The purpose of handwriting with tablet-computers and smartpens in mathematical group work over distance

THE PURPOSE OF HANDWRITING WITH TABLET-COMPUTERS AND SMARTPENS IN MATHEMATICAL GROUP WORK OVER DISTANCE

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In traditional group work in mathematics, handwriting is a relevant element to enable reasoning, for instance, by supporting the generation of ideas or the storing of information. However, as COVID-19 has forced students to learn mathematics over distance, trad½itional handwriting cannot be used anymore during group work. To address this issue, this exploratory study investigated the question of how students can use handwriting in a mobile-learning setting via Zoom, in which students use tablets and smartpens to collaborate over distance. It was found that, compared to traditional group work with pen-and-paper, the distance collaboration setup allows for handwriting to become a synchronous collaboration tool, for example, enabling the individual development of ideas that can be extended by peers. More research is needed to investigate the particular epistemic role of handwriting and, particularly, the role of handwriting with smartpens in distance collaboration settings.

Keywords: Distance collaboration, mathematical reasoning, mobile learning, writing.

INTRODUCTION

Writing in university mathematics has different functions. Firstly, as a means of documenting and consolidating work (Heintz, 2000), secondly, to communicate with peers, and thirdly, to publish in academic journals (Burton & Morgan, 2000). During collaborative activities where learners negotiate mathematics, writing has a communicative purpose, as it allows learners to realize mathematical objects through symbolic, graphical, or concrete representations (Duval, 2006) and also through vernacular language (Sfard, 2008). For instance, the area of a triangle is not a tangible entity but can be realized as an object through the symbolic representation $A = \frac{1}{2} \cdot a \cdot h$ or by drawing an arbitrary triangle. In this form, writing is mainly happening in the form of handwriting to document mathematical work. Accordingly, handwriting in mathematics is a reasoning tool that allows learners to document their work with mathematical objects, and through this, make this work applicable for negotiations and reasoning.

Traditionally, mathematical handwriting is based on pen-and-paper. As such, it was found that blackboards and paper are very relevant for communicating mathematics in a face-to-face situation, as they provide material links to previous mathematical reasoning, which helps to avoid communicational breakdowns (Misfeldt, 2006). It was also found that computers do not help in this same way in face-to-face communications (Misfeldt, 2006).

With the ubiquity of tablet computers, which provide smart pen functionality, this stance towards computers for facilitating handwriting in communication needs to be revisited. Particularly, with the COVID-19 pandemic, students were forced to collaborate over distance, being relegated to use computers for communicational purposes, even though these might be counterproductive for mathematical communication. In fact, the notion of face-to-face communication changes now that distance collaboration tools such as MS Teams or Zoom are being used frequently, which allows face-to-face communication over distance. Yet, there is little research on how writing functions in these "new" face-to-face collaborations over distance in a university mathematics setting.

THEORY

With respect to the functions of handwriting in mathematics, Misfeldt (2006) provides a categorization of the purposes of traditional handwriting in mathematics, of which the following are relevant for the here investigated issue:

1. *Heuristic treatment:* Learners heuristically come up with ideas, try them out and make connections between them.
2. *Control treatment*: Learners engage in a deeper investigation of their heuristic ideas. Control treatment can take the form of an investigation of a proposition or an open-ended investigation, for example, by means of performing a calculation.
3. *Information storage*: Learners write in order to save information for later access and use.
4. *Communication:* Learners use handwriting for communication in various forms, such as annotating existing text or commenting on ideas (Misfeldt, 2006, p. 27).

In particular, Misfeldt found that the communicative function of handwriting can come in the form of public or private communication. The public function of handwriting consists of students using written signs to communicate an idea or previously written signs as a deictic or gestural reference in oral communication. The private function of handwriting consists of learners using writing to create a private space for developing new ideas on their own, without immediately making these ideas public to the rest of the group (Misfeldt, 2006).

With respect to hybrid collaboration, the incongruence between computer code for writing formulas/diagrams and the conventionalized mathematical formulas that can be easily used during handwriting can lead to a breakdown of communication, hindering learners from using computer writing for public and private communication (Misfeldt, 2006). In other words, hybrid collaborations with traditional computers without touch functionality can hinder students' collaboration in mathematics, as it does not allow for convenient use of writing for the above-described four functions of mathematical writing, and in particular, writing for public communication purposes.

Yet, mobile technology has changed the notion of computers, allowing for new forms of collaborative learning (Schuck et al., 2017). Mobile technology enables hybrid forms of communication, where students can collaborate over distance, as learners can see each other, screens can be shared (Pegrum et al., 2013), material can be distributed, or questions can be discussed via social media (Simonova, 2016) or tools such as MS Teams/Zoom. This form of hybrid collaboration is further supported by having immediate access to digital resources through a mobile network connection. As a result, learning can occur at any place or at any time, in collaboration with peers or even experts all over the world (El-Hussein & Cronje, 2010; Pegrum et al., 2013).

Thus, with tablet computers and smartpens, the problem of the inconvenience of using writing for public communication purposes could be alleviated, as it provides a convenient way for students to use handwriting in distance collaboration setups. However, there is a lack of research that addresses the issue of how university students use handwriting during distance collaboration with tablet computers and smartpens.

Therefore, this paper discussed the following research question:

With what purposes do students use handwriting in distance collaboration settings, where the distance collaboration is implemented with tablet computers and smartpens?

METHODOLOGY

In an exploratory case study, five groups of two university students (Groups A–E) volunteered to take part in the study presented here. The students were asked to work collaboratively on a proof in vector geometry, which was a familiar topic for students. They were recruited from a first-year course on linear algebra, which was taught in English as Medium of Instruction at a technical university in the Netherlands. The students were mostly Bachelor students of applied mathematics or computer science. They were asked to collaborate over distance in a Zoom meeting, using iPads and smartpens. Students were asked to work in English, but Dutch-speaking students sometimes used Dutch during their work. The students reported that they experienced this setup as highly relevant because at the time of the study, they experienced a hard lockdown with limited opportunities for collaboration.

For the purpose of this study, a variety of tools to enable Distance Collaboration was used:

- *Web conferencing tool:* Zoom (iPad app).
- *Tablet:* Apple iPad tablets.
- *Smart Pen:* Apple Pencils.
- *Online whiteboard:* Students worked on a shared PDF-file, which acted as a whiteboard for enabling handwriting.
- *Keyboard:* Apple smart keyboards (to enable proof-writing).
- *Further resources:* The lecture script from the linear algebra course was given to students in the form of a PDF that they could access on the iPad.

The distance collaboration was simulated in a laboratory setting by asking students to collaborate from different rooms on campus. The data collection was realized with the video recording function of the Zoom app, resulting in videos where students' conversations and their writing on the white-board were captured simultaneously. Zoom was chosen because it provided the functionality of screen recording for data analysis purposes, the possibility to use a PDF as a whiteboard so that the task could be displayed on the students' writing space, and the integration of handwriting via a smartpen.

The students' collaborations were supervised by an interviewer, who provided students with help for using Zoom as well as content-related hints to ensure students' continued engagement with the proving task. The students were asked to prove the theorem shown in Figure 1.

Definition: A medial triangle of a triangle *ABC* is the triangle with vertices at the midpoints of the triangle's sides AB, AC and BC.

Theorem: The orthocenter of the medial triangle *DFE* of an arbitrary triangle *ABC* is the circumcenter of triangle *ABC*.

Figure 1. Theorem that students were asked to prove

Afterwards, the video data was transcribed. The transcripts were analyzed using inductive content analytical methods (Mayring, 2015). Firstly, in a segmentation step, instances of use of handwritings were identified, and the associated episodes inventoried. Secondly, by constant comparing and contrasting of these episodes with respect to the writing purpose (who is being addressed in the text), two main categories were found (general vs. mathematical purpose). Thirdly, all episodes falling under one category were further analyzed, resulting in a typology of episodes with similar usages of handwriting during distance collaboration. In this step, the above-described list of purposes of writing in mathematics was used as sensitizing concepts. Accordingly, the episodes below represent idealisations of purposes of handwriting, with some groups utilizing multiple purposes at different times in their work. The different types of uses of handwriting are presented in the following.

RESULTS

Overall, the analysis found two different purposes for handwriting during distance collaboration on mathematical proving. Firstly, students used handwriting with smartpens for the general purpose to enable or manage the process of collaboration (see General Purposes 1 and 2). Secondly, students used handwriting with smartpens for a decidedly mathematical purpose, namely the purpose of mathematical problem solving, in line with notions that conceptualize proving as a problem-solving process (see Mathematical Purposes 1 to 3).

4.1. General Purpose 1: Explicit Reference Tool to Enable Communication in the Collaboration Process

It was found that handwriting can be a general reference tool to capture ideas in the process of collaboration, that is, with a general purpose. During this use of handwriting, students simultaneously use handwriting and oral communication, verbalizing their writing while they write. This use of handwriting seems to support information storage and public communication at the same time, allowing the other student to follow the thinking process of the other student. However, there was only one instance of this use in the data.

Group B, Minutes 38:13 – 39:44

Dirk: Yeah, the green ones are easier, so [refers to green lines in a previously drawn triangle]

Carl: [writes more and reads aloud what he writes]

Dirk: Good point. How do you find CDE?

Carl: You have to find the intersection point of AF and then … let's take BD [carries on writing and reading loud]. I have to make the line BD, right?

In the episode, a student represents the vertices of a triangle, using handwriting to express vertices in their vector representation. Here, the use of handwriting as a reference tool allows the student Dirk to follow the other student Carl. This enables both students to ask each other questions and to answer them. Here, students also refer to their drawing, in this case as "the green ones" with which he refers to green elements in their drawing.

4.2. General Purpose 2: Structuring the Collaboration Process

In this category, students use handwriting with the explicit general purpose of structuring their collaboration process. In these episodes, the students talk about how to best capture their work. In other words, the students work on the metalevel of how to best use handwriting during their proof writing. As the transcript below illustrates, the student Esha explicitly names the function of handwriting for accomplishing a "more concrete idea of what we are doing".

Group C: Minutes 43:59 – 44:09

Esha: Yeah, I think you can start to write it down, so we have a more concrete idea of what we are doing.

Faiza: Wait, I want to first draw [erases]. I am going to draw a non-conventional triangle. [starts drawing and writing]

This episode reminds of the function of handwriting for heuristic treatment, as the student Esha explicitly mentions that they intend to form a more concrete idea.

4.3. Mathematical Purpose 1: Explanation of Mathematical Thinking to Partner

With a specific mathematic purpose, handwriting was used to explain a mathematical concept to the other student. This purpose seems to be a form of public communication. The case below highlights this purpose on the example of the definition of the orthocenter. In the episode, the student Hendrik makes use handwriting to make a drawing of the geometrical situation in the given proving task, in order to explain his thinking process to his partner Gemma.

Group D: Minutes 19:24 – 20:56

Hendrik:	Yeah, I could not find orthocenter either. Ah, yes, okay. I found it. The orthocentre is the point … wait, I will just draw it. [draws]
Hendrik:	Yeah, the altitudes of the triangle passing through a common point. So it is… [draws] and, that one [draws]. The orthocenter of the medial triangle, the circumcentre of the triangle ABC… the medial triangle. Circumcenter. Perpendicular bisector. The line should draw the perpendicular bisector, right?

Notably, in this episode, the student also uses the digital environment to connect different sources for meaning-making, namely the lecture script to look up a definition of orthocenter and handwriting to realize the definition in their drawing of a triangle. Hence, handwriting and the digital environment contribute to each other for the benefit of the students' collaborative mathematical reasoning.

The presented function reminds of the purpose of handwriting for public communication, as proposed by Misfeldt (2006). Here, this purpose of handwriting is tightly ingrained into the overall reasoning process of using established knowledge to generate ideas, enabled by a digital resource (the lecture script).

4.4. Mathematical Purpose 2: Visualization of Mathematical Processes

In the second mathematical purpose, students use handwriting to generate a drawing that represents the situation described in the proving task. As can be seen below, initiated by talk to structure the reasoning process (Marc in turn 1), the student Marc begins to draw a triangle. During this process, the students try to understand the concepts in the task description (orthocenter, medial triangle, circumcenter). As the student Marc is thinking aloud, his partner Lisa can contribute to this process (Turn 4).

Group E, Minutes 16:14 – 16:59

Marc:	First, let's draw a triangle? [draws]
Lisa:	Yeah.
Marc:	The definition here. [writes] I think. The orthocenter of the medial triangle DFE of an arbitrary [reads]. Erm, what is an orthocenter?
Lisa:	Orthocenter, I think it's orthogonal.
Marc:	Ah! Of the medial triangle DFE. So the medial triangle is so [draws]. Is the circumcenter of triangle…

Similar to the previous episode, the students use handwriting for public communication and for storing information. Particularly, the students use the drawing process as a means to understand the given task. This public communication ensures that the partner who is not writing can contribute to the process and possibly check whether the concepts in the task have been adequately realized in the other student's writing.

4.5. Mathematical Purpose 3: Semi-Private Reasoning

It was expected that in the distance collaboration setting, there would be few opportunities for students to use handwriting to create private spaces. Yet, contrary to this expectation, handwriting in the distance collaboration setup was also used for creating room for individual reasoning. The following episode highlights how the student Esha developed her reasoning, supported by handwriting. The other student, Faiza, gave Esha room to develop her thoughts. At the same time, as Esha's reasoning was not private in the actual sense (that is, the other student can see the writing), Faiza can build on Esha's reasoning afterwards.

Group C: Minutes 18:47–21–57

Esha: You want to compare the angles? I don't [think it] will actually work. So, basically, if we take the center to be O… So far, let's just assume that this is the circumcenter. So we have to [incomprehensible] the orthocenter is also the circumcenter. The orthocenter of DFE is the circumcenter of ABC. So, we know that [starts writing] OA = OB. It's also obvious if you take the triangle OAB because OA = OB because it is the midpoint. So we have that, but how do we prove that the definition of the orthocenter is the perpendicular bisector? How do we prove that? Can we prove [incomprehensible] Yes, OK.

Faiza: It would be easier to [incomprehensible] the perpendicular bisector if we, for some reason, know it is an equilateral triangle. Because we know… let's say it was not in any case … it would not go to the same point for all of the…

Thus, similar to creating private spaces in traditional group work settings, handwriting can support the creation of individual lines of reasoning in distance settings. Possibly, handwriting functions as a signal to the other student to give some room for developing such a line of reasoning. Interestingly, in contrast to a traditional setup, the distance collaboration ensures that handwriting cannot be completely private, giving the other student the opportunity to extend or build on the student's line of reasoning. Therefore, in distance collaboration setups, handwriting does not support the creation of actual private spaces, but the creation of semi-private spaces for individual thought that, at that moment, is independent of the partner but can easily be taken up by the partner later on.

CONCLUSION AND DISCUSSION

This paper investigated the question of with what purposes do students use handwriting in distance collaboration settings, where the distance collaboration is implemented with tablet computers and smartpens. Overall, it was found that one can distinguish between two different purposes: a general purpose to structure the process of collaboration in the distance setting (see section 4.1. and 4.2.), and a specific mathematical purpose to support mathematical reasoning and fulfilling the mathematical task at hand (see section 4.3. to 4.5).

The second purpose (section 4.3–4.5), which highlight particularly mathematical functions of handwriting, does remind of the heuristic functions found by Misfeldt (2006), but also showcase differences due to the distance setting (for example, private space in contrast with semi-private space).

Overall, the main finding presented in this paper is that, in contrast to the traditional use of handwriting in pen-and-paper setups, the distance collaboration setup allows for handwriting to become a synchronous collaboration tool. This collaborative function is probably enabled by the fact that handwriting is always public, that is, visible to the other group members. If implemented over a longer time, handwriting could become a fully utilized collaborative tool in distance collaboration, possibly fulfilling similar functions as oral communication. However, compared to traditional setups, handwriting has some limitations here, as deixis or gestures cannot be used to reference previously written elements. This limitation could well be an advantage, as it forces students to make implicit connections explicit in their reasoning process, e.g., by highlighting written elements or by color-coding elements (as Mathematical Purpose 1).

Following research in writing didactics, mathematical writing can be understood as a problem-solving process requiring writers to make decisions about how to represent mathematical objects and their manipulations (Kruse & Ruhmann, 2006). The mathematical purposes of writing found here support such a conceptualization of mathematical writing as a reasoning process. Accordingly, similar to findings in the secondary school context where mathematical writing was found to be beneficial for consolidating and reviewing knowledge (e.g., Colonnese et al., 2018), it can be suspected that mathematical writing, and particularly 'forced' public writing, can have similar benefits for collaboratively consolidating or reviewing knowledge in the process of proving. Hence, there is a further need to investigate the epistemic role of handwriting in learning mathematics. Such research could also address the question of whether handwriting on tablet computers could have further benefits compared to traditional handwriting in mathematics, as tablet computers allow students to integrate other resources into the writing process, such as the lecture script or online searches.

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