# An effective controlled islanding method for power grid through a sequence of optimization problems

# Effective Controlled Islanding Method for Power Grids Solving a Sequence of Optimization Problems

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***Abstract*—Controlled islanding is an important approach to prevent instability in power grids. In this paper, a novel approach is proposed for power system separation, which consists of two steps: 1) Finding multiple islanding scenarios; 2) Choosing the best option to obtain the most desirable island. In the first step, different islanding solutions are determined by a proposed hierarchical clustering method. In this algorithm, which is based on a minimum active power flow disruption objective function, the generator coherency constraints are considered in the clustering process. In the second step, the best separation scenario is chosen based on an arbitrary objective function. Particularly, in this paper, the amount of load shedding and the voltage profile deviation after separation are considered as the final criteria to select the best solution among available options. In so doing, the degree of load importance is also taken into account. The proposed two-step method is applied on an IEEE 9-bus test system and it is also evaluated on an IEEE 39-bus grid. The simulation results on the IEEE 39-bus grid and the comparative analysis with a state-of-the-art method confirm that the final islanding solution is more optimized based on the secondary criteria, which have not been addressed in the existing approaches. Moreover, the proposed method is computationally efficient and can be employed in real-scale power grids.**

***Index Terms*—Graph theory, hierarchical spectral clustering, power system separation, power system instability, two-step approach.**

## NOMENCLATURE

### A. Acronyms

| | |
|---|---|
| AC | Alternating Current. |
| MPI | Minimum Power Imbalance. |
| MPFD | Minimum Power Flow Disruption. |
| PMU | Phasor Measurement Unit. |
| VSC-HVDC | Voltage Source Converter-based High Voltage Direct Current. |
| WAMS | Wide Area Measurement System. |

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### B. Symbols

| | |
|---|---|
| $\psi_i$ | $i^{\text{th}}$ eigenvector of $\boldsymbol{L}_N$. |
| $\text{cut}(S)$ | The boundary of the subgraph. |
| $\text{dens}(S)$ | The density of the subgraph. |
| $\text{ratio}(S)$ | Quality of a partition. |
| $\text{index}(S)$ | Quality index of islanding to $n$ partitions. |
| $\eta_{\text{G}}(k)$ | The general objective function of separation. |
| $\lambda_i$ | $i^{\text{th}}$ eigenvalue. |
| $\mathbb{R}^k$ | $k$-dimensional Euclidean space. |
| $\Vert.\Vert$ | Euclidean distance. |
| $a_i$ | $i^{\text{th}}$ vertex in cluster $A$. |
| $A$ | Cluster $A$. |
| $b_j$ | $j^{\text{th}}$ vertex in cluster $B$. |
| $B$ | Cluster $B$. |
| $C_j$ | $j^{\text{th}}$ coherent group. |
| $C_p$ | $p^{\text{th}}$ coherent group. |
| $d_i$ | Weighted degree of the $i^{\text{th}}$ vertex ($v_i$). |
| $\text{dist}(v_i, v_j)$ | The similarity between the two vertices $v_i$ and $v_j$. |
| $D$ | A diagonal matrix of ($d_i$) elements. |
| $\text{Dist}_{N\times N}$ | Similarity matrix. |
| $E$ | Edges of the graph. |
| $f(V_i)$ | Voltage deviation index. |
| $F$ | The secondary objective function. |
| $G$ | Graph representation of the power grid. |
| $G_a$ | Generators inside cluster $A$. |
| $G_b$ | Generators inside cluster $B$. |
| $G_i$ | $i^{\text{th}}$ generator. |
| $k$ | Number of coherent groups. |
| $k_a$ | Number of vertices in cluster $A$. |
| $k_b$ | Number of vertices in cluster $B$. |
| $k_1$ | The weighting factor for $P_{\text{shed}}$ in $F$. |
| $k_2$ | The weighting factor for $f(V_i)$ in $F$. |
| $\boldsymbol{L}$ | Laplacian matrix. |
| $L_i$ | The busbar load shedding coefficient. |
| $\boldsymbol{L}_N$ | Normalized Laplacian matrix. |
| $M$ | The number of busbars which need load shedding. |
| $n$ | Number of partitions. |
| $N$ | Number of buses. |
| $N_{\text{G}}$ | Number of generators. |
| $P_{\text{shed}}(i)$ | The amount of load shedding related to the $i^{\text{th}}$ busbar. |

| | |
|---|---|
| $P_{ij}$ | Active power flow between the two buses $i$ and $j$. |
| $u_i$ | Normalized vector of $x_i$. |
| $v_i$ | $i^{\text{th}}$ vertex. |
| $V_{i\max}$ | Maximum acceptable voltage. |
| $V_{i\min}$ | Minimum acceptable voltage. |
| $V$ | Vertices of the graph. |
| $w_{ij}$ | Weight of the edge between the two vertices $v_i$ and $v_j$. |
| $W$ | Weights of the edges in the graph. |
| $x_i$ | Coordinates of $i^{\text{th}}$ vertex in the new $k$-dimensioned Euclidean space. |

## I. Introduction

NOWADAYS, power grids are susceptible to instability. The reports of recent blackouts throughout the world show that a short-circuit event in a transmission line sometimes leads to cascading failures and global blackouts in the power grid. Usually, because of time limitations, it is hard to manually perform the remedial actions in the case of successive failures. Therefore, employing an automatic protection and control system is inevitable to accurately and quickly perform the necessary actions in order to reduce the consequences of an incident [1], [2].

The existing methods are primarily based on graph theory and try to decrease the computational burden. They usually utilize one of the following objective functions: 1) minimum power imbalance (MPI) in the islands, and 2) minimum power flow disruption (MPFD). The methods in the first category seek the separation points that result in the minimum power imbalance in the partitions [3], [4]. Since these methods use searching techniques, they are usually time-consuming. The other methods (based on the second objective function) would detect the separation points to minimize the variation of power flowing through the remaining lines [1]. These methods lead to better transient stability and they reduce the overload of electrical equipment; making the grid restoration easier [1]. Moreover, the implementation of this objective function is simpler in graph partitioning theory [1]. On the other hand, as the minimum power imbalance concern is not explicitly considered in the optimization procedure, some complementary actions, such as load/generation shedding, may be required after separation in order to have stable islands.

In [1] a two-step islanding algorithm was proposed based on MPFD. This method utilized the spectral clustering algorithm to determine the coherency grouping of the generators and find the separation lines accordingly. The aforementioned method should be executed recursively to obtain more than two islands. The drawback of this approach is its high computational burdens [5], [6]. Later on, in [5] a new algorithm was proposed based on MPFD, which is more efficient in terms of computation time. One main drawback of this approach is the fixed number of islands (equal to the number of coherent groups of generators). In [7], hierarchical spectral clustering was employed, which is computationally more efficient; however, the contribution of this paper was primarily on the mathematical theory and practical power grid separation requirements, such as generator coherency, were not considered. In [8], the k-medoids algorithm was used in spectral clustering, which provided only one islanding solution similar to [5]. In [9], the power system separation approach was introduced for the hybrid AC/VSC-HVDC grids. In [10], a multi-layer clustering method was employed to minimize both the active and reactive power disruption. In [11], the correction coefficients between bus frequency components were also added to the algorithm. The aforementioned methods utilized the k-mean algorithm for clustering, which leads to only one islanding scenario. In addition, as the reactive power is usually controlled locally, complicating the method in [10] does not result in remarkable advancement in the islanding results.

The presented literature survey shows that the MPFD objective function is the common approach used in the power system islanding methods. The main advantages of the methods based on this objective function are that they can be easily implemented using the graph partitioning theory and they are fast and suitable for real-time and practical applications. On the other hand, in these methods, other practical requirements, e.g. power imbalance in the islands, are not directly considered.

To overcome this limitation, an effective two-step approach is introduced in this paper. In the first step, several islanding solutions are provided using the proposed hierarchical clustering algorithm. Then in the second step, the best separation scenario is chosen based on a desirable objective function, which would lead to minimum load shedding in the islands. Moreover, other potential practical requirements that cannot to be considered in the first step can be taken into account in the second objective function. The initial section of the proposed method is based on the minimum active power flow disruption objective function through the hierarchical spectral clustering theory. In fact, the conventional hierarchical spectral clustering theory is developed to support the generator coherency constraint in the power system separation. In addition, to defeat the restrictions of the primary objective function and reach the most desirable islanding solution, a secondary objective function can be defined based on the arbitrary criteria. Particularly, in this paper, this objective function is defined in such a way as to reduce load shedding cost and decrease voltage profile deviation after islanding. The proposed two-step algorithm is applied on an IEEE 9-bus test grid and the procedure of the algorithm is scrutinized. Simulation results on the IEEE 39-bus test grid confirm that the availability of the number of islanding scenarios in the proposed method and utilizing the secondary objective function lead to the most desirable islanding solution. Meanwhile, low computational burden of the proposed method makes it appropriate to be used in real-scale power grids.

In summary, this paper includes three main contributions:

1) A two-step approach in the power grid separation context is introduced: According to the limitation of the MPFD as the popular objective function, this paper utilized the hierarchical clustering algorithm to obtain simultaneously several islanding solutions. Hence, the other power system requirements can also be considered in a desirable secondary objective function to reach more sustainable islands.
2) In the first step, a multi-solution method is proposed based on the hierarchical clustering algorithm: In so do-

ing, new similarity criteria are defined in the conventional hierarchical clustering algorithm to consider the generator coherency constraints in the clustering. Table I shows the comparison between the proposed hierarchical clustering algorithm and the existing methods.

3) A secondary objective function is defined in the second step: In order to consider the other power system requirements, which are not possible to be taken into account in the main objective function, a secondary criterion is defined based on load shedding cost and voltage deviations after islanding. Hence, the most desirable islanding solution can be chosen among the available options of the first step.

The rest of this paper is organized as follows. Section II presents the power system separation concept based on graph theory. Section III discusses the graph spectral clustering algorithms. In Section IV, the proposed method is presented, and Section V evaluates the method using IEEE standard test systems. In Section VI, a comparative analysis with a state-of-the-art method is performed. Finally, Section VI concludes the paper.

## II. Power System Separation Based on Graph Theory

### A. Power System Graph Representation

$N$-bus electrical grids can be represented with an undirectional weighted graph as $G=(V,E,W)$. In this graph, $V$ and $E$ denote the vertices and edges respectively, which represent the buses and transmission lines in the power grid. Hence:

$$v_i \in V, \qquad i=1,2,\cdots,N \tag{1}$$

$$e_{ij} \in E \subset V \times V, \qquad i,j=1,2,\cdots,N \tag{2}$$

Based on the nature of the power system, this graph is a simple type without multiple edges and loops. $W$ denotes the weights of the edges, which are the values of power flow in the branches. Assuming no network losses, $w_{ij}=|P_{ij}|=|P_{ji}|$, where $|P_{ij}|$ is the active power flow between the two buses $i$ and $j$.

### B. The Objective Function of Islanding

In this paper, minimum power disruption is selected as the objective function due to its easy implementation and low computational burden. To realize this end, it is necessary to find the largest islands, which have strong connections between their nodes, i.e., with maximum power flow in the relevant branches, and weak connections with other islands, i.e., with minimum power flow in the tie-lines. To measure the quality of the clustering scenario, two quantities are defined in the graph theory including the boundary and the denseness of the subgraph. The boundary of the subgraph is the summation of weights of the edges between vertices in $S$ and vertices out of $S$, as follows [12]:

$$\mathrm{cut}(S)=\sum_{i\in S, j\notin S} w_{ij} \tag{3}$$

where the subgraph $S$ is defined as a set of vertices of the original graph. In addition, $i \in S$ indicates an individual vertex in the subgraph. It should be noted that $\mathrm{cut}(S)$ represents the sum of power flowing through the tie-lines connected to the island.

The denseness of a subgraph is defined as the sum of weighted degrees of its vertices as [12]:

$$\mathrm{dens}(S)=\sum_{i\in S} d_i \tag{4}$$

where $d_i$ is the weighted degree of the $i^{\text{th}}$ vertex ($v_i$), which should be calculated as [12]:

$$d_i=\sum_{j=1}^{N} w_{ij} \tag{5}$$

In the power system, $d_i$ is equivalent to the sum of absolute values of input and output powers of the $i^{\text{th}}$ bus. In addition, $\mathrm{dens}(S)$ represents the internal power flow of the island plus boundaries.

In order to measure the quality of an island, the following quantity is defined [5].

$$\mathrm{ratio}(S)=\frac{\mathrm{cut}(S)}{\mathrm{dens}(S)} \tag{6}$$

The lower value of $\mathrm{ratio}(S)$ for an island means that the sum of tie-lines power flowing is much lower than the total power of the lines inside the island. Therefore, the best island is the biggest one from which low-power transmission lines are interrupted. For separation of the power system into $n$ independent islands, the total quality of islanding ($\mathrm{index}(S)$) can be defined by the maximum quality of each island, as:

$$\mathrm{index}(S)=\max\{\mathrm{ratio}(S_i)\},\ \text{ for } i=1,\cdots,n \tag{7}$$

TABLE I
The Comparison between the Proposed Hierarchical Clustering Algorithm and the Existing Methods

| Reference | Main objective function | Consideration of generator coherency grouping | Computational burden | The flexible choice of the number of islands | Multi-solution |
|---|---|---|---|---|---|
| [1] | MPFD | Yes | Low | No | No |
| [3] | MPI | Yes | High | No | No |
| [4] | MPI | Yes | Moderate | No | No |
| [5] | MPFD | Yes | Low | No | No |
| [7] | MPFD | No | Low | Yes | No |
| [8] | MPFD | Yes | Low | No | No |
| [10] | MPFD | Yes | Moderate | No | No |
| [11] | MPFD | Yes | Moderate | No | No |
| The proposed hierarchical clustering algorithm | MPFD | Yes | Low | Yes | Yes |

To find an optimal solution, it is required to calculate the index$(S)$ for all possible islanding scenarios, where each scenario includes $n$ independent islands, and choose the scenario that achieves the lowest value of index$(S)$. Hence, the general objective function for the power system islanding is defined as:

$$\eta_{\mathrm{G}}(k) = \min\left\{\max_{i=1,\cdots,n}\{\text{ratio}(S_i)\}\right\} \tag{8}$$

It should be noted that in power grid separation, the generators inside each island should be coherent to reach stable islands. Hence, this constraint shall be included in the optimization process.

Obtaining the optimal solution for such a large power system graph is not computationally feasible and it is an NP-hard problem [12]. Accordingly, to achieve an acceptable approximate solution, the use of spectral clustering and Cheeger inequality have been proposed [6].

## III. Graph Spectral Clustering

The graph partitioning aims to find a group of vertices, which have a stronger connection with each other and a weaker connection with the vertices of the other groups. One of the efficient graph partitioning methods is spectral clustering. This method is based on the eigenvalues and eigenvectors of the graph Laplacian matrix [12]. The method is described in this section in detail.

### A. Spectral Clustering Theory

The Laplacian matrix ($\boldsymbol{L}$) is used extensively in graph analysis. For $G = (V, E, W)$, the Laplacian is an $N \times N$ matrix in which $N$ is the number of vertices [12]. It is defined as:

$$[\boldsymbol{L}]_{i,j} = \begin{cases} d_i, & \text{if } i = j; \\ -w_{ij}, & \text{if } i \neq j \text{ and } (i,j) \in E; \\ 0, & \text{otherwise.} \end{cases} \tag{9}$$

Moreover, the normalized Laplacian is [12]:

$$\boldsymbol{L}_N = D^{\frac{-1}{2}} L D^{\frac{-1}{2}} \tag{10}$$

where $D$ is a diagonal matrix in which the diagonal elements ($d_i$) are non-zero. The normalized Laplacian matrix is scale-independent and it is more advantageous for clustering applications [6].

In order to solve the optimization problem in (8), spectral clustering provides an approximate solution using the smallest $k$ eigenvalues ($\lambda_i$) and their corresponding eigenvectors ($\psi_i$) of the normalized Laplacian matrix. The advantage of the approach compared to the direct analytical method is its much lower computational complexity [6].

Cheeger inequality shows how close the approximate solution is to the optimal level [6]:

$$\frac{\lambda_k}{2} \leq \eta_{\mathrm{G}}(k) \leq O(k^2)\sqrt{\lambda_k} \tag{11}$$

Therefore, selecting smaller $\lambda_k$ leads to a smaller value for $\eta_{\mathrm{G}}(k)$ and thus better islanding quality can be obtained.

The spectral clustering uses $k$ eigenvectors of the Laplacian matrix to find the vertices in a k-dimensional Euclidean space of $\mathbb{R}^k$, which is known as spectral k-embedding. The previous studies show that using the normalized Laplacian in spectral clustering leads to more appropriate solutions [6]. In the next step, the vertices should be clustered using a proper algorithm in Euclidean space, usually utilizing the k-mean or the k-medoids method. Despite all the benefits of these algorithms, they have some limitations: 1) the number of clusters should be known, and 2) connections of vertices in the graph are not considered [7]. To overcome these limitations, hierarchical spectral clustering has been used [13], which is described in the following subsection.

### B. Hierarchical Spectral Clustering Theory

Hierarchical clustering is based on the creation of a hierarchy of clusters. Among various approaches, the agglomerative method could be appropriate for power grid islanding. This approach is “bottom-up”, which is described as follows:

- In a graph with $N$ vertices, the two most similar vertices (based on the similarity criteria) are selected to create a cluster. Therefore, a new graph containing $N-1$ clusters is formed. In other words, two vertices would be merged to form a new cluster, and each of the other clusters contains one vertex.
- In the new graph, the two most similar clusters are merged and form a new cluster. In such a condition, the new graph contains $N-2$ clusters.
- This procedure continues so that new clusters are formed in higher orders.

The result of hierarchical spectral clustering is generally shown as a dendrogram, as illustrated in Fig. 1. The dendrogram visualization has several advantages. First, it is possible to change the number of clusters without extra calculations. Second, it gives a general perspective on the similarity between the clusters [14].

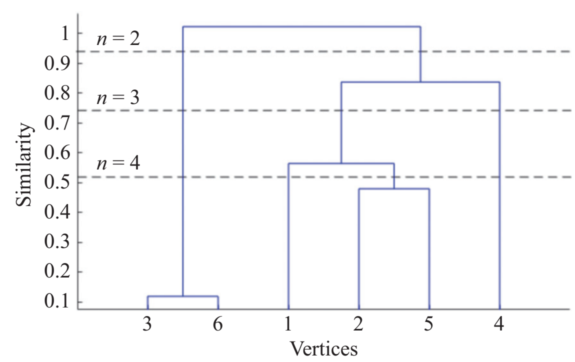


Fig. 1. Illustrating a typical dendrogram.

## IV. The Proposed Islanding Method

The proposed islanding method, which is based on the WAMS, consists of the following sequences (Fig. 2):

1) Finding the islanding scenarios through a proposed hierarchical clustering algorithm.
2) Evaluating the islanding scenarios based on a proposed secondary objective function and choosing the best one.

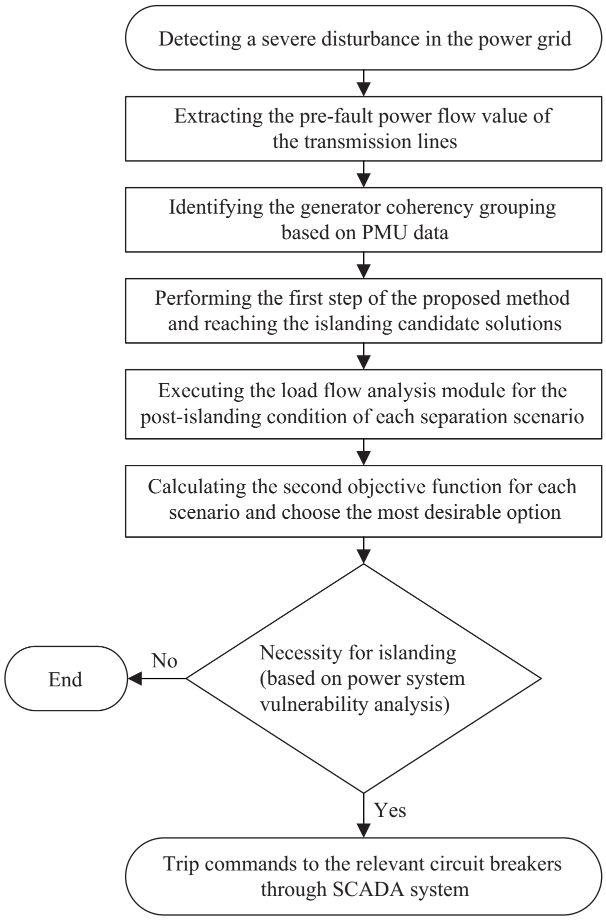


Fig. 2. The flowchart of the proposed method.

### A. The Proposed Hierarchical Clustering Algorithm

In this paper, to reach a proper multi-solution approach, the hierarchical spectral clustering algorithm is extended to satisfy the generator coherency constraint in the islands. It is assumed that all electrical quantities of the power grid are available through the WAMS and the generator coherency grouping is also identified by the PMU-based existing methods [15].

To adapt the hierarchical spectral algorithm to the power system islanding context, the similarity criteria and merging rule are defined as follows:

- Calculating the initial similarity between vertices
  - The coordinates of vertices ($x_i$) are calculated in $\mathbb{R}^k$ space, where $k$ is the dimension of the embedding space which is assumed to be equal to the number of coherent groups of generators in the whole power grid.
  - The similarity is defined as follows for two adjacent vertices in the graph:

$$\text{dist}(v_i, v_j) = \|u_i - u_j\| \tag{12}$$

    where $v_i$ is the $i^{\text{th}}$ vertex, $u_i \in \mathbb{R}^k$ is the normalized coordinate of $v_i$, and $\|.\|$ indicates the Euclidean distance.
  - For two non-adjacent vertices, the similarity is defined as the shortest path between them, which is calculated based on the Floyd-Warshall algorithm [16], [17], as:

$$\text{dist}(v_i, v_j) = \text{minpath}(v_i, v_j) \tag{13}$$

- Calculating the similarity between clusters
  - The similarity between two clusters is defined as:

$$\text{dist}(A, B) = \max\{\text{dist}(a_i, b_j)\},\\ i = 1, \cdots, k_a,\ j = 1, \cdots, k_b \tag{14}$$

    where $A$, $B$ denote two clusters, $a_i$ is the $i^{\text{th}}$ vertex in cluster $A$, $b_j$ is the $j^{\text{th}}$ vertex in cluster $B$, and $k_a$ and $k_b$ are the number of vertices in clusters $A$ and $B$, respectively.
- The merging rule
  In the conventional form of the hierarchical clustering algorithm, two most similar vertices/clusters (with minimum similarity) are merged to create a new cluster. To consider the generator coherency constraint, another restriction is added to the hierarchical process as described in the following. It is assumed that there are $N_{\text{G}}$ generators in the power grid in $k$ coherent groups. The coherency group of each generator can be determined based on transient stability studies or using data of PMUs available in the power grid. This can be defined as:

$$G_i \in C_j, \begin{cases} i = 1, \cdots, N_{\text{G}} & j, p = 1, \cdots, l, \cdots, k \\ C_j \cap C_p = \varnothing & \forall j \neq p \end{cases} \tag{15}$$

  where $G_i$ is the $i^{\text{th}}$ generator, $C_j$ and $C_p$ are the $j^{\text{th}}$ and $p^{\text{th}}$ coherent groups, respectively.
  To merge two clusters $A$ and $B$, their generators must be in the same coherent group that can be stated as:

$$(G_a \cup G_b) \subset C_l, \forall G_a \in A,\ \forall G_b \in B \tag{16}$$

  where $G_a$ and $G_b$ represent the generators inside clusters $A$ and $B$, respectively.

### B. The Proposed Secondary Objective Function

The existing methods provide only one separation scenario, in which the number of islands is equal to the number of coherent generators [5], [8], [10], [11]. On the other hand, restrictions, including power imbalance and overload of equipment, are not directly considered in the optimization procedure. Therefore, some complementary actions, such as load/generation shedding, are required after islanding to have stable islands. These actions increase the cost of consumer/equipment outage. The proposed hierarchical spectral clustering algorithm leads to several independent islanding scenarios without further computations. Thus, other power system requirements can also be considered in the algorithm by using a proper secondary objective function. This function should be defined based on desirable criteria to achieve the best scenario. In this paper, the amount of load rejection and bus voltage deviation from the rated value after islanding are the main concerns, which are taken into account in the proposed secondary objective function as follows:

$$F = k_1 \times \sum_{i=1}^{\text{M}} L_i \times P_{\text{shed}}(i) + k_2 \times \sum_{j=1}^{N} f(V_j) \tag{17}$$

where $P_{\text{shed}}(i)$ denotes the amount of load shedding related to the $i^{\text{th}}$ busbar, $M$ is the number of busbars which need load shedding, and $f(V_i)$ represents the grid voltage deviation, which is calculated as:

$$f(V_i) = \begin{cases} 10^{5(V_{i\,\min} - V_i)} - 1 & V_i < V_{i\,\min} \\ 0 & V_{i\,\min} < V_i < V_{i\,\max} \\ & \text{or } V_i = 0 \\ 10^{15(V_i - V_{i\,\max})} - 1 & V_{i\,\max} < V_i \end{cases} \tag{18}$$

where $V_{i\,\min}$ and $V_{i\,\max}$ are assigned to be 90% and 105% respectively.

The coefficients, $k_1$ and $k_2$ are weighting factors that are considered equal to $1/(0.1 \times P_{\text{GT}})$ and 0.1, respectively based on the comprehensive studies for different power systems. Meanwhile, $P_{\text{GT}}$ is the average of the power generation in the power plants in normal operating conditions. $L_i$ denotes the busbar load shedding coefficient, which is defined by the system operator regarding the degree of load importance. It should be noted that the proposed method employs a load flow study module. After executing the first step and providing the islanding solutions, the load flow study should be performed for each option based on the post-islanding configuration of the power grid. Hence, the voltage profile for each scenario would be available. The required load shedding is also calculated based on the load flow results and the generation limitation of each island.

It is worth mentioning that the proposed secondary objective function is just defined to show the effectiveness of the multi-solution concept and could be improved in future studies in this context.

### *C. The Practical Considerations*

Nowadays, almost all bulk power grids are monitored and controlled by the main control system, which is responsible for the stability of the power grid. In so doing, the main control system employs several software modules e.g. load flow, state estimation, etc. Hence, the power system controlled islanding module can also be added to this system. This module can manage the separation of the power grid based on available data of the network (from PMUs) according to the flowchart of Fig. 2.

## V. Simulation Results

In this section, the proposed islanding method is described on the model of the IEEE 9-bus test grid. Meanwhile, to verify the computational efficiency of the method and show the effectiveness of the multi-solution approach, simulation of the IEEE 39-bus and IEEE 118-bus test grids are also performed. All numerical calculations are carried out using MATLAB software [18] on a PC with 2.00 GHz Core i7 CPU and 6 GB RAM.

### *A. IEEE 9-bus Test Grid*

To scrutinize the proposed method, the dynamic model of the IEEE 9-bus test grid in DIgSILENT software is used [19]. As depicted in Fig. 3, this grid contains three generators, six transmission lines, three loads, and three power transformers.

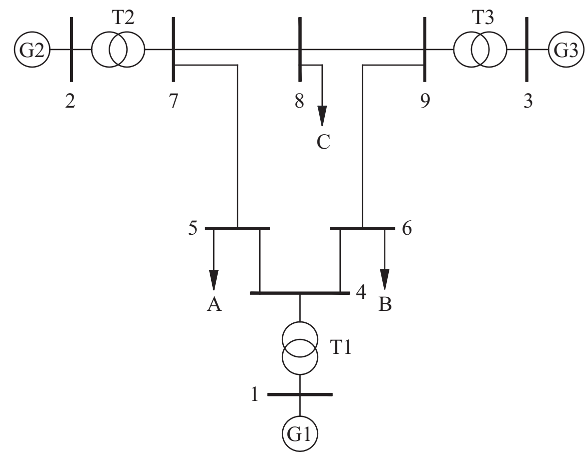


Fig. 3. Single line diagram of the IEEE 9-bus test grid.

The corresponding graph of the system under study is shown in Fig. 4.

It is assumed that a three-phase short-circuit fault on line 5–7 takes place, which is cleared after 200 ms by a protective relay. In this case, severe oscillations occur in the grid and the system will go toward instability in the absence of any proper control action. Under such a condition, vulnerability analysis shows the necessity of power system separation as the last resort to prevent wide area instability.

To determine separation points, the proposed approach can be employed as follows:

1) After separation of the line between buses 5 and 7 by protective relays, an updated graph should be generated, which is shown in Fig. 5.
2) The weighted adjacency matrix of the graph ($W$) is equal to:

$$W = \begin{bmatrix} 0 & 0 & 0 & 72 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 163 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 85 \\ 72 & 0 & 0 & 0 & 41 & 30 & 0 & 0 & 0 \\ 0 & 0 & 0 & 41 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 30 & 0 & 0 & 0 & 0 & 60 \\ 0 & 163 & 0 & 0 & 0 & 0 & 0 & 76 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 76 & 0 & 24 \\ 0 & 0 & 85 & 0 & 0 & 60 & 0 & 24 & 0 \end{bmatrix} \tag{19}$$

3) Based on the results of simulation-based transient stability studies, the generators are classified into two coherent groups after the disturbance as:

$$C_1 = \{G_1\},\ C_2 = \{G_2, G_3\} \tag{20}$$

4) The normalized Laplacian matrix of the graph is calculated as follows:

$$L_{\text{N}} = \begin{bmatrix} 1 & 0 & 0 & -0.708 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -0.826 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -0.709 \\ -0.708 & 0 & 0 & 1 & -0.534 & -0.269 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.534 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.269 & 0 & 1 & 0 & 0 & -0.485 \\ 0 & -0.826 & 0 & 0 & 0 & 0 & 1 & -0.492 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.492 & 1 & -0.185 \\ 0 & 0 & -0.7086 & 0 & 0 & -0.485 & 0 & -0.185 & 1 \end{bmatrix} \tag{21}$$

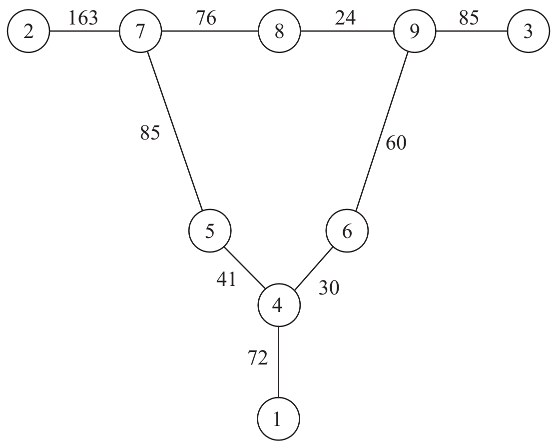


Fig. 4. Graph representation of the IEEE 9-bus test system.

5) The eigenvalues of the normalized Laplacian matrix are 0, 0.0531, 0.1903, 1, 1, 1, 1.8097, 1.9469, and 2. The two smallest eigenvalues ($k = 2$) are considered and their corresponding eigenvectors are:

$$\psi_1 = \begin{bmatrix} +0.2549 \\ +0.3844 \\ +0.2776 \\ +0.3602 \\ +0.1923 \\ +0.2869 \\ +0.4656 \\ +0.3015 \\ +0.3918 \end{bmatrix}, \ \psi_2 = \begin{bmatrix} -0.3615 \\ +0.4317 \\ -0.1076 \\ -0.4839 \\ -0.2729 \\ -0.2109 \\ +0.4952 \\ +0.2290 \\ -0.1437 \end{bmatrix} \tag{22}$$

6) The coordinates of vertices in the new Euclidean space are calculated as follows:

$$x_1 = \begin{bmatrix} +0.2549 \\ -0.3615 \end{bmatrix}, \cdots, x_9 = \begin{bmatrix} +0.3918 \\ -0.1437 \end{bmatrix} \tag{23}$$

7) The normalized coordinates are:

$$u_1 = \begin{bmatrix} +0.5762 \\ -0.8173 \end{bmatrix}, \cdots, u_9 = \begin{bmatrix} +0.9388 \\ -0.3445 \end{bmatrix} \tag{24}$$

8) The initial similarity matrix based on the proposed method is calculated using (12) and (13):

$$\mathrm{Dist}_{9\times 9} = \begin{bmatrix} 0 & 1.7565 & 0.6210 & 0.0258 & 0.0517 & 0.3216 & 1.7294 & 1.5629 & 0.6031 \\ 1.7565 & 0 & 1.1714 & 1.7307 & 1.7565 & 1.4349 & 0.0271 & 0.1936 & 1.1534 \\ 0.6210 & 1.1714 & 0 & 0.5952 & 0.6210 & 0.2994 & 1.1443 & 0.9778 & 0.0180 \\ 0.0258 & 1.7307 & 0.5952 & 0 & 0.0258 & 0.2958 & 1.7036 & 1.5371 & 0.5772 \\ 0.0517 & 1.7565 & 0.6210 & 0.0258 & 0 & 0.3216 & 1.7294 & 1.5629 & 0.6031 \\ 0.3216 & 1.4349 & 0.2994 & 0.2958 & 0.3216 & 0 & 1.4078 & 1.2413 & 0.2814 \\ 1.7294 & 0.0271 & 1.1443 & 1.7036 & 1.7294 & 1.4078 & 0 & 0.1665 & 1.1263 \\ 1.5629 & 0.1936 & 0.9778 & 1.5371 & 1.5629 & 1.2413 & 0.1665 & 0 & 0.9598 \\ 0.6031 & 1.1534 & 0.0180 & 0.5772 & 0.6031 & 0.2814 & 1.1263 & 0.9598 & 0 \end{bmatrix} \tag{25}$$

9) After merging vertices based on the similarity matrix and considering the coherency constraint, the tree shown in Fig. 6 is obtained.

10) For different values of $n$, separation results are summarized as:

$$n = 2: \{4, 1, 5\}, \{7, 2, 8, 9, 3, 6\}$$
$$n = 3: \{4, 1, 5\}, \{7, 2, 8\}, \{9, 3, 6\}$$
$$n = 4: \{4, 1, 5\}, \{7, 2, 8\}, \{9, 3\}, \{6\}$$

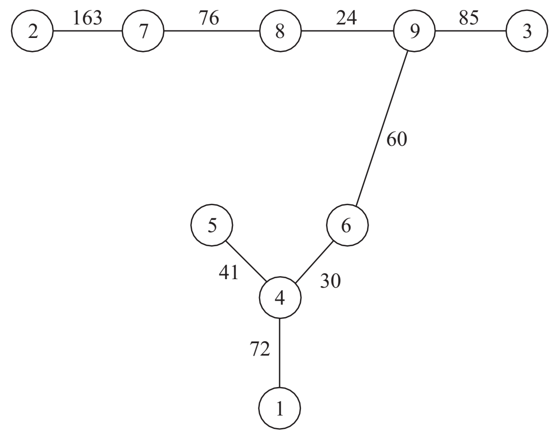


Fig. 5. Graph representation of the IEEE 9-bus system after separation of the line between buses 5 and 7.

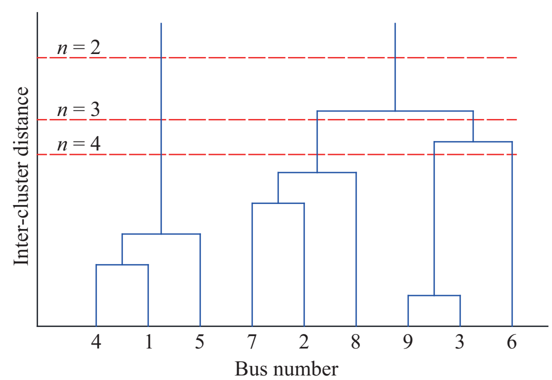


Fig. 6. The hierarchical tree of the clustering.

11) The load-flow study is performed for different values of $n$ and the secondary objective function is calculated based on (17). Table II shows the summary of these studies ($L_i$ is considered equal to one for all busbars).

TABLE II
THE SIMULATION RESULTS FOR IEEE 9-BUS TEST GRID

| Numbers of islands ($n$) | Splitting branches | $k_1 \times \sum L_i \times P_{\text{shed}}(i)$ | $k_2 \times \sum_{i=1}^{N} f(V_i)$ | $F$ |
|---|---|---|---|---|
| 2 | 4–6 | 0 | 0 | 0 |
| 3 | 4–6, 8–9 | 0 | 0 | 0 |
| 4 | 4–6, 8–9, 6–9 | 8.29 | 0 | 8.29 |

12) As shown in Table II, splitting into two or three islands leads to zero load shedding and the voltage deviations are in an acceptable margin. Hence, the value of the secondary objective function ($F$) is zero. Furthermore, regarding the integrity of the power system and ease of restoration, the two-island option is the best solution for the system under study.

### *B. IEEE 39-bus Test Grid*

In this part, the proposed method is evaluated on the IEEE 39-bus test system. The dynamic parameters of the system have been completely expressed in [20]. According to the simulation performed in DIgSILENT software, after short circuit occurrence in the transmission line between the 16$^{\text{th}}$

and 17th buses and fault clearance, the generators oscillate in four groups. Due to the necessity of islanding, the proposed method is applied to find the proper separation points.

The simulation results of the 1st step of the proposed method are summarized in Table III. According to the table, the run time of the algorithm is about 9 ms. As the computational burden of the spectral clustering algorithms is approximately proportional to the cubic of the number of vertices ($N^3$) [14], the proposed method could be utilized for a real-scale power system.

To show the importance of the second objective function in the proposed method, two cases are defined for load shedding coefficients: 1) All busbars are considered to be uniform regarding their load shedding importance, 2) two busbars are considered to be more important with a higher load shedding coefficient. According to the calculation results, which are shown in Table IV, the five-island's solution is the best scenario for the first case, while the separation of the power grid into six islands leads to the most desirable solution for the second case.

TABLE III
THE SIMULATION RESULTS RELATED TO THE 1ST STEP OF THE PROPOSED METHOD FOR THE IEEE 39-BUS TEST GRID

| Numbers of islands ($n$) | Splitting branches | Simulation time (ms) |
|---|---|---|
| 4 | 3-4, 14–15, 3–18, 2–25, 9–39 | 9 |
| 5 | 3-4, 14–15, 3–18, 2–25, 9–39, 22–23, 16–24 | |
| 6 | 3-4, 14–15, 3–18, 2–25, 9–39, 22–23, 16–24, 8–9 | |

TABLE IV
THE CALCULATION RESULTS RELATED TO THE 2ND STEP OF THE PROPOSED METHOD FOR THE IEEE 39-BUS TEST GRID

| Load shedding cases | Numbers of islands ($n$) | $k_1 \times \sum L_i \times P_{\text{shed}}(i)$ | $k_2 \times \sum_{i=1}^{N} f(V_i)$ | $F$ |
|---|---|---|---|---|
| $L_i = 1$ for all busbars | 4 | 0.813 | 0.148 | 0.96 |
| | 5 | 0.813 | 0.134 | **0.95** |
| | 6 | 1.030 | 0.134 | 1.16 |
| $L_i = 1$ for busbar #4 and #8 $L_i = 0.5$ for the other busbars | 4 | 0.715 | 0.148 | 0.86 |
| | 5 | 0.715 | 0.134 | 0.85 |
| | 6 | 0.515 | 0.134 | **0.65** |

## C. IEEE 118-bus Test Grid

To verify the scalability of the proposed method in a large power grid, the model of an IEEE 118-bus test gird is also used [21]. The separation results of this power grid after a short circuit event in the transmission line between the 23nd and 25th buses are summarized in Table V.

## VI. Comparative Analysis with a State-of-the-art Method

In this section, the proposed algorithm is compared with a state-of-the-art method presented in [11]. The aforementioned algorithm is based on a multi-layer spectral clustering algorithm, which utilizes the active and reactive power flow and the correlation coefficient between frequency components. The k-mean algorithm used in [11] leads to only one islanding solution. The number of islands is equal to the number of coherent groups.

To perform the comparative analysis, the IEEE 39-bus test grid was simulated and the two methods were implemented with the same initial assumptions. The results are summarized in Table VI. According to the table, both methods lead to the same separation results for the bisectional case. However, the availability of the multi solutions in the first step of the proposed method leads to a more desirable islanding scenario (lower $F$) based on the secondary objective function. It should be specified that the state-of-the-art method is much more complicated than the first step of the proposed method, which is only based on an active power flow.

## VII. Conclusion

This paper proposes a novel two-step approach for intentional islanding that can be utilized in modern power grids

TABLE V
THE SEPARATION RESULTS OF THE IEEE 118-BUS TEST GRID

| Assumptions | | Separation results | | | | | |
|---|---|---|---|---|---|---|---|
| Initial fault | Generator coherency grouping | Number of islands ($n$) | Line outage | $k_1 \times \sum L_i \times P_{\text{shed}}(i)$ $L_i = 1$ for all busbars | $k_2 \times \sum_{i=1}^{N} f(V_i)$ | $F$ | Simulation time (ms) |
| Line 23–25 | $V_{G1} = \{v_{10}, v_{12}, v_{25}, v_{26}, v_{31}\}$ $V_{G2} = \{v_{46}, v_{49}, v_{54}, v_{59}, v_{61}, v_{65}, v_{66}, v_{69}, v_{80}, v_{87}, v_{89}, v_{100}, v_{103}\}$ | 2 | 15-33, 19–34, 30–38, 24–70, 24–72 | 0 | 0.597 | **0.60** | 180 |
| | | 3 | 15-33, 19–34, 30–38, 24–70, 24–72, 23–24 | 0.279 | 0.597 | 0.88 | |
| | | 4 | 15-33, 19–34, 30–38, 24–70, 24–72, 23–24, 77–82, 80–96, 96–97, 80–99, 98–100 | 0.430 | 0.597 | 1.03 | |

TABLE VI
COMPARATIVE ANALYSIS WITH A STATE-OF-THE-ART METHOD [11]

| The method | Assumptions | | Separation results | | | | |
|---|---|---|---|---|---|---|---|
| | Initial fault | Generator coherency grouping | Number of islands ($n$) | Line outage | $k_1 \times \sum L_i \times P_{\text{shed}}(i)$ $L_i = 1$ for all busbars | $k_2 \times \sum_{i=1}^{N} f(V_i)$ | $F$ |
| The proposed method Based on P | Line 13–14 Line 16–17 | $V_{G1} = \{v_{30}, v_{31}, v_{32}, v_{37}, v_{38}, v_{39}\}$ $V_{G2} = \{v_{33}, v_{34}, v_{35}, v_{36}\}$ | 2 | 14–15 | 2.441 | 0.022 | 2.46 |
| | | | 3 | 14–15, 4–14 | 2.441 | 0.000 | **2.44** |
| | | | 4 | 14–15, 4–14, 3–4, 4–5 | 8.779 | 0.101 | 8.88 |
| [11] Based on P, Q and f | | | 2 | 14-15 | 2.441 | 0.022 | 2.46 |

(with DGs) as well as traditional power systems. In the first step, several islanding solutions are provided based on the proposed hierarchical spectral clustering theory. Then the best islanding solution can be selected based on an arbitrary second objective function in the second step. The proposed clustering algorithm is based on the minimum power flow disruption objective function and generator coherency constraints are considered in it. The availability of several islanding scenarios helps one to prevail with the limitation of the main objective function and choose the best option based on the secondary objective function. This function should be defined based on the desired criteria of the system designer. In this paper, the amount of load shedding and the voltage profile deviation of the busbars were considered as the main concerns in the proposed objective function. Meanwhile, the degree of load importance was also reflected in the cost function. The two-step method was applied on the IEEE 9-bus grid and the procedure of the algorithm was scrutinized. Simulation results on the IEEE 39-bus grid and the comparative analysis with a state-of-the-art method confirmed that the availability of numbers of islanding scenarios in the proposed method and utilizing a secondary objective function lead to a better islanding solution. It should be mentioned that the low computational burden of the method makes it proper for real-scale applications in power system islanding.

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