A Priori Error Bounds for Model Reduction of Interconnected Systems

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Lars Janssen, TU/e*
Email: l.a.l.janssen@tue.nl

Rob Fey, TU/e*
Email: r.h.b.fey@tue.nl

Bart Besselink, RUG†
Email: b.besselink@rug.nl

Nathan van de Wouw, TU/e*
Email: n.v.d.wouw@tue.nl

1 Introduction

Many complex models of dynamic (multi-)physical systems are naturally based on an interconnection of subsystems. The models of interconnected systems often have a high number of states, which makes controller synthesis, simulation and analysis computationally expensive. This motivates the need for model order reduction (MOR) techniques applicable to such systems. *Subsystem reduction*, where each of the subsystems is reduced individually, is a completely modular approach. Hence, the interconnection structure of the high-order model is preserved and the computationally challenging reduction of one very high-dimensional model is avoided by dividing the problem into multiple smaller problems. However, although subsystem reduction methods will lead to accurate reduced-order representations of the high-order subsystems, this does not directly mean that the reduced-order interconnected system is also accurate. Based on [1], we introduce a method which gives an a priori error bound on the frequency response of the reduced interconnected system based on the a priori error bounds of the reduced subsystems.

2 Preliminaries

We study systems of k linear time-invariant (LTI) subsystems that can be aggregated and represented by block-diagonal transfer function $G = \text{diag}(G_1, \dots, G_k)$. The subsystems are interconnected via matrix K. The external inputs u and outputs y are connected to the subsystems via external input and output matrix L and M, respectively. In subsystem reduction, each of the subsystems G_j is reduced to an approximate reduced-order subsystem $\hat{G}_j$ and the aggregate is given by $\hat{G} = \text{diag}(\hat{G}_1, \dots, \hat{G}_k)$. The high-order interconnected system G_c and the reduced-order interconnected system $\hat{G}_c$ with $\hat{y} = \hat{G}_c u$ (Figure 1), are then

$$G_c = MG(I-KG)^{-1}L,\ \hat{G}_c = M\hat{G}(I-K\hat{G})^{-1}L. \quad (1)$$

3 Methodology

For balanced truncation, a widely used MOR method, an upper bound on the $\mathscr{H}_\infty$-norm of the difference between the high-order and reduced-order systems is available a priori.

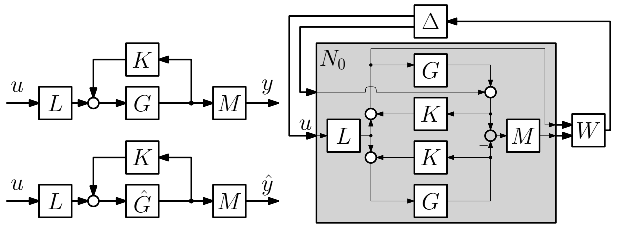

Figure 1: Block diagrams of G_c (top left), $\hat{G}_c$ (bottom left), and $G_c - \hat{G}_c$ represented by N_0, W and Δ (right).

In this work, we extend a priori knowledge of subsystem errors $\|G_j - \hat{G}_j\|_\infty$ to knowledge on $\|G_c - \hat{G}_c\|_\infty$. To do so, we define all subsystem reduction errors as an uncertain system $\Delta := G - \hat{G}$. Then, we can model $G_c - \hat{G}_c$ as a closed loop between N and Δ as shown in Figure 1. Here, the nominal system $N := N_0 W$ is a function of $N_0(G,K,L,M)$ and weighting matrix $W := \text{diag}(w_1 I, \dots, w_k I, w_c I)$. Using this framework, the structured singular value μ, a powerful tool from robust control, can be used. Namely, if N is internally stable, then

$$\left.\begin{array}{l} \|G_c - \hat{G}_c\|_\infty \leq w_c^{-1}, \text{and} \\ \|G_j - \hat{G}_j\|_\infty \leq w_j \ \forall\ j = 1, \dots, k \end{array}\right\} \Leftrightarrow \mu_\Delta(N) < 1. \quad (2)$$

Given that an error bound w_j is available a priori for each subsystem, this relation can be used to find an error bound w_c^{-1} on the interconnected system. A less conservative frequency-dependent version of (2) can be used at the cost of increased computation time. In [1], these bounds are illustrated using an example.

4 Conclusion

With (2), we can directly compute the impact of errors in subsystems to the error of the interconnected system. This result is also the first step towards determining a model accuracy specification for the individual subsystems guaranteeing a required overall system accuracy. Therefore, the method can be used as a significant means for the development of accurate modeling of interconnected systems within a completely modular setting.

References

[1] Janssen, L.A.L. et al., "A Priori Error Bounds for Model Reduction of Interconnected Linear Systems using Robust Performance Analysis", ACC 2022 (accepted for publication).

*Dynamics and Control, Mechanical Engineering, Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands

†Bernoulli Institute for Mathematics, Computer Science and Artificial Intelligence, University of Groningen, P.O. Box 407, 9700 AK Groningen, The Netherlands