Modular Model Reduction of Interconnected Systems

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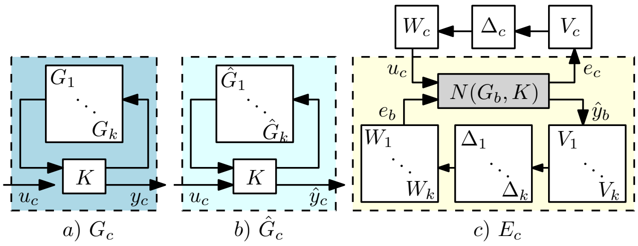

Fig. 1: Transfer functions of a) the high-order, b) the reduced-order interconnected models, and c) the RP perspective.

1 Introduction

Complex models of dynamic (multi-)physical systems are often based on an interconnection of subsystems with a high number of states. For such systems, model order reduction (MOR) is required to make controller synthesis, simulation and analysis computationally feasible. In modular MOR, each subsystem model is reduced individually. This preserves the interconnection structure of the model, and dividing the problem into multiple smaller problems avoids the computationally challenging reduction of one high-dimensional model. However, although modular MOR leads to accurate subsystem models, it does not guarantee the accuracy of the interconnected reduced-order model (ROM). We introduce a top-down method that allows translation of accuracy requirements on the interconnected ROM to a set of accuracy requirements for subsystem ROMs which allows for a completely modular approach.

2 Methodology

To model the system, we combine k linear time-invariant subsystems G_j in the transfer function $G_b = \mathrm{diag}(G_1, \dots, G_k)$. Inputs u_b and outputs y_b of the subsystems are interconnected via the coupling matrix K. All subsystems are reduced to ROMs given by $\hat{G}_b = \mathrm{diag}(\hat{G}_1, \dots, \hat{G}_k)$. The high-order interconnected model G_c and interconnected ROM $\hat{G}_c$ are then both given by a feedback of K with G_b and $\hat{G}_b$, respectively, external input u_c, and output y_c, as shown in Fig. 1a and 1b. In this work, we relate $E_j = G_j - \hat{G}_j$ to $E_c = G_c - \hat{G}_c$ by reformulating the problem such that the structured singular value μ, a tool from robust control [1], can be used. We define a weighted uncertain system for which $E_j = W_j \Delta_j V_j$. Then, we model E_c as a feedback of VNW and Δ as shown in Fig. 1c. Nominal system N is a function of G_b and K. $V = \mathrm{diag}(V_1, \dots, V_k, V_c)$, $W = \mathrm{diag}(W_1, \dots, W_k, W_c)$ and $\Delta = \mathrm{diag}(\Delta_1, \dots, \Delta_k, \Delta_c)$.

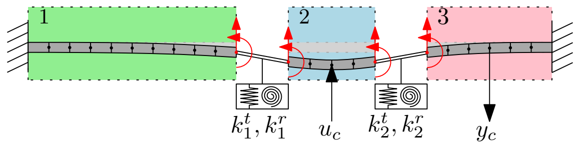

Fig. 2: : Example system: Three interconnected beams.

Theorem 1 *From robust performance (RP) [1], let $\omega \in \mathbb{R}$, for the system in Fig. 1c,*

$$\begin{aligned} &|W_c(\omega)E_c(i\omega)V_c(\omega)| < 1, \textit{ for all } E_j(i\omega) \textit{ such that} \\ &|W_j^{-1}(\omega)E_j(i\omega)V_j^{-1}(\omega)| \leq 1, \textit{ if and only if} \\ &\mu_\Delta(V(\omega)N(i\omega)W(\omega)) < 1. \end{aligned} \tag{1}$$

Given that $\mu_\Delta(V(\omega)N(i\omega)W(\omega)) < 1$ can be verified computationally, this relation can be used, given requirements on the interconnected ROM, to find a set of subsystem requirements in terms of $V_j(\omega)$ and $W_j(\omega)$ for any $\omega \in \mathbb{R}$. Then, the subsystems can be reduced completely individually, i.e., modularly. To illustrate how Theorem 1 can be used for modular MOR, we apply it to the system illustrated in Fig. 2. In this case, the subsystems are reduced as much as possible within the computed subsystem requirements using balanced truncation. The results are shown in Tab. 1.

Tab. 1: # states in the high-order model and for a ROM satisfying given accuracy requirements for all $30 \leq \omega \leq 10{,}000$.

Subsystem	G_1	G_2	G_3	G_c
High-order model	200	84	120	404
ROM: $\lvert E_c(i\omega)\rvert < 0.01 \lvert G_c(i\omega)\rvert$	21	19	15	55
ROM: $\lvert E_c(i\omega)\rvert < 0.1\ \lvert G_c(i\omega)\rvert$	19	13	12	44

3 Conclusion

Modular MOR is a computationally efficient method that allows for the computation of ROMs of interconnected (multidisciplinary and multi-physical) subsystems. We introduce a mathematical relation between the accuracy of subsystem ROMs and the accuracy of the interconnected ROMs that allows for modular MOR with accuracy guarantees. We apply this to an example model, in which we reduce the interconnected system using modular MOR while guaranteeing different levels of accuracy for the ROM.

References

[1] Packard, A., & Doyle, J. (1993). The complex structured singular value. Automatica, 29(1), 71-109

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