

#### Entropically secure encryption with faster key expansion

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ENTROPICALLY SECURE ENCRYPTION WITH
 FASTER KEY EXPANSION
 Mehmet Hüseyin Temel and Boris Škorić

Eindhoven University of Technology

m.h.temel@tue.nl, b.skoric@tue.nl



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### **Entropic Security**

An encryption scheme is called *perfect* if the ciphertext reveals no information whatsoever about the plaintext. For perfect encryption of *classical* plaintexts the length of the key needs to be at least the entropy of the plaintext, and the simplest cipher is the One-Time Pad (OTP) or Vernam cipher. In the *quantum* setting, perfect encryption of an *n*-qubit plaintext state requires a key length of 2n bits, and the simplest cipher achieving this kind of encryption is the Quantum One-Time Pad (QOTP) [1, 2, 3].

If one does not aim for *perfect* security, it is possible to get information-theoretic guarantees about the encryption even with shorter keys, as long as a lower bound is known on the min-entropy of the plaintext. The notion of  $(t, \varepsilon)$ -entropic security has been introduced [4, 5], stating that the adversary's advantage in guessing any function of the plaintext is upper bounded by  $\varepsilon$  if the min-entropy of the plaintext (conditioned on Eve's side information) is at least t. It can be seen as an information-theoretic version of semantic security. It has been shown that  $(t, \varepsilon)$ -entropically secure encryption of an n (qu)bit plaintext can be achieved with key length  $n - t + 2\log \frac{1}{\varepsilon}$  [5, 6, 7]. In the quantum case the t can become negative when Eve's quantum memory is entangled with the plaintext state.

We introduce a new key expansion method for entropically secure encryption, both classical and quantum [8]. The main idea is to *postfix* a pseudorandom string f(k) to the short key k, instead of creating an entirely new string from k. For the computation of f(k) we use finite-field multiplication with a public random string. Our key expansion is faster than previous schemes.

### 1. The Quantum One-Time Pad

Let  $\mathcal{H}_2$  denote the Hilbert space of a qubit. Let Z and X be single-qubit Pauli operators, in the standard basis given by  $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$  and  $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ . The simplest way to encrypt an n-qubit state  $\varphi \in \mathcal{D}(\mathcal{H}_2^{\otimes n})$  is to encrypt each qubit independently. The key is  $\beta = (\beta_1, \ldots, \beta_n) \in \{0, 1\}^{2n}$ , with  $\beta_i = (s_i, t_i)$ .

$$F_{\beta}(\varphi) = U_{\beta}\varphi U_{\beta}^{\dagger}$$
 where  $U_{\beta} = \bigotimes_{i=1}^{n} X^{s_i} Z^{t_i}$ . (1)

If the input state is entangled with the Eve's state i.e.  $\varphi^{AE} \in \mathcal{D}(\mathcal{H}_A \otimes \mathcal{H}_E)$ , then the effect of QOTP encryption is

$$\varphi^{AE} \quad \mapsto \quad F_{\beta}(\varphi^{AE}) = (U_{\beta} \otimes \mathbb{1}^{E})\varphi^{AE}(U_{\beta}^{\dagger} \otimes \mathbb{1}^{E}).$$
(2)

It holds that  $2^{-2n} \sum_{\beta \in \{0,1\}^{2n}} F_{\beta}(\varphi^{AE}) = \mathbb{1}/2^n \otimes \varphi^E$  for any  $\varphi^{AE} \in \mathcal{D}(\mathcal{H}_A \otimes \mathcal{H}_E)$ .

### 3. Our scheme

### Message state: $\varphi^A \in \mathcal{D}(\mathcal{H}_2^{\otimes n})$

**Key:**  $k \in \{0,1\}^{\ell}$ The construction shown below is for the case  $\ell > n$ . The case  $\ell < n$  is similar.

**Random public strings:**  $u \in \{0, 1\}^{\ell}$  and  $v \in \{0, 1\}^{2n-\ell}$ 

**Expanded key:**  $b(k, u, v) = k ||(uk + v)_{lsb}|$ 

**Encryption:**  $\operatorname{Enc}(k, \varphi^A) = \left(u, v, F_{b(k,u,v)}(\varphi^A)\right)$ 

## 2. Entropic Security in the Quantum Setting

Entropic security has been generalized to the fully quantum setting where both the plaintext and ciphertext are quantum states. Desrosiers [6] introduced definitions of entropic security and entropic indistinguishability for quantum ciphers.

**Definition: Strong entropic security in the quantum setting** (Def.4 in [7]). An encryption system R is called strongly  $(t, \varepsilon)$ -entropically secure if for all states  $\varphi^{AE}$  satisfying  $\mathsf{H}_{\min}(A|E)_{\varphi} \geq t$ , all interpretations  $\{(p_i, \sigma_i^{AE})\}$  of  $\varphi^{AE}$ , all adversaries  $\mathcal{A}$  and all functions f, it holds that

 $\left| \Pr[\mathcal{A}(R(\sigma_i^{AE})) = f(i)] - \Pr[\mathcal{A}(R(\varphi^{A}) \otimes \sigma_i^{E}) = f(i)] \right| \leq \varepsilon.$ 

Here 'interpretation' means  $\varphi^{AE} = \sum_{i} p_i \sigma_i^{AE}$ .

**Definition: Entropic indistinguishability in the quantum setting** (Def.3 in [7]). An encryption system  $R : \mathcal{D}(\mathcal{H}_A) \to \mathcal{D}(\mathcal{H}_{A'})$  is called  $(t, \varepsilon)$ -indistinguishable if

$$\exists_{\Omega^{A'} \in \mathcal{D}(\mathcal{H}_{A'})} \quad \mathsf{H}_{\min}(A|E)_{\varphi} \ge t \implies \left\| R(\varphi^{AE}) - \Omega^{A'} \otimes \varphi^{E} \right\|_{1} \le \varepsilon.$$

$$(4)$$

Similar to the classical setting, these definitions are equivalent up to parameter changes. **Theorem 1 in [7]**:  $(t - 1, \varepsilon/2)$ -entropic indistinguishability implies strong  $(t, \varepsilon)$ -entropic security for all functions.

Descriptions also introduced a scheme with a key length of  $n - t + 2 \log \frac{1}{\varepsilon}$  using a similar key expansion method as [5]. Here t is the min-entropy of the quantum state. The analysis in [6] applies only if Eve is not entangled with the plaintext. Description Description [7]



# 4. Results

If the key length is set as  $\ell = n - t + 2\log \frac{1}{\varepsilon} + 3$  then our scheme is  $(t, \varepsilon)$ -entropically secure.

• Our key expansion is faster than all previous constructions, while achieving the shortest known key length. In particular, a factor 2 in speed is gained in the unentangled quantum case without further assumptions on Eve.

- The scheme works both for quantum and classical one-time pads.
- Our security proofs are a bit more straightforward.
- We make slightly weaker assumptions on the plaintext, working with collision entropy

generalized the analysis, with conditional quantum min-entropy as defined by Renner [9], and showed that the results hold even with entanglement. They also proved a minimum required key length of n - t - 1. instead of min-entropy.

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(3)

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