Stock control in practice: about incorrect formulas, infeasible solutions, and mathematics on quicksand

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With a title like the one above, I hope to attract attention. This attention should lead to an interest in a field, inventory management, where there is still much to be gained when it comes to inventing the mathematics that makes it possible to replace heuristics with limited usability with exact analyses, which enable deeper insights. These insights can lead to numerical methods that, through their implementation, can lead to better operational decisions, which ultimately imply higher profitability of companies, but also less waste by reducing the number of rush orders, reducing material rot, and better utilization of scarce resources. The good news is that the empirical validity of mathematical models for stock control issues has been demonstrated, more or less as we do in physics. But the mathematical analysis of these models is far from finished. Good news, too, so.

This paper is structured as follows. We introduce the concept of material coordination and formulate a basic model for this. That basic model makes it possible to formulate the objective of material coordination, and to indicate the mathematical hurdles to be overcome. The essence of these hurdles are the stochastic processes we face. Even for simple stock control situations, exact methods are already numerically complex and therefore time-intensive. This still makes large-scale application difficult, even though computers are getting faster and faster. We discuss the typical "short-cuts" that are currently used within the so-called Enterprise Resource Planning (ERP) systems, which are used at almost every company. Here the wrong formulas and infeasible solutions pop up. These have implications for the daily work of planners, schedulers and managers. We argue that there are essentially solutions available, which have been over for quite some time, but are still not commonplace. Next, we discuss a number of laws that are of use when looking for new results. And finally, the mathematical challenges that are worth exploring further.

Material coordination

Inventory control involves monitoring the quantity in stock of an "item" at a location, so that customer demand can be met and orders can be placed in a timely manner at the delivery of the item to replenish the stock. The latter is called order release. Material coordination is the coordination of order release decisions of items used in the production and distribution of products. This includes raw materials, components, ingredients, semi-finished products, and end products at various locations in order to meet market demand in a timely manner. Coordinating suggests alignment. This alignment should consist of making order release decisions enforceable and preparing the decisions taken today for what is decided tomorrow and beyond. You could say that the decision today should be possible, but so should tomorrow's decision. The current material coordination concepts make this visible by showing the scheduler the course of its stock over time for each controlled material, as well as the order release decisions in the time leading up to this inventory turnover. It is usually the case that an order release decision has two faces. It generates a future replenishment in the receiving inventory point and an immediate decrease at the supplying inventory points. The latter is important to remember: one order release decision means that the desired quantity must be available from multiple materials. It is precisely from this that an important part of the coordination functionality of a concept should exist, because usually the quantities of material actually available are not precisely matched due to all kinds of causes. And then manually determining the right quantities is practically impracticable. Especially since solving a problem in one place seems to lead to a new problem in another place. In material coordination,

[1] Of this article, an extended Dutch version is available, wherein more information is given on the analysis of stochastic models, on http://home.kpn.nl/tondekok/

everything seems to be related to everything. And not only does that seem like it, it really does. To clarify this, we formulate a basic model for material coordination. We hereby assume that capacity is always sufficient to deliver this quantity to the stock point of the item after order release of one quantity for an item, this quantity within a set time, the delivery time.

Basic model

In our basic model, we assume discreet time. The basic model is defined by the items and their interrelationships, the standard for the time needed to produce an item from its parts, the exogenous demand process of the finished products and any spare parts, and the cost structure. The decision variables concern the order releases. The net inventory is defined as the physical inventory minus backorders.

N.	Number of items
E.	Collection of items with exogenous demand
a_{ij}	Number of items i needed to create one item j, i=1,...,N, , j=1,...,N
T	Decision horizon
$D_i(t)$	Exogenous demand for item i in period t, t =1,...,T, i=1,...,N
$F_{t,i}(t+s)$	Prediction made at beginning of period t of exogenous demand for item i in period $t+s$, t =1,...,T, s=0,...,$T-t$, i=1,...,N
L_i	Delivery time of item i, i=1,...,N
$X_i(t)$	Net stock of item i at the end of period t, t =0,...,T, i=1,...,N
$r_i(t)$	Quantity released from item i at the beginning of period t, t =1,...,T, i=1,...,N
h_i	Inventory cost per item i in stock at the end of a period, i=1,...,N
p_k	Penalty costs per item k shortfall at the end of a period, $k \in E$
υ_i	Safety stock of item i, i=1,...,N

Then we formulate the model below. We want to minimize the sum of inventory costs and penalty costs over a finite horizon T, making the right decisions on $\left\{r_i(t) \middle| 1 \le i \le N, 1 \le t \le T\right\}$ this horizon, taking into account the availability of so-called *child items,* the items needed to create an item. And taking into account the assumption that what is now released is available after the delivery time. It is important that the delivery time is the *standard time* at the time of order release, after *all necessary* child items have been allocated to the *order,* and the moment of receipt of the order in the stock point of the item.

$$\min_{\left\{r_i(t) \middle| 1 \le i \le N, 1 \le t \le T\right\}} \sum_{t=1}^{T} \sum_{i=1}^{N} h_i X_i^+(t) + \sum_{t=1}^{T} \sum_{k \in E} p_k X_k^-(t)$$

$s.t.$

$$\sum_{j=1}^{N} a_{ij} r_j(t) \le X_i(t-1), i = 1,...N, t = 1,...,T \qquad \text{material availability}$$

$$X_i(t) = X_i(t-1) + r_i(t-L_i) - \sum_{j=1}^{N} a_{ij} r_j(t) - D_i(t), i = 1,...N, t = 1,...,T \qquad \text{inventory balance}$$

$$r_i(t) \ge 0$$

This basic model already appears to be usable in practice, as any capacity constraints can be solved with short-term measures, such as adjustments to routings and priorities of orders already released, deployment of additional personnel and means of production, outsourcing, and overtime. Usability is based on two adjustments:

1. Replacing $D_i(t+s)$ the forecast of demand at $F_{t,i}(t+s)$ time t. After all, the above model is not solvable without mathematical assumptions about **future demand.** $D_i(t+s)$ With this adjustment we assume that we know the future question exactly!
2. Introducing a safety stock υ_i to absorb uncertainty in demand

This leads to the following LP-model:

$$\min_{\{r_i(t)|1\le i\le N,1\le t\le T\}} \sum_{s=0}^{T-1}\sum_{i=1}^{N} h_i\left(X_i\left(t+s\right)-\upsilon_i\right)^+ + \sum_{s=1}^{T-1}\sum_{k\in E} p_k\left(X_k\left(t+s\right)-\upsilon_k\right)^-$$

$$s.t.$$

$$\sum_{j=1}^{N} a_{ij} r_j\left(t+s\right) \le X_i\left(t+s-1\right), i=1,...N, s=0,...,T-1$$

$$X_i\left(t+s\right) = X_i\left(t+s-1\right) + r_i\left(t+s-L_i\right) - \sum_{j=1}^{N} a_{ij} r_j\left(t+s\right) - F_{t,i}\left(t+s\right), i=1,...N, s=0,...,T-1$$

$$r_i\left(t+s\right) \ge 0, i=1,...,N, s=0,...,T-1$$

Although planning problems are often formulated as (MI)LP, after which a solution is calculated with an optimization method or heuristics, this is not the way in which the material coordination problem is usually solved in practice. This uses a much simpler algorithm, known as the Material Requirements Planning (MRP) algorithm. In this case, a safety stock is again assumed for each item, which is chosen as the target stock at the end of the period in which the order arrives, i.e.

$$X_i\left(t+L_i+s\right) = \upsilon_i, i=1,...,N, s=0,...,T-t-L_i . \tag{1}$$

Then a rolling plan is calculated for time t by recursively calculating the order release, starting with the items, which have no successors, and then the direct predecessors. Once again, the prediction of demand is being used, whether the quantities of items with exogenous demand are to be made to meet exogenous demand. The latter is called the Master Production Schedule (MPS). The rolling plan is therefore determined as follows.

$$r_i\left(t\right) = \sum_{s=0}^{L_i} F_{ti}\left(t+s\right) + \sum_{s=0}^{L_i}\sum_{j=1}^{N} a_{ij} r_j\left(t+s\right) + \upsilon_i - \sum_{s=0}^{L_i-1} r_i\left(t+s-L_i\right), i=1,...,N \tag{2}$$

It is not difficult to see that this algorithm leads to *infeasible solutions* , as the material availability edge conditions are not taken into account. There are two reasons why this algorithm is still the most widely used planning algorithm in the industry:

1. The compute complexity is linear in the number of items, so it can be applied to any company.
2. Around this algorithm and the systems that ensure the input and export, a professional training program has been set up by the American Production and Inventory Control

Society (APICS),with which almost every logistics professional has been trained since 1970, directly or indirectly (including the author).

When the "MRP system" was introduced in the 1970s at large companies such as Philips, it was a big leap forward, because the computer took over a large part of the administration, which is necessary in material management. With the increase in computing power of computers, the frequency of "running" the MRP algorithm was increased from weekly to daily to real-time. In fact, the MRP algorithm has turned against the user with this. After all, every time a customer order comes in, the "equilibrium situation" is disturbed: the comparison (1) no longer applies. And then the MRP system will immediately generate custom order releases for many items. However, because comparison (2) does not take into account the material availability of *child items,* the proposed order release plan is often not allowed and manual intervention is required. And when it is finally in order, the next customer order comes in, or the sales plans change. To this day, it does not seem permissible to tackle the problem at its root: replacing an overly simplistic algorithm that does "work". All manual intervention is considered normal work of a planner.

It has to be said, (MI)LP is not yet a good alternative, because run time is too long given the size of the material coordination problem: for planning, recalculation must be possible within 10 seconds, because one wants to check whether manual adjustments, which we cannot get out of (after all, we use a model of reality),are effective. Typical adjustments include accelerating or delaying already outstanding production and customer orders, increasing or lowering sales plans. But there does appear to be an alternative, which requires about as much computing time as the MRP I algorithm and has already proven itself as an effective planning algorithm in practice, see De Kok et al. (2005). This algorithm is a by-product of research aimed at determining optimal strategies for order release in multi-echelon stock systems, as described above, under stochastic stationary demand. De Kok (2018) provides insight into the so-called *Synchronized Base Stock* (SBS) strategies, which make it possible to control general value networks, generating in a "split second" an feasible plan for a realistic value network, and which have been shown through numerical studies to significantly lower (approx. 10%) costs compared with (MI)LP-based plans. Schouten (2018) has shown that the SBS strategies can also be adapted for low-volume production, as in high-tech, where we cannot ignore the order releases are integer. In a *real-life experiment,* she shows that the SBS strategies without any human intervention lead to at least the same performance as in practice with a lot of human intervention.

Practical insights into the control of value networks

Demand at item level is indistinguishable from stationary
The basis for the application of stochastic models is that the demand for items is almost indistinguishable from stationary demand due to the relatively high value of the standard deviation from the average. As a result, a possible underlying pattern is drowned out by noise. The use of stochastic models does require careful modelling and model analysis, which unfortunately in the practice of stock control software is lacking: *incorrect formulas.*

Stochastic multi-echelon stock models are empirically valid
For the analysis and "optimization" of value networks, the ChainScope software has been developed, supported by a STW Valorisation Grant. Although Chainscope has not got off the ground, the software has been frequently used in MSc projects since 2009 (cf. De Kok (2018)). The basis of the software consists of the calculation of so-called Synchronized Base Stock (SBS) strategies, but in order to be able to use the software at each company, *heuristics* have been added to take into account series size limitations, maximum stock restrictions (for example, in the case of

tanks and silos, or instability of an intermediate product), and product rejection (for example, in the semiconductor industry). Although the mathematical analysis made extensive use of *discrete event simulation*, it turned out that building a simulation model for general value networks with all the characteristics mentioned is extremely complex and as far as I know not available, still. This problem was overcome by *empirical validation*.

Customer service is determined by average inventory and reorder frequency
Now we have already indicated above that people are frequently intervening in the order release proposals of systems. We find empirical validity of the models used. The only explanation for this finding is that the customer service performance of a value network is determined by the average inventory and the average order release quantity (or average order release frequency). In De Kok (2018), this finding is supported by comparing the performance of different one-product-one-location inventory strategies, all of which have the same average stock and order quantity. Under the assumption of stationary demand, these strategies appear to provide comparable customer service, unless the average order size is large compared to the average demand per unit of time.

Optimal control creates a flow of goods
When calculating the *optimal stock capital distribution* in value networks, three important insights appear to emerge in all practical case situations:

1. Most of the stock capital is at the so-called customer order decoupling point , the most downstream, demand-driven, inventory points.
2. From items with low value, but long delivery times, a significant stock is also maintained *in time.*
3. Most of the items are held in little or no stock- with which many items in the value network never end up in stock, other than as Work In Progress.

It's insight 3. which poses the greatest challenge in the implementation of the insights obtained in concrete control parameters for existing material coordination methods such as MRP-I. We know that the MRP-I algorithm generates infeasible solutions, which need to be adjusted manually. The MRP-I algorithm does not synchronize the orders of child items, nor does it have an allocation mechanism. And it is precisely this latter mechanism that plays a crucial role when we want to implement optimal strategies. So as long as we stick to the MRP-I algorithm, we are forced to hold too high stocks in the chain in places where they do not directly contribute to real customer service.

Open problems

I have indicated above that at the moment there are only heuristics in the form of "proprietary knowledge" in the ChainScope software for value networks with, among other things, series size restrictions and product rejection. Developing strategies that can be used effectively in practical situations , where uncertainty in supply and demand is the norm, is a major challenge. Although the current modelling and analysis "works", it can safely be said that relaxations of preconditions and unjustified assumptions of stochastic independence, lead to mathematical quicksand.

An important missing aspect is capacity limitation. It may take the form of a maximum output on a means of production per unit of time , or of a maximum quantity of work in progress. This latter type of restriction is common in the high-tech industry, for example in the case of clean rooms and other types of so-called setups where a capital asset is assembled and tested.

Of course, it is possible to use MILP formulations to take into account all possible generic and specific preconditions. But we know that SBS strategies lead to better performance than LP formulations within a rolling plan approach. Endogenously, the uncertainty in the model of reality is apparently crucial. In addition, as with SBS strategies compared to LP, the computational complexity of the developed strategies can also be much lower than with MILP, while control parameters may be optimized. It will undoubtedly require a lot of effort and patience to turn the challenges into actionable planning software, but the contribution to more effective and efficient use of scarce resources and materials for eternity does not seem to me to be a bad reward.

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