

Structure Preserving Reduction of Interconnected Structural **Dynamics Models**

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Introduction & objectives

High-tech machines often consist of modules, which require accurate dynamical models for design and control purposes. Even after reduction using state-of-the-art CMS methods, these models are too large to evaluate in acceptable time. In this project, a structure preserving balanced truncation (SPBT) method is presented to reduce more effectively, by reducing the subsystems such that

- the I/O behaviour of the coupled reduced system, accurately approximates the original I/O behaviour,
- the subsystems retain their second-order structure.

System representation

This work treats the reduction of *k* interconnected structural dynamics models Σ_i , respresented in second-order form by

$$\mathbf{\Sigma}_{j} : \begin{cases} M_{j}\ddot{q}_{j}(t) + L_{j}\dot{q}_{j}(t) + K_{j}q_{j}(t) = B_{j}v_{j}(t), \\ z_{j}(t) = C_{j}q_{j}(t), \end{cases} \quad j = 1, \dots, k, \quad (1)$$

where M_j , L_j , K_j , B_j and C_j are the mass, damping, stiffness, input and output matrices, respectively. These substructure models Σ_j are subsequently coupled by their inputs and outputs, v_j and z_j . The resulting interconnected, or coupled, model Σ_c has inputs u, outputs y and Degrees of Freedom (DoF) $q^T = [q_1^T, ..., q_k^T]$.

Methodology

Balancing is a type of reduction, where the states of the system are transformed to have equal controllability and observability. Once the states are *balanced*, the state components that are both difficult to control and difficult to observe are truncated. Unfortunately, regular balancing is not applicable to second-order systems, without destroying the secondorder structure, which is important for interpretation.

In this project, a structure preserving balancing method is used to reduce the subsystems Σ_j , using a transformation based on the entire coupled system Σ_c . This way, each subsystem model is mapped to a subspace, which is relevant for the coupled I/O behaviour.

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A schematic representation of the steps in the reduction procedure is shown in Figure 1. Only the block-diagonal elements of the controllability and observability Gramians, P_c and Q_c , are used to generate the block diagonal elements of the transformation matrix T, using a standard procedure in balancing. With regular balancing, the full Gramians are balanced, resulting in a full transformation matrix T.



Figure 1: Structure preserving balanced reduction, by balancing the block-diagonal elements of Gramians P_c and Q_c .

Results

The SPBT method is tested on a simple SISO system, representing two flexibly interconnected Euler beams with 5% modal damping, as shown in Figure 2. Its performance is compared to balanced truncation (BT) of Σ_c , CMS reduction and second-order balanced truncation (SOBT) of separate Σ_j . All methods are compared by means of the Modal Assurance Criterion (MAC), which indicates how well modes correspond (Figure 3a, 1 means perfect, 0 means no correspondence) and the relative error frequency response magnitude (Figure 3b).



Figure 2: Two flexibly coupled Euler beams.

Conclusion & future work

SPBT already seems promising, and is currently further improved to remain statically exact and to select the reduced order per subsystem more smartly.



Figure 3: MAC matrices (a) and relative error in the frequency response magnitude (b) of differently reduced models.

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