^N - \left.\frac{\partial \underline{X}}{\partial \underline{\beta}^T}\right|_N \Delta\underline{\beta}\right\}^* + \lambda\Delta\underline{\beta} = \underline{0} \tag{3.22}$$

Note that the only difference between (3.13) and (3.22) is the term $\lambda\Delta\underline{\beta}$.
In analogy to the Gauss-Newton method one may derive an expression for $\Delta\underline{\beta}$:

$$\underline{\beta}^{N+1} = \underline{\beta}^{N} + \Bigg\{\underbrace{2\,\mathrm{re}\left[\frac{\partial \underline{X}^H}{\partial \underline{\beta}} \cdot \left.\frac{\partial \underline{X}}{\partial \underline{\beta}^T}\right|_N\right]}_{\text{matrix A}} + \lambda I\Bigg\}^{-1} \cdot \left\{2\,\mathrm{re}\left[\left.\frac{\partial \underline{X}^H}{\partial \underline{\beta}}\right|_N (\underline{Y}-\underline{X}^N)\right]\right\} \tag{3.23}$$

It is even possible to compute an optimal λ per iteration, but this consumes to much time, and moreover λ depends on the more or less arbitrary choice of r. So the exact computation of λ is not relevant.
The algorithm of Marquardt gives a powerful solution to this problem (ref.6).

Let $s > 1$; let $\lambda^{(N)}$ denote the value of λ of the previous iteration; initially let $\lambda^{(0)} = 0.01$ (This depends on the non-linearity of $\underline{X}$, with regard to the parameters, and on the scaling of matrix A).
Let $PI^{(N)}$ be the value of $(\underline{Y}-\underline{X})^H(\underline{Y}-\underline{X})$, if $\underline{X}$ is the output of the model with parameter set $\underline{\beta}^{(N)}$; and let $PI(\lambda)$ be the Performance Index, which follows if the model parameter set equals $\underline{\beta}^{(N)}$ + a correction step, computed from (3.23).
Then:

i   if $PI(\lambda^{(N)}/s) < PI^{(N)}$, then $\lambda^{(N+1)} = \lambda^{(N)}/s$ (3.24)

ii   if $PI(\lambda^{(N)}/s) > PI^{(N)}$ and $PI(\lambda^{(N)}) < PI^{(N)}$

then $\lambda^{(N+1)} = \lambda^{(N)}$ (3.25)

iii   if $PI(\lambda^{(N)}) > PI^{(N)}$ increase λ by successive multiplication with s until for some smallest (integer) w holds:
$PI(\lambda^{(N)}.s^w) < PI^{(N)}$

then let: $\lambda^{(N+1)} = \lambda^{(N)}.s^w$ (3.26)

Remarks:

1 One may find in ref.6 and 7 more extended information on Marquardt's method. Ref.7 gives some graphical interpretations of the method.

2 Marquardt scales in each iteration matrix A by replacing for each term a_{ij} of the A matrix:

$$a_{ij_{scaled}} = \frac{a_{ij}}{\sqrt{a_{ii} \cdot a_{jj}}} \qquad (3.27)$$

This may have some advantages with regard to numerical aspects. Let us see:

$$A = 2\mathrm{re}\left[\frac{\partial \underline{x}^H}{\partial \beta}\cdot\frac{\partial \underline{x}}{\partial \beta^T}\right] \qquad \text{(follows from(3.23))} \tag{3.28}$$

Then $$a_{ii} = 2\sum_{k=-h}^{h}\left|\frac{\partial X_k}{\partial \beta_i}\right|^2 \tag{3.29}$$

For the non-diagonal terms of A we may write:

$$a_{ij} = 2\mathrm{re}\left\{\sum_{k=-h}^{h}\left(\frac{\partial X_k}{\partial \beta_i}\right)^*\cdot\left(\frac{\partial X_k}{\partial \beta_j}\right)\right\} \leqslant 2\left|\sum_{k=-h}^{+h}\left(\frac{\partial X_k}{\partial \beta_i}\right)^*\cdot\left(\frac{\partial X_k}{\partial \beta_j}\right)\right| \tag{3.30}$$

And with the inequality of Schwartz follows:

$$2\left|\sum_{k=-h}^{+h}\left(\frac{\partial X_k}{\partial \beta_i}\right)^*\cdot\left(\frac{\partial X_k}{\partial \beta_j}\right)\right| \leqslant 2\sqrt{\sum_{k=-h}^{+h}\left|\frac{\partial X_k^*}{\partial \beta_i}\right|^2}\cdot\sqrt{\sum_{k=-h}^{+h}\left|\frac{\partial X_k}{\partial \beta_j}\right|^2} \tag{3.31}$$

Hence, after scaling of matrix A (see (3.27)), the terms of A_{scaled} will be, in absolute value, smaller than or equal to1

$$|a_{ij}| \leqslant 1 \tag{3.22}$$

Because of the way of scaling the diagonal elements a_{ii} are always 1. The whole scaling manipulation gives a numerically good conditioned matrix. Thereby it has the particular advantage that in:

$$A_{scaled} + \lambda I$$

λI is compared with a matrix which is in each iteration about the same and has diagonal elements equal 1.
So one is able to see if the PI-surface is in a certain region of the parameter space quadratic or not; namely if it is quadratic λ will decrease and otherwise it will increase.

4. PARAMETER SENSITIVITY AND PHYSIOLOGICAL RELEVANT CONSTRAINTS

This chapter deals with the parameter choice, the sensitivity of the 8-sections model (section 2.2) for these parameters and the parameter scaling resulting from the sensitivity investigation.
Moreover it derives physiological relevant boundary values with respect to the parameters.

4.1 The choice, sensitivity and scaling of parameters

In model equation (2.11) γ and Z_0 are the most obvious parameters. But γ and Z_0 are frequency dependent and therefore they are not suited to be the parameters which will be adjusted to bring $Pout_m[k]$ for all k as good as possible in accordance with $Pout[k]$ [1] (in Least-Squares sense).
In other words, parameters are necessary, which are constant with respect to k.
Repeating (2.9), (2.10), (2.15) and (2.16), it is clear that:

$$Z_o = \sqrt{Z_1 Z_2} = \sqrt{(Rs1 + j\omega L1)(Rp1 + 1/j\omega C1)}$$

$$\gamma = \sqrt{\frac{Z_1}{Z_2}} = \sqrt{\frac{Rs1 + j\omega L1}{Rp1 - j/\omega C1}}$$

Rs1, L1, Rp1 and C1 satisfy this requirement.
So the last are suited as parameters, when the error criterion PI is not too wild a function of this parameter set, for then the adjustment with hill climbing techniques will be very difficult (see section 3.3). Parameter sensitivity studies and parameter estimation results have proved that it is not necessary to choose other parameters.
Hence, from now on when we speak about "the parameters", we mean:

Rs1, L1, Rp1 and C1.

------

[1] Note that $Pout_m[k]$ is the kth harmonic of the model output pressure just as $Pout[k]$ the kth harmonic of the process output pressure.

It is very important to investigate the sensitivity of the model for the different parameters. Great difference for example may cause a very slow convergence of the estimation procedure. This is very clear in the case of Steepest-Descent. See fig 4.1a and 4.1b.
Moreover, when the contour lines are very narrow and long ellipsoids, it is possible that the minimum can not be found by an estimation procedure. The truncation errors made by the digital computer namely, may cause such stepsizes $|\Delta\beta|$ that each time the method will jump over the minimum.
Hence it is sensible to scale the parameters, on the base of the sensitivity investigation, in such a way, that the sensitivity for these scaled parameters is about the same.

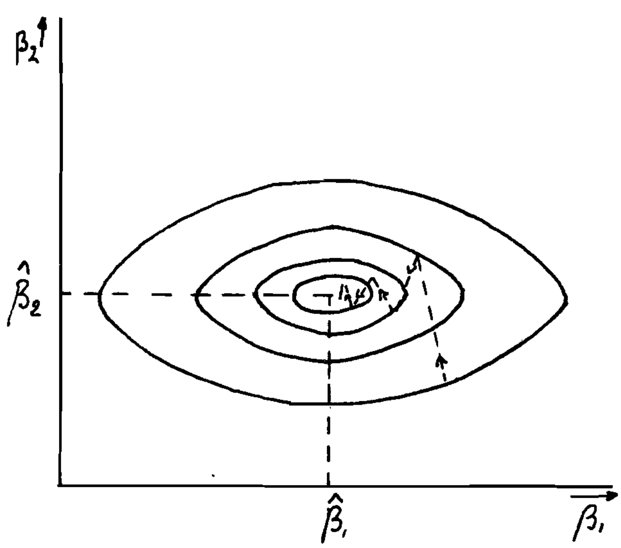


fig 4.1a Map of PI-contour lines before scaling

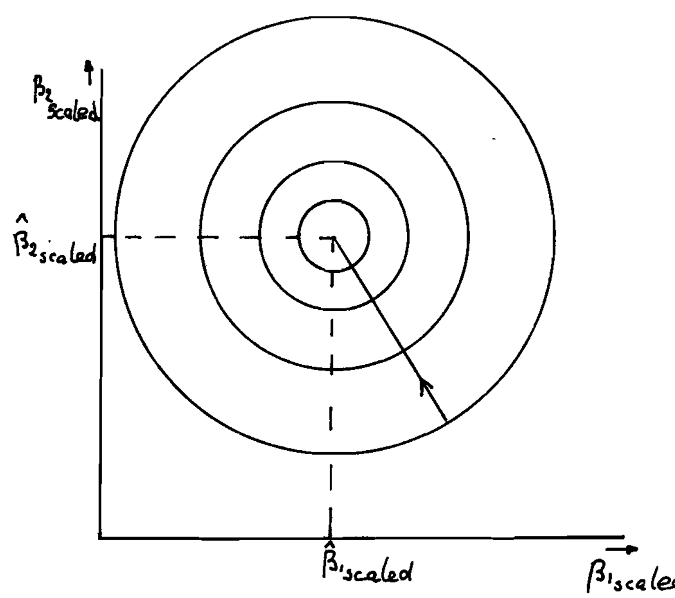


fig 4.1b Map of PI-contour lines after scaling

When the parameters are not scaled the Steepest-Descent adjustment path will be very long and the adjustment very slow (fig 4.1a) But when the ellips axes are parallel with the parameter axes, it is possible to transform by proper scaling, the contour lines to circles. In that case (fig 4.1b) the Steepest-Descent method reaches the minimum $(\hat{\beta}_1, \hat{\beta}_2)$ in one step.

The sesitivity investigation gives,besides information about the way of scaling, information about the shape of the PI-surface. This will become clear in the following discussion.

In the sesitivity study subsequently all possible combinations of two parameters , so Rsl-L1, Rsl-Rpl, Rsl-C1, L1-Rpl, L1-C1, Rpl-C1, are chosen to be the variable parameters. The other two then are fixed. The variable ones, β_i and β_j , then are varied with step kx $\Delta\beta_i$, lx $\Delta\beta_j$ respectively, for k =0, ± 1, ± 2,....$\pm$ n and l = 0, ± 1, ± 2, ..., $\pm$m. The corresponding PI-values are computed. This yields a matrix(fig 4.2) or when plotted 3-dimensionally, the PI_surface. When properly placed in the matrix, it is possible to draw contourlines through the PI-values of equal size. These lines are more or less ellipsoids, the lengths of their short and long axes depend on the size of $\Delta\beta_i$ and $\Delta\beta_j$.

It is the intention of this sensitivity study to find the scaling factors for the paramters, so that the contourlines of PI as function of β_i^* and β_j^* [1]) are almost circles, and the PI(β_i^*, β_j^*)-surface almost a symmetrical beehive. In that case namely, the model is equally sensitive for β_i^* and β_j^*.

We have searched for these scaling factors varying $\Delta\beta_i$ and $\Delta\beta_j$; for all possible combinations of β_i and β_j , until we had a step($\Delta\beta_i$, $\Delta\beta_j$) where a change $\Delta\beta_i$ in parameter β_i gave about an equal change ΔPI in PI as a step $\Delta\beta_j$ in β_j does. Those sets ($\Delta\beta_i$, $\Delta\beta_j$) depend on the value of β_i and β_j and of the others. However in the neighbourhood of the PI-minimum, a step ΔL1 in L1 gives about an equal change in PI as a step ΔRsl = 6ΔL1 in Rsl, a step ΔRpl = 25000ΔL1 in Rpl and a step ΔC1 = 200ΔL1 in C1. Or expressed in percents with respect to nominal parameter values [2]) : ΔL1 = 1% of $L1_{nominal}$ agrees with ΔRsl = 9,8% of $Rsl_{nominal}$, with ΔRpl = 50% of $Rpl_{nominal}$ and ΔC1 = 3,7% of $C1_{nominal}$.

------

1) β_i^* means scaled parameter β_i.

2) The investigation has been performed with respect to the 8-sections model. One may find the nominal values in the left column of table 2.1.

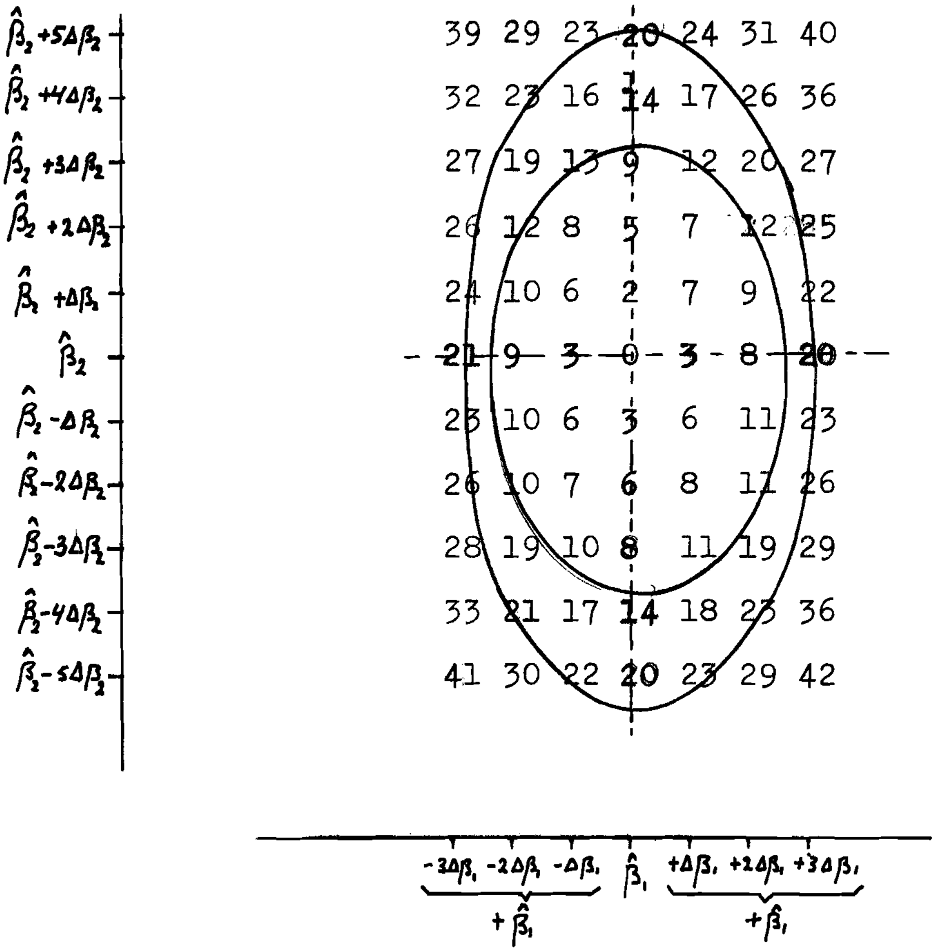


FIG 4.2 Example of a printed matrix of PI-values as function of β_1 and β_2. In this case the contour lines may be transformed into circles by increasing $\Delta\beta_2$ or decreasing $\Delta\beta_1$.

So with this definition, the model is most sensitive for L1, subsequently followed by Rsl, Cl, and Rpl. The model is very insensitive for Rpl.

With this knowledge parameter scaling is possible:
We have chosen for scaling without origin displacement. This has the advantage that the relative errors $\frac{\beta_{i_{optimal}} - \beta_{i_{nominal}}}{\beta_{i_{nominal}}}$ and $\frac{\beta^*_{i_{optimal}} - \beta^*_{i_{nominal}}}{\beta^*_{i_{nominal}}}$ 1) are the same.

-----

1) β_i^* is the scaled parameter β_i.

With proportional scaling: $Rsl = QU1 \times Rsl^*$ (4.1)

$$L1 = QU2 \times L1^* \qquad (4.2)$$

$$Rpl = QU3 \times Rpl^* \qquad (4.3)$$

$$Cl = QU4 \times Cl^* \qquad (4.4)$$

setting $L1^*_{nominal} = 1000$, (4.5)

and with the results above mentioned, holds for QU1, QU2, QU3, QU4 respectively:

$$^{1)}QU2 = \frac{L1_{nominal}}{L1^*_{nominal}} = \frac{.274 \times 10^{-3}}{1000} = 2.74 \times 10^{-7} \qquad (4.6)$$

$$QU1 = 6 \times QU2 \qquad (4.7)$$

$$QU3 = 25000 \times QU2 \qquad (4.8)$$

$$QU4 = 200 \times QU2 \qquad (4.9)$$

The nominal values of these scaled parameters are (see table 2.1):

$Rsl^*_{nominal} = 101.58$ $\qquad Rpl^*_{nominal} = 20$

$L1^*_{nominal} = 1000$ $\qquad Cl^*_{nominal} = 267.88$

To see the effect of the scaling, and the shape of the surface, PI (β_i^*, β_j^*) has been plotted and printed for all combinations of β_i^* and β_j^*.

fig	par1 horizontal	par2 backwards	stepsize in par1,2	min. value par1	max. value par1	min. value par2	max. value par2
4.3	$L1^*$	Rsl^*	10	940	1080	41.58	181.58
4.4	Rpl^*	Rsl^*	10	10	150	10	150
4.5	Cl^*	Rsl^*	10	177.9	317.9	10	150
4.6	Rpl^*	$L1^*$	10	10	150	940	1080
4.7	Rpl^*	$L1^*$	1	13	27	1011	1025
4.8	Cl^*	$L1^*$	10	210	350	940	1080
4.9	Cl^*	Rpl^*	10	210	350	10	150

Table 4.1 List of figures of PI-surfaces after scaling of the parameters

------

matrix of scaled PI-values

GESCHAALDE H-MATRIX:

$L1^* \rightarrow$

Rsl		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
		940	950	960	970	980	990	10^3	1010	1020	1030	1040	1050	1060	1070	1080
41.6	1	64	51	41	32	24	19	15	13	13	14	17	22	28	37	47
51.6	2	59	46	36	27	20	14	10	8	8	9	12	17	23	31	41
61.6	3	55	42	32	23	16	10	7	5	4	5	8	13	19	27	37
71.6	4	52	40	29	20	13	8	4	2	2	3	6	10	17	24	34
81.6	5	50	38	28	19	12	7	3	1	0	1	4	9	15	23	32
91.6	6	49	37	27	19	12	6	3	0	0	1	[illegible]	8	15	22	32
101.6	7	50	38	28	19	12	7	3	1	1	2	5	9	15	23	32
111.6	8	51	39	29	21	14	9	5	3	3	4	7	11	17	25	34
121.6	9	53	41	32	23	17	11	8	6	6	7	9	14	20	27	37
131.6	10	56	45	35	27	20	15	12	10	9	11	13	18	24	31	40
141.6	11	60	49	39	31	25	20	16	14	14	15	18	22	28	36	45
151.6	12	65	54	45	37	30	25	22	20	20	21	24	28	34	42	51
161.6	13	71	60	51	43	36	32	28	27	26	28	31	35	41	48	57
171.6	14	78	67	58	50	44	39	36	34	34	35	38	42	48	56	65
181.6	15	85	74	65	58	52	47	44	42	42	43	46	51	57	64	73

PI-surface

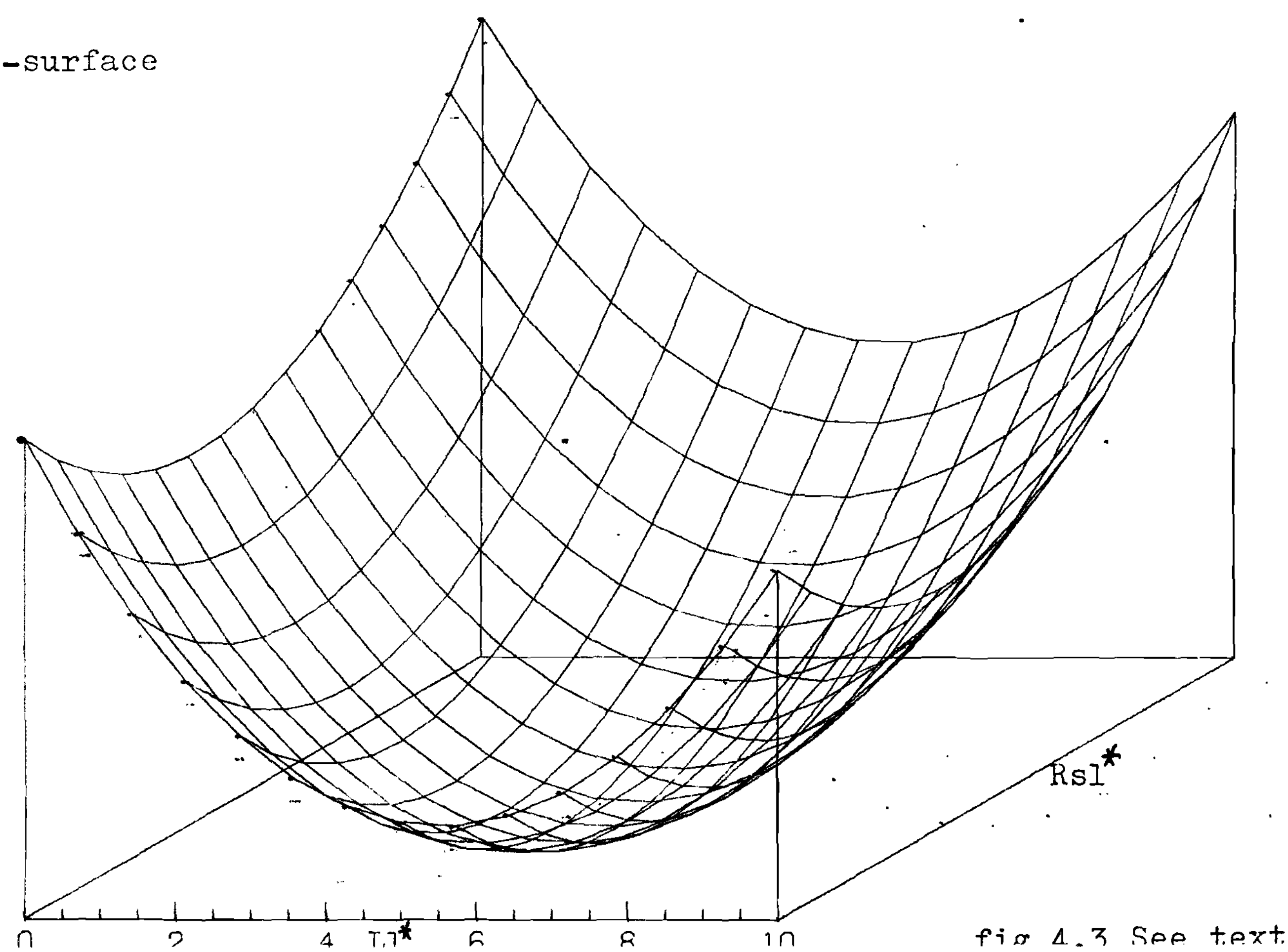


fig 4.3 See text.

Rpl →

Rsl* ↓	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150
10	61	55	52	50	49	49	50	50	51	52	52	53	54	55	56
20	47	41	38	37	37	37	38	39	40	41	42	43	44	46	47
30	35	30	27	26	26	27	28	29	31	32	34	35	37	38	40
40	25	21	18	18	18	19	20	22	24	26	27	29	31	33	35
50	17	13	11	11	12	13	15	17	19	21	23	25	27	29	32
60	11	7	6	6	7	9	11	13	15	18	20	23	25	28	30
70	7	3	2	3	4	6	8	11	14	16	19	22	25	27	30
80	4	1	0	1	3	5	8	10	13	16	20	23	26	29	32
90	3	0	0	1	3	6	9	12	15	18	22	25	28	32	35
100	4	1	1	3	5	8	11	14	18	22	25	29	33	36	40
110	6	4	4	6	8	12	15	19	23	26	30	34	38	42	46
120	9	8	8	10	13	17	20	24	29	33	37	41	45	50	54
130	14	13	14	16	19	23	27	32	36	40	45	49	54	59	63
140	21	20	21	24	27	31	36	40	45	49	54	59	64	69	73
150	29	28	29	32	36	40	45	50	55	60	65	70	75	80	85

PI-surface

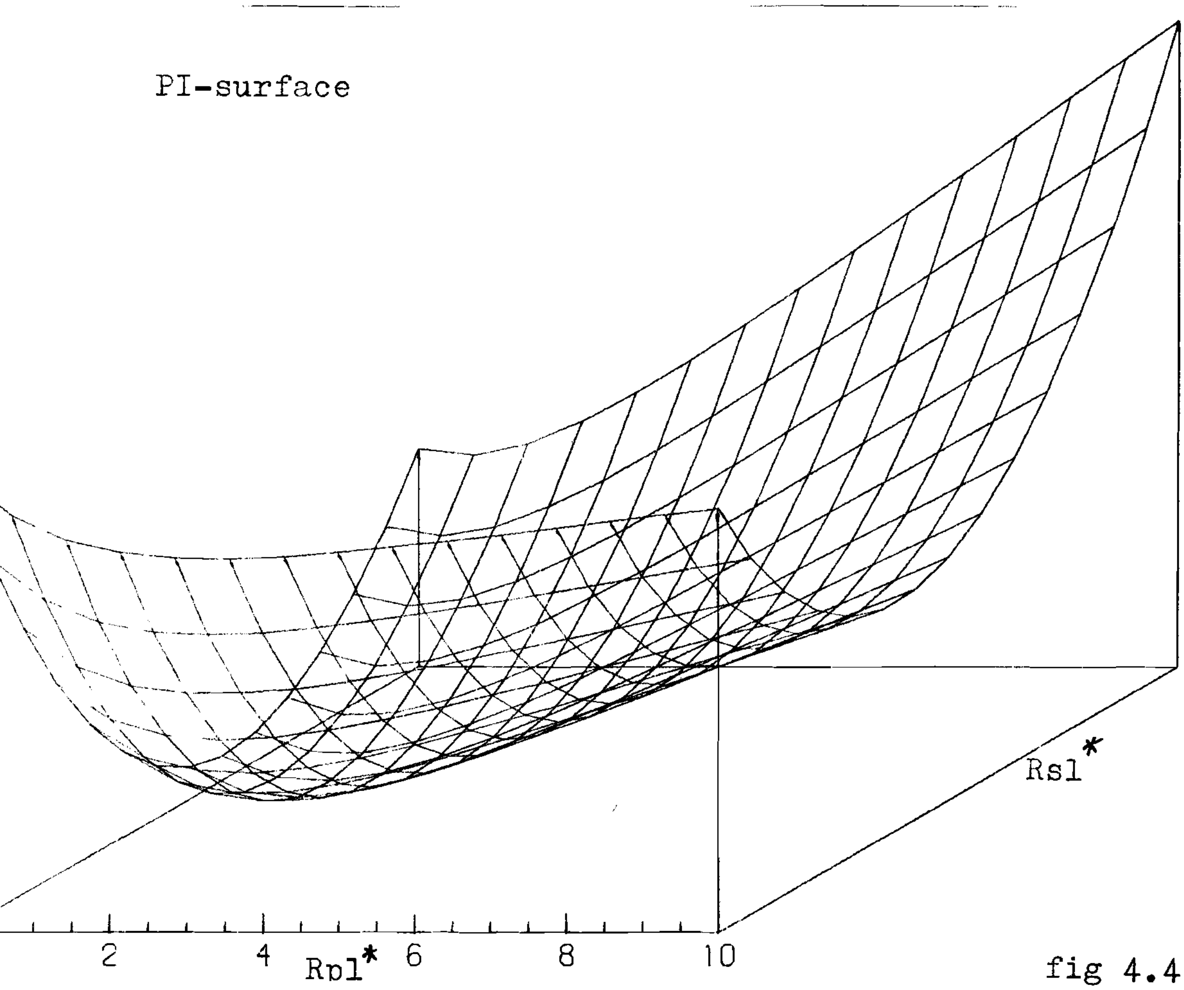


fig 4.4 See text.

GESCHAALDE H-MATRIX:

Rsl* ↓	1 (178)	2 (188)	3 (198)	4 (208)	5 (218)	6 (228)	7 (238)	8 (248)	9 (258)	10 (268)	11 (278)	12 (288)	13 (298)	14 (308)	15 (318)
10	63	54	46	38	32	28	24	22	21	21	23	26	31	37	44
20	61	52	43	36	29	24	20	18	16	16	18	20	24	30	36
30	60	50	42	34	27	21	17	14	12	12	13	15	19	24	30
40	60	50	41	32	25	19	15	11	9	9	9	11	14	19	24
50	60	49	40	32	24	18	13	9	7	6	6	7	10	14	19
60	60	50	40	31	24	17	12	8	5	4	4	5	7	11	15
70	61	51	41	32	24	17	11	7	4	2	2	3	5	8	12
80	63	52	42	32	24	17	11	7	3	1	1	1	3	6	10
90	65	54	43	34	25	18	12	7	4	1	0	0	2	4	8
100	67	56	45	36	27	19	13	8	4	2	0	0	1	3	7
110	70	58	48	38	29	21	15	9	5	2	1	0	1	3	6
120	73	61	50	40	31	24	17	11	7	4	2	1	2	4	6
130	77	65	54	44	34	26	19	14	9	6	4	3	3	5	7
140	81	69	57	47	38	29	22	17	12	8	6	5	5	6	9
150	85	73	61	51	42	33	26	20	15	11	9	8	8	9	11

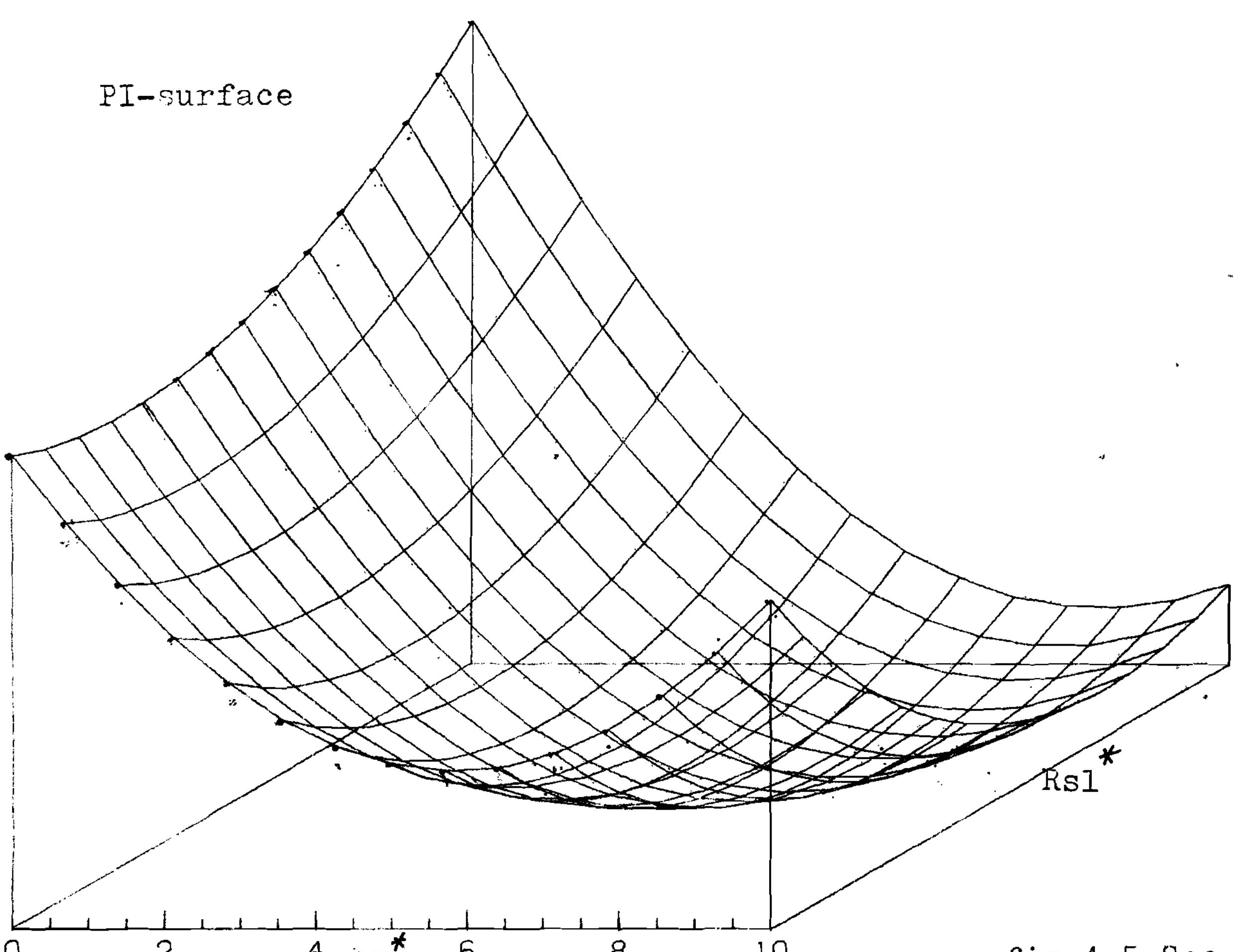


fig 4.5 See text.

matrix of scaled PI-values Rpl →

Ll* ↓		10	20	30	40	50	60	70	80	90	100	110	120	130	140	150
940	1	59	55	52	51	51	51	52	53	54	56	57	59	60	62	64
950	2	45	41	40	39	39	40	41	42	44	45	47	49	51	52	54
960	3	34	30	29	28	29	30	31	33	35	36	38	40	42	45	47
970	4	24	21	19	20	20	22	23	25	27	29	32	34	36	38	41
980	5	15	13	12	13	14	15	17	19	22	24	26	29	31	34	36
990	6	9	7	7	7	9	11	13	15	18	20	23	26	28	31	34
1000	7	5	3	3	4	6	8	10	13	16	18	21	24	27	30	32
1010	8	2	0	1	2	4	7	9	12	15	18	21	24	27	30	33
1020	9	1	0	1	2	5	7	10	13	17	20	23	26	29	32	35
1030	10	2	1	2	4	7	10	13	16	20	23	26	30	33	36	39
1040	11	5	4	6	8	11	14	17	21	24	28	31	35	38	42	45
1050	12	10	9	11	13	16	20	23	27	31	34	38	42	45	49	52
1060	13	16	16	18	21	24	28	31	35	39	43	47	50	54	58	62
1070	14	25	25	27	30	33	37	41	45	49	53	57	61	65	69	72
1080	15	35	35	38	41	44	48	52	57	61	65	69	73	77	81	85

PI-surface

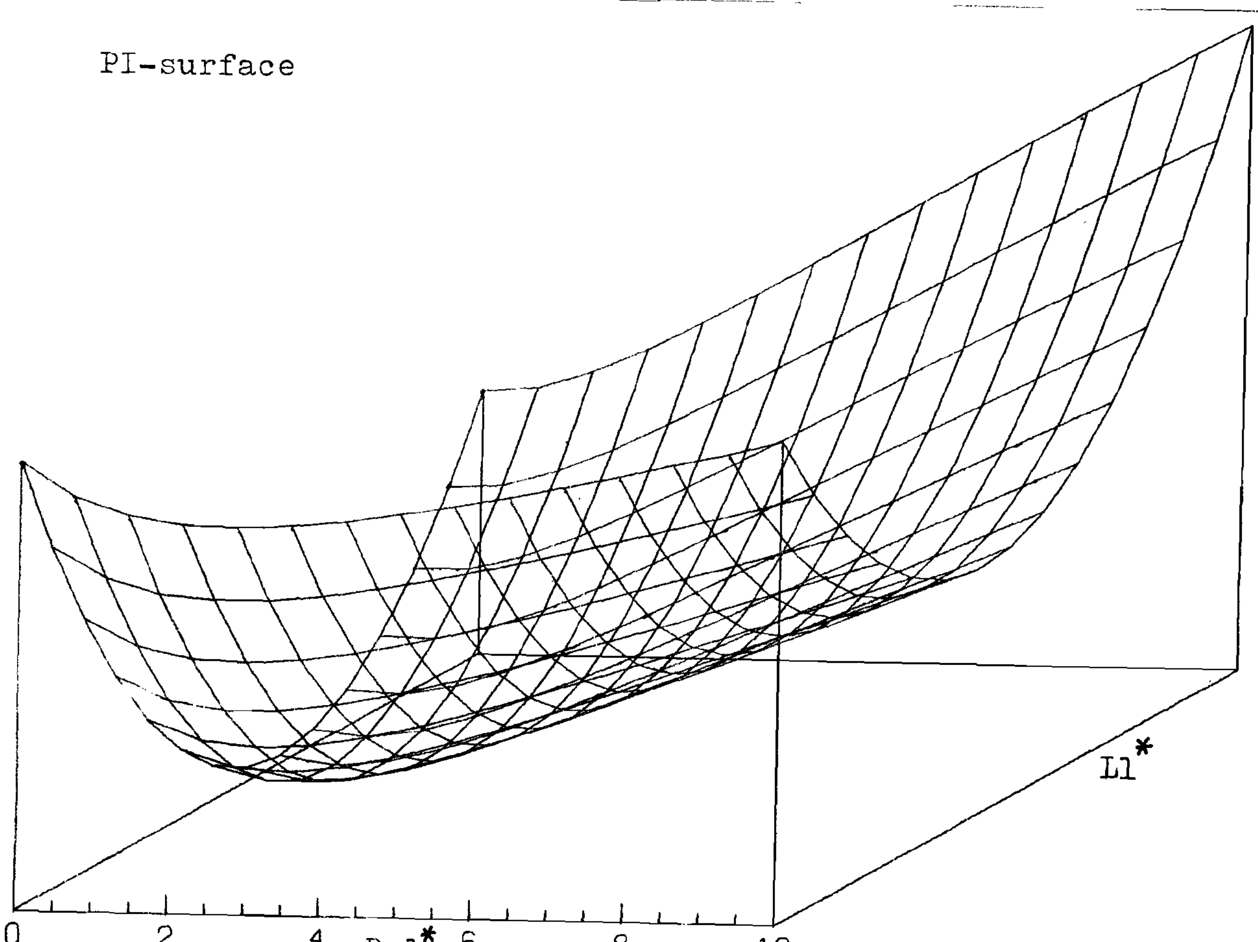


fig 4.6 See text.

Matrix of scaled PI-values Rpl →

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
		13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
1011	1	85	72	61	52	44	38	33	30	28	27	28	29	32	35	40
1012	2	75	62	52	42	35	29	25	22	20	20	20	22	25	29	33
1013	3	66	54	43	35	27	22	18	15	13	13	14	16	19	23	28
1014	4	59	47	36	28	21	16	[illegible]	9	8	8	9	11	15	19	24
1015	5	53	41	31	23	16	11	7	5	4	4	6	8	12	16	22
1016	6	48	36	27	19	12	7	4	2	1	2	3	6	10	15	20
1017	7	45	33	24	16	10	5	2	0	0	1	3	6	10	14	20
1018	8	43	32	22	15	9	5	2	0	0	1	3	6	10	16	22
1019	9	42	31	22	15	9	5	3	1	1	3	5	8	13	18	25
1020	10	43	32	23	16	11	7	5	4	4	5	8	12	16	22	29
1021	11	45	34	26	19	14	11	8	8	8	10	13	16	21	27	34
1022	12	48	38	30	23	18	15	13	13	13	15	18	22	28	34	41
1023	13	53	43	35	29	24	21	19	19	20	22	26	30	35	42	49
1024	14	59	49	42	36	31	28	27	27	28	30	34	39	44	51	58
1025	15	66	57	50	44	40	37	36	36	37	40	44	49	54	61	69

L1* ↓

PI-surface

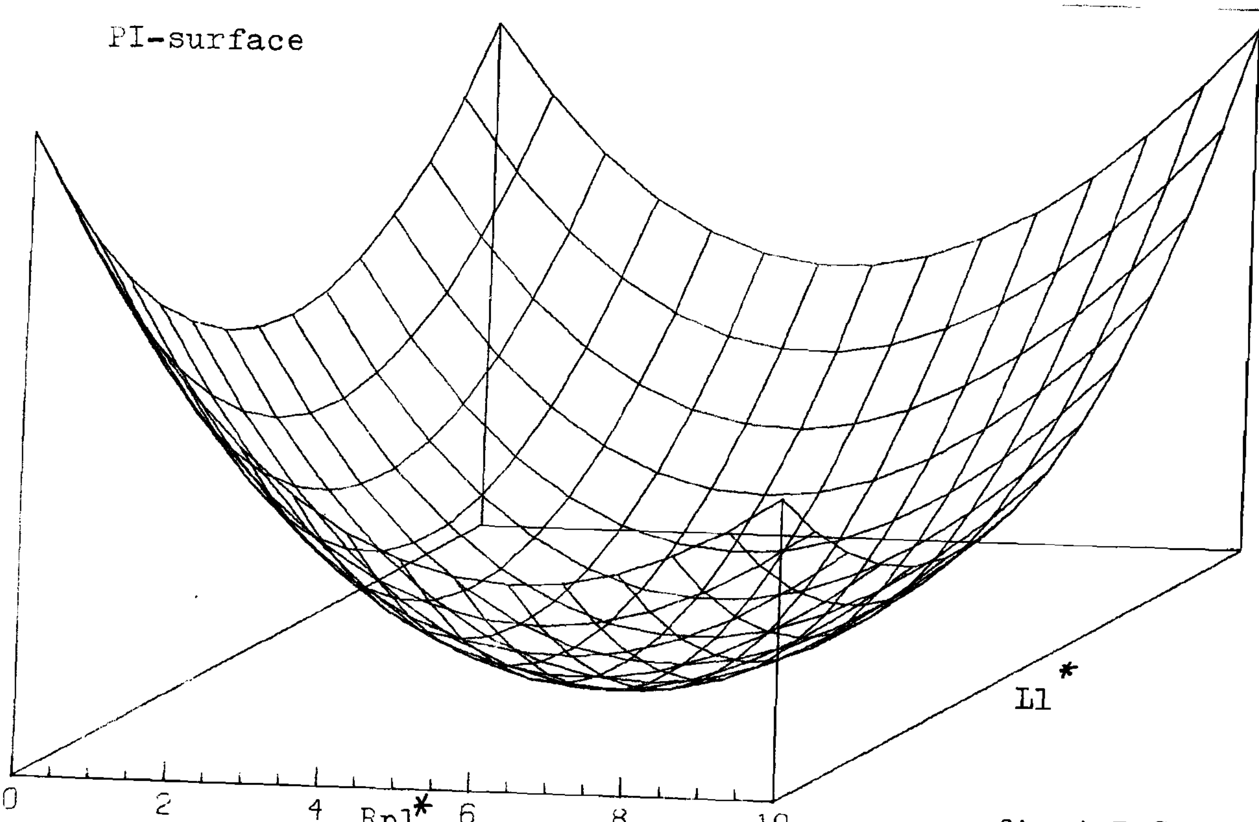


fig 4.7 See text.

matrix of scaled PI-values CI* →

L1* ↓		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
		210	220	230	240	250	260	270	280	290	300	310	320	330	340	350
940	1	66	57	48	40	33	27	21	17	13	10	9	7	7	8	9
950	2	58	49	41	33	27	21	16	12	9	7	6	5	6	7	9
960	3	51	42	34	27	21	16	12	8	6	4	4	4	5	7	9
970	4	44	36	29	22	16	12	8	5	3	2	2	3	5	7	11
980	5	38	30	23	17	12	8	5	3	1	1	2	3	6	9	13
990	6	33	25	19	13	9	5	3	1	0	1	2	4	8	12	16
000	7	28	21	15	10	6	3	1	0	0	1	3	6	10	15	21
010	8	24	17	12	7	4	2	0	0	1	2	5	9	13	19	25
020	9	20	14	9	6	3	1	0	1	2	4	8	12	18	24	31
030	10	18	12	8	4	2	1	1	2	4	7	11	17	23	30	38
040	11	15	10	6	4	2	2	2	4	7	11	16	22	29	37	46
050	12	14	9	6	4	3	3	4	7	11	15	21	28	36	44	54
060	13	13	9	6	5	4	5	7	11	15	20	27	35	43	53	63
070	14	12	9	7	6	7	8	11	15	20	26	34	42	52	62	74
080	15	13	10	9	8	10	12	15	20	26	33	41	51	61	73	85

PI-surface

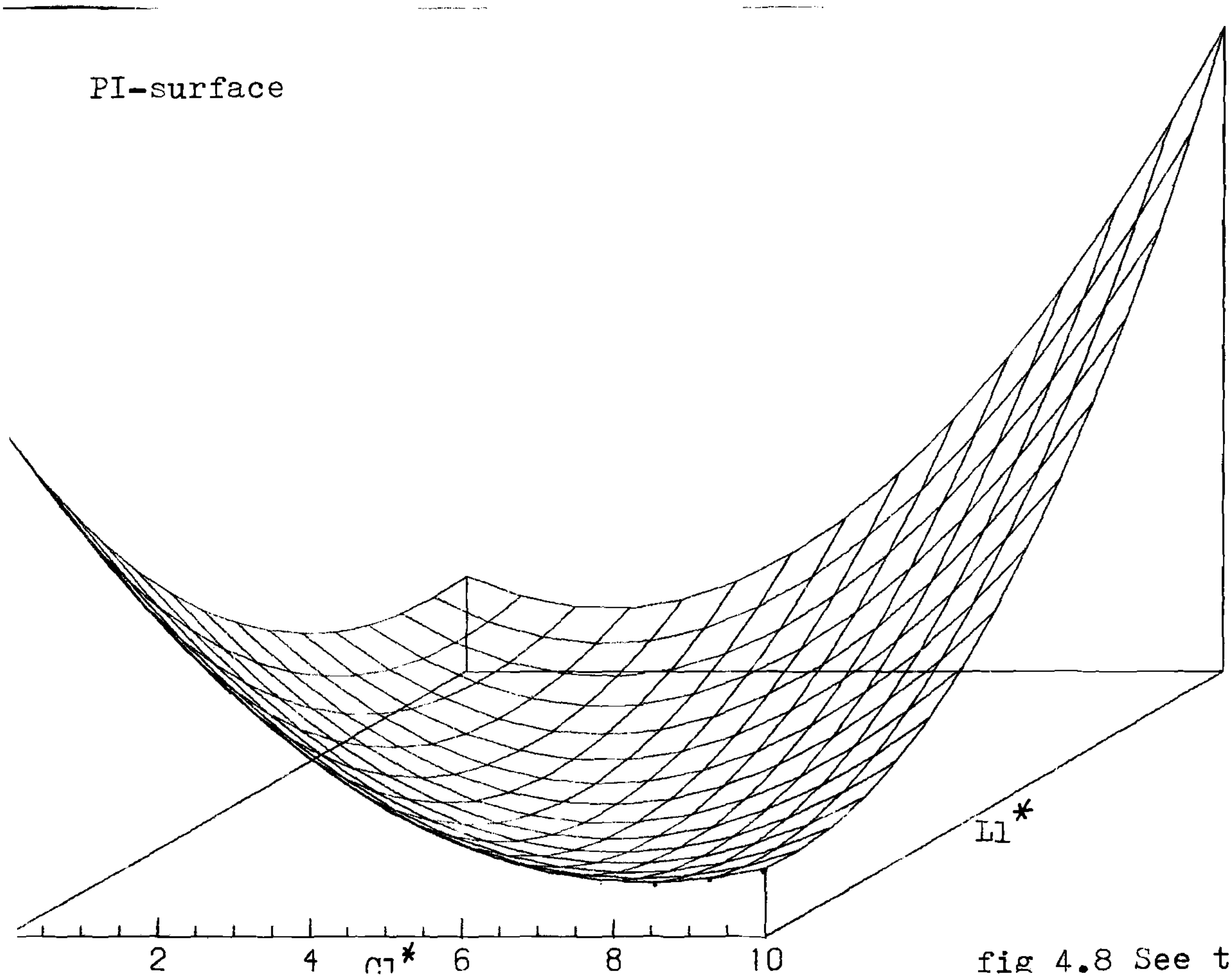


fig 4.8 See text.

matrix of scaled PI-values Cl* ⟶

Rpl* ↓

	1 210	2 220	3 230	4 240	5 250	6 260	7 270	8 280	9 290	10 300	11 310	12 320	13 330	14 340	15 350
1 0	85	65	47	32	20	11	5	2	2	5	11	20	32	46	64
2 0	80	60	43	28	17	8	3	0	0	3	9	17	28	42	58
3 0	76	57	41	27	16	8	3	0	1	3	9	17	28	41	56
4 0	74	56	40	27	17	9	4	2	2	5	10	18	28	41	56
5 0	73	55	40	28	18	11	6	4	5	7	13	20	30	42	57
6 0	73	56	41	29	20	13	9	7	7	10	15	23	32	44	58
*7 0	73	57	43	31	22	16	12	10	10	13	18	26	35	46	60
8 0	74	58	45	34	25	19	15	13	14	17	22	29	38	49	63
9 0	75	60	47	36	28	22	18	16	17	20	25	32	41	52	65
10 0	76	61	49	38	30	25	21	20	20	23	28	35	44	55	68
11 0	78	63	51	41	33	28	24	23	24	27	32	39	47	58	71
12 0	79	65	53	44	36	31	28	27	27	30	35	42	51	62	74
13 0	81	67	56	46	39	34	31	30	31	34	39	46	54	65	77
14 0	83	70	58	49	42	37	34	33	34	37	42	49	58	68	81
15 0	85	72	61	52	45	40	38	37	38	41	46	53	61	72	84

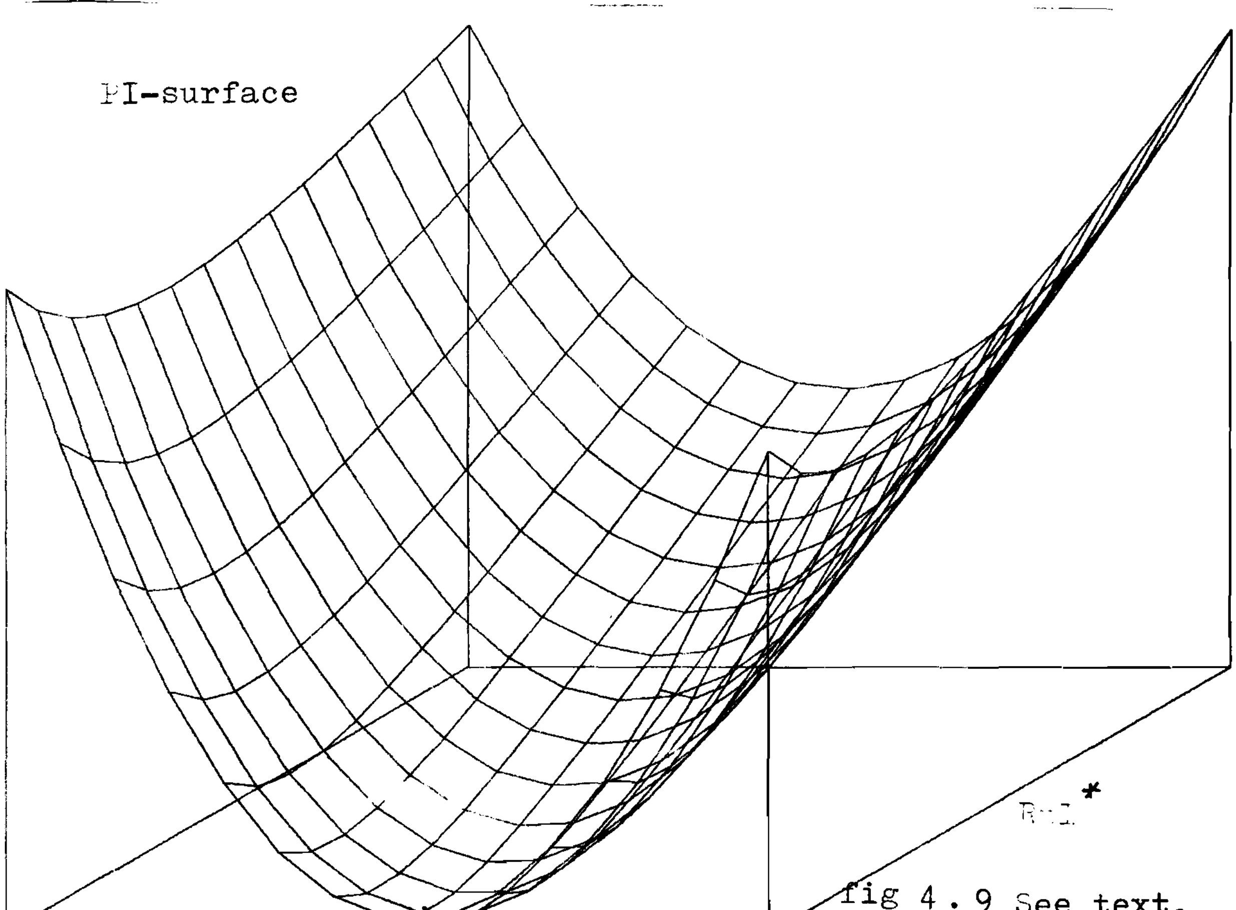


fig 4.9 See text.

From these figures, it is clear, that the sensitivity of the model for the four scaled parameters is about the same, at least in the neighbourhood of the optimal parameter set (corresponding with minimal PI value).
There are several figures e.g.: fig 4.5 and fig 4.8, where the PI contourlines still are ellipsoids, but it is clear that it is not possible by only properly scaling to convert these contour lines in circles, because the axes of the ellipses are not parallel to the parameter axes.
From e.g.: fig 4.6 in comparison with fig 4.7 it is clear that this sensitivity is a function of the parameters, because further away from the minimum the sensitivity for Rpl^* is much lower than for Ll^* in this case.

In spite of this, we want to scale just once, using the transformation given by (4.1) - (4.9). We will use this scaling too, estimating on the rubber tube. From now on there is a new set of parameters:

Rsl^*, Ll^*, Rpl^* and Cl^*.

4.2 Physiological relevant bounds to the parameter space

Of course, the most optimal parameter set varies from process to process, from patient to patient. Fig 4.3 till 4.9 show the PI surface (2-variable parameters) for a limited region around the optimal parameter set of the 8-sections model, and in those drawings the surface appears to be good for estimation with the Gauss-Newton method (almost quadratic surface).
PI curves (of one variable parameter) of a parameter region with a greater extent, show however, that there are local maxima in these curves. See for instance fig 4.10. We have plot many of these curves and it appears that only those curves where Ll, respectively Cl is the variable parameter, have local maxima.

From these curves one may gain some insight in the position of these local maxima and intermediate inflection points(see fig3.3 but this position depends greatly on the values of the other 3 parameters and on the position of the optimal parameter set, so it depends on the kind of process.

MODEL TO MODEL *
PATIENTWAARDEN:RS1 * 1.0158E+02 L1* 1.0000E+03 RP1* 2.0000E+01 C1* 2.6788E+02

STARTWAARDEN: RS1 * 1.0158E+02 L1* 1.0000E+04 RP1* 2.0000E+01 C1* 2.6788E+02 DELTA 50 AIT 200

PI:	L1:
3.2105E+03	1.0000E+00
3.3187E+03	5.1000E+01
3.3104E+03	1.0100E+02
3.1570E+03	1.5100E+02
2.9509E+03	2.0100E+02
2.7140E+03	2.5100E+02
2.4514E+03	3.0100E+02
2.1724E+03	3.5100E+02
1.8885E+03	4.0100E+02
1.6090E+03	4.5100E+02
1.3404E+03	5.0100E+02
1.0874E+03	5.5100E+02
8.5476E+02	6.0100E+02
6.4700E+02	6.5100E+02
4.6772E+02	7.0100E+02
3.1871E+02	7.5100E+02
1.9999E+02	8.0100E+02
1.1036E+02	8.5100E+02
4.3116E+01	9.0100E+02
1.1690E+01	9.5100E+02
4.8216E-03	1.0010E+03
1.2626E+01	1.0510E+03
4.9791E+01	1.1010E+03
1.1238E+02	1.1510E+03
2.0181E+02	1.2010E+03
3.1997E+02	1.2510E+03
4.6903E+02	1.3010E+03
6.5134E+02	1.3510E+03
8.6933E+02	1.4010E+03
1.1253E+03	1.4510E+03
1.4213E+03	1.5010E+03
1.7589E+03	1.5510E+03
2.1390E+03	1.6010E+03
2.5617E+03	1.6510E+03
3.0258E+03	1.7010E+03
3.5293E+03	1.7510E+03
4.0685E+03	1.8010E+03
4.6389E+03	1.8510E+03
5.2343E+03	1.9010E+03
5.8480E+03	1.9510E+03
6.4723E+03	2.0010E+03
7.0990E+03	2.0510E+03
7.7201E+03	2.1010E+03
8.3277E+03	2.1510E+03
8.9147E+03	2.2010E+03
9.4747E+03	2.2510E+03
1.0003E+04	2.3010E+03
1.0494E+04	2.3510E+03
1.0947E+04	2.4010E+03
1.1360E+04	2.4510E+03

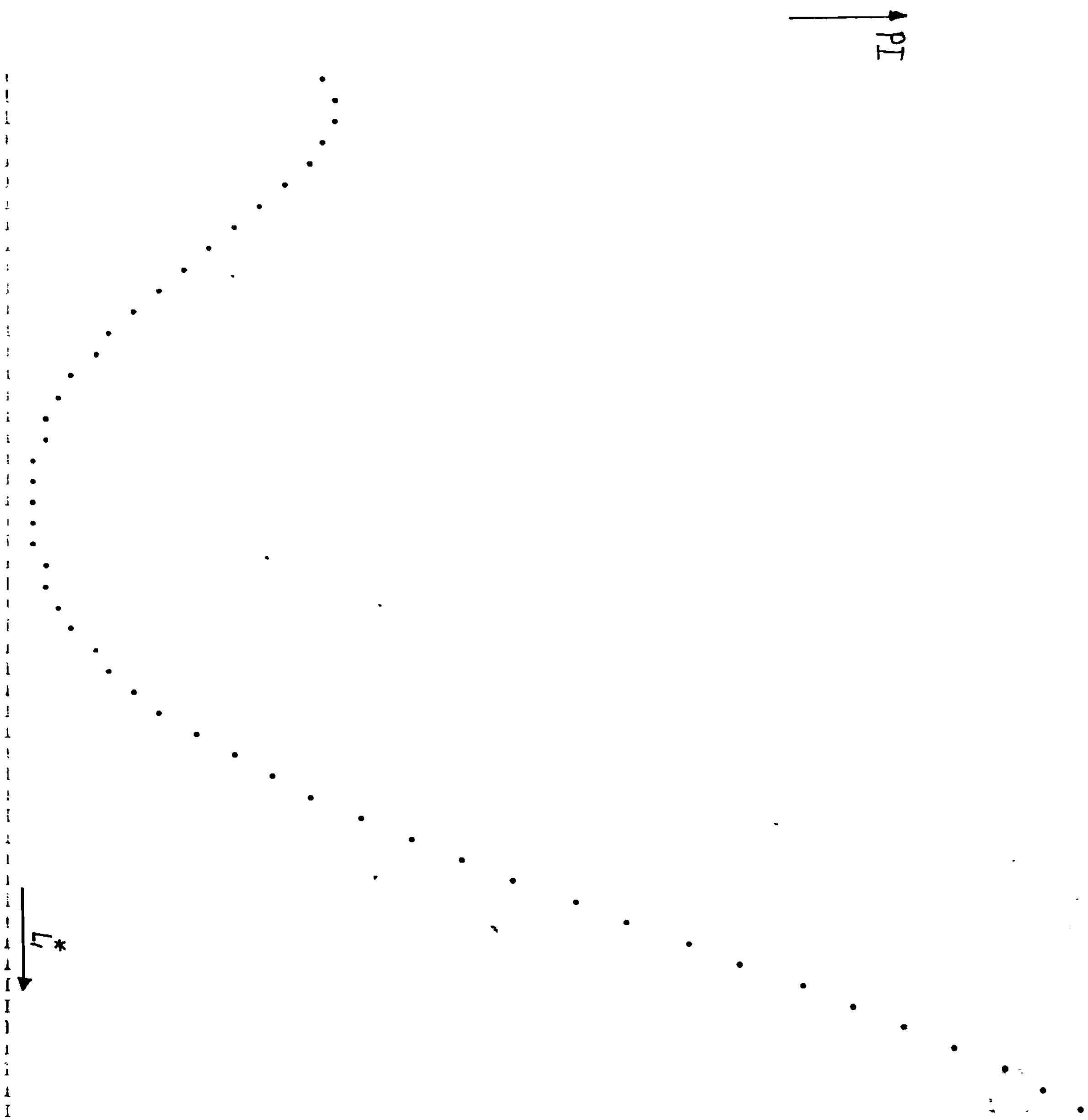
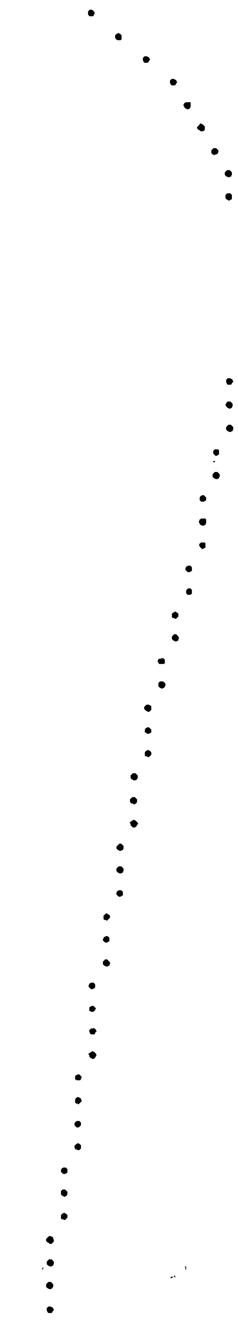


Fig 4.10 PI-curve as function of L1, continued on page 50.

Continuation of Fig 4.10.

[illegible]	[illegible]
1.2061@+04	2.5510@+03
1.2352@+04	2.6010@+03
1.2605@+04	2.6510@+03
1.2821@+04	2.7010@+03
1.3004@+04	2.7510@+03
1.3156@+04	2.8010@+03
1.3279@+04	2.8510@+03
1.3376@+04	2.9010@+03
1.3450@+04	2.9510@+03
1.3503@+04	3.0010@+03
1.3537@+04	3.0510@+03
1.3555@+04	3.1010@+03
1.3559@+04	- 3.1510@+03
1.3550@+04	3.2010@+03
1.3531@+04	3.2510@+03
1.3503@+04	3.3010@+03
1.3466@+04	3.3510@+03
1.3424@+04	3.4010@+03
1.3376@+04	3.4510@+03
1.3323@+04	3.5010@+03
1.3268@+04	3.5510@+03
1.3209@+04	3.6010@+03
1.3149@+04	3.6510@+03
1.3088@+04	3.7010@+03
1.3026@+04	3.7510@+03
1.2963@+04	3.8010@+03
1.2901@+04	3.8510@+03
1.2840@+04	3.9010@+03
1.2779@+04	3.9510@+03
1.2720@+04	4.0010@+03
1.2662@+04	4.0510@+03
1.2605@+04	4.1010@+03
1.2551@+04	4.1510@+03
1.2498@+04	4.2010@+03
1.2447@+04	4.2510@+03
1.2399@+04	4.3010@+03
1.2352@+04	4.3510@+03
1.2307@+04	4.4010@+03
1.2265@+04	4.4510@+03
1.2224@+04	4.5010@+03
1.2185@+04	4.5510@+03
1.2148@+04	4.6010@+03
1.2112@+04	4.6510@+03
1.2077@+04	4.7010@+03
1.2042@+04	4.7510@+03
1.2007@+04	4.8010@+03
1.1973@+04	4.8510@+03
1.1938@+04	4.9010@+03
1.1902@+04	4.9510@+03
1.1865@+04	5.0010@+03
1.1828@+04	5.0510@+03
1.1789@+04	5.1010@+03
1.1748@+04	5.1510@+03
1.1707@+04	5.2010@+03
1.1664@+04	5.2510@+03
1.1621@+04	5.3010@+03
1.1577@+04	5.3510@+03
1.1533@+04	5.4010@+03

L^*



For Gauss-Newton one has to constrain to a parameter region between the two inflection points. For Marquardt, however, the parameter space has to be limited by the local maxima. Because we do not know the exact position of these points, we may only hope, that a physiological relevant parameter region will be positioned between these two inflection points, maxima respectively so that parameter estimation with Gauss-Newton or Marquardt will be possible.

To find physiologically relevant bounds to the parameter space, the following expressions for Rsl, Ll, Rpl and Cl may be derived (ref.1):

$$Rsl = K_2 \frac{8\eta}{\pi r_0^4} \quad (4.10)$$

$$Ll = K_1 \frac{\rho}{\pi r_0^2} \quad (4.11)$$

$$Rpl = \frac{2\eta_0 h}{3\pi r_0^3} \quad (4.12)$$

$$Cl = \frac{3\pi r_0^3}{2E_1 h} \quad (4.13)$$

Where r_0 = inner radius of the vessel, ρ = mass density of the blood η = viscosity of the blood, E_1 = elasticity modulus of the wall, η_0 = viscosity of the wall, h = the thickness of the wall and K_1 and K_2 are functions of $a_1 r_0$ where $a_1 = \frac{\omega\rho}{\eta}$ and ω = angular frequency.

When it is possible to find physiologically justified bounds to these new variables, it is possible to compute with (4.10)-(4.13), the corresponding bounds in Rsl, Ll, Rpl and Cl.

Stating:

$$.75 \leqslant r_0 \leqslant 1.5 \quad (4.14)$$

$$K_1 = 1.1 \text{ is constant} \quad (4.15)$$

$$\rho = 1.05 \text{ g/cm}^3 \text{ and constant} \quad (4.16)$$

$$.02 \leqslant \eta \leqslant .06 \text{ dyne.sec/cm}^2 \quad (4.17)$$

$$K_2 = 2.25 \text{ when } r_0 = .75 \text{ cm} \quad (4.18a)$$

$$K_2 = 4.20 \text{ when } r_0 = 1.5 \text{ cm} \quad (4.18b)$$

Then the constraints for Rsl and Ll are:

$$3.17_{10}{-5} = \frac{4.20}{1334.6} x \frac{8x.02}{(1.5)^4} \leqslant Rsl \leqslant \frac{2.25}{1334.6} x \frac{8x.06}{(.75)^4} = 8.14_{10}{-4} \quad (4.19)$$

$$1.22_{10}{-4} \leqslant Ll = \frac{1.1}{1334.6} x \frac{1.05}{r_0^2} \leqslant 4.90_{10}{-4} \quad (4.20)$$

These values are for the unscaled parameters and are expressed in medical units (therefore factor 1334.6 in (4.19) and (4.20), for converting from giorgi system to medical units).

Justification of (4.14) till (4.18b):

1 The human aorta radius will vary from person to person from .75 - 1.5 cm.

2 K_1 is almost constant for these r_0's (see De Pater, ref. 1).

3 The mass density of the erythrocytes is almost the same as that of water. The mass density of blood with 50% erythrocytes disagrees with only .05 g/cm^3 from water. So we are stating that ρ stays the same.

4 The visco elasticity of the blood depends on the hematocrit. When this quantity varies from 15 till 65%, η will vary from 2 till 6 centipoise (fig 4.11).

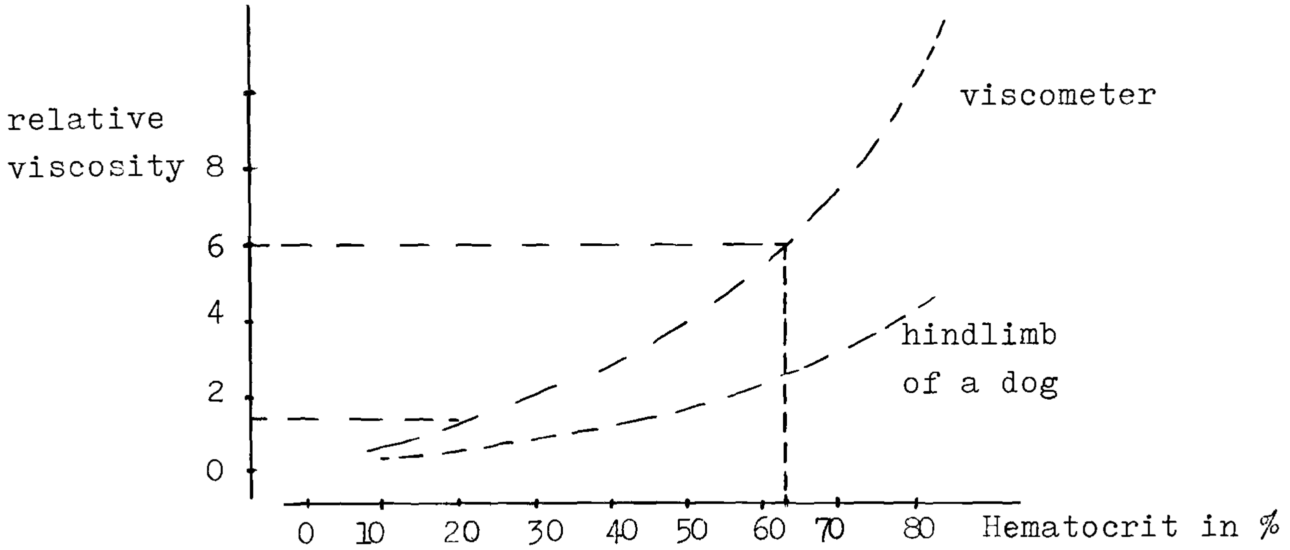


fig 4.11 Blood viscosity as function of the hematocrit measured with a viscometer and the line below is measured using the vascular bed of the hind limb of a dog (from A.C. Burton, ref. 8).

5 K_2 varies with r_0 (see De Pater, ref. 1 page 86).

Because very little is known about the elasticity of the wall it is difficult to compute bounds for Cl with formula (4.12). Instead of (4.12) we will use:

$$c_{p0} \simeq \frac{1}{\sqrt{L1.C1}} \text{ cm/sec}$$

Here, c_{p0} is the pulse wave velocity.

Actually, this equation is not right, but the attenuation has no great effect on c_{p0} (ref.1, page 67).
Following Burton (ref.8), the pulse wave velocity may vary from 400 till 1000 cm/sec. So the lower bound of Cl is given by:

$$C1 = \frac{1}{L1 \cdot {}_{10}6} \text{ cm}^2\text{/mmHg} \tag{4.21a}$$

and the upper bound by:

$$C1 = \frac{1}{L1x16_{10}4} \text{ cm}^2\text{/mm Hg} \tag{4.21b}$$

So these bounds are functions of Ll, namely hyperbolas.
The only constraint on Rpl is, that this parameter may not be negative.

unscaled parameter	lower bound	upper bound	scaled parameter	lower bound (rounded off)	upper bound (rounded off)
Rsl	$3.17_{10}-5$	$8.14_{10}-4$	Rsl*	19	495
Ll	$1.22_{10}-4$	$4.90_{10}-4$	Ll*	445	1800
Rpl	$6.78_{10}-4$	----	Rpl*	1	---
Cl	$\frac{1}{L1_{10}6}$	$\frac{1}{L1x16_{10}4}$	Cl*	1)	2)

1) $\frac{1}{L1^*.QU2QU4_{10}6}$ 2) $\frac{1}{L1^*.QU2QU4x16_{10}-4}$

Table 4.2 Summary of the respective bounds.

These bounds are implemented in the estimation procedure. It appears that estimations with Gauss-Newton from several start points (of the parameters) are possible (see chapter 4).

However, the parameters of the rubber tube will lay beyond this region (see chapter 5). Because we just want to test the estimation procedure on this tube, we had to change these bounds. With

$$0,6 \leqslant r_0 \leqslant 1,5 \text{ cm} \tag{4.22}$$

$$K_1 = 1.1 \tag{4.23}$$

$$\rho = 1 \tag{4.24}$$

$$.02 \leqslant \eta \leqslant .06 \tag{4.25}$$

$$K_2 = 1.95 \text{ when } r_0 = .6 \text{ cm} \tag{4.26a}$$

$$K_2 = 4.20 \text{ when } r_0 = 1.5 \text{ cm} \tag{4.26b}$$

$$200 \leqslant c_{p0} \leqslant 1000 \text{ cm/sec} \tag{4.27}$$

we obtain the bounds summarized in table 4.3

Justification of (4.22)-(4.27)

1 The inner radius r_0 of the tube is .65 cm
2 The fluid in the rubber tube is water, hence ρ=1
3 K_2 = 1.95 when r_0 = .6 cm (see De Pater ref.1 page 86)
4 The lower bound of c_{p0} has decreased because at the beginning of the tube a lumped C is placed and this C has a value which in combination with L of the tube will give a c_{p0} (see (4.20)) of about 250 cm/sec.

unscaled parameter	lower bound	upper bound	scaled parameter	lower bound	upper bound
Rsl	$1.58_{10}-5$	$1.72_{10}-3$	Rsl*	10	1050
Ll	$1.16_{10}-4$	$8.59_{10}-4$	Ll*	420	2640
Rpl	$6.78_{10}-3$	----	Rpl*	1	---
Cl	(4.21a)	(4.21b)	Cl*	1)	2)

1) $\frac{1}{\text{L1QU2QU4}_{10}6}$ 2) $\frac{1}{\text{L1QU2QU4.4}_{10}4}$

Table 4.3 Parameter bounds to be able to estimate the tube parameters.

These bounds are implemented in the Marquardt estimation procedure.

5. ADJUSTMENT PROCEDURES AND ESTIMATION RESULTS

The last chapter summarizes some practical information about the programming of the methods of Gauss-Newton and Marquardt. It describes the results of these methods applied to model-to-model, model-to-8-sections model and model-to-tube adjustments.

5.1 Practical information about the Gauss-Newton procedure

Fig 5.1 gives the scheme for the Gauss-Newton estimation procedure. It will be explained in the next pages.

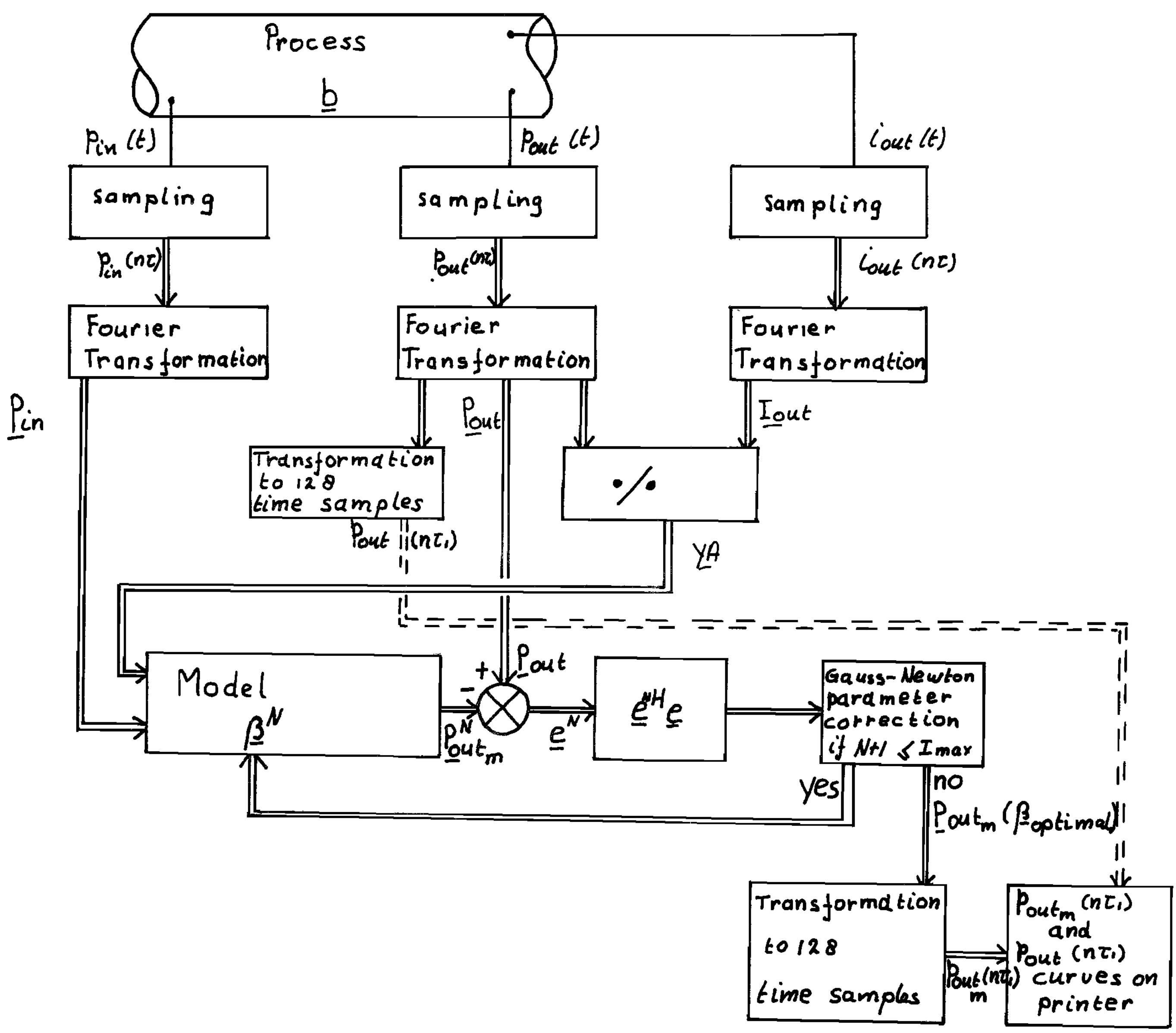


Fig 5.1 Diagram for process parameter estimation with Gauss-Newton.

In this diagram $p_{in}(t)$, $p_{out}(t)$ and $i_{out}(t)$ are continuous process signals. The Analog -Digital-Converter gives the samples $p_{in}(n\tau)$, $p_{out}(n\tau)$ and $i_{out}(n\tau)$; here $f_s = \frac{1}{\tau}$ is the sampling frequency. These samples punched on data cards are the input of the estimation program.

The basic frequency of "the heart", viz.: the function generator of the 8-sections model and the pump of the rubber tube, is very accurately adjustable; in our case it is adjusted to 1 Hz. This property enables us to choose the number of samples in one period. This number, namely, is then very well matched with the sampling frequency.

Fourier transformation performed on the samples of one period (250 when f_s = 250Hz) delivers the vectors $\underline{P}in$, $\underline{P}out$ and $\underline{I}out$ with:

$$\underline{P}in = \left\{ Pin[-h], \ldots, Pin[0], \ldots, Pin[h] \right\}^T \tag{5.1}$$

$$\underline{P}out = \left\{ Pout[-h], \ldots, Pout[0], \ldots, Pout[h] \right\}^T \tag{5.2}$$

$$\underline{I}out = \left\{ Iout[-h], \ldots, Iout[0], \ldots, Iout[h] \right\}^T \tag{5.3}$$

The elements of these vectors are the complex Fourier coefficients of the respective signals[1]. Here h is the highest relevant harmonic (13 in our case).

After this transformation $\underline{Y}A$ is determined from $\underline{P}out$ and $\underline{I}out$, and then $\underline{P}in$ and $\underline{Y}A$ are the input of the "model". This is a procedure which computes the model output pressure $\underline{P}out_m$ for a certain parameter set β^N (N indicates the Nth iteration) with equation (2.20).

$$\underline{X}^N = \underline{P}out_m^N = \left\{ Pout_m^N[-h], \ldots, Pout_m^N[0], \ldots, Pout_m^N[h] \right\} \tag{5.4}$$

According to paragraph 3.2 we will write $\underline{X}$ instead of $\underline{P}out_m$ and $\underline{Y}$ instead of $\underline{P}out$ (process output).

------

1) The coefficients for positive and negative frequency are related, because their inverse transform, the original time signals, are real.
Hence, it is required that $F^*[-k] = F[k]$.

The following step is the comparison of process and model output. This gives the error vector:

$$\underline{e}^N = \underline{P}out - \underline{P}out_m^N = \underline{Y} - \underline{X}^N \qquad (5.5)$$

Then the Performance Index is determined: $PI^N = (\underline{e}^N)^H \underline{e}^N$ (see (3.2)) and compared with MINPI; the minimal PI-value within (N - 1) iterations. When PI^N is smaller than MINPI, and there are no local PI-minima besides the global one in the admitted parameter region, then the set $\underline{\beta}^N$ is closer to the optimal one, than all others. In that case $\underline{\beta}^N$ will be stored.

According to (3.16), repeated on the next page, a correction step $\Delta\underline{\beta}$ is computed from $\underline{\beta}^N$. It is possible that the new set, viz.: $\underline{\beta}^{N+1} = \underline{\beta}^N + \Delta\underline{\beta}$ is worse than $\underline{\beta}^N$ itself. But one has to accept this "wrong" step, because the Gauss-Newton method according to formula (3.16)[1], can find from a certain set $\underline{\beta}^N$ only one new set $\underline{\beta}^{N+1}$ [2].

With this new parameter set again the model output $\underline{X}^{N+1}$ is computed, as well as the $\underline{e}^{N+1}$ and the PI^{N+1} etc. One such a cycle: Model output → error → PI → parameter updating → Model output, is called one evaluation.

The stop criterion of the estimation procedure is determined by Imax; the maximum number of evaluations. After some estimations on the same process, it is possible to choose a reasonable Imax.

After the Imax evaluations the model output $\underline{X}$optimal with the best set is computed and after this, $\underline{X}$optimal is converted with Fast Fourier Transform to 128 samples in the time domain.

Because we want to compare model and process output graphically, it is useful to give the process output $p_{out}(t)$ also in 128 time samples. This can easily be done, limiting the frequency spectrum of $p_{out}(t)$ with a digital filter at 64Hz.

Hence, the block "Transformation to 128 time samples" in fig 5.1, which performs the inverse transformation of Pout[k], the Fourier coefficients of $p_{out}(t)$, with only 64 components.
If f_s is smaller than 128Hz, the frequency spectrum Pout[k] is filled up with zero's from $k = \frac{1}{\tau}+1$ until 64; if f_s is greater than 128Hz, the digital filter sets component 64 on zero, while the higher ones are cut off.

------

[1] Note that we have not used a Gauss-Newton version with an extra parameter to vary the size of the correction step. [2] See next page

For easy reference we repeat (3.16): the Gauss-Newton correction - step:

$$\Delta\underline{\beta} = \underline{\beta}^{N+1} - \underline{\beta}^{N} = \left[2\mathrm{re}\left(\frac{\partial \underline{X}^H}{\partial \underline{\beta}} \cdot \frac{\partial \underline{X}}{\partial \underline{\beta}^T} \right) \Bigg|_{\underline{\beta}^N} \right]^{-1} \times \left[2\mathrm{re}\left\{ \frac{\partial \underline{X}^H}{\partial \underline{\beta}} \Bigg|_{\underline{\beta}^N} (\underline{Y} - \underline{X}^N) \right\} \right] \tag{3.16}$$

The following discussion deals with the determination of the matrix of partial derivatės

$$\frac{\partial \underline{X}^H}{\partial \underline{\beta}} \Bigg|_{\underline{\beta}^N} \quad \text{and} \quad \frac{\partial \underline{X}}{\partial \underline{\beta}^T} \Bigg|_{\underline{\beta}^N}$$

Because the second matrix equals the transposed and complex conjungated first matrix, we want to restrict the discussion to $\frac{\partial \underline{X}^H}{\partial \underline{\beta}}$.

$$\frac{\partial \underline{X}^H}{\partial \underline{\beta}} \Bigg|_{\underline{\beta}^N} = \begin{pmatrix} \frac{\partial X^*_{-13}}{\partial \beta_1} & \cdots\cdots\cdots & \frac{\partial X^*_{+13}}{\partial \beta_1} \\ & & \\ \frac{\partial X^*_{-13}}{\partial \beta_4} & \cdots\cdots\cdots & \frac{\partial X^*_{+13}}{\partial \beta_4} \end{pmatrix}^{1)} \Bigg|_{\underline{\beta}^N} \tag{5.6}$$

Since x(t) is real it is required that:

$$X^*[-k] = X[k] \text{ or } X^*[k] = X[-k]. \tag{5.7}$$

Continuation of remark [2] on page 57.

[2] In the case that $\underline{\beta}^{N+1}$ is outside the admitted parameter region (table 4.2 and table 4.3) $\underline{\beta}^{N+1}$ becomes the border value.

------

[1] The *-symbol indicates the complex conjungate.

With this relation the number of the partial derivatives, that must be determined, may elegantly be restricted.
In virtue of (5.7) namely holds:

$$\frac{\partial X_{-k}}{\partial \beta_j} = \frac{\partial X_k^*}{\partial \beta_j} = \left(\frac{\partial X_k}{\partial \beta_j}\right)^* \qquad (5.8)$$

So the number of different derivatives is 4x14=56 instead of 4x27=108.

There are various ways for determining $\frac{\partial X_k}{\partial \beta_j}$.
First there is the analytic expression for it, derived from (2.20)[1]
For the 4 parameters the analytic expressions are given in (5.9) until (5.12).

$$\frac{\partial X_k}{\partial Rs1} = \frac{-\left\{\frac{d}{2Z_0}.\sinh(\gamma d)+YA\left[\frac{1}{2\gamma}.\sinh(\gamma d)+\frac{d}{2}.\cosh(\gamma d)\right]\right\} Pin[k]}{\left\{\cosh(\gamma d) + Z_0.\sinh(\gamma d),YA\right\}^2} \qquad (5.9)$$

$$\frac{\partial X_k}{\partial L1} = j\omega\frac{\partial X_k}{\partial Rs1} \qquad (5.10)$$

$$\frac{\partial X_k}{\partial Rp1} = \frac{-\left\{-\frac{\gamma d}{2Z_2}.\sinh(\gamma d)+YA\left[\frac{\gamma}{2}\sinh(\gamma d)-\frac{dZ1}{2Z2}.\cosh(\gamma d)\right]\right\} Pin[k]}{\left\{\cosh(\gamma d) + Z_0.\sinh(\gamma d).YA\right\}^2} \qquad (5.11)$$

$$\frac{\partial X_k}{\partial C1} = \frac{j}{\omega C1^2}.\frac{\partial X_k}{\partial Rp1} \qquad (5.12)$$

These are very troublesome expressions, especially when we have in mind the numerical unstability of the less complicated formulas of paragraph 2.3.

------

1) Of course one may derive such expressions also from (2.17) or (2.18).

We have used (5.9) - (5.12)in this form in combination with the method of Marquardt (the subject in paragraph 5.4), applied to model-to-model adjustments. The results were very bad. We think this happens because of numerical instability . Maybe the equations must be written in another way, maybe approximated. We have not investigated this any further.

It is more clever to make use of formula (2.20) from which we know that it will be stable in the relevant frequency range. If we approximate:

$$\frac{\partial X_k}{\partial \beta_j}$$

by:

$$\left. \frac{X_k(\beta_j + \Delta\beta_j) - X_k(\beta_j)}{\Delta \beta_j} \right|_{\substack{\beta_i^N \\ i \neq j}} \qquad (5.13)$$

or,by:

$$\left. \frac{X_k(\beta_j + \Delta\beta_j) - X_k(\beta_j - \Delta\beta_j)}{2\Delta \beta_j} \right|_{\substack{\beta_i^N \\ i \neq j}} \qquad (5.14)$$

then we only have to perform some function evaluations. Of course (5.14) is a better approximation than (5.13) because it is symmetrical. And it is also clear that $\Delta\beta_j$ must be as small as possible for a good approximation. On the other hand, a very small $\Delta\beta_j$ may cause such a little difference in the two terms of the numerator of (5.13) and (5.14) that there will be a great loss of digits. It is clear that the computed correction steps will be very badly determined, if the remaining digits after the subtraction of the two terms are affected by truncation errors.

If $\underline{\beta}$ means the scaled parameter vector, in accordance with paragraph 4.1, we may take the value of $\Delta\beta_j$ the same for all j. Then the sensitivity is almost the same for all parameters. It appears from the Gauss-Newton method, applied to model-to-model and model-to-8-sections model estimations that the choice $\Delta\beta_j = 1$ is a reasonable one.

In the case that $\Delta\beta_j < 1$, more wrong steps (see page 57) have to be accepted. With $\Delta\beta_j > 1$ and using (5.13), it appears that the parameters do not want to stay "at the minimum", once coming there.

5.2 Model-to-model parameter estmation results

Model-to model adjustments have the advantage that the estimation procedure is not affected by the sampling noise, the noise of the process, the measurement noise etc. Moreover the model is an exact representation of the process, because the last one is the model with a certain parameter set $\underline{\beta} = \underline{b}$, the "patient parameter set".

In this situation $\underline{P}$in and $\underline{Y}$A (see fig 5.1) are obtained from the 8-sections model, because those signals, which are similar to physiological ones, are already at our disposal.
The process output equals $\underline{X}(\underline{b}) \equiv \underline{P}out_m(\underline{b})$.

The intention is to find patient set $\underline{b}$ with Gauss-Newton, starting from $\underline{\beta}^o$. Table 5.1 gives the estimation results for 6 different situations.

In the first three situations, the "patient set" $\underline{b}$ has been chosen on the borders of the admitted parameter region, which is bounded according to the constraints given in table 4.2. The starting points $\underline{\beta}^o$ are the same in these situations, namely the parameter set of the 8-sections model (see table 2.1).
In situation 4 and 5, $\underline{b}$ is a border setting, while the starting points $\underline{\beta}^o$ are positioned at the opposite bounds. In situation 6 patient set $\underline{b}$ equals the parameter set of the 8-sections model, while β^o now is on the border of the admitted region.

no	PATIENT VALUES				START VALUES				*)	START	MINPI after 20 evaluation
	Rsl	L1	Rpl	C1	Rsl	L1	Rpl	C1		PI	
1	$3.124\ 10^{-5}$	$4.932\ 10^{-4}$	$6.850\ 10^{-3}$	$2.192\ 10^{-3}$	$1.670\ 10^{-4}$	$2.740\ 10^{-4}$	$1.370\ 10^{-1}$	$1.468\ 10^{-2}$	9	$1.37\ 10^{3}$	$6.6\ 10^{-24}$
2	$3.124\ 10^{-5}$	$1.219\ 10^{-4}$	$6.850\ 10^{-3}$	$8.494\ 10^{-3}$	$1.670\ 10^{-4}$	$2.740\ 10^{-4}$	$1.370\ 10^{-1}$	$1.468\ 10^{-2}$	6	$1.93\ 10^{3}$	$4.5\ 10^{-20}$
3	$3.124\ 10^{-5}$	$4.932\ 10^{-4}$	3.425	$2.192\ 10^{-3}$	$1.670\ 10^{-4}$	$2.740\ 10^{-4}$	$1.370\ 10^{-1}$	$1.468\ 10^{-2}$	7	$1.36\ 10^{3}$	$1.1\ 10^{-19}$
4	$8.220\ 10^{-4}$	$4.932\ 10^{-4}$	3.083	$1.260\ 10^{-2}$	$3.124\ 10^{-5}$	$1.206\ 10^{-4}$	$6.850\ 10^{-3}$	$8.494\ 10^{-3}$	9	$4.92\ 10^{3}$	$5.8\ 10^{-20}$
5	$3.124\ 10^{-5}$	$1.219\ 10^{-4}$	$6.850\ 10^{-3}$	$8.494\ 10^{-3}$	$8.220\ 10^{-4}$	$4.932\ 10^{-4}$	3.083	$1.260\ 10^{-2}$	8	$4.90\ 10^{3}$	$6.6\ 10^{-24}$
6	$1.670\ 10^{-4}$	$2.740\ 10^{-4}$	$1.370\ 10^{-1}$	$1.468\ 10^{-2}$	$3.288\ 10^{-5}$	$1.219\ 10^{-4}$	$6.850\ 10^{-3}$	$2.192\ 10^{-3}$	11	$1.91\ 10^{3}$	$4.1\ 10^{-20}$

*) patient set $\underline{b}$ is reached in ... iterations[1].

Table 5.1 Estimation results model-to-model, the estimated values are exactly the same as the "patient values".

------

[1] Note the difference between evaluation and iteration: an

Fig 5.2 shows those situations, indicated by a "cross" and subscript "$\underline{\beta}^{o}_{sit}$...." or "$\underline{b}_{sit}$...." in the case we mean a starting point or patientset of situation.... respectively. The admitted parameter region is given in two diagrams.

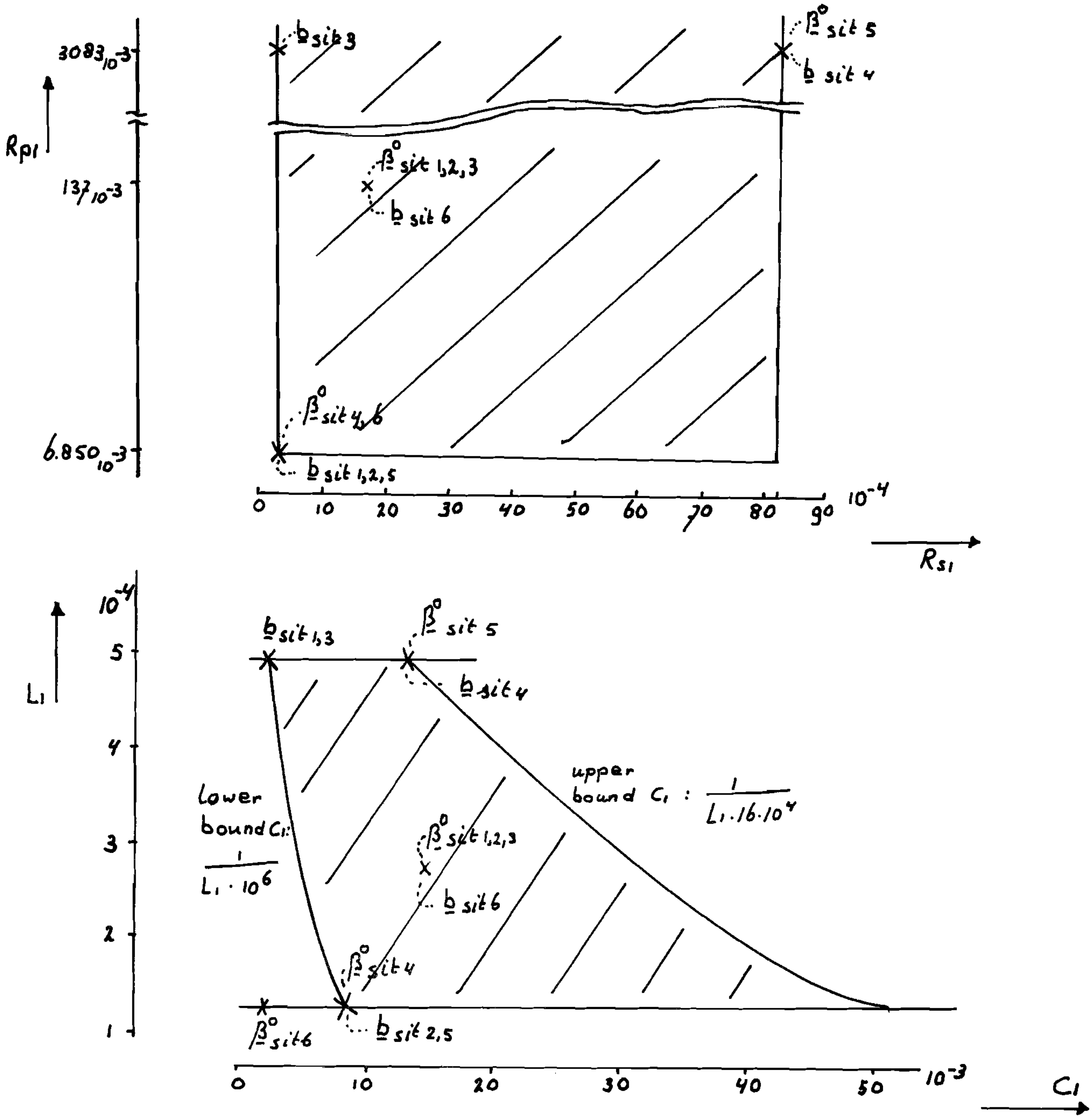


Fig 5.2 The 6 situations of table 5.1. Start and patient values are indicated by a "cross".

Continuation of remark 1) of page 61.
an evaluation is every computing cycle which leads to a updated parameter set, while an evaluation which yields a new set closer to the minimum than all the others, is called an iteration.

In all situations the estimated parameter values are, within the 5 printed digits, the same as the "patient parameters",

Appendix 3 shows the path followed by the Gauss-Newton procedure in situation number 1.

According to these very good results, Gauss-Newton appears to be very suited for the parameter estimation within the constraint region.

5.3 Model-to-8-sections model adjustment

Instead of using the model output $\underline{P}out_m(\underline{\beta})$, when $\underline{\beta}=\underline{b}$ is a certain "patient-set", as "patient signal", we consider now the 8-sections model output as process output. So this 8-sections model will serve as "patient aorta", while its parameters (see paragraph 2.2) are the "patient parameters".

In this set-up, the model is no exact description of the process, and the sampling noise on the signals may effect the estimations.

We have performed estimations on this "patient", again with Gauss-Newton, while the parameters had to stay within the admited parameter region of table 4.2 (see fig 5.2).

Table 5.2 gives the results of these adjustments.

There is one "patient-set" $\underline{b}$, printed in the first four columns of the lowest line of the table, and there are 17 different starting points. In the first 16 situations, these points are positioned on all different boundary points of the admitted region (see fig 5.2). In the 17th situation, the starting point is on the "patient-set" itself.

The results of these estimations are in all 17 situations the same, they are once printed in this table.

NO	STARTING VALUES				*)	PI at the starting points
	Rsl	L1	Rpl	C1		
1	$3.124_{10}{-5}$	$1.219_{10}{-4}$	$6.850_{10}{-3}$	$8.494_{10}{-3}$	9	$2.11_{10}{3}$
2	$3.124_{10}{-5}$	$1.219_{10}{-4}$	$6.850_{10}{-3}$	$5.069_{10}{-2}$	9	$2.28_{10}{3}$
3	$3.124_{10}{-5}$	$1.219_{10}{-4}$	3.425	$8.494_{10}{-3}$	8	$1.86_{10}{3}$
4	$3.124_{10}{-5}$	$1.219_{10}{-4}$	3.425	$5.069_{10}{-2}$	7	$1.01_{10}{3}$
5	$3.124_{10}{-5}$	$4.932_{10}{-4}$	$6.850_{10}{-3}$	$2.192_{10}{-3}$	7	$1.29_{10}{3}$
6	$3.124_{10}{-5}$	$4.932_{10}{-4}$	$6.850_{10}{-3}$	$1.260_{10}{-2}$	6	$3.70_{10}{3}$
7	$3.124_{10}{-5}$	$4.932_{10}{-4}$	3.425	$2.192_{10}{-3}$	7	$1.28_{10}{3}$
8	$3.124_{10}{-5}$	$4.932_{10}{-4}$	3.425	$1.260_{10}{-2}$	9	$3.48_{10}{3}$
9	$8.220_{10}{-4}$	$1.219_{10}{-4}$	$6.850_{10}{-3}$	$8.494_{10}{-3}$	7	$1.67_{10}{3}$
10	$8.220_{10}{-4}$	$1.219_{10}{-4}$	$6.850_{10}{-3}$	$5.069_{10}{-2}$	9	$7.15_{10}{2}$
11	$8.220_{10}{-4}$	$1.219_{10}{-4}$	3.425	$8.494_{10}{-3}$	10	$1.59_{10}{3}$
12	$8.220_{10}{-4}$	$1.219_{10}{-4}$	3.425	$5.069_{10}{-2}$	10	$1.58_{10}{3}$
13	$8.220_{10}{-4}$	$4.932_{10}{-3}$	$6.850_{10}{-3}$	$2.192_{10}{-3}$	9	$1.37_{10}{3}$
14	$8.220_{10}{-4}$	$4.932_{10}{-4}$	$6.850_{10}{-3}$	$1.260_{10}{-2}$	8	$2.39_{10}{3}$
15	$8.220_{10}{-4}$	$4.932_{10}{-4}$	3.425	$2.192_{10}{-3}$	11	$1.36_{10}{3}$
16	$8.220_{10}{-4}$	$4.932_{10}{-4}$	3.425	$1.260_{10}{-2}$	9	$2.48_{10}{3}$
17	$1.670_{10}{-4}$	$2.740_{10}{-4}$	$1.370_{10}{-2}$	$1.468_{10}{-2}$	3	6.14

THE ESTIMATED PARAMETER VALUES

Rsl	L1	Rpl	C1
$1.580_{10}{-4}$	$2.786_{10}{-4}$	$1.691_{10}{-1}$	$1.536_{10}{-2}$

8-SECTIONS MODEL PARAMETER VALUES

$1.670_{10}{-4}$	$2.740_{10}{-4}$	$1.370_{10}{-1}$	$1.468_{10}{-2}$

The PI-value after 20 evaluations, is the same for all 17 situations: 1.14057.

Table 5.2 Estimation results of model-to-8-sections model adjustment. The optimal parameter values are the same, starting from 17 different starting points.

*) Optimal set reached in .... iterations.

From those results, it is clear that the minimum of the PI-surface can be found fast and inambiguous with the Gauss-Newton procedure. The values of the optimal parameter set, however, do not agree exactly with the "patient values".
This may be caused by the following effects:

1. Bias in the estimations due to sampling noise on the signals. It must be mentioned here that we used in the 17 situations just one series of signals of the 8-sections model, so we had the same noise contribution in all 17 situations.
2. The model is no exact description of the process.
3. To a lesser extent because of the inaccurate potmeter settings on the analog computer.

The investigation of the effect of noise on the estimations is a good suggestion for further work.

Appendix 4 gives the path followed by the estimation procedure, performed in situation no.1.The table in that appendix shows in the first three lines the heading and the starting values of the parameters and the Performance Index. DELPAR is the stepsize $\Delta\beta_j$, used in the determination of the partial derivates $\frac{\partial x_k}{\partial \beta_j}$. In the next lines the scaled and unscaled updated parameters are printed. The "#" character means that the parameter under this symbol has exceeded the bounds. In that case that parameter becomes the boundary value. When a "wrong" step is done, in other words when a $PI^{(N+1)}$ is found which is larger than MINPI (the minimal PI-value up to the Nth iteration), this has been indicated by omission of the words "ITER" and "DELPAR".
Furthermore appendix 4 gives, on 3 pages, the process and model output as function of time; "*"-character indicates the process or "patient"signal, the "#"-character indicates the model output after optimal parameter setting. The "S"-symbol is used when for a certain $t = n\tau$, model and process output are the same.

5.4 Practical information about the method of Marquardt

The expression for the correction step in the case of Marquardt's method is almost the same as it is by the Gauss-Newton method. See (3.23) and (3.16), which are repeated here:

$$\underline{\beta}^{N+1} - \underline{\beta}^{N} = \Bigg[\underbrace{2\mathrm{re}\left\{\frac{\partial \underline{X}^{H}}{\partial \underline{\beta}}\cdot\frac{\partial \underline{X}}{\partial \underline{\beta}^{T}}\right\}\Bigg|_{N}}_{A}\Bigg]^{-1} \times \Bigg[2\mathrm{re}\left\{\frac{\partial \underline{X}^{H}}{\partial \underline{\beta}}\Bigg|_{N}(\underline{Y}-\underline{X}^{N})\right\}\Bigg] \qquad (3.16)$$

$$\underline{\beta}^{N+1} - \underline{\beta}^{N} = \Bigg[\overbrace{2\mathrm{re}\left\{\frac{\partial \underline{X}^{H}}{\partial \underline{\beta}}\cdot\frac{\partial \underline{X}}{\partial \underline{\beta}^{T}}\right\}}\Bigg|_{N} + \lambda I\Bigg]^{-1} \times \Bigg[2\mathrm{re}\left\{\frac{\partial \underline{X}^{H}}{\partial \underline{\beta}}\Bigg|_{N}(\underline{Y}-\underline{X}^{N})\right\}\Bigg] \qquad (3.23)$$

The only difference is the matrix λI, which is added to matrix A before the inversion takes place.

The extra parameter λ, however, makes it possible to vary the correction step $\Delta\underline{\beta}$, from a Gauss-Newton step, if $\lambda=0$, to a very small Steepest-Descent step if λ is very large.

Hence, in contrast with Gauss-Newton, it is possible with Marquardt's method to compute from one parameter point $\underline{\beta}^{N}$ more then one new point $\underline{\beta}_{1}^{N+1}$, and, following paragraph 3.3, it is even possible, to find from $\underline{\beta}^{N}$ always a "better" set $\underline{\beta}^{N+1}$, that means a set closer to the optimal one, if λ is large enough. So we do not have to admit wrong steps.

Fig 5.3 gives a flow diagram of Marquardt's procedure. One can see in this figure, that a new set $\underline{\beta}_{1}^{N+1}(\lambda,\underline{\beta}^{N})$ only is accepted if it is closer to the minimum. Only in that case set $\underline{\beta}$ becomes $\underline{\beta}_{1}^{N+1}$.

The stop criterion is determined by the total number of admitted evaluations Imax (see section 5.1). The partial derivatives are determined in the same way as they were using Gauss Newton's method.

In section 3.3 a remark is made about the scaling, advised by Marquardt. In our procedure we only apply the parameter scaling in accordance with paragraph 4.1, for, after this scaling, Marquardt's method gives no further improvement.

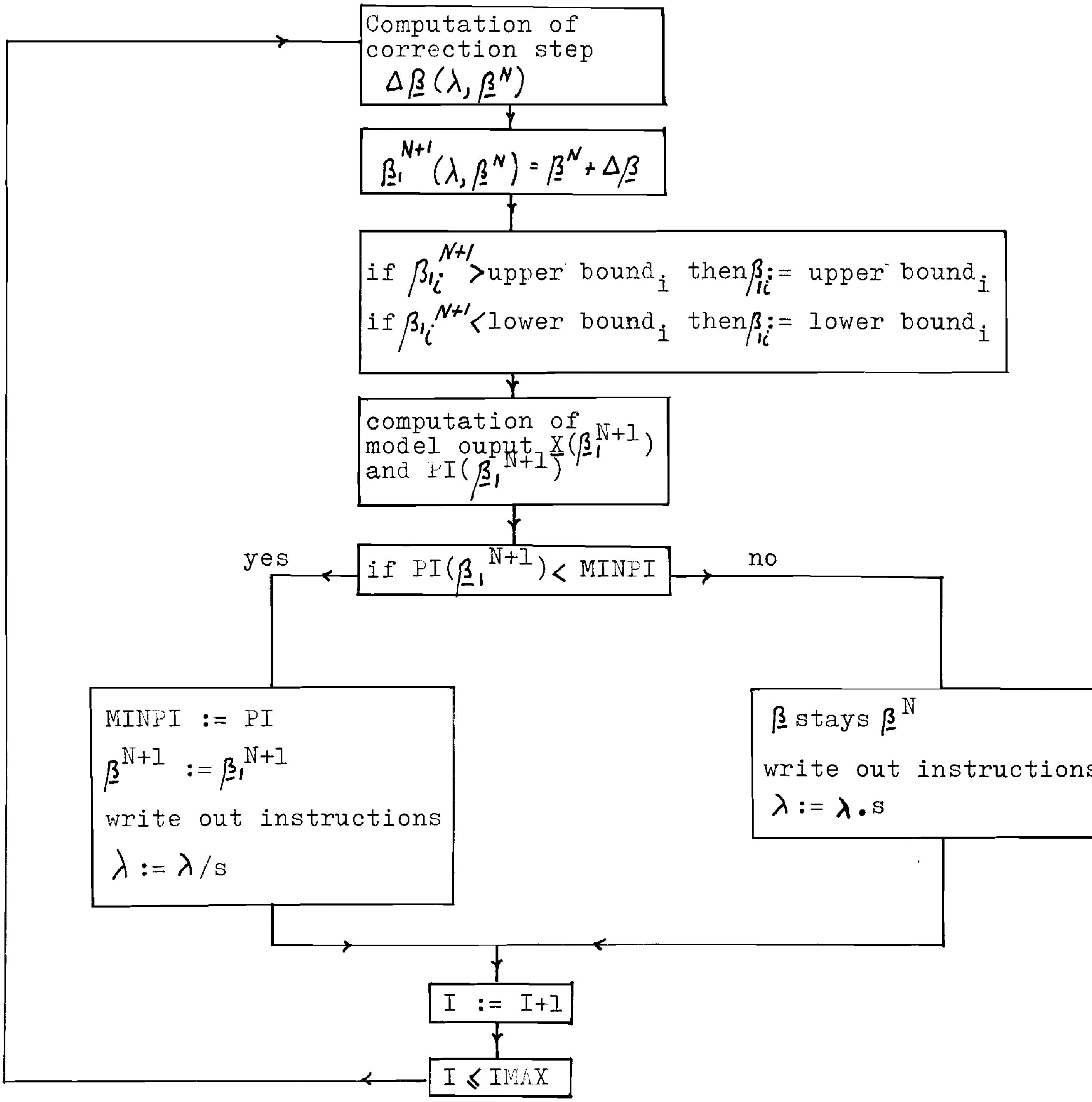


fig 5.3 Flow diagram the Marquardt procedure.

In analogy with the Gauss-Newton procedure, one evaluation is defined as the cycle:

1. Computation of the new set $\beta_1^{N+1}(\lambda,\beta^N)$
2. Computation of model ouput and PI-value
3. Determination whether $PI(\beta_1^{N+1})$ is smaller or lager than MINPI
   · updating of λ, and/or β and MINPI.

5.5 Model-to-tube estimation results

The Marquardt estimation procedure is first applied to model-to-8-sections model adjustment. Starting from $\underline{\beta}^{o}$ of of situation 1 of table 5.2, with the same constraints to the parameter region as in that situation, this method gives the same optimal set as Gauss-Newton does.
Remarkable however, is the difference in the numbers of boundary transgressions. When using Marquardt's method, this number is much lower than in the case of the Gauss-Newton adjustment. See for example, appendix 4 and 5. Appendix 4 gives the estimation path follwed by the Gauss-Newton method, and appendix 5 the path followed by Marquardt's procedure. Starting points and constraints in both situations are equal. Gauss-Newton gives 10 boundary transgressions (number of "#" symbols), Marquardt only 4 (indicated by "bg" or "og" before the parameter value; bg= upper bound, og = lower bound). Of course,this effect is originated by the extra constraint to the parameter space in the case of Marquardt's method. For, with a choice of λ a r is connected, that means the radius of the hypersphere, in which the method will search after a minimal PI-value (paragraph 3.3).

Furthermore the Marquardt method is applied to model-to-tube adjustment. So the necessary signals $\underline{P}$in, $\underline{Y}$A and $\underline{P}$out for the estimating procedure have been measured on the rubber tube.

The set up of this physical aorta model has been made by Leliveld (ref.4). It consist of a mechanical pump, a bottle at the beginning of the tube as lumped capacitance and a rather stiff rubber tube. Because our distributed model is excited by the input pressure of the rubber tube, although measured before the bottle, it is not possible in our case to consider the lumped C as distributed over the tube, for, our method can not "see" this capacitance with the pressure as input. The opposite was true if flow had been used as input signal.

In this situation of model-to-tube adjustment, the parameter space is bounded by the border values of the parameters, printed in table 4.3, with the exception of the lower bound of C1. We expect namely a very low C1-value, because the tube is stiff, and that is why we have chosen a constant C1 lower bound on $5.48_{10}{-7}\frac{ml}{mmHg}\cdot\frac{1}{cm}$, in stead of the hyperbola function.

Table 5.3 gives the estimation results of 4 situations; the model parameters, namely, have been adjusted with the Marquardt procedure, starting from the parameter values of the 8-sections model, using four different sample series of signals, i.e. with signals with various noise contribution. Fig 5.4 gives in two maps Rsl - Rpl and Ll - Cl the starting points $\underline{\beta}^0$ and the optimal sets $\hat{\underline{\beta}}$ of these situations.

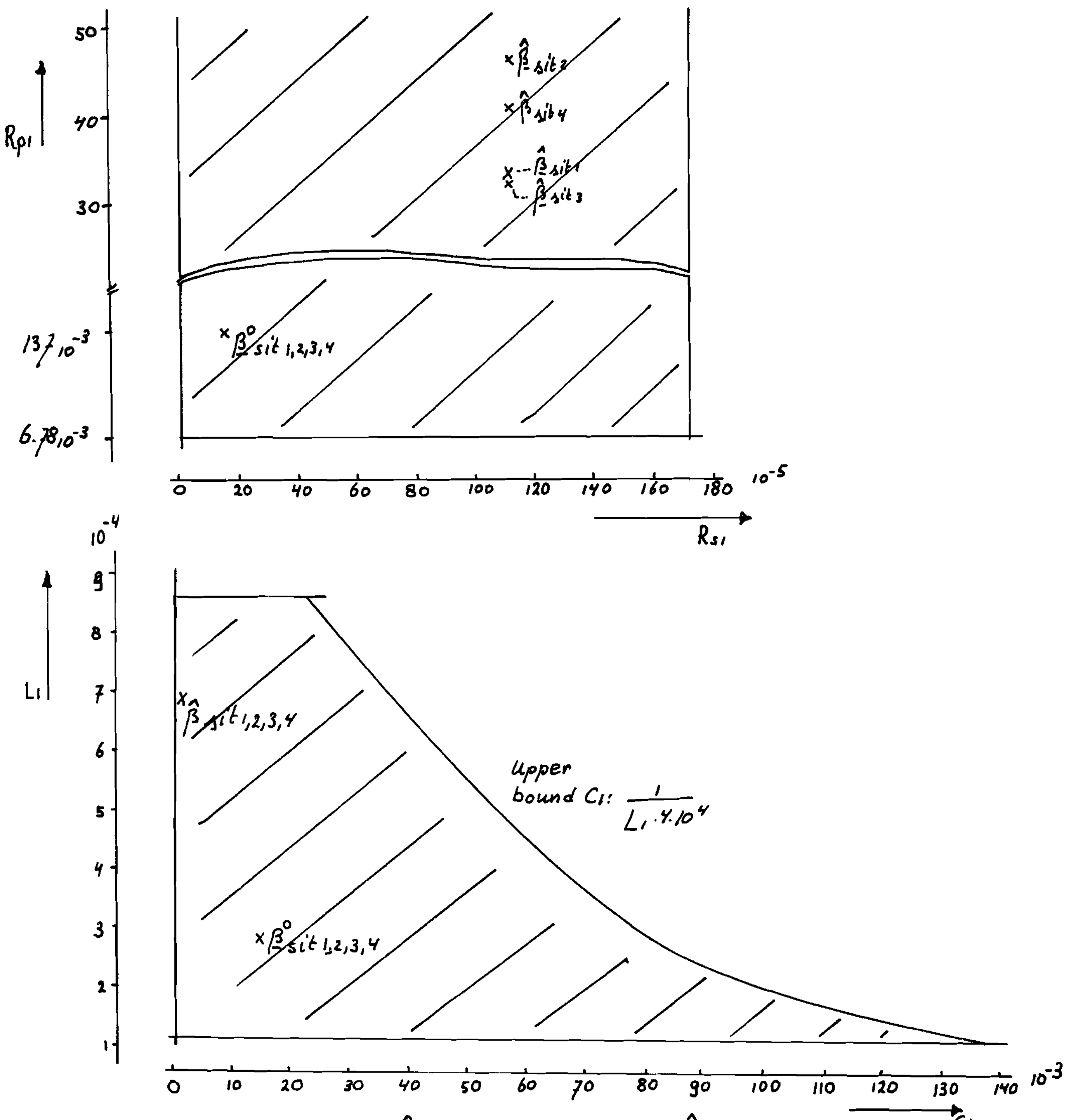


Fig 5.4 Starting points $\underline{\beta}^0$ and optimal sets $\hat{\underline{\beta}}$ in the bounded parameter region.

ARTING POINT: $Rsl=1.67_{10}-4$, $Ll=2.74_{10}-4$, $Rpl=1.37_{10}-1$, $Cl=1.468_{10}-2$						
itu-ion	Estimated parameter values within 30 evaluations				START PI	MINPI in 30 evaluations
	Rsl	Ll	Rpl	Cl		
1	$1.173_{10}-3$	$6.873_{10}-4$	$3.343_{10}+1$	$1.278_{10}-3$	$1.13_{10}+2$	$3.06_{10}-1$
2	$1.118_{10}-3$	$6.857_{10}-4$	$4.836_{10}+1$	$9.512_{10}-4$	$8.04_{10}+1$	$3.29_{10}-1$
3	$1.147_{10}-3$	$6.806_{10}-4$	$3.313_{10}+1$	$1.443_{10}-3$	$6.41_{10}+1$	$3.51_{10}-1$
4	$1.106_{10}-3$	$6.844_{10}-4$	$4.134_{10}+1$	$1.228_{10}-3$	$6.51_{10}+1$	$3.40_{10}-1$

Table 5.3 Estimation results of model to tube adjustment, using 4 different sample series of the signals p_{in}, p_{out} and i_{out}.

The adjustment paths, followed by the estimation procedure, in the 4 situations are given in Appendix 6,7,8,9 respectively. The table in those appendices shows: the PI-value, λ, scaled and unscaled parameters (paragraph 4.1) of each evaluation. When a new parameter updating leads to a worse PI, this is indicated by the omission of the words ITER and LAMBDA. In those cases the performance of the method of Marquardt becomes quite clear.
See for instance Appendix 6 after ITER 10. A worse set is found with $\lambda = 10^{-12}$, the λ increases, till λ is large enough to give a better set. So between ITER 10 and 11, 5 evaluations have been performed, before a better set was found.
In those appendices also are given the process and model output after the adjustment as function of time. "*"-symbols indicates process output pressure, while "#"-symbolindicates the optimal model output. The "S"-character was printed when the two outputs were equally large.

The means and standard deviations of the four estimations are given in table 5.4. One may also find there Rsl and Ll values measured at the tube by Leliveld (ref.4), all in medical units.

	mean of estimations	measured values	standard deviations
Rsl	$1.136_{10}-3$	$1.30_{10}-3$	$2.6_{10}-5$
Ll	$6.845_{10}-4$	$6.75_{10}-4$	$2.5_{10}-6$
Rpl	$3.907_{10}+1$	-----	6.3
Cl	$1.225_{10}-3$	-----	$1.8_{10}-4$

Table 5.4 Mean and standard deviation and measured parameter values.

With regard to Rsl and Ll the results appears to be very reasonable. The estimations of Rpl and Cl, however, are very inaccurate. This becomes quite clear, comparing the results of situation no.1 in table 5.3 and those given in table 5.5. There a situation is given, where the same data set is used, but where another starting point is chosen. Appendix 10 gives the estimation path and time signals.

	Rsl	Ll	Rpl	Cl	PI
START:	$3.124_{10}-5$	$1.219_{10}-4$	$6.850_{10}-3$	$8.494_{10}-3$	$7.75_{10}+1$
OPTIMAL:	$1.133_{10}-3$	$6.409_{10}-4$	$4.348_{10}+1$	$3.901_{10}-2$	$2.74_{10}-1$

Table 5.5 Estimation result when the same data set is used as in sit.no.1 of table 5.3, but starting from another point.

From the MINPI value ($2.74_{10}-1$) it appears that the optimal set must be closer to the "patient" set (=parameter set of the tube) in this case, than in the situation 1 of table 5.3. In the best iteration (ITER 18) , however, parameter Cl is bounded by the upper bound, although we expect a small Cl, on account of the stiff tube. Furthermore the optimal Cl-value in this case is very different from the one in table 5.3, in spite of the little difference in MINPI-value. So it appears that, in spite of the performed scaling, here the sensitivity for Cl and in a lesser degree for Rpl is much smaller.

This last remark becomes clearer by looking at appendix 10. It appears that the correction steps, especially with regard to Rpl and Cl, sometimes are very badly determined. See for instance ITER 17: in the following updating , Rpl jumps from a large value to the lower bound, while Cl jumps from upper bound to lower bound.

The correction steps are greatly dependent on the determination of the partial derivative (see(3.23)). Having in mind the way we approximate these derivatives (5.14), then these bad correction steps can only be explained with the occurence of a great loss of digits, what happens when the computer subtracts two almost equal numbers. This means that the $\Delta\beta_j$ for determining the derivativ is too small for Rpl and Cl, and presumably not for Rsl and Ll. This means however, that the scaling, based on the sensitivity studies, which worked well in the model-to-8-sections model adjustment, is not sufficient for this problem. Of course, this is very well possible, for, the sensitivity of the model for the parameters is a function of these parameters, and it was investigated in the neighbourhood of the parameter setting of the 8-sections model.
The parameters of the tube, however, are far away from this set. This problem may be solved in two ways:

1. Check the relative difference

$$v = \frac{X_k(\beta_j + \Delta\beta_j) - X_k(\beta_j - \Delta\beta_j)}{X_k(\beta_j)}$$

   in every iteration. Make $\Delta\beta_j$ larger,if v is, for instance smaller than 10^{-8}. In fact this means, if wanted, a new scaling every iteration. An unlimited enlargement of $\Delta\beta_j$, however, is not always desirable, because it may give rise to a worse derivative approximation. Because there are some estimations for the tube parameters, it is also possible to investigate the sensitivity of the model for parameter values in the neighbourhood of these tube parameters, and to base on this investigation a new scaling.

2. Computation of $X_k(\beta_j \pm \Delta\beta_j)$ in double precision.
   Then $\Delta\beta_j$ may still stay small enough.

We think that a combination of both methods will give the best solution; however, it is possible that only double precision computation is sufficient.

There is another possibility, namely, the search for other, more sensitive parameters.

6. CONCLUSIONS

This report deals with a linear model of a tubelike aorta. It is a mathematical description of the pressure-flow relations in the vessel, represented in the frequency domain.
Properties of this model are:

1. The model is not lumped but distributed in the parameters. So, the already highly simplified description, derived e.g. by De Pater, of the pressure-flow relationships in the vessel, is not further approximated.
2. The model output pressure equation uses a known terminal admittance. Its structure needs not to be known, because the admittance can be determined by simply dividing the process output flow through the pressure.
3. The model output pressure equation needs two inputs: input pressure and terminal admittance. This means that three process signals have to be measured (see 2.). These signals are sufficient for the model adjustment.
4. The frequency range is important. Because, in general, the model is not an exact representation of the process, the frequency range may affect the position of the minimum of the Performance Index. In our case a frequency range 0 - 13Hz seems to be enough, i.e. a frequency band of 13Hz will give no other estimates, than a model with a frequency range of more than 13Hz.
5. The model can be programmed on a digital computer.

Because of Parseval's theorem, there is a simple relation between Least-Squares estimators in time and Least-Squares estimators in frequency domain. This makes the estimations in time and frequency domain equivalent. The expressions of Gauss-Newton and Marquardt parameter correction steps in the frequency domain are not essentially more difficult then those with regard to the time domain.
Conclusions concerning the estimation results:

1. From model-to-model and model-to-8-sections model estimation results, Gauss-Newton appears to be a suited method for finding the PI-minimum, when only a strictly bounded parameter region is admitted. Physiological relevant values are within this admitted region.

2. From model-to-tube estimations it appears that it is very important to test a new set up in a more realistic "situation", realistic in the sense of more measurement noise, more disagreement between model and process etc. The disadvantage of the tube we used, however, is, that its parameters are beyond the physiological important region. This has caused a far reaching change in the estimation procedure.
   At first the admitted parameter region had to be enlarged and at the second place Gauss-Newton's method had to be replaced by the method of Marquardt. Moreover, it appeared that in the case that Rpl andCl are far away from the 8-sections parameter set, the sensitivity of the model for these parameters is much lower, then has been found in paragraph 4.1.
3. Because of the inaccurate determination of the derivative, due to the lower sensitivity, model-to-tube estimations do not give the same optimal sets, when starting in different points of the parameter space, although they use the same signal samples.
   This is contrary to the model-to-8-sections model adjustments. This remark refers mainly to the estimation of Rpl and Cl.
4. The estimation with this model is very fast. In 80 seconds 30 evaluations (parameter updating → model output → parameter-updating)have been performed.

From this investigation we are now in a position to give some suggestions for further work:

1. The approximation of $\frac{\partial X_k}{\partial \beta_j}$ may be refined and more accurately computed, when working in double precision.
   Note, however, that only the model output has to be computed in double precision. After subtraction of the $X_k(\beta_j + \Delta\beta_j)$ with $X_k(\beta_j - \Delta\beta_j)$, single precision computation is sufficient.
   This operation may give much better results.
2. The above mentioned method may be combined with a sensitivity dependent scaling.

3. It may be useful to study the sensitivity of the model for the parameters in a region around the tube parameter set.
   It is probable, that for those parameter values, and with the tube as process, the PI-curves and surfaces are different, from those obtained in the case that the 8-sections model serves as process and only a restricted region around the 8-sections parameter set is investigated.
   Maybe it will be possible to conclude from these curves, how accurate Rsl,Ll, Rpl and Cl may be estimated. When only inaccurate estimations are possible, perhaps combinations of Rsl, Ll, Rpl and Cl have to be chosen as "the parameters".
   It must be mentioned again, however, that the tube we used, has no physiological relevant parameter values.
4. After these refinements an investigation of the effect of noise on the signals, on the parameter estimations will be useful, to get an idea about possible bias on the estimates. This will be important when the model will be used to estimate parameters from real physiological signals, or in general from signals with much noise.

After these improvements, it will be possible to estimate on a human aorta with this model, or on another vessel, if it is well described by the linear pressure-flow relations, which were the basis for this model. Furthermore it will be possible to describe a model, based on the same principles, but including the tapering of the vessel, (ref.9).

REFERENCES

1. De Pater, L., An Electrical Analoque of the Human Circulatory System, diss. Groningen (1966)

2. Tomesen, H.H., Linear and Nonlinear Modelling of a Tubelike aorta, M.Sc. Thesis, Eindhoven Technological University, The Netherlands (1971)

3. Timmer, H.W., Parameter Estimation on a Model of the Aorta with a Hybrid Computer, (in Dutch), M.Sc. Thesis, Eindhoven Technological University, The Netherlands (1972)

4. Leliveld, W.H., A Mock Circulation System M.Sc. Thesis, Eindhoven Technological University, The Netherlands (1972)

5. Damen, A.A.H. Estimations on the Depolarisation Wave through both Heart Ventricles,(in Dutch M.Sc. Thesis, Eindhoven Technological University, The Netherlands (1970)

6. Marquardt, D.W., An Algorithm for Least-Squares Estimation of Nonlinear Parameters, J. Soc. Indust. Appl. Math. Vol. II, no.2, June, 1963 p.p. 431-441

7. Wijnakker, W.H.H. Parameter Estimation on the Human Circulatory System by a Hybrid Computer M.Sc. Thesis, Eindhoven Technological University, The Netherlands (1970)

8. Burton, A.C., Physiology and Biophysics of the Circulation, Year Book Med. Publ. Inc. Chicago (1968)

9. Strano, J.J.; Welkowitz, W.; Fich, S., Measurement and Utilization of In Vivo Blood-Pressure Transfer Fuctions of Dog and Chicken Aortas, IEEE, Tr. on Bio. Med. Eng. Vol. BME 19, no.4, July, 1972 p.p. 261-271

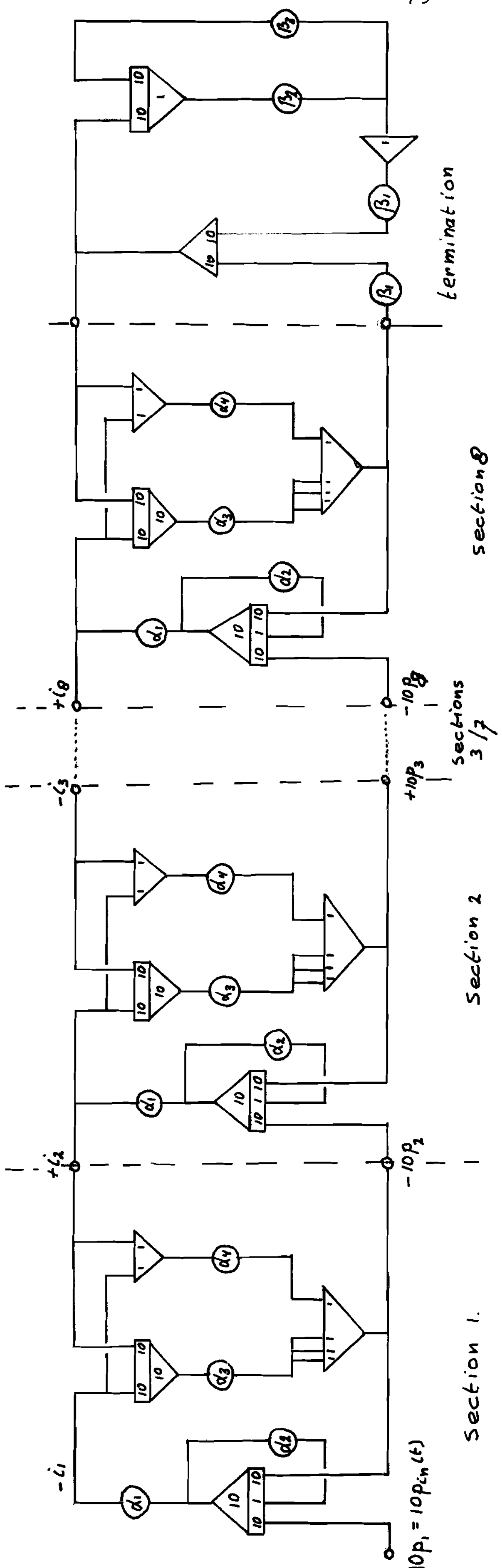
10. Phillipson, G.A., Identification of Distributed Systems, American Elsevier Publ. Comp. Inc., New York 1971

11. Tewari, H.P.; Sunderam, K., Digital Computer Simulation of Pulse Wave Transmission in Arteries, Med. Biol. Eng., Vol.9, Pergamon Press, 1971, Great Brittain p.p. 297-304

12. Cooley, J.W.; Lewis, P.A.W.; Welch, P.D., Application of the Fast Fourier Transform to Computation of Fourier Integrals, Fourier Series, and Convolution Integrals, IEEE, Tr. on Audio. El. Ac., Vol. AU 15, no.2, June, 1967 p.p. 79-85

APPENDIX 1

The analog scheme
of the 8-sections model.
1 section represents
5 cm of the aorta.



$\alpha_1 = \frac{1}{1000L} = .73$

$\alpha_3 = \frac{1}{30C} = .45$

$\alpha_2 = \frac{R_s}{10L} = .061$

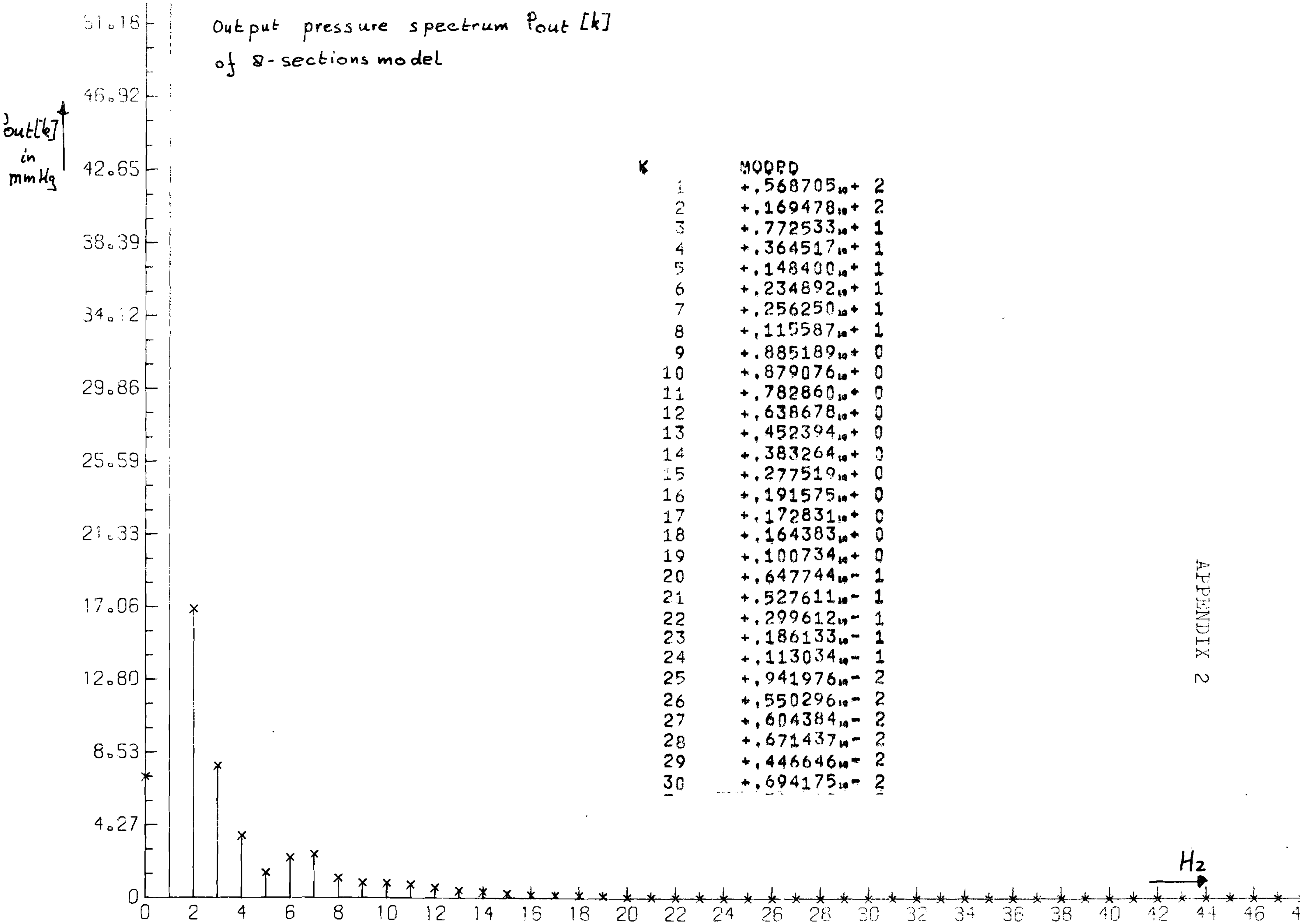
$\alpha_4 = 10R_p = .274$

$\beta_1 = \frac{1}{100 R_{sa}} = .125$

$\beta_2 = \frac{1}{C_{sa}} = 1$

$\beta_3 = \frac{1}{R_{per}} = .633$

APPENDIX 2



K	MODPD
1	+.568705$_{10}$+ 2
2	+.169478$_{10}$+ 2
3	+.772533$_{10}$+ 1
4	+.364517$_{10}$+ 1
5	+.148400$_{10}$+ 1
6	+.234892$_{10}$+ 1
7	+.256250$_{10}$+ 1
8	+.115587$_{10}$+ 1
9	+.885189$_{10}$+ 0
10	+.879076$_{10}$+ 0
11	+.782860$_{10}$+ 0
12	+.638678$_{10}$+ 0
13	+.452394$_{10}$+ 0
14	+.383264$_{10}$+ 0
15	+.277519$_{10}$+ 0
16	+.191575$_{10}$+ 0
17	+.172831$_{10}$+ 0
18	+.164383$_{10}$+ 0
19	+.100734$_{10}$+ 0
20	+.647744$_{10}$- 1
21	+.527611$_{10}$- 1
22	+.299612$_{10}$- 1
23	+.186133$_{10}$- 1
24	+.113034$_{10}$- 1
25	+.941976$_{10}$- 2
26	+.550296$_{10}$- 2
27	+.604384$_{10}$- 2
28	+.671437$_{10}$- 2
29	+.446646$_{10}$- 2
30	+.694175$_{10}$- 2

APPENDIX 3

MODEL TO MODEL

				Scaled parameters GENORMEERDE PARAMETERS RS1*	L1*	RP1*	C1*	unscaled parameters NIET-GENORMEERDE PARAMETERS: RS1	L1	RP1	C1
START:	PI: 1.36637@+03	DELPAR=	1.0	1.0158@+02	1.0000@+03	2.0000@+01	2.6788@+02	1.670@-04	2.740@-04	1.370@-01	1.468@-02
ITER: 1	PI= 2.90550@+02	DELPAR=	1.0	3.4577@+02	1.5361@+03	1.0164@+02	1.1896@+02	5.685@-04	4.214@-04	6.962@-01	6.519@-03
ITER: 2	PI= 1.71761@+01	DELPAR=	1.0	1.9000@+01	1.7967@+03	3.4084@+02	6.6554@+01	3.124@-05	4.923@-04	2.335@+00	3.647@-03
ITER: 3	PI= 5.49597@+00	DELPAR=	1.0	1.9000@+01	1.8000@+03	6.4115@+02	3.7000@+01	3.124@-05	4.932@-04	4.392@+00	2.028@-03
ITER: 4	PI= 1.76340@+00	DELPAR=	1.0	1.9000@+01	1.8000@+03	1.0000@+00	3.7000@+01	3.124@-05	4.932@-04	6.650@-03	2.028@-03
ITER: 5	PI= 1.64223@-01	DELPAR=	1.0	1.9000@+01	1.8000@+03	2.1594@+01	3.9674@+01	3.124@-05	4.932@-04	1.479@-01	2.174@-03
ITER: 6	PI= 1.31429@-03	DELPAR=	1.0	1.9115@+01	1.7999@+03	1.0000@+00	4.0076@+01	3.142@-05	4.932@-04	6.850@-03	2.196@-03
ITER: 7	PI= 5.72409@-05	DELPAR=	1.0	1.9000@+01	1.8000@+03	1.3454@+00	3.9999@+01	3.124@-05	4.932@-04	9.216@-03	2.192@-03
ITER: 8	PI= 4.14250@-08	DELPAR=	1.0	1.9000@+01	1.8000@+03	9.9071@-01	4.0000@+01	3.124@-05	4.932@-04	6.786@-03	2.192@-03
ITER: 9	PI= 3.34529@-13	DELPAR=	1.0	1.9000@+01	1.8000@+03	1.0000@+00	4.0000@+01	3.124@-05	4.932@-04	6.850@-03	2.192@-03
ITER:10	PI= 9.84174@-15	DELPAR=	1.0	1.9000@+01	1.8000@+03	1.0000@+00	4.0000@+01	3.124@-05	4.932@-04	6.850@-03	2.192@-03
ITER:11	PI= 9.1-941@-18	DELPAR=	1.0	1.9000@+01	1.8000@+03	1.0000@+00	4.0000@+01	3.124@-05	4.932@-04	6.850@-03	2.192@-03
ITER:12	PI= 3.86428@-20	DELPAR=	1.0	1.9000@+01	1.8000@+03	1.0000@+00	4.0000@+01	3.124@-05	4.932@-04	6.850@-03	2.192@-03
	PI= 1.38658@-18		1.0	1.9000@+01	1.8000@+03	1.0000@+00	4.0000@+01	3.124@-05	4.932@-04	6.850@-03	2.192@-03
	PI= 1.37247@-18		2.0	1.9000@+01	1.8000@+03	1.0000@+00	4.0000@+01	3.124@-05	4.932@-04	6.850@-03	2.192@-03
ITER:13	PI= 1.49151@-20	DELPAR=	4.0	1.9000@+01	1.8000@+03	1.0000@+00	4.0000@+01	3.124@-05	4.932@-04	6.850@-03	2.192@-03
ITER:14	PI= 6.61744@-23	DELPAR=	4.0	1.9000@+01	1.8000@+03	1.0000@+00	4.0000@+01	3.124@-05	4.932@-04	6.850@-03	2.192@-03
	PI= 7.27919@-23		4.0	1.9000@+01	1.8000@+03	1.0000@+00	4.0000@+01	3.124@-05	4.932@-04	6.850@-03	2.192@-03
ITER:15	PI= 6.61744@-24	DELPAR=	8.0	1.9000@+01	1.8000@+03	1.0000@+00	4.0000@+01	3.124@-05	4.932@-04	6.850@-03	2.192@-03
	PI= 6.61744@-24		8.0	1.9000@+01	1.8000@+03	1.0000@+00	4.0000@+01	3.124@-05	4.932@-04	6.850@-03	2.192@-03
	PI= 6.61744@-24		16.0	1.9000@+01	1.8000@+03	1.0000@+00	4.0000@+01	3.124@-05	4.932@-04	6.850@-03	2.192@-03
PATIENTWAARDE = "Patient set":				1.9000@+01	1.8000@+03	1.0000@+00	4.0000@+01	3.124@-05	4.932@-04	6.850@-03	2.192@-03

Model-to-model adjustment with Gauss-Newton; situation no.1 of table 5.1.
DELPAR = $\Delta\beta_j$, The step for the derivative determination.

				scaled parameters: RS1*	L1*	RP1*	C1*	*unscaled parameters:* RS1	L1	RP1	C1
START:	PI: 2.10999E+03	DELPAR=	1.0	1.9000E+01	4.4500E+02	1.0000E+00	1.5500E+02	3.124E-05	1.219E-04	6.850E-03	8.494E-03
				#							
							#				
ITER: 1	PI= 8.84669E+02	DELPAR=	1.0	5.0000E+02	7.9000E+02	1.3654E+02	8.4303E+01	8.220E-04	2.165E-04	9.353E-01	4.620E-03
				#							
							#				
ITER: 2	PI= 2.03045E+02	DELPAR=	1.0	1.9000E+01	1.3851E+03	8.7733E+02	4.8081E+01	3.124E-05	3.795E-04	6.010E+00	2.635E-03
	PI= 9.80477E+02		1.0	1.9333E+02	6.9902E+02	4.9322E+03	4.3986E+02	3.179E-04	1.915E-04	3.379E+01	2.410E-02
							#				
	PI= 4.18603E+02		1.0	2.9689E+02	1.2941E+03	1.7409E+04	5.1464E+01	4.881E-04	3.546E-04	1.193E+02	2.820E-03
				#							
					#						
						#					
							#				
	PI= 2.28259E+03		1.0	1.9000E+01	4.4500E+02	1.0000E+00	9.3538E+02	3.124E-05	1.219E-04	6.850E-03	5.126E-02
	PI= 5.50514E+02		1.0	1.6485E+02	5.6933E+02	5.3162E+00	6.3699E+02	2.710E-04	1.560E-04	3.642E-02	3.491E-02
	PI= 2.93437E+02		1.0	1.9046E+02	9.0337E+02	1.2605E+01	1.5979E+02	3.131E-04	2.475E-04	8.635E-02	8.756E-03
ITER: 3	PI= 3.67316E+01	DELPAR=	1.0	3.1835E+01	1.1269E+03	1.4136E+02	1.8113E+02	5.234E-05	3.088E-04	9.683E-01	9.926E-03
						#					
ITER: 4	PI= 1.19537E+01	DELPAR=	1.0	1.2042E+02	9.7089E+02	1.0000E+00	3.1869E+02	1.980E-04	2.660E-04	6.850E-03	1.746E-02
ITER: 5	PI= 1.53777E+00	DELPAR=	1.0	1.0013E+02	1.0143E+03	1.6776E+01	2.8437E+02	1.646E-04	2.779E-04	1.149E-01	1.558E-02
ITER: 6	PI= 1.14333E+00	DELPAR=	1.0	9.6182E+01	1.0170E+03	2.3903E+01	2.8026E+02	1.581E-04	2.787E-04	1.637E-01	1.536E-02
ITER: 7	PI= 1.14058E+00	DELPAR=	1.0	9.6095E+01	1.0168E+03	2.4658E+01	2.8031E+02	1.580E-04	2.786E-04	1.689E-01	1.536E-02
ITER: 8	PI= 1.14057E+00	DELPAR=	1.0	9.6103E+01	1.0168E+03	2.4679E+01	2.8033E+02	1.580E-04	2.786E-04	1.691E-01	1.536E-02
ITER: 9	PI= 1.14057E+00	DELPAR=	1.0	9.6103E+01	1.0168E+03	2.4680E+01	2.8033E+02	1.580E-04	2.786E-04	1.691E-01	1.536E-02
ITER:10	PI= 1.14057E+00	DELPAR=	1.0	9.6103E+01	1.0168E+03	2.4680E+01	2.8033E+02	1.580E-04	2.786E-04	1.691E-01	1.536E-02
	PI= 1.14057E+00		1.0	9.6103E+01	1.0168E+03	2.4680E+01	2.8033E+02	1.580E-04	2.786E-04	1.691E-01	1.536E-02
	PI= 1.14057E+00		1.0	9.6103E+01	1.0168E+03	2.4680E+01	2.8033E+02	1.580E-04	2.786E-04	1.691E-01	1.536E-02
	PI= 1.14057E+00		1.0	9.6103E+01	1.0168E+03	2.4680E+01	2.8033E+02	1.580E-04	2.786E-04	1.691E-01	1.536E-02
	PI= 1.14057E+00		1.0	9.6103E+01	1.0168E+03	2.4680E+01	2.8033E+02	1.580E-04	2.786E-04	1.691E-01	1.536E-02
	PI= 1.14057E+00		1.0	9.6103E+01	1.0168E+03	2.4680E+01	2.8033E+02	1.580E-04	2.786E-04	1.691E-01	1.536E-02
PATIENTWAARDE:= "*patient set*" =				1.0158E+02	1.0000E+03	2.0000E+01	2.6788E+02	1.670E-04	2.740E-04	1.370E-01	1.468E-02

Model-to-8-sections model adjustment with Gauss-Newton; situation no.1 of table 5.2. DELPAR = $\Delta\beta_j$, the step for the derivative determination.

APPENDIX 4.2

*8-sections model
output pressure p_{out} (nτ)

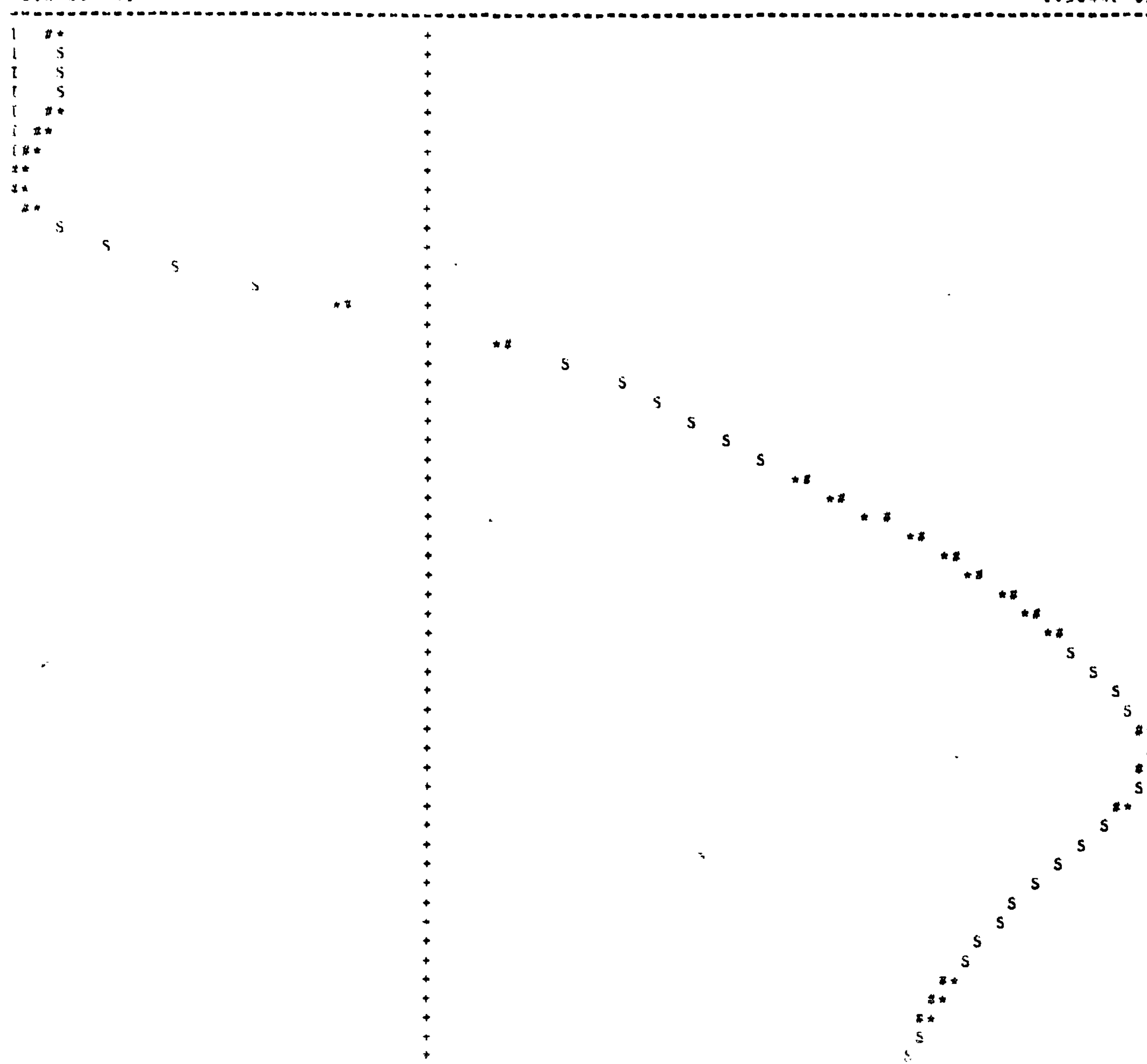
PATIENTUITGANGSDRUK
IN MMHG

#distributed model
output pressure p_{out_m} (nτ)

MODELUITGANGSDRUK
IN MMHG

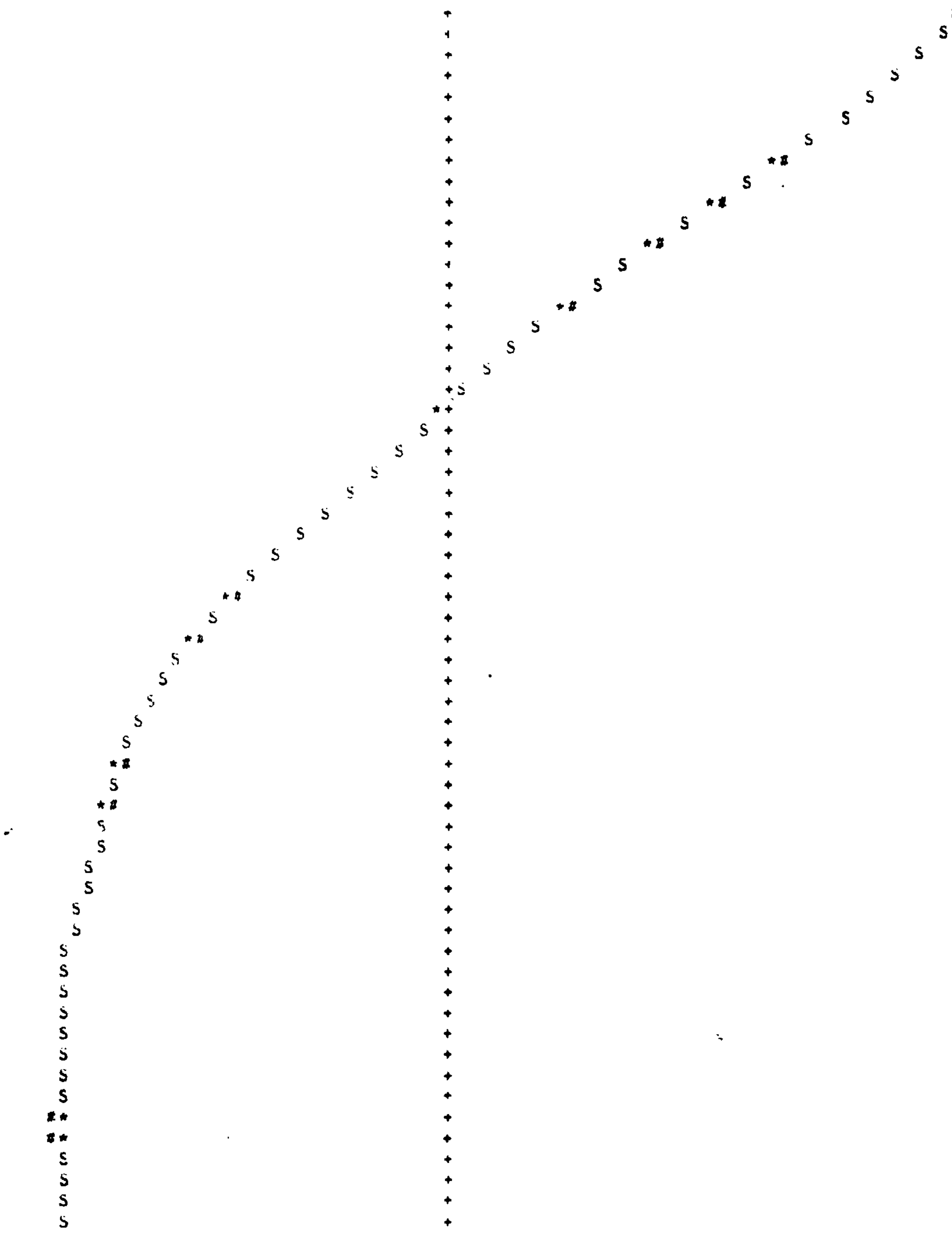
1 POSITIE IS 2.45307@+00MMHG

-8.6865@+01 … 1.5644@+02



PATIENTUITGANGSDRUK IN MMHG	MODELUITGANGSDRUK IN MMHG
-7.9330@+01	-8.0236@+01
-7.6993@+01	-7.9870@+01
-7.6675@+01	-7.9651@+01
-7.8835@+01	-8.0064@+01
-7.9837@+01	-8.1435@+01
-8.1735@+01	-8.3726@+01
-8.4137@+01	-8.6421@+01
-8.6189@+01	-8.8551@+01
-8.6712@+01	-8.8865@+01
-8.4462@+01	-8.6127@+01
-7.8461@+01	-7.9446@+01
-6.8280@+01	-6.8535@+01
-5.4204@+01	-5.3840@+01
-3.7213@+01	-3.6464@+01
-1.8784@+01	-1.7933@+01
-5.4923@-01	1.5831@-01
1.6064@+01	1.6501@+01
3.0142@+01	3.0339@+01
4.1436@+01	4.1567@+01
5.0326@+01	5.0655@+01
5.7627@+01	5.8422@+01
6.4295@+01	6.5745@+01
7.1133@+01	7.3295@+01
7.8603@+01	8.1373@+01
8.6718@+01	8.9889@+01
9.5167@+01	9.8471@+01
1.0346@+02	1.0665@+02
1.1116@+02	1.1403@+02
1.1799@+02	1.2047@+02
1.2398@+02	1.2604@+02
1.2932@+02	1.3100@+02
1.3433@+02	1.3565@+02
1.3922@+02	1.4020@+02
1.4404@+02	1.4465@+02
1.4853@+02	1.4878@+02
1.5246@+02	1.5223@+02
1.5521@+02	1.5459@+02
1.5644@+02	1.5555@+02
1.5596@+02	1.5497@+02
1.5382@+02	1.5293@+02
1.5031@+02	1.4970@+02
1.4587@+02	1.4563@+02
1.4096@+02	1.4110@+02
1.3602@+02	1.3643@+02
1.3131@+02	1.3184@+02
1.2700@+02	1.2744@+02
1.2311@+02	1.2332@+02
1.1963@+02	1.1951@+02
1.1651@+02	1.1607@+02
1.1373@+02	1.1303@+02
1.1125@+02	1.1040@+02
1.0900@+02	1.0810@+02
1.0685@+02	1.0597@+02
1.0461@+02	1.0379@+02

APPENDIX 4.3



1.0203E+02	1.0129E+02
9.8872E+01	9.8227E+01
9.4971E+01	9.4445E+01
9.0258E+01	8.9907E+01
8.4793E+01	8.4694E+01
7.8742E+01	7.8975E+01
7.2330E+01	7.2944E+01
6.5781E+01	6.6773E+01
5.9266E+01	6.0571E+01
5.2882E+01	5.4382E+01
4.6657E+01	4.8208E+01
4.0582E+01	4.2045E+01
3.4644E+01	3.5917E+01
2.8858E+01	2.9892E+01
2.3269E+01	2.4066E+01
1.7932E+01	1.8532E+01
1.2885E+01	1.3340E+01
8.1107E+00	8.4683E+00
3.5338E+00	3.8237E+00
-9.7047E-01	-7.3235E-01
-5.5381E+00	-5.3382E+00
-1.0271E+01	-1.0084E+01
-1.5201E+01	-1.4983E+01
-2.0276E+01	-1.9968E+01
-2.5375E+01	-2.4915E+01
-3.0349E+01	-2.9695E+01
-3.5063E+01	-3.4211E+01
-3.9435E+01	-3.8426E+01
-4.3451E+01	-4.2363E+01
-4.7146E+01	-4.6074E+01
-5.0575E+01	-4.9610E+01
-5.3778E+01	-5.2977E+01
-5.6752E+01	-5.6130E+01
-5.9454E+01	-5.8990E+01
-6.1825E+01	-6.1472E+01
-6.3820E+01	-6.3533E+01
-6.5447E+01	-6.5199E+01
-6.6777E+01	-6.6564E+01
-6.7926E+01	-6.7766E+01
-6.9025E+01	-6.8940E+01
-7.0172E+01	-7.0167E+01
-7.1395E+01	-7.1451E+01
-7.2648E+01	-7.2718E+01
-7.3828E+01	-7.3849E+01
-7.4820E+01	-7.4739E+01
-7.5543E+01	-7.5337E+01
-7.5990E+01	-7.5678E+01
-7.6226E+01	-7.5866E+01
-7.6362E+01	-7.6037E+01
-7.6512E+01	-7.6301E+01
-7.6739E+01	-7.6693E+01
-7.7027E+01	-7.7159E+01
-7.7293E+01	-7.7574E+01
-7.7419E+01	-7.7799E+01
-7.7311E+01	-7.7739E+01
-7.6950E+01	-7.7398E+01
-7.6408E+01	-7.6882E+01
-7.5838E+01	-7.6368E+01
-7.5411E+01	-7.6036E+01
-7.5255E+01	-7.5997E+01

```
-7.5399@+01      S                       +                                                      -7.6246@+01
-7.5754@+01      S                       +                                                      -7.6658@+01
-7.6152@+01      S                       +                                                      -7.7041@+01
-7.6407@+01      S                       +                                                      -7.7213@+01
-7.6402@+01      S                       +                                                      -7.7087@+01
-7.6144@+01      S                       +                                                      -7.6719@+01
-7.5769@+01      S                       +                                                      -7.6296@+01
-7.5501@+01      S                       +                                                      -7.6066@+01
-7.5555@+01      S                       +                                                      -7.6239@+01
-7.6040@+01      S                       +                                                      -7.6889@+01
-7.6905@+01     #*                       +                                                      -7.7903@+01
-7.7933@+01     S                        +                                                      -7.9015@+01
-7.8826@+01     S                        +                                                      -7.9901@+01
-7.9323@+01    #*                        +                                                      -8.0320@+01
               ---------------------------------------------------------------------------

* DUIDT OP PATIENTUITGANGSDRUK EN # DUIDT OP MODELUITGANGSDRUK BY OPTIMALE INSTELLING
S DUIDT OP HET SAMENVALLEN VAN 2 PUNTENVAN DE RESP. GRAFIEKEN


SIGNAAL-SIGNAALVERSCHIL VERHOUDING= 3.80311@+0103
```


APPENDIX 4.4

Model-to-8-sections model adjustment with Marquardt.

			scaled parameters				unscaled parameters			
			RS1*	L1*	RP1*	C1*	RS1	L1	RP1	C1
START:	PI: 2.10999@+03		1.5000@+01	4.4500@+02	1.0000@+00	1.5500@+02	3.124@-05	1.219@-04	6.850@-03	8.494@-03
ITER: 1	PI= 1.02628@+03	LAMBDA=1.0@-02	1.7098@+02	6.5654@+02	5.6614@+01 0G	1.0144@+02	2.811@-04	1.799@-04	3.878@-01	5.559@-03
ITER: 2	PI= 2.25729@+02	LAMBDA=1.0@-03	1.8633@+02	1.0992@+03	1.6406@+02	7.4438@+01	3.063@-04	3.012@-04	1.124@+00	4.079@-03
ITER: 3	PI= 6.75057@+01	LAMBDA=1.0@-04 0G	1.5000@+01	1.1321@+03	5.0914@+02	1.5531@+02	3.124@-05	3.102@-04	3.488@+00	8.511@-03
	PI= 7.74843@+01	1.0@-05	1.2336@+02	8.4780@+02 0G	1.0000@+00	4.2088@+02	2.028@-04	2.323@-04	6.850@-03	2.306@-02
ITER: 4	PI= 3.74116@+01	LAMBDA=1.0@-04	7.0514@+01	8.7772@+02	2.3083@+02	3.8308@+02	1.159@-04	2.405@-04	1.581@+00	2.099@-02
	PI= 1.92294@+02	1.0@-05	1.2196@+02	1.2836@+03	2.2436@+01 0G	5.1884@+01	2.005@-04	3.517@-04	1.537@-01	2.843@-03
ITER: 5	PI= 1.64304@+01	LAMBDA=1.0@-04	1.1022@+02	1.0051@+03	8.3445@+01	2.5722@+02	1.812@-04	2.754@-04	5.716@-01	1.410@-02
ITER: 6	PI= 4.48864@+00	LAMBDA=1.0@-05	9.9085@+01	1.0161@+03	3.8736@+00	2.8447@+02	1.629@-04	2.784@-04	2.653@-02	1.559@-02
ITER: 7	PI= 1.30076@+00	LAMBDA=1.0@-06	9.6554@+01	1.0188@+03	1.9014@+01	2.7911@+02	1.587@-04	2.791@-04	1.302@-01	1.529@-02
ITER: 8	PI= 1.14148@+00	LAMBDA=1.0@-07	9.6023@+01	1.0171@+03	2.4277@+01	2.7999@+02	1.579@-04	2.787@-04	1.663@-01	1.534@-02
ITER: 9	PI= 1.14058@+00	LAMBDA=1.0@-08	9.6097@+01	1.0168@+03	2.4666@+01	2.8031@+02	1.580@-04	2.786@-04	1.690@-01	1.536@-02
ITER:10	PI= 1.14057@+00	LAMBDA=1.0@-09	9.6103@+01	1.0168@+03	2.4680@+01	2.8033@+02	1.580@-04	2.786@-04	1.691@-01	1.536@-02
ITER:11	PI= 1.14057@+00	LAMBDA=1.0@-10	9.6103@+01	1.0168@+03	2.4680@+01	2.8033@+02	1.580@-04	2.786@-04	1.691@-01	1.536@-02
	PI= 1.14057@+00	1.0@-11	9.6103@+01	1.0168@+03	2.4680@+01	2.8033@+02	1.580@-04	2.786@-04	1.691@-01	1.536@-02
	PI= 1.14057@+00	1.0@-10	9.6103@+01	1.0168@+03	2.4680@+01	2.8033@+02	1.580@-04	2.786@-04	1.691@-01	1.536@-02
	PI= 1.14057@+00	1.0@-09	9.6103@+01	1.0168@+03	2.4680@+01	2.8033@+02	1.580@-04	2.786@-04	1.691@-01	1.536@-02
	PI= 1.14057@+00	1.0@-08	9.6103@+01	1.0168@+03	2.4680@+01	2.8033@+02	1.580@-04	2.786@-04	1.691@-01	1.536@-02
	PI= 1.14057@+00	1.0@-07	9.6103@+01	1.0168@+03	2.4680@+01	2.8033@+02	1.580@-04	2.786@-04	1.691@-01	1.536@-02
	PI= 1.14057@+00	1.0@-06	9.6103@+01	1.0168@+03	2.4680@+01	2.8033@+02	1.580@-04	2.786@-04	1.691@-01	1.536@-02
	PI= 1.14057@+00	1.0@-05	9.6103@+01	1.0168@+03	2.4680@+01	2.8033@+02	1.580@-04	2.786@-04	1.691@-01	1.536@-02

APPENDIX 5

APPENDIX 6.1

Model-to-tube adjustment with Marquardt; situation no.1 of table 5.3

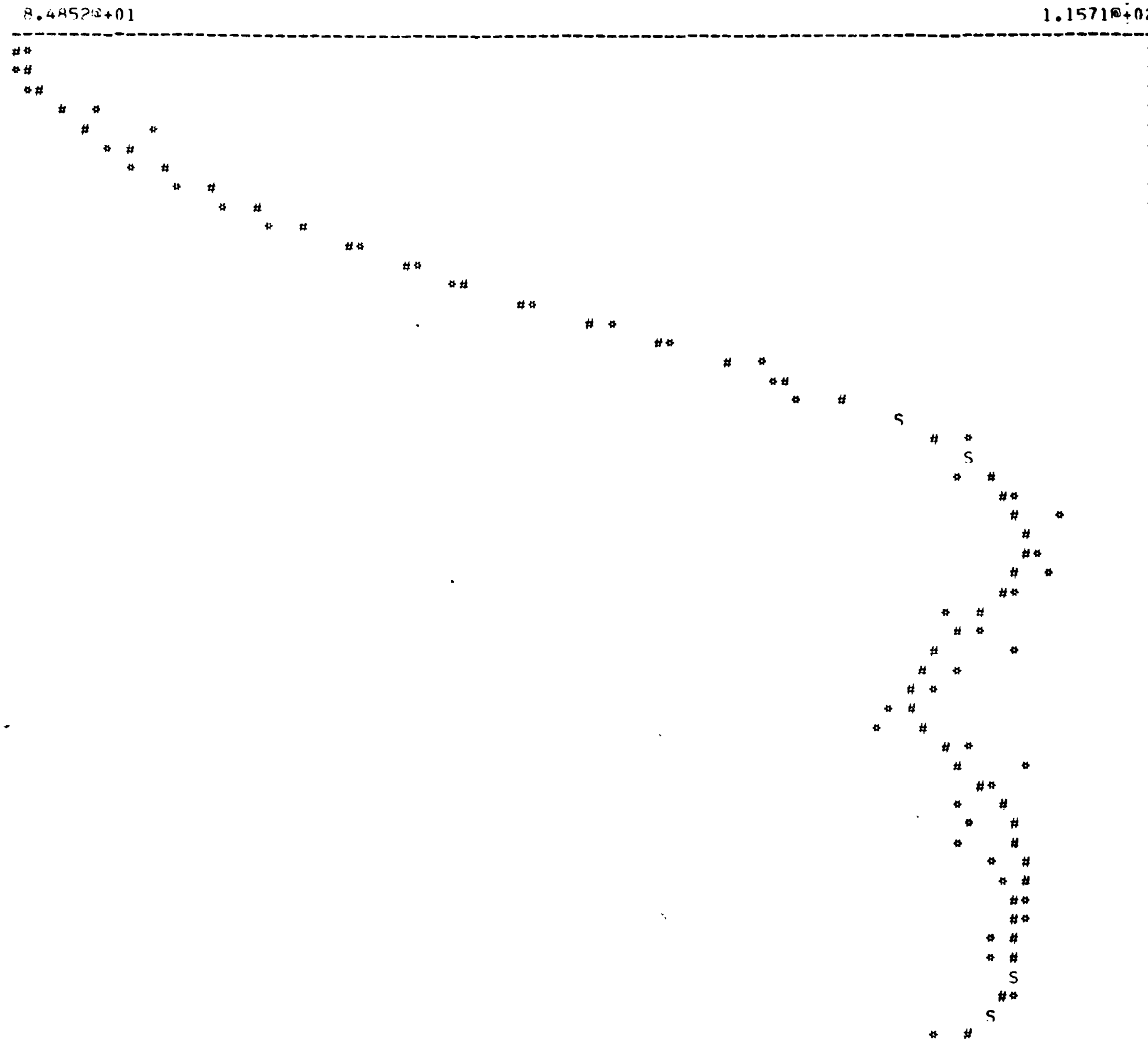
			scaled parameters				unscaled parameters			
			RS1*	L1*	RP1*	C1*	RS1	L1	RP1	C1
START:	PI: 1.13054@+02		1.0158@+02	1.0000@+03	2.0000@+01	2.6788@+02	1.670@-04	2.740@-04	1.370@-01	1.468@-02
ITER: 1	PI= 8.52743@+01	LAMBDA=1.0@-02	1.1323@+02	1.0070@+03	6.6981@+01	2.6701@+02	1.862@-04	2.759@-04	4.588@-01	1.463@-02
ITER: 2	PI= 4.75065@+01	LAMBDA=1.0@-03	1.8960@+02	1.0378@+03	1.9168@+02	2.5066@+02	3.117@-04	2.844@-04	1.313@+00	1.374@-02
ITER: 3	PI= 1.66600@+01	LAMBDA=1.0@-04	5.1726@+02	1.1651@+03	4.0322@+02	2.0910@+02	8.504@-04	3.192@-04	2.762@+00	1.146@-02
ITER: 4	PI= 5.36633@+00	LAMBDA=1.0@-05	8.3950@+02	1.5229@+03	7.6594@+02	1.4842@+02	1.380@-03	4.173@-04	5.247@+00	8.134@-03
ITER: 5	PI= 2.17761@+00	LAMBDA=1.0@-06	8.2267@+02	2.0541@+03	1.2366@+03	8.7176@+01	1.352@-03	5.628@-04	8.471@+00	4.777@-03
ITER: 6	PI= 5.09575@-01	LAMBDA=1.0@-07	7.5139@+02	2.4780@+03	1.4240@+03	2.9497@+01	1.235@-03	6.790@-04	9.755@+00	1.616@-03
ITER: 7	PI= 3.44943@-01	LAMBDA=1.0@-08	7.1601@+02	2.5219@+03	2.6536@+03	2.3947@+01	1.177@-03	6.910@-04	1.818@+01	1.312@-03
ITER: 8	PI= 3.07451@-01	LAMBDA=1.0@-09	7.1538@+02	2.5039@+03	4.4521@+03	2.4927@+01	1.176@-03	6.861@-04	3.050@+01	1.366@-03
ITER: 9	PI= 3.06345@-01	LAMBDA=1.0@-10	7.1360@+02	2.5097@+03	4.7716@+03	2.3178@+01	1.173@-03	6.877@-04	3.269@+01	1.270@-03
ITER:10	PI= 3.06340@-01	LAMBDA=1.0@-11	7.1356@+02	2.5081@+03	4.9045@+03	2.3388@+01	1.173@-03	6.872@-04	3.360@+01	1.282@-03
	PI= 3.06393@-01	1.0@-12	7.1325@+02	2.5117@+03	4.7943@+03	2.2656@+01	1.173@-03	6.882@-04	3.284@+01	1.242@-03
	PI= 3.06393@-01	1.0@-11	7.1325@+02	2.5117@+03	4.7944@+03	2.2656@+01	1.173@-03	6.882@-04	3.284@+01	1.242@-03
	PI= 3.06391@-01	1.0@-10	7.1325@+02	2.5116@+03	4.7961@+03	2.2658@+01	1.173@-03	6.882@-04	3.285@+01	1.242@-03
	PI= 3.06376@-01	1.0@-09	7.1325@+02	2.5115@+03	4.8105@+03	2.2673@+01	1.173@-03	6.881@-04	3.295@+01	1.242@-03
ITER:11	PI= 3.06338@-01	LAMBDA=1.0@-08	7.1323@+02	2.5108@+03	4.8642@+03	2.2730@+01	1.173@-03	6.880@-04	3.332@+01	1.246@-03
	PI= 3.06358@-01	1.0@-09	7.1355@+02	2.5079@+03	4.9255@+03	2.3423@+01	1.173@-03	6.872@-04	3.374@+01	1.284@-03
ITER:12	PI= 3.06334@-01	LAMBDA=1.0@-08	7.1357@+02	2.5083@+03	4.8903@+03	2.3389@+01	1.173@-03	6.873@-04	3.350@+01	1.282@-03
	PI= 3.06361@-01	1.0@-09	7.1327@+02	2.5112@+03	4.8152@+03	2.2729@+01	1.173@-03	6.881@-04	3.298@+01	1.246@-03
ITER:13	PI= 3.06333@-01	LAMBDA=1.0@-08	7.1326@+02	2.5107@+03	4.8579@+03	2.2775@+01	1.173@-03	6.879@-04	3.328@+01	1.248@-03
	PI= 3.06352@-01	1.0@-09	7.1355@+02	2.5080@+03	4.9205@+03	2.3407@+01	1.173@-03	6.872@-04	3.371@+01	1.283@-03
ITER:14	PI= 3.06330@-01	LAMBDA=1.0@-08	7.1356@+02	2.5084@+03	4.8846@+03	2.3373@+01	1.173@-03	6.873@-04	3.346@+01	1.281@-03
	PI= 3.06351@-01	1.0@-09	7.1329@+02	2.5111@+03	4.8195@+03	2.2768@+01	1.173@-03	6.880@-04	3.301@+01	1.248@-03
ITER:15	PI= 3.06328@-01	LAMBDA=1.0@-08	7.1328@+02	2.5106@+03	4.8566@+03	2.2807@+01	1.173@-03	6.879@-04	3.327@+01	1.250@-03

	PI= 3.06345@-01	1.0@-09	7.1354@+02	2.5081@+03	4.9157@+03	2.3385@+01	1.173@-03	6.872@-04	3.367@+01	1.281@-03
ITER:16	PI= 3.06326@-01	LAMBDA=1.0@-08	7.1355@+02	2.5085@+03	4.8818@+03	2.3351@+01	1.173@-03	6.873@-04	3.344@+01	1.280@-03
	PI= 3.06343@-01	1.0@-09	7.1330@+02	2.5109@+03	4.8236@+03	2.2799@+01	1.173@-03	6.880@-04	3.304@+01	1.249@-03
ITER:17	PI= 3.06325@-01	LAMBDA=1.0@-08	7.1329@+02	2.5105@+03	4.8567@+03	2.2834@+01	1.173@-03	6.879@-04	3.327@+01	1.251@-03
	PI= 3.06339@-01	1.0@-09	7.1353@+02	2.5082@+03	4.9113@+03	2.3361@+01	1.173@-03	6.873@-04	3.364@+01	1.280@-03
ITER:18	PI= 3.06323@-01	LAMBDA=1.0@-08	7.1354@+02	2.5086@+03	4.8801@+03	2.3330@+01	1.173@-03	6.873@-04	3.343@+01	1.278@-03
	PI= 3.06337@-01	1.0@-09	7.1332@+02	2.5108@+03	4.8274@+03	2.2827@+01	1.173@-03	6.880@-04	3.307@+01	1.251@-03

*tube output pressure $P_{out}(n\tau)$
PATIENTUITGANGSDRUK
IN MMHG

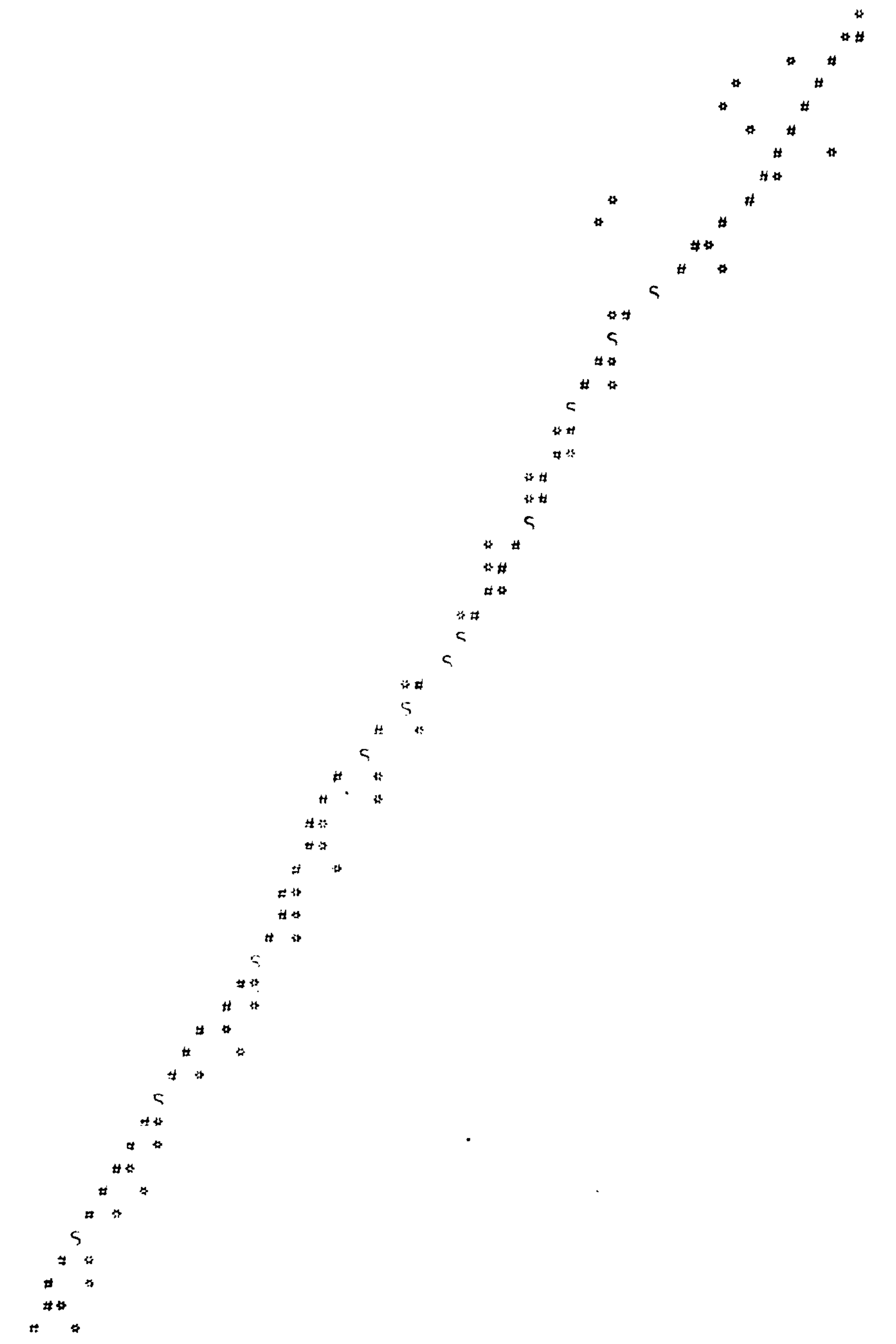
if $P_{out}(n\tau) = P_{out_m}(n\tau)$ then the "S"-symbol has been printed.
1 POSITIE IS 3.08604@-01MMHG

#distributed model output pressure $P_{out_m}(n\tau)$
MODELUITGANGSDRUK
IN MMHG



PATIENTUITGANGSDRUK IN MMHG	MODELUITGANGSDRUK IN MMHG
8.5025@+01	8.4943@+01
8.4852@+01	8.5100@+01
8.5275@+01	8.5473@+01
8.6879@+01	8.6070@+01
8.3442@+01	8.6871@+01
8.7362@+01	8.7836@+01
8.7849@+01	8.8918@+01
8.9080@+01	9.0076@+01
9.0431@+01	9.1288@+01
9.1773@+01	9.2558@+01
9.4261@+01	9.3906@+01
9.5793@+01	9.5362@+01
9.6845@+01	9.6947@+01
9.8928@+01	9.8662@+01
1.0092@+02	1.0048@+02
1.0248@+02	1.0233@+02
1.0517@+02	1.0416@+02
1.0549@+02	1.0586@+02
1.0605@+02	1.0739@+02
1.0897@+02	1.0870@+02
1.1086@+02	1.0978@+02
1.1062@+02	1.1063@+02
1.1038@+02	1.1130@+02
1.1182@+02	1.1179@+02
1.1316@+02	1.1212@+02
1.1571@+02	1.1229@+02
1.1244@+02	1.1227@+02
1.1292@+02	1.1206@+02
1.1211@+02	1.1166@+02
1.1021@+02	1.1113@+02
1.1112@+02	1.1052@+02
1.1204@+02	1.0994@+02
1.1030@+02	1.0948@+02
1.0987@+02	1.0924@+02
1.0854@+02	1.0925@+02
1.0825@+02	1.0952@+02
1.1082@+02	1.0997@+02
1.1228@+02	1.1053@+02
1.1142@+02	1.1109@+02
1.1053@+02	1.1157@+02
1.1069@+02	1.1190@+02
1.1036@+02	1.1209@+02
1.1138@+02	1.1215@+02
1.1176@+02	1.1215@+02
1.1228@+02	1.1211@+02
1.1242@+02	1.1209@+02
1.1145@+02	1.1207@+02
1.1122@+02	1.1202@+02
1.1188@+02	1.1190@+02
1.1187@+02	1.1165@+02
1.1119@+02	1.1125@+02
1.0994@+02	1.1070@+02
1.0882@+02	1.1004@+02

APPENDIX 6.4



1.0846@+02	1.0932@+02
1.0828@+02	1.0864@+02
1.0685@+02	1.0803@+02
1.058[illegible]@+02	1.0755@+02
1.0533@+02	1.0719@+02
1.0617@+02	1.0690@+02
1.080[illegible]@+02	1.0664@+02
1.0673@+02	1.0634@+02
1.030[illegible]@+02	1.0596@+02
1.0252@+02	1.0549@+02
1.0513@+02	1.0494@+02
1.0548@+02	1.0435@+02
1.0372@+02	1.0379@+02
1.0302@+02	1.0326@+02
1.0284@+02	1.0284@+02
1.0289@+02	1.0252@+02
1.0298@+02	1.0229@+02
1.0197@+02	1.0203@+02
1.0180@+02	1.0191@+02
1.0189@+02	1.0172@+02
1.0107@+02	1.0150@+02
1.0110@+02	1.0125@+02
1.0104@+02	1.0098@+02
1.0023@+02	1.0070@+02
1.0011@+02	1.0041@+02
1.0047@+02	1.0012@+02
9.9484@+01	9.9808@+01
9.9439@+01	9.9466@+01
9.9327@+01	9.9079@+01
9.8102@+01	9.8644@+01
9.8209@+01	9.8172@+01
9.8615@+01	9.7685@+01
9.7411@+01	9.7214@+01
9.7503@+01	9.6790@+01
9.7481@+01	9.6434@+01
9.6284@+01	9.6153@+01
9.6290@+01	9.5935@+01
9.6745@+01	9.5757@+01
9.5730@+01	9.5585@+01
9.5716@+01	9.5386@+01
9.5629@+01	9.5136@+01
9.4723@+01	9.4827@+01
9.4786@+01	9.4464@+01
9.4770@+01	9.4065@+01
9.4050@+01	9.3652@+01
9.4383@+01	9.3249@+01
9.3600@+01	9.2870@+01
9.2745@+01	9.2518@+01
9.2777@+01	9.2191@+01
9.2795@+01	9.1880@+01
9.2153@+01	9.1575@+01
9.2363@+01	9.1272@+01
9.1807@+01	9.0975@+01
9.0901@+01	9.0692@+01
9.1133@+01	9.0434@+01
9.1153@+01	9.0207@+01
9.0323@+01	9.0011@+01
9.0620@+01	8.9834@+01
9.0015@+01	8.9655@+01

APPENDIX 6.5

```
0.9112E+01                     *#                                    0.9451E+01
0.9410E+01                     #*                                    0.9199E+01
0.9205E+01                   #*                                      0.8891E+01
0.8514E+01                  S                                        0.8529E+01
0.8913E+01                 #  *                                      0.8129E+01
0.8376E+01              #  *                                         0.7720E+01
0.7470E+01             S                                             0.7327E+01
0.7763E+01            #  *                                           0.6972E+01
0.7388E+01          #  *                                             0.6661E+01
0.6976E+01         #  *                                              0.6388E+01
0.7498E+01        #     *                                            0.6135E+01
0.6741E+01      #    *                                               0.5883E+01
0.5906E+01     #*                                                    0.5622E+01
0.6403E+01     #    *                                                0.5361E+01
0.6057E+01   #    *                                                  0.5127E+01
0.5898E+01 #    *                                                    0.4970E+01
           ---------------------------------------------------------
```

* DUIDT OP PATIENTUITGANGSDRUK EN # DUIDT OP MODELUITGANGSDRUK BIJ OPTIMALE INSTELLING
S DUIDT OP HET SAMENVALLEN VAN 2 PUNTEN VAN DE RESP. GRAFIEKEN

Model-to-tube adjustment with Marquardt; situation no.2 of table 5.3.

			scaled parameters				unscaled parameters			
			RS1*	L1*	RP1*	C1*	RS1	L1	RP1	C1
START:	PI= 8.0[illegible]799E+01		1.0155E+02	1.0000E+03	2.0000E+01	2.6788E+02	1.670E-04	2.740E-04	1.370E-01	1.468E-02
ITER: 1	PI= 6.44[illegible]E+01	LAMBDA=1.0E-02	1.1122E+02	1.0047E+03	5.5112E+01	2.6354E+02	1.828E-04	2.753E-04	3.775E-01	1.444E-02
ITER: 2	PI= 3.88[illegible]E+01	LAMBDA=1.0E-03	1.7808E+02	1.0307E+03	1.6391E+02	2.4514E+02	2.928E-04	2.824E-04	1.123E+00	1.343E-02
ITER: 3	PI= 1.4748[illegible]	LAMBDA=1.0E-04	4.8117E+02	1.1531E+03	3.5786E+02	1.9775E+02	7.911E-04	3.159E-04	2.451E+00	1.084E-02
ITER: 4	PI= 4.7[illegible]E+00	LAMBDA=1.0E-05	7.9844E+02	1.5411E+03	7.0476E+02	1.2404E+02	1.313E-03	4.223E-04	4.828E+00	6.797E-03
ITER: 5	PI= [illegible].7656[illegible]	LAMBDA=1.0E-06	7.7086E+02	2.1553E+03	1.2225E+03	5.8234E+01	1.267E-03	5.905E-04	8.374E+00	3.191E-03
ITER: 6	PI= [illegible].1[illegible]110E-01	LAMBDA=1.0E-07	7.2538E+02	2.3555E+03	1.5949E+03	6.3810E+01	1.193E-03	6.454E-04	1.093E+01	3.497E-03
ITER: 7	PI= 5.4122[illegible]-01	LAMBDA=1.0E-08	7.1004E+02	2.4026E+03	2.1606E+03	5.3377E+01	1.167E-03	6.583E-04	1.480E+01	2.925E-03
ITER: 8	PI= 3.80274E-01	LAMBDA=1.0E-09	6.934[illegible]E+02	2.4741E+03	2.9473E+03	3.0759E+01	1.140E-03	6.779E-04	2.019E+01	1.686E-03
ITER: 9	PI= 3.43210E-01	LAMBDA=1.0E-10	6.8887E+02	2.4706E+03	4.3410E+03	2.8614E+01	1.132E-03	6.769E-04	2.974E+01	1.568E-03
ITER:10	PI= 3.36632E-01	LAMBDA=1.0E-11	6.8278E+02	2.5082E+03	4.8923E+03	1.8077E+01	1.122E-03	6.873E-04	3.351E+01	9.906E-04
	PI= 3.37816E-01	1.0E-12	6.8504E+02	2.4744E+03	7.5438E+03	2.4892E+01	1.126E-03	6.780E-04	5.167E+01	1.364E-03
	PI= 3.37798E-01	1.0E-11	6.8505E+02	2.4744E+03	7.5385E+03	2.4890E+01	1.126E-03	6.780E-04	5.164E+01	1.364E-03
	PI= 3.37615E-01	1.0E-10	6.8506E+02	2.4748E+03	7.4866E+03	2.4870E+01	1.126E-03	6.781E-04	5.128E+01	1.363E-03
ITER:11	PI= 3.36134E-01	LAMBDA=1.0E-09	6.8515E+02	2.4774E+03	7.0618E+03	2.4712E+01	1.126E-03	6.788E-04	4.837E+01	1.354E-03
	PI= 4.07424E-01	1.0E-10	6.7484E+02	2.6178E+03	3.9365E+0306	1.0000E-02	1.109E-03	7.173E-04	2.696E+01	5.480E-07
	PI= 4.01283E-01	1.0E-09	6.744[illegible]E+02	2.6067E+03	4.9494E+0306	1.0000E-02	1.109E-03	7.142E-04	3.390E+01	5.480E-07
	PI= 3.93282E-01	1.0E-08	6.7392E+02	2.5886E+03	6.5645E+0306	1.0000E-02	1.108E-03	7.093E-04	4.497E+01	5.480E-07
	PI= 3.90464E-01	1.0E-07	6.7402E+02	2.5801E+03	7.0053E+0306	1.0000E-02	1.108E-03	7.069E-04	4.799E+01	5.480E-07
	PI= 3.86141E-01	1.0E-06	6.7595E+02	2.5530E+03	7.0568E+03	9.7901E-01	1.111E-03	6.995E-04	4.834E+01	5.365E-05
	PI= 3.40142E-01	1.0E-05	6.8062E+02	2.4974E+03	7.0615E+03	1.2106E+01	1.119E-03	6.843E-04	4.837E+01	6.634E-04
ITER:12	PI= 3.30935E-01	LAMBDA=1.0E-04	6.8414E+02	2.4789E+03	7.0618E+03	1.8573E+01	1.125E-03	6.792E-04	4.837E+01	1.018E-03
ITER:13	PI= 3.29780E-01	LAMBDA=1.0E-05	6.8167E+02	2.4871E+03	7.0617E+03	1.8548E+01	1.121E-03	6.815E-04	4.837E+01	1.016E-03
ITER:14	PI= 3.29462E-01	LAMBDA=1.0E-06	6.8009E+02	2.5026E+03	7.0602E+03	1.6554E+01	1.118E-03	6.857E-04	4.836E+01	9.072E-04

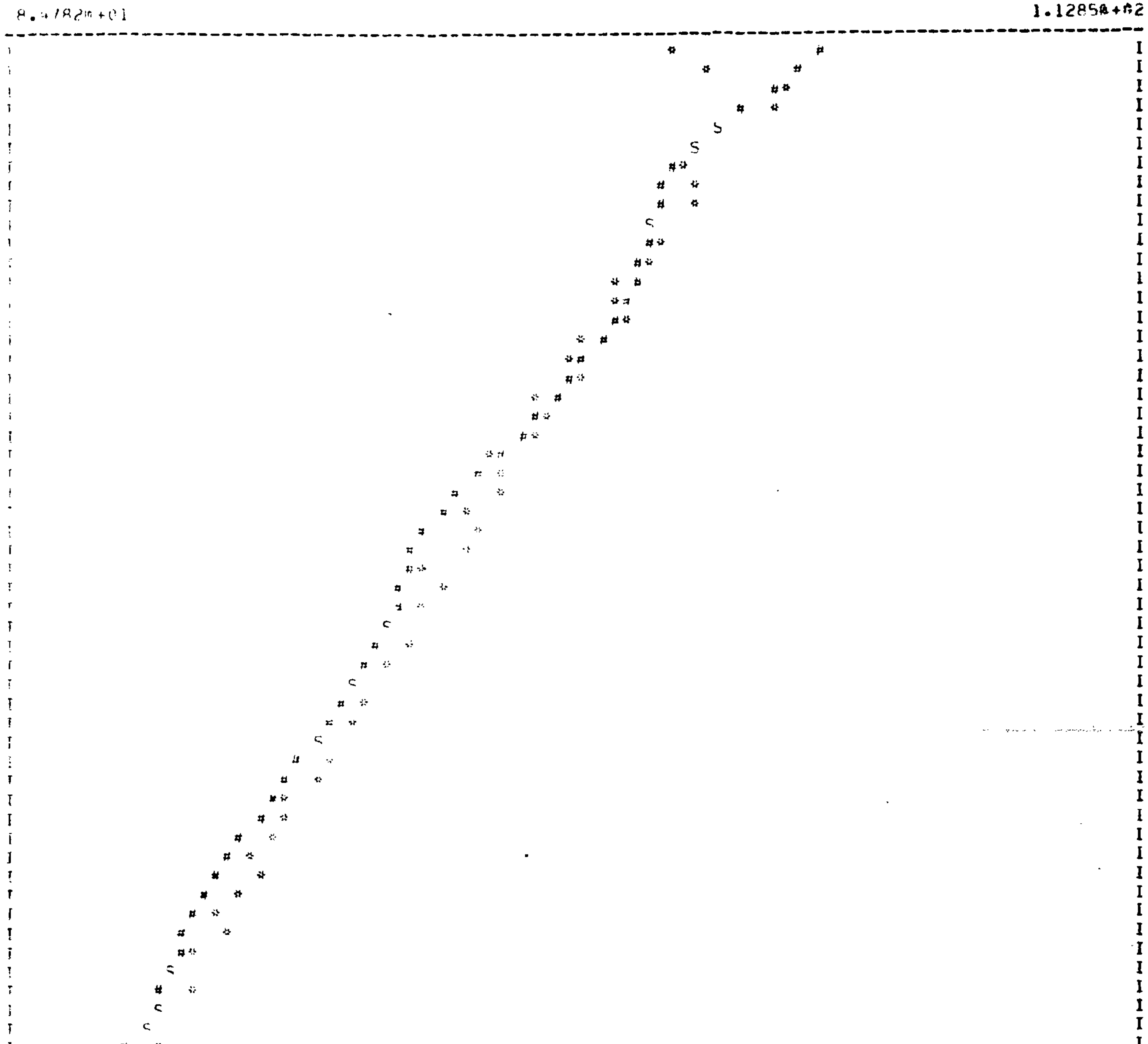
	FI= 3.29814E-01	1.0E-07	6.8152E+02	2.4949E+03	7.0626E+03	1.9534E+01	1.120E-03	6.836E-04	4.836E+01	1.070E-03
	FI= 3.29729E-01	1.0E-06	6.8141E+02	2.4961E+03	7.0604E+03	1.9399E+01	1.120E-03	6.839E-04	4.836E+01	1.063E-03
	FI= 3.29489E-01	1.0E-05	6.8096E+02	2.5001E+03	7.0602E+03	1.8886E+01	1.119E-03	6.850E-04	4.836E+01	1.035E-03
ITER:15	FI= 3.29233E-01	LAMBDA=1.0E-04	6.8028E+02	2.5024E+03	7.0602E+03	1.8137E+01	1.118E-03	6.857E-04	4.836E+01	9.939E-04
	FI= 3.29244E-01	1.0E-05	6.8021E+02	2.5030E+03	7.0601E+03	1.7025E+01	1.118E-03	6.858E-04	4.836E+01	9.330E-04
ITER:16	FI= 3.29172E-01	LAMBDA=1.0E-04	6.8031E+02	2.5024E+03	7.0602E+03	1.7357E+01	1.118E-03	6.857E-04	4.836E+01	9.512E-04
	FI= 3.29178E-01	1.0E-05	6.8060E+02	2.5014E+03	7.0601E+03	1.7988E+01	1.119E-03	6.854E-04	4.836E+01	9.857E-04

APPENDIX 7.2

"tube output pressure $P_{out}(n\tau)$
PATIENTUITGANGSDRUK
IN MMHG

if $P_{out}(n\tau) = P_{out_m}(n\tau)$, the "s"-symbol has been printed.
1 POSITIE IS 2.80709@-01MMHG

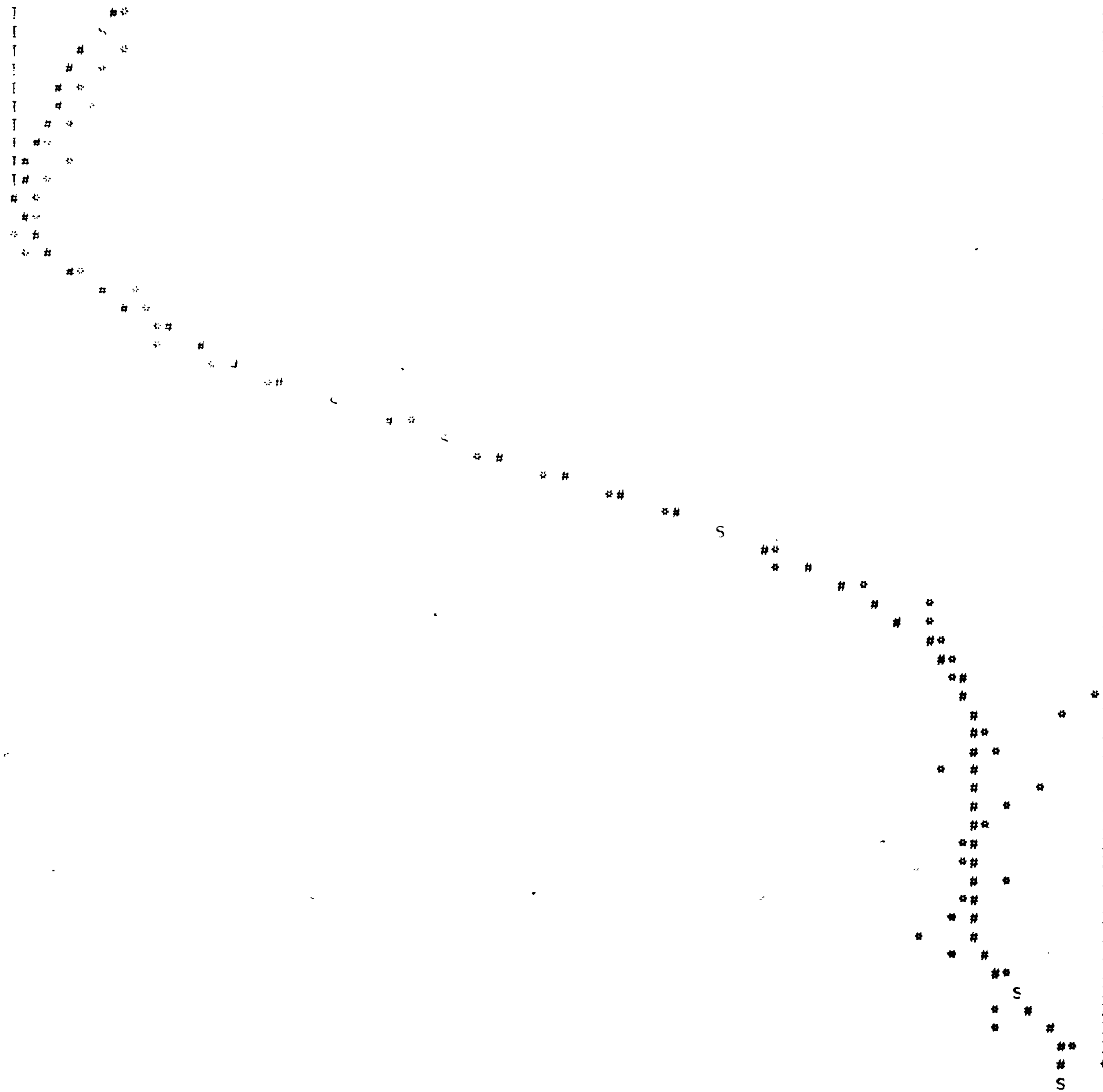
#distributed model output pressure $P_{out_m}(n\tau)$
MODELUITGANGSDRUK
IN MMHG



1.0135@+02	1.0478@+02
1.020[illegible]@+02	1.0428@+02
1.0400@+02	1.0367@+02
1.0379@+02	1.0301@+02
1.0250@+02	1.0236@+02
1.0190@+02	1.0179@+02
1.0149@+02	1.0134@+02
1.0180@+02	1.0103@+02
1.0171@+02	1.0083@+02
1.0070@+02	1.0070@+02
1.0086@+02	1.0059@+02
1.0079@+02	1.0045@+02
9.9674@+01	1.0026@+02
9.9763@+01	1.0002@+02
9.997[illegible]@+01	9.9722@+01
9.8966@+01	9.9399@+01
9.8772@+01	9.9063@+01
9.8837@+01	9.8714@+01
9.7883@+01	9.8346@+01
9.[illegible]135@+01	9.7942@+01
9.741[illegible]@+01	9.7492@+01
9.6745@+01	9.6997@+01
9.7057@+01	9.6476@+01
9.7098@+01	9.5961@+01
9.6028@+01	9.5492@+01
9.6412@+01	9.5100@+01
9.6017@+01	9.4801@+01
9.4893@+01	9.4588@+01
9.5426@+01	9.4436@+01
9.5064@+01	9.4304@+01
9.4006@+01	9.4153@+01
9.4679@+01	9.3956@+01
9.4215@+01	9.3700@+01
9.3325@+01	9.3393@+01
9.3469@+01	9.3053@+01
9.3278@+01	9.2698@+01
9.2426@+01	9.2343@+01
9.2691@+01	9.1992@+01
9.2326@+01	9.1639@+01
9.1473@+01	9.1276@+01
9.1610@+01	9.0898@+01
9.1259@+01	9.0512@+01
9.0683@+01	9.0134@+01
9.0491@+01	8.9788@+01
9.0426@+01	8.9494@+01
8.9916@+01	8.9263@+01
9.0077@+01	8.9085@+01
8.9308@+01	8.8939@+01
8.8774@+01	8.8791@+01
8.9235@+01	8.8606@+01
8.8607@+01	8.8363@+01
8.8111@+01	8.8057@+01
8.8561@+01	8.7703@+01

APPENDIX 7.3

APPENDIX 7.4



8.7546@+01	8.7327@+01
8.7078@+01	8.6958@+01
8.7582@+01	8.6619@+01
8.709[illegible]@+01	8.6317@+01
8.6512@+01	8.6046@+01
8.6964@+01	8.5793@+01
8.6100@+01	8.5546@+01
8.5648@+01	8.5307@+01
8.6335@+01	8.5095@+01
8.5624@+01	8.4946@+01
8.5362@+01	8.4900@+01
8.5[illegible]09@+01	8.4997@+01
8.4782@+01	8.5260@+01
8.4928@+01	8.5689@+01
8.6448@+01	8.6268@+01
8.7804@+01	8.6967@+01
8.8183@+01	8.7757@+01
8.8461@+01	8.8620@+01
8.8496@+01	8.9558@+01
8.9852@+01	9.0585@+01
9.129[illegible]@+01	9.1723@+01
9.2920@+01	9.2986@+01
9.3117@+01	9.4374@+01
9.3768@+01	9.5858@+01
9.6593@+01	9.7394@+01
9.8282@+01	9.8923@+01
1.0023@+02	1.0039@+02
1.0154@+02	1.0175@+02
1.0300@+02	1.0298@+02
1.043[illegible]@+02	1.0409@+02
1.0426@+02	1.0509@+02
1.0669@+02	1.0600@+02
1.0824@+02	1.0683@+02
1.0820@+02	1.0757@+02
1.0865@+02	1.0821@+02
1.0883@+02	1.0871@+02
1.0876@+02	1.0908@+02
1.1243@+02	1.0930@+02
1.1158@+02	1.0940@+02
1.0983@+02	1.0941@+02
1.1003@+02	1.0937@+02
1.0854@+02	1.0934@+02
1.1111@+02	1.0931@+02
1.1043@+02	1.0932@+02
1.0961@+02	1.0933@+02
1.0928@+02	1.0935@+02
1.0916@+02	1.0934@+02
1.1032@+02	1.0932@+02
1.0913@+02	1.0932@+02
1.0892@+02	1.0935@+02
1.0814@+02	1.0948@+02
1.0884@+02	1.0971@+02
1.1040@+02	1.1005@+02
1.1056@+02	1.1048@+02
1.1004@+02	1.1094@+02
1.0997@+02	1.1137@+02
1.1187@+02	1.1169@+02
1.1285@+02	1.1185@+02
1.1180@+02	1.1183@+02

* DUIDT OP PATIENTUITGANGSDRUK EN # DUIDT OP MODELUITGANGSDRUK BIJ OPTIMALE INSTELLING
S DUIDT OP HET SAMENVALLEN VAN 2 PUNTEN VAN DE RESP. GRAFIEKEN

APPENDIX 7.5

Model-to-tube adjustment with Marquardt; situation no.3 of table 5.3.

			scaled parameters				*unscaled parameters*			
			RS1*	L1*	RP1*	C1*	RS1	L1	RP1	C1
START:	PI: 6.40528D+01		1.015[illegible]+02	1.0000D+03	2.0000D+01	2.6788D+02	1.670D-04	2.740D-04	1.370D-01	1.468D-02
ITER: 1	PI= 5.57383D+01	LAMBDA=1.0D-02	1.1028D+02	1.0023D+03	4.1397D+01	2.5476D+02	1.813D-04	2.746D-04	2.836D-01	1.396D-02
ITER: 2	PI= 3.64442D+01	LAMBDA=1.0D-03	1.7590D+02	1.0282D+03	1.2889D+02	2.1772D+02	2.892D-04	2.817D-04	8.829D-01	1.193D-02
ITER: 3	PI= 1.44006D+01	LAMBDA=1.0D-04	4.7389D+02	1.1730D+03	3.0291D+02	1.5447D+02	7.791D-04	3.214D-04	2.075D+00	8.465D-03
ITER: 4	PI= 4.42556D+00	LAMBDA=1.0D-05	7.8564D+02	1.6589D+03	5.8771D+02	7.2427D+01	1.292D-03	4.545D-04	4.026D+00	3.969D-03
ITER: 5	PI= 1.24421D+00	LAMBDA=1.0D-06	7.6932D+02	2.2761D+03	5.0320D+02	5.5677D+01	1.265D-03	6.236D-04	3.447D+00	3.051D-03
ITER: 6	PI= 9.26101D-01	LAMBDA=1.0D-07	7.1768D+02	2.5020D+03	5.7138D+02	3.9847D+01	1.180D-03	6.855D-04	3.914D+00	2.184D-03
ITER: 7	PI= 8.63075D-01	LAMBDA=1.0D-08	7.1805D+02	2.5255D+03	7.0040D+02	4.5670D+01	1.180D-03	6.920D-04	4.798D+00	2.503D-03
ITER: 8	PI= 7.95498D-01	LAMBDA=1.0D-09	7.1423D+02	2.5198D+03	8.9244D+02	4.1438D+01	1.174D-03	6.904D-04	6.113D+00	2.271D-03
ITER: 9	PI= 6.91586D-01	LAMBDA=1.0D-10	7.1842D+02	2.5044D+03	1.0802D+03	4.7068D+01	1.181D-03	6.862D-04	7.400D+00	2.579D-03
ITER:10	PI= 5.52839D-01	LAMBDA=1.0D-11	7.1553D+02	2.4874D+03	1.4410D+03	4.4642D+01	1.176D-03	6.816D-04	9.871D+00	2.446D-03
ITER:11	PI= 4.64176D-01	LAMBDA=1.0D-12	7.1404D+02	2.4667D+03	1.9009D+03	4.4471D+01	1.174D-03	6.759D-04	1.302D+01	2.437D-03
ITER:12	PI= 3.97033D-01	LAMBDA=1.0D-13	7.0714D+02	2.4714D+03	2.4966D+03	3.6635D+01	1.163D-03	6.772D-04	1.710D+01	2.008D-03
ITER:13	PI= 3.64752D-01	LAMBDA=1.0D-14	7.0280D+02	2.4726D+03	3.3129D+03	3.2226D+01	1.155D-03	6.775D-04	2.269D+01	1.766D-03
ITER:14	PI= 3.52350D-01	LAMBDA=1.0D-15	6.9820D+02	2.4901D+03	3.9432D+03	2.5591D+01	1.148D-03	6.823D-04	2.701D+01	1.402D-03
ITER:15	PI= 3.52019D-01	LAMBDA=1.0D-16	6.9802D+02	2.4797D+03	4.8332D+03	2.7075D+01	1.148D-03	6.794D-04	3.311D+01	1.484D-03
	PI= 3.80406D-01	1.0D-17	6.9335D+02	2.5273D+03	4.0244D+03	1.5249D+01	1.140D-03	6.925D-04	2.757D+01	8.357D-04
	PI= 3.80406D-01	1.0D-16	6.9335D+02	2.5273D+03	4.0244D+03	1.5249D+01	1.140D-03	6.925D-04	2.757D+01	8.357D-04
	PI= 3.80408D-01	1.0D-15	6.9335D+02	2.5273D+03	4.0244D+03	1.5249D+01	1.140D-03	6.925D-04	2.757D+01	8.357D-04
	PI= 3.80408D-01	1.0D-14	6.9335D+02	2.5273D+03	4.0244D+03	1.5249D+01	1.140D-03	6.925D-04	2.757D+01	8.357D-04
	PI= 3.80408D-01	1.0D-13	6.9335D+02	2.5273D+03	4.0244D+03	1.5249D+01	1.140D-03	6.925D-04	2.757D+01	8.357D-04
	PI= 3.80403D-01	1.0D-12	6.9335D+02	2.5273D+03	4.0245D+03	1.5250D+01	1.140D-03	6.925D-04	2.757D+01	8.357D-04
	PI= 3.80358D-01	1.0D-11	6.9334D+02	2.5272D+03	4.0259D+03	1.5252D+01	1.140D-03	6.925D-04	2.758D+01	8.358D-04
	PI= 3.79918D-01	1.0D-10	6.9334D+02	2.5271D+03	4.0391D+03	1.5273D+01	1.140D-03	6.924D-04	2.767D+01	8.370D-04

	PI= 3.76379E-01	1.0E-09	6.9329E+02	2.5257E+03	4.1511E+03	1.5454E+01	1.140E-03	6.920E-04	2.844E+01	8.469E-04
	PI= 3.66185E-01	1.0E-08	6.9312E+02	2.5207E+03	4.5504E+03	1.6106E+01	1.139E-03	6.907E-04	3.117E+01	8.826E-04
	PI= 3.61254E-01	1.0E-07	6.9308E+02	2.5169E+03	4.7923E+03	1.6610E+01	1.139E-03	6.896E-04	3.283E+01	9.102E-04
	PI= 3.57544E-01	1.0E-06	6.9360E+02	2.5100E+03	4.8292E+03	1.7620E+01	1.140E-03	6.877E-04	3.308E+01	9.656E-04
ITER:16	PI= 3.51821E-01	LAMBDA=1.0E-05	6.9550E+02	2.4903E+03	4.8329E+03	2.0644E+01	1.143E-03	6.823E-04	3.311E+01	1.131E-03
ITER:17	PI= 3.51238E-01	LAMBDA=1.0E-06	6.9760E+02	2.4840E+03	4.8372E+03	2.6340E+01	1.147E-03	6.806E-04	3.313E+01	1.443E-03
	PI= 3.56370E-01	1.0E-07	6.9359E+02	2.5118E+03	4.8038E+03	1.8069E+01	1.140E-03	6.882E-04	3.291E+01	9.902E-04

APPENDIX 8.2

*tube output pressure $P_{out}(nT)$

PATIENTUITGANGSDRUK
IN MMHG

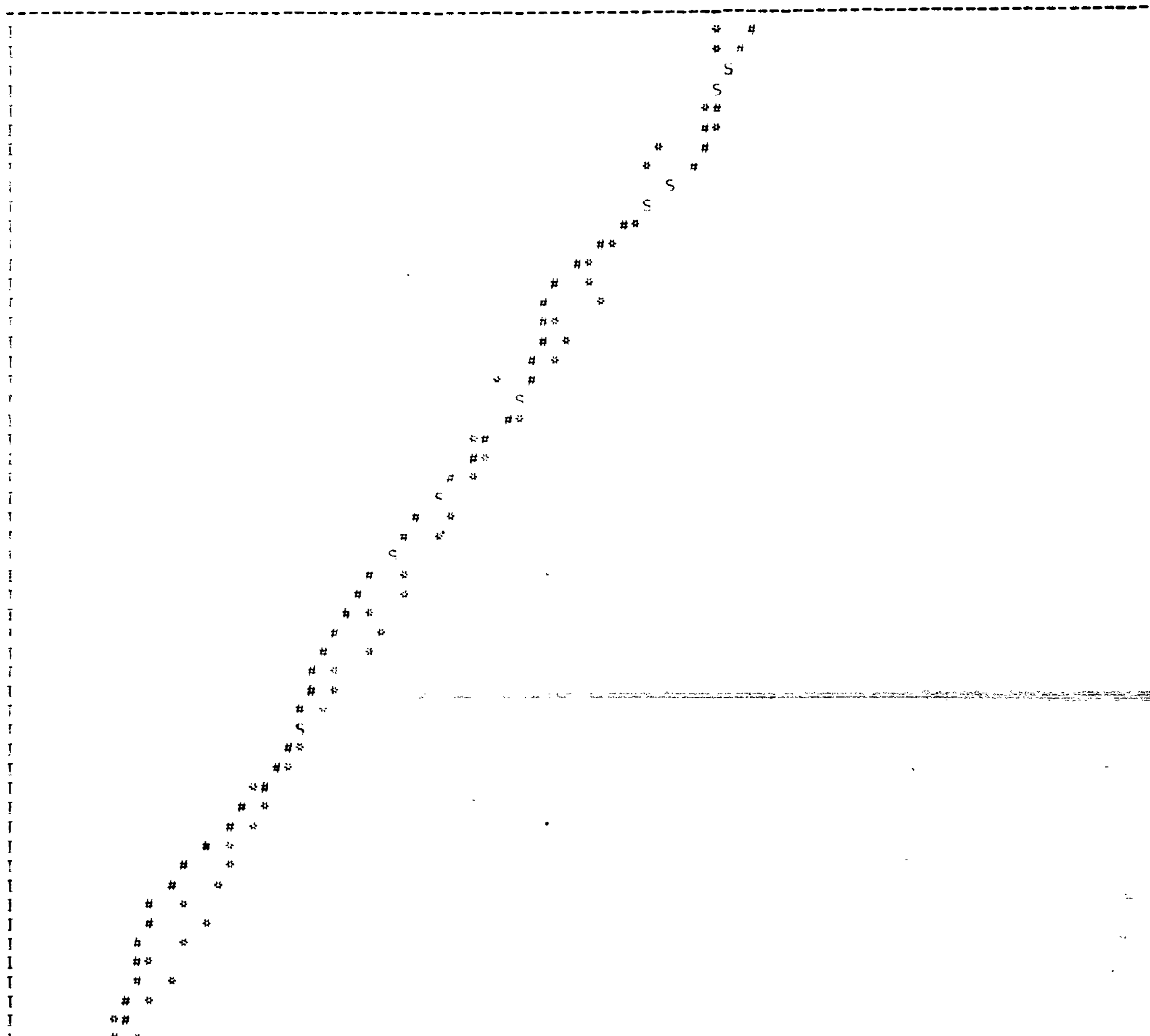
"S" symbol if $P_{out}(nT) = P_{out_m}(nT)$

1 POSITIE IS 2.83976E-01MMHG

#distributed model output pressure $P_{out_m}(nT)$

MODELUITGANGSDRUK
IN MMHG

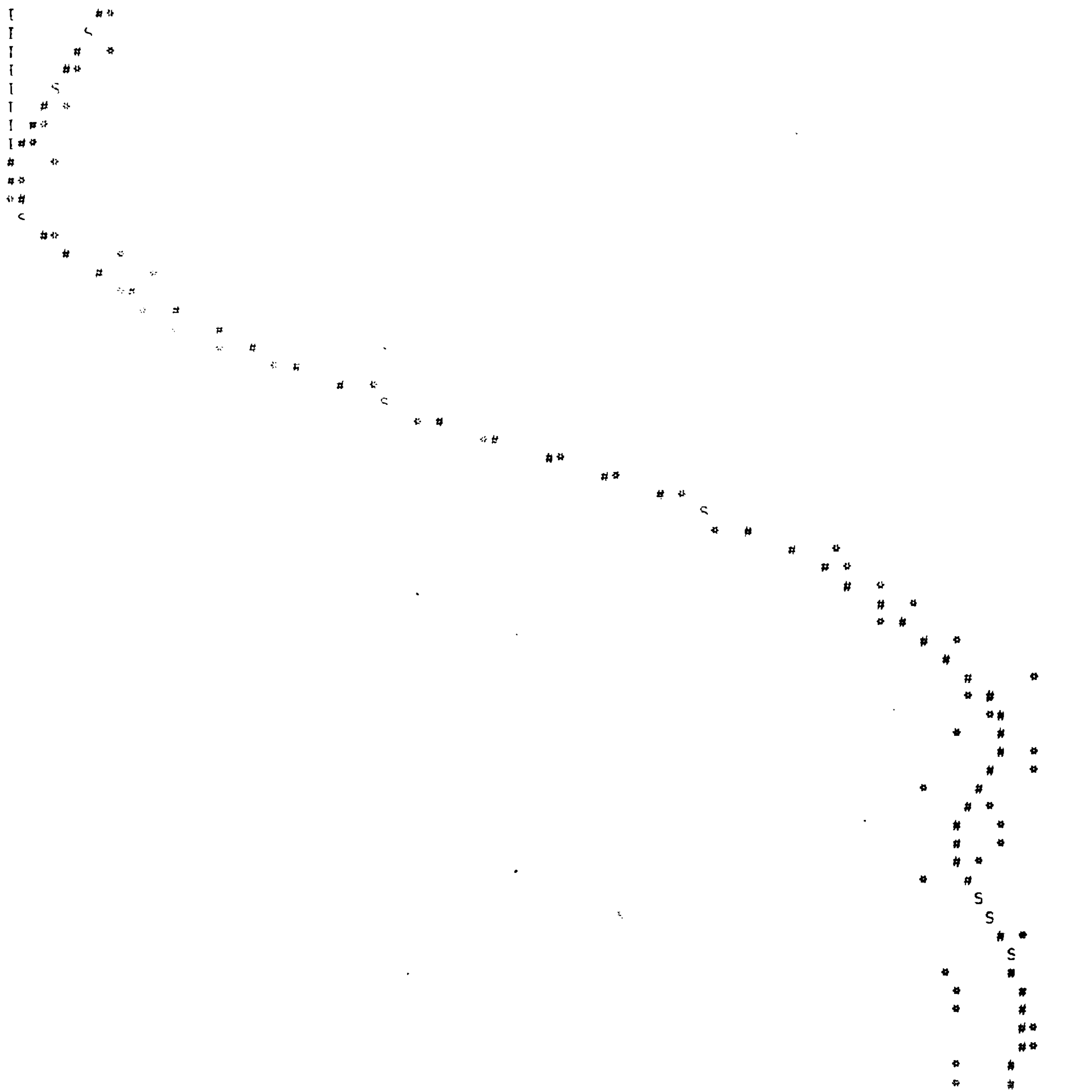
8.5864E+01 1.1426E+02



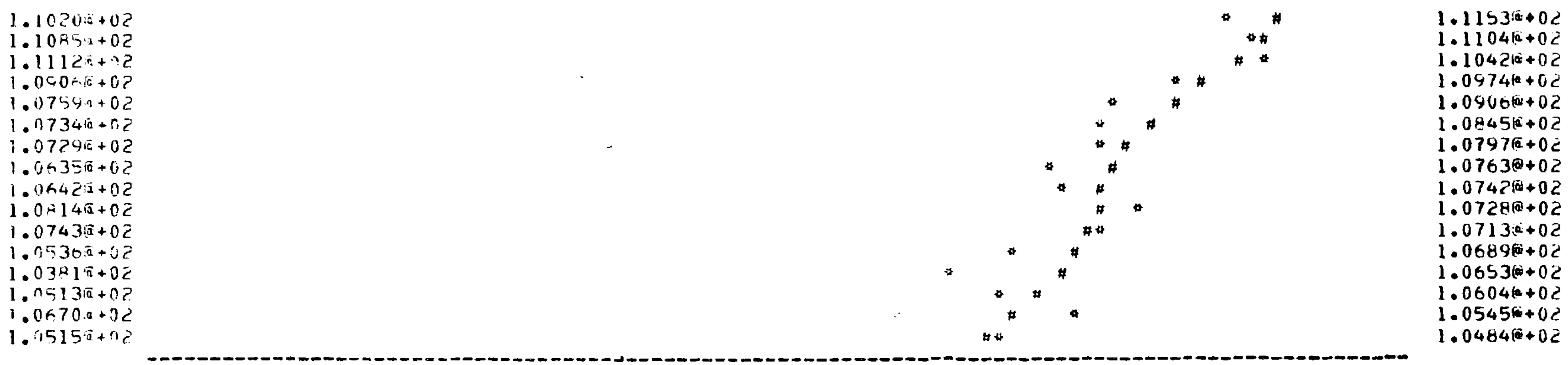
PATIENTUITGANGSDRUK IN MMHG	MODELUITGANGSDRUK IN MMHG
1.0331E+02	1.0427E+02
1.0326E+02	1.0382E+02
1.0373E+02	1.0352E+02
1.0327E+02	1.0334E+02
1.0310E+02	1.0324E+02
1.0337E+02	1.0315E+02
1.0204E+02	1.0297E+02
1.0173E+02	1.0267E+02
1.0227E+02	1.0221E+02
1.0175E+02	1.0163E+02
1.0132E+02	1.0099E+02
1.0092E+02	1.0036E+02
1.0011E+02	9.9816E+01
1.0023E+02	9.9407E+01
1.0047E+02	9.9143E+01
9.9347E+01	9.9000E+01
9.9521E+01	9.8922E+01
9.9247E+01	9.8844E+01
9.7996E+01	9.8707E+01
9.8355E+01	9.8476E+01
9.8363E+01	9.8142E+01
9.7350E+01	9.7724E+01
9.7595E+01	9.7258E+01
9.7424E+01	9.6784E+01
9.6507E+01	9.6334E+01
9.6800E+01	9.5923E+01
9.6439E+01	9.5551E+01
9.5307E+01	9.5206E+01
9.5736E+01	9.4875E+01
9.5682E+01	9.4548E+01
9.4815E+01	9.4226E+01
9.4929E+01	9.3922E+01
9.4689E+01	9.3652E+01
9.3843E+01	9.3427E+01
9.3911E+01	9.3249E+01
9.3725E+01	9.3102E+01
9.3035E+01	9.2956E+01
9.3153E+01	9.2773E+01
9.2688E+01	9.2520E+01
9.1854E+01	9.2175E+01
9.2109E+01	9.1739E+01
9.1833E+01	9.1235E+01
9.1201E+01	9.0706E+01
9.1392E+01	9.0201E+01
9.1052E+01	8.9766E+01
9.0309E+01	8.9429E+01
9.0627E+01	8.9196E+01
9.0083E+01	8.9054E+01
8.9287E+01	8.8966E+01
8.9786E+01	8.8890E+01
8.9268E+01	8.8790E+01
8.8532E+01	8.8640E+01
8.8876E+01	8.8431E+01

APPENDIX 8.3

APPENDIX 8.4



8.8352@+01	8.8168@+01
8.7818@+01	8.7867@+01
8.8319@+01	8.7546@+01
8.7601@+01	8.7221@+01
8.7123@+01	8.6905@+01
8.7352@+01	8.6605@+01
8.6638@+01	8.6335@+01
8.6548@+01	8.6110@+01
8.7030@+01	8.5958@+01
8.6031@+01	8.5913@+01
8.5864@+01	8.6012@+01
8.6256@+01	8.6286@+01
8.7018@+01	8.6752@+01
8.8757@+01	8.7404@+01
8.9557@+01	8.8218@+01
8.8834@+01	8.9154@+01
8.9334@+01	9.0168@+01
9.0303@+01	9.1224@+01
9.1409@+01	9.2302@+01
9.2779@+01	9.3404@+01
9.5200@+01	9.4546@+01
9.5536@+01	9.5755@+01
9.6557@+01	9.7049@+01
9.8072@+01	9.8429@+01
1.0008@+02	9.9871@+01
1.0160@+02	1.0133@+02
1.0348@+02	1.0274@+02
1.0379@+02	1.0404@+02
1.0428@+02	1.0519@+02
1.0748@+02	1.0617@+02
1.0768@+02	1.0701@+02
1.0840@+02	1.0773@+02
1.0929@+02	1.0839@+02
1.0859@+02	1.0903@+02
1.1062@+02	1.0966@+02
1.1426@+02	1.1028@+02
1.1241@+02	1.1084@+02
1.1083@+02	1.1129@+02
1.1139@+02	1.1158@+02
1.1063@+02	1.1168@+02
1.1246@+02	1.1160@+02
1.1255@+02	1.1138@+02
1.0954@+02	1.1108@+02
1.1137@+02	1.1080@+02
1.1179@+02	1.1061@+02
1.1175@+02	1.1055@+02
1.1101@+02	1.1063@+02
1.0965@+02	1.1084@+02
1.1103@+02	1.1112@+02
1.1136@+02	1.1141@+02
1.1232@+02	1.1168@+02
1.1196@+02	1.1189@+02
1.1011@+02	1.1203@+02
1.1065@+02	1.1213@+02
1.1047@+02	1.1219@+02
1.1255@+02	1.1221@+02
1.1260@+02	1.1218@+02
1.1064@+02	1.1208@+02
1.1059@+02	1.1187@+02



* DUIDT OP PATIENTUITGANGSDRUK EN # DUIDT OP MODELUITGANGSDRUK BIJ OPTIMALE INSTELLING
$ DUIDT OP HET SAMENVALLEN VAN - PUNTEN VAN DE RESP. GRAFIEKEN

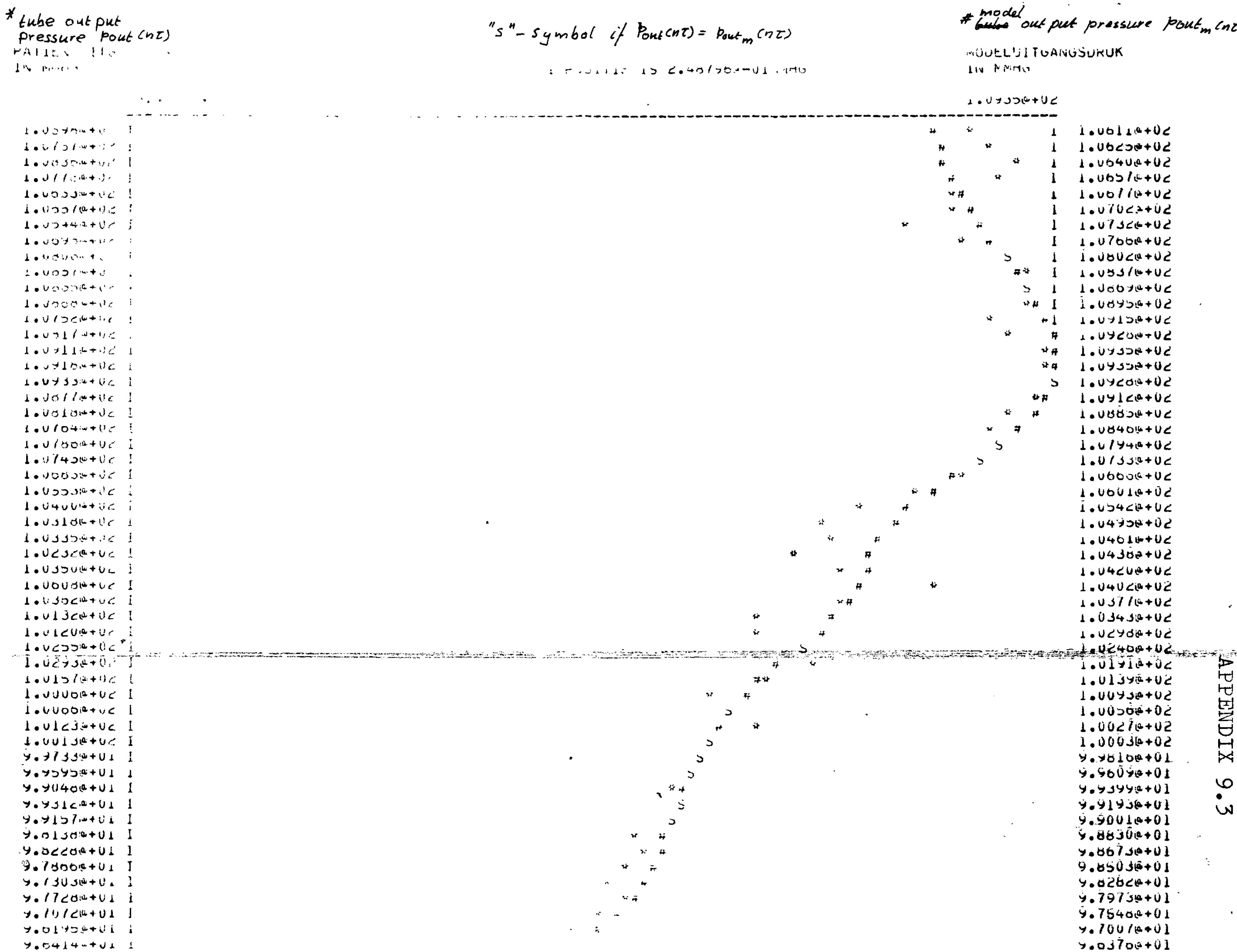
Model-to-tube adjustment with Marquardt; situation no. 4 of table 5.3

		scaled parameters				unscaled parameters			
		[illegible]*	[illegible]*	RP1*	C1*	RS1	L1	RP1	C1
START:	PI: [illegible]		[illegible]	2.0000E+01	2.6788E+02	1.570E-04	2.740E-04	1.370E-01	1.468E-02
ITER: 1	PI= [illegible]		[illegible]	5.2657E+01	2.6729E+02	1.800E-04	2.752E-04	3.006E-01	1.465E-02
ITER: 2	PI= [illegible]		[illegible]	[illegible]	2.5693E+02	2.791E-04	2.818E-04	1.041E+00	1.408E-02
ITER: 3	PI= [illegible]		[illegible]	[illegible]	2.2123E+02	7.525E-04	3.123E-04	2.272E+00	1.212E-02
ITER: 4	PI= [illegible]		[illegible]	[illegible]	1.6711E+02	1.284E-03	4.035E-04	4.500E+00	9.157E-03
ITER: 5	PI= [illegible]		[illegible]	[illegible]	6.2150E+01	1.248E-03	5.636E-04	8.675E+00	4.501E-03
ITER: 6	PI= 4.54[illegible]		[illegible]	1.925[illegible]+03	4.7237E+01	1.152E-03	6.552E-04	1.318E+01	2.569E-03
ITER: 7	PI= 3.75[illegible]		[illegible]	[illegible]	3.9465E+01	1.128E-03	6.728E-04	1.653E+01	2.164E-03
ITER: 8	PI= 3.5[illegible]		[illegible]	[illegible]	2.9371E+01	1.117E-03	6.824E-04	2.299E+01	1.610E-03
ITER: 9	PI= 3.4[illegible]		[illegible]	[illegible]	2.3225E+01	1.113E-03	6.811E-04	3.014E+01	1.547E-03
ITER:10	PI= 3.4[illegible]		[illegible]	[illegible]	2.0009E+01	1.107E-03	6.891E-04	3.082E+01	1.097E-03
	PI= 3.43505[illegible]		[illegible]	[illegible]	2.6379E+01	1.110E-03	6.804E-04	4.425E+01	1.446E-03
	PI= 3.43491[illegible]		[illegible]	[illegible]	2.6377E+01	1.110E-03	6.804E-04	4.422E+01	1.445E-03
	PI= 3.43[illegible]		[illegible]	[illegible]	2.6353E+01	1.110E-03	6.805E-04	4.389E+01	1.444E-03
ITER:11	PI= 3.4[illegible]		[illegible]	[illegible]	2.0165E+01	1.110E-03	6.812E-04	4.134E+01	1.434E-03
	PI= [illegible]		[illegible]	[illegible]	5.3359E+00	1.100E-03	7.029E-04	2.810E+01	2.924E-04
	PI= 3.6[illegible]		[illegible]	[illegible]	6.4056E+00	1.100E-03	7.012E-04	3.206E+01	3.511E-04
	PI= 3.604[illegible]		[illegible]	[illegible]	8.3162E+00	1.099E-03	6.981E-04	3.902E+01	4.557E-04
	PI= 3.633[illegible]		[illegible]	5.9932E+03	9.2102E+00	1.099E-03	6.967E-04	4.107E+01	5.047E-04
	PI= 3.554[illegible]		[illegible]	6.0309E+03	1.1610E+01	1.101E-03	6.929E-04	4.131E+01	6.472E-04
ITER:12	PI= 3.41537[illegible]		[illegible]	6.0342E+03	1.7846E+01	1.105E-03	6.844E-04	4.133E+01	9.780E-04
ITER:13	PI= 3.4[illegible]		[illegible]	6.0353E+03	2.3885E+01	1.108E-03	6.832E-04	4.134E+01	1.309E-03
	PI= 3.44151[illegible]		[illegible]	6.0083E+03	1.5405E+01	1.102E-03	6.909E-04	4.116E+01	8.442E-04
	PI= 3.4244[illegible]		[illegible]	6.0327E+03	1.6427E+01	1.103E-03	6.892E-04	4.132E+01	9.002E-04

APPENDIX 9.2

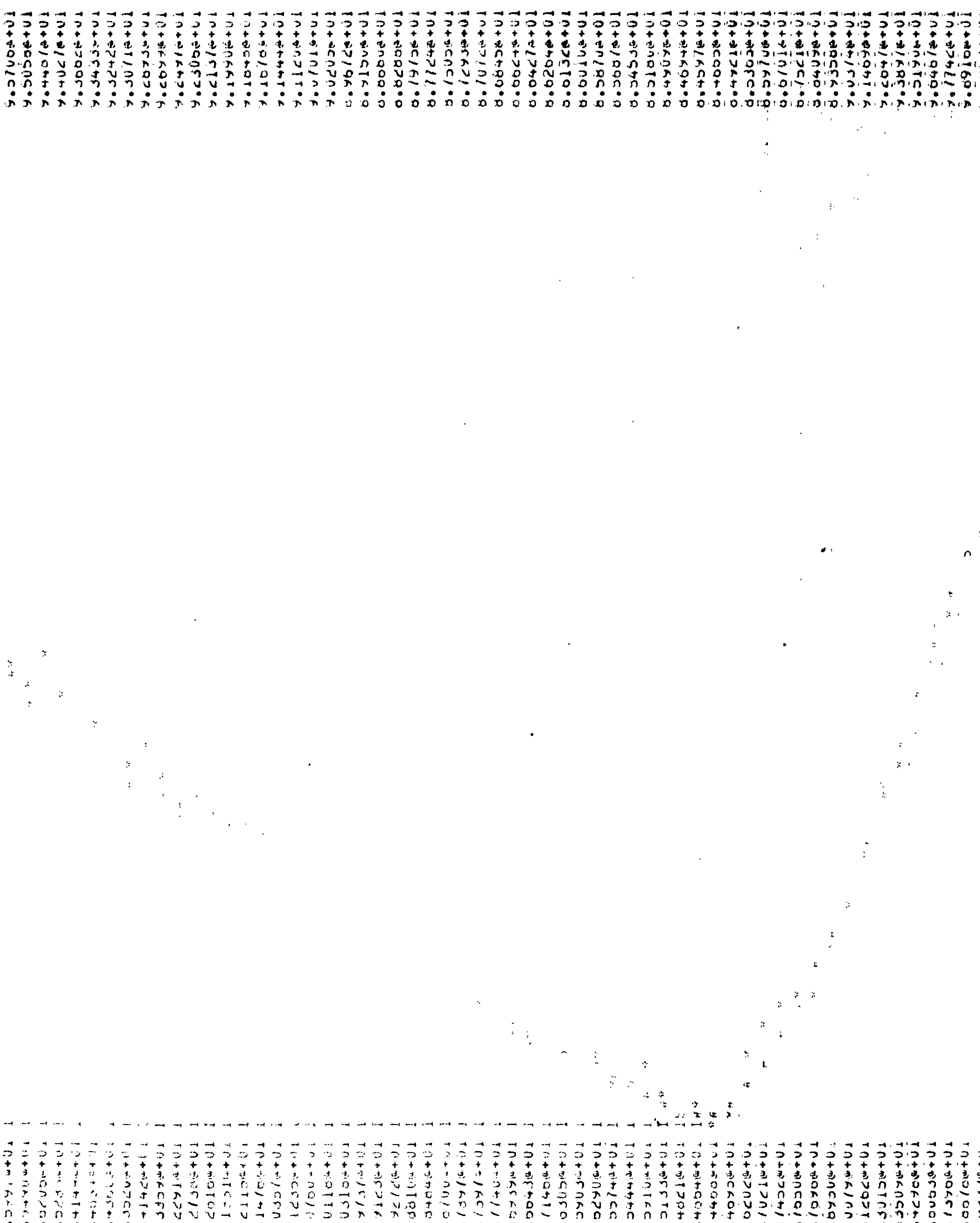
ITER:14 RI= 3.4 [illegible]	[illegible]	5.7551e+03	1.9122e+01	1.105e-03	6.850e-04	4.134e+01	1.048e-03
ITER:15 RI= [illegible]	[illegible]	6.0554e+03	2.2960e+01	1.107e-03	6.839e-04	4.134e+01	1.258e-03
RI= 3. [illegible]	[illegible]	6.0136e+03	1.7430e+01	1.103e-03	6.869e-04	4.119e+01	9.552e-04
RI= 3. [illegible]	[illegible]	6.0333e+03	1.8037e+01	1.114e-03	6.679e-04	4.133e+01	9.884e-04
ITER:16 RI= 3. [illegible]	[illegible]	6.0552e+03	1.9706e+01	1.105e-03	6.852e-04	4.134e+01	1.080e-03
ITER:17 RI= [illegible]	[illegible]	6.0051e+03	2.2402e+01	1.106e-03	6.844e-04	4.134e+01	1.226e-03
RI= 3. [illegible]	[illegible]	6.0159e+03	1.8524e+01	1.104e-03	6.879e-04	4.122e+01	1.015e-03

APPENDIX 9.3

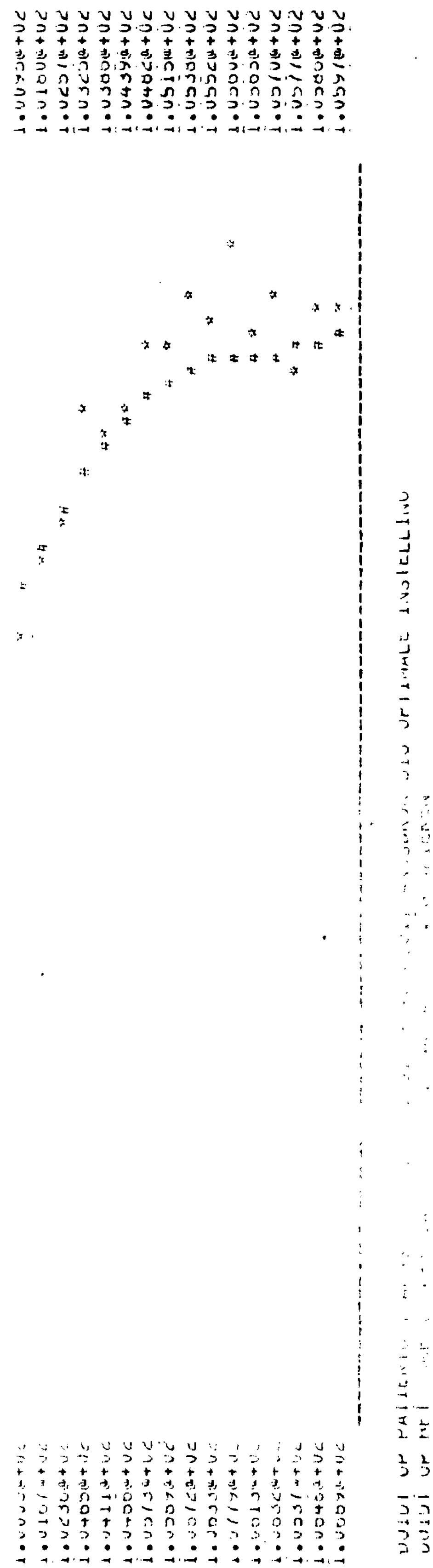


-104-

APPENDIX 9.4



APPENDIX 9.5



Model-to-tube adjustment with Marquardt; situation given in table 5.5.

			scaled parameters				unscaled parameters			
			RS1*	L1*	RP1*	C1*	RS1	L1	RP1	C1
START:	PI: 7.75258E+01		1.9000E+01	4.4500E+02	1.0000E+00	1.5500E+02	3.124E-05	1.219E-04	6.850E-03	8.494E-03
ITER: 1	PI= 7.05112E+01	LAMBDA=1.0E-02	2.8060E+01	4.4804E+02	9.6251E+00	1.3588E+02	4.613E-05	1.228E-04	6.593E-02	7.446E-03
ITER: 2	PI= 5.52122E+01	LAMBDA=1.0E-03	9.7844E+01	5.0206E+02	2.9269E+01	8.3382E+01	1.609E-04	1.376E-04	2.005E-01	4.569E-03
ITER: 3	PI= 2.25198E+01	LAMBDA=1.0E-04	4.3336E+02	8.3636E+02	2.8874E+01	2.8302E+00	7.124E-04	2.292E-04	1.978E-01	1.551E-04
ITER: 4	PI= 5.08740E+00	LAMBDA=1.0E-05	7.6204E+02	1.6221E+03	2.9003E+01	OG1.0000E-02	1.253E-03	4.445E-04	1.987E-01	5.480E-07
ITER: 5	PI= 5.92597E-01	LAMBDA=1.0E-06	7.3426E+02	2.3975E+03	2.9003E+01	7.8094E-01	1.207E-03	6.569E-04	1.987E-01	4.280E-05
ITER: 6	PI= 4.47331E-01	LAMBDA=1.0E-07	6.9931E+02	2.5563E+03	2.9458E+01	4.4167E+00	1.150E-03	7.004E-04	2.018E-01	2.420E-04
ITER: 7	PI= 4.36846E-01	LAMBDA=1.0E-08	6.9671E+02	2.5586E+03	2.7339E+02	2.4569E+00	1.145E-03	7.011E-04	1.873E+00	1.346E-04
ITER: 8	PI= 4.36181E-01	LAMBDA=1.0E-09	6.9720E+02	2.5596E+03	6.9636E+02	3.0189E+00	1.146E-03	7.013E-04	4.770E+00	1.654E-04
ITER: 9	PI= 4.32743E-01	LAMBDA=1.0E-10	6.9702E+02	2.5582E+03	3.7704E+03	2.7949E+00	1.146E-03	7.009E-04	2.583E+01	1.532E-04
ITER:10	PI= 3.98102E-01	LAMBDA=1.0E-11	6.9852E+02	2.5556E+03	1.6277E+04	4.4494E+00	1.148E-03	7.002E-04	1.115E+02	2.438E-04
	PI= 3.99950E-01	1.0E-12	7.1338E+02	2.4921E+03	2.6138E+04	2.4477E+01	1.173E-03	6.828E-04	1.790E+02	1.341E-03
ITER:11	PI= 3.94960E-01	LAMBDA=1.0E-11	7.1340E+02	2.4935E+03	2.3550E+04	2.4416E+01	1.173E-03	6.832E-04	1.613E+02	1.338E-03
	PI= 8.58949E+01	1.0E-12	6.8589E+02	1.8786E+03	OG1.0000E+00	3.9762E+02	1.128E-03	5.147E-04	6.850E-03	2.179E-02
	PI= 6.80157E+01	1.0E-11	6.8678E+02	2.0221E+03	OG1.0000E+00	3.1519E+02	1.129E-03	5.541E-04	6.850E-03	1.727E-02
	PI= 3.62168E+00	1.0E-10	6.8852E+02	2.3014E+03	1.2287E+03	1.5484E+02	1.132E-03	6.306E-04	8.416E+00	8.485E-03
	PI= 3.98475E-01	1.0E-09	6.8908E+02	2.3913E+03	2.0869E+04	1.0323E+02	1.133E-03	6.552E-04	1.430E+02	5.657E-03
	PI= 4.03696E-01	1.0E-08	6.8921E+02	2.4028E+03	2.3277E+04	9.6553E+01	1.133E-03	6.584E-04	1.594E+02	5.291E-03
	PI= 4.00978E-01	1.0E-07	6.8974E+02	2.4089E+03	2.3523E+04	9.2618E+01	1.134E-03	6.600E-04	1.611E+02	5.075E-03
ITER:12	PI= 3.87166E-01	LAMBDA=1.0E-06	6.9318E+02	2.4397E+03	2.3548E+04	7.1851E+01	1.140E-03	6.685E-04	1.613E+02	3.937E-03
ITER:13	PI= 3.77952E-01	LAMBDA=1.0E-07	6.8819E+02	2.4648E+03	2.3521E+04	2.4537E+02	1.131E-03	6.754E-04	1.611E+02	1.345E-02
ITER:14	PI= 3.75523E-01	LAMBDA=1.0E-08	6.9189E+02	2.4745E+03	2.3241E+04	5.8070E+02	1.137E-03	6.780E-04	1.592E+02	3.182E-02
ITER:15	PI= 3.66234E-01	LAMBDA=1.0E-09	6.9203E+02	2.4652E+03	2.0406E+04	BG6.7538E+02	1.138E-03	6.755E-04	1.398E+02	3.701E-02
	PI= 1.86750E+02	1.0E-10	6.8937E+02	2.3547E+03	OG1.0000E+00	BG7.0708E+02	1.133E-03	6.452E-04	6.850E-03	3.875E-02

ITER#16	PI= 3.51527@-01	LAMBDA=1.0@-09	6.9168@+02	2.4488@+03	1.6928@+04BG6.7993@+02		1.137@-03	6.710@-04	1.160@+02	3.726@-02
	PI= 1.36382@+02	1.0@-10	6.8874@+02	2.2834@+03DG1.0000@+00		5.2244@+02	1.132@-03	6.256@-04	6.850@-03	2.863@-02
ITER#17	PI= 3.23800@-01	LAMBDA=1.0@-09	6.9090@+02	2.4176@+03	1.2415@+04BG6.8870@+02		1.136@-03	6.624@-04	8.504@+01	3.774@-02
	PI= 1.13474@+00	1.0@-10	6.8851@+02	2.1774@+03DG1.0000@+00DG1.0000@-02			1.132@-03	5.966@-04	6.850@-03	5.480@-07
ITER#18	PI= 2.73690@-01	LAMBDA=1.0@-09	6.8946@+02	2.3389@+03	6.3475@+03BG7.1187@+02		1.133@-03	6.409@-04	4.348@+01	3.901@-02
	PI= 6.89558@-01	1.0@-10	6.8833@+02	2.1170@+03	2.3002@+03	7.1313@+02	1.132@-03	5.801@-04	1.576@+01	3.908@-02
	PI= 3.21301@-01	1.0@-09	6.8835@+02	2.1495@+03	3.1090@+03	7.4702@+02	1.132@-03	5.890@-04	2.130@+01	4.094@-02

APPENDIX 10.2

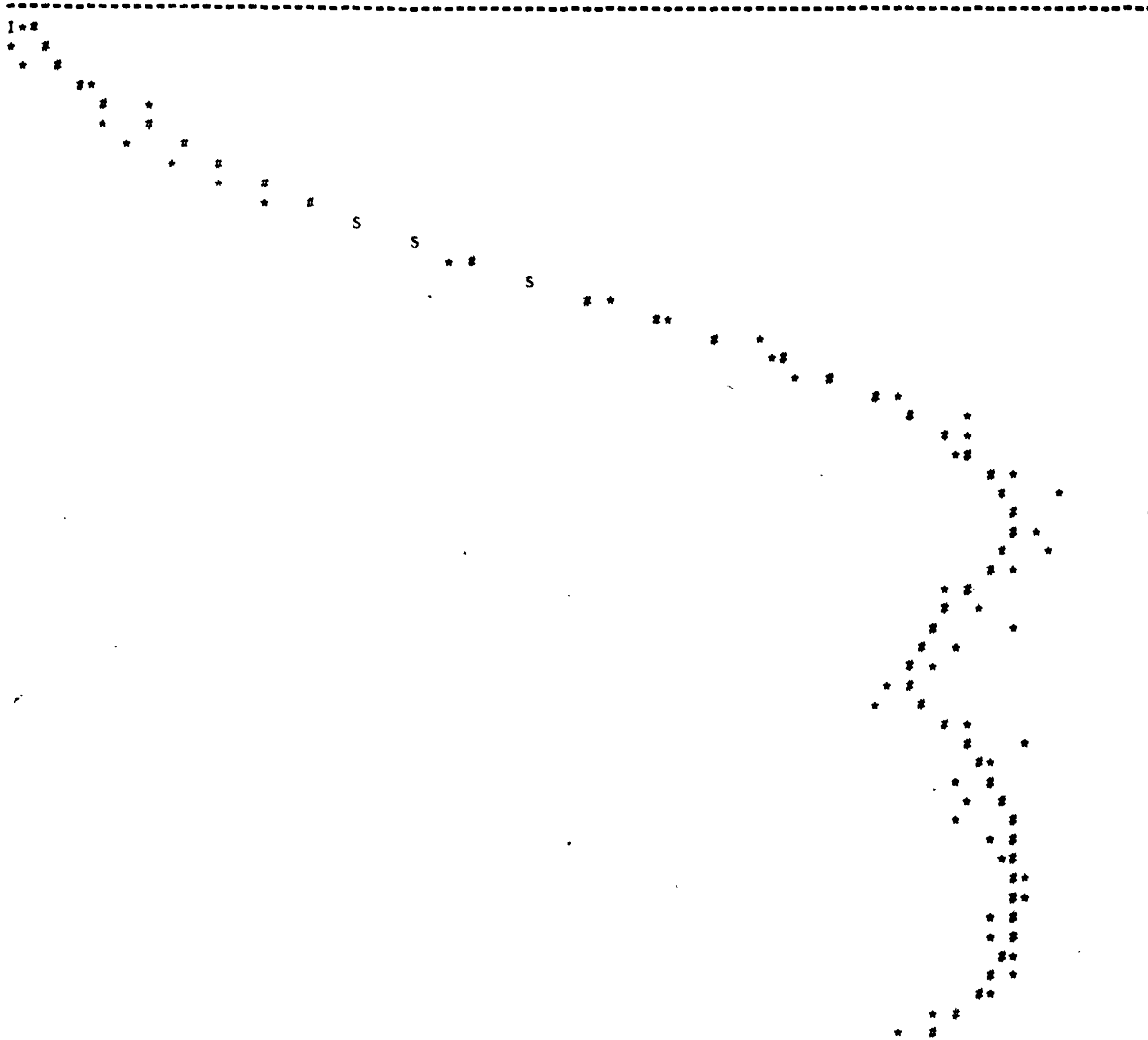
*tube output pressure $P_{out}(n\tau)$ — PATIENTUITGANGSDRUK IN MMHG

"S"-symbol if $P_{out}(n\tau) = P_{out_m}(n\tau)$

#distributed model output pressure $P_{out_m}(n\tau)$. — MODELUITGANGSDRUK IN MMHG

1 POSITIE IS 3.08604@-01MMHG

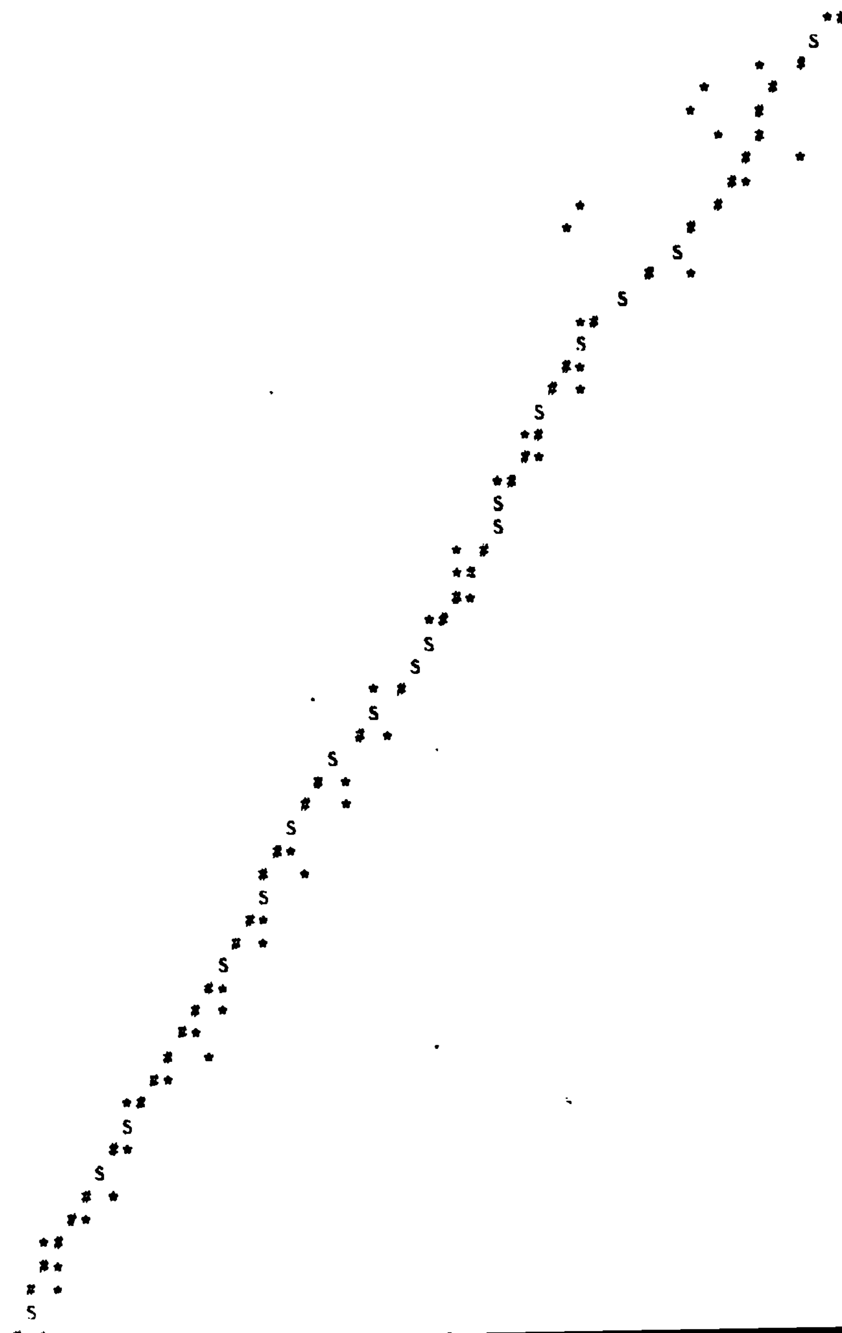
8.4852@+01 … 1.1571@+02



PATIENTUITGANGSDRUK IN MMHG	MODELUITGANGSDRUK IN MMHG
8.5025@+01	8.5455@+01
8.4852@+01	8.5645@+01
8.5275@+01	8.6055@+01
8.6879@+01	8.6680@+01
8.8442@+01	8.7492@+01
8.7362@+01	8.8447@+01
8.7849@+01	8.9498@+01
8.9080@+01	9.0611@+01
9.0431@+01	9.1771@+01
9.1773@+01	9.2987@+01
9.4261@+01	9.4283@+01
9.5793@+01	9.5686@+01
9.6845@+01	9.7211@+01
9.8928@+01	9.8851@+01
1.0092@+02	1.0057@+02
1.0248@+02	1.0232@+02
1.0517@+02	1.0401@+02
1.0549@+02	1.0560@+02
1.0605@+02	1.0701@+02
1.0897@+02	1.0824@+02
1.1086@+02	1.0927@+02
1.1062@+02	1.1011@+02
1.1038@+02	1.1080@+02
1.1182@+02	1.1133@+02
1.1316@+02	1.1170@+02
1.1571@+02	1.1189@+02
1.1244@+02	1.1190@+02
1.1292@+02	1.1170@+02
1.1211@+02	1.1132@+02
1.1021@+02	1.1080@+02
1.1112@+02	1.1024@+02
1.1204@+02	1.0972@+02
1.1030@+02	1.0935@+02
1.0987@+02	1.0919@+02
1.0854@+02	1.0927@+02
1.0825@+02	1.0958@+02
1.1082@+02	1.1004@+02
1.1228@+02	1.1057@+02
1.1142@+02	1.1107@+02
1.1053@+02	1.1148@+02
1.1069@+02	1.1176@+02
1.1036@+02	1.1191@+02
1.1138@+02	1.1196@+02
1.1176@+02	1.1197@+02
1.1228@+02	1.1196@+02
1.1242@+02	1.1195@+02
1.1145@+02	1.1194@+02
1.1122@+02	1.1187@+02
1.1188@+02	1.1171@+02
1.1187@+02	1.1140@+02
1.1119@+02	1.1095@+02
1.0994@+02	1.1035@+02
1.0882@+02	1.0968@+02

APPENDIX 10.3

APPENDIX 10.4



1.0846E+02	1.0898E+02
1.0828E+02	1.0835E+02
1.0685E+02	1.0781E+02
1.0586E+02	1.0741E+02
1.0533E+02	1.0711E+02
1.0617E+02	1.0688E+02
1.0804E+02	1.0665E+02
1.0679E+02	1.0637E+02
1.0307E+02	1.0600E+02
1.0252E+02	1.0554E+02
1.0513E+02	1.0500E+02
1.0548E+02	1.0443E+02
1.0372E+02	1.0388E+02
1.0302E+02	1.0338E+02
1.0288E+02	1.0296E+02
1.0289E+02	1.0262E+02
1.0298E+02	1.0235E+02
1.0197E+02	1.0211E+02
1.0160E+02	1.0189E+02
1.0189E+02	1.0167E+02
1.0107E+02	1.0144E+02
1.0110E+02	1.0119E+02
1.0104E+02	1.0095E+02
1.0023E+02	1.0071E+02
1.0011E+02	1.0047E+02
1.0047E+02	1.0022E+02
9.9484E+01	9.9946E+01
9.9439E+01	9.9620E+01
9.9327E+01	9.9236E+01
9.8102E+01	9.8795E+01
9.8209E+01	9.8313E+01
9.8615E+01	9.7819E+01
9.7411E+01	9.7345E+01
9.7503E+01	9.6922E+01
9.7481E+01	9.6569E+01
9.6284E+01	9.6291E+01
9.6290E+01	9.6074E+01
9.6745E+01	9.5892E+01
9.5730E+01	9.5715E+01
9.5716E+01	9.5515E+01
9.5629E+01	9.5272E+01
9.4723E+01	9.4982E+01
9.4786E+01	9.4651E+01
9.4770E+01	9.4295E+01
9.4050E+01	9.3932E+01
9.4383E+01	9.3576E+01
9.3600E+01	9.3235E+01
9.2745E+01	9.2908E+01
9.2777E+01	9.2589E+01
9.2795E+01	9.2270E+01
9.2153E+01	9.1947E+01
9.2363E+01	9.1623E+01
9.1807E+01	9.1307E+01
9.0901E+01	9.1011E+01
9.1133E+01	9.0748E+01
9.1153E+01	9.0524E+01
9.0323E+01	9.0333E+01

APPENDIX 10.5

```
8.9112P+01                * #                   8.9788P+01
8.9410P+01                 S                    8.9545P+01
8.9205P+01                S                     8.9251P+01
8.8514P+01              *#                      8.8914P+01
8.8913P+01              #*                      8.8551P+01
8.8376P+01             S                        8.8187P+01
8.7470P+01          * #                         8.7843P+01
8.7763P+01           S                          8.7530P+01
8.7368P+01          S                           8.7249P+01
8.6976P+01         S                            8.6986P+01
8.7498P+01        # *                           8.6724P+01
8.6741P+01       #*                             8.6446P+01
8.5906P+01     *#                               8.6153P+01
8.6403P+01     # *                              8.5863P+01
8.6057P+01    # *                               8.5614P+01
8.5898P+01    #*                                8.5460P+01
            ------------------------------
```

* DUIDT OP PATIENTUITGANGSDRUK EN # DUIDT OP MODELUITGANGSDRUK BIJ OPTIMALE INSTELLING
S DUIDT OP HET SAMENVALLEN VAN 2 PUNTEN VAN DE RESP. GRAFIEKEN