Exact time and Galerkin space formulation for circuit coupling problems with pulse excitation

Exact time and Galerkin space formulation for circuit coupling problems with pulse excitation

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Summary. This paper presents a coupled electric circuit and finite element formulation for windings with increased exposure to losses due to pulsed-wave modulation (PWM) operation. It exploits the exact solution of a first-order ODE system and allows to decouple the spatial and temporal problems aiming to reduce the computational burden. A test case is implemented with the proposed approach and compared with a finite element formulation.

1 Introduction

The control of the energy flow in an Electrical Energy Conversion Unit (EECU) by modulating the voltage and current sources with power electronic switches is widely used. The parasitic effects such as significant winding losses, noise, accelerated ageing of insulation, to name only a few, become dominant. It is crucial to account for these effects at the design stage. The continuously increasing switching frequency is challenging the EECU models. Customised basis functions, parallel-in-time simulations are several approaches to compute these models more efficiently [1, 2]. Already at the switching frequency of tens kHz, thin conductors such as litz wire and foil are used to minimise the effect of skin depth and maximise the useful area of the conductor cross-section [3]. For many EECU applications, these phenomena are dominating in contrast with magnetic saturation or core losses.

This paper proposes an efficient method aiming at accurately resolving the eddy currents in conductors. To this end, the voltage is assumed constant and material properties linear during a switching interval. This allows expressing the coupled finite element model with the electrical circuit as a first-order ODE system for which the exact solution is known. The results are compared with a finite element approach for the obtained ODE system. The reflection in terms of problem size and computational complexity is provided.

2 Coupling of vector potential and circuit

The test case is depicted in Fig. 1. The geometry of the coil is considered sufficiently long in $\vec{z}$ direction to model it in a two-dimensional problem. The Dirichlet

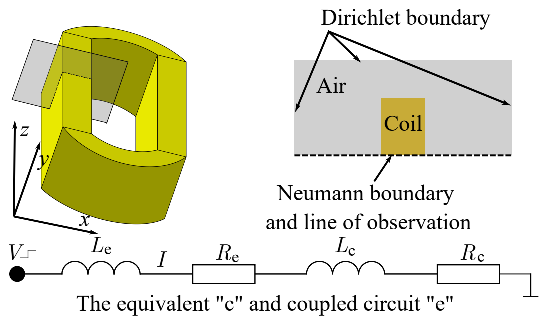

Fig. 1. The graphics of the coil, its two-dimensional simplification and the coupled equivalent circuit

and Neumann boundary conditions are applied to account for symmetry. The coupling of the vector magnetic potential and the equivalent circuit of the coil together with external elements of the circuit is implemented with the AVI formulation [4]. For the 2D case, only the $\vec{z}$ component of the magnetic vector potential A_z is considered

$$\nabla(\nu\nabla A_z) = \sigma\left(\frac{V_c}{D} - \frac{\partial A_z}{\partial t}\right) \tag{1}$$

where ν is the reluctivitty, σ is the conductivity and D is the depth of the domain. The voltage V_c arises from the scalar potential and couples with external circuits. The PWM voltage source V, external elements R_e and L_e are introduced as shown in Fig. 1

$$V = V_c + R_e I + L_e \frac{\partial I}{\partial t}. \tag{2}$$

Lastly, the total current is formed by the induced eddy currents and the current emerging from the voltage drop of the conducting region

$$I = \frac{V_c}{D}\int_{\Omega_{\text{coil}}} \sigma \, \mathrm{d}s - \int_{\Omega_{\text{coil}}} \sigma \frac{\partial A_z}{\partial t} \mathrm{d}s. \tag{3}$$

2.1 Numerical formulations

A spatial discretization of equation 1 combined with equations 2 and 3 has the following structure:

$$M\frac{\partial u}{\partial t} = Su + f, \tag{4}$$

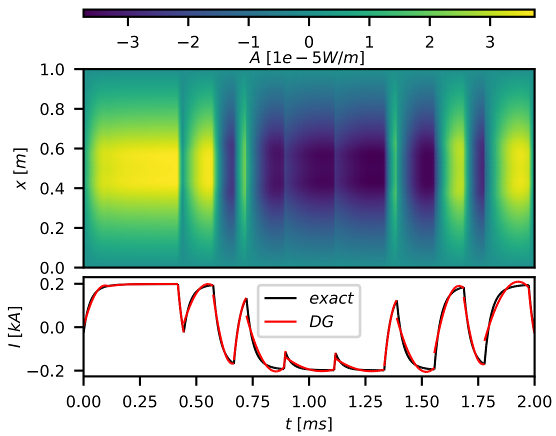

Fig. 2. Obtained results with 9 pulses and $N = 5$ elements per spatial dimension compared with DG approach

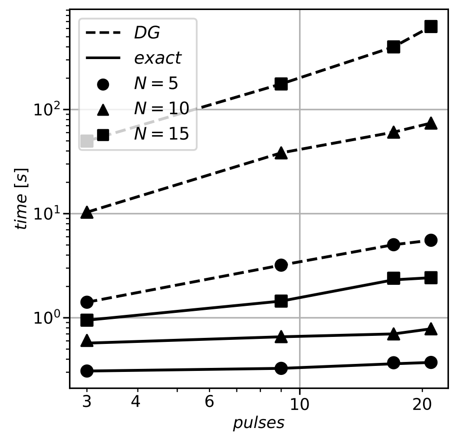

Fig. 3. Computational burden of switching frequency and spatial refinement

where u is the vector of unknowns, M and S are constant matrices and f is a vector that is constant per switching interval.

Exact solution

We separate u, M, S and f into time-dependent (subscript n) and time-independent (subscript z) components and rewrite equation (4) as

$$M_{nn}\frac{\partial u_n}{\partial t} = S_{nn}u_n + S_{nz}u_z + f_n, \quad (5)$$

$$M_{zn}\frac{\partial u_n}{\partial t} = S_{zn}u_n + S_{zz}u_z + f_z. \quad (6)$$

By expressing u_z from (6) into (5) the following reduced system of equations is obtained, depending only on u_n:

$$M_r\frac{\partial u_n}{\partial t} = S_r u_n + f_r. \quad (7)$$

The analytical solution of (7) starting at $t = t_0$ until f changes, is given by

$$u_n = Q\exp(\Lambda(t - t_0))w - S_r^{-1} f_r. \quad (8)$$

The vector w depends on the initial condition. The diagonal matrix Λ contains eigenvalues and matrix Q the eigenvectors satisfying the generalised eigenvalue problem.

Discontinuous Galerkin

To compare the performance of the proposed approach, we use a Discontinuous Galerkin discretisation of equation (4) with an upwind flux and Legendre basis functions spanning the switching intervals.

3 Preliminary results and reflections

The AVI formulation detailed in section 2 is applied to the test-case introduced in this paper. The results obtained with exact time solution are shown in Fig. 2, where A_z is captured from the observation line as shown in Fig. 1. The resulting current is compared with DG implementation from 2.1. Both methods use standard second order polynomial shape functions for spatial discretisation. The heavy eigenvalue decomposition is performed once and used in an optimisation problem to find the initial solution vector w in each duty cycle. In contrast, DG describes the solution using basis functions in time, scaling the model with the degree of the basis. The scaling of the computational burden with the number of pulses and mesh elements is shown in Fig. 3. The preliminary results show that the proposed approach has a significant performance advantage over DG. In the full paper, a detailed analysis in terms of dimension of the problem, computation burden and necessary pre-computations required will be provided.

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