# Joint Physics-Based and Kernel-Regularized LPV Feedforward

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## 1 Background

Increasing demands for motion control result in a situation where Linear Parameter-Varying (LPV) dynamics have to be taken into account. Inverse-model feedforward for LPV systems is challenging since the inverse is often dynamically dependent. The aim of this paper is to develop an identification approach that directly identifies dynamically scheduled feedforward controllers for LPV systems from data.

## 2 Problem Formulation

The goal is to create LPV feedforward controller $F_{LPV}$ to reduce tracking error $e = r - y$ for LPV system $G_{LPV}$. The closed-loop setup considered is shown in Figure 1.

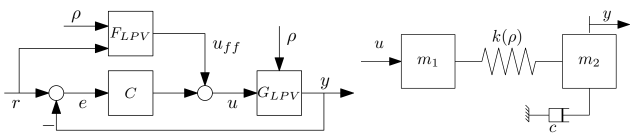


**Figure 1:** Left: Feedforward structure considered. Right: Example system that can be represented as (1).

The class of LPV systems considered are statically dependent on the scheduling variable with the following representation

$$G_{LPV}: \sum_{i=-2}^{n_a} a_i(\rho(t)) y^{(i)}(t) = \sum_{j=0}^{n_b} b_j(\rho(t)) \frac{d^j}{dt^j} \iint u(t)\, dt^2, \quad (1)$$

with $\cdot^{(i)}$ the $i$'th derivative if $i \geq 0$ or the $i$'th integral if $i < 0$. The inverse dynamics, i.e., the dynamics from $y$ to $u$, are dynamically dependent on $\rho$, due to the second integral.

**Example 1.** Consider the system in Figure 1, where the input-output behavior from input $u$ to output $y$ is given by

$$\left(m_2 m_1 \tfrac{d^2}{dt^2} + c m_1 \tfrac{d}{dt} + (k(\rho)(m_1 + m_2))\right) y + k(\rho) c \int y\, dt = k(\rho) \iint u\, dt^2.$$

The inverse dynamics are given by

$$u = \tfrac{m_2 m_1}{k(\rho)} y^{(4)} + \tfrac{c m_1}{k(\rho)} y^{(3)} + (m_1 + m_2)\ddot{y} + c\dot{y} + \frac{\frac{2\dot{\rho}^2 k'^2(\rho)}{k(\rho)} - \dot{\rho}^2 k''(\rho) - \ddot{\rho} k'(\rho) - 2\dot{\rho} k'(\rho)}{k^2(\rho)} f(y, \dot{y}, \ddot{y}),$$

with $f(y, \dot{y}, \ddot{y}) = m_1 m_2 \ddot{y} + c m_1 \dot{y} + k(\rho)(m_1 + m_2) y$ and $k'$ and $k''$ the derivatives of $k$ with respect to $\rho$. ■

## 3 Approach

The approach uses a basis function approach with LPV feedforward parameters, where the parameters are learned using data with kernel-regularized least-squares. The feedforward controller is parameterized as

$$F_{LPV}: w_{ff} = \sum_{i=1}^{n_\theta} \theta_i(\rho) \psi_i\left(\frac{d}{dt}, I\right) r, \quad (2)$$

where the second integral is applied such that the dynamic dependence on the scheduling sequence is compensated for. The input applied to the system is calculated as $u_{ff} = \frac{d^2}{dt^2} w_{ff}$. The parameters $\theta_i(\rho)$ are identified using kernel-based regularization similar to [1] as follows

$$\hat{\Theta} = \arg\min_{\Theta} \|\bar{w} - \Phi\Theta\|^2 + \gamma \|\Theta\|^2_{\mathcal{H}}, \quad (3)$$

where $\Theta$, $\bar{w}$ and $\Psi$ are matrices or vectors of $\theta_i$, $w$ and $\psi_i$ and $\mathcal{H}$ denotes the squared induced norm on the reproducing kernel Hilbert space, where $\|\Theta\|^2_{\mathcal{H}} = \Theta^\top K^{-1} \Theta$. The kernel $K$ incorporates prior knowledge of the feedforward parameters, such as smoothness or periodicity.

## 4 Results

Tracking performance of the system in Figure 1 with $k(\rho) = EA/(\rho(L - \rho))$ is compared for traditional LTI feedforward and the developed approach and is seen in Figure 2.

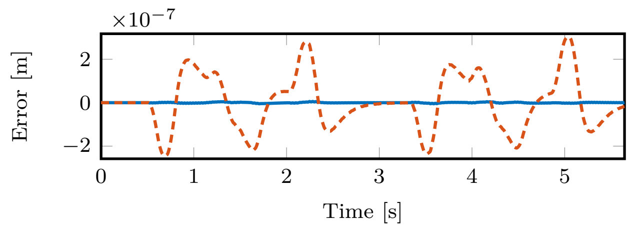


**Figure 2:** Error for LTI (- -) and developed (—) feedforward.

Significant tracking performance is achieved by the developed approach compared to LTI feedforward. The error 2-norm $\|e\|_2$ reduces from $4.2 \cdot 10^{-6}$ m to $5.8 \cdot 10^{-8}$ m.

### Acknowledgments

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### References

[1] G. Pillonetto, F. Dinuzzo, T. Chen, G. De Nicolao, and L. Ljung, "Kernel methods in system identification, machine learning and function estimation: A survey," *Automatica*, vol. 50, no. 3, pp. 657–682, 2014.