# Structure Preserving Reduction of Interconnected Structural Dynamics Models

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## Introduction & objectives

High-tech machines often consist of modules, which require accurate dynamical models for design and control purposes. Even after reduction using state-of-the-art CMS methods, these models are too large to evaluate in acceptable time. In this project, a structure preserving balanced truncation (SPBT) method is presented to reduce more effectively, by reducing the subsystems such that

- the I/O behaviour of the coupled reduced system, accurately approximates the original I/O behaviour,
- the subsystems retain their second-order structure.

## System representation

This work treats the reduction of $k$ interconnected structural dynamics models $\boldsymbol{\Sigma}_j$, respresented in second-order form by

$$\boldsymbol{\Sigma}_j : \begin{cases} M_j\ddot{q}_j(t) + L_j\dot{q}_j(t) + K_j q_j(t) = B_j v_j(t), \\ z_j(t) = C_j q_j(t), \end{cases} \quad j = 1, \dots, k, \quad (1)$$

where $M_j, L_j, K_j, B_j$ and $C_j$ are the mass, damping, stiffness, input and output matrices, respectively. These substructure models $\boldsymbol{\Sigma}_j$ are subsequently coupled by their inputs and outputs, $v_j$ and $z_j$. The resulting interconnected, or coupled, model $\boldsymbol{\Sigma}_c$ has inputs $u$, outputs $y$ and Degrees of Freedom (DoF) $q^T = [q_1^T, \dots, q_k^T]$.

## Methodology

Balancing is a type of reduction, where the states of the system are transformed to have equal controllability and observability. Once the states are *balanced*, the state components that are both difficult to control and difficult to observe are truncated. Unfortunately, regular balancing is not applicable to second-order systems, without destroying the second-order structure, which is important for interpretation.
In this project, a structure preserving balancing method is used to reduce the subsystems $\boldsymbol{\Sigma}_j$, using a transformation based on the entire coupled system $\boldsymbol{\Sigma}_c$. This way, each subsystem model is mapped to a subspace, which is relevant for the coupled I/O behaviour.

A schematic representation of the steps in the reduction procedure is shown in Figure 1. Only the block-diagonal elements of the controllability and observability Gramians, $P_c$ and $Q_c$, are used to generate the block diagonal elements of the transformation matrix $T$, using a standard procedure in balancing. With regular balancing, the full Gramians are balanced, resulting in a full transformation matrix $T$.

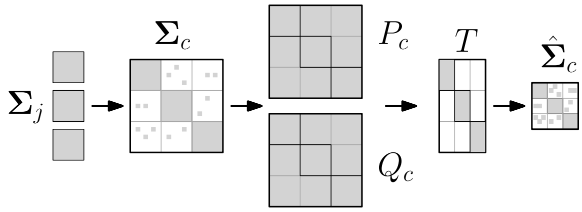



Figure 1: Structure preserving balanced reduction, by balancing the block-diagonal elements of Gramians $P_c$ and $Q_c$.

## Results

The SPBT method is tested on a simple SISO system, representing two flexibly interconnected Euler beams with 5% modal damping, as shown in Figure 2. Its performance is compared to balanced truncation (BT) of $\boldsymbol{\Sigma}_c$, CMS reduction and second-order balanced truncation (SOBT) of separate $\boldsymbol{\Sigma}_j$. All methods are compared by means of the Modal Assurance Criterion (MAC), which indicates how well modes correspond (Figure 3a, 1 means perfect, 0 means no correspondence) and the relative error frequency response magnitude (Figure 3b).

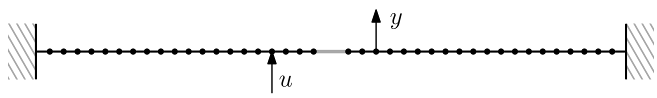
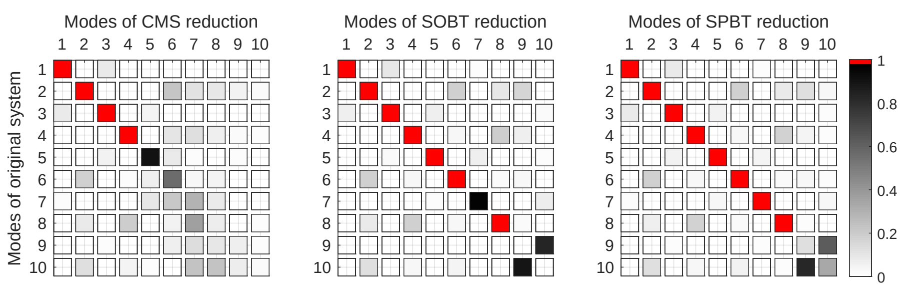


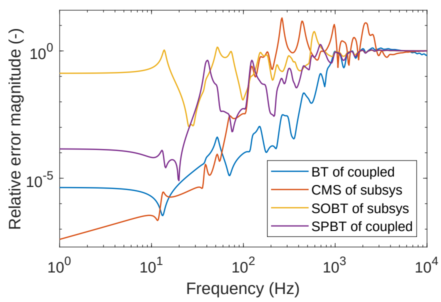

Figure 2: Two flexibly coupled Euler beams.

## Conclusion & future work

SPBT already seems promising, and is currently further improved to remain statically exact and to select the reduced order per subsystem more smartly.




(a)




(b)

Figure 3: MAC matrices (a) and relative error in the frequency response magnitude (b) of differently reduced models.

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