Probabilistic Constellation Shaping Algorithms: Performance vs. Complexity Trade-offs

Citation for published version (APA):
Gültekin, Y. C., & Alvarado, A. (2022). *Probabilistic Constellation Shaping Algorithms: Performance vs. Complexity Trade-offs: Performance vs. Complexity Trade-offs*. Paper presented at 42nd WIC Symposium on Information Theory and Signal Processing in the Benelux, SITB 2022, Louvain-la-Neuve, Belgium.

Document status and date:
Published: 27/07/2022

Document Version:
Publisher’s PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
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Probabilistic Constellation Shaping Algorithms: Performance vs. Complexity Trade-offs

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***Abstract*—We review the recent advances in the design of probabilistic shaping algorithms. We investigate the implementation complexity of these algorithms in terms of required storage and computational power. We show that (1) the optimum performance can be achieved via different algorithms creating a trade-off between storage and computational complexities, and (2) a significant reduction in complexity can be achieved via the recently-proposed *shift-based band-trellis enumerative sphere shaping* if a slight degradation in performance is tolerated.**

***Index Terms*—Probabilistic Amplitude Shaping, Enumerative Coding, Implementation Complexity.**

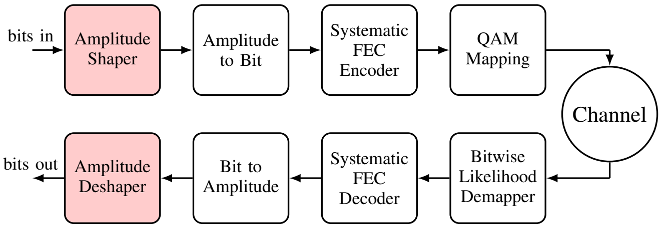

Fig. 1. PAS block diagram. Red blocks are the focus of this paper.

I. EXTENDED ABSTRACT

Probabilistic amplitude shaping (PAS) [1] combines an amplitude shaper with a forward error correction (FEC) code, and achieves the capacity of the additive white Gaussian noise (AWGN) channel [2], [3]. The function of the amplitude shaping block is to generate the amplitudes of the channel inputs while a systematic FEC encoder determines their signs as shown in Fig. 1. Popular amplitude shaping algorithms which are optimum for the AWGN channel include constant composition distribution matching (CCDM) [4], enumerative sphere shaping (ESS) [5], multiset-partition distribution matching (MPDM) [6], shell mapping (SM) [7], etc. This optimality is in the sense that the resulting channel input distribution approaches the Gaussian distribution for large shaping blocklength N and large constellation cardinality M.

The objective when designing an amplitude shaper is to obtain a certain characteristic (e.g., fixed composition, small average energy, small energy variation, low kurtosis, etc.) for the channel input sequences with (1) low storage complexity, and (2) low computational complexity. For the AWGN channel, this objective is to obtain a (sampled) Gaussian-like channel input distribution, i.e., the Maxwell–Boltzmann (MB) distribution. CCDM, for instance, generates amplitude sequences with a fixed composition which is obtained by quantizing the MB distribution. On the other hand, ESS and SM, both sphere shaping algorithms, generate amplitude sequences such that the resulting signal space has an N-spherical shape, which in turn indirectly induces an MB-like distribution.

For other types of communication channels, different input distributions or signal space structures may be more advantageous to improve performance. As an example, the nonlinear interference generated during the propagation of the channel input waveform over the nonlinear fiber channels has been shown to depend on the fourth-order standardized moment of the input distribution (i.e., kurtosis) [8], [9], or on the energy variations in the input waveform [10], [11]. Accordingly, we have recently proposed a modified version of ESS, kurtosis-limited ESS (K-ESS), to generate shaped input sequences with low kurtosis [12]. Then in [13], we have proposed another modified version of ESS, band-trellis ESS (B-ESS), to generate sequences with small energy variations. We have demonstrated that K-ESS and B-ESS provide higher signal-to-noise ratios (SNRs) and increased achievable rates concerning uniform signaling and AWGN-optimal shaping.

On the practical side, a bounded-precision (BP) implementation method was proposed for ESS and SM in [14] to decrease their high storage and computational full-precision (FP) complexities, resp. This method can be applied to ESS, K-ESS, B-ESS, and in fact, to any enumerative-coding-based shaping algorithm. In [15], a finite-precision (FiP) implementation was proposed for arithmetic-coding-based DM algorithms. This technique can be applied to CCDM, MPDM, and in fact, to any DM algorithm that has an underlying arithmetic encoder. Then in [16], an on-the-fly (OtF) computation method was proposed for ESS, creating a trade-off between its storage and computational complexities. In [17], a logarithmic-domain implementation was introduced for arithmetic-coding-based CCDM such that high-precision multiplications and divisions required in the algorithm are replaced with low-precision addi-

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The work of Y.C. Gültekin and A. Alvarado has received funding from the ERC under the EU's H2020 programme via the Starting grant FUN-NOTCH (ID: 757791) and via the Proof of Concept grant SHY-FEC (ID: 963945).

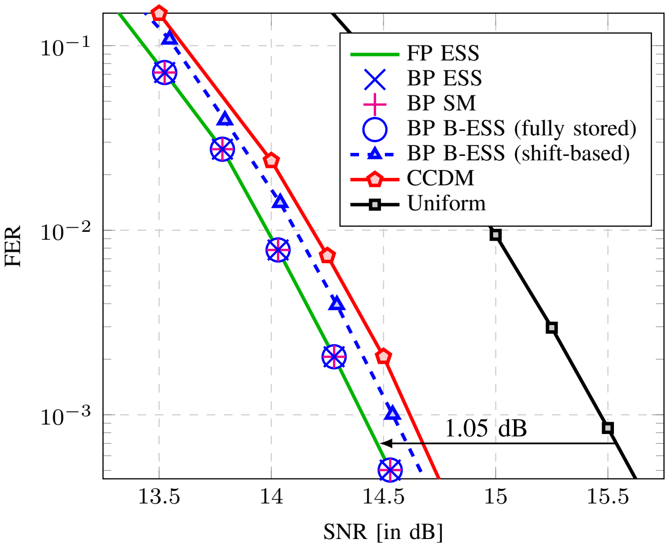

Fig. 2. FER vs. SNR for 64-QAM at the PAS transmission rate of 4 bit/2-D. All shaping schemes use a blocklength of $N = 216$. FEC is based on IEEE 802.11's LDPC codes.

tions and subtractions. Finally, in [18], the implementation of B-ESS was discussed and an OtF computation technique was provided based on binary shifts such that the required storage is independent of the shaping blocklength. The theses [19], [20] provide a good overview of the implementation of DM algorithms, while [21] provides a discussion on the complexity of various shaping algorithms.

In this work, we investigate the performance vs. complexity trade-offs of some of the above-mentioned shaping algorithms. We show in Fig. 2 that the optimum performance can be obtained with different algorithms from different parts of the storage vs. computational complexity spectrum as shown in Fig. 3. We also show that if a slight performance loss is tolerated, our shift-based B-ESS has significantly-reduced complexity concerning other enumerative algorithms.

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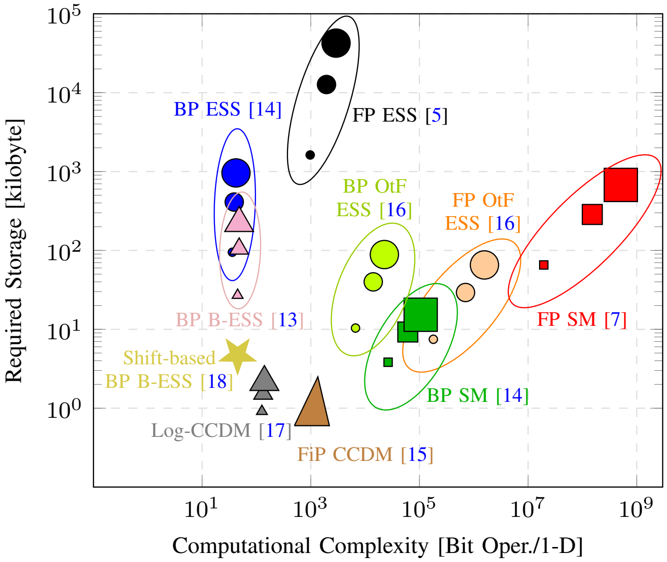

Fig. 3. Maximum computational complexity vs. maximum required storage of amplitude shaping, modified and extended from [21, Fig. 12]. Size of the markers are proportional to the corresponding blocklength $N \in \{216, 432, 648\}$.

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