

# Energy optimal coordination of fully autonomous vehicles in urban intersections

**Citation for published version (APA):**

Pelosi, C., Padilla, G. P., & Donkers, M. C. F. (2020). Energy optimal coordination of fully autonomous vehicles in urban intersections. *IFAC-PapersOnLine*, 53(2), 15090-15095. <https://doi.org/10.1016/j.ifacol.2020.12.2031>

**DOI:**

[10.1016/j.ifacol.2020.12.2031](https://doi.org/10.1016/j.ifacol.2020.12.2031)

**Document status and date:**

Published: 01/01/2020

**Document Version:**

Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

**Please check the document version of this publication:**

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
- The final author version and the galley proof are versions of the publication after peer review.
- The final published version features the final layout of the paper including the volume, issue and page numbers.

[Link to publication](#)

**General rights**

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal.

If the publication is distributed under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license above, please follow below link for the End User Agreement:

[www.tue.nl/taverne](http://www.tue.nl/taverne)

**Take down policy**

If you believe that this document breaches copyright please contact us at:

[openaccess@tue.nl](mailto:openaccess@tue.nl)

providing details and we will investigate your claim.

# Energy Optimal Coordination of Fully Autonomous Vehicles in Urban Intersections

C. Pelosi G.P. Padilla M.C.F. Donkers

*Dept Electrical Eng, Eindhoven University of Technology, Netherlands  
(e-mail: c.pelosi@student.tue.nl, {g.p.padilla.cazar,m.c.f.donkers}@tue.nl)*

**Abstract:** This paper provides a solution to conflict resolutions between Autonomous Vehicles (AV) crossing an urban intersection. The conflict resolution problem is formulated as an optimal control problem, where the objective is to minimize the energy consumption of all the vehicles, while avoiding collisions. Since the problem has a combinatorial nature, it is tackled through a sequential mixed-integer quadratically constrained programming approach. Simulation results show that since the AVs do not need to follow specific driving rules, the intersection crossing order is chosen to optimize the overall energy consumption. The research outcome underlines the benefits of moving towards fully autonomous systems which will allow for higher traffic throughput. Furthermore, the proposed formulation is the starting point for future explorations towards real-time implementation.

Copyright © 2020 The Authors. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0>)

*Keywords:* Autonomous Vehicles, Eco-Driving, Optimal Control, Mixed-Integer Programming

## 1. INTRODUCTION

A major challenge in modern transportation is to achieve zero accidents while at the same time emit as little greenhouse gasses and harmful pollutants as possible. The European Union aims to cut emissions by 40% with respect to 1990 levels by 2030 (European Commission, 2019). One of the key tasks is the reduction of the total energy consumption in different environmental problems including urban transportation, which accounts for almost a quarter of Europe's greenhouse gasses emissions. One of the main causes can be attributed to traffic congestion and idling time of vehicles at signalised intersections (Schrang et al., 2019; Kural et al., 2014).

To this regard, Electric Vehicles (EV) and Autonomous Vehicles (AV) can play a principal role, since they have shown advantages in terms of being environmentally friendly and energy efficient (Xu et al., 2015; Grauers et al., 2012; Litman, 2019). For instance, as shown in Tate et al. (2018), fully automated road transport systems will lead to energy consumption reductions of 55% - 66%. Moreover, thanks to the latest technological developments, it is nowadays possible to create a communication network between the different agents of an urban crossroad. Communication Infrastructure-to-Vehicle (I2V) and Vehicle-to-Infrastructure (V2I) will enable the intersections to be more efficient in terms of time and energy consumption.

The topic of coordination of vehicles along an intersection has been addressed in the literature by different researchers. For instance, Campos et al. (2015) propose a decentralized problem formulation where each agent solve a local optimization problem. However, the intersection decision order considers heuristics for priority assignments which might lead to an energy sub-optimal solution. In

Colombo and Vecchio (2015), a scheduling-based approach is proposed, where the researchers focus on the feasibility of a crossing sequence, where a supervisor controller acts when necessary to maintain safety. Unfortunately, this approach do not guarantee energy optimal solutions.

On the other hand, optimal control problem formulations allow explicit performance objectives such as energy efficiency. However, while it is frequently stated that energy minimization is the objective, this target is commonly not explicitly included in the cost function (de Campos et al., 2014, 2015; Zhang et al., 2016). Hult et al. (2018) propose an economic model predictive control formulation using an objective function which directly captures both energy consumption and travel time. However, rear-end collision avoidance is not taken into account, i.e., scenarios where multiple vehicles proceed in the same direction after crossing the intersection.

This paper proposes an approach which aims to fill the gap noticed in the literature by proposing an optimal control problem formulation able to solve any kind of intersection conflict scenarios between AVs aiming to cross an intersection, i.e., vehicles coming from each direction can proceed straight or make a turning maneuver and multiple vehicles can proceed along the same path once they have crossed the intersection, with the objective to minimize the global energy consumption of the vehicles. Moreover, an alternative modelling framework is proposed in order to simplify the formulation of the problem and, finally, a Sequential Quadratic Programming (SQP) formulation is adopted in order to solve the problem.

## 2. PROBLEM FRAMEWORK

In this paper, the coordination of an urban intersection scenario of  $N_v$  AVs is probed. Each vehicle  $n \in \mathcal{N}_v =$

$\{1, \dots, N_v\}$  follows a pre-defined route such that a collision could occur if no control action is applied. The desired route of each AV is considered to be *a priori* given, i.e., by using a high-level path planning algorithm, which is outside the scope of this study. Hence, the objective is to control the velocity of each vehicle along its trajectory such that the energy consumption is minimized and the vehicles positions are mutually exclusive, i.e, a control agent will modify the desired velocity profiles of the vehicles in order to let them cross safely while minimizing energy losses.

Specifically, a four-way perpendicular crossroad is considered in this analysis since, firstly, it represents a typical urban crossroad and, secondly, because it allows to perform analysis on complex intersection scenarios, which will be detailed in Section 4. Nevertheless, the work carried out in this paper can be adapted for scenarios with different cross angles and number of road segments, the latter at the cost of an increase in the number of variables. Finally, all vehicles are considered to be equipped with V2V, V2I communication systems and at most one vehicle per lane is approaching the intersection, i.e., cases with multiple vehicles reaching the crossroad from the same direction are not taken into account. This last assumption reduces the problem complexity as it excludes rear-end collisions before the intersection joint. However, this simplification can be removed effortlessly imposing a distance constraint between vehicles reaching the intersection from the same lane.

### 2.1 Mapping from 2D to 1D

The geometry of the crossroad is depicted in Fig. 1 where we define as Intersection Zone (IZ) the area where two or more routes might intersect, i.e., where a side collision could occur. The IZ is defined by a set of four coordinates  $\mathcal{IZ} = \{(x_l^I, y_l^I)\}_{l \in \{1, \dots, 4\}}$  with respect to the absolute Cartesian coordinate system  $OXY$  which define the edges of the IZ, where the superindex  $I$  refers to Intersection. Moreover, the absolute position of the  $N_v$  vehicles is defined with respect to  $OXY$  and the initial and final conditions on position and velocity are known and defined as  $\mathcal{P}_n^o := \{(x_n^o, y_n^o)\}_{n \in \mathcal{N}_v}$ ,  $\mathcal{P}_n^f := \{(x_n^f, y_n^f)\}_{n \in \mathcal{N}_v}$ ,  $\bar{V}_n^o, \bar{V}_n^f$ , for all  $n \in \mathcal{N}_v$ , respectively. The relative position of each

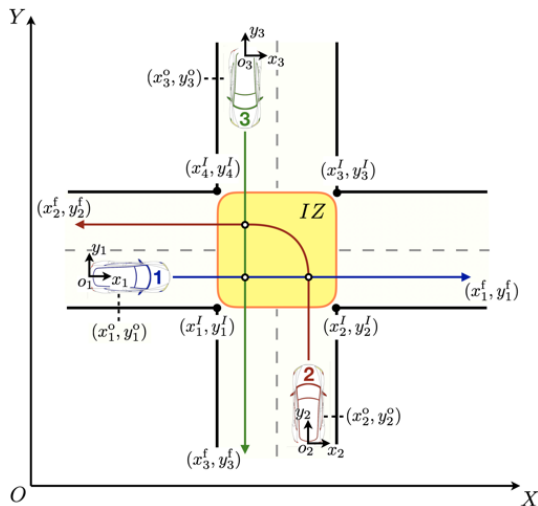


Fig. 1. Crossroad schematic & conflict graph definition

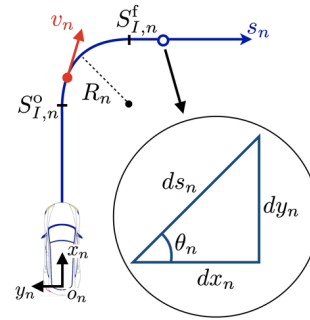


Fig. 2. Vehicle's trajectory definition

vehicle with respect to each other is defined through a  $o_n x_n y_n$  coordinate system, coplanar to the absolute one and rigid to the associated vehicle  $n \in \mathcal{N}_v$ . The motion of each AV in the two-dimensional space can be described by the following equations of motion:

$$\frac{d}{ds_n} x_n = \cos \theta_n \tag{1a}$$

$$\frac{d}{ds_n} y_n = \sin \theta_n \tag{1b}$$

$$\frac{d}{ds_n} \theta_n = K_n \tag{1c}$$

where  $\theta_n$  defines the orientation of the vehicle with respect to the initial configuration, and derivatives are taken with respect to the trajectory  $s_n$  (cf. Fig. 2). The curvature of the vehicle  $K_n$ , which depends on the trajectory, is equal to 0 for all  $t \geq 0$  for vehicles having a straight trajectory and is defined as

$$K_n(s_n) = \begin{cases} \frac{1}{R_n} & \text{for } S_{I,n}^o \leq s_n \leq S_{I,n}^f, \\ 0 & \text{otherwise,} \end{cases} \tag{2}$$

for vehicles making a turn, where  $R_n$  is the radius of turn and  $S_{I,n}^o, S_{I,n}^f$  define the beginning and the end of the intersection, respectively.

To simplify the analysis and the mathematical formulation detailed in Section 3, the trajectory of each vehicle is considered to be straight, however still keeping the curvature information of the vehicle as shown in Fig. 3. This formulation allows to simplify the problem into single dimensional while still having knowledge of which vehicle is performing a turning maneuver inside the IZ, information which importance will be addressed in Section 3. Finally, in order to define the vehicles and intersection information with respect to an absolute one-dimensional reference system, the information defined for each vehicle  $n$  on the respective trajectory  $s_n$  are mapped to the absolute coordinate  $s^*$  as shown in Fig. 3, setting  $S_1^o = S_2^o = \dots = S_{N_v}^o = S^o$ .

The single dimension coordinate reformulation is justified from the fact that vehicle's trajectories are imposed and cannot be modified and, therefore, the tangential velocity, which is the desired variable to be regulated, can be modified just in terms of intensity, i.e., its modulus can be varied but not its direction. Therefore, using a 1D framework will allow to ease the problem formulation, reducing the number of control variables, which will need to be defined just with respect to the single dimensional trajectory coordinate  $s^*$ .

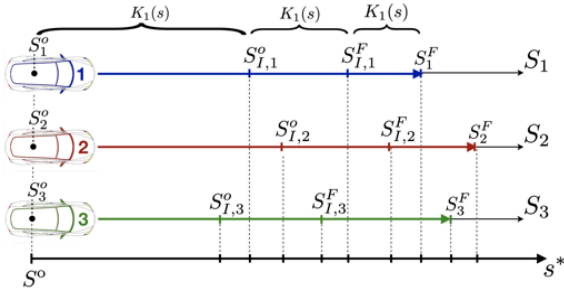


Fig. 3. Definition of a global one-dimensional reference

## 2.2 Conflicting Points

Two vehicles crossing the intersection define a Conflicting Point (CP) if their trajectories intersect. Therefore, for each vehicle  $j \in \mathcal{N}_V$  we define a set  $\mathcal{A}^j = \{n \in \mathcal{N}_V : \|\begin{bmatrix} x_n(s_1) \\ y_n(s_1) \end{bmatrix} - \begin{bmatrix} x_n(s_2) \\ y_n(s_2) \end{bmatrix}\| = 0 \text{ for some } s_1, s_2 \in \mathcal{R}_+\}$ . Moreover, since the CP for vehicle  $n$  with vehicle  $j$  coincides with the CP of vehicle  $j$  with vehicle  $n$ , in order to remove redundant CPs, the conflicting points set is defined as  $\mathcal{CP} = \{(i, j) \in \mathcal{N}_V \times \mathcal{N}_V : i \in \mathcal{A}^j, \text{ with } j > i\}$ .

## 3. OPTIMAL CONTROL PROBLEM FORMULATION

In the previous section, we have presented a representation for the vehicle coordination problem as a one-dimensional motion, in which possible conflicts are represented by conflicting points. This representation will be used to define the energy optimal vehicle coordination problem, which is formulated as an optimal control problem in this section. The problem can be seen as an extension of the eco-driving problem of Padilla et al. (2018) towards multiple vehicles, while at the same time *i*) including additional constraints representing the avoidance of conflicts on the intersection, *ii*) avoiding rear-end collisions and *iii*) incorporating energy-losses during cornering, which is discussed in more detail in Padilla et al. (2020).

The objective in the eco-driving problem as discussed in Padilla et al. (2018) is to minimize the power  $P(t)$  required from a vehicle in order to cover a given distance  $S^f - S^o$  over a provided time interval  $t^f - t^o$ , knowing the velocity and acceleration bounds  $v(t) \in [\underline{v}, \bar{v}]$ ,  $a(t) \in [\underline{a}, \bar{a}]$ , respectively, the boundary conditions on position and velocity and subject to longitudinal vehicle dynamics.

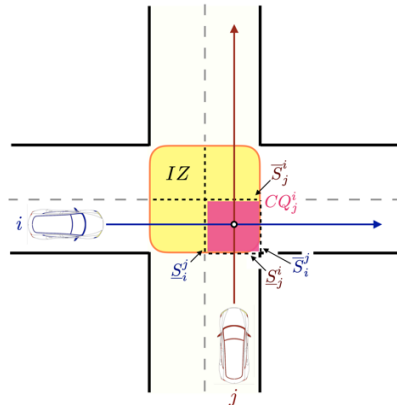


Fig. 4. Conflicting Quadrant definition

## 3.1 Intersection Constraint

As introduced in Section 2.2, a conflicting point is defined between each pair of AVs for which the trajectories intersect. Mathematically, if we consider AVs as point masses, in order to prevent collisions between vehicles, the condition  $S_i(t) \neq S_j(t)$  has to hold for all  $(i, j) \in \mathcal{CP}$  and for all  $t \geq 0$ . However, since the trajectories of two vehicles can cross only inside the intersection, the constraint needs to be active just when one of the two vehicles resides inside the crossroad and does not need to exist when at least one of the two vehicles has already left it. Moreover, since each vehicle has a length and a width, the aforementioned constraint does not guarantee safety and, hence, it has to be slightly modified to accommodate for the length of the vehicles. Thus, we consider to divide the IZ in four quadrants as shown in Fig. 4 and we define a conflicting quadrant  $CQ_j^i \forall (i, j) \in \mathcal{CP}$ .

Furthermore, we consider that each vehicle can reside entirely in one quadrant. Therefore, for each pair  $(i, j) \in \mathcal{CP}$  we define two bounds  $\underline{S}_j^i, \bar{S}_j^i$  which define the beginning and the end respectively of the  $CQ_j^i$  for vehicle  $i$  and, similarly for vehicle  $j$ ,  $\underline{S}_i^j, \bar{S}_i^j$ . In Fig. 4 an example as been reported in order to clarify the concept. Therefore, in order to guarantee a collision-free scenario, the constraint has been defined as follow:

$$\begin{cases} s_j \leq \underline{S}_j^i(1 - \delta_c) + S_j^f \delta_c \\ s_j \geq \bar{S}_j^i \delta_c \end{cases} \quad (3)$$

with  $\delta_c \in \{0, 1\}$ , for  $c \in \mathcal{C} = \{1, \dots, C\}$  with  $C = \dim(\mathcal{CP})$ , being the Intersection Decision Variable (IDV) which will decide the crossing order between the two vehicles. The IDV is defined as a binary variable which will impose vehicle  $j$  to leave the intersection before vehicle  $i$  if  $\delta_c = 1$ , or await until vehicle  $i$  has exit the crossroad if  $\delta_c = 0$ .

## 3.2 Rear-End Constraint

In the case where two or more vehicles proceed in the same direction after crossing the intersection, an additional constraint has to be imposed to prevent rear-end collisions. In order to prevent collisions among vehicles  $i$  and  $j$ , with  $i, j \in \mathcal{N}_V$  and  $i \neq j$ . This can be imposed by requiring that either

$$s_{r,i}(t) - s_{r,j}(t) \geq \epsilon \quad \text{if } s_{r,i}(t) > s_{r,j}(t) \quad (4a)$$

or

$$s_{r,j}(t) - s_{r,i}(t) \geq \epsilon \quad \text{if } s_{r,j}(t) > s_{r,i}(t) \quad (4b)$$

holds for  $t \geq t_{I,i}^f$ , in which  $s_{r,i}(t) = s_i(t) - S_{I,i}^f$  and  $s_{r,j}(t) = s_j(t) - S_{I,j}^f$  are the positions of the vehicles measured from exit point of the IZ,  $t_{I,i}^f$  being the time instant at which the  $i$ -th vehicle exits the IZ and  $\epsilon$  is a positive constant that guarantees a safety distance among the two vehicles. Note that (4) can be expressed as the product between the two inequalities, leading to the following quadratic (concave) inequality constraint:

$$(s_{r,i}(t) - s_{r,j}(t))^2 \geq \epsilon^2 \quad (5)$$

As a remark, (5) has to be active only for those vehicles that proceed in the same direction and just when the  $i$ -th vehicle exits the intersection. Therefore, (5) is required for

all  $(i, j) \in \mathcal{R}_{\mathcal{E}} = \{(i, j) : s_i(t_i) = s_j(t_j)\}$  for some  $t_i, t_j \in \mathbb{R}^+$  and  $t_i \geq t_{1,i}^f$ .

### 3.3 Dynamical Model

Typically, traditional eco-driving formulations neglect lateral dynamics and only consider longitudinal vehicle dynamics defined by the difference between the traction force in the longitudinal direction  $F_u(t)$  and the dissipative forces which, for a vehicle proceeding on a trajectory with no slope, are the aerodynamical drag force  $F_{air}$  and rolling resistance  $F_{roll}$ . Therefore, the vehicle dynamics are defined by Newton's second law as

$$mu = F_u - \underbrace{\sigma_d v^2}_{F_{air}} - \underbrace{mgc_r}_{F_{roll}} \quad (6)$$

where  $m$  represents the equivalent mass of the vehicle,  $u(t) = \frac{dv}{dt}$  is the vehicle acceleration,  $v(t)$  defines the vehicle velocity,  $g \approx 9.81m/s^2$  is the gravitational acceleration constant,  $c_r$  is the rolling force coefficient and  $\sigma_d = \frac{1}{2}c_d\rho_a A_f$ , with  $\sigma_d$  the drag coefficient,  $\rho_a$  the air density and  $A_f$  the frontal area of the vehicle. However, as shown in (Padilla et al., 2020), this model is conservative for vehicles while cornering, since it neglects the effects of the friction force which has a substantial impact on the energy losses.

Therefore, the dynamical model considered in this paper is defined as follow

$$mu = F_u - \sigma_d v^2 - mgc_r - \hat{F}_{fr} \quad (7)$$

where the last term on the right-hand side represents the component of the friction force acting on the longitudinal direction of the vehicle.

In this work, a kinematic bicycle model is considered, i.e., the velocity vectors on the front and rear wheels are aligned with their respective longitudinal directions, which is a reasonable assumption for low vehicle motion speed ( $\leq 5 m/s$ ), Rajamani (2012). Moreover, we assume rear-wheel traction vehicles with front-wheels-only steering systems, implying that the rear wheel will be aligned with the longitudinal axis of the vehicle for the entire route. On the other hand, the front wheel is able to change orientation and his steering angle is defined by  $\delta_f \in (-\frac{\pi}{2}, \frac{\pi}{2})$ . The radius of curvature  $R$  is defined from the instantaneous center of rotation  $O$  and the center of gravity  $C$  of the vehicle. We assume  $C$  to be located at distances  $l_r$  and  $l_f$  from the rear and front wheel axis, respectively. The friction force supplies the centripetal force  $F_c$  which is applied in  $C$ , and points toward  $O$ . Note that the velocity vector of the vehicle is tangential to the trajectory and forms an angle  $\beta$  with respect to the longitudinal vehicle axis given by

$$\beta = \arcsin(l_r K). \quad (8)$$

Therefore, the component acting on the longitudinal axis of the vehicle can be obtained as

$$\hat{F}_{fr} = F_c \cos(\frac{\pi}{2} - \beta) = ml_r v^2 K^2. \quad (9)$$

The reader can refer to (Padilla et al., 2020) for further details about the derivation of (9).

### 3.4 Cost Function Extension

The objective of the problem in Padilla et al. (2018) is to minimize the power consumption  $P(t)$  of a vehicle, which

is assumed to be a quadratic function of the form

$$P(v, F_u) = \beta_0 v^2 + \beta_1 v F_u + \beta_2 F_u^2 \quad (10)$$

for some parameters  $\beta_0, \beta_1, \beta_2 \geq 0$ . The quadratic form is a realistic assumption for EVs, since it properly captures mechanical friction losses and Ohmic losses. As done in Padilla et al. (2018), the dynamical model of the vehicle defined in (7), which represent an equality constraint for the problem, is substituted in (10) in order to obtain a simplified yet equivalent formulation. Thus, the cost function for a generic AV becomes:

$$P(v, s, u) = \beta_0 v^2 + \beta_1 \eta(s) v^3 + \beta_2 (mu)^2 + \beta_2 (\eta(s) v^2 + c_r mg)^2 \quad (11)$$

with  $\eta(s) = \sigma_d + ml_r K(s)^2$ .

The goal of our problem is to minimize the global energy consumption of  $N_v$  AVs approaching from multiple lanes. We therefore take the cost function of the optimal control problem as the arithmetic sum of the cost function defined for each vehicle, i.e.,

$$P(\{v_n, s_n, u_n\}_{n \in \mathcal{N}_v}) = \sum_{n=1}^{N_v} P(v_n, s_n, u_n) \quad (12)$$

in which now every vehicle can have different  $\beta_{0,n}$ ,  $\beta_{1,n}$  and  $\beta_{2,n}$ .

### 3.5 Cornering Constraint

The introduction of a further restriction while cornering arises from the desire to guarantee safe driving conditions for all the vehicles involved in the intersection control problem. As introduced in Section 3.3, in order to make a turn, the centripetal force needs to be applied to the vehicle. However, in order to avoid vehicle's slip, the total force applied to the vehicle must be lower than the maximum friction force during normal operation defined as

$$(mu(t))^2 + m^2 v(t)^4 K(s(t))^2 \leq (m\mu_s g)^2 \quad (13)$$

where the terms in the left-hand side of (13) represent the resultant force applied to the vehicle in the tangential direction and the centripetal force, respectively, and the term on the right-hand side is the maximum friction force, with  $\mu_s > 0$  being the friction coefficient.

### 3.6 Optimal Control Problem

After introducing and motivating all the constraints required for energy optimal coordination of  $N_v$  AVs crossing an intersection, we can now formalize this problem as the following optimal control problem:

$$\min_{s_n(t), v_n(t), u_n(t), \delta_{i,j}(t)} \int_{t_0}^{t_f} \sum_{n=1}^{N_v} P(v_n(t), s_n(t), u_n(t)) dt \quad (14a)$$

subject to

$$\begin{aligned} \frac{d}{dt}s(t) &= v(t), & \frac{d}{dt}v(t) &= u(t) & (14b) \\ s_n(t^o) &= S_n^o, & s_n(t^f) &\geq S_n^f & (14c) \\ v_n(t^o) &= V_n^o, & v_n(t^f) &= V_n^f & (14d) \\ \underline{v}_n &\leq v_n(t) \leq \bar{v}_n & & & (14e) \\ \underline{u}_n &\leq u_n(t) \leq \bar{u}_n & & & (14f) \\ u_n(t)^2 + v_n(t)^4 K_n^2 &\leq (\mu_{s,n}g)^2 \text{ for all } S_{I,n}^o \leq s_n(t) \leq S_{I,n}^f & & & (14g) \end{aligned}$$

$$s_j(t) \leq \underline{S}_j^i [1 - \delta_{i,j}(t)] + S_j^f \delta_{i,j}(t) \quad \text{for all } (i, j) \in \mathcal{CP} \text{ and } \underline{S}_i^j \leq s_i(t) \leq \bar{S}_i^j \quad (14h)$$

$$s_j(t) \geq \bar{S}_j^i \delta_{i,j}(t), \quad \text{for all } (i, j) \in \mathcal{CP} \text{ and } \underline{S}_i^j \leq s_i(t) \leq \bar{S}_i^j \quad (14i)$$

$$(s_i(t) - s_j(t) - S_{I,i}^f + S_{I,j}^f)^2 \geq \epsilon^2 \quad \text{for all } (i, j) \in \mathcal{RE} \text{ and } s_i(t) \geq S_{I,i}^f \quad (14j)$$

and  $\delta_{i,j} \in \{0, 1\}$ , where (14a) is defined as in (12), the time evolution of velocity and position are defined by (14b), (14c) and (14d) define initial and final conditions on position and velocity, respectively, and boundary conditions on velocity and acceleration are defined by (14e) and (14f), respectively. Eq.(14g) defines a constraint imposed on the overall acceleration of the vehicle which has been obtained from (13). As a remark, (14g) becomes inactive when the vehicle is proceeding on a straight trajectory ( $K_n = 0$ ), since it is assumed that  $\bar{u} \leq \mu_{s,n}g$ . Finally, (14h-14i) describe the intersection constraint and (14j) prevents rear-end collisions. As a last remark, notice that in order to not force the crossing order between vehicles proceeding toward the same direction, the condition on the final position in (14c) has to be imposed in terms of inequality constraint.

The optimal control problem (14) belongs to the non-linear mixed-integer programming category of problems, since (14g) is non-linear and because the control inputs of the problem,  $u_n(t)$  and  $\delta_c(t)$ , are continuous and discrete, respectively. In order to solve the optimal control problem using a static optimization technique, (14) is discretized at times  $t_k = k\tau + t_0$ ,  $k \in \mathcal{K} = \{0, \dots, K-1\}$  with time step

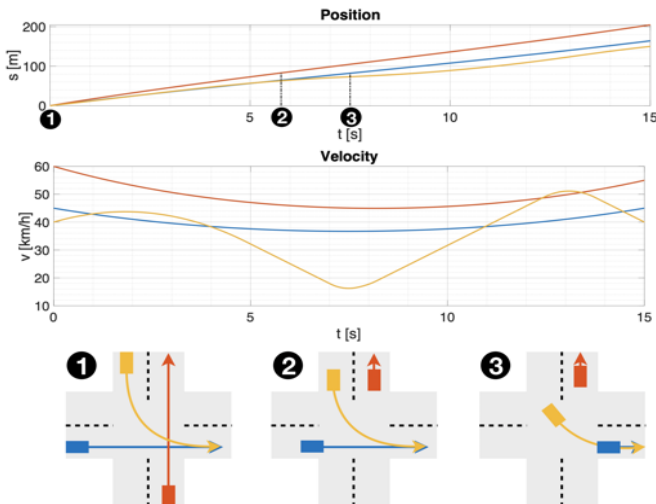


Fig. 5. Position, velocity profiles & order resolution of  $I_s = [1, 1, 0, 2]$

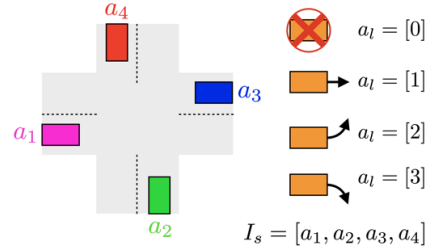


Fig. 6. Intersection Scenario ( $I_s$ ) Definition

$\tau = \frac{t_f - t_0}{K}$  using a forward Euler discretization method. Moreover, since the objective function (14a) is non-convex and constraint (14g) is non-linear, a sequential mixed-integer quadratically constraint programming approach has been used, linearizing (14g) and convexifying (14a), in order to find the solution.

#### 4. SIMULATION STUDY AND COMPARISONS

In this section, several simulations are reported in order to show the benefits of relying on AVs. Comparisons between different intersections configurations are made in order to present additional remarks on the proposed control problem reported in this paper.

For this comparisons, we consider a four-way intersection scenario where roads are enumerated as  $a_l$  for  $l = [1, \dots, 4]$ , in an anti-clockwise manner starting at the left, as depicted in Fig. 6. Moreover, in order to define the trajectory of each vehicle, as reported in Fig. 6, for each  $a_l$  a value is associated depending on the trajectory of the vehicle approaching, i.e., [0] if the vehicle is absent, [1] if the vehicle follows a straight trajectory, [2] if it makes a left turn and [3] if it turns to the right. Therefore, we define as Intersection Scenario ( $I_s$ ) the vector  $I_s = [a_1, a_2, a_3, a_4]$  with for each  $a_l$  a numeric value as defined above.

##### 4.1 Simulation Results for Different Situations

This case study is reported in order to show how the solutions among similar cases can differ from each other

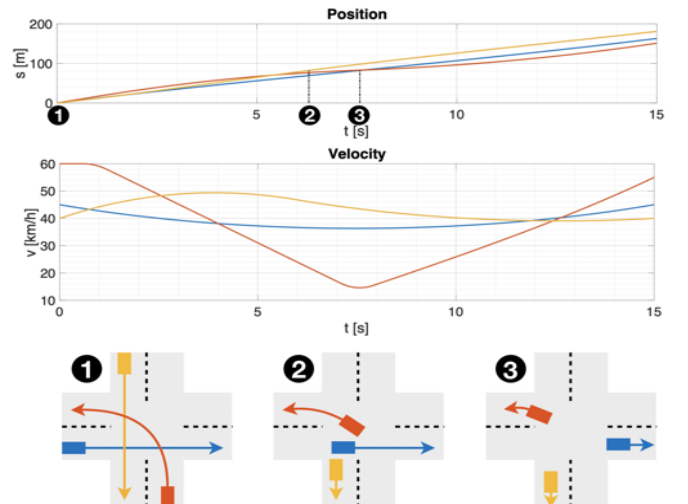


Fig. 7. Position, velocity profiles & order resolution of  $I_s = [1, 2, 0, 1]$

depending on the trajectories and number of AVs approaching the intersection. To do so, let us compare the bottom-left corner of Fig. 7 with the bottom-left corner of Fig. 5. As can be seen, in both scenarios two vehicles follow a straight trajectory while a third one makes a left turn. However, the vehicles arrival directions are different. The initial and final conditions for vehicles coming from the same direction are equal. As can be noticed comparing the position plots between the two cases (Fig. 7-5 top), the distances covered from the vehicles vary in order to obtain the optimal solution for the different cases. Moreover, it can be noticed that even though the vehicles are coming from the same direction in both cases, the different trajectories affect the intersection resolution order in the two scenarios (Fig. 7-5 bottom). As a last remark, even though the two scenarios are similar to each other, the computational time differs significantly since the solver requires  $\approx 20s$  to compute the solution for the scenario in Fig. 7 and  $\approx 120s$  for the case in Fig. 5.

The considerable difference in terms of computational time is due to the fact that (14) is solved as a sequential mixed-integer quadratically constrained program, in which successive quadratic approximations of the nonlinear mixed-integer programs are solved. In the case of Fig. 7, the quadratic constraint (14j) is inactive, while for Fig. 5, the quadratic constraint (14j) is active between vehicle 4 and 1 since they aim to proceed towards the same direction. This leads to a significantly higher computational complexity. Lastly, Table 1 reports the energy consumption ( $P$ ), the time ( $t_s$ ) required to compute the solution and the number of iterations ( $n^\circ itr$ ) to achieve convergence for different intersection scenarios ( $I_s$ ) defined accordingly to Fig. 6 as previously explained.

Table 1. Energy Consumption ( $P$ ), Algorithm Computational time ( $t_s$ ) and Iterations ( $n^\circ itr$ ) for Different Intersection scenarios ( $I_s$ )

$I_s$	$P[MW]$	$t_s[s]$	$n^\circ itr$
[3,0,0,0]	5.26	1.52	5
[1,1,0,0]	7.07	5.56	3
[1,2,0,0]	7.91	15.8	6
[2,2,0,0]	9.64	94.01	33
[1,0,0,2]	8.71	15.4	2
[1,0,0,3]	8.95	10.24	5
[1,1,0,2]	11.6	176.59	6
[1,2,0,1]	10.8	19.71	7
[1,1,1,1]	14.3	24.3	5

## 5. CONCLUSIONS

In this paper, we have proposed a solution to conflict resolutions between autonomous vehicles (AVs) crossing an urban intersection. The conflict resolution problem is formulated as an optimal control problem, where the objective is to minimize energy consumption of all the vehicles, while avoiding collisions. The first contribution of this paper has been the proposal of a one-dimensional problem framework in order to simplify the modeling to handle conflicts between AVs in urban crossroads. The intersection control problem presented in this paper is combinatorial since, depending on the number of vehicles reaching the crossroad and their respective trajectories, the problem's nature changes. Simulation results point out

how relying on AVs can increase the traffic throughput and allow for more efficient solutions in terms of energy consumption due to the unnecessary constriction on the intersection crossing order. Even though the methodology is not yet suitable for real-time implementation, the simulation example shows that the vehicle coordination problem can be solved as an optimal control problem. The proposed solution strategy might serve as a benchmark for more heuristic solution to the vehicle coordination problem.

## REFERENCES

- Campos, G.R., Falcone, P., and Sjoberg, J. (2015). Traffic safety at intersections: a priority based approach for cooperative collision avoidance. In *Proc Int Symp Future Active Safety Technology*.
- Colombo, A. and Vecchio, D.D. (2015). Least restrictive supervisors for intersection collision avoidance: A scheduling approach. *IEEE Trans Autom Control*.
- de Campos, G.R., Falcone, P., and Wymeersch, H. (2015). An approximate solution to the optimal coordination problem for autonomous vehicles at intersections. In *Proc American Control Conf*.
- de Campos, G.R., Falcone, P., Wymeersch, H., Hult, R., and Sjoberg, J. (2014). Cooperative receding horizon conflict resolution at traffic intersections. In *Proc Conf Decision and Control*.
- European Commission (2019). Management plan 2019 - climate action.
- Grauers, A., Sarasini, S., and Karlstrom, M. (2012). Why electromobility and what is it? In *Systems Perspective on Electromobility*. Chalmers University of Technology.
- Hult, R., Zanon, M., Gros, S., and Falcone, P. (2018). Energy-optimal coordination of autonomous vehicles at intersections. In *Proc European Control Conf*.
- Kural, E., Parrilla, A.F., and Grauers, A. (2014). Traffic light assistant system for optimized energy consumption in an electric vehicle. In *Proc Int Conf Connected Vehicles and Expo*.
- Litman, T. (2019). *Autonomous Vehicle Implementation Predictions: Implications for Transport Planning*. Victoria transport institute.
- Padilla, G., Pelosi, C., Beckers, C., and Donkers, M. (2020). Eco-driving for energy efficient cornering of electric vehicles in urban scenarios. In *Proc IFAC World Congress*.
- Padilla, G., Weiland, S., and Donkers, M. (2018). A global optimal solution to the eco-driving problem. *IEEE Control Systems letters*.
- Rajamani, R. (2012). *Vehicle Dynamics and Control*. Springer.
- Schrank, D., Lomax, T., and Eisele, B. (2019). 2019 urban mobility report.
- Tate, L., Hochgreb, S., Hall, J., and Bassett, M. (2018). Energy efficiency of autonomous car powertrain. *SAE Int*.
- Xu, G., Zheng, C., Zhang, Y., Xu, K., and Liang, J. (2015). *Energy Efficiency of Electric Vehicles – Energy Saving and Optimal Control Technologies*. IntechOpen.
- Zhang, Y.J., Malikopoulos, A.A., , and Cassandras, C.G. (2016). Optimal control and coordination of connected and automated vehicles at urban traffic intersections. In *Proc American Control Conf*.