Simulation Supported Bayesian Network Approach for Performance Assessment of Infrastructure Systems

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Simulation Supported Bayesian Network Approach for Performance Assessment of Infrastructure Systems

Mohsen Jafari Songhori

Department of Industrial Engineering & Innovation Science, Eindhoven University of Technology, The Netherlands, E-mail: M.jafari.songhori@Tue.nl

Claudia Fecarotti

Department of Industrial Engineering & Innovation Science, Eindhoven University of Technology, The Netherlands, E-mail: C.fecarotti@Tue.nl

Geert-Jan van Houtum

Department of Industrial Engineering & Innovation Science, Eindhoven University of Technology, The Netherlands, E-mail: G.J.v.Houtum@Tue.nl

We present a simulation supported Bayesian Network modeling approach to evaluate the performance of bridge networks with respect to both infrastructure owner's cost and users' travel time based on bridge level maintenance decisions. By combining system decomposition, simulation and Bayesian Networkm (BN) modelling, our approach enables the construction of a BN model of bridge networks where probabilistic information resulting from simulation are used to populate the conditional probability tables. Our approach is therefore useful when access to actual conditions of bridges and their monitoring is difficult, and the conditional dependencies accross different networks elements are not easily quantifiable. Once built, the BN can be used by infrastructure managers as a scenario analysis tool to assess how maintenance decisions on individual bridges affect maintenance costs and travel time for the whole network. The approach is presented on a small-scale bridge network for demonstration purposes.

Keywords: Bridge networks, simulation, dependability, travel time, maintenance costs, system decomposition

1. Introduction

Bridge networks are an essential part of transportation systems which contribute to the urban and economic development in the area of their location. Unavailability of bridges may lead to long diversions and congestions, thus affecting travel time (Orcesi and Cremona, 2010) and air pollution, with consequences on the quality of life. With ageing of bridges, increasing traffic demand and higher user expectations, infrastructure managers seek tools to support maintenance decisions to achieve a trade-off between service performance requirements and maintenance costs.

To address the above needs, this paper presents a simulation supported Bayesian network approach (SSBN) for system level assessment of bridge networks performance with respect to maintenance costs and expected travel time. The resulting model can be used for scenario analysis to support maintenance decisions.

We model the network as a multi-state system which can operate at different performance levels depending on the conditions of its constitutive bridges. To account for dependencies between bridges, we take a network-based approach for the dependeability assessment of bridge networks based on the recently introduced concept of SSBN (El-Awady and Ponnambalam, 2021). We use simulation to obtain probabilistic information to quantify these dependencies. Simulation is an established approach to approximate the performance of complex systems particularly when there is a lack of data. For instance, Huseby and Natvig (2013) developed a discrete-event simultion model to evaluate the availability and criticality of components for a multi-state network flow system of repairable components.

In the next sections, we demonstrate our methodology via application to the illustrative bridge network in Figure 1.

2. Proposed Methodology

We first decompose the system into three layers of system resolution, each corresponding to a layer of the BN model shown in Figure 2. The top layer, L=0, corresponds to the whole system; two system performance variables are placed at the top layer of the BN to indicate the total maintenance costs and the expected travel time for the whole system (based on an arbitrary traveller who goes from a road crossing to another crossing in the network). The next layer of system resolution is layer L=1, where the system is broken down into individual road sections, each containing a bridge. For each road section, we define two random variables indicating the maintenance costs and availability of the road section. Finally, layer L=2 collects the root variables which represent the maintenance options for the bridges, expressed in terms of maintenance rates. The directed arcs between the nodes of the BN indicate the causality relationships between the random variables. As the structure progresses downward from the top layer L=0 to the bottom layer L=2, it will model the dependencies between performance across the three layers from system through road sections down to maintenance options. To quantify these dependencies between the random variables, we need to provide the conditional probabilities associated with each random variable in the BN. To this aim, we resort to simulation. Specifically, we conduct simulations for each random variable at layers L = 0; 1, and their results are used to elaborate condictional dependence of the corresponding random variables (e.g., conditional dependence of “road 1-2 availability” on “maintenance rate 1” in Figure 2).

Overall, the methodology consists of the following three steps. (1) *Step 1*- Model bridge network and, evaluate availability & maintenance costs of each road section with a bridge. (2) *Step 2*- Calculate travel time and costs of the whole network: we estimate system level maintenance costs and expected travel time by means of simulation based on input from *Step 1*. (3) *Step 3*- Build the BN of the bridge network: the simulation results

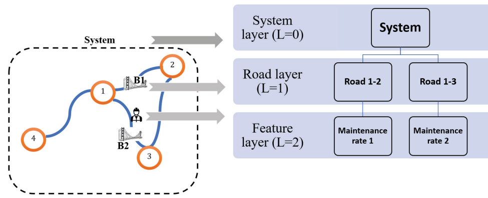

Fig. 1. Th focal bridge network & its corresponding three layers of system resolution.

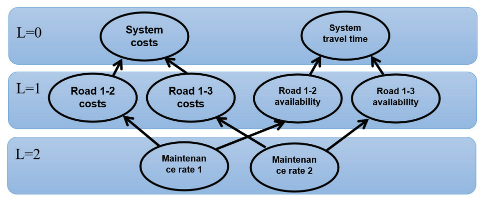

Fig. 2. The BN structure of the focal bridge network in three layers which have direct correspondence with system resolution layers.

from *Steps 1 and 2* are used to derive the conditional probability tables of the BN.

3. *Step 1:* Model bridge network and evaluate maintenance costs & availability of road sections

We model the bridge network as a directed graph $G = \{V, A\}$ with nodes (set V) and edges (set A) representing the road crossing points and the road sections respectively. The graph of the focal bridge network in Figure 1 has four crossing points and four road sections, two of which (sections 1-2 and 1-3) contain one bridge each.

The condition of the bridge affects the availability and maintenance costs associated to the corresponding road section. For the network in Figure 1, bridges $B1$ and $B2$ insist on road sections 1-2 and 1-3 respectively. We model each bridge as a multi-state system and describe the degradation and maintenance process as a Continuous Time Markov Chain (CTMC). The state of the bridge is discrete in space and continuous in time. The bridge evolves through 4 possible states which we indicate with index $z = 1, 2, 3, 4$, each representing a different operative conditions. The state-space diagram of the aformentioned Markov

Chain is shown in Figure 3. State $z = 1$ corresponds to good conditions (full capacity), from which the bridge can deteriorate either to state $z = 2$ with rate $\lambda_{12} = \frac{1}{30}$ where the bridge operates at partial capacity. From degraded state $z = 2$, the bridge can either fail with rate $\lambda_{23} = 0.05$ (and move to state $z = 3$) or jump to state $z = 4$ where the bridge is under maintenance but it is still operated at a partial capacity. From this state bridge conditions are restored to "good" ($z = 1$) with a rate λ_{41}. From the failed state the bridge remains closed until maintenance is executed thus improving bridge conditions to either state $z = 2$ or $z = 1$ with rates $\lambda_{32} = 0.2$ and $\lambda_{31} = 0.1$, respectively. The maintenance rates depend on the value taken by the random variables in layer $L = 2$ of the BN. As maintenance of a bridge on average can take three months (Saydam et al., 2013), in this example, we define maintenance rate 1 associated to road section $1-2$ (indicated by λ_{24}^1) as follows. It can take either of the two values indicated in vector $\mathcal{RM}_1 = \begin{pmatrix} 0.2 \\ 0.5 \end{pmatrix}$: value 0.20 with probability of 0.6, and otherwise, a value of 0.50. Similarly, maintenance rate 2 for road section $1 - 3$, λ_{24}^2 is defined such that it takes one of the two values in vector $\mathcal{RM}_2 = \begin{pmatrix} 0.15 \\ 0.4 \end{pmatrix}$, with a similar probabilities.

With the given definitions, the matrices $\mathbb{Q}_1, \mathbb{Q}_2$ indicate the transition rate matrix of the corresponding CTMC of roads $1 - 2$ & $1 - 3$, respectively. In these matrices, cell $\lambda_{zz'}$ represents the transition rate from state z to state z' given that the current state is state z. In addition, the holding time of a bridge in state z is exponentially distributed with parameter λ_z (the diagonal element of row z is $-\lambda_z$, and $\lambda_z = \sum_{z'} \lambda_{zz'}$).

$$\mathbb{Q}_1 = \begin{pmatrix} \lambda_1 = \frac{-1}{30} & \lambda_{12} & 0 & 0 \\ 0 & \lambda_2 & \lambda_{23} & \lambda_{24}^1 \\ \lambda_{31} & \lambda_{32} & \lambda_3 = -0.3 & 0 \\ 1 & 0 & 0 & \lambda_4 = -1 \end{pmatrix}$$

$$\mathbb{Q}_2 = \begin{pmatrix} \lambda_1 = \frac{-1}{30} & \lambda_{12} & 0 & 0 \\ 0 & \lambda_2 & \lambda_{23} & \lambda_{24}^2 \\ \lambda_{31} & \lambda_{32} & \lambda_3 = -0.3 & 0 \\ 1 & 0 & 0 & \lambda_4 = -1 \end{pmatrix}$$

We assume that the performance of a road section can settle at different levels of its nominal capacity depending on the operative conditions of the bridge. We model the performance of a road section as its operational capacity normalized with respect to its nominal capacity, thus taking values between 0 (closure of the road section) and 1 (full capacity). Therefore for $z = 1$ the road section operates at its full capacity, while in states $z = 2$ and $z = 4$ we assume that the normalized *operational capacity* is 0.6 and 0.4 respectively. We define the steady state availability A_{i-j} of a road section as the steady-state probability that its normalized *operational capacity* as defined above is greater than a given threshold, $A_{i-j} \geq \widehat{A_{i-j}}$. For the sake of illustration here, we assume $\widehat{A_{i-j}} = 0.4$.

As we consider fixed time horizon, we use Monte Carlo simulation to simulate a large number of life histories over a given time horizon $T = 600$ (months) for each bridge (with a warm-up period of 500 during which simulation runs without collecting results). During each run, which is an "observation" of the bridge state evolution over time, we record the realization of the following random variables: number of times a repair transition occurs and the percentage of time that the bridge is in each possible state. By multiplying the first quantity by the cost of rehabilitation, we obtain the maintenance costs for each "observation". The availability of the road sections is expressed as the sum of the percentage of time the bridge is in state $z = 2$ and $z = 4$, weighted by the corresponding normalized *operational capacity*. By doing this for each run, and conducting $n_{sim} = 1000$ runs, we obtain the cost and availability vectors for each road section RC_{i-j} and RV_{i-j}, respectively (where elements of those vectors show observed total costs & availability of each road).

4. *Step 2:* Analyze whole network expected travel time and maintenance costs

I. Markov Chain Traffic Assignment- We first implement the Markov Chain Traffic Assignment approach developed in (Salman and Alaswad, 2018) to model traffic dynamics and calculate the ex-

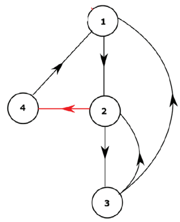

Fig. 3. State space diagram of an bridge with $n = 4$ states.

pected travel time based on the availability vector RV_{i-j} obtained in *step 1*. Indeed the traffic dynamics can be modeled using Markov Chains. First we define the dual graph of G such that every road becomes a node of the dual network; two nodes in the dual network are connected with an edge if their corresponding roads are connected in the bridge network (see the dual graph of the focal bridge network in Figure 4).

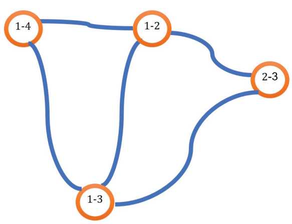

Fig. 4. The dual network of the focal bridge network. The nodes labelled in order to clarify the corresponding road (e.g., node "$1-2$" represents road section $1-2$).

The transition probability matrix $\hat{\mathbb{P}}$ of the Markov Chain (MC) associated to the dual network, is built based on the average travel time of the road sections and the junction turning probabilities as follows (Crisostomi et al., 2011). We define the turning probability from road section a to road section a' as $d^{a,a'}$ (whose value can be estimated based on collected data), and the average travel time as $t^{a,a'}$. The set of average travel times on all road sections is normalized such that the smallest one is 1. The components of the transition probability matrix $\hat{\mathbb{P}}$ are then calculated as follows:

$$\hat{p}_{a,a} = \frac{t^{a,a} - 1}{t^{a,a}}, \tag{1}$$

$$\hat{p}_{a,a'} = (1 - \hat{p}_{a,a}) \times d^{a,a'} \tag{2}$$

where $\hat{p}_{a,a}$ corresponds to the probability of a loop and U-turn, while $\hat{p}_{a,a'}$ indicates transition from roads section a to a'. In this example we assume that traffic enters and leaves the network through road section $1-4$. We consider traffic scenario 1 with the following travel times in minutes: $\{(1-2, 2), (1-3, 4), (1-4, 4), (2-3, 6)\}$[a]. In addition, we assume values of the junction turning probabilities $d^{a,a'}$ and collect them in matrix $\hat{\mathbb{D}}$ (cells across a row sum up to one as they show probabilities of turning from its corresponding road to that of the other rows):

$$\hat{\mathbb{D}} = \begin{array}{c} \begin{matrix} 1-2 & 1-3 & 1-4 & 2-3 \end{matrix} \\ \begin{pmatrix} 0 & 0.2 & 0.4 & 0.4 \\ 0.3 & 0 & 0.3 & 0.4 \\ 0.4 & 0.6 & 0 & 0 \\ 0.7 & 0.3 & 0 & 0 \end{pmatrix} \end{array} \begin{matrix} 1-2 \\ 1-3 \\ 1-4 \\ 2-3 \end{matrix}$$

Then, using equations 1 and 2, transition probability matrix $\hat{P}$ will be obtained:

$$\hat{\mathbb{P}} = \begin{array}{c} \begin{matrix} 1-2 & 1-3 & 1-4 & 2-3 \end{matrix} \\ \begin{pmatrix} 0 & 0.2 & 0.4 & 0.4 \\ 0.15 & 0.5 & 0.15 & 0.2 \\ 0.2 & 0.3 & 0.5 & 0 \\ 0.23 & 0.1 & 0 & 0.67 \end{pmatrix} \end{array} \begin{matrix} 1-2 \\ 1-3 \\ 1-4 \\ 2-3 \end{matrix}$$

The Kemeny constant of this MC provides an estimation of the performance of the bridge network in terms of average expected travel time. Indeed it indicates the average expected travel time from an arbitrary road to a destination chosen randomly. Kemeny constant KK is given by

$$KK = \sum_{a'} m_{a,a'} \pi_{a'} \qquad a \in A \tag{3}$$

where $m_{a,a'}$ is the mean first passage time, and $\pi_{a'}$ is the long run fraction of time that the chain will be in state a'. The mean first passage time from road a to a' is the expected number of transitions that the MC process needs to make to reach road a' (for the first time) when it starts from road a. Note that each road section corresponds to a state in the corresponding MC model, and the

[a]Here, average travel time from two directions on a road (e.g., $2 \to 1, 1 \to 2$) are assumed to be equal.

Kemeny constant is independent of the choice of a[b], see (Crisostomi et al., 2011).

To account for the availability of road sections when calculating the expected travel time in the network, we simply recalculate the entries of matrix $\hat{P}$ corresponding to the road sections with reduced availability as follows. If v'_a is the availability of road sections a', with $v'_a < 1$, then the entries in the corresponding row of matrix $\hat{\mathbb{P}}$ are updated as follows ($\bar{p}_{a,a}$, $\bar{p}_{a,a'}$ indicate updated cells in that row)[c].

(1) If $\hat{p}_{a,a} = 0$, then,

- $\bar{p}_{a,a} = (1 - v'_a)$,
- $\bar{p}_{a,a'} = \hat{p}_{a,a'} \times v'_a \qquad \forall p'_{a,a'} > 0$

(2) If $\hat{p}_{a,a} \neq 0$, then,

- $\bar{p}_{a,a} = \hat{p}_{a,a} + (1 - \hat{p}_{a,a}) \times (1 - v'_a)$,
- $\bar{p}_{a,a'} = \hat{p}_{a,a'} \times v'_a \qquad \forall p'_{a,a'} > 0$

For traffic scenario 1, with road section $a = 1 - 2$ as reduced availability $v'_a = 0.9$, then we obtain the new transition matrix which we call $\bar{\mathbb{P}}$:

$$\bar{\mathbb{P}} = \begin{matrix} & \begin{matrix} 1-2 & 1-3 & 1-4 & 2-3 \end{matrix} & \\ & \begin{pmatrix} 0.1 & 0.18 & 0.36 & 0.36 \\ 0.15 & 0.5 & 0.15 & 0.2 \\ 0.2 & 0.3 & 0.5 & 0 \\ 0.23 & 0.1 & 0 & 0.67 \end{pmatrix} & \begin{matrix} 1-2 \\ 1-3 \\ 1-4 \\ 2-3 \end{matrix} \end{matrix}$$

To model uncertainty with respect to the incoming & outgoing traffic flows, we can consider multiple traffic scenarios. That is, we assume average travel time t on road $1 - 4$ (as a channel through which traffic flows into & leaves from the focal bridge network) can take different values on a finite set of possible scenarios according to a probability distribution. So, in addition to scenario 1, we define another traffic scenario, called "scenario 2" which average travel time on roads (in minutes) in Figure 1 are defined similar to scenario 1 as follows: $\{(1-2, 2), (1-3, 4), (1-4, 8), (2-3, 6)\}$. With these values and using equations 1-2, transition probability matrix will be:

$$\hat{\mathbb{P}}' = \begin{matrix} & \begin{matrix} 1-2 & 1-3 & 1-4 & 2-3 \end{matrix} & \\ & \begin{pmatrix} 0 & 0.2 & 0.4 & 0.4 \\ 0.15 & 0.5 & 0.15 & 0.2 \\ 0.1 & 0.15 & 0.75 & 0 \\ 0.23 & 0.1 & 0 & 0.67 \end{pmatrix} & \begin{matrix} 1-2 \\ 1-3 \\ 1-4 \\ 2-3 \end{matrix} \end{matrix}$$

[b]So, instead of writing $KK(a)$ with $a \in A = \{1-2, 1-3, 1-4.2-3\}$, we write KK.

[c]Note that we have $\sum_{a' \in A} \bar{p}_{a,a'} = \sum_{a' \in A} p'_{a,a'} = 1$.

Next, we model uncertainty of incoming & outgoing traffics, and use that in simulation & calculation of Kemeny constant (expected travel time within the system). Here, we assume that the rate of having scneario 1 follows a Bernoulli distribution with $r = 0.6$ (and scenario 2 occurs with probability $1 - r$ probability). Therefore, if over three simulation experiments, if scenario 1 occurs twice, then for two runs, travel time is estimated by Kemeny constant of the MC with $\hat{\mathbb{P}}$, and only one simulation experiment, it is estimated by the MC with $\hat{\mathbb{P}}'$.

II. Simulation Process- Here we use vectors RC_{i-j} and RV_{i-j} obtained in *Step 1* for each road section as "observed data" from which empirical distributions are constructed, and simulate the bridge network as a system with uncertain inputs using the bootstrap resampling method in (Barton and Schruben, 2001). Formally, we sample $n_b = 300$ values with replacement from that vector, order those resampled values, and call them $\{v_{(1)}, v_{(2)}, .., v_{(n_b)}\}$. Next using the ordered resampled values, we construct a distribution function with $\hat{F}(x_0) = 0$, $\hat{F}(x_{n_b+1}) = 1$, and for intervals $x_{(i)} \leq x \leq x_{(i+1)}$, $\hat{F}(x) = \alpha \times \frac{i}{n_b+1} + (1-\alpha) \times \frac{i+1}{n_b+1}$, where $\alpha = \frac{x_{(i+1)} - x}{x_{(i+1)} - x_{(i)}}$.

For a given traffic scenario, we therefore (1) build the empirical distributions from vectors RC_{i-j} and RV_{i-j} to calculate the estimated availability and maintenance costs (shown by RC^*_{i-j}) for each road section, (2) We specify the incoming/outgoing traffic flow scenario, and apply the MCTA method, where we account for the estimated availability of road sections as seen before to calculate the *Kemeny constant* representing an estimate of the expected travel time within the network, (3) We sum the costs estimate accross all road sections in the network to obtain the maintenance costs for the entire network, and add a stochastic term to account for stochasticity of the coordination costs as $RC^*_{1-2} + RC^*_{1-3} + 2 * Beta(2,5) * 0.1 * (RC^*_{1-2} + RC^*_{1-3})$. In this added term, $Beta(2,5)$ is a random number generated from beta distribution, and multiplication of terms by 2 (i.e., the length of the shortest path between roads $1-2$ & $1-3$ over the bridge network) is included in the equation to represent economic dependencies among maintenance costs of the two roads. We repeat this procedure for 400 runs of simulations, with each run providing "observed" values of travel time and maintenance costs for the network, which are then collected into the vector of expected travel time RT and expected maintenance costs RC for the entire network.

5. *Step 3:* Building Bayesian Network model

We use the results of simulation in *Step 2*, namely vectors RT and RC as a source information to materialize the conditional dependencies among the random variables of the BN model. A Bayesian Network (BN) is a probabilistic acyclic graph with nodes representing random variables and directed arcs representing causal relationships between variables. Formally, a Bayesian Network is a 3-triple $(\mathbb{XB}, \mathbb{GB}, \mathbb{PB})$ where $\mathbb{XB} = \{X^0, X_1^1, X_2^1, \bar{X}^0, \bar{X}_1^1, \bar{X}_2^1, X_1^2, X_2^2\}$ is a set of nodes which represent random variables.

Variables $X^0, \bar{X}^0$ indicate system costs & travel time at layer $L = 0$ (the superscript number of a variable shows the layer to which it belongs). Similarly, variables X_1^1, X_2^1 $(\bar{X}_1^1, \bar{X}_2^1)$ represent costs (availability) of roads $1-2$ & $1-3$, respectively. Lastly, variables X_1^2, X_2^2 indicate maintenance rates 1 and 2.

Also, $\mathbb{GB}$ is a set of directed edges and, set $\mathbb{PB}$ provides conditional probability densities that characterise conditional dependence among variables,
$\mathbb{PB} = \{f(X^0|X_1^1, X_2^1), f(X_1^1|X_1^2), f(X_2^1|X_2^2), f(\bar{X}^0|\bar{X}_1^1, \bar{X}_2^1), f(\bar{X}_1^1|\bar{X}_1^2), f(\bar{X}_2^1|X_2^2)\}$.

As mentioned in the previous section, the components of vectors RT and RC are treated as "realizations" of the random variables $X^0, \bar{X}^0$ which indicate system costs and travel time at layer $L = 0$ of the BN. Similarly, the components of vectors RC_{i-j} and RV_{i-j} are treated as "realizations" of the maintenance costs and availability of road section $i - j$. To identify discretization intervals for each random variable in a wise manner, we fit a probability density function $\hat{g}$ to these vectors based on Maximum Likelihood method. For each of the obtained density functions, we calculate the High Density Region (HDR) as follows (Hyndman, 1996). The $100(1-\beta)\%$ HDR is the subset $J(\hat{g})$ of the sample space of random variable X such that $J(\hat{g}) = \{x : \hat{g}(x) \geq g_\beta\}$, where $\hat{g}_\beta$ is the largest constant such that $P(X \in J(\hat{g})) \geq 1 - \beta$ with $\beta = 0.6$.

For the sake of illustration, we then divide the sample space of each random variables at layers $L = 0, 1$ into $n_d = 3$ intervals. We divide subset $J(\hat{g})$ of the sample space into $n_d - n_x$ intervals where n_x is the number of segments/intervals between regions of the identified subset $J(\hat{g})$ $(n_d >> n_x)$. For example, if $J(\hat{g}) = (0.1, 0.4) \cup (0.8, 0.9)$ for random variable $\bar{X}_1^1$ with sample space of $x \in [0, 1]$, then, $n_x = 3$ (as there are three intervals among HDR intervals, $(0.0, 0.1]$, $[0.4, 0.8]$, and $[0.9, 1)$).

For each random variable we use the discretization intervals to define the corresponding discretised data vector. We define function $\mathcal{MD}$ mapping each random variable to its discretized data vector $(u = 1, 2)$: $\mathcal{MD}(X^0) = \mathcal{RC}$, $\mathcal{MD}(\bar{X}^0) = \mathcal{RT}$, $\mathcal{MD}(X_u^1) = \mathcal{RC}_u$, $\mathcal{MD}(\bar{X}_u^1) = \mathcal{RV}_u$. Moreover, using the identified discretization intervals for variables $X_1^1, X_2^1, \bar{X}_1^1, \bar{X}_2^1$, function $\mathcal{MD}'$ maps these variables to the relevant discretised input vactor of step 2 $(u = 1, 2)$: $\mathcal{MD}'(X_u^1) = \mathcal{RC}'_u$, $\mathcal{MD}'(\bar{X}_u^1) = \mathcal{RV}'_u$.

We can now estimate the conditional probabilities by using maximum likelihood estimators (see page 299 Pousi et al., 2013) as follows:

$$P(X^0 = \mathfrak{d}|X_1^1 = \mathfrak{d}_1, X_2^1 = \mathfrak{d}_2) = \begin{cases} \frac{\mathcal{NN}}{\mathcal{N}} & \mathcal{NN}, \mathcal{N} \geq 0 \\ \frac{1}{n_d} & \mathcal{N} = \mathcal{NN} = 0 \end{cases}$$

where $\mathfrak{d}, \mathfrak{d}_1, \mathfrak{d}_2 \in \{0, 1, 2, .., n_d - 1\}$ are integers which we use to refer to the interval within which the corresponding random variable takes its value (e.g. $X_1^1 = \mathfrak{d}_1 = 2$ means that random variable X_1^1 takes value within the third interval). $\mathcal{N}$ indicates the number of simulation records within vectors $\mathcal{MD}'(X_1^1), \mathcal{MD}'(X_2^1)$ with $X_1^1 = \mathfrak{d}_1, X_2^1 = \mathfrak{d}_2$, while $\mathcal{NN}$ indicates the number of simulation records within vectors $\mathcal{MD}(X^0), \mathcal{MD}'(X_1^1), \mathcal{MD}'(X_2^1)$ with $X^0 = \mathfrak{d}, X_1^1 = \mathfrak{d}_1, X_2^1 = \mathfrak{d}_2$. Furthermore, in the case of having $\mathcal{N} = \mathcal{NN} = 0$, then, we consider lack of information about that conditional probability, and therefore, each of the n_d possible values for the random variable conditioned on the other variables are considered to have an equal likelihood. This approach is used for calculation of other conditional probabilities.

6. Results and discussion

The results obtained are illustrated in Table 1 and Figures 5-6. Table 1 summarizes the intervals within which each random variable in the BN can take its value when $n_d = 3$. Figures 5-6 summarize the structure (see Figure 2) and the marginal probabilities of all the values taken by the corresponding nodes. As the value of cell λ_2 in matrix $\mathbb{Q}_1$ is higher than that of $\mathbb{Q}_2$ (elements of vector $\mathcal{RM}_1$ have higher values than those of $\mathcal{RM}_2$), we expect bridge $B1$ to have a shorter holding time in maintenace than bridge $B2$ (mean of exponential distribution with paramter λ is $\frac{1}{\lambda}$). In this line, while the BN variables related to road $1-2$ $(X_1^1, \bar{X}_1^1)$ and that of road $1-3$ $(X_2^1, \bar{X}_2^1)$ have similar marginal distribution, we observe higher availability & lower cost intervals for the former group than those of the latter ones. In addition, one may note that the derived intervals in Table 1 are subset of possible intervals for a variable (e.g., for $\bar{X}_1^1$, $(0.81, 1] \subset [0, 1]$) as they are obtained from simulation. Lastly, the reader may note that the shown travel times are in the number of steps of the MC associated with the dual network, and since travel times on roads are normalized, and the shortest was 2 minutes, then, the shown travel times should be multiplied by 2 to get travel times in minutes.

As mentioned earlier, the developed BN model can be used for scenario analysis. For instance, we used the "cpquery" function in "bnlearn" library in R to estimate the conditional probability of having system travel time be less than 7.47*2 minutes ($\bar{X}^0 \in 0, 1$) and cost be in range $(2873.52, 5747.04)]$ ($X^0 = 1$) given that the desired periodic preventive maintenance rate 1 is 0.2 ($X_1^2 = 0$). Those estimations of $P(\bar{X}^0 \in 0, 1 | X_1^2 = 0)$, $P(X^0 = 1 | X_1^2 = 0)$ based on 1000 samples of the conditional probabilities are shown in Figure 7.

Table 1. BN variables, intervals & values.

Var	Interval	$\partial, \partial_1, \partial_2$
Road 1-3, availability: $\bar{X}_2^1$	(0.78, 0.86]	0
	(0.86, 0.93]	1
	(0.93, 1.0]	2
Road 1-2, availability: $\bar{X}_1^1$	(0.81, 0.88]	0
	(0.88, 0.94]	1
	(0.94, 1.0]	2
Road 1-3, cost: X_2^1	(0.0, 1962.93]	0
	(1962.93, 3925.87]	1
	(3925.87, 5888.8]	2
Road 1-2, cost: X_1^1	(0.0, 1475.44]	0
	(1475.44, 2950.88]	1
	(2950.88, 4426.32]	2
System time: $\bar{X}^0$	(5.6, 6.53]	0
	(6.53, 7.47]	1
	(7.47, 10.41]	2
System cost: X^0	(0.0, 2873.52]	0
	(2873.52, 5747.04]	1
	(5747.04, 8620.56]	2

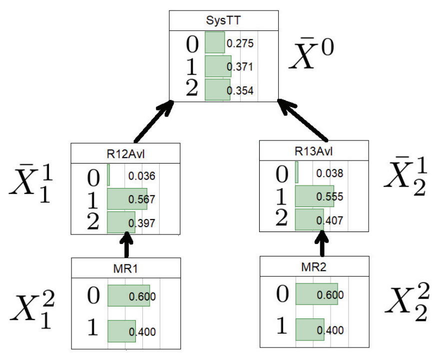

Fig. 5. BN for travel time with related marginal distributions.

7. Conclusions

This paper presents a simulation supported Bayesian Network modeling approach to assess the performance of bridge networks. The approach has been demonstrated on a small scale bridge network. It can be used by infrastructure managers as a scenario analysis tool to support maintenance decisions. Once built, the BN allows to model how the effects of different maintenance rates at bridge level, propagate accross the network thus affecting travel time and maintenance costs. For applicability to large scale networks,

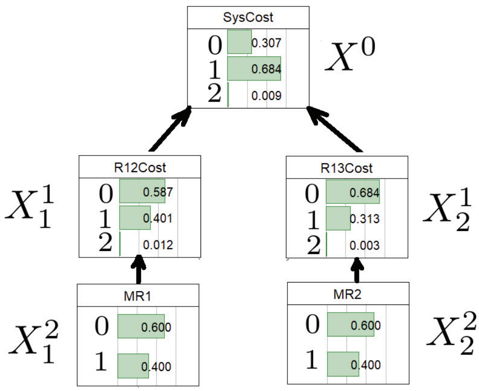

Fig. 6. BN for maintenance costs and related marginal distributions.

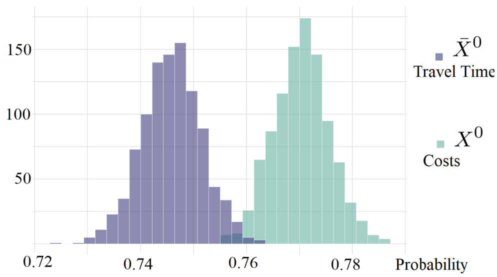

Fig. 7. Estimated probabilities of $\bar{X}^0 \in 0, 1$ & $X^0 = 1$, conditioned on $X_1^2 = 0$.

we will further introduce an additional step to the methodology which uses MCTA and the Kemeny constant to identify subnetworks which can be considered as independent according to their traffic dynamics. In the BN model this will translate into an additional layer between the "system" layer (whole network, currently layer $L = 0$) and the road sections layers (currently $L = 1$).

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