

## The coupled market

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## The coupled market

Unlocking the potential of distributed energy resources in the electricity market

## Mana Farrokhseresht





## The Coupled Market

Unlocking the Potential of Distributed Energy Resources in the Electricity Market

PROEFSCHRIFT

ter verkrijging van de graad van doctor aan de Technische Universiteit Eindhoven, op gezag van de rector magnificus prof.dr.ir. F.P.T. Baaijens, voor een commissie aangewezen door het College voor Promoties, in het openbaar te verdedigen op vrijdag 12 februari 2021 om 16:00 uur

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## Summary

A high penetration of distributed energy resources (DERs) brings benefits and opportunities for the power system. However, in the current electricity market, there are challenges for the participation of DERs. These include scalability and the complexity of many relatively small units participating in one central market, neglecting distribution system constraints in the market clearing process, and inaccessibility of ancillary service markets and their high transaction fees for DERs. The existing market model is unable to deal with the participation of DERs in the market in the most efficient way and has therefore given rise to new ways of looking at the electricity market. In this thesis, a new market model, the coupled market, that allows full participation of DERs in the day-ahead and balancing markets is proposed. In this market model, DERs can participate in a local market operated by the distribution market operator (DMO). Distribution system constraints are included during the local market clearing process to keep the system within its operating limits. Moreover, the proposed market model introduces an ancillary service market where DERs can trade their capacity. To ensure interoperability between the local and central market a coordination scheme is introduced: At each of the day-ahead or balancing markets, the DMO first solves a preliminary scheduling problem where the total cost of the local market is minimized while the distribution system constraints are taken into account. Through this optimization, the DMO can estimate the local market price for energy or balancing services. Based on those prices, he will bid into the central market and acquire/sell additional energy. First, the coupled market is studied from the systems perspective. Total cost comparison between the coupled and the current market is conducted. The comparison shows that the coupled market is cost-comparable with the current market and capable of adequately enforcing the distribution system constraints. More in-depth research has been performed to analyse the proposed coupled market from the DMOs and DERs perspective. Studying the coupled market from the DMOs perspective investigates the DMO participation in the central market. In the preliminary scheduling of the coupled market model, the DMO must estimate the central market prices, as this step happens before the central market clearing. The DMO must deal with the uncertainty in the central market prices in such a way that he can minimize the total cost in its area. To handle this uncertainty, either scenario or regret based approach can be considered where the scenario-based is preferable for the DERs

and regret based approach is most beneficial from the view of the distribution system operator or the energy supplier. Finally, in studying the coupled market from the DERs perspective, what the bidding strategy of DERs should be to maximise their profit within the coupled market is addressed. DERs participating in the coupled market can have market power despite their small size, as the DMO market size is smaller and the location of the DERs can matter. The strategic operation of DERs is investigated through a wind farm. As storage systems bring benefits for renewable-based DERs, the wind farm is equipped with a storage system. The proposed strategic bidding of this wind farm with a storage system is modelled through a bi-level optimization showing that a wind farm with a storage system can exercise its market power in a local market to generate higher revenue. Moreover, the distribution system parameters, e.g. cable resistance and loads, can affect the ability to exercise market power by the wind farm with a storage system in the local market. Lastly, generating power from the wind farm is significantly higher in the coupled market compared with the amounts cleared in the current market showing that the coupled market can better unlock the potential of the renewable-based DERs which want to participate in the market.

## Nomenclature

The main nomenclature used in this thesis is listed in Tables 1-3. Other symbols and abbreviations are defined where they first appear.

Table 1: Sets and Indices					
c(C)	Index(set) of combined wind farm and energy storage				
	systems.				
e(E)	Index(set) of energy storage systems.				
g(G)	Index(set) of generators.				
i(I)	Index(set) of sending nodes.				
j(J)	Index(set) of receiving nodes.				
l(L)	Index(set) of line.				
$L_D$	Set of distribution lines.				
$L_T$	Set of transmission lines.				
$N_D$	Set of distribution nodes.				
$N_T$	Set of transmission nodes.				
$N_{D-T}$	Set of interface nodes in distribution system.				
$N_{T-D}$	Set of interface nodes in transmission system.				
s(S)	Index(set) of scenarios.				
t(T)	Index(set) of time steps.				
w(W)	Index(set) of wind farms.				

	Table 2: Parameters
$b_i$	Nodal susceptance [p.u.].
$B_l$	Line shunt susceptance of transmission line l [p.u.].
$E_e^{ini}$	Initial energy value for the energy storage system [MWh].
$E_e^{max}$	Maximum ESS state-of-charge [MWh].
$E_e^{min}$	Minimum ESS state-of-charge [MWh].
$G_i$	Distribution nodal admittance [p.u.].
$M_{\_}$	A large positive number.
$O_{e,t}^{Edis/Ech}$	Energy (upward/downward) offer price by the storage in
-,-	the balancing market $[\in/MWh]$ .
$O_{a,t}^E$	Energy offer price of generators in day-ahead market
5,7	[€/MWh].
$O_{a,t}^{RUP}$	Upward reserve offer price of generators $[\in/MW]$ .
$O_{a,t}^{RDN}$	Downward reserve offer price of generators $[\in/MW]$ .
$O_{at}^{EUP/EDN}$	Energy (upward/downward) offer price of generators in bal-
$g,\iota$	ancing market [€/MWh].
$P_{c}^{ch,max}$	Maximum charging power of ESS [MW].
$P_e^{dis,max}$	Maximum discharging power of ESS [MW].
$P_{i,t}^{load}/Q_{i,t}^{load}$	Active/reactive power load demand [MW/MVAr].
$P_{w,t,s}^{Wact}$	Actual wind farm power production [MW].
$P_w^{W,v,s}$	Installed wind farm power [MW].
$P_{at}^{gmax}/P_{at}^{gmin}$	Maximum/minimum active power of generator g [MVAr]
$Q_{g,t}^{gmax}/Q_{g,t}^{gmin}$	Maximum/minimum reactive power of generator g [MVAr]
$r_l$	Resistance of a distribution line.
$S_{g,t}$	Rated apparent power of generator g [MVA].
$S_{l,t}$	Rated apparent power of line l [MVA].
$SI_{t,s}$	Total system imbalance in scenario s and time t [MW].
$TC_l$	Transmission line capacity.
$V_i^{max}/V_i^{min}$	Maximum/minimum voltage of bus $i \in N_D$
$x_l$	Reactance of a distribution line.
$\alpha_{Imb}$	Coefficient for total system imbalance in distribution sys-
	tem.
$\alpha_T$	Coefficient for total reserve capacity requirement in trans-
. 1	mission system.
$\eta^{cn}/\eta^{ais}$	Charging/discharging efficiency of the ESS [p.u.].
$\lambda_{t,s}^{ID}$	Wholesale day-ahead market price in scenario s and time t $[\notin /MWh]$
\+/-	[\(\/ M M II]).
$\Lambda_{t,s}$	Forecasted positive/negative impaiance prices $[\mathbf{t}/\mathbf{MWh}]$ .
$\pi_s$	Scenario probability.

	Table 3: Decision Variables
$E_{e,t}$	Energy stored (state-of-charge) in the ESS [MWh].
$f_{l,t}^p/f_{l,t}^q$	Active/reactive power over line l [MW/MVAr].
$I_{l,t}$	Square current over line l [A].
$\widehat{o}_{c.t}^{\dot{E}}$	Energy offer price of WF-ESS [ $\in$ /MWh].
$\widehat{o}_{e,t}^{Rech}$	Downward reserve offer price of ESS $[\in/MW]$ .
$\widehat{o}_{e \ t}^{Redis}$	Upward reserve offer price of ESS $[\in/MW]$ .
$P_{e,t}^{ch/dis}$	Charging/discharging rate of ESS in day-ahead energy market [MW].
$\widehat{P}_{e,t}^{ch/BL}$	Energy quantity bid/offer ESS in balancing market (downward regulation) [MW].
$\widehat{P}_{e,t}^{dis/BL}$	Energy quantity bid/offer ESS in balancing market (upward regulation) [MW].
$P_{e,t}^{ch/BL}$	Charging rate of ESS in balancing market(downward regulation) [MW].
$P_{e,t}^{dis/BL}$	Discharging rate of ESS in balancing market (upward regulation) [MW].
$\widehat{P}^{DA}_{c,t}$	Energy quantity bid/offer by WF-ESS at time t, [MW].
$P_{c,t}^{DA}$	Scheduled energy from the WF-ESS in day-ahead energy market [MW].
$P_{i,t}^{DMO}$	Real power injection in interface node $i \in N_{D-T}$ as DMO's energy bid in the TO market [MW].
$P_{i,t}^{DMO/DN}/P_{i,t}^{DMO/UP}$	Downward/upward regulation in the balancing mar- ket at $N_{D-T}$ node at the distribution system as the
	TMO market [MW]
$P_{g,t}/Q_{g,t}$	Scheduled active/reactive power output from gener- ator g [MW/MVAr]
$P^{DA}_{L}$	Scheduled wind power in day-ahead market [MW].
$P_{TD}^{TD}/Q_{TD}^{TD}$	Real/reactive power injection in T-D interface node
- i,t / Vi,t	i representative of TMO's bid in the local market [MW/MVAr].
$P_{\rm exp}^{TD/DN} / O_{\rm exp}^{TD/UP}$	Downward/upward regulation in the balancing mar-
$i_{i,t} / \mathfrak{C}_{i,t}$	ket at T-D interface node i representative of TMO's bid in the level merket [MW/MWAr]
$D^{DN/UP}$	Dig in the local market [WWW/WWAT].
$P_{g,t}$	Downward/upward regulation from generator g in
$P_{c,t,s}^{Total real time}$	Actual power produced by the WF-ESS in scenario s and time t [MW].

$R_{g,t}^{DN/UP}$	Scheduled downward/upward reserve capacity of the generator $\sigma$ [MW]						
$R_{e,t}^{ch/dis}$	Charging/discharging rate of ESS in reserve market (down-						
$\hat{D}ch/dis$	ward/upward reserve) [MIW].						
$R_{e,t}$	Upward/ downward reserve bid/oner by storage system e at						
	time t, $[\mathbf{M}\mathbf{W}]$ .						
$R_{i,t}^{DMO/DN}$	Aggregated downward reserve at node						
	$i \in N_{D-T}$ as DMO's upward reserve bid in the TMO market						
	[MW].						
$R_{i,t}^{DMO/UP}$	Aggregated upward reserve at node						
$\iota,\iota$	$i \in N_{D-T}$ as DMO's upward reserve bid in the TMO market						
	[MW].						
$R_{i,i}^{DER}$	Aggregated reserve capacity from DERs at T-D node [MW].						
$u_{0,t}$	Binary variable related to the charging state of storage.						
$V_{i,t}$	Square bus voltage [p.u.] at node $i \in N_D$						
11	Binary variable defines the positive and negative imbalance of						
$g_{l,s}$	WE-ESS						
7.4	Binary variable related to the imbalance direction of WF-ESS						
$\Delta_{i,s}$	Imbalance of WF-ESS in scenario s and time t [MW]						
$\Delta^+$	Positive imbalance of WE-ESS and time t [MW]						
$\Delta^{c,t,s}$	Negative imbalance of WE-ESS and time t [MW]						
$\Delta_{c,t,s}$	Impalance of wind form and time t [MW]						
$\Delta_{w,t,s}$	Desitive indulation of sind from and time t [MW].						
$\Delta_{w,t,s}$	Positive imbalance of wind farm and time t [MW].						
$\Delta_{w,t,s}$	Negative imbalance of wind farm and time t [MW].						
$\theta_{i,t}$	Transmission bus angle.						

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# Introduction

This chapter is an introduction to this thesis. Section 1.1 explains the main challenges that distributed energy resources are facing in the current electricity markets. These challenges are the main motivation doing the research work presented in this thesis. In section 1.2, the main research question followed by the sub-questions which are addressed in this thesis, are described. Finally, in section 1.3, the thesis outline and the content of each chapter are explained. It is shown how each chapter addresses the formulated research sub-questions.

#### 1.1 DER's participation in electricity markets

The penetration of distributed energy resources (DERs) such as onshore wind turbines, solar systems, electrical vehicles, storage systems, CHPs, is increasing in the distribution systems. DERs are defined as behind the meter technologies and are small-scale power generation (normally in the range of 1 kW to 10 kW) providing an alternative to or an enhancement of the centralized generator. In Europe, DERs are increasing too. Figure 1.1 shows the share of onshore wind and solar systems in the total electricity generation in Europe based on data from [1]. As the figure shows, both onshore wind and solar have sharply increasing trends at their contributions in the total produced electricity. In contrast to the increase in the share of DERs at the distribution system, the share of centralized generators connected to the transmission system in producing electricity stays either constant or faces a decreasing trend [2].



Figure 1.1: Total share of wind and solar in electricity generation [%], based on data from [1]

These changes in distribution and transmission systems due to increasing DERs and reducing centralized generators, however, are so far not followed by any change in the electricity market structure. Current electricity markets have been designed for large centralized generators and cannot offer a promising solution to deal with the aforementioned changes. Current market rules and the administrative requirements for market participants may hurt DERs to access markets since market rules mainly have been set-up based on technical characteristics of centralized generators. High minimum bid size requirement, for example, is one of the market rules which can prevent the participation of DERs in electricity markets. In day-ahead markets, the minimum volume increment is 0.1 MW, while in balancing markets the minimum bid size could be as high as 50 MW, with increments of 10 MW [3], since balancing markets have traditionally been designed for large generators. Even though some balancing markets have reduced

the minimum bid size in the range of 1-5 MW, however for DERs it still can be difficult to enter into those markets.

In addition to existing limitations in market rules, there are also technical barriers that can cause DERs to face more difficulties to enter the electricity market. These technical barriers are based on the fact that DERs are connected to the distribution system. Therefore, activating DER's in the current market in which the distribution network constraints are not considered, can lead to security issues such as under/over-voltages, congestions and so on within the distribution system [4]. In the current electricity market which usually clears at the transmission level, it is difficult to include all the technical and security characteristics of the distribution systems in one centralized market-clearing. Because first of all, market operators have little knowledge about the distribution network. Second, the computational complexity in one centralized market-clearing algorithm which incorporates all technical constraints of transmission and distribution grids is huge, and solving that algorithm can be unattractive.

Moreover, the security of the distribution system depends on the power flowing over the network. Connecting DERs to the distribution network makes the power flow more difficult to be predictable. This is explained in figure 1.2. Figure 1.2.(a) shows a traditional distribution network without having DERs. As shown in the figure, only loads are connected to the distribution network. This causes the power flows only from the transmission to the distribution networks, as shown by red arrows in figure 1.2.(a). Given the uni-directional power flow, the prediction of power flows over the distribution network becomes easier. However, in a distribution network with a high number of DER's integrated, unidirectional power flow from the transmission to the distribution system is no longer guaranteed. Instead, as shown in figure 1.2.(b), bi-directional power flows over the



Figure 1.2: Power flow in distribution network

distribution system happen which makes the prediction of power flowing over the distribution network more difficult. The situation can be even worse when more renewable-based DERs, such as wind turbines or solar systems with uncertain power generation are connected to the distribution network. Consequently, the direction of the power flow over the interface transformer between the transmission and distribution systems becomes unknown for system and market operators in the current electricity market, and this can lead to an insecure power system i.e. a system in which the technical performance indicators such as voltage, frequency, and loading of the network assets operate out of their standard operational limits. Current electricity market is a centralized market model with a single central [national] market where all market participants participate.

Therefore, by knowing the aforementioned market and technical barriers for participation of DERs in electricity markets, the question becomes emerged as to how to develop an electricity market to facilitate the participation of DERs into the electricity market while the distribution network can still work in its secure operational limits. The related research questions are given in the following section.

#### 1.2 Research questions

The changes in the distribution system due to increasing the number of DERs combined with the fact that current electricity market is not able to fully cope with increasing DERs, give rise to the question of how the electricity market can be optimized. This leads to the main research question:

How should the current market designs be adapted to enable full participation of DERs in an economically efficient and system-secure way?

To answer the main research question the following studies need to be done. First, it should be investigated what are limitations and deficits, and the main barriers to the participation of DERs in the current electricity markets. Thereafter, a solution in terms of a new market design needs to be proposed to overcome the aforementioned limitations and enable the participation of DERs in the market. To evaluate the applicability of the proposed market design, it should be investigated from different perspectives. First, from the system's perspective to understand if the proposed market design is conducive to economic efficiency and system security/reliability in comparison with the current electricity markets. Second, from the main stakeholders perspective to see how the proposed market design affects their position. That means, to answer the main research question, it can be divided into a number of sub-questions: 1) What are the potential limitations for DERs to participate in a network-secure and economically efficient way in the current electricity markets?

First, we need to clarify what are those limitations in current electricity markets. To answer this question, an extensive literature review is required to have a thorough insight into current market models and to understand their advantages and disadvantages, and consequently to formulate the research goal associated with the required changes in the current market designs.

## 2) How could the electricity market design be changed in order to overcome the discovered limitations?

After finding the answer to sub-question 1 about the research goal associated with changes required for the current electricity market, sub-question 2 is seeking a solution to overcome those limitations and to implement the required changes by adapting current markets. Hence, the answer to this question will lead to proposing a new market design which avoids the limitations found in sub-question 1.

## 3) Do the changes in the market design result in economic and secure operation of the power system?

This question looks at the proposed market design from the system's perspective. To answer this question one needs to evaluate the main features of the proposed market design which are focused on improving the current limitations. Therefore, this question is answered by comparing the network security and economic efficiency aspects of the proposed market design with the current market design.

4) Do the changes in the market design result in an economic and secure operation from the perspective of stakeholders within the market?

This question looks at the proposed market design form the perspective of the main stakeholders. To answer this question, the effect of the proposed market design on the performance and objectives of the main stakeholders within the market, i.e. both market players and network operators, needs to be evaluated.

#### 1.3 Thesis outline

The research work of this thesis is based on the flowchart shown in the left side of figure 1.3 and the thesis outline is on the right side.



Figure 1.3: The thesis research work and its outline

Chapter 2 provides the necessary background and definitions of key concepts and their interactions in electricity markets. Chapter 3 mainly reviews existing literature on DERs market participation to discover the research gaps. Understanding the basic concepts and finding out the research gaps lead to the design of a new market for DERs in the second step. The new market design called "the coupled market" is described in chapter 4. The coupled market is evaluated from two main perspectives of the system and the main stakeholders in the third step. Evaluating the coupled market from a system perspective is shown in chapter 5 and from main stakeholders perspectives is shown in chapters 6 and 7. Finally, the conclusions of evaluating the coupled market from different perspectives are drawn in the fourth step and are shown in chapter 8.

The research questions posed in the previous section are addressed in the main five chapters of this thesis. In each of the chapters 3-7, one of the sub-questions

is assessed. This is presented below together with a more detailed explanation of each chapter's contents.

#### Chapter 2

This chapter explains the fundamental concepts in electricity markets which are important for the research work of this thesis. The main topics of this chapter include how electricity markets work, the main actors and components in the current electricity market and the interaction between them, electricity market design, different products, and how they are traded in electricity markets, market power and different ways of imposing market power by market players, and finally, how electricity markets operate in Europe.

#### Chapter 3

This chapter answers the first sub-question and is mainly a literature review on existing research works on electricity market designs for market participation of DERs. The main goal of this chapter is first, to give an insight into existing solutions in literature for overcoming the current challenges of DERs market participation. The second goal is to show the research gaps in the literature which are going to be addressed through this thesis. The chapter begins with an introduction to DERs and their potential in providing energy and services. Thereafter, in the rest of the chapter, the participation of DERs in the electricity market is discussed. The barriers for DERs in current market are given. Then the possible solutions of DER market integrations through aggregators and local electricity markets are explained. The different market models in the existence of local markets are described. Finally, DERs revenue including different pricing and exercising market power in local markets are explained.

#### Chapter 4

This chapter answers the second sub-question by proposing the so-called *coupled* market. In this market design, there is a local electricity market that enables the full participation of DERs in day-ahead and balancing markets. This local market is operated by the distribution market operator (DMO). The DMO takes into account distribution system security in the market clearing algorithm. There is an interaction between this local market and the (national) central market which is explained in this chapter. The chapter explains the main features of the coupled market including the market organization, main stakeholders and their roles and interactions within the market, the interaction between the local and the

central market, pricing mechanism, and market timing. Finally, the mathematical formulation associated with the market-clearing algorithm is presented.

#### Chapter 5

This chapter answers the third sub-question by comparing the performance of the proposed coupled market with the state-of-the-art centralized market model to investigate the network-security and economic efficiency aspects of the coupled market. At the beginning of the chapter, the centralized market model is explained and its difference with the coupled market in terms of market organization, pricing, scalability, and market timing are described. The quantitative comparison between the coupled market and the centralized market model, in terms of total system costs, while including or ignoring the distribution network security constraints, is shown followed by the corresponding numerical results.

#### Chapter 6

This chapter answers the fourth sub-question which is about studying the coupled market from the perspectives of the main stakeholders. One of the main stakeholders, within the coupled market, is the DMO. Since the DMO is a new market actor, it is important to look into its performance in more detail. This chapter looks into the coupled market from the DMO's perspective. As explained above, in the coupled market there is an interaction between the local and the central markets which means the DMO, on behalf of DERs, can participate in the central market. Therefore, this chapter mainly looks into DMO's different bidding strategies in the national market. First, at the beginning of the chapter, the DMO's roles and responsibilities and its interaction with other stakeholders in the coupled market are explained. Thereafter, the quantitative study including the numerical results is presented, which shows how different DMO's bidding strategies in the central market affect the performance of the local market including the costs and the market participation of DERs.

#### Chapter 7

This chapter also answers the fourth sub-question but from the DER's perspective. As the main research question is toward enabling the DER's market participation, therefore, it is important to study the coupled market from DER's perspective too. The main goal of this chapter is to show how in the coupled market model, the DER's participation in the local market can raise its revenue in comparison with its revenue in the current centralized market model. For this, first, the problem of revenue maximization of DERs in the coupled market is formulated. Then a comparison is made between DERs revenue in the coupled market versus the current centralized market model. Thereafter, the associated numerical results are presented.

#### Chapter 8

With the discussion of all these topics, the thesis will be concluded with a recapitulation of the research sub-questions presented in section 1.2. Through this review, an answer to the main research question is formulated and the conclusions which can be drawn from it are given in the final chapter of this thesis. Subsequently, a list of the main contributions in the research of this thesis is presented. Finally, some recommendations as future works for this thesis are given.

## 2 Electricity Markets

In this chapter, an overview of the most important concepts in the area of electricity markets which are used in this thesis is presented. First, in section 2.1, an introduction to electricity markets is given. This section shows how electricity markets work by explaining the development of the electricity market after the deregulation of the energy sector, and describing the different interactions between buyers and sellers. Section 2.2 shows the main actors and components in the current electricity market and the interaction between them. In section 2.3, electricity market design, different products, and how they are traded in the various markets, are presented. Market power and different ways of imposing market power by market players are also discussed in this section. Section 2.4 explains how electricity markets operate in Europe. As examples, two countries, Germany and the Netherlands are chosen to illustrate similarities and differences among electricity markets in Europe. Since the renewable-based distributed energy resources are becoming more integrated into the power systems, the last discussion of this chapter is assigned to the participation of renewable energy resources (RES) in electricity markets and their situations in the Netherlands and Germany. Finally, this chapter ends with some conclusions in section 2.5, summarizing the current status, with a focus on the degree of market integration of RES.

#### 2.1 Introduction to electricity markets

Following deregulation of the energy sector, electricity, in the same way as any other commodity, is traded in a market and this market is called the *electricity* market. However, there are fundamental differences between electricity and other commodities. The first difference is the capability of storing (electrical) energy. Storing a high amount of electrical energy in storage units (e.g. batteries and hydro reservoirs) is more difficult in comparison with storing other commodities. This makes the balance between supply and demand in the power system more challenging compared with other systems. The second difference is transport. Electricity is transported via the electrical infrastructure, or the grid. The generators and consumers of electricity are connected at all times to the the electrical grid, and together they form the physical power system. Therefore, unlike other commodities, transportation is an essential and inseparable for electricity. Finally, the balance between supply and demand must be held at any instant and based on second by second interactions. Otherwise, the whole system collapses. The electricity system can cover an entire country, or even an entire continent. The social, economic and even political consequences of such a large blackout are completely intolerable for the national economy. Therefore, a lack of supply for electricity can be a disaster. These differences create new structures and organizations in the electricity market which may not exist in other commodity markets. Therefore, it is important to first understand how electricity markets work. This is explained in the next section.

#### How does the electricity market work?

Traditionally, there was a monopoly in electricity markets, where generators, transmission, and distribution companies were owned by one vertically-integrated utility public. This model is shown in Figure 2.1.a. Generators are all owned by the national utility. Therefore, there is no competition at any level in this monopolistic model. After a while, a single-buyer model was developed. This model is shown in Figure 2.1.b. In this model, generators are no longer owned by transmission utilities. Generators sell their energy to a single-buyer transmission and distribution utility. The transmission utility then sells the purchased energy to the distribution units. It should be noted that all agreements are only made between the single buyer and generators and there is no direct contract between distributors and generators. This model is the beginning of competition in the electricity market.

In the 1990s, liberalization happened which caused the power system to become unbundled. It means that generators, transmission and distribution utilities are no longer owned by one entity. Figure 2.1.c shows this model. New entities that emerged out of this unbundled power system are named "energy suppliers".



Figure 2.1: Developing electricity market

They buy energy from generators and sell it to consumers. Moreover, the new structure creates competition among generators and energy suppliers and brings new opportunities to electricity markets [5].

#### Interaction between buyers and sellers in the electricity market

Same as any market, the electricity market is an environment where buyers and sellers interact with each other and agree on transactions. The interaction between supply and demand leads to an equilibrium in which the market price is created. In an ideal free market, this equilibrium can be reached on a bilateral direct contract or over-the-counter (OTC) contract basis between the buyers and the sellers. Consumers buy their required energy from large generators or energy suppliers. Beforehand, they must predict their consumption pattern. However, the predicted consumption might not be the same as the one occurring in realtime. Moreover, generators might not be able to produce the exact amount of energy agreed in the contract. An unpredicted (partial) outage can prevent them to deliver the same contracted energy. All in all, unpredictable events can create a mismatch between supply and demand. To avoid this, consumers and suppliers must be notified about the imbalances on a short-time basis. This needs trading of large amounts of information since there are a high number of transactions in this open free market. As a result, it is a question of whether this interaction system is fast enough to prevent imbalances before the entire system collapses [6].

Therefore, instead of bilateral contracts, the power system needs also a managed pool-based electricity market which is competitive enough to supply the electricity to consumers with the least cost. In the power pool model, all electricity trades occur through a central market place. The power pool operator receives the bids and offers from sellers and buyers and clears the market. Under this model, the power is traded only through the central market and there are no direct trades between buyers and sellers [7].

Finally, the third way of interaction between the buyers and sellers is a hybrid model which is a combination of bilateral contracts and centralized power exchange. Consumers can decide to participate in a power exchange or to negotiate directly with generators through a bilateral contract. The benefit of this model is the maximum flexibility for the end-users. Note that the difference between the pool-based market and the power exchange is that participation in power exchange is voluntary while in the power pool it is mandatory [7].

#### 2.2 The three layers in the electricity market

In electricity markets, there are different actors. If each actor is assumed to be located in one layer, there can be three different layers in total: the physical layer, the control layer, and the market layer. Each layer has specific roles and responsibilities in the electricity market. Figure 2.2 shows these three layers with their components and interactions which are explained below.

#### 2.2.1 Physical layer

As Figure 2.2 shows, the bottom layer is the physical layer. This layer contains distribution and transmission grids, their network components and infrastructure and also the consumers and generators connected to the distribution and transmission grids.

The main quantity exchanged within this layer is power. Traditionally, the (large) generation is connected to the transmission grid. In such a system, the power will flow from the transmission to the distribution grid hence there is a unidirectional power flow. Distribution grids are passive and they mainly connect consumers, also traditionally known as loads. However, the introduction of distributed energy resources (DER) creates a paradigm shift in power systems. The high integration level of DERs changes the passive distribution network into an active network in which there are loads together with (small) generators and storage systems. Moreover, the power flow is no longer uni-directional and can also flow from the distribution to the transmission grid in case of excessive local generation in the distribution grid. In other words, bidirectional power flow can happen between



Figure 2.2: Main components of the electricity market, showing the three layers and main actors

transmission and distribution grids, though of course not at the same time. This evolution adds higher complexity in the management of distribution systems, but also brings new possibilities to optimize the overall power system by allowing distribution systems to participate actively in the system operation and by allowing DERs to participate actively in the distribution system management [3].

As Figure 2.2 shows, the interaction between the physical layer with its upper layers is limited to the information and cash flows. The control layer must monitor the power flow within the physical layer to guarantee the security of the power system. Therefore, there should be a continuous information flow between the physical layer and the control layer regarding the status of the grids, injection or withdrawal of power by generators and loads and so on. The interaction between the physical layer and the market layer is related to the cash flow. After clearing the market by the market operator, generators and consumers are being paid or pay through the market layer. A detailed explanation for the control layer and the market layer can be found below.

#### 2.2.2 Control layer

The middle layer in Figure 2.2 is the control layer. As it is mentioned earlier, the main task of this layer is to control and monitor the power flow within the physical

layer in which infrastructures of distribution and transmission grids together with their components are located. The main actors in the control layer are the distribution system operator (DSO) and the transmission systems operator (TSO). Their roles and responsibilities are briefly described here.

The role of the TSO is to transport energy in a given region from centralized generators to dispersed industrial consumers and DSOs over its high-voltage grid [8]. "Transmission System Operator is a natural or legal person responsible for operating, ensuring the maintenance of and, if necessary, developing the transmission system in a given area and, where applicable, its interconnections with other systems, and for ensuring the long-term ability of the system to meet reasonable demands for the transmission of electricity." This definition is provided by Directive (EU) 2019/944 of the European Parliament and of the Council on common rules for the internal market for electricity and amending Directive 2012/27/EU, art. 2(35). The TSO protects the systems long-term transport adequacy and is responsible for operating the power system on a larger often national scale. The main focus of the TSO is maintaining power balance, subject to limits on the high-voltage network capacity. The TSO can also, at times, face congestion issues related to the transmission of electricity, for example over interconnection corridors with other countries, or where there are capacity issues as a result of maintenance. etc.

As mentioned above, the passive distribution network has changed into an active distribution system. This leads to having DSOs instead of distribution network operators in the distribution grid. Consequently, new roles and responsibilities for the DSO are created. "DSO is responsible for operating, ensuring the maintenance of the distribution grid and its facilities and, if necessary, developing the distribution system in a given area and, where applicable, its interconnections with other systems and for ensuring the long term ability of the system to meet reasonable demands for the distribution of electricity or gas". This definition is provided for by Directive 2007/72/EC, art. 2 (6) with regards to electricity and by Directive 2007/73/EC, art. 2 (6) with regards to gas [9].

#### 2.2.3 Market layer

The uppermost layer in Figure 2.2 is the market layer. The main actor in this layer is the market operator (MO). The MO is the one who manages the electricity market and its main responsibility is to make sure that aggregated supply and demand are matched over a designated time interval called market time-unit.

When the MO finds the equilibrium between demand and supply, i.e. clears the market, the market price is determined. Any market operator must be independent of any market participant and energy supplier for two major reasons: (1) to operate the market fairly and impartially and (2) to utilize the resources in the

most efficient and least-cost manner. Usually, MOs are non-profit entities that are also independent of any buyers and sellers participating in the market. They are also independent of the ownership of any other profit-making entities. These MOs are governed by an independent board of directors and comply with the rules set by national regulators [10].

In addition to the market operator, there is another actor in the market layer which is called the energy supplier. The interaction between energy suppliers and the physical layer is related to the cash flow. The energy supplier buys energy from the bulk generators and sells it to the consumers which are connected to the distribution grid in the retail market. The role of the energy supplier is especially important for the grid when there are lots of passive market participants (i.e. market participants which do not participate on their own but rather through energy supplier).

#### 2.3 Electricity market design

As mentioned earlier, the electricity market must be reliable and fast enough to overcome unpredictable events hence to guarantee the balance between supply and demand. Electricity markets should perform economically efficient hence must be designed in a way that trading commodities help to increase its performance. For that purpose, some technical problems specific to power systems must be overcome, such as managing forecast inaccuracies in supply and demand, and the resulting real-time imbalances.

Electricity markets are multi-commodity markets including wholesale energy, reserve capacity and balancing energy. Energy is the main product. However, the reserve is an important product too that guarantees that enough backup resources is available in case of equipment failure, fluctuations of production from renewable energy sources, and (sudden) demand forecast changes. Each of these products can be traded in a different market. As mentioned in section 2.1, electricity can be traded in a pool-based market. The pool consists of a day-ahead market and shorter-term markets known as adjustment markets which are also called intra-day markets. The balancing market that ensures the real-time balance between supply and demand is another type of market that is pool-organized and is settled ex-post, while other markets are ex-ante. The energy traded in the pool is mostly negotiated in the day-ahead market, while adjustment markets such as intra-day and balancing markets are used to make adjustments to the energy cleared in the day-ahead market [11].

Besides the pool-based and bilateral markets, there exists a futures market, i.e. an auction market in which participants buy and sell physical or financial products for delivery on a specified future date. The main feature of futures markets is that



Figure 2.3: A pool-based market showing the sequence of clearing multiple markets

they allow trading physical or financial products in the future at today's prices. Thus, futures markets are useful if the price of electricity is highly uncertain in the pool, which is the case in pool-based electricity markets [11]. Note that futures markets and bilateral contracts are not of any interest in this thesis. Therefore, only a more detailed explanation for day-ahead, intra-day, balancing and reserve markets are provided below.

#### 2.3.1 Day-ahead market

Figure 2.3 shows the clearing sequence of different markets for a pool-based electricity market. As it is shown, the day-ahead market for day D typically is cleared before noon of the previous day, day D-1.

In the day-ahead, producers submit energy volumes and their corresponding minimum selling prices for every hour of the market horizon and every production unit. At the same time, retailers and consumers submit energy volumes and their corresponding maximum buying prices for every hour of the market horizon. The market operator collects bids and offers, aggregates them in the so-called bidding curves for supply and demand, and clears the market using a market-clearing procedure. A market-clearing procedure results in market-clearing prices for every hour, as well as production and consumption schedules. If the transmission network constraints are not considered in the market-clearing procedure, the resulting market-clearing price is identical for all market participants. On the other hand, if the transmission network is taken into account during clearing the market, a locational marginal price (LMP) is associated with each node of the power system which may differ from node to node due to line losses and line congestion. The LMP is in fact an extreme case when the network is modelled in great detail. If a transmission line is congested, more expensive generation is needed to be dispatched on the downstream side of the congested line. This increase in expensive generation yields an increase in the market-clearing prices in those nodes placed on the downstream side of the congested line [11].
The deadline for the market participants to submit their offer price (currency per MWh) and the quantity (in megawatt-hour (MWh)) can differ from country to country. These bids are input to a complex market clearing algorithm to calculate the clearing price. Normally, this market-clearing procedure takes less than few hours (depending on regulations) and determines which bids are accepted or rejected. Thereafter is the settlement phase where the financial and physical transactions are being established. The output of the market-clearing algorithm is multiple time series of prices. For example in Europe, these prices are normally between -500  $\in$ /MWh and 3000  $\in$ /MWh, and traded quantities for each bidding zone and for the time windows of 24 hours for the next day [12]. The deadline for the market participants to submit their bids is at 12h00 CET and the delivery of the accepted bid begins by the sellers at the cleared price from 00h00 CET of the next day (day D).

#### 2.3.2 Intra-day market

As shown in Figure 2.3, the intra-day market for day D is cleared every few hours (e.g., every four hours) once the day-ahead market has been cleared. The intra-day electricity market is a short-term market offering flexible continuous trading opportunities on the same day as the physical delivery. Intra-day trading is largely used to cover expected energy imbalances before a real-time balancing market takes over. This is important for market participants since they can avoid paying high imbalance costs in the real-time balancing market. Intra-day markets have been developed in recent years, due to integrating more renewable energy resources into electricity grids. For example, a renewable energy producer might lose production due to unexpected shortfall which is mainly caused by weather fluctuations. The intra-day market allows the producer to buy the deficit energy to maintain its energy balance.

Besides trading in the pool-based market, the intra-day market offers also its participants various opportunities and flexibility based on their trading strategies. Market participants can submit single or block contracts with different validity and execution restrictions [13] [14]. Participants have a specific motivation to use a specific contract type. A contract type is one of the fundamental building blocks for trading strategies and thus trading algorithms. Therefore, one main difference to day-ahead trading is the pricing on the intra-day market. While dayahead trades are related to market-clearing price principles, where the marginal bid sets the price for all transactions, the prices in intra-day trading are set in a "pay-as-bid" process. This is why bid prices are often used in intra-day trading. The result is that there are no fixed prices for products on the intra-day market [15].The intra-day market is not applied in this thesis and is presented only for the sake of completeness, therefore, a more detailed explanation is not given.

# 2.3.3 Balancing market

The balancing or real-time market is a single-buyer auction-based market which is cleared each hour or several times within an hour. This market aims to provide energy to compensate deficit and excess generation to keep the supply-demand balance in the market up to the last moment before delivery time. In the case of deficit of generation, producers and consumers submit their offers to be accepted by the market operator in an increasing-price order. Subsequently, in the case of excess generation, the offers to reduce the generation are accepted on a decreasing-price basis.

Usually, the TSO is responsible for real-time buying and selling electricity in the balancing market to assure that the control area they are responsible for is in balance. For this, the TSO can outsource balancing services to the so-called Balance Responsible Parties (BRPs), to ensure the balance in the control area is maintained. A BRP is a private legal entity that has a view of the balance of one or multiple access points to the transmission grid. BRPs portfolio is also called the balancing group. A balancing group is a number of feed-in and feed-out points within a control area and is owned by a BRP. The sum of the (day-ahead and intra-day) transactions for each BRP is called an energy program (e-program) and BRP composes a balanced portfolio by combining injection, offtake, exchange with other BRPs and possibly import or export from or to another control area [16]. Each generator in the grid is obliged to have a contract with a BRP. Alternatively, they can be their own balancing responsible party. For example, in Figure 2.2, bulk generators and consumers which are connected to the transmissions grid can act as BRPs in the market. An energy supplier can also be a BRP. If a BRP deviates from its e-program, it has to pay the imbalance cost to the network operator. The network operator procures the necessary balancing energy from a balancing service provider (BSP). A BRP can be also a BSP if the generator of the BRP are active in the balancing market.

The price in the balancing market is called "imbalance price". If a generator or consumer generates less or consumes more energy than their scheduled energy in the day-ahead market, they cause an imbalance surplus in the system. On the contrary, if a generator or a consumer generates more or consumes less energy than the one scheduled in the day-ahead market, they cause an imbalance shortage. Depending on the status of the system's imbalance and pricing mechanism in the balancing market, they have to pay or be paid for their shortage or surplus.

Generally speaking, there are two ways for the imbalance pricing mechanism, single pricing, and dual pricing. Before explaining these two pricing mechanisms, it should be mentioned that in the balancing there can be two situations for the system: if the system requires additional indeed or reduced withdrawal of power, the system is called "short" and if the system requires higher withdrawal or lower infeed of power, the system is called "long". The corresponding imbalance prices in these two systems are called short and long imbalance prices, respectively. Under a single pricing approach, the long and short imbalance prices are identical. It means that market participants receive the imbalance price if they have a generation surplus and have to pay this same price if they have a shortage in the balancing market [17]. However, in the dual pricing approach, the long and short imbalances have different prices, depending on the direction of the market participant's imbalance with respect to the direction of the system's imbalance. In other words, if their imbalance is in the opposite direction of the system imbalance, they usually have to pay a price equal to the day-ahead price. If they are in the same direction with the net system imbalance, they have to pay the imbalance price based on the marginal price of the last balancing unit deployed, which is usually higher than the day-ahead price.

Generally, the difference between the imbalance price and the day-ahead price encourages market participants to provide beneficial contributions and hold them back from adverse contributions to the system's balance. Hence, the imbalance price is generally higher than the day-ahead price in a short system, but lower in a long system. Germany, Belgium, and the UK are countries that apply a single pricing scheme. The dual pricing approach is applied in the Netherlands, France, Spain and the Nordic countries [18][19]. The dual pricing approach usually creates more incentive for market participants to net their imbalances [20]. In this thesis, dual pricing is applied.

#### 2.3.4 Reserve market

To guarantee that enough balancing resources are available during the real-time operation of the power system, the system operator, i.e. the TSO, allocates reserve capacity in advance. For this, the capacity reservation happens through the reserve market while the reserve activation happens through the imbalance market. Scheduling of the reserve capacity requires generators to operate less than their maximum capacity. Deployment of the reserve capacity, however, needs re-dispatching of units committed in the day-ahead market, curtailing loads and/or running extra generators, and increasing loads and/or cutting-off the generators, to compensate unexpected shortage/surplus of energy in real-time.

The reserve market is cleared either jointly with the day-ahead market or immediately following it by the system operator as it is shown in Figure 2.3. In European markets, simultaneous energy and reserve market clearing are not applied. This type of joint market clearing is more used in the US. However, simultaneous arrangement of energy and reserve capacity gives the authority to market players to arbitrate between both on shorter notice and consequently increases the utilisation of their flexibility in the market. Therefore, joint energy and reserve capacity markets are being investigated in the current market organisation of the European markets [21]. There are three types of reserve products in the reserve market; Primary reserve or Frequency Containment Reserve (FCR), secondary reserve or Frequency Restoration Reserve (FRR) consisting of aFRR and mFRR corresponding to automatic and manual activation, respectively; and tertiary reserve or Replacement Reserve (RR). Through activating these reserve products, so-called "ancillary services" are being provided to the TSO [22][23] [24]. Ancillary service is defined as a service necessary for the operation of a transmission or distribution system[25]. In chapter 3, this term is explained in more detail.

The sources of the reserve capacity market may be from the supply side as well as the demand side. The main criteria are to fulfil all the technical requirements to deliver specific reserve products. However, up to now, the supply side such as thermal and hydro-power plants are the dominant sources for the reserve capacity. All thermal power plants like gas, coal and nuclear are capable of producing all three reserve products, their difference is in the amount of the reserve they can deliver due to their different ramp rates. Moreover, electrical storage systems have high ramp rates hence they are ideal to deliver reserve capacity, too. New emerging decentralized generations such as small-scale CHPs can also deliver reserve capacity, i.e. when they are pooled in virtual power plants [26].

### 2.3.5 Market power

A typical assumption in the electricity market operation is that market participants cannot influence the market price through their actions. However, this assumption is true only if there are a large number of market participants and none of them can control (large amounts of) the consumption or production portfolio hence cannot manipulate market prices. In other words, the electricity market is "perfectly competitive". In a perfectly competitive market, if a generator sells with a price higher than its marginal price or a consumer offers a price less than the market price, it will be dropped out of the market and will be simply replaced by other generators or consumers. In the perfect competition, the market price is set through the interaction of the cumulative buyer's and seller's offers and bids. Each generator can only increase their bidding price up to the market price. Under this competitive market, buyers and sellers are called "price-takers".

However, in the real world, a perfectly competitive market is hardly possible and the market is rather an "imperfect market". In such an imperfect market, a generator might affect the market price by considering how much quantity to produce to affect the market price and/or at what price to offer to affect this quantity. In this situation, the generator is exercising market power. Market power is defined as the ability of a generator or group of generators to raise the market price higher than the price determined by the perfect competition or limit the access to the market for their competitors [27]. The generator with market power withholds its power below than its actual capacity or raises its offer prices to increase the market price and consequently increase its profit.

The generators with market power that influence the market price, are called "price-makers". Market power is more likely to happen in electricity markets when price elasticity of demand is low or systems have low reserve levels or when parts of the transmission system are heavily loaded. The latter increases the risk of locational market power due to network congestion [28] [29] [30]. The result of exercising market power by generators is to transfer the profits from buyers to generators which means in the end, that it is the consumer who has to pay the cost of raising the market price. There are more negative consequences of exercising market power, among others, obstructing the quality of service, innovation, and market competition by replacing cheaper with more expensive generators [31] [32].

There are several ways for a generators to perform market power in the electricity market. These ways, according to [33] are: using price bidding strategies to raise market prices independently of changes in underlying supply and demand conditions; exploiting market power resulting from local transmission network constraints; capacity withholding to increase market prices, in particular by manipulating the capacity payment mechanism under the existing trading arrangements; and manipulation of complex market rules to increase prices and earn excessive profits. Reference [34] summarises the market power behaviour of generating companies in three strategies: financial withdrawal (price increase), physical withdrawal (volume reduction), and physical withdrawal with free bilateral contracts. Reference [35] classifies the exercise of market power in electricity markets in two broad strategies: They can withhold their capacity from the market and this strategy is called "physical withholding" [36]. Or, they can submit inflated supply offers that reflect monopoly or oligopoly power and this strategy is called "economic withholding" [37]. Below, these two strategies are explained.

#### Physical withholding

In any market, a dominant market player can affect the market price. In electricity markets, a large generator can withhold part of its capacity to affect the market price. Physical withholding or capacity withholding by a generator is a mechanism to exercise market power. This phenomenon is explained by a simple example. Figure 2.4.a shows a supply bidding ladder by generators where the market price is  $30 \in /MWh$ . If generator A (Gen A) withholds part of its capacity, the ladder will shift to the right as shown in Figure 2.4.b. As a result, the market price goes higher to  $35 \in /MWh$ .



Figure 2.4: Physical withholding by a generator

#### Economic withholding

In a power system, in addition to the size of the generator, the location of the generator can also influence its ability to exercise market power. In this case, economic withholding by a generator occurs when it offers a price that is above its marginal cost. There are many reasons which cause economic withholding in electricity markets. A more common reason is transmission constraints [31].

# 2.4 European electricity markets

Till now, some important characteristics of electricity markets in general have been introduced. In this section, first, the application of electricity markets in Europe is introduced. Thereafter, the electricity market in two countries, the Netherlands and Germany, are explained in more detail.

# 2.4.1 Market clearing in Europe

The European Power Exchange (EPEX) is a neutral mediator active in the Central-Western European countries that operates the market and provides clearing services. This market platform includes consumers (non-retail) and big market players such as utilities, system operators (TSOs), power plants, solar and wind farm owners, aggregators, energy traders, and suppliers and financial service providers in the energy sector [38].

The European electricity system is administrated in two essential phases separated by the so-called gate closure. Decentralized decisions are characteristic for the first phase before gate closure while central coordination prevails after gate closure. In the first phase, the day-ahead spot market has the crucial function to align the demand expectations and the system-wide scheduling decisions of the various generators. Physical energy is traded for a single hour (or blocks of hours) of the following day for all area [26]. After the EPEX day-ahead spot market is held, all market participants have to submit their one-quarter-hourly energy schedules for the following day to the TSOs. At this stage, the system is theoretically in the balance as all expected generation schedules must equal the expected consumption schedules. The TSOs monitor the schedule balance and the feasibility of the expected power flows. These schedules can be adjusted until gate closure of the intra-day market, 45 minutes to one hour ahead of delivery. There is a continuous intra-day spot market to facilitate physical energy transactions necessary for those re-schedulings. At the EPEX hourly and one-quarter-hourly products are traded.

These are the last market-based transactions and decentralized decisions before the energy schedules are fixed and financially binding. After gate closure, the second phase starts and the TSOs take over the responsibility for any further action. If necessary, the TSOs call the contracted reserve capacity based on the imbalance price merit order to balance the system in real-time.

Each county slightly differs in how their electricity markets operate. To show this difference, the Netherlands and Germany are considered as two examples of the West European countries. Their electricity markets are explained below.

# 2.4.2 The Netherlands

This section gives some information concerning the generation mix, the application of day-ahead, intra-day, reserve and balancing markets, and the participation of renewable energy resources (RES) in the electricity market in the Netherlands.

# Generation mix

The dominant source of the power generated in the Netherlands, in 2018, was gas. Although it is going to decline at 2.51% compound annual growth rate (CAGR) by 2030, still gas remains a dominant power generation with about 30% share. On the other hand, wind and solar photovoltaic (PV) are drastically increasing by 2030, with an expected cumulative share of 60% of the total capacity. Coal power generation is expected to be phased out by 2030, according to the report of GlobalData [39]. This report also predicts a high share of renewable generation in the Netherlands, so that the PV and wind will increase by 13% and 12% (CAGR) out of a total installed generation fleet of 40 GW by 2030.

# Day-ahead market

As mentioned in section 1.3.1, in the day-ahead market, electricity is traded one day before actual delivery on an hourly-based auction. This holds for the Netherlands as well. Before 2015, the Netherlands had its exchanges which were called APX [40]. In 2015, APX together with BELPEX, the Belgian exchange, merged into the EPEX spot market. Buyers and sellers can participate in the EPEX. Besides that, they can also trade bilaterally.

Base-load, peak and off-peak energy are traded on the day-ahead market. Bids are submitted electronically. Thereafter, the equivalent between supply and demand is settled and the market price is calculated for each hour of the following day. Hourly contracts and flexible block contracts can be traded. The minimum bid size requirement for participation in the day-ahead market is 0.1 MW [38].

In the Netherlands, currently, the electricity market is a single bidding zone although it has been proposed by the Dutch TSO TenneT, the electricity market to be divided into three different bidding zones. In each zone, a market participant can freely exchange electricity without any capacity allocation but it is also possible to trade cross-bidding zone [41]. In each zone, the electricity market price is equal for all market participants.

The day-ahead market is the market with the highest trading volumes and number of participants and therefore the price from the day-ahead market is most often referred to as the electricity price. In 2013, 47.3 TWh were traded on the day-ahead market. Prices can range from -500 to  $3000 \in /MWh$ . The average base price is  $51.9 \in /MWh$ , and the price level is relatively stable [42].

# Intra-day market

From 2015, there is a slight decrease in the trading volume in the day-ahead market in the Netherlands and a shift toward the intra-day market. One reason is indeed increasing RES share in the Netherlands. The intra-day market happens closer to the actual delivery time thus RES generation forecast can be more realistic [43].

As it was said in section 1.3.2, in the intra-day market, electricity is traded on the delivery day itself and this happens in the Netherlands as well. The intra-day market enables market participants to correct for deviations in their day-ahead contracts, because of a better estimation in the intra-day markets for RES generation, loads, unexpected power plant outages, etc.

In the Netherlands, intra-day hourly products are traded on both Nord Pool (only for cross-border trading between the Netherlands and Norway) and EPEX spot. However, the intra-day market in the Netherlands is organized differently.

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As discussed in section 1.3.2, the intra-day market is an auction-based platform. However, the Dutch intra-day market does not perform through an auction. Moreover, the Dutch continuous intra-day market contains standardised hourly products [44]. Participants continuously trade power products in hourly intervals. Prices can range from -99,999.90 to 99,999.90  $\in$ /MWh [42].

#### **Balancing** market

The Dutch TSO, TenneT, operates the balancing market in the Netherlands and is the single-buyer for regulating and reserve power. There is an obligation to participate in the balancing market for generators larger than 60 MW in the form of bids. The minimum bid size requirement is 4 MW and the maximum bid size is 200 MW. Currently, small generators cannot participate in the balancing market in the Netherlands, however, there is a discussion to reduce the minimum bid size in the balancing market from 5 to 1 MW. Thus, it becomes easier for smaller generators to participate in this market [42]. Bids also contain minimum activation time, location (for re-dispatch use) and regulation rate. Bids are valid for at least four 15-minute settlement periods[45].

BRPs inform TenneT daily about their planned transactions for the next day and the networks that they will use for transporting the electricity. Dutch DSOs, Enexis, Stedin, Aliander, Rendo, Westland, Enduris, and Cogas, inform TenneT about how much electricity each BRP actually has consumed and produced. The difference between the amounts in the e-program and the actual measured values is the imbalance. The Dutch imbalance pricing mechanism is quite elaborate. In principle, single imbalance pricing is applied which enables "passive balancing". Passive balancing gives the opportunity to the BRP to gain profit in imbalance settlement if its imbalance is opposite to the system imbalance [18]. However, depending on the regulation state dual pricing may be applied (based on both the regulation prices). Roughly, dual pricing is applied to program time units in which both upward and downward regulation has been activated.

#### **Reserve** markets

In the Netherlands, in the reserve capacity markets, the TSOs procure reserve capacity via one-sided auctions some time ahead of its contingent use. The contracted capacity is called in real-time as required to balance the system when a difference between the planned energy schedule and the required load arises in real-time. In other words, there is not a separate market for real-time balancing and whoever was called has to deliver energy in the real-time balancing. The option-like character of reserve capacity is reflected in the two-part pricing. The provision of reserve capacity is remunerated with a reservation price [€/MW] for

reserving the capacity, and a reserve energy price  $[\in/MWh]$  is paid for exercising the reserve option to generate the required energy in real-time [26]. The reserve market applied in the rest of the thesis has resemblance to the Netherlands and German reserve markets.

The minimum bid size for the total FCR is 1 MW and is provided by all synchronously connected TSOs inside the synchronous grid of Europe. It needs to be activated within 30 seconds and maintained for up to 15 minutes. For the FRR (aFRR and mFRR), the total minimum bid size is 4 MW [46]. The activation is direct and automatic by the affected TSO. It must be activated within 5 minutes and should last between 30 seconds and 15 minutes. Finally, the minimum bid size for the RR is 5 MW which can be arranged by telephone and schedule-based requests of the affected TSO. It needs to be activated from 15 minutes for up to four quarter hours or up to several hours in the event of several disturbances [42].

#### Participation of RES in electricity markets

As mentioned above in Generation mix, the RES share in the electricity sector in the Netherlands is increasing. The Dutch electricity market is equipped with an intra-day market and a short gate closure set 45 minutes to one hour before delivery which can have an advantage for RES to participate in the market [47]. As discussed in section 1.3.2, the intra-day market helps RES producers to adjust their power predictions closer to real-time.

The support scheme for RES in the Netherlands, is based on a sliding premium that balances the risk of varying electricity prices [48]. Based on [49], the SDE+ scheme "grants a premium on top of the market price to the producers of renewable energy in order to compensate for the difference between the wholesale price of electricity from fossil fuel sources and the price of electricity from renewable sources". In other words, RES generators are exposed to a lower upside and downside risk, as they can expect to receive always at least the base tariff if their generation profile is such that it can earn the average electricity price. Therefore, RES generators are not exposed to fluctuations in the annual average electricity price. RES generators need to sell their output on the day-ahead market and have full balancing responsibility. However, the most output is sold under long term contracts, so that short- term market signals do not feed through to generators. Currently in the Netherlands, a major assessment is in the legislative process that will concentrate support on the cheapest RES technologies. However, in terms of RES market integration, no major investigation and changes are expected [47]. However, there is a phasing out scheme for subsidies in NL, between now and 2030 according to [49].

### 2.4.3 Germany

This section gives some information concerning the generation mix, the application of day-ahead, intra-day, balancing markets, and the participation of RES in the electricity market in Germany. The reserve market in Germany is similar with the reserve market in the Netherlands.

#### Generation mix

In Germany, electricity generated from renewable energies is about 65% of the total generation mix by 2030.

GlobalData's latest report reveals that in 2018, renewable energy dominated Germany's power capacity mix followed by thermal, hydro and nuclear power with 53.4%, 36.9%, 5.2% and 4.4% shares respectively. In the non-hydro renewable energy mix, wind contributed a 51.4% share while solar photovoltaic (PV) a 39.4% share [50].

Despite RES increasing to a share of 65% of electricity generation, Germany is still heavily reliant on fossil fuels. In 2018, approximately 35% of electricity was generated by coal, the most polluting of energy sources. However, the German government says it will abandon nuclear energy by 2022 and is planning for a long-term exit from coal. Also, electricity generation from fossil fuels has dropped as green power is given priority entrance to the German power system, and power demand has declined due to mild weather and ongoing efficiency drives [51].

#### Day-ahead market

The day-ahead market in Germany takes place in EPEX too. The auction takes place at noon every day for delivery the following day, in 24-hour intervals. Prices are between -500 and 3000  $\in$ /MWh. The average price in 2013 is 37.8  $\in$ /MWh base, 43.1  $\in$ /MWh peak. Volume traded in 2013 is 245 TWh which is 41% of gross electricity consumption, which is 599.8 TWh. As mentioned above, RES share is increasing in the electricity sector in Germany. This can affect the wholesale market prices. Figure 2.5 shows the day-ahead market prices in Germany versus in the Netherlands, which occurred on March 3rd 2020. The data are from [52]. As the figure shows, in Germany, day-ahead prices remain negative for a few hours around noon and early afternoon. This phenomenon results from a market distortion caused by renewable support mechanisms [53]. Prices in the day-ahead markets are between -500 and 3000  $\in$ /MWh. The volume traded is 245 TWh on auction and 260 TWh traded OTC.



Figure 2.5: Day-ahead market price in Germany vs. the Netherlands. Data from March 3rd 2020 based on [52]

#### Intra-day market

In Germany, the same as in the Netherlands, the intra-day market trades occur as more reliable information becomes available such as new RES forecasts, plant outages or changed demand situations.

The German intra-day market consists of two parts, in addition to the possibility for OTC trades. Firstly, there is a daily intra-day auction at 15.00 on the previous day, which functions similarly to the day-ahead market except that quarter-hourly products instead of hourly products are traded. Secondly, there are two continuous intra-day markets: one operated by EPEX Spot with quarter-hourly, 30-minute (since 2017) and hourly products, and one operated by Nord Pool Spot. Nord Pool Spot offers 15-minute, 30-minute, hourly and block products. The fact that quarter-hourly products are traded in the intra-day market, in contrast to the hourly products in the day-ahead market, enables market participants to have a better approximation of the real demand ramps and generation variability (e.g. from solar or wind power generation) [43].

Prices in the intra-day markets are between  $+-3000 \in /MWh$ . The volume traded is 19 TWh on auction and 29 TWh traded OTC [42]. Comparing these numbers with the day-ahead market, one can conclude that the German intra-day market is sufficiently liquid <sup>1</sup>.

<sup>&</sup>lt;sup>1</sup>Based on [54], "market liquidity describes the extent to which an asset can be bought and sold quickly and at stable prices. It is a measure of how many buyers and sellers are present in a market, and whether transactions can take place easily."

#### **Balancing** market

Unlike the day-ahead market which is transaction-based, the balancing mechanism in Germany is an accounting procedure and administered by the German TSOs – TenneT, 50Hertz, Amprion, and TransnetBW [26].

The tariff system for the settlement of imbalances is a single pricing system where prices for deviations are calculated on a 15-minute basis. Prices are determined by summing the TSOs payments for, or revenues from, FRR and RR control. The costs for balancing energy are distributed among the market participants responsible for imbalances [42].

There are notable differences between the German and Dutch balancing system. One key difference with a major impact is that TenneT provides market participants in the Netherlands live updates on reserve activation volumes and prices, while German market participants do not receive such updates. By providing these updates, TenneT financially stimulates Dutch market participants to deviate from their portfolio if this reduces the overall system imbalance, a mechanism which is called "passive balancing". Another key difference is that the Dutch system allows free bids for the FRR, in contrast to Germany. This means that only contracted market participants of balancing capacity can provide FRR energy in Germany, while in the Netherlands, non-contracted market participants can also bid-in for balancing energy bids [26].

#### Participation of RES in electricity market

Generally, due to the Renewable Energy Sources Act (EEG) mechanism, which guarantees fixed feed-in tariffs (FIT), electricity generated in RES power plants is usually fed-in regardless of market prices (which typically reflect the current demand and supply) [55]. But with an increasing share of RES in the German electricity mix, discussions on market integration of RES started. Incumbents argued that RES should be forced to leave the sheltered niche of their fixed feed-in tariffs (FITs) and instead should take responsibility and be offered to the markets just like conventional electricity [47]. However, selling electricity from RES directly to the electricity markets would require completely new business models and actor cooperation, since marketing strategies between RES and conventional power differ substantiallyfor instance RES are not able to hedge price risks on futures markets unless be part of a larger portfolio of an energy producer. In this case, they can use other assets to hedge price risks.

In Germany, same as in the Netherlands, several features have been introduced in the market design which are favourable for RES integration: there is a relatively short gate-closure time as well as an intra-day market where traded volumes have been increasing as is the case in the Netherlands. However, due to the FIT



Figure 2.6: The PDF for the ratio of imbalances to the peak-load in the Netherlands vs. Germany. Data from 2019 based on [56]

support scheme, there is not much long-lasting experience with direct market participation of RES.

To further promote the market integration of RES in Germany, the EEG provides a number of approaches [47], which are listed briefly as follows:

- the possibility for RES producers to temporarily refuse the feed-in tariff scheme to directly participate in the market.
- RES producers can optionally choose to benefit from a technology-specific market premium on top of the revenues which they gain from direct marketing.
- the EEG gives authorisation to the German Government for introducing financial incentives parallel to the feed-in tariffs and to change the preconditions for participation of RES in the balancing market.

Finally, to have a comparison between the total imbalances due to RES integration in the power systems of Germany and the Netherlands, Figure 2.6 is presented. This figure shows the probability density function (PDF) for the ratio of the yearly system imbalance to the peak-load in Germany and the Netherlands, in 2019. The data for the peak-load and system imbalances of both countries are from [56]. Since the horizontal axis shows the ratio of the imbalance to the peakload, 0 on this axis depicts a zero-imbalance system. Therefore, a zero-imbalance system in the Netherlands has a significantly higher probability to occur than the one in Germany. In other words, in Germany, system imbalances have higher chance to occur which can be due to high RES integration and market distortions.

# 2.5 Conclusions

This chapter attempts to give an overview of the electricity market and its main elements. How electricity markets work, what are the main actors, products and how different products are traded in electricity markets are shown in this chapter. Also, the European electricity market is discussed and as an example, the electricity markets in the Netherlands and Germany are presented and compared. For each country, some statistics about generation mix, day-ahead, intra-day, balancing, and reserve markets are presented. Lastly, the current participation of renewable energy resources in the electricity market for the Netherlands and Germany are reviewed. Statistics show that currently, in Germany, RES hardly can participate in the electricity market, as they are benefiting from support schemes and feed-in tariffs. In the Netherlands also the participation of RES in the electricity market is limited. Although strategies toward RES integration differ, however, in both Germany and the Netherlands, the most attention has been given to policy measures to increase the amount of RES in the system rather than generating a new market structure in which RES can flourish on their own. Consequently, continuing an increasing trend for RES in the future cannot be reached by RES market integration efforts alone and it will require an adaptation of electricity market structure to the capabilities of RES.

Chapter 2: Electricity Markets

3

# DERs in electricity markets

This chapter presents an overview of the possibilities for participation of distributed energy resources (DERs) in the electricity markets<sup>1</sup>. DERs are smallscale power generation that provide an alternative to or an enhancement of centralized generators in producing energy. A short introduction about different types of DERs and their shares in electricity production in the Netherlands and Germany is given in section 3.1. Besides producing energy, DERs are also able to provide ancillary services which may be provided through market mechanisms. Ancillary services are services necessary for the operation of a transmission or distribution system. In section 3.2, different ancillary services provided by DERs are explained. Section 3.3 gives examples of DERs trading their energy and ancillary services in electricity markets. Section 3.4 discusses the main barriers which DERs are currently facing in electricity markets. Section 3.5 and section 3.6 are about aggregators and local electricity markets as potential solutions to deal with those barriers. Also, a literature review on local electricity markets is presented in this section. Section 3.7 explains the transmission-distribution systems coordination from technical and market perspectives. DER's revenues including pricing in local markets and performing market power by DERs are discussed in section 3.8. Finally, the conclusions for this chapter is given in section 3.9.

<sup>&</sup>lt;sup>1</sup>This chapter is based on the published works in [57][58][59]

# 3.1 What is a DER?

Generally speaking, DERs can be defined as: "electricity-producing resources or controllable loads that are connected to a local distribution system or connected to a host facility within the local distribution system" based on [60]. Therefore, DERs can include small and medium-sized power sources connected to the distribution network such as solar panels, combined heat and power (CHP) units, electricity storage, small natural gas-fuelled generators, electric vehicles (EV) and controllable loads. In this thesis, DER technologies can be divided into:

- Distributed generation (DG): power generating technologies in the distribution grids which can be classified as dispatchable generators such as cogeneration units, CHP, combined cycle gas turbines (CCGT) and stochastic or variable renewable energy resources (VRES) such as wind turbines and PV systems which are dependent on the weather condition.
- Energy storage systems: DERs are not only energy producing resources and they can also store energy in technologies such as battery energy storage systems (BESS) or flywheels which can store electricity hence have the capability of both supplying and demanding electricity to and from the grid.
- Active/controllable loads or demand response: based on the aformnetiond definition of DERs, a controllable load can also be seen as a DER. A controllable load means increasing or decreasing energy usage by endusers and in response to price signals to reduce their energy costs or even gain some profits. Controllable loads are not applied in this thesis.

DERs can benefit the environment in several ways. First of all, the source of energy in some DERs such as wind farms and PV systems is clean energy without  $CO_2$  emissions. The use of DERs can reduce the energy generation by centralized fossil-fuel power plants, hence reduce the negative environmental footprint of these generators. Moreover, some DER technologies can harness energy that might be wasted otherwise. Lastly, DERs can be used to generate electricity locally close to consumers which can significantly avoid wasting energy through the transport losses in transmission and distribution grids.

Besides these environmental benefits of DERs, high penetration of DERs brings other benefits and opportunities for power systems, among others, increasing the affordability and reliability and decreasing network costs: consumers who have access to DER assets such as storage systems can gain some profits by selling back energy to the grid during peak hours and also deliver so-called ancillary services to the grid. These services are not only affordable for DERs but also help to balance the grid and in this way increase the reliability. Moreover, ancillary



Figure 3.1: The electricity produced by onshore-wind and PV-systems from 2004-2018, the Netherlands. Data based on [1]



Figure 3.2: The electricity produced by onshore-wind and PV-systems from 2004-2018, Germany. Data based on [1]

services provided by DERs can result in delaying network expansion and grid reinforcement and hence reduced network costs.

Currently, the trend of integrating DERs in distribution grids is increasing. Figure 3.1 and 3.2 show a significant increasing trend for both wind turbine and PV systems during 2004-2018 in the Netherlands and Germany, respectively. Figure 3.1 shows that in this period, in the Netherlands, electricity generation from both wind turbines and PV systems has been increased by rate of 400%, which is quite comparable with the situation in Germany. Installed capacity, however, is significantly larger in Germany than in the Netherlands.

As mentioned earlier, in addition to energy production, DERs have the capability of providing the ancillary services to the grid. Moreover, knowing the fact that the number of DERs in the distribution grids is increasing, brings the necessity to study DER's potential for providing ancillary services to the grid which is discussed in detail in the next section.

# 3.2 Energy and ancillary service provision by DERs

As explained in chapter 2, the main actors in power systems who control and manage transmission and distribution grids are TSOs and DSOs. The TSO is responsible for the system-level balancing and security, while the DSO controls the security of its local grid.

The Directive 2009/72/EC [25] defined ancillary service as "a service necessary for the operation of a transmission or distribution system." Legislative Proposal for a Directive of the European Parliament and the Council on the internal market for electricity on common rules for the internal market in electricity of 30 November 2016 (COM(2016) 864 final 2016/0380 (COD) - the element of the documentation known as the Winter Energy Package [61]) added to the above legal definition the following words: "including balancing and non-frequency ancillary services but not congestion management." However, based on the definition by ENTSO-e in [62], services for congestion management are sometimes also mentioned as part of ancillary services. The Dutch TSO, TenneT, uses these ancillary services for various purposes in accordance with its regulated tasks and obligations as a TSO: balancing reserves, reactive power, redispatch, black start facility, compensation of losses and sustainability of grid losses through Guarantees of Origin [63]. In Germany, ancillary services include: operational management, balancing service, instantaneous reserve, reactive power, short-circuit current contribution, system restoration [64].

Traditionally, ancillary services have been provided by large centralized generators and industrial load shedding and the TSO was responsible to deliver those services from the transmission to all connected parties, to guarantee the security of the whole power system. However, the paradigm has shifted due to integrating more DERs in distribution systems. Consequently, the sole dependency of the power system on centralized generators for producing ancillary services has changed. This happens because of several reasons. First, the penetration of VRES with variable outputs has increased in power systems. This leads to an increase in uncertainty and variability in the system as well. This uncertainty and variability cause a higher chance of mismatch between supply and demand, hence increased requirement for balancing power and energy. In a future with many connected VRES to the power systems, centralized generators may not be adequate to fulfil the balancing need of the system due to uncertainties in the supply. Moreover, DERs (normally non-renewable-based DERs since it can be more difficult for the system operator to rely on stochastic renewable-based DERs) can also have a capability for ancillary service procurement and can complement the ancillary services currently provided by conventional large generators. Generally speaking, the ancillary services from DERs can be classified for different purposes [65] [3]:

- System-oriented services: this services are mainly utilized by the TSO. There are three different applications for the system-oriented services: a) Frequency response and system balancing: TSOs can use ancillary services from DERs for the procurement and activation of the reserve for balancing purposes. DERs provide operating reserve value when they can be used to increase supply or reduce demand on the grid instead of central generators. Based on definition in [66], this service consists of three different parts: Frequency containment reserve (FCR), Frequency restoration reserves (FRR) and Replacement reserve (RR). b) Congestion management: TSOs can use DER's ancillary services as corrective actions for congestion management. c) System adequacy: DER's ancillary services can contribute to system adequacy for example, via a reduction of peak-demand or increasing the availability of storage systems.
- Grid-oriented services: This services are utilized locally by the DSO. There are three applications for DERs ancillary services in the local grid: a) Transportation adequacy: the DSO can use DERs ancillary services as an alternative to grid reinforcements, especially in those cases where capacity limits are reached only for a few hours per year (incidental congestion). b) Quality of supply services: DERs help the DSO to improve supply quality to customers connected at the low-voltage distribution level. c) Voltage and power flow management services: the DSO can use DERs ancillary services to manage the distribution grid and to solve local problems, such as feeder or transformer over-loading, over/under voltage, etc.
- Balancing services: This service is mainly utilized by BRPs and/or aggregators. BRPs and/or aggregators can use ancillary services from DERs to correct unbalanced positions in their portfolios, optimize their positions in electricity markets and contracting ancillary services to hedge risks. Also, DERs can be used by prosumers to optimize their generation and consumption profiles thus reducing their energy bills.

There are several concrete examples of DERs providing ancillary services to the system. These examples can be classified based on different DER technologies:

• Distributed generation: Combined Cycle Gas Turbines (CCGT) and wind farms are technologies for the provision of TSO frequency response services whereas CCGTs, Diesel standby generators and perhaps even micro-CHPs are best placed to provide reserve services. Moreover, VRES can also provide various ancillary services through their active power control which is an adjustment of VRES power production in various time frames to deliver balancing services and/or congestion management. PV systems and wind turbines, for example, can deliver a fast response to regulation signals hence provide up and down regulations. Downward regulation is obtained through curtailment and upward regulation is the result of operating units at a power point below their maximum generation possible at a given time and increasing to the maximum value when needed. However, due to the stochastic nature of VRES, usually, there are some regulations and market barriers for VRES to deliver ancillary services e.g. reserve and balancing services.

- Energy storage systems: Li-ion battery systems are an example of energy storage systems, which have been developing quite fast recently. The possible applications of Li-ion BESS in term of delivering ancillary services to the grid, can be subdivided into three main groups: front-of-the-meter (FTM), behind-the-meter (BTM), and microgrids. FTM is by far the category with most potential use-cases, including all grid services, large-scale renewable integration and support on transmission systems as well as distribution systems. Furthermore, large-scale BESS are most likely to play a relevant role within these applications. Next, BTM use-cases are centred on smaller batteries for commercial & industrial (C&I) usage, which can be up-scaled using aggregation. The UK is the first European country that has opened a fast frequency response market as part of its grid services (EFR enhanced frequency response) to mitigate frequency deviations, which are more of an issue with an island system as compared to continental Europe. This proves to be very favourable for BESS since EFR requires asset activation within 1 second. Therefore, more than 200 MW of equivalent battery systems have been awarded in tenders for EFR in 2018. Furthermore, the primary frequency regulation service (FFR - firm frequency response) also requires a faster activation time in UK than the rest of Europe (10 seconds vs. 30 seconds)[59] which makes it a good market for batteries.
- Active loads or demand response: demand response can enable active participation of electricity consumers in the balancing process. Moreover, the ancillary services provided by demand response can include congestion management and various frequency-response reserves [67].

# 3.3 DERs in electricity markets

So far, different DER's characteristics, energy, and potential ancillary services from DERs have been discussed. Another important aspect of studying DERs is their participation in the electricity market. This is the topic of discussion for the rest of this chapter.

As shown in section 3.1, recent climate and environmental policies lead to integration of more clean-energy DERs into power systems. A key factor that incentives provision of energy and ancillary services by DERs is their exposure to market signals. Electricity markets, therefore, can be an effective transaction platforms for small DERs to actively provide their energy and system- and grid- oriented ancillary services. Electricity markets, as discussed in chapter 2, include various time-frame markets such as day-ahead, intra-day, reserve and balancing markets in which DERs can trade their energy and system- and/or grid-oriented ancillary services.

Table 3.1 classifies energy and different system/grid-oriented ancillary services provided by DERs in electricity markets by taking some practical examples of implementing DERs in energy and ancillary service markets. This classifications is based on 1) Product: different products from DERs including energy, frequency responses and system balancing, and congestion management, 2) Trading time frame: different markets for trading the specific product including day-ahead, intra-day, reserve, and balanign markets 3) Notification before realtime: the required time for delivering the product before the real-time, 4) Suited DER type: examples for DERs which are able to deliver the particular product, and finally 5) Examples for countries where those DERs can already deliver that particular energy and ancillary services.

Product	Trading timeframe	Notification before real-	Suited DER type	Examples		
		time				
Energy	Intra-day market/day- ahead market	1-24 hours/24- 48 hours	Aggregated loads and gen- eration (wind turbines,PV systems)	Nordic region: opened to demand response France: The French Block Exchange Notifi- cation of Demand Response (NEBEF) mecha- nism allows trading of demand response as well		
FCR	Reserve mar- ket	30 seconds	Aggregated EVs, commer- cial and residential loads, electrical heating, storage systems	UK: Demand response with dynamically con- trolled refrigerators.		
FRR (aFRR, mFRR)	Reserve mar- ket	15 minutes	Aggregated EVs, residen- tial continues loads, elec- trical heating, storage sys- tems	USA: EVs and stationary batteries for fre quency regulation in PJM.		
Balancing mechanism (RR)	Balancing market	15 minutes	EV's, storage systems, CHP units	Germany: industrial loads participate in bal ancing mechanism, Belgium: Since 2014 aggre gation of distribution-connected DR resource possible in tertiary reserve.		
Passive balanc- ing	Real-time	-	Commercial aggregators	The Netherlands		
Transmission & distribu- tion congestion management	Ancillary ser- vice market	30 minutes- hours	Aggregated EVs, energy storage and combined heat and power (CHP)	France: congestion management is traded in balancing market and the Voltalis load manage ment of residential heating devices. the Nether lands: the GOPACS platform launched by Ten- neT where market-based congestion manage ment solutions are posed through intra-day bi lateral trading.		

Table 3.1:	Energy and	system-oriented	ancillary	services	provided b	v DERs in	electricity mar	kets
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# 3.4 Barriers for DER's market integrations

Although Table 3.1 shows possibilities for DER's market integration, however, participation of DERs in current electricity markets faces many barriers. For example, the current market design has been built based on the needs of central generators, or, the regulatory framework established by the national regulators often lacks appropriate incentives for the use of DERs. There are also more barriers to the participation of DERs in the market which are mainly established by the grid operators or are inherent to the renewable-based DERs. Here, the three of these barriers are discussed below, namely, bid size, location and variability.

- **Bid size:** in the current wholesale markets, before the actual procurement begins, the "pre-qualification" of market players must be checked. The pre-qualification rules are important to ensure the quality of service as the TSO relies on those services in critical situations. The pre-qualification process checks if energy and services supplied by market players meet the quality requirements in a given market, e.g. minimum technical and administrative requirements. However, these requirements can impose high market entry barriers for some small market players like DERs [68]. However, in recent years, the minimum bid size and bid increments have been lowered substantially in day-ahead markets. Currently, the threshold in day-ahead markets is typically at 0.1 MW, which implies good possibilities for DER participation. While such a decrease has taken place also to access to some balancing markets (in Germany for example, the minimum bid size requirement to access to FCR, FRR, and RR markets (i.e. ancillary service markets) are 1 MW, 5 MW, and 5 MW, respectively [69]), minimum bid size and bid increments still remain high in many markets, for example, in the Netherlands, the minimum bid size requirement for the secondary reserve market can be as high as 20 MW [69].
- Location: DERs are connected to the low and medium voltage levels of the grid. Hence, the TSO cannot check if the procurement of energy and ancillary services by DERs is safe for the distribution network. Neglecting distribution network constraints in the current market clearing process can jeopardize the security of the distribution network. Therefore, by default, there can be a limit for DERs to deliver energy and/or ancillary services in the current market to avoid further disturbance in the distribution network. However, this limit can avoid full integration of DERs into market.
- Variability: DERs such as wind turbines and PV systems are renewablebased energy resources and are dependent on the weather conditions hence their energy production can vary over time. The uncertainty in their output may cause an imbalance between demand and supply in power systems.

Moreover, dealing with all those small and energy-variable DERs in one central (national) market can be very complicated. As explained in section 3.2, although technically VRES-based DERs can also provide up and down regulation, however, the stochastic nature of VRES can prevent them to participate in some particular markets such as reserve and balancing markets. Because the system operator can hardly rely on these stochastic resources to provide certain services at critical moments. As a result, variability can limit the participating of DERs in the current electricity market.

In sections 3.5 and 3.6, possible solutions to overcome the aformnetiond barriers are introduced: the role of aggregators and local electricity markets in smoothing those barriers for DERs in the electricity market are discussed.

# 3.5 DER's market integration through aggregators

A common solution to cope with the aforementioned barriers for participation of DERs in the current electricity markets is aggregating DERs through a new type of market player called an aggregator. The aggregation of small and medium-sized resources facilitates access to the market, with the aggregator acting as an intermediary. This section includes definitions of an aggregator, aggregator's role and responsibilities, literature review, and the limitations which aggregators are facing within electricity markets.

### 3.5.1 What is an aggregator?

Based on the definition in [70], the aggregator is "a market participant that combines multiple customer loads or generated electricity for sale, for purchase or auction in any organized energy market." In the last European Directive on Energy Efficiency [71], the aggregator is essentially defined as a demand-response provider, i.e., as a means to gather short-duration customer load that is otherwise unable to participate in any organized energy market. Here, the focus is on the demand-response as a tool to empower customers and promote energy efficiency. In [72], the aggregator is defined in a general way as" a company who acts as an intermediary between electricity end-users and DER owners and the power system participants who wish to serve this end-user or exploit the services provided by these DERs". Aggregators are market players inherently. Their aim is the commercially successful operation of their connected units, be it in the form of energy-schedule optimization or the form of ancillary services. The aggregator can optimize the value of services from DERs by selling it to the entity which has the most immediate need for those services and hence is prepared to pay competitively. Moreover, aggregators can mitigate the risk of non-delivery of services from a single DER by aggregating a large number of them and consequently guaranteeing the provision of energy and ancillary services [73].

# 3.5.2 Roles and responsibilities

As mentioned above, the main responsibility of an aggregator is to collect energy and/or ancillary services from its associated customers, aggregate it into bids, and if it is possible trade them in electricity markets, otherwise through contracts considering its own and the customer's best interest [74][75]. However, in addition to aggregation, there are various other responsibilities that an aggregator can take. Aggregators may operate in different parts of the electricity network and utilize the units in their portfolio for trading in electricity and ancillary service markets. Depending on the market design, an aggregator can also be understood as an entity that coordinates the units in a certain area of the network [73]. The aggregator and DERs should agree on commercial terms and conditions for procuring energy and ancillary services. Accordingly, aggregators in electricity markets can be categorized into three types with different responsibilities:

- **Production aggregator:** aggregates small generators to enable them to access the electricity market. An example is the virtual power plant concept. The aggregator builds a flexible portfolio as part of its business model. Other market players in the market can also utilize these aggregated services.
- **Demand aggregator:** acts as an arbitrator between small resources and other market players in electricity markets. For example, they can have an agreement with residential or commercial electricity consumers to aggregate their capability for shifting the loads and building a Demand Response Service. Therefore, they make a single resource of many small active loads and sell their services in the market. Energy suppliers can also be as this type f aggregators.
- **Commercial aggregator:** Aggregators may also take the roles of energy retailers or BRPs. They act as a balance responsible entity and monitor their power balance to be sure that the power purchased and generated matches the power sold and consumed within its portfolio (see Table 3.1).

Depending on these different roles, aggregators can bring different types of added value for the system; small-scaled DERs can access the marketplace, as aggregators remove the complex burden of market participation of DERs by enabling them to deal with only a single entity [76]. Balance responsible parties (BRP) can use aggregators to optimize their portfolios to mitigate risks of deviating from their day-ahead schedules. Finally, DSOs and TSOs have more options to balance the system, as it is expected that the aggregator would be responsible for the imbalances resulting between its actual and forecasted demand and generation. Thus, a more effective balancing operation will be achieved if the aggregator acts as a BRP. Moreover, the overall efficiency of this arrangement will be higher because of the economies of scale involved [72].

### 3.5.3 Literature review on aggregators

According to literature, several types of research have been done on the performance of a virtual power plant (VPP) or energy management systems as aggregators to facilitate the participation of DERs in the wholesale energy trading and system balancing. In this part, some of that literature are reviewed.

Reference [77] develops the concept of VPP as an aggregator for the participation of DERs in the wholesale market and/or ancillary services to provide energy and/or services for the transmission system and to enhance the visibility and control of DERs to system operators and other market actors by providing an interface between these components. In [78], an optimization algorithm based on direct load control is proposed to manage a VPP composed of a large number of customers who participate in the real-time balancing market. Reference [79] considers a price-taker VPP which sells and purchases electricity in both the wholesale day-ahead and the balancing markets seeking to maximize its expected profit. In [80], another profit-maximizing VPP problem is addressed in which a weekly self-scheduling of the VPP is taken into account. The VPP fulfils its long-term bilateral contracts, while it acts in the wholesale day-ahead market trying to maximize its overall profit. In [81], a probabilistic Price Based Unit Commitment (PBUC) approach is employed to model the uncertainty in market price and generation sources, for optimal bidding of a VPP in a day-ahead electricity market. Reference [82] presents an algorithm to optimize the day-ahead scheduling of a VPP while taking into account the actual location of each DER in the network and their specific capability.

Reference [83] elaborates on an efficient algorithm for the energy management system inside a residential energy hub with the goal of minimizing the operating cost in a competitive electricity market. Reference [84] presents an optimization method for balancing in the Universal Smart Energy Framework (USEF). The algorithm acts on multiple levels within the hierarchical energy market structure of the framework. Distribution network constraints are not taken into account during market clearing process, however, there is an iterative interaction between the BRP, the aggregators, and the DSO to avoid overloading in the grid. Reference [85] designs a hierarchical control framework focusing on both wholesale energy trade functions and ancillary services that enables the provision of flexibility services through aggregation entities.

# 3.5.4 Limitations

Beside literature, there are also some real-life examples for companies who work as aggregators such as Restore in Belgium, Powerhouse in the Netherlands, and Next in Germany.

In the above-mentioned example as well as in the literature, the participation of DERs in the central market is happening through an aggregator. The DSO is mainly a grid operator and the distribution networks are not taken into account during the market clearing process. Aggregators do not take into account distribution network constraints as they have no incentive to do so. Also, aggregators do not usually have information about the geographical location of DERs in the distribution network. The control actions that they impose on the connected units may influence the transformer and line loadings of one or several distribution systems.

Consequently, dispatching of DERs can lead to over/under voltages or congestion problems in the distribution grid. Thus, care has to be taken if relevant distribution-grid constraints exist in the area of operation of an aggregator. Some earlier publications have taken into account the grid constraints into the aggregator's optimization portfolio, however, their applied solutions are somehow complicated and time-consuming [86].

Another disadvantage of an aggregator is that assigning the role of retailing to the aggregator could allow them to exercise market power. Aggregators can deliberately create bids in the day-ahead market that would result in network congestions, which then forces the DSO to activate their aggregated flexibility [87]. However, avoiding the issue of market power is rather difficult even if the aggregator is not a retailer (for example, congestions that can be alleviated only by one aggregator).

Possible measures to mitigate such issue might be long term contracts, flexibility price caps [88], and efficient monitoring of irregular market bids by comparing them to DSO forecasts from expected power flow over the distribution grid. Also, the DSOs use of flexibility depends on its economic value compared to reinforcing the grid. Therefore, aggregators will always be inclined to present flexibility prices in the allowed price range of DSOs.

Lastly, according to [65], the role of the aggregator is not even defined in most European countries, and there is no legislation regarding how this new role will be embedded in the electricity market framework. The aggregation service is still not completely allowed for short-term markets. Reference [65] lists the European countries where there are limitations for participation of aggregators into dayahead market, intra-day market and ancillary service markets (FCR, FRR, RR).

# 3.6 DER's market integration through local electricity markets

In the previous section, the aggregator entity as a solution for the participation of DERs in the electricity market is introduced. However, as discussed, the aggregator alone cannot overcome violating distribution network constraints due to activating DERs in the context of current electricity markets. Recently, a new concept of the local electricity market (LEM) has been emerging in power systems which can be seen as an alternative to overcome the aforementioned limitations of aggregators. This section mainly reviews the concept of LEM in various sources from the literature. First, the LEM is defined and then a literature review on the concept of the LEM is presented.

### 3.6.1 What is a local electricity market?

An alternative for the participation of DERs in the central energy and balancing market is a local electricity market (LEM) where DER's services could be procured and the DSO (or a new entity) is involved as an active player [89]. European regulatory bodies added the Local Energy Community (LEC) definition in Article 16 of the proposal for a directive on common rules for the internal market of electricity [90] and define it as an efficient way to manage energy at the community level. In [91], LEM is defined as an institutional framework that allows the purchase of local energy and ancillary services. When resources are located within the distribution grid, the DSO should oversee the schedule of operation of the relevant resources connected to its network. Since the distribution grid is local, the contracting of ancillary services in this context can be referred to as a "local market".

Markets can be defined in terms of the entire market and component sub-markets [5]. An entire market typically consists of a set of closely related end-product markets and the intermediate-product markets that feed into them; sub-markets for electricity include the wholesale spot market, wholesale forward markets, and markets for ancillary services. According to [5], a market can be defined as an environment that allows potential buyers, sellers, and retailers of a given economic product to engage in trade. Besides the obvious need to agree on the quality, quantity, and price of the goods, three other important matters must be decided when a buyer and a seller arrange a trade: the date of delivery of the goods, the mode of settlement, and the transaction conditions. This definition is also true for the concept of the LEM and consequently, a local market is defined by its spatial specifications and so it can be thought of as a new sub-market for energy and ancillary services [88].



Figure 3.3: The possible areas for a local electricity market

Figure 3.3 shows possible limited areas for local electricity markets. As shown in Figure 3.3, customers and DERs located in area A and area B can belong to a local electricity market with a specific DSO, and a specific voltage level. It is possible that more than one DSO is being active in the geographic area belonging to a LEM. For the deployment of ancillary services, the most relevant aspect is the management of the resources by the DSO that manages the grid, at a certain level, and the retailer that manages the consumer. However, the geographic area where the consumer is located might influence contracting procedures, tariff structures, and taxes that consumers will pay [92]. Sellers in the LEM can include all generators and loads connected to the distribution systems. The LEM can have a same configuration as of the central market meaning generators can be sellers and loads can be buyer or there can be a single buyer market. This configuration has significant effect on the TSO-DSO coordination which will be discussed in more details in section 3.7.

Based on [88], the local market is a central platform-based system and it fits partially in the centralized optimization category with direct control signals. Hence, a LEM can be in opposition with similar approaches like Transactive Energy (TE) systems. Reference [93] defines the TE as "a system of economic and control mechanisms that allows the dynamic balance of supply and demand across the entire electrical infrastructure using value as a key operational parameter." Contracts in the local market specify available periods, cost per device and specific characteristics like control type which is requested by DSO to ensure the appropriate response to attend their demands. However, the TE approach could be less attractive for DSO because there is no central entity responsible for meeting the DSO request, and multiple negotiations are needed.

### 3.6.2 Literature review on local electricity market

In section 3.2, the capability of DERs in providing ancillary services has been discussed. Research works in literature, however, extensively study the ability of DERs in delivering the so-called flexibility in the local market. The definition of flexibility can differ and depending on specifications it can be translated into similar market products as ancillary services. Generally speaking, flexibility can be defined as " the modification of generation injection and/or consumption patterns in reaction to an external signal (price signal or activation) in order to provide a service", based on [94]. Note that not all DERs can be flexible enough in the sense of being able to increase or decrease their generation/consumption as a result of price signals or direct controls, among other solar systems. Flexible resources such as electrical vehicles, storage systems and demand-response are kinds of DERs which are capable of being flexibility providers. This section presents some existing literature where the LEM has been studied either practically or theoretically.

In [95], presents an analysis of four organizational models for flexibility management. The case studies have been categorized as multi-objective optimization, dynamic pricing, local aggregator and local integrated utility. The paper shows that the different approaches impose new roles on traditional actors, especially on the DSO and also it shows that both the local aggregator and dynamic pricing present potentials for retail competition and feasibility of up-scaling in Europe. Another example in which the concept of LEM has been developed is [91] which shows a general description of local flexibility markets as a market-based management mechanism for aggregators and the needed interactions between all local market stakeholders. There are some European pilot projects where the idea of LEM has been extended. The De-Flex Market developed by the German Association of Energy Market Innovators (BNE) is an example which provides an instrument for the DSO to solve local capacity constraint using DER flexibility [96]. Another example is the FLECH market, a Danish project which tries to solve the congestion that happened in the distribution system with a high number of DERs [97].

The following papers studying the LEM form different aspects. In [98], a local energy market design is investigated to realize market-based control for the integration of PV generation and residential energy storage. In [91], the LEM is

applied in which individual users can sell their excess electricity either to other users in their neighbourhood or to suppliers, based on a system of bidding. Reference [99] studies the application of LEM in the peer-to-peer trading for electricity storage systems. It investigates the value of prosumer batteries and the market features by which the economic potential of end-user batteries can be extracted optimal. Reference [100], introduces a decentralized implicit interaction framework for trading flexibility available from proactive end-users (prosumers) in the day-ahead and intra-day market which are operated by a local flexibility market operator. The second part of the paper focuses on establishing a strategy for the DSO to procure the flexibility it needs from the day-ahead and intra-day markets, as well as through real-time dispatching, at the lowest possible cost.

Reference [101] promotes the local markets by showing that flexibility potential can be sold on markets on all voltage levels and also the flexibility from DERs can help the DSO to maintain grid operation constraints and postpone or avoid network reinforcement. References [102] and [103] propose an algorithm to optimize the total operational cost of the DSO in congestion management using demand-side flexibility. In [104], demand flexibility tool is used for optimal congestion management and minimizing the DSO's total costs and also for deferring physical network expansions. Reference [105] proposes a new method, namely swap, to employ the flexibility service from electric vehicles and heat pumps for real-time congestion management.

In [88], a decision-making problem for a new aggregator type called Smart Energy Service Provider (SESP) is proposed to schedule flexible energy resources. This aggregator operates a local electricity market with high penetration of distributed energy resources. In [106], a network-constrained transactive control method is developed to integrate DERs into a power distribution system to optimize the operational cost of DERs and power losses of the distribution network, as well as preventing grid problems including power transformer congestion and voltage violations. In this method, a price coordinator is introduced to facilitate the interaction between the DSO and the aggregators. Also, in [107] this networkconstrained transactive control method is extended to develop a new modelling technique that allows the congestion price to be directly interpreted as locational marginal pricing in the system.

Reference [108] designs a market to solve network problems and aid in balancing actions at a specific location through contracting flexibility from DERs. Market design is studied in terms of temporal, spatial, contractual, and price-clearing dimensions. Reference [87] proposes a flexibility market led by the DSO and aimed at solving distribution grid congestions. Another local market design is proposed in [98] in which a market-based control for the coordination of distributed energy resources via an efficient local electricity market is developed. Reference [109] developed a day-ahead micro-market supervised by the micro-market operator who executes the auction algorithm. Advantages and befits of the LEM including serving as an ancillary services provider, reducing the burden on the systemlevel balancing market required for meeting power demand, etc. are enumerated in [110]. An ancillary service market framework addressing voltage control in multi-microgrid systems is proposed in [111].

In short, based on the above-mentioned literature, there are several advantages to having a LEM for DERs. First from the DER's perspective, aside from facilitating the participation of DERs in electricity markets, the LEM is a mechanism through which the flexibility and other ancillary services (e.g. voltage regulation, reserve capacity, etc.) from DERs can be fully extracted. Moreover, through the LEM, the issues regarding scalability and integration of large numbers of DERs into the distribution system, which is expected to happen shortly, can be remarkably reduced. From the distribution system's perspective, the DSO would be the first beneficiary of energy and ancillary services which will be directly delivered to the distribution system through the LEM. Furthermore, the dependency of the distribution system on the transmission system would be reduced through the LEM and consequently, the resilience of the entire power system would be improved. However, it can have some consequences for the TSO, for example, the TSO might receive less money from the DSO. Finally, from the transmission system's perspective, besides eliminating the complexity of scheduling many DERs in one central market, the market operator of the LEM can also help the Balance Responsible Party (BRP) by keeping the supply and demand balance in its jurisdiction grid.

## 3.6.3 Summarizing the literature

In Table 3.2, literature about the participation of DERs in the electricity market which have been reviewed in sections 3.5.3 and 3.6.2 are classified. This classification is as follows: The first column indicates if DERs participate in the LEM at the distribution level or the central market (CM) at the transmission level. The second column shows the time frame, whether the market is a day-ahead (DA) and/or intra-day market (ID) and/or real-time balancing market (BL) and/or capacity market (CPM)<sup>2</sup>. The third column is whether or not DERs participate through an aggregator in the market. The fourth column belongs to the role of the DSO in the market if it is a grid operator (GO) or a market operator (MO) or a independent distribution system operator (IDSO)<sup>3</sup>. The fifth column indicates if there is coordination between TSO and DSO during the market clearing process at the central and/or local level. The sixth column shows if during the market clearing process, there is a coordination between the LEM and the central market. The seventh column indicates whether or not distribution network

 $<sup>^{2}</sup>$ CPM is a market to ensure sufficient reliable capacity is available by providing payments to encourage investment in new capacity or for existing capacity to remain open [112]

 $<sup>^3</sup>$  The IDSO is responsible for distribution grid operation together with providing market mechanisms in the distribution system [113].

Although Table 3.2 shows that the concept of LEM has been covered in literature, there is lack of research on existing LEMs in literature which are scalable and feature the coordination between TSO-DSO and taking into account the distribution network security during market-clearing process. This thesis, therefore, focuses on elaborating literature's missing points in developing a LEM.

Ref.	Market	Time	Aggre.	DSO	TSO-DSO	Dist.net.	Scalability	Comput.	VRES
		frame		role	Coord.	const.		complex.	
[77]	CM	DA	$\checkmark$	GO	×	×	High	Med.	×
[78]	CM	BL	$\checkmark$	GO	$\checkmark$	×	High	Med.	×
[79]	CM	DA&BL	$\checkmark$	GO	×	×	High	High	$\checkmark$
[80][81][82]	CM	DA	$\checkmark$	GO	×	×	High	High	$\checkmark$
[83]	CM	DA	Х	GO	×	×	Low	Med.	×
[84]	CM	BL	$\checkmark$	GO	×	×	High	High.	$\checkmark$
[102][103]	CM	DA	$\checkmark$	GO	×	×	Med.	Med.	×
[104]	CM	BL	$\checkmark$	GO	×	×	High	High	×
[105]	CM	BL	×	GO	×	×	Low	High	×
[114]	CM	DA&BL	$\checkmark$	GO	×	×	High	High	×
[115]	CM	DA&ID	Х	GO	×	×	Low	Low	×
[107]	TE	DA	$\checkmark$	IDSO	×	$\checkmark$	High	High	×
[106]	TE	DA	$\checkmark$	IDSO	$\checkmark$	$\checkmark$	High	High	×
[100]	LEM	DA&ID	$\checkmark$	GO	×	×	High	Med.	×
[95]	LEM	DA&BL	$\checkmark$	GO&MO	×	×	High	Med.	×
[116]	LEM	CPM	$\checkmark$	GO&MO	×	×	High	High	×
[96]	LEM	ID&BL	$\checkmark$	GO	$\checkmark$	×	High	High	×
[97]	LEM	DA&BL	$\checkmark$	GO	×	×	High	High	×
[91]	LEM	DA&BL	$\checkmark$	GO	×	×	High	Med.	×
[88]	LEM	DA&BL	$\checkmark$	GO	×	×	High	High	×
[108]	LEM	DA&BL	$\checkmark$	GO	$\checkmark$	×	High	High	×
[87]	LEM	DA	$\checkmark$	GO&MO	$\checkmark$	×	High	Low	×
[98]	LEM	DA&BL	$\checkmark$	GO&MO	×	×	High	Med.	×
[109]	LEM	DA	$\checkmark$	GO	×	×	High	Med.	×
[110]	LEM	DA	$\checkmark$	GO	$\checkmark$	×	High	High	×
[111]	LEM	DA	$\checkmark$	GO	×	×	High	Med.	×

Table 3.2: Literature review on participation of DERs in electricity market
# 3.7 TSO-DSO coordination

The increase of DERs and the emergence of the concept of local electricity markets both create a more decentralized electricity system and change the traditional dynamic between local distribution systems and the system-wide transmission system. This creates a need to investigate the impact of these dynamic changes, particularly at the transmission-distribution interface. Hence, collaboration and coordination between DSOs and TSOs are required to ensure the local markets can effectively forecast and have visibility of DERs activity, benefit from the provision of ancillary services, and explore opportunities to incorporate them into electricity markets. This section studies the TSO-DSO coordination from two perspectives; First, technical aspects of the TSO-DSO coordination are discussed. Next, the effect of the TSO-DSO coordination on forming different electricity market models is shown.

## 3.7.1 Technical perspective

Increasing penetration of DERs into the grid will impact both distribution and transmission networks as well as local and central (national) market operations. Although new technologies and smart grid architectures enable two-way communication, it can still be difficult to create a direct connection between a large centralized generator and a customer's appliances and that may need a third party, especially if energy is traded. Information sharing and coordination of DERs by the DSOs and TSOs are required to realize efficiency gains from coordination and integrated operation between TSOs and DSOs, as well as integrated price formation at the central and local levels [117].

In a report by International Energy Agency (IEA) [118], the TSO-DSO coordination is studied under the following uses cases: congestion management, balancing, use of energy and ancillary services from DERs, real-time control and supervision, and network planning. In the case of energy and ancillary service usage, it is pointed out the need for coordination to ensure bids are activated and do not cause problems anywhere in the grid. Finally, in 2015 EU guidelines with ENTSO-E participation [119] set up rules and responsibilities for the TSOs, DSOs and grid users coordination and data exchange in operational planning. The TSO-DSO coordination is also required in System Operation Guidelines (SO-GL), Electricity Balancing Guidelines (EB-GL), and Network Code on Emergency and Restoration (NCER) [120][121][122]. Reference [123] indicates several key aspects for the TSO-DSO coordination:

- Clear specifications of TSO and DSO observability needs.
- TSO oversight of any active power action with an impact on balancing or on the transmission grid constraints.
- Operational planning and before real-time TSO-DSO coordination.
- Definition of DER controllability procedures to identify the mutual impact of TSO and DSO service activations in emergencies, and development of system operation agreements under these emergencies.
- TSO-DSO coordination for efficient and non-discriminatory usage of DER's energy and ancillary service.
- Structural data exchanges (demand forecasts, generation forecasts, dynamic data models, single line diagrams of the planned network, etc.) for planning purposes.

Also, reference [124] investigates current TSO-DSO coordination mechanisms in different countries and suggests future improvements or coordinated procedures for each of the following identified challenges: congestion of transmissiondistribution interfaces, congestion of transmission lines, system balancing, voltage support (TSO-DSO), islanding or anti-islanding, re-synchronization black-start, and coordinated protection.

The traffic light concept developed in [124] can be utilized for regulating the interaction between TSOs, DSOs and market players focusing on ancillary services from DERs to solve distribution network problems. Moreover, the traffic light concept could be used to support the information exchange between TSO and DSO in terms of energy and ancillary service activation. There are three colour phases in the traffic light method. The green colour indicates that the network situation is not critical hence no market restrictions. At the yellow phase, there is a potential for network issues to happen. Therefore DSO has to call DERs to deliver their services to fix the issue. Finally, at the red phase, the system stability and security are in danger and the DSO is allowed to execute emergency actions including overriding contracts or performing direct control on a generator or consumer unit.

Reference [125] introduces a new interaction model between TSO, DSO, BRPs, producers, and retailers based on dynamic access bounds to the network, changing throughout the day and preventing the activation of DERs leading to congestions. BRPs request to the DSO a power range access to the grid. The DSO computes each BRP safe access range, in such a way that if every BRP is in its safe range no grid congestions can occur. Contracts between BRPs and DSO are then specified with a full access range, where the BRP can operate without any constraint or obligation and where the DSO can impose restrictions on the production or

consumption if necessary. Moreover, to avoid congestion due to simultaneous activation of DERs, in a day by day operation, dynamic ranges are used through two old and new baselines: BRPs provide baseline proposals inside their service ranges; based on these baselines, the DSO computes dynamic ranges for each BRP, so that its network is secure; BRPs submit to the DSO and the TSO new baselines which are used as a reference for the provision of ancillary services. If a BRP violates its dynamic range, it is penalized at a regulated tariff higher but of the same order of magnitude as the imbalance price. These dynamic access ranges change multiple times throughout the day.

Finally, it should be mentioned here that the TSO-DSO coordination requires additional developments in the following two areas: (a) conceptual model for data exchange and energy and ancillary service activation, as well as a suitable information and communication technology platform; (b) tools and algorithms for joint service management, and for forecasting and control of active/reactive nodal injections in primary substations and interface transformers [126].

### 3.7.2 Market perspective

From a market perspective, the coordination between the TSO and DSO is also important in forming different market models as are classified in [127]. With the existence of a local market, the central market at the transmission level, and the local markets at the distribution level, can influence each other. Consequently, the coordination between these two markets makes it possible for the TSO and all connected DSOs to balance supply and demand system-wide while resolving voltage and congestion issues locally. Furthermore, this coordination provides opportunities to use DERs for the provision of ancillary services, not only for the distribution grid but also for the benefit of the entire power system. In addition, studying the TSO-DSO coordination from a market perspective can help to better investigate the aforementioned technical aspects of the TSO-DSO coordination, among other controlling congestions and active/reactive power flow over the interface transformer between transmission and distribution grids.

Generally speaking, the coordination between the TSO and the DSO can lead to three different market schemes for the participation of DERs in the market: the global electricity market model, the independent local market model and the local market with a TSO-DSO coordination model. These models together with examples from the literature are discussed below.

#### Global market model

In this model, there is no local market at the distribution level. Instead, there is only one TSO-operated central market for both resources connected at transmis-



Figure 3.4: Centralized market

sion and distribution levels. It means that the TSO is the only buyer of energy and ancillary services services from both large generators and DERs. The TSO contracts DERs directly from the distribution grid, possibly via an aggregator. The role of the DSO is limited to impose pre-qualification, i.e. a process to ensure that activation of resources from DERs by the TSO does not jeopardize the security and stability of the distribution grid [128]. Note that, participation in this market model is not compulsory and there is also bilateral contracts between the sellers and buyers. Nevertheless, the schematic market model shown in the figures don't picture those who have bilateral contracts and only show the market participation. In this scheme, there are two sub-models as explained below.

#### Centralized market

There is only one central market which is a transmission-level market operated by a central market operator (CMO). The market operator of the central market is the only entity that has access to all the resources at the distribution and transmission system to be used for the entire power system balancing [129]. In the market clearing process in this set-up, distribution network constraints are not taken into account and the DSO is not involved in the procurement of energy and ancillary services from DERs. The advantage of this model is the relatively simple process of its market-clearing. Since the distribution grid constraints are not taken into account during market clearing, the pre-qualification has to be imposed by the DSO to secure the distribution grid. This model is shown in Figure 3.4.

#### Common TSO-DSO market

This set-up could be seen as an extension of the centralized market model. The difference is that in this model, in the market clearing process, a common opti-



Figure 3.5: Common TSO-DSO market

mization is considered, which satisfies the needs for both distribution and transmission grids and minimizes the total cost of procuring energy and ancillary services. Therefore, distribution constraints are simultaneously taken into account during the central market clearing. Collaboration and sharing of data between TSO and DSO are necessary for this market model to be implemented in practice. However, due to the combination of all constraints in the whole system in one optimization problem, the market-clearing involves a heavy computation process. In [130] this model is used. This model is shown in Figure 3.5. This market model can be similar to the situation prior to deregulation in which a vertically integrated utility running an optimal power flow.

#### Independent Local Market

In this market set-up, there is a local market which is operated by the DSO. This local market can be mandatory for the DERs which means they cannot participate into the central market, though there can be still bilateral contracts between the sellers and buyers within the local area. The TSO is the operator of the central market.

However, unlike previous market models, the TSO cannot access DERs and utilize their energy and ancillary services for its purpose. As in this model the balancing of the distribution grid is carried completely by the local market, this model is called Shared Balancing Responsibility in [128]. The efficiency of resource allocation is relatively low and BRPs may face a higher cost in this model, as both TSO and DSO have limited access to resources outside their jurisdiction area. However, the operational process of market clearing is relatively less complicated. In [131] this market architecture is applied. In Figure 3.6, this model is shown.



Figure 3.6: Independent local market

#### Local market with TSO-DSO coordination

In this set-up, there is a local market that is operated and cleared by the local market operator (LMO). DERs offer their energy and ancillary services first to this local market. Local market operator is the buyer of their services and its priority is to solve local problems, by activating DERs connected to its jurisdiction area. The remaining non-used energy and ancillary service capacity from DERs can be aggregated and transferred to the transmission network. There are not that many examples in the literature showing a complete transmission-distribution coordination together with the existence of a local market. Few examples of research works in literature which can be similar to this market model are discussed below, although they still don't reflect all the potential features of this market model regarding the security of distribution network constraints during the market clearing process.

Depending on whether or not the local market operator participates in the central market with these non-used aggregated DERs, there can be two types of local markets which are in coordination with the central market: non-strategic local market operator and strategic local market operator schemes.

#### Non-strategic local market operator

In the approach proposed in [132][133], the DSO (here considered as a local market operator) clears the local market first. Then, if the demands cannot be fulfilled by local resources, or supply is not completely used, the DSO imports or exports electricity from higher voltage grid levels. The DSO has the priority over the TSO for the allocation of DERs from the distribution grid. After solving



Figure 3.7: Local market with non-strategic LMO

local grid constraints, DSO aggregates and offers the remaining bids (perhaps the most expensive ones) to the TSO. This corresponds to Figure 3.7. The concern about non-strategic DSOs is that there is no guarantee that the resources are used efficiently throughout the whole system.

#### Strategic local market operator

Reference [134] proposes a methodology to optimize the trading strategies of a profit-maximizing proactive distribution company (PDISCO) by mobilizing the demand response. A separate entity from the DSO could take on the role of PDISCO to manage DERs, and coordinate with the DSO to respect network constraints and provide congestion management services. The PDISCO submits continuous offers and bids strategically to the transmission-level central market. Modelling of PDISCO together with profit-maximizing distributed generators is presented in [135]. The PDISCO in this framework is an aggregator that acts in the wholesale market by finding the best-aggregated offer based on the offers received from the distributed generators. The upper-level problem is distributed generators profit maximization and the lower level problem the PDISCOs offers to day-ahead and balancing markets [135]. This model corresponds to Figure 3.8.

Table 3.3 summarizes the most important features of the aforementioned market models. The first column indicates the role of the DSO in the model; local market operator (LMO) and/or grid operator (GO) and/or Aggregator. The second column shows if the market model has the potential to take into account distribution network constraints during the market clearing process. The third



Figure 3.8: Local market with strategic LMO

column shows the role of the TSO, which is always central market operator. The fourth column shows if there is any TSO-DSO coordination and exchange of information during the market clearing process. The fifth column is the expected total cost of the system. The sixth column indicates the resource allocation efficiency. Finally, the seventh column belongs to the complexity in the market model i.e. computational and organizational/administrative complexity in terms of required metering and data collection infrastructure.

Note that Table 3.3 shows the potentials of having or not having a specific feature for different market models. It does not necessarily mean that in the aforementioned literature belonging to these market models, these features were included. For example, the table shows that for the non-strategic and strategic DSO, there is the potential of taking into account the distribution network constraints during the market clearing process. However, the corresponding literature in [132]-[135], did not include the security of the distribution network in their optimization algorithm to avoid more complexity in their models. From this table, one can conclude that the local market models in which the LMO is considered either strategic or non-strategic have the highest capability of being in coordinate with the TSO. This coordination, therefore, leads to a higher resource allocation efficiency in these market models. Moreover, in these market models, the security of the distribution network constraints are considered during the market clearing process. Taking into account the distribution network constraint can improve the security of the distribution system, however, it can increase the complexity of the model.

All in all, according to conclusions from Table 3.3 and what is shown earlier in Table 3.2 about the research gap in existing literature, it can be concluded that scalability, security of distribution network constraints, renewable variability, allocation efficiency, and coordination between transmission and distribution level

	DSO role	Network con- straints	TSO role	TSO- DSO coordi- nation	Exp. cost	Alloca- tion effi- ciency	Complexity
Non-	LMO	$\checkmark$	СМО	$\checkmark$	Low	High	High
$\mathbf{strategic}$	GO						
LMO							
Strategic	LMO	$\checkmark$	CMO	$\checkmark$	Low	High	Med.
$\mathbf{LMO}$	GO						
	Agg.						
Indep.	LMO	$\checkmark$	CMO	×	Low	Low	Low
local	GO						
market							
Central.	GO	×	CMO	×	Low	Low	Low
$\mathbf{market}$							
Common	GŌ	$\checkmark$	CMO	Х	High	High	High
market							

Table 3.3: TSO-DSO coordination schemes (LMO = local market operator, GO = grid operator, CMO = central market operator)

markets are important features for designing a local electricity market. Therefore, among other, the local market model with TSO-DSO coordination can better reflect these important features.

# 3.8 DER's revenue in electricity markets

In addition to an efficient market model for the participation of DERs, defining a proper revenue maximization strategy is also important for encouraging more DERs to deliver their energy and ancillary services in the appropriate markets. This section splits into two parts: In the first part, the pricing mechanism in the LEM is described together with the corresponding literature review. The second part is about market power in the LEM.

# 3.8.1 Pricing mechanism

For a long time, green support schemes have imposed a fixed feed-in tariff for DERs and it has been extensively applied in literature. For example, reference [116] proposes congestion management mechanisms for price-responsive electric

vehicle demand in electricity distribution networks. This paper emphasizes the importance of a correct pricing mechanism for EVs connected to the distribution network and concludes that fixed tariffs do not solve congestion efficiently and may not be effective as they influence the economic signal of the wholesale electricity price, leading to unnecessarily high electricity costs for EV charging. Reference [136] tries to answer how the utility of the future should charge for the use of the electricity distribution network. It concludes that there is no one single retail tariff that will suit all customers. Instead, the ideal tariff for each customer will differ in the degree of exposure to price signals they are prepared to tolerate, together with the degree of remote control of their local devices and appliances. Hence, the goal for policy-makers is not to determine the best tariff, but to create a regulatory framework in which customers are offered a range of retail contracts from which customers can choose the retail contract that best meets their needs, and which promotes overall economic efficiency.

For promoting the utilization of DERs, delivering more ancillary service by DERs, decreasing the real power losses, grid access, preventing congestions and overloading a correct pricing mechanism for energy and services at the distribution network level is important, therefore, market pricing in local markets have recently raised more attention. For example, the EcoGrid pilot project in [137] proposes a real-time market-platform which provides a mechanism to encourage the participation of small-scale DERs. This pilot project gives attention to the importance of a correct pricing mechanism for market clearing at the distribution level similar to the locational marginal price (LMP). However, distribution network constraint are not considered in the market clearing algorithm.

Moreover, in the central transmission-level markets, a lossless power flow can be applicable to determine the LMP. In contrast, in the distribution grid, power losses in cables and shunts cannot be neglected, and voltage and reactive power have to be taken into account in the power flow calculations. Therefore there are differences between transmission and distribution levels in terms of products and markets [130]. In the distribution level, the real power, reserve and reactive power need to be traded by DERs while respecting the distribution network constraints. Therefore, it is important to compute a distribution locational marginal price (DLMP) in the distribution level similar to the one in the wholesale market, while the security limits in the distribution networks, i.e. voltage and line limits, are being taken into account.

In [138], an integrated DLMP method designed to alleviate congestion induced by electric vehicles is proposed. The DSO determines DLMPs by solving a non-linear social welfare optimization in the local DSO market. Reference [139] presents a DLMP method through quadratic programming (QP) designed to alleviate the congestion that might occur in a distribution network with high penetration of flexible demands. DSO calculates dynamic tariffs and publishes them to the aggregators, who make the optimal energy plans for their flexible demands. The

DLMP method using QP solves the multiple solution issue of the aggregator optimization, which may cause decentralized congestion management by DLMP to fail. It is proven in this paper, using convex optimization theory, that the aggregators optimization problem through QP is strictly convex and has a unique solution. The distribution network has a radial topology hence analysing the DLMP is simpler compared with the transmission network. However, the non-linearity arising from accounting for losses through power flow calculations in distribution grids, makes the computation of the DLMPs more complicated. Therefore, reference [140] tries to overcome this problem by duality analysis of a second-order conic program (SOCP) relaxation of the optimal power flow. The paper explains mathematically how congestion, voltage constraints, and real power losses affect the formation of DLMPs. In this thesis, SOCP and DLMP is applied to model the distribution network in market clearing algorithm and calculting the local market price, respectively. This will be explained in more details in chapter 4.

## 3.8.2 Bidding strategies

As discussed in chapter 2, a dominant assumption in electricity market research is that there is perfect competition. However, in practice, the market is imperfect which can lead to exercising market power by market players. In this section, we review some examples from the literature that study the revenue maximization of DERs in central markets. To show how this content is covered in literature, the publications containing DERs profit maximization (with or without market power) are reviewed.

#### • DERs as price-takers

To encourage integrating more DERs in power systems, several research works have tackled revenue maximization of (aggregated) DERs e.g microgrids or virtual power plants in the electricity market. In [141], an approach combining robust optimization and stochastic programming is proposed to define the day-ahead charging schedule for an electric vehicle fleet. Reference [142] develops a risk-constrained scenario-based stochastic programming framework to determine the optimal hourly bids that the micro-grid aggregator submits to the day-ahead market to maximize its profit. In [143], an energy management system for a micro-grid is optimized to maximize its profit in the day-ahead market while abiding by system constraints and regulatory rules. Also, reference [144] proposes a revenue maximization problem for an electrical vehicle charging scheduling mechanism in a micro-grid that respects both local and global peak constraints.

Reference [145] studies a techno-economic impact of the massive integration of small generators and demands into virtual power plants both on the system functioning and on the outcome of demands and generators within the virtual power plants. Also, in [146], the optimal operation of a virtual power plant considering the risk factors affecting its daily operation profits in day-ahead and balancing markets is studied. Reference [79] defines a twostage stochastic mixed-integer linear programming model that maximizes the virtual power plant expected profit in both day-ahead and balancing markets. In [147], a probabilistic model using a modified scenario-based decision-making method for optimal day-ahead scheduling of electrical and thermal energy resources in a virtual power plant is developed. Reference [148] applies a stochastic chance-constrained planning method to build a multi-objective optimization model for virtual power plant scheduling in day-ahead markets. Reference [149] formulates a virtual power plant as a service-centric aggregator that enables the market integration of DERs in day-ahead markets and simultaneously supports cooperation with the distribution system operator in addressing the issue of network usage. Reference [80] considers the virtual power plant which sells and purchases electricity in both the day-ahead and the balancing markets seeking to maximize its expected profit. Reference [82] presents an algorithm to optimize the day-ahead thermal and electrical scheduling of a large-scale virtual power plant while taking into account the actual location of each DER in the public network and their specific capability.

In the aformnetiond literature, the bidding price of the price-taking market players are equal to their marginal cost. Marginal cost of generator measures the cost incurs when producing one more MW power and includes the the variable costs due to fuel and the other variable operating and maintenance costs [150]. Price-taking market players can offer a price higher than marginal cost if they have opportunity cost. An opportunity cost is the revenue a generator would get from offering its power to an alternative use, e.g. selling it in a different location where he can earn higher revenue. If the average cost of generating power is constant, a generator's marginal cost can also be constant if it is equal to average cost. In this thesis, a price-taking market player is assumed to have an infinite ramp rate and its marginal cost is also considered to be constant.

#### • DERs as price-makers

There is still a limited amount of literature in which the strategic behaviour of DERs in the local electricity market is studied. Strategic bidding of market players in the centralized electricity market, however, has been studied thoroughly in the literature, some of which is reviewed here. The existing literature is classified in terms of different strategic approaches. In [151], the bidding behaviour of the storage system which acts strategically through economic withholding is studied. Reference [152] is another example where the storage system is exercising market power, however, by withholding its capacity. However, the effect of transmission constraints on strategic bidding of market players is mostly discarded, such as in [153]. Reference [154] also proposes a profit maximization problem for a wind turbine operating in a traditional wholesale market without considering system constraints. However, there is some literature such as [155] and [156] where transmission system constraints are considered through DC and AC power flow, respectively. Reference [157] is another example that proposes a problem with mathematical programming with equilibrium constraints-based procedure for calculating oligopolistic price equilibria for an electricity market while taking into account transmission constraints. Reference [158] studies the strategic behaviour of a wind turbine through a bi-level model for the jointly cleared wholesale energy and reserve markets where the transmission system is modelled through a DC power flow. This bi-level approach is also used in [159] which addresses the optimal bidding strategy problem of a commercial virtual power plant seeking to maximize its profit in the day-ahead market.

Performing market power by DERs also can happen in the LEM, as the local markets may easily become highly concentrated, creating challenges related to liquidity and market power. DERs can have a significant impact on the value of energy and services, leading to situations where the market price goes extremely high locally. However, there is not enough literature showing the impact of performing market power by DERs on the performance of the local market. This topic will be elaborated in chapter 7.

# **3.9 Conclusions**

This chapter gives an overview of the distributed energy resources (DERs) in electricity markets. Technical characteristics of DERs and their potential for providing energy and ancillary services are presented. The main barriers to the participation of DERs in the current electricity markets are discussed. Aggregators as a solution for the participation of DERs in electricity markets are introduced and their advantages and disadvantages are explained. To cope with limitations in the aggregator setup, local markets are presented, as a promising alternative for the participation of DERs in electricity markets. Accordingly, the local electricity market and its features is introduced with a corresponding literature review. In the context of participation of DERs in local markets, it is important to study the coordination between the DSO and the TSO. The coordination has been studied from two perspectives; the technical and the market perspectives. From the market perspective, the effect of TSO-DSO coordination in forming different market structures is shown. Lastly, the revenue of DERs in the LEM is discussed and it is shown that a correct pricing mechanism in local markets is important to unlock the utilization of more energy and ancillary services from DERs.

The consensus is that the current electricity market design has limitations when it comes to the participation of DERs in the electricity market. The existing literature has tried to address those limitations. However, some limitations remain, among others, the absence of respecting distribution network constraints and neglecting the technical aspects of the TSO-DSO coordination during the market-clearing process even with the introduction of local markets. Therefore, an alternative option is to implement market-based initiatives to deal with these research gaps and finally, to unlock the full potential of DERs in electricity markets. This will be covered in the next 4 chapters.

# The coupled market

In chapter 3, challenges that DERs are facing with in their participation in current electricity markets are introduced. To overcome these challenges including minimum bid size requirements and neglecting distribution system constraints in the current market, a coupled market model is proposed. In this coupled market<sup>1</sup>, there is a local market in which distributed energy resources can participate while distribution system constraints are fully taken into account. Section 4.1 introduces the coupled market model and its main elements and features including the market organization, pricing mechanism, transmission-distribution network interaction, and market timing. In section 4.2, the mathematical formulations related to the coupled market model are explained. First, the market-clearing algorithm for the coupled market is presented in detail, and next the mathematical formulation regarding modelling of the distribution network is explained, including the applied second-order conic power flow and calculation of the distribution locational marginal pricing. Finally, the conclusions for this chapter are given in section 4.3.

<sup>&</sup>lt;sup>1</sup>This chapter is based on: Farrokhseresht, M., Paterakis, N., Gibescu, M., and Slootweg, J.G. (2020). Enabling market participation of distributed energy resources through a coupled market design, *IET Renewable Power Generation*, 14, 4.

# 4.1 Coupled market model

The proposed coupled market is a new market structure that allows full participation of DERs in the day-ahead and balancing markets. In the coupled market, there is a local electricity market in which the local resources can participate. This local market is operated by the distribution market operator (DMO) who can also participate in the central market which is operated by the transmission market operator. Therefore, the distribution market operator can sell the excess energy in the local market to the transmission market operator or buy the deficit energy from the transmission market operator market to fulfil the demand in the local market in the most economic way. The distribution market operator enforces the distribution network constraints in the market clearing algorithm, and in this way helps to keep the distribution system within the safe and secure limits.

The local day-ahead market is a joint energy and reserve market. Beside that, there is also a local balancing market. Note that the balancing is a system-wide service and is part of the responsibility of the transmission system operators (TSOs). The reason for having a local balancing market is explained as follow. First, if full participation of DERs in the balancing market would be allowed, the scalability issue for the TSO with regards to the many balancing resources available still exists. Therefore, having some sort of aggregation for DERs is important in the system-wide balancing market. Moreover, in the coupled market, the local balancing market acts as an aggregator. The DMO participates in the central balancing market, therefore, balancing is not fully performed locally and still is system-wide. Lastly, the proposed coupled market in this paper belongs to a future with a lot of (renewable-based) DERs in the distribution system, and there can be that balancing in distribution systems becomes part of the responsibility of the local market. It means that in the future, the local market can go toward being more independent of the upstream system.

Note that, the DERs, connected to the downstream interface transformer cannot participate through another market than the local market run by the DMO. This set-up is chosen to allow for checking of the network constraints within the distribution network as well as the small area of the local grid doesn't allow to have competing local markets. As the DMO has considerable power in its role as sole market operator within the local grid/market area the DMO needs to be tightly regulated. A more detailed discussion of the DMO is given in chapter 6.

The main elements and features of the coupled market model are introduced in the following subsections. First, in subsection 4.1.1, the market organization, roles, and responsibilities of the main market players are explained. How to price the energy, reserve and balancing services in the coupled market is explained in subsection 4.1.2. In subsection 4.1.3, the interaction between the DMO and TMO in the distribution and transmission network levels, respectively, is described. Finally, in subsection 4.1.4, market timing is elaborated on.

# 4.1.1 Market organization

This section gives an overview of how the coupled market model works. Figure 4.1 shows this market scheme. In this market model, there is one central market operated by the transmission market operator (TMO) and there can be multiple local markets operated by the distribution market operators (DMO). The main stakeholders in this scheme are DERs, prosumers, consumers, energy suppliers, bulk generators and large consumers connected to the transmission network, the distribution system operator (DSO), the transmission system operator (TSO), and the market operators: the DMO and the TMO. As the figure shows, there are three types of interactions between different stakeholders; money, electrical power/energy, and information which can differ depending on to whom the interaction belongs. Below, these interactions between main market players in the coupled market are explained.

#### • DERs, Prosumers, and Consumers

The main market players in the local market are DERs such as wind turbine and PV systems and prosumers such as responsive demand, households with rooftop PV systems. A prosumer is considered as a consumer who owns a DER such as wind turbine or a PV system or an EV. As shown in Figure 4.1, there are two types of interactions between the DMO-operated local market and DERs, exchanging information and money. Information exchange between the DMO and DERs/prosumers mainly consists of their bid prices and quantities for energy and services. If it is a day-ahead market, DERs and prosumers submit their bids before the central day-ahead market starts. If it is a balancing market, DERs and prosumers are called by the DMO, they can enter the local market, and the DMO will ensure that DERs are remunerated for their produced energy and services. The DMO as the operator of markets facilitates trading and exchanging money between the sellers and buyers.

Those (passive) consumers such as small-scale households who are not participating in the local market have a long-term contract with an energy supplier. The interaction between DERs, prosumers, and consumers with the DSO is mainly due to the power flow to make sure that security constraints of the distribution system are met after activating the local energy resources.

DERs and prosumers expect transparent, efficient, and undiscriminating market access. Transparency in the market means clarity as to which prod-



Figure 4.1: Market organization

ucts and services are available, in what quantity and price, and where. Transparency is important in a market since it is one of the essential conditions required for a free market to be efficient. Discrimination in the market means charging market participants different prices for the same product or service.

### • Energy suppliers

Energy suppliers are companies that buy the energy on behalf of consumers on the energy market and charge their clients for the amount of energy they consume. It is also common that the energy suppliers own some generation assets themselves. Energy deregulation gives the consumers the ability to choose an energy supplier themselves.

As shown in Figure 4.1, the energy supplier has two contracts: one with consumers (e.g. small households) connected to the distribution system and one with the generators and consumers connected to the transmission system. In the transmission system, the energy supplier can have a bilat-

eral contract with the bulk generators to sell their energy to the large-scale consumers connected directly to the transmission system. Also, the energy supplier can actively bid into the central market and buy the required energy for the consumers who do not want to participate in the central markets.

The role of the energy supplier at the distribution system is, however, slightly different than the one in the transmission system. As mentioned earlier, not all the consumers connected to the distribution system willing to participate in the local market in order to avoid price volatility and/or other market risks. Instead, they prefer to have a contract with an energy supplier and pay the energy supplier for providing their energy. The energy supplier gives this information about MWh energy required by his clients to the DMO. The DMO takes into account this required energy in the local market clearing algorithm. This means the DMO has to fulfil the required energy for the energy supplier's clients and consequently, the energy supplier has to pay money to the DMO. In other words, the energy supplier acts as an intermediary between the local market and the consumers who do not want to participate in there. Therefore, those consumers who have a contract with the energy supplier don't experience price fluctuations in the market and can benefit from a fixed-price contract for their energy consumption. This interaction is explained in more detail in chapter 6.

Those consumers at the local distribution system who has contracts with energy suppliers do not have to face price volatility in markets, instead, they can have a fixed-price long-term contract with an energy supplier. In other words, the benefit of the energy suppliers in the market is that they can bring price stability for their clients. The money that the energy supplier pays to the DMO is the money which consumers pay the energy supplier minus some fees which are related to its administrative tasks and the price stability it provides for households.

#### • Distribution system operator

As explained in chapter 3, the distribution system operator is mainly responsible for managing the distribution system. As mentioned earlier, the interaction between the DSO and the local resources connected to the distribution network level is mainly related to the power injected or withdrawn to or from the distribution network. The DSO and the TSO exchange power flow either from the transmission to the distribution network or vice versa. As Figure 4.1 shows, the interaction between the DSO with the DMO is regarding the market results which show the (future) activation of local resources. DMO-DSO interaction will be studied in more detail in chapter 6.

#### • Distribution market operator

As mentioned earlier, the DMO is the local market operator to whom DERs offer their energy and/or balancing services. The DMO can be considered as

the distribution network equivalent of the market operator, which is responsible for managing the electricity market and scheduling power transfers to achieve the secure operation of the local distribution network. The DMO should be responsible for balancing the supply and demand locally, be able to receive bids and offers from DERs, and participate in the central market.

The DMO can be part of the DSO or an independent entity. In the latter case, DMO and DSO should exchange information regarding the distribution network security status and activation of the local resources. The DMO has two major roles: first, it is a market operator when it comes to collecting individual bids received from local prosumers and DERs in its jurisdiction and clear the local market. Second, it is a market player when it comes to participating in the TMO-operated central market to buy or sell the deficit or excess energy from the distribution system. The concepts and background literature related to the DMO together with a detailed exploitations for the DMO's roles and responsibilities and its interactions in the coupled market will be discussed in more detail in chapter 6 section 6.1-6.2.3.

#### • Transmission system operator

In chapter 3, the TSO is introduced as the entity that manages the transmission system and is responsible to protect the security of the transmission network. As Figure 4.1 shows, the interaction between the TSO and the TMO market is regarding market results showing the information about the activated generators connected to the transmission system or the amount of consumption at nodes in the transmission system. The TSO should also be aware of how much power flows over the interface transformers between the transmission and distribution systems.

### • Transmission market operator

Similar to the DMO, the TMO is the equivalent market operator that manages the central market including the wholesale energy and balancing markets where the DMO, energy suppliers, and bulk generators and consumers participate. The TMO can be considered similar to the Power Exchange in the European electricity markets or the ISO in the United States. The TMO collects bids and offers from market participants at the transmission system and accordingly, clears the market. The TMO and the TSO should exchange information regarding the security of the transmission network during the market clearing processes. The interaction between the TMO and the DMO-operated local market is explained in detail later in section 4.1.3.

Note that in Europe, for example, the TSO is responsible for balancing and is a different entity than the power exchange who operates the dayahead market. In the coupled market, one TMO could be assigned for the day-ahead market operation and one other entity for the balancing market, separately. However, for the sake of simplicity, one TMO is introduced which operates both the balancing and day-ahead markets. Having one TMO in the market scheme does not affect results in comparison with the situation of two separate TMO organizations.

There are several benefits for market players in the coupled market. From the DER's perspective, aside from generally facilitating the participation of DERs in electricity markets, the local market is an opportunity where the flexibility and ancillary services (e.g. voltage regulation, reserve capacity, etc.) from DERs can be fully extracted. Moreover, through the local electricity market, problems regarding the scalability and integration of numerous amounts of DERs into the distribution system, can be remarkably reduced. Consequently, this can reduce the complexity of handling many stochastic market players in one central market. From the distribution system's perspective, the DSO would be the first beneficiary of flexibility and ancillary services which are directly delivered to the distribution system through the local electricity market. Furthermore, the dependency of the distribution system on the transmission system would be reduced through the local electricity market and consequently, the resilience of the entire power system would be improved. Finally, from the transmission system's perspective, besides eliminating the complexity of scheduling many DERs in one central market, the local electricity market aggregation can also act as a Balance Responsible Party (BRP), helping the transmission system in balancing the supply and demand at the system level.

In short, reducing the complexity of direct scheduling of DERs in the wholesale market, solving scalability problems, improving grid resilience by reducing the dependency on the TSO are among the beneficial functions that the DMO-operated local electricity market can provide to the power system [160][161].

## 4.1.2 Pricing mechanism

As discussed in chapter 3, locational marginal pricing (LMP)-based approach can indicate the price of energy buys and sales in the wholesale electricity market. In the coupled market, the LMP is applied to calculate the wholesale market price. LMP takes into account the effect of actual operating conditions on the transmission system in determining the price of electricity at different locations in the grid. In general, prices in LMP-based wholesale markets vary by location and time and reflect the incremental cost of meeting demand at a given location and point in time. LMPs are made up of three components: energy price, congestion cost, and losses. Note that, the grid tariffs include the cost of losses plus costs for grid investment, maintenance and etc. The latter are assumed to be fixed cost, therefore they havent been included in the objective function. Hence, the LMP is only reflecting the price for energy, losses and congestion. Day-ahead LMPs represent prices in day-ahead markets that let market participants buy and sell wholesale electricity a day before the operating day. Balancing LMPs represent prices in real-time markets that let participants buy and sell power during the day of operation [162].

Distribution locational marginal pricing (DLMP) and its applications in distribution system are also discussed in chapter 3. In the coupled market, DLMPs are used for both day-ahead and balancing prices in the local market. Moreover, the imbalance pricing mechanism is based on dual pricing which is explained in chapter 2 and will be more elaborated in chapter 7.

Moreover, in chapter 3, it is shown that in the literature detailed and complete modelling of the distribution network security constraints during the market clearing process of local electricity market schemes is absent. Detailed modelling of distribution network constraints refers to a full AC power flow representing the distribution grid, as the DC optimal power flow cannot fully reflect the nature of the distribution network, for reasons stated in chapter 3. The DC optimal power flow does not consider the voltage and reactive power flows, which are critical features in ensuring the transport of real power in distribution system operations especially when subjected to voltage problems [163]. Therefore it remains an open question how to develop a local electricity market to facilitate the participation of DERs into the electricity market while the distribution network can still work within its secure and stable operational limits. To deal with the computational complexity challenge, a convexified AC optimal power flow (second-order conic optimal power flow) based on second-order cone relaxation [164] [165] [166] is applied in this thesis for the market clearing at the local market level.

# 4.1.3 TMO-DMO interaction

In addition to the introduction of a local electricity market at the distribution level, it was also discussed in chapter 3 that there should be coordination between the central TMO market and the local DMO market. Otherwise, the efficiency of resource allocation is lower and BRPs may face a higher balancing cost, as both the TSO and DSO have limited access to resources outside their jurisdictions and there is no guarantee that the resources are used efficiently throughout the whole system. Additionally, if the local market is relatively small, it might not have enough resources to ensure adequate security of supply. Thus, coordination between the central market operator and the local market operator is essential to unlock the full potential of DERs for the provision of flexibility and ancillary services to the benefit of the entire power system. Finally, it is discussed that a coordinated approach can better balance supply and demand system-wide while resolving voltage and congestion issues locally. This will be also shown in the results sections in chapter 5. One of the main challenges of the local electricity market is TMO-DMO interaction. In general, there are two types of TMO-DMO interactions regarding the power flow over the interface transformer between transmission and distribution grids: a) unidirectional and b) bidirectional power flow. Distribution systems for many years have been designed based on the assumption of unidirectional power flow [167]. In this case, the power can only flow from HV to the MV level. In contrast, in the bidirectional power flow, the power can be imported or exported from or to the local market, at any given time. Therefore, in case that DER's capacity is not enough to supply the load of the distribution level, energy from the transmission level will be imported, and when there is an excess of local resources and too little demand, excess energy from the distribution level will be exported.

Not only the power flow but also the information flow between the DMO and the TMO can be classified as a unidirectional or a bidirectional information flow. In the unidirectional information flow, there is a static limit for the power flow over the interface transformer. This means, during the market clearing process at a local or the wholesale level, there is no exchange of information between the TMO and the DMO regarding the interface power flow. This way of exchanging information can be seen in an independent local market model which has been discussed in chapter 3. In this independent local market model, the local market operator operates the local market based on a predefined profile for the power flow over the transmission-distribution interface transformer. On the contrary, in the bidirectional information flow, there is a dynamic limit for the power flow over the interface transformer which means there is an exchange of information between the TMO and the DMO during the market-clearing processes. The focus of this coupled market is on the bidirectional flow for both power and information. This way of interacting between the TMO and the DMO is the reason for calling this model a "coupled TMO-DMO market".

Figure 4.2 describes the concept of bidirectional information flow between the TMO and the DMO. First, the DMO through preliminary scheduling solves an optimization problem where the total system cost within its jurisdiction is minimized. In this optimization, the distribution network constraints are taken into account and the DMO considers the bids from DERs and the TMO market. The bids of the TMO market, however, must be predicted by the DMO since this step happens before the TMO-operated central market clearing. The results of this preliminary scheduling are the DMO bidding in term of price (depending on the market, either day-ahead energy and reserve or balancing market prices) and an initial limit for the power flow over the interface transformer between the transmission and distribution systems which is basically equivalent with the DMO bidding quantities. The local market price is calculated based on the DLMP approach explained in the previous part. The information which pass from the DMO to the TMO market included the DMO bidding price and quantity in the TMO market in which the DMO will participate as a market player. In the



Figure 4.2: TMO-DMO bidirectional interaction

next step, the DMO together with other generators and consumers at the transmission system participates in the central market. The TMO-operated central market sends the accepted bids in terms of cleared market price and quantities to market players including the DMO. The final value for the power flow over the interface transformer (meaning the accepted volume for the DMO in the central market) will be sent back to the DMO. Finally, in the third step, the DMO by knowing how much power is sold or purchased to or from the TMO market clears the local (day-ahead or balancing) market. In this step, in the local market clearing algorithm, the distribution network constraints are also taken into account. The reason to take into account the distribution network constraint again is due to the change in the amount of power flowing over the interface transformer.

## 4.1.4 Market timing

As mentioned earlier, the coupled market model allows full participation of DERs in the day-ahead and balancing markets. In Figure 4.3, the time sequence of the



Figure 4.3: Timing in the coupled market model

coupled TMO-DMO market model is shown. The three steps of the bidirectional power flow shown in Figure 4.2 happen in both the day-ahead and balancing markets, as can be seen in Figure 4.3. In total there are six steps in Figure 4.3. Step I is the preliminary scheduling for the local day-ahead market. The dayahead market is considered as a joint energy and reserve capacity market. The reasons for introducing a reserve capacity market, joint with the day-ahead, are as follows. Firstly, in the coupled market, the TSO relies on the DMO market for extra balancing resources. As the TSO does not have control over DERs and the distribution grid, there is a chance that in the balancing market, there will be a lack of resources. To avoid this situation, a reserve market should be created to guarantee that there will be enough energy available for the balancing phase. The provision of reserve capacity is remunerated with a reservation price  $(\in/MW)$  for reserving the capacity. Secondly, the European regulators are paying more attention to the reserve market and the simultaneous alignment of energy provision and reserve capacity as a more efficient market design [168]. Currently, a simultaneous market clearing for energy and reserve market does not happen in European electricity markets, but it is quite widespread in US electricity markets as explained in [169].

Step I happens in D-1, the day before the delivery time, and at a time prior to the clearing time of the day-ahead wholesale market. Through this step, the DMO solves an optimization problem for determining its bidding in the central market. Step II is the TMO-operated central day-ahead market, which clears in the day before delivery time (day D-1) with the time resolution of one hour and over a 24-hour time-horizon. Same as the local market, the TMO-operated central day-ahead market is a joint energy and reserve capacity market. The results for this step are the central day-ahead market price and the scheduled power for market participants including bulk generators and consumers at the transmission system and the DMO. After clearing the day-ahead wholesale market, the IOMO in step III.

The results of this market will be sent to the local balancing market preliminary scheduling in step IV.

The interaction betwen the local markets and the central market in the balancing market is similar to that of the day-ahead market. The difference is the duration of the scheduling interval, which is 15 minutes for the balancing market. Step IV happens before real-time in the day of the delivery time (day D) and is preliminary scheduling for the local balancing market. Through this step, the DMO bidding price and quantities in the central balancing market is calculated. In step V (real-time), the TMO clears the central real-time balancing market according to the scheduled energy and reserve of market players. The TMO will send back the final value for the interface transformer power flow to the DMO. Finally, in step VI, the local balancing market is cleared by the DMO, based on the updated interface power flow from step V and the DER scheduled energy and reserve from step III.

It is assumed that only the market entities which have been accepted in the reserve market are allowed to participate in the balancing market where the actual activation of the balancing energy will happen. The procured balancing energy is limited to the scheduled reserve in the reserve capacity market. However, the generators which have been chosen in the reserve capacity market, will not necessarily have to deliver energy in the balancing phase. Therefore, the balancing market is needed in order to make sure that at any time the balance between supply and demand in the system is achieved in the most economically efficient manner.

As explained earlier in chapter 2, the reserve and balancing markets in the coupled market model follows the Dutch market. In the coupled market, there are separate markets for balancing and reserve capacity. The option-like character of reserve capacity in the coupled market is reflected in the two-part pricing. The provision of reserve capacity is remunerated with a reservation price ( $\in$ /MW) for reserving the capacity, and a reserve energy price ( $\in$ /MWh) is paid for exercising the reserve option to generate the required energy in real-time. Therefore, there are separate prices for the capacity reservation ( $\in$ /MW) and for balancing energy ( $\in$ /MWh).

# 4.2 Mathematical formulations

In this section, the mathematical formulations applied in the coupled market are explained. As discussed in chapter 3, in the current local market concrete modelling of the distribution network in the market-clearing algorithm is missing. In the coupled market, however, the distribution systems with their main security elements are modelled. Moreover as explained in subsection 4.1.2, the pricing mechanism in the coupled market is based on the DLMP approach. In this section, the mathematical formulations corresponding to the six steps of the coupled market clearing algorithm which have been explained in subsection 4.1.4, are presented in the following subsections.

#### Step I. Preliminary scheduling by the DMO

In this step, the DMO first collects all the bids and offers from DERs by solving a preliminary scheduling problem where the objective function is minimizing the total cost of energy and reserve capacity. The objective function is shown in (4.1).

$$\min \left[ \sum_{t \in T} \sum_{g \in G_D} (O_{g,t}^E P_{g,t} + O_{g,t}^{RUP} R_{g,t}^{UP} + O_{g,t}^{RDN} R_{g,t}^{DN}) + \sum_{t} \sum_{i \in N_{D-T}} \sum_{s} \pi_s \lambda_{t,s}^{TD} P_{t,i}^{TD} \right]$$

$$(4.1)$$

The objective function in (4.1) minimizes the total cost of generation and reserve capacity of DERs plus the expected cost of buying/selling energy from/to the TMO market. The cost of this energy is the day-ahead price in the wholesale market. This price has to be estimated by the DMO. There are different ways for the DMO to deal with this uncertain price. One of those ways is a stochastic, scenario-based approach which means that the price is estimated based on a set of scenarios and their associated probabilities of occurrence. How to deal with the uncertainty in the TMO market price is studied in more detail in chapter 6. Note that the demand is assumed to be inelastic, therefore, the objective function in (4.1) can be equivalent with maximizing the social welfare.

As mentioned earlier, in this step, the technical constraints of the distribution network are considered. The distribution network is based on the branch flow model and represented through a second-order cone programming (SOCP) relaxation, which is tight for radial distribution networks [165]. In Appendix B, the distribution network modelling is described in more detail. Given a power flow from distribution node  $i \in N_D$  to node j, i refers to the from-node and j refers to the to-node.

$$(\theta_{i,l,t}): \quad V_{i,t} = V_{A_i,t} + 2(r_l f_{l,t}^p + x_l f_{l,t}^q) - I_{l,t}(r_l^2 + x_l^2) \quad l(A_i,i) \in L_D, t \in T$$

$$(4.2)$$

$$(\lambda_{i,t}^{ED}): \sum_{l=(A_{i},i)} (f_{l,t}^{p} - I_{l,t}r_{l}) + \sum_{g \in G_{D} \in i} P_{g,t}$$
  
=  $P_{i,t}^{load} + \sum_{l=(i,j)} f_{l,t}^{p} + G_{i}V_{i,t} \quad i \in N_{D}, t \in T$  (4.3)

$$(\lambda_{i,t}^{E}) : \sum_{l=(A_{i,i})} (f_{l,t}^{p} - I_{l,t}r_{l}) + P_{i,t}^{TD} = \sum_{l=(i,j)} f_{l,t}^{p} + G_{i}V_{i,t} \quad i \in N_{D-T}, t \in T$$

$$(4.4)$$

$$(\mu_{i,t}): \sum_{l=(A_{i},i)} (f_{l,t}^{q} - I_{l,t}x_{l}) + \sum_{(g \in G_{D}) \in i} Q_{g,t}$$
  
=  $Q_{i,t}^{load} + \sum_{l=(i,j)} f_{l,t}^{q} - b_{i}V_{i,t} \quad i \in N_{D}, t \in T$  (4.5)

$$(\mu_{i,t}^{0}): \sum_{l=(A_{i},i)} (f_{l,t}^{q} - I_{l,t}x_{l}) + Q_{i,t}^{TD} = \sum_{l=(i,j)} f_{l,t}^{q} - b_{i}V_{i,t} \quad i \in N_{D-T}, t \in T$$

$$(4.6)$$

$$(\lambda_t^{RUP}): \quad \sum_{g \in G_D} R_{g,t}^{UP} \ge \alpha_D \sum_{g \in G_D} P_g^{gmax} \quad t \in T$$

$$(4.7)$$

$$(\lambda_t^{RDN}): \quad \sum_{g \in G_D} R_{g,t}^{DN} \ge \alpha_D \sum_{g \in G_D} P_g^{gmax} \quad t \in T$$

$$(4.8)$$

$$(\varphi_{g,t}^+): \quad P_{g,t} + R_{g,t}^{UP} \le P_g^{gmax} \quad g \in G_D, t \in T$$

$$(4.9)$$

$$(\varphi_{g,t}^{-}): \quad P_{g,t} - R_{g,t}^{DN} \ge P_g^{gmin} \quad g \in G_D, t \in T$$

$$(4.10)$$

$$(\xi_{l,t}): \quad (f_{l,t}^p)^2 + (f_{l,t}^q)^2 \le V_{i,t}I_{l,t} \quad i \in N_D, l(A_i, i) \in L_D, t \in T$$
(4.11)

$$(\zeta_{l,t}): \quad (f_{l,t}^p)^2 + (f_{l,t}^q)^2 \le S_{l,t}^2 \quad i \in N_D, l(A_i, i) \in L_D, t \in T$$
(4.12)

$$(\vartheta_{i,t}): \quad (P_{i,t}^{TD})^2 + (Q_{i,t}^{TD})^2 \le V_{i,t}I_{l,t} \quad i \in N_{D-T}, l(A_i, i) \in L_D, t \in T$$
(4.13)

$$(\phi_{g,t}): (P_{g,t})^2 + (Q_{g,t})^2 \le S_{g,t}^2 \quad g \in G_D, t \in T$$
 (4.14)

$$(\sigma_{i,t}^+, \sigma_{i,t}^-): \quad V_i^{min} \le V_{i,t} \le V_i^{max} \quad i \in N_D, t \in T$$

$$(4.15)$$

$$(\varrho_{i,t}^+, \varrho_{i,t}^-): \quad I_l^{min} \le I_{l,t} \le I_l^{max} \quad l \in L, t \in T$$

$$(4.16)$$

$$(\delta_{g,t}^+, \delta_{g,t}^-): \quad Q_i^{gmin} \le Q_{g,t} \le Q_i^{gmax} \quad g \in G_D, t \in T$$

$$(4.17)$$

$$(\beta_{g,t}^+, \beta_{g,t}^-): \quad P_g^{gmin} \le P_{g,t} \le P_g^{gmax} \quad g \in G_D, t \in T$$

$$(4.18)$$

Constraint (4.2) accounts for the voltage difference which is induced by the power flow over a line. Constraints (4.3) and (4.4) are active power balance equations of the distribution system. Constraints (4.5) and (4.6) are reactive power balance equations. In (4.7) and (4.8) the minimum amount of total upward and downward reserves that must be procured from DERs is considered, respectively. The  $\alpha_D$ is a percentage of the total generation capacity in the distribution network. This constraint guarantees that a certain amount of the total installed capacity from dispatchable generators is available for balancing purposes. Later in the balancing market (step IV and step VI) and during the deployment of reserve capacities, distribution network constraints are included. Constraints (4.9) and (4.10) are limits for the total capacity of generators.Constraints (4.11) and (4.13) show the relation between voltage and current and active and reactive power flow over a line and represent the conic equation of the distribution grid. Note that in the optimal power flow formulations in which minimizing the generation cost is the objective, the inequality (4.13) becomes binding. In the presented formulation, although the active power load is not explicitly included in the optimization objective, constraint (4.13) becomes binding at the optimal solution since active power losses represent additional load that has to be covered by the generating units at an increased cost. A more detailed explanation about the second-order conic AC power flow is presented further in Appendix B.

Constraint (4.12) imposes the congestion limit for the distribution lines. Constraint (4.14) is related to the generation capability curves and is linearised by the method explained in [170]. Constraints (4.15)-(4.18) impose limits on the involved decision variables ( $P_{g,t}, Q_{g,t}, V_{i,t}, I_{l,t}$ ). Note that the optimization above includes a general form of DERs as generators and DERs like storage systems have not yet been considered in this optimization. However, the technical constraints for these DERs like storage systems can be easily included in the optimization too, as will be shown in chapter 7.

Through this optimization process, the local market price for energy and reserve will be determined based on the DLMP approach. The energy price  $(\lambda_{i,t}^E)$  is the Lagrange multiplier of (4.4) at the interface node between the TSO and DSO, which can be determined by deriving the Karush-Kuhn-Tucker (KKT) conditions of the above convex optimization problem. For any convex optimization problem any points that satisfy the KKT conditions are primal and dual optimal. In other words, the KKT conditions provide necessary and sufficient conditions for optimality. More detailed explanations for KKT conditions can be found in [171].

Moreover, the Lagrangian multipliers of (4.7) and (4.8) symbolized with  $\lambda_t^{RUP}$ and  $\lambda_t^{RDN}$ , respectively, which are considered as the price for the upward and downward reserve capacity in the distribution system. How to calculate the DLMP is explained in Appendix B. In Appendix A, full equations of the KKT conditions are shown. The power injected at the interface node  $(\widetilde{P_{i,t}^{TD}})$ , upward and downward reserve capacity of DERs  $(\widetilde{R_{g,t}^{UP/DN}})$ , energy price  $(\lambda_{i,t}^{E})$  and reserve prices  $(\lambda_t^{RUP}, \lambda_t^{RDN})$  are outputs of this step. Based on  $\lambda_{i,t}^{E}, \lambda_{i,t}^{RUP}, \lambda_t^{RDN}$ and  $\widetilde{P_{i,t}^{TD}}$ , the DMO participates in the TMO-operated central day-ahead market in step II.

## Step II. Central day-ahead market clearing

In this step, the wholesale day-ahead joint energy and reserve capacity market is cleared by the TMO. The DMO and generators connected to the transmission network participate in this market. The objective of this market is maximizing social welfare, however, since in this paper the demand is considered to be inelastic, social welfare is equivalent to minimizing the total generation cost. More explanations for calculating of the social welfare can be found in [172].

$$\min \left[ \sum_{t \in T} \sum_{g \in G_T} (O_{g,t}^E P_{g,t} + O_{g,t}^{RUP} R_{g,t}^{UP} + O_{g,t}^{RDN} R_{g,t}^{DN}) + \sum_{t \in T} \sum_{i \in N_{T-D}} (\lambda_{i,t}^E P_{i,t}^{DMO} + \lambda_t^{RUP} R_{i,t}^{DMO/UP} + \lambda_t^{RDN} R_{i,t}^{DMO/DN}) \right]$$
(4.19)

The first term in (4.19) consists of the cost of energy and reserve capacity procured by the transmission-connected generators. The second term accounts for the total costs of energy and reserve capacity procured from the DMO.

For a transmission network, the error in using a linearised, DC power flow is less than that of a distribution network, therefore the well-established DC power flow approximation can be used to model the transmission network constraints:

$$f_{l,t}^p = B_l(\theta_{i,t} - \theta_{j,t}) \quad (i,j) \in l \in L_T, t \in T$$

$$(4.20)$$

$$-TC_l \le f_{l,t}^p \le TC_l \quad l \in L_T, t \in T \tag{4.21}$$

$$\sum_{g \in G_T} P_{g,t} + P_{i,t}^{DMO} + \sum_{(A_i,i) \in l} f_{l,t}^p$$
  
=  $P_{i,t}^{load} + \sum_{(i,A_i) \in l} f_{l,t}^p \quad i \in N_T, l \in L_T, t \in T$  (4.22)

$$R_{i,t}^{DMO/UP} + \sum_{g \in G_T} R_{g,t}^{UP} \ge \alpha_T \sum_{g \in G_T} P_g^{gmax} \quad i \in N_{T-D}, t \in T$$

$$(4.23)$$

$$R_{i,t}^{DMO/DN} + \sum_{g \in G_T} R_{g,t}^{DN} \ge \alpha_T \sum_{g \in G_T} P_g^{gmax} \quad i \in N_{T-D}, t \in T$$

$$(4.24)$$

$$P_{g,t} + R_{g,t}^{UP} \le P_g^{gmax} \quad g \in G_T, t \in T$$

$$(4.25)$$

$$P_{g,t} - R_{g,t}^{DN} \ge P_g^{gmin} \quad g \in G_T, t \in T$$

$$(4.26)$$

$$P_g^{gmin} \le P_{g,t} \le P_g^{gmax} \quad g \in G_T, t \in T$$

$$(4.27)$$

$$0 \le P_{i,t}^{DMO} \le \widetilde{P_{i,t}^{TD}} \quad i \in N_{T-D}, t \in T$$

$$(4.28)$$

$$0 \le R_{i,t}^{DMO/UP} \le \sum_{g \in G_D} \widetilde{R_{g,t}^{UP}} \quad i \in N_{T-D}, t \in T$$

$$(4.29)$$

$$0 \le R_{i,t}^{DMO/DN} \le \sum_{g \in G_D} \widetilde{R_{g,t}^{DN}} \quad i \in N_{T-D}, t \in T$$

$$(4.30)$$

Constraint (4.20) considers the power flow over a transmission line and (4.21) imposes a limit on this power flow based on the transmission line capacity. In (4.22), the power balance equation is shown. Constraints (4.23) and (4.24) are the required reserve capacity in the transmission level which is a percentage of the total generation directly connected to the transmission network. Constraints (4.25) and (4.26) correspond to the capacity limits of generators in the transmission grid. Constraint (4.27) limits the generation of the transmission generators to their maximum capacity. Constraints (4.28)-(4.30) limit the scheduled energy and reserve capacity of the DMO in the central market to its bidding quantities. Constraints (4.29) and (4.30) limit the upward and downward reserves from the DMO. Energy price ( $\lambda_{i,t}^{ET}$ ) and reserve price ( $\lambda_{i,t}^{RT/UP}$ ,  $\lambda_{i,t}^{RT/DN}$ ) are the Lagrangian multipliers of (4.22) and (4.23,4.24), respectively. After clearing this market, the DMO is informed about the allocated power flow over the interface transformer ( $\widetilde{P_{i,t}^{DMO}}$ ) and the required reserve capacity ( $R_t^{DMO/UP}$ ,  $R_t^{DMO/DN}$ ).

#### Step III. Local day-ahead market clearing

In this step, the DMO clears the day-ahead joint energy and reserve capacity market. This local day-ahead market is cleared by solving the optimization problem defined by constraints (4.31)-(4.35).

$$min \quad \left[\sum_{t \in T} \sum_{g \in G_D} (O_{g,t}^E P_{g,t} + O_{g,t}^{RUP} R_{g,t}^{UP} + O_{g,t}^{RDN} R_{g,t}^{DN})\right] \tag{4.31}$$

Subject to:

$$\sum_{l=(A_{i},i)} (f_{l,t}^{p} - I_{l,t}r_{l}) + \sum_{g \in G_{D}} P_{g,t} + \widetilde{P_{i,t}^{DMO}}$$
  
=  $P_{i,t}^{load} + \sum_{l=(i,A_{i})} f_{l,t}^{p} + G_{i}V_{i,t} \quad i \in N_{D}, t \in T$  (4.32)

$$\sum_{g \in G_D} R_{g,t}^{UP} \ge R_t^{\widetilde{DMO/UP}} \quad i \in N_{D-T}, t \in T$$

$$(4.33)$$

$$\sum_{g \in G_D} R_{g,t}^{DN} \ge R_t^{\widetilde{DMO/DN}} \quad i \in N_{D-T}, t \in T$$

$$(4.34)$$

$$(4.2), (4.9) - (4.12), (4.14) - (4.18) \tag{4.35}$$

The objective function in (4.31) minimizes the total generation cost of DERs in the distribution system. The constraints in this step are mostly similar to Step I. The main differences are in the power balance equation in (4.32) and in the required reserve capacity in (4.33) and (4.34). In (4.32),  $\widetilde{P_{i,t}^{DMO}}$  is a parameter symbolizing the power flow injected at the interface node of the distribution system. The outputs of this step, regarding the reserve capacity  $(\widetilde{R_{g,t}^{UP}})$  and  $(\widetilde{R_{g,t}^{DN}})$  and the scheduled energy of DERs  $(\widetilde{P_{g,t}})$ , are inputs for the balancing market which is explained further.

#### Step IV. Balancing preliminary scheduling by the DMO

In this step, the DMO estimates the local balancing market price by which it will participate in the central real-time balancing market. This optimization is modelled through stochastic programming where the objective function minimizes the expected balancing service cost of the distribution network.

$$\min \left[ \sum_{g \in G_D} (O_{g,t}^{EUP} P_{g,t}^{UP} - O_{g,t}^{EDN} P_{g,t}^{DN}) + \sum_{i \in N_{D-T}} \sum_{s} \pi_s (\lambda_{t,s}^+ P_{i,t}^{TD/UP} - \lambda_{t,s}^- P_{i,t}^{TD/DN}) \right]$$
(4.36)

The objective function in (4.36) consists of the cost of balancing services procured from DERs and imported from the transmission grid. Similar to Step 1, the price for the balancing service from the transmission grid is considered as the central balancing market price and is based on scenarios. The distribution network constraints are also taken into account in this step.

$$\begin{aligned} (\lambda_{i,t}^{EBL}) : & \sum_{l=(A_{i},i)} (f_{l,t}^{p} - I_{l,t}.r_{l}) + \sum_{g \in i} (\widetilde{P_{g,t}} + P_{g,t}^{UP} - P_{g,t}^{DN}) \\ & + \sum_{i \in N_{D-T}} (P_{i,t}^{TD/UP} - P_{i,t}^{TD/DN}) = \alpha_{Imb}SI_{t,s} + P_{i,t}^{load} \\ & + \sum_{l=(i,j)} f_{l,t}^{p} + G_{i}.V_{i,t} \quad i \in N_{D} \\ & \sum_{l=(A_{i},i)} (f_{l,t}^{q} - I_{l,t}.x_{l}) + \sum_{g \in i} Q_{g,t}^{UP} - Q_{g,t}^{DN} \\ & = Q_{i,t}^{load} + \sum_{l=(i,j)} f_{l,t}^{q} - b_{i}V_{i,t} \quad i \in N_{D} \end{aligned}$$
(4.38)

$$\widetilde{P_{g,t}} + P_{g,t}^{UP} \le P_g^{gmax} \quad g \in G_D \tag{4.39}$$

$$\widetilde{P_{g,t}} - P_{g,t}^{DN} \ge P_g^{gmin} \quad g \in G_D \tag{4.40}$$

$$P_{g,t}^{UP} \le \widetilde{R_{g,t}^{UP}} \quad g \in G_D \tag{4.41}$$

$$P_{g,t}^{DN} \le \widetilde{R_{g,t}^{DN}} \quad g \in G_D \tag{4.42}$$

$$(4.2), (4.13) - (4.18) \tag{4.43}$$

Constraint (4.37) is the power balance equation. Symbol  $\alpha_{Imb}$  is the fraction of the total system imbalance at the distribution system. Constraint (4.38) is the reactive power balance equation. Constraints (4.39)-(4.42) limit the upward and downward balancing regulations. The rest of the constraints including the relation between the voltage and current, active and reactive power are similar to the ones in step I. The output of this step is the local balancing market price  $(\lambda_{i,t}^{EBL})$ , the Lagrangian multiplier of (4.37). With this price, the DMO will participate in the TMO balancing market in Step V.

#### Step V. Central balancing market clearing

In this step, the TMO clears the real-time central balancing market. Generators connected to the transmission grid and the DMO participate in this market. The objective function is as follows:

$$\min \left[ \sum_{g \in G_T} (O_{g,t}^{EUP} P_{g,t}^{UP} - O_{g,t}^{EDN} P_{g,t}^{DN}) + \sum_{i \in N_{T-D}} \lambda_{i,t}^{EBL} (P_{i,t}^{DMO/UP} - P_{i,t}^{DMO/DN}) \right]$$

$$(4.44)$$

In (4.44), the cost of deployed balancing services from all the resources is taken into account. The first term is related to the cost of balancing services from the transmission generators. In the second term,  $\lambda_{i,t}^{EBL}$  is the price of balancing services from the aggregated DERs by the DMO. The network constraints of the transmission system are enforced as:

$$\sum_{g \in i} (\widetilde{P_{g,t}} + P_{g,t}^{UP} - P_{g,t}^{DN}) + (\widetilde{P_{i,t}^{DMO}} + P_{i,t}^{DMO/UP} - P_{i,t}^{DMO/DN}) + \sum_{(A_{i},i) \in l} f_{l,t}^{p} = SI_{t,s_{w}} + P_{i,t}^{load} + \sum_{(i,A_{i}) \in l} f_{l,t}^{p} \quad i \in N_{T}, l \in L_{T}, t$$

$$\widetilde{P_{g,t}} + P_{g,t}^{UP} \leq P_{g}^{gmax} \quad g \in G_{T}$$
(4.46)

$$\widetilde{P_{g,t}} - P_{g,t}^{DN} \ge P_g^{gmin} \quad g \in G_T$$
(4.47)

$$P_{g,t}^{UP} \le \widetilde{R_{g,t}^{UP}} \quad g \in G_T \tag{4.48}$$

$$P_{g,t}^{DN} \le \widetilde{R_{g,t}^{DN}} \quad g \in G_T \tag{4.49}$$

$$P_{i,t}^{DMO/UP} \le R_{i,t}^{\widetilde{DMO/UP}} \quad i \in N_{T-D}$$

$$(4.50)$$

$$P_{i,t}^{DMO/DN} \le R_{i,t}^{DMO/DN} \quad i \in N_{T-D}$$

$$\tag{4.51}$$

$$(4.20), (4.21) \tag{4.52}$$

Equation (4.45) is the power balance equation in which  $SI_{t,s_w}$  is the total system imbalance. Note that  $s_w$  shows the realization of a scenario. Constraint (4.46)-(4.51) limit the upward and downward balancing by transmission generators and the DMO. The rest of the constraints are similar as the ones in step II. The results of this step, which will be passed on to the DMO, are  $P_{i,t}^{\widetilde{DMO}/UP}$  and  $P_{i,t}^{\widetilde{DMO}/DN}$  indicating the deployed balancing energy from transmission to

the distribution system.

### Step VI. Local balancing market clearing

This is the final step where DMO clears the local balancing market. The objective function minimizes the cost of balancing services deployed by DERs:

$$min \quad \left[\sum_{g \in G_D} (O_{g,t}^{EUP} P_{g,t}^{UP} - O_{g,t}^{EDN} P_{g,t}^{DN})\right] \tag{4.53}$$

Subject to:

$$(\lambda_{i,t}^{EBL}) : \sum_{l=(A_{i},i)} (f_{l,t}^{p} - I_{l,t}r_{l}) + \sum_{g \in G_{D} \in i} (\widetilde{P_{g,t}} + P_{g,t}^{UP} - P_{g,t}^{DN})$$
  
$$= (\widetilde{P_{i,t}^{DMO/UP}} - \widetilde{P_{i,t}^{DMO/DN}}) + P_{i,t}^{load} + \sum_{l=(i,A_{i})} f_{l,t}^{p} + G_{i}V_{i,t} \quad i \in N_{D}$$
  
(4.54)

$$\widetilde{P_{g,t}} + P_{g,t}^{UP} \le P_g^{gmax} \quad g \in G_D \tag{4.55}$$

$$\widetilde{P_{g,t}} - P_{g,t}^{DN} \ge P_g^{gmin} \quad g \in G_D \tag{4.56}$$

$$P_{g,t}^{UP} \le \widetilde{R_{g,t}^{UP}} \quad g \in G_D \tag{4.57}$$

$$P_{g,t}^{DN} \le \widehat{R_{g,t}^{DN}} \quad g \in G_D \tag{4.58}$$

$$(4.13), (4.13) - (4.18) \tag{4.59}$$

In the power balance equation (4.54),  $P_{i,t}^{\widetilde{DMO}/UP}$  and  $P_{i,t}^{\widetilde{DMO}/DN}$  are the upward and downward regulation actions from transmission level to the distribution level, respectively, which have been calculated in step V. Constraints (4.55)-(4.58) are limits enforced for the upward and downward schedule adjustments of DERs. The rest of the constraints in (4.59) are similar with the constraints in step I. The output of this step is the cleared local balancing market price  $(\lambda_{i,\tau}^{DBL})$  and the quantities  $(P_{g,t}^{UP}, P_{g,t}^{DN})$  for the resources connected to the distribution network.

# 4.3 Conclusions

This chapter gives an overview of the proposed coupled market design and introduces the main elements and features of this new market model. The importance of this market model is explained subsequently. First, the market organization is described in which the main market stakeholders, including distributed energy resources, prosumers and consumers, energy suppliers, the distribution system operator, the transmission system operator, the distribution market operator, and transmission market operator and their interactions are defined, accordingly. Then, the pricing mechanism is explained. Thereafter, the interaction between the transmission market operator and the distribution market operator is explained in detail. Lastly, the sub-section on market timing shows how different market time-frames (day-ahead and balancing) work in the coupled market. In the next section, the corresponding mathematical formulations of the marketclearing algorithm are presented. The objectives and constraints at each step of the coupled market are defined and explained.

This chapter shows how the coupled market can be a solution to overcome the existing challenges with DER participation in current markets. In the next chapters, the coupled market is analysed from different perspectives to fully understand and quantify the advantages of this new market model.

Chapter 4: The coupled market
5

## The coupled market: System's perspective

In the previous chapter, it was shown that the coupled market model allows distributed energy resources (DERs) to participate in day-ahead and balancing markets. A joint local market for energy and reserve capacity in the day-ahead market was introduced, operated by the distribution market operator (DMO). The DMO can participate in the central market on DER's behalf. The distribution network are included in the local market clearing process. The question remains whether the coupled market model is an efficient market design when a large amount of DERs are present. In this chapter<sup>1</sup>, this question is answered by comparing the coupled market model with the centralized market model. The centralized market model is compatible with the current market design in Europe, in which there is no local market for DERs. In section 5.1, the centralized market model is introduced together with its main features including market organization, pricing mechanism, market timing, scalability, and mathematical formulations. In section 5.2, the coupled market is compared with the centralized market model from the viewpoint of total system cost. The case studies, input data, and assumptions are explained in subsection 5.2.1. Results and discussion are shown in subsection 5.2.2. This chapter ends with conclusions in section 5.3.

<sup>&</sup>lt;sup>1</sup>The results of this chapter is based on the published work in: Farrokhseresht, M., Paterakis, N., Gibescu, M., and Slootweg, J.G. (2020), Enabling market participation of distributed energy resources through a coupled market design, *IET Renewable Power Generation*, 14, 4.



Figure 5.1: Centralized market model (benchmark)

## 5.1 Centralized market model (benchmark)

A scheme consisting of centralized day-ahead and balancing markets is considered as the benchmark model for our analysis. This has the most compatibility with the current electricity market regulation. In this model, there are no DMOoperated local markets, and distribution network constraints are not taken into account during the market-clearing process. DERs are considered to be connected at the interface node of the transmission network. The TMO operates both day-ahead and balancing markets for all DERs and large-scale generators in the transmission system. In the following subsections, the difference between the coupled market and the centralized market model is explained in terms of market organization, pricing mechanism, market timing, and scalability.

## 5.1.1 Market organization

As the market organization in 5.1 shows, the main stakeholders in the centralized market model are the same as the ones in the coupled market, except that the

DMO-operated local market is absent in the centralized market model. The roles and responsibilities of these market players remain the same as are explained in chapter 4. As Figure 5.1 shows, DERs directly participate in the TMO market. Note that this way of DERs participation in the central market is in contrast with what happens in reality in the current markets. In reality, DERs participate in the central market indirectly through an supplier (e.g. aggregator) as discussed in chapter 3. The direct way of participation of DERs in the central market as shown in Figure 5.1 is, however an idealized situation in which there are no market requirements for DERs, i.e. the DERs do not have to pay market transactions fees and there are no minimum bid size requirements for market participants, etc.

In this TMO-operated market, there is no interaction between the TMO and the DSO, meaning that the distribution network constraints are not taken into account during the market-clearing process (only the TSO-DSO interface node is taken into account). Therefore, there is a chance that the activation of DERs after the TMO market-clearing, leads to an operation violation of the distribution network. The DSO is responsible for managing the distribution network and ensuring its security which happens through a post-market-clearing re-dispatch. The TSO is responsible for the security of the transmission system. The TSO's interaction with the TMO, bulk generators, consumers are the same as in the coupled market. Same as in the coupled market, the energy supplier may have contracts with household consumers at the distribution system. However, energy suppliers may only interact with the TMO market.

#### 5.1.2 Pricing mechanism

The pricing mechanism in the centralized market model is also based on locational marginal pricing (LMP), as explained in chapter 4. In the TMO market clearing algorithm, the transmission network constraints are taken into account. However, the distribution network is not considered, therefore the market prices are not reflecting e.g. losses and congestions in the distribution network except for the transmission-distribution interface node.

#### 5.1.3 Market timing

In the centralized market model, the full participation of generators and DERs in day-ahead and balancing markets is considered. Like in the coupled market, the day-ahead market is a joint energy and reserve market. The market timing that belongs to the centralized market model is shown in Figure 5.2. Since the DMO-operated local market does not exist in the centralized market model, steps belonging to the local market in the coupled market are discarded here. Therefore, the centralized market model merely consists of step II and step V



Figure 5.2: Market timing for the centralized market model

in the coupled market model. DERs and prosumers at the distribution together with other generators and consumers at the transmission level can simultaneously participate in the market. Therefore, DERs don't have to submit their bids earlier as in the case of the coupled market.

Market participants submit their bids in the day-ahead market at around noon on day D-1, one day before the delivery time. The TMO clears the day-ahead joint energy and reserve market. Subsequently, market participants are informed about their scheduled energy and reserve power. The balancing market is cleared every 15 minutes before real-time in day D, day of the delivery. Note that this chapter is from the system's perspective, hence the ex-post imbalance settlement in which the market participants pay or are being paid for their imbalances is not considered. The imbalance settlement is included in the balancing market in chapter 7 with a focus on the coupled market from the DER's perspective.

#### 5.1.4 Scalability

It has been stated in chapter 4 that one of the advantages of the coupled market model is its scalability. This paragraph proves this assertion. Assume that n is the total number of DERs at the distribution network and m is the number of DMOs with equal sizes. Therefore, one DMO has  $\frac{n}{m}$  DERs in its jurisdiction area. Only the day-ahead market is considered in this scalability proof. In a centralized market model where all DERs are dispatched in the wholesale market, the total number of DERs being activated or not (can be also seen as for example unit commitment) is  $2^n$ . However, as there are three steps in the day-ahead coupled market model, the total combination for DER's being activated or not is the summation of the combinations in steps I, II, and III. Since there are m DMOs at the distribution network level, the total combination of DER's commitment for all DMOs is calculated as:  $\{m.2^{\frac{n}{m}} + m.2^{\frac{n}{m}} + 2^m\}$  and if  $m \ll n$ , this term is equivalent with  $2^{\frac{n}{m}}$  which is smaller than  $2^n$ , the total combination in the centralized model. It means that, if the number of DERs rises in the distribution network, the total combination of DER's commitments in the centralized market model raises at a faster rate than in the coupled market model. Therefore, the scalability in the coupled market compared with the centralized market model is better. This is simply due to spatial decomposition into m distribution networks, each with its own smaller combinatorial problem.

#### 5.1.5 Mathematical formulations

In this section, the mathematical formulation applied in the centralized marketclearing algorithm is explained. As mentioned earlier in section 5.1.3, the centralized market model only consists of step II and step V from the coupled market clearing process which are shown in Figure 4.3 in chapter 4. The mathematical formulation for the day-ahead and balancing market in the centralized market model are described in the following subsections.

#### Day-ahead market

The day-ahead joint market of the centralized market model is quite similar to step II in the coupled model. The difference is that, for the objective function in the centralized model, the DMO bidding prices for energy and reserve ( $\lambda_{i,t}^{ED}$  and  $\lambda_{i,t}^{RD}$  in the objective function of step II, shown in chapter 4) are equal to zero. Moreover,  $P_{g,t}$  and  $R_{g,t}^{UP}$  and  $R_{g,t}^{DN}$  represent energy and upward and downward reserves for all generators including DERs in the distribution system and large generators in the transmission system. Therefore, the objective function and constraints are as follows:

$$min \quad \left[\sum_{t \in T} \sum_{g \in G_T} (O_{g,t}^E P_{g,t} + O_{g,t}^{RUP} R_{g,t}^{UP} + O_{g,t}^{RDN} R_{g,t}^{DN})\right] \tag{5.1}$$

Subject to:

$$f_{l,t}^p = B_l(\theta_{i,t} - \theta_{j,t}) \quad (i,j) \in l \in L_T, t \in T$$

$$(5.2)$$

$$-TC_l \le f_{l,t}^p \le TC_l \quad l \in L_T, t \in T$$

$$(5.3)$$

$$\sum_{g \in G_T} P_{g,t} + \sum_{w \in i} P_{w,t}^{DA} + \sum_{(A_i,i) \in l} f_{l,t}^p$$
  
=  $P_{i,t}^{load} + \sum_{(i,A_i) \in l} f_{l,t}^p \quad i \in N_T, l \in L_T, t \in T$  (5.4)

$$\sum_{g \in G_T} R_{g,t}^{UP} \ge \alpha_T \sum_{g \in G_T} P_g^{gmax} \quad i \in N_{T-D}, t \in T$$
(5.5)

$$\sum_{g \in G_T} R_{g,t}^{DN} \ge \alpha_T \sum_{g \in G_T} P_g^{gmax} \quad i \in N_{T-D}, t \in T$$
(5.6)

$$P_{g,t} + R_{g,t}^{UP} \le P_g^{gmax} \quad g \in G_T, t \in T$$

$$(5.7)$$

$$P_{g,t} - R_{g,t}^{DN} \ge P_g^{gmin} \quad g \in G_T, t \in T$$

$$(5.8)$$

$$P_g^{gmin} \le P_{g,t} \le P_g^{gmax} \quad g \in G_T, t \in T$$
(5.9)

The network constraints are similar to the constraints in step II of the coupled market model. The only difference is in the power balance equation (5.4) where  $P_{i,t}^{load}$  belongs to the loads in both the transmission and distribution networks. Also,  $P_{g,t}$  counts for all the generators and DERs in the system. Constraint (5.5) and (5.6) show the total required upward and downward reserves for the system, respectively. Constraints (5.7)-(5.9) limit the up/downward reserve and generation from the market participants. Constraints (5.2) and (5.3) belong to the DC power flow in transmission network. Similar to the coupled market, energy and reserve market prices are the Lagrangian multipliers of (5.4) and (5.5)-(5.6), respectively.

#### **Balancing** market

The balancing market component of the centralized model is cleared in a similar way as step V of the coupled market model. However, the term  $\lambda_{i,t}^{DBL}$  in the objective function of step V equals to zero. The term  $\Delta P_{g,t,s}$  represents the power adjustment (upward or downward) for all generators including DERs connected to the distribution system and generators in the transmission system.

$$min \quad \left[\sum_{g \in G_D} (O_{g,t}^{EUP} P_{g,t}^{UP} - O_{g,t}^{EDN} P_{g,t}^{DN})\right] \tag{5.10}$$

Subject to:

$$\sum_{g \in i} (\widetilde{P_{g,t}} + P_{g,t}^{UP} - P_{g,t}^{DN}) + \sum_{w \in i} P_{w,t,s_n}^{Wact} + \sum_{(A_i,i) \in l} f_{l,t}^p$$

$$= SI_{t,s_w} + P_{i,t}^{load} + \sum_{(i,A_i) \in l} f_{l,t}^p \quad i \in N_T, l \in L_T$$
(5.11)

$$\widetilde{P_{g,t}} + P_{g,t}^{UP} \le P_g^{gmax} \quad g \in G_T$$
(5.12)

$$\widetilde{P_{g,t}} - P_{g,t}^{DN} \ge P_g^{gmin} \quad g \in G_T$$
(5.13)

$$P_{g,t}^{UP} \le \widetilde{R_{g,t}^{UP}} \quad g \in G_T \tag{5.14}$$

$$P_{g,t}^{DN} \le \widetilde{R_{g,t}^{DN}} \quad g \in G_T \tag{5.15}$$

$$P_{i,t}^{DMO/UP} \le R_{i,t}^{\widetilde{DMO}/UP} \quad i \in N_{T-D}$$

$$(5.16)$$

$$P_{i,t}^{DMO/DN} \le R_{i,t}^{\widetilde{DMO/DN}} \quad i \in N_{T-D}$$

$$(5.17)$$

$$(5.2), (5.3)$$
  $(5.18)$ 

In the objective function in (5.10), the index g belongs to all transmission-level generators and DERs in the underlying distribution systems. It minimizes the total cost of balancing services from all market participants. The equation in (5.11) shows the power balance equation in the TMO balancing market. Constraints (5.12)-(5.17) limit the upward and downward balancing by transmission-level generators and DERs. The rest of the constraints are similar to the ones in the day-ahead market. Similar to step V in the coupled market, the balancing market price is the Lagrangian multiplier of the power balance equation in (5.11).

## 5.2 Coupled versus centralized market model

In chapter 4, the coupled market model is introduced and it is shown how this coupled market model enables the participation of DERs in a joint energy and reserve day-ahead and balancing markets while at the same time, distribution network constraints are considered. The main question which is going to be answered in this section is whether the coupled market model is an efficient market design when a large amount of DERs are present. This question is answered by comparing the total system cost for the coupled market and the centralized market models, while including or ignoring the distribution network constraints. First, the input data, case studies, and assumptions are described. Next, the results are presented and discussed.

#### 5.2.1 Input data and case studies

The proposed coupled TMO-DMO market model is tested using a radial 30-bus medium voltage Dutch distribution system and the IEEE-24 bus transmission system [173]. The data for the offer prices of distributed generators are from [174]. Appendix C shows data for generators connected to the transmission and distribution grids. A wind turbine in the system is located at bus number 18 (at the end of the feeder) of the distribution grid. The residential loads in the distribution system are generated with the method described in [175]. For the industrial loads, the data from the NEDU profiles [176] is used. The day-ahead and imbalance market prices are obtained from the Belgian TSO Elia [177]. Another set of data from the ENTSO-e transparency platform [56] for the day-ahead and balancing market prices in the Netherlands are also applied. Wind power generation, day-ahead, and imbalance market prices are stochastic and depend on scenarios and their corresponding data are presented in Appendix C. The time resolution of the day-head market is one hour with a time horizon of 24-hours and the balancing market is cleared every 15 minutes. Parameters  $\alpha_D$  and  $\alpha_T$  are considered as 30% of the total installed generation installed at the distribution and transmission system levels, respectively.

The mathematical optimization models are formulated in the General Algebraic Modelling System (GAMS) and solved with the solvers CPLEX and MOSEK on a computer with CPU E5-2697 v3@2.6GHz.The following assumptions are taken into account:

- Demands connected to both transmission and distribution systems are inelastic.
- DERs consists of stochastic generators and dispatchable generators. In this analysis, the stochastic generator is a wind generator. Dispatchable generators can participate in day-ahead energy and reserve capacity markets and the balancing market. However, stochastic generators can only participate in the day-ahead energy market.
- Wind forecast errors are assumed to be the cause of real-time system imbalances. These possible imbalances are represented via a set of scenarios.
- Here, the storage system has not been included among DERs at the distribution system. However, the storage system and its technical constraints can be included in the optimization problem as shown in chapter 7.
- The reserve capacity market presented in this paper ensures that enough balancing energy will be available in the balancing market. During the actual deployments of the reserve in the balancing market, the network constraints in both local and central markets are taken into account.

## 5.2.2 Results and discussions

To understand the effect of considering the distribution network constraints during the market-clearing on the total cost and security of the system, the results of two cases are studied. Case (1) is the TMO-DMO coupled market model with the distribution network constraints set as inactive. Case (1) with no network constraints has been performed to have a fair comparison between the centralized and the coupled market models (since in the centralized market model which is compatible with the current market structure, the distribution network constraints are ignored). This resembles the case in which the distribution network has enough capacity so that its constraints will not be violated in any situation, regardless of the demand and generation profiles. To show that the coupled market model is indeed capable of respecting the network constraints, case (2) has been implemented, where distribution network constraints have been added.

Table 5.1 shows the total system costs ( $k \in$ ) of the coupled market model in case (1) and the centralized market model, both with inactive distribution network constraints. In the table, it is indicated for each number which step (I-VI) it belongs to, from the coupled market clearing steps explained in chapter 4. In the coupled market model, the total system cost is the cost of the TMO market. As can be seen from the table, the total system cost in the coupled market model with inactive distribution network constraints is only slightly higher than that of the centralized market model. This low-cost difference shows that the coupled market model can theoretically be operated with little additional cost (economic inefficiency) when compared to the current centralized market model. However, the reason for this cost difference is related to the DMO's overestimation or underestimation of the wholesale market price. To explain this, we consider only the day-ahead market and the objective function in equation (1). If the DMO overestimates the wholesale market price, the more expensive DERs can be dispatched in the local market while the DMO could have imported cheaper energy from the wholesale market instead. In contrast, if the DMO underestimates the wholesale market price, the cheaper DERs will not be dispatched and the DMO has to buy more expensive energy from the wholesale market. The DMO bidding strategy in the TMO market will be explained in more detail in chapter 6.

In the first set of data, the bidding prices by the generator in the day-ahead and balancing markets are considered the same. Therefore the expected value of the cost in the day-ahead market is just slightly higher than the one in balancing market. This should resemble a market with high RES penetration, where

		Coupled market		Centralized market
		Local	TMO	
		$\mathbf{market}$	market	
System cost	Day-ahead	0.445	37.445	36.280
		(step III)	(step II)	
	Balancing	0.213	36.757	35 949
	Dataliening	(step VI)	(step V)	00.040

Table 5.1: Market results (k $\in$ ) for the coupled market (case (1)) versus centralized market models

		Coupled market		Centralized market
		Local market	TMO market	
System cost	Day-ahead	5.163 (step III)	60.195 (step II)	58.042
	Balancing	2.582 (step VI)	37.512 (step V)	25.520

Table 5.2: Market results  $(k \in)$  for the coupled market (case (1)) versus centralized market model based on data from [56]

marginal prices are low and variability is high. Currently, the cost of the balancing market is usually a fraction of that in the day-ahead market. Therefore, an additional simulation for case (1) is run but with new data for day-ahead and balancing market prices which are closer to the current reality. The market results for this new set of data are shown in Table 5.2. For the TMO market, the day-ahead and balancing market prices of the Dutch system available in the ENTSO-e transparency platform are now used. The data are shown in Appendix C. As Table 5.2 shows, with this set of new data, the coupled market with inactive distribution network constraints is still cost-comparable with the centralized market model.

Table 5.3 shows the total system costs of the coupled market model in case (2). In case (2), the distribution network constraints have been activated. The table shows that the additional cost of taking into account the distribution network constraints to the local market clearing process in the coupled market model will increase the system cost significantly, to 131 % compared to case (1) where the distribution network constraints are inactive. Due to the activation of constraints in case (2) which limits the network, the cheaper DERs cannot be used all the time. Therefore, the lower cost of case (1) could only be obtained if the distribution network would be reinforced. Table 5.4 shows results for a similar simulation set-up but is run for the new set of input data used in Table 5.2. The results in Table 5.4 also show a significantly higher cost for the coupled market compared with the centralized market model, when distribution network constraints become active during market clearing.

Figures 5.3 and 5.4 depict relative comparisons between the coupled and centralized market models based on results shown in Tables 5.2 and 5.4 (with input data from the Dutch day-ahead and imbalance markets available in the ENTSO-e transparency platform). Figure 5.3 belongs to case (1), in which the distribution network constraints are neglected in the DMO-operated local market clearing. From this figure, it is clear that the coupled market is cost-comparable with

		Coupled market		Centralized
				$\mathbf{market}$
		Local	$\mathbf{TMO}$	
		$\mathbf{market}$	market	
	Day-ahead	2.143	47.379	36.280
System cost		(step III)	(step II)	
	Delensing	1.120	45.608	25.040
Dataticing	(step VI)	(step V)	55.949	

Table 5.3: Market results (k€) for the coupled market (case (2)) versus centralized market model

Table 5.4: Market results ( $k \in$ ) for the coupled market (case (2)) versus centralized market model based on data from [56]

		Coupled market		Centralized market
		Local	TMO	
		$\operatorname{market}$	$\operatorname{market}$	
System cost Day-al	Day aboad	24.15	74.08	58.042
	Day-alleau	(step III)	(step II)	
	D - 1	11.448	42.534	05 500
	Dalancing	(step VI)	(step V)	20.020

the centralized market models, although it still has a slightly higher cost that is the result of the imperfect estimation by the DMO of the TMO market prices. Figure 5.4 shows the comparison between the results of the coupled market in case (1) versus case (2). Activating distribution network constraint in the DMOoperated local market significantly raises the cost compared with case (1) which neglects the distribution network. Consequently, the coupled market becomes more expensive than the centralized market model.

Figure 5.5 shows the single line diagram of the distribution network after dispatching DERs in case (1). The red lines in the figure indicate which feeders are overloaded during at least a single time instance over the 24-hour horizon of the simulation. In case (2), the feeders are never overloaded as the implementation of the distribution network constraints in the local market-clearing is preventing the dispatch of DERs which will induce overloading.

Finally, a sensitivity analysis has been done to study the effect of an increasing share of DERs in the system. Figure 5.6 shows a comparison between the system cost in the coupled and centralized market models when the share of DERs is increased. The coupled market model here refers to case (1) which has been



Figure 5.3: System cost in coupled versus centralized market models for case (1)



Figure 5.4: System cost in case (1) versus case (2)

explained above. For this sensitivity analysis, the simulation input data stays the same but the amount of DER in the system has changed in such a way that the same connection points are used but the max installed power is scaled up or down. The  $\alpha$  shown in Figure 5.6 is a ratio indicating the total share of DERs compared with the maximum peak load in the system. The market results shown in Table 5.1 are belonged to  $\alpha = 1$ .  $\alpha = 0.75$  and  $\alpha = 1.25$ , for example, stand



Figure 5.5: Overloading in the distribution system when grid constraints are not taken into account



Figure 5.6: Increasing DERs in coupled vs. centralized market models,  $\alpha$ : a ratio indicating the total share of DERs with respect to the maximum peak load

for the situations where the total DERs in the system are 75% and 125% of the maximum peak load, respectively. As the figure shows, while increasing the share of DERs in the system, the coupled market and the centralized market model still remain stay cost-comparable. By increasing the share of DERs to a higher value, e.g. the double of maximum peak load ( $\alpha$ =2), the relative difference between coupled and centralized market models is slightly increasing.

## 5.3 Conclusions

This chapter studied the proposed coupled market design from the system's perspective. To this end, a centralized market model is used as a benchmark to compare the results of the two models. At the beginning of this chapter, the currently applied centralized market model is presented, and its difference with the coupled market is explained in terms of market organization, pricing mechanism, market timing, mathematical formulations and scalability. Thereafter, the coupled market and centralized market models are compared to show whether the coupled market is an efficient market design to enable the participation of DERs. This question is answered by comparing the total system cost for the coupled market and the centralized market models while including or ignoring the distribution network constraints.

The results show that the coupled market model without the distribution network constraints is cost-comparable with the centralized market model even in the situations where the share of DERs in the system has been increased. To strengthen this conclusion, input data are varied. The results show that the system cost in the coupled market with inactive distribution network constraints is comparable with that of the centralized market model. Moreover, the coupled market seems to be a more scalable model compared with the current centralized market model. Additionally, for certain market conditions, the distribution network constraints can be violated and assets in the distribution network become overloaded. Adding the distribution network constraints to the coupled market model will alleviate the overloading while raising the system cost even further. However, to have a more accurate overview of the consequences of this cost increase, it is needed to compare the increase of the system cost due to adding the distribution network constraints in the market clearing process, to the cost of accelerated ageing or even a blackout in the system due to the overloading of network assets.

# The coupled market: DMO's perspective

The distribution market operator (DMO), the market operator in the local market, has been introduced in chapter 4. The current chapter<sup>1</sup> aims to look deeper into the DMO and its role and interactions with other stakeholders in the coupled market and will also investigate quantitatively the DMO's different bidding strategies in the wholesale day-ahead market. For this, the DMO's dealing with uncertainties within the wholesale market prices is studied through two main approaches, the scenario-based approach to minimize the expected cost and the min-max regret as a robust risk-averse approach. The results from these two approaches are compared with the deterministic and perfect-knowledge approaches as limits of the worst and best performances of the DMO, respectively. In section 6.1, the DMO is introduced. In section 6.2, DMO's roles in the local market, its interaction with main market players are studied, and conflicting interests are discussed as well. In section 6.3, the DMO bidding strategy in the wholesale market is studied through the aforementioned four approaches: scenario-based, minmax regret, deterministic, and perfect-knowledge. Input data and case studies are described in section 6.4. Section 6.5 presents numerical results and discusses their implications. Finally, conclusions are derived in section 6.6.

<sup>&</sup>lt;sup>1</sup>This chapter is based on: Farrokhseresht, M., Slootweg, J.G., and Gibescu, M. (2020), Day-ahead bidding strategies of a distribution market operator in a coupled local and central market, *submitted*.

## 6.1 What is a DMO?

As discussed earlier in chapter 3, due to the increasing penetration of DERs, the complexity of planning and operating the distribution network has increased. The paradigm has changed, from a unidirectional to bi-directional power flow, depending on the local generation and load conditions. This change leads to the emergence of new functions and roles for distribution network operators, which are now becoming distribution system operators (DSOs). One of the new functions is that the DSO needs to provide local resilience capability<sup>2</sup> [178] and reduce dependence on the TSO for providing i.e. balancing services, so the distribution system can maintain service to its customers when the rest of the system is in an abnormal condition [179]. Consequently, the DSO can fulfil multiple roles including network operating and short and long-term planning to address bi-directional flows in the grid. Besides, there can be two other roles for the DSO: distribution market operator and distributed energy resources manager.

The DMO can be seen as part of the existing/emerging DSO concepts, rather than being a separate entity independent from the DSO. An independent DMO can manage a market which can consist of different areas and each area belongs to a separate DSO. However, if the DMO is part of the DSO, the DMO can only manage the local area which belongs to its DSO hence there are separate markets for each DSO's jurisdiction. Both schemes, an independent DMO or a united DMO and DSO, have their pros and cons. In one hand, a DMO being part of the DSO leads to easier exchange information between the DMO and the DSO in such a way that less administrative tasks are involved. On the other hand, a DMO being part of the DSO can raise the risk that the DMO operates the local market in a way which is in favour of the DSO. This is indeed in conflict with a DMO as a neutral market operator and therefore to avoid that, there is a need for more strict regulations.

Regardless of who manages the local market – a DMO who is part of the DSO or who is independent– the two vital responsibilities including managing the local market at the distribution level and operating the distribution system must be performed in such a way as to avoid possible disturbances in the distribution system. Therefore, in case of an independent DMO, it is essential to have constant communication between the DMO and the DSO, to maintain the security of the distribution system [180].

 $<sup>^{2}</sup>$ Resilience capability at the distribution system means the grid can work continuously despite adverse events happened at the up-stream system.

#### The DMO and similar entities in the literature

In this section, the literature in which the DMO or similar concepts have been discussed is reviewed. The DMO is not extensively used in literature, however, similar concepts to the DMO such as virtual power plants or micro-grid operators have been widely discussed. Therefore, the literature review section contains two parts. The first part is about literature in which there are similar entities to the DMO trying to facilitate the market participation of DERs, and the second part is about literature in which there are some entities (possibly using different names) with the same responsibilities as the DMO.

The virtual power plant is a distributed power plant that aggregates production data from DERs to scale up their generation and to trade the resulting volume on the electricity market. The concept of the virtual power plant and its participation in electricity markets is widely studied in the literature. Some variations on this concept are reviewed here. References [148] [147] [82] presents an algorithm to optimize the day-ahead scheduling of a large scale virtual power plant. References [146] and [145] studies the techno-economic impact of the massive integration of small generators and demand into virtual power plants both on the system functioning and on the outcome for demands and generators within the virtual power plants. Reference [79] defines a two-stage stochastic mixed-integer linear programming model that maximizes the virtual power plant's expected profit in both day-ahead and balancing markets. In [149] a virtual power plant as a service-centric aggregator is proposed, which enables the market integration of DERs in day-ahead markets and simultaneously supports cooperation with the distribution system operator (DSO) in addressing the issue of network utilization. Another emerging concept in power systems is micro-grids. The micro-grid is defined as a "localized group of electricity sources and loads that normally operates connected to and synchronous with the traditional wide area synchronous grid, but can also disconnect to island mode and function autonomously as physical or economic conditions dictate" [181]. The micro-grid, similar to the virtual power plant, can ease the participation of DERs in electricity markets as shown in research presented in [141][142][144][143].

However, there are important differences between the DMO used in the proposed coupled market model and the virtual power plants and micro-grids elsewhere in the literature. Firstly, the virtual power plant and micro-grid operators are profit-based while the DMO is a non-profit entity although it can participate in the TMO-operated central market. However, the money that the DMO gains in the central market is for later distributing among DERs and not for his own benefit. Therefore, the virtual power plant and micro-grid operator can merely be considered as market players while the DMO is not only a market player participating in the central market but also acts as a local market operator. Secondly, constant information exchange between the DMO and the DSO is essential to guarantee the security of the distribution system. However, respecting (external) distribution system constraints is not the concern of the virtual power plant or micro-grid operator. More discussion about the DMO's roles and responsibilities follows in section 6.2.

There are few concepts used in the literature that have similar definitions to the one for the DMO in the coupled market. One of them is the Distributed System Platform Provider (DSPP). The DSPP has been developed in a pilot program, by the UK National Grid, to create a transactive energy marketplace for owners of energy resources. DSPP can be created as a new entity or be part of the currently existing electric utility [182]. The role of the DSPPs is to create markets, tariffs, and operational systems to monetize DER's products and services that provide value for the system. Another example is called Smart Energy Service Provider (SESP) which can be found in the research work of EMPOWER project [183]. The SESP can also be seen as an aggregator participates in the wholesale market as well as a service provider for a trading floor for energy and energy-related services. Another similar entity is the independent distribution system operator (IDSO) proposed in [113]. The IDSO is responsible for distribution grid operation together with providing market mechanisms in the distribution system, enabling open access, and ensuring safe and reliable electricity services.

The term distribution market operator, has been applied in [180] which proposes an optimal scheduling model for a micro-grid participating in a local electricity market in interaction with a DMO. The DMO administers the established electricity market at the distribution level, sets electricity prices, determines the amount of the power exchange among market participants, and interacts with the independent system operator at the transmission level. The role of the DMO is similar to an aggregator which aggregates the bids and offers from DERs and participates on behalf of them in the wholesale market. In [180], however, the DMO can be seen merely as an aggregator. The security of the distribution network constraints is not included in the DMO's optimization profile. The DMO's interaction with the DSO and the wholesale market is not qualitatively studied nor quantitatively.

In short, in the aforementioned literature which include the DMO or similar entities as DMO, a detailed quantitative study of the DMO's (or a similar entity's) participation in the TMO central market has not been performed, and its roles and interactions with other market participants have not been discussed. Therefore, this chapter covers these topics by looking at the DMO's roles, responsibilities and interactions with the central market and other main stakeholders qualitatively as well as quantitatively.

## 6.2 Roles and interactions

In this section, the main role and responsibilities of the DMO in managing the local electricity market are discussed. Furthermore, the DMOs interaction with the main market participants is discussed. Based on these possible conflicts of interest are identified.

## 6.2.1 DMO's roles & responsibilities

The DMO is one of the most important actors in the local market. The DMO facilitates the local electricity market and allows for the participation of small resources in the central electricity market. The responsibilities and functions of the DMO are twofold: market-related and system-related. These two functions are explained in more detail below.

#### Market related

The DMO provides a market platform for DERs and prosumers to trade their energy and services. This platform enables receiving bids from DERs and prosumers as well as clearing and settlement on a day-ahead basis, and also for real-time balancing. Regarding this role, there are two major responsibilities for the DMO:

#### • Represent local resources in the TMO market

As explained in chapter 4, in the preliminary scheduling step, through an optimization algorithm, the DMO finds the mismatch between supply and demand (including from the upstream network). The DMO offers/bids this mismatch later to the TMO market. When participating in the TMO market, the DMO acts as a market player e.g. an aggregator.

#### • Optimal power flow in the local market

After the TMO market is cleared, the aggregate of the assigned energy and/or services to the DMO and prosumers are known. Then, the DMO clears the local market and assigns to each DER and prosumer their corresponding energy and services. In this function, the DMO acts as a market operator.

#### System related

As discussed in chapter 2, the DSO is responsible for preventing over/under voltages and overloads in the distribution system. The DMO clears the local

market and dispatches the local resources, hence it has all the information about the amounts of power injected or withdrawn to/from the distribution system at specific moments. As the DMO needs to enforce the distribution network constraints during the market clearing, the DMO and the DSO need to be in constant communication with each other. The DMO shares the information on the loading of the network with the DSO, while the DMO enforces the distribution network constraints in the market clearing algorithm, based on information from the DSO. In this way, it is helping the DSO to maintain the security of the distribution system. Therefore, besides operating and managing the local market, the DMO contributes to the distribution system operation as well.

#### 6.2.2 DMO's interactions

Since the role of the DMO as proposed for the coupled market is novel, it is important to discuss how the DMO would interact with other stakeholders. To facilitate this discussion, a schematic overview of the interactions between the DMO and the main market participants and entities in the coupled market including DERs, prosumers, consumers, the DSO, energy suppliers, and the TMO is shown in Figure 6.1. Two types of interactions between the DMO and other stakeholders are considered; exchange of information, and exchange of money. Each interaction of the DMO is discussed in more detail in the following subsections.

#### • Interaction with a DER or prosumer

In this interaction, what the DMO expects from DERs and prosumers to avoid using market power is they submit their true bidding price which is equal to their marginal cost. Nevertheless, DERs might perform market power - as discussed in chapter 3 - and therefore, it is important to investigate DERs behaviour in the local market. This topic is covered in more detail in chapter 7. Moreover, the DMO needs to rely on DERs and prosumers that they are being committed to what they promise in the market and they are capable to deliver the amount of energy and services as scheduled. The DER or prosumer, on the other hand, needs to be able to rely on the DMO to be represented in the central market. Therefore, these stakeholders need the DMO to gain the most value from the TMO market. Moreover, these resources need to have confidence in the clearing of the market by the DMO in an honest way. As the market participation of the DERs and prosumers goes via the DMO. If these actors do not believe the DMO operates independently, they would have less of an incentive to participate in the market.



Figure 6.1: DMO interactions

#### • Interaction with the energy supplier

As Figure 6.1 shows, the interaction between the DMO and the energy supplier is twofold, exchanging information, and money. The information exchange is related to the energy being contracted between the energy supplier and its clients at the distribution system. This information helps the DMO to know how much energy (average power over a market clearing interval) is withdrawn, hence how much power flows over the distribution system. Note that in the DMOs market algorithm, all consumers who participate in the local market either directly or via an energy supplier, are included hence the DMO can enforce the distribution network constraints. The money that the energy supplier pays to the DMO is the money which its consumers pay the energy supplier minus some fees which are related to its administrative tasks and the price stability it provides for households.

#### • Interaction with the DSO

The main interaction between the DMO and the DSO is an information exchange, as shown in Figure 6.1. This information is mainly related to the security of the distribution system. Since distribution network constraints are included in the DMO's market clearing algorithm, the DSO must be confident that the network constraints are enforced by the DMO. The DMO, on the other hand, must also be confident that the DSO gives him accurate information about the distribution system characteristics. If, for instance, the DSO gives information about the capacity of the system with a very high safety margin taken into account, it can significantly impact the market outcome. Moreover, the DMO perhaps wants the DSO to invest in the distribution system where those constraints are activated on a structural basis. The reason is that the DMO wants to prevent performing market power by DERs due to network constraints, as discussed earlier in chapter 3, and demonstrated with simulations, later in chapter 7.

#### • Interaction with the TMO

Although the interaction between the DMO and the TMO is explained in detail in section 4.1.3 of chapter 4, here a supplementary explanation are given. As Figure 6.1 shows, the interaction between the DMO and TMO-operated market is twofold: information exchange and money exchange. The TMO-DMO interaction can be seen as an ordinary interaction between an energy supplier/aggregator and a market operator in the current market models. The similarity is that the DMO, like any other market participant, expects the TMO to operate a transparent and competitive market which is fair and easy to access for all the market participants and there is no discrimination or market power. The difference is that the DMO is a market operator itself and participates in the TMO market based on the energy supplied by DERs and prosumers. What the TMO expects from the DMO is that same as any conventional market participant, the DMO be trustworthy and able to deliver the energy and services they committed in the market.

#### 6.2.3 Conflicting interests

The role of the DMO has been designed in such a way that the conflicts of interest are being kept to a minimum. The DSO, for instance, wants to limit the flow to avoid overloading the interface transformer. In some situations, this would decrease the profit of the DER. The DMO is therefore implemented as a separate entity from the DSO. As the DMO is a market operator it adjudicates between buyers and sellers. The buyers would like a price as low as possible and the sellers would like a price as high as possible. The algorithm of the DMO should therefore be transparent and not seem to favour one or the other. For this, the DMO must be very transparent in the algorithm it uses to facilitate the market. Therefore, it is important to study the participation of the DMO in the TMO market in more detail. This is discussed in the next section.

## 6.3 DMO's bidding strategies in the TMO market

In the previous section, the interactions between the DMO and other actors active in electricity markets are studied. It is shown that the DMO is not only a market operator in the local market but also can be a market player when it comes to its participation in the TMO market. Therefore, it is important to study different approaches for the DMO in creating the preliminary schedules to see its effect on the system's cost, energy bought/sold in the TMO market, and the performance of the (day-ahead) local market in general. Another motivation for studying the participation of the DMO in the TMO market is that – as shown in chapter 5 – the total system cost in the coupled market model is higher than that in the centralized market model. The main reason for that was briefly discussed as due to the imperfect knowledge of the DMO about the TMO market price. Therefore, this participation of the DMO in the TMO market is an important topic, which will be further elaborated in this section.

The focus of this section is only on the day-ahead market which is shown in Figure 6.2 and therefore, the balancing market is not taken into account. The results of the optimization in which the DMO chooses different approaches are independent of the market time-frame, hence the conclusion for the day-ahead market can be applied for the balancing market as well. As explained in chapter 4, in the preliminary scheduling (the first step in the day-ahead coupled market) the DMO solves an optimization problem in which the TMO market price is considered as an uncertain parameter. Through this optimization, the DMO bidding, in terms of price and quantity, is calculated. Finding an optimized solution depends on how the DMO deals with the TMO market price in its optimization algorithm in the preliminary scheduling. Moreover, how the uncertainty in the wholesale market price is handled can affect the local market price and the total system cost, as well as the energy bought/sold from the DMO in the TMO-operated market.

Four different approaches are considered for anticipating the wholesale market price by the DMO while participating in the TMO market: deterministic, scenario-based, min-max regret, and perfect-knowledge. The deterministic and perfect-knowledge approaches are considered as (theoretical) limits for the worst and best performance of the market, respectively. To cope with the uncertainty in the TMO market prices in preliminary scheduling, the scenario-based and minmax regret are applied, as two alternative strategies. Generally, to handle the uncertainty, there should be specified limits to up- or down-side risks. However, since the cost of over- or underbidding in the market is relatively small, only limited up-side and down-side risks exist. Therefore the robust approach is not necessary and applicable here. Instead, scenario-based approaches are considered. The first one is a stochastic approach in which the average of the scenarios is taken into account. The second one is a risk-measured approach, the min-max



Day D-1

Figure 6.2: Day-ahead market in the coupled market model

regret. In the following subsections, the four different approaches of the DMO to cope with the TMO market prices are explained.

## 6.3.1 Handling uncertainty by deterministic approach

Most decision-making problems can be formulated as optimization problems, and if the input data of the optimization problem are well-defined and deterministic, its optimal solution (optimal decision) is achieved by solving the problem. This is guaranteed indeed, only if the objective function and constraint space are convex. The decision is then implemented to attain the best outcome. The deterministic approach is the simplest (most naive) way for the decision-maker to deal with the uncertainty in input parameters. When the occurrence of an uncertain parameter depends on different probabilistic scenarios, the deterministic approach considers the impact of only a single scenario hence treats the probability of an event as finite and equal to one.

The deterministic approach typically models scenarios, where the input values are known and the outcome is observed. There is an overlap in deterministic and probabilistic modelling. For example, probabilistic modelling (i.e. running multiple scenarios at different probabilities of occurrence) can be used to generate a deterministic scenario; typical scenarios might include: worst-case, best-case, most "likely" scenario [184]. There are a number of issues with a deterministic approach, including the fact that it does not consider the full range of possible outcomes, and does not quantify the likelihood of each of these outcomes. Consequently, the deterministic approach may actually be underestimating the potential risk involved in decision-making [185].

As discussed in chapter 4, the day-ahead component of the coupled market consists of three steps: the preliminary scheduling, the TMO day-ahead market, and the local day-ahead market. These three steps, with the DMO choosing a deterministic approach in the preliminary scheduling, are explained as follows.

#### Step I. Day-ahead preliminary scheduling by the DMO

As explained in chapter 4, the DMO first aggregates all the bids and offers from DERs by solving a preliminary scheduling problem where the objective function is minimizing the total cost of energy and reserve capacity. The objective function is shown in (6.1).

$$\min \left[ \sum_{t \in T} \sum_{g \in G_D} (O_{g,t}^E P_{g,t} + O_{g,t}^{RUP} R_{g,t}^{UP} + O_{g,t}^{RDN} R_{g,t}^{DN}) + \sum_{t} \sum_{i \in N_{D-T}} \lambda_t^{DA} P_{t,i}^{TD} \right]$$
(6.1)

Equation (6.1) minimizes the total cost of generation and reserve capacity of DERs in the local market plus the expected cost of buying/selling energy and reserve from/to the TMO market.  $\lambda_t^{DA}$  is an estimated wholesale market price by the DMO and can be accounted for as an average of wholesale market prices in previous days for that same time-slot. The constraints for the objective function are similar to constraints in (4.2)-(4.18) in chapter 4.

The DMO bidding energy and reserve quantities are power injected at the interface node  $(\widetilde{P_{i,t}^{TD}})$ . The reserves of DERs  $(\widetilde{R_{g,t}^{UP/DN}})$ , energy price  $(\lambda_{i,t}^{E})$  and reserve price  $(\lambda_{i,t}^{RUP} \text{ and } \lambda_{i,t}^{RDN})$  are outputs of this step. Note that the tilde above the symbol is the sign for being output of the previous step. Based on  $\lambda_{i,t}^{E}, \lambda_{i,t}^{RUP}, \lambda_{i,t}^{RDN}$  and  $\widetilde{P_{i,t}^{TD}}$ , the DMO participates in the day-ahead market at TMO-operated central level in step II.

#### Step II. Central day-ahead market clearing

In this step, the DMO participates in the TMO day-ahead market based on energy and reserve prices and quantities calculated in step I. The objective of the TMO day-ahead market is shown in 6.2

$$\min \left[ \sum_{t \in T} \sum_{g \in G_T} (O_{g,t}^E P_{g,t} + O_{g,t}^{RUP} R_{g,t}^{UP} + O_{g,t}^{RDN} R_{g,t}^{DN}) + \sum_{t \in T} \sum_{i \in N_{T-D}} (\lambda_{i,t}^E P_{i,t}^{DMO} + \lambda_t^{UP} R_{i,t}^{DMO/UP} + \lambda_t^{DN} R_{i,t}^{DMO/DN}) \right]$$

$$(6.2)$$

The first line in (6.2) consists of the cost of energy and reserve capacity procured from the transmission-level generators. The second line accounts for the total costs of energy and reserve capacity procured by DERs. The objective function is subject to constraints which are similar as shown in (4.20)-(4.30) in chapter 4.

After clearing this market and assigning market prices  $(\lambda_{i,t}^{TE}, \lambda_{i,t}^{TR})$  and quantities to each market player, the DMO is informed about the allocated power flow over the interface transformer  $(P_{i,t}^{DMO})$  and the required reserve capacity from DERs  $(R_{i,t}^{DMO/UP} \text{ and } R_{i,t}^{DMO/DN})$ .

#### Step III. Local day-ahead market clearing

In this step, the DMO clears the local day-ahead market with the objective function as:

$$\min \left[ \sum_{t \in T} \sum_{g \in G_D} (O_{g,t}^E P_{g,t} + O_{g,t}^{RUP} R_{g,t}^{UP} + O_{g,t}^{RDN} R_{g,t}^{DN}) + \sum_{t} \sum_{i \in N_{D-T}} (\lambda_{i,t}^{TE} \widetilde{P_{i,t}^{DMO}} + \lambda_{i,t}^{TR} (\widetilde{R_{i,t}^{DMO/UP}} + \widetilde{R_{i,t}^{DMO/DN}})) \right]$$
(6.3)

The objective function in (6.3) is minimizing the total generation cost of DERs in the distribution system plus the revenue/cost received/paid from/to the TMO's day-ahead market. The constraints in this step are similar to (4.32)-(4.35) shown in chapter 4.

The outputs of this step are the reserve capacity  $(R_{g,t}^{UP/DN})$  and the scheduled energy of dispatchable generators  $(\widetilde{P_{g,t}})$ .

#### 6.3.2 Handling uncertainty by scenario-based approach

Generally speaking, the deterministic approach is a conservative approach that can lead to a high cost of operating the distribution system and also may not consider all the possible scenarios which can also lead to an insecure system operation [186]. Instead of a deterministic model, a stochastic model can be adopted in which different TMO market prices are simulated as different scenarios. Stochastic programming is an important approach to address parameter uncertainty in modelling. By analysing the historical data of the uncertain parameters, their probability distribution functions can be estimated, and accordingly, a set of scenarios can be generated [187].

A stochastic optimization problem can be formulated by implicitly weighting (with the probabilities of occurrence) the individual solutions associated with each set of input data to achieve a single solution that is the best for all sets of input data. Implementing the solution obtained by solving the stochastic problem places the decision-maker in the best possible situation when considering all possible input data sets correctly weighted by their respective probabilities. This solution is not the best for each individual set of input data but it is the best if all of them, weighted with their probabilities of occurrence, are simultaneously considered [11]. Below, the general formulation of the stochastic problem is explained.

As mentioned earlier, stochastic programming is used to formulate and solve problems with uncertain parameters. Within a stochastic programming context, each uncertain parameter is modelled as a random variable. In stochastic programming, random variables are usually represented by a finite set of realizations or scenarios. For instance, random variable X can be represented by X(s), s = 1, ..., N, where s is the scenario index, N is the number of scenarios considered and is the cardinality of the set of scenarios. We denote by X the set of possible realizations of random variable X, i.e., X = X(1), ..., X(N). Each realization X(s) is associated with a probability  $\pi_s$  defined as  $\pi(s) = P(s|X = X(s))$ , where  $\sum_x \pi(s) = 1$ .

A random variable whose value develops over time is known as a stochastic process. Thus, a stochastic process is made up of a set of dependent random variables sequentially arranged in time. For each period, the corresponding random variable (e.g., prices for the next 24 hours) depends on the other random variables (e.g., prices in previous hours or days). Stochastic processes in a given time horizon can be represented by scenarios. For instance, stochastic process SP can be represented by vectors SP(s), s = 1, ..., N, Note that SP contains the set of dependent random variables constituting the stochastic process, i.e., SP = SP(1), ..., SP(N). Each realization SP(s) is associated with a probability  $\pi(s)$ . For example, if SP represents the 24 hourly electricity prices of tomorrow, SP(s) is a  $24 \times 1$  vector representing one possible realization of these prices. The general modelling of a scenario-based optimization associated with a minimization problem is as follows:

$$\min \quad [f_1(y) + \sum_s \pi(s) f_2(x(s))]$$

$$subject \quad to: \quad g(y) \le 0$$

$$h(x(s)) \le 0, \forall s \in \{1, ..., N\}$$

$$x(s) \in X, y \in Y$$

$$(6.4)$$

The three steps in the day-ahead coupled market model when the DMO chooses the scenario-based approach are explained as follows.

#### Step I. Day-ahead preliminary scheduling by the DMO

The difference between objective functions of this step in the deterministic and the scenario-based approach is related to the wholesale market price symbolized by  $\lambda_t^{DA}$  in (6.1). Here, this price depends on scenarios and therefore is shown by  $\lambda_{t,s}^{TD}$ . From the point of accuracy, it is preferable to have as many scenarios as possible, however additional scenarios have diminishing returns, while the computational complexity increases. Therefore 10 scenarios for wholesale market prices are used, as this is a good trade-off between computational complexity and accuracy. The objective function of the preliminary scheduling step through scenario-based approach is:

$$\min \left[ \sum_{t \in T} \sum_{g \in G_D} (O_{g,t}^E P_{g,t} + O_{g,t}^{RUP} R_{g,t}^{UP} + O_{g,t}^{RDN} R_{g,t}^{DN}) + \sum_{t} \sum_{i \in N_{D-T}} \sum_{s} \pi_s \lambda_{t,s}^{TD} P_{t,i}^{TD} \right]$$
(6.5)

In this new formulation, the wholesale market price,  $\lambda_{t,s}^{TD}$ , is scenario-based and  $\pi_s$  is the probability of occurrence for each scenario. The constraints for the objective function in (6.5) are similar to the ones for (6.1).

The output is the DMO bidding prices and quantities for energy and reserve by which it participates in the TMO day-ahead market.

#### Step II. Central day-ahead market clearing

In this step, the central day-ahead market is cleared by the TMO. The objective function and constraints are the same as the objective function in (6.2) and its constraints. The outputs of this step for the DMO are the cleared energy and reserve prices and quantities from this market.

#### Step III. Local day-ahead market clearing

In this step, the local day-ahead market is cleared by the DMO and energy and reserve prices and quantities are assigned to corresponding DERs. The objective function and constraints are similar to the objective function in (6.3) and its constraints.

#### 6.3.3 Handling uncertainty by min-max regret approach

The min-max regret approach is a strategy that minimises the maximum regret of a decision, and is useful for a risk-averse decision-maker. Regret in this context is defined as the opportunity loss through having made the wrong decision. The min-max regret criteria are often used to obtain solutions hedging against parameter uncertainty and are aimed at constructing a solution that minimizes the worst-case regret over all possible scenarios while ensuring system robustness.

That means, by using the min-max regret approach for the same given uncertainty set, a risk-averse market participant will get the minimum regret as compared to the stochastic optimization approaches, under the worst-case scenario [188]. Relatively speaking, the min-max regret approach can provide a less conservative objective value than a worst-case robust optimization approach and a more robust result than the stochastic optimization approach for the same given uncertainty set [189]. The general modelling of the min-max regret optimization associated with a minimization problem is explained as follows.

The min-max regret associated to a minimization problem OP has as input a finite set S of scenarios where each scenario  $s \in S$  is represented by a vector SP = SP(1), ..., SP(N).

Assume that f(x, s) is the value of solution  $x \in X$  under scenario  $s \in S$ , and  $x_s^*$  is an optimal solution under scenario s, and  $f_s^* = f(x_s^*, s)$  is the corresponding optimal value. In other words,  $f_s^*$  is the optimum value under perfect information, i.e., the optimum obtained if the decision-maker had known the information before making any self-scheduling decisions.

Considering a solution  $x \in X$ , its regret R(x, s), under scenario  $s \in S$  is defined as the  $f(x, s) - f_s^*$ . The maximum regret  $R_{max}(x)$  of solution x is then defined as  $R_{max}(x) = max_{s \in S}R(x, s)$ . Consequently, the min-max regret corresponding to minimization problem OP is:

$$minR_{max}(x = min_{x \in X} max_{s \in S}[f(x, s) - f_s^*])$$

$$(6.6)$$

Equation (6.6) basically shows that under the min-max regret criterion, the objective function minimizes the largest regret value over all possible scenarios. The

three steps in the day-ahead coupled market model with the DMO choosing the min-max regret approach are explained as follows.

#### Step I. Day-ahead preliminary scheduling by the DMO

In this step, for the preliminary scheduling, the DMO deals with the uncertainty in the wholesale market prices by the min-max regret strategy. According to the general min-max regret model explained above, OP the minimization problem becomes:

$$OP: min \quad [\sum_{t \in T} \sum_{g \in G_D} (O_{g,t}^E P_{g,t} + O_{g,t}^{RUP} R_{g,t}^{UP} + O_{g,t}^{RDN} R_{g,t}^{DN}) \\ + \sum_{t} \sum_{i \in N_{D-T}} \sum_{s} \pi_s \lambda_{t,s}^{TD} P_{t,i}^{TD}] \qquad (6.7)$$

$$subject \quad to: \quad (4.2) - (4.18)$$

where f(x,s) is equal to the objective function in (6.7), with  $x = \{P_{g,t}, R_{g,t}^{UP}, R_{g,t}^{DN}, P_{i,t}^{TD}\}$  and  $s = \{s_1, s_2, ..., s_{10}\}$ . Consequently,  $f(s_n)^*$  equals to  $f(x^*, s_n)$  is an optimal value of f(x, s) when  $s = s_{n \in \{1...10\}}$  and  $x = \{P_{g,t}^*, R_{g,t}^{UP}^*, R_{g,t}^{DN}^*, P_{i,t}^{TD}^*\}$ . If the value for the regret function, R(x, s), equals to  $f(x, s) - f(s_n)^*$ , and the maximum of regret function is shown by  $R_{max}(x)$ , the min-max regret,  $minR_{max}$ , corresponding to the optimization problem OP consists of the following equations:

$$\min_{x \in X} \max_{s \in S} [f(x, s) - f(s_n)^*]$$
subject to: (4.2) - (4.18) (6.8)

(6.9)

which equals to:

$$\begin{split} \min R_{max} &= \min_{x \in X} \max_{s \in S} [ [\sum_{t \in T} \sum_{g \in G_D} (O_{g,t}^E P_{g,t} + O_{g,t}^{RUP} R_{g,t}^{UP} + O_{g,t}^{RDN} R_{g,t}^{DN}) \\ &+ \sum_t \sum_{i \in N_{D-T}} \lambda_{t,s}^{TD} P_{t,i}^{TD}] - [\sum_{t \in T} \sum_{g \in G_D} (O_{g,t}^E P_{g,t}^* \\ &+ O_{g,t}^{RUP} R_{g,t}^{UP*} + O_{g,t}^{RDN} R_{g,t}^{DN*}) \\ &+ \sum_t \sum_{i \in N_{D-T}} \lambda_{t,s_n}^{TD} P_{t,i}^{TD*}] ] \\ subject \quad to: \quad (4.2) - (4.18) \end{split}$$

The optimization problem in (6.9) is not easy to solve as it can converge to nonlocal-minimax points. For this, a new variable,  $\delta$ , is introduced to change the  $minR_{max}$  optimization problem into the following set of equations:

$$\begin{split} \min R_{max} &= \min \ \delta \\ subject \ to: \ \delta &\geq \left[ \sum_{t \in T} \sum_{g \in G_D} (O_{g,t}^E P_{g,t} + O_{g,t}^{RUP} R_{g,t}^{UP} + O_{g,t}^{RDN} R_{g,t}^{DN}) \right. \\ &+ \sum_t \sum_{i \in N_{D-T}} \lambda_{t,s}^{TD} P_{t,i}^{TD} \right] - \left[ \sum_{t \in T} \sum_{g \in G_D} (O_{g,t}^E P_{g,t}^* \\ &+ O_{g,t}^{RUP} R_{g,t}^{UP*} + O_{g,t}^{RDN} R_{g,t}^{DN*}) \\ &+ \sum_t \sum_{i \in N_{D-T}} \lambda_{t,s_n}^{TD} P_{t,i}^{TD*} \right] \end{split}$$
(6.10)

and (4.2) - (4.18)

The results of the optimization problem in (6.10) is the DMO bidding in the TMO day-ahead market, in terms of prices and quantities.

#### Step II. Central day-ahead market clearing

This step, clearing the TMO day-ahead market, is the same as the one explained for deterministic and scenario-based approaches. The objective function and constraints are same as the ones in (6.2) and (4.20)-(4.30) in chapter 4. The output of this step is cleared day-ahead market prices and quantities assigned by the TMO to all market players, including the DMO.

#### Step III. Local day-ahead market clearing

The local day-ahead market is cleared by the DMO in this step in an optimization expressed by (6.3) and constraints (4.32)-(4.35) in chapter 4.

#### 6.3.4 Handling uncertainty by perfect-knowledge

In this approach, it is assumed that the DMO has perfect-knowledge about the wholesale market prices, hence it solves a self-scheduling problem to determine its most beneficial actions for given prices. Therefore, this approach can be seen as the (ideal) benchmark for comparing the results of other approaches.

As the DMO has the perfect knowledge about the wholesale market price, only one step needs to be done which is equivalent to the local day-ahead market clearing in step III. The only difference with step III in previous approaches is that here, in the DMO's optimization, the  $P_{t,i}^{TD}$  is no longer a parameter and is considered as one of the decision variables. The objective function of this optimization is:

$$\min \quad [\sum_{t \in T} \sum_{g \in G_D} (O_{g,t}^E P_{g,t} + O_{g,t}^{RUP} R_{g,t}^{UP} + O_{g,t}^{RDN} R_{g,t}^{DN}) \\ + \sum_{t} \sum_{i \in N_{D-T}} \lambda_t^{DA} P_{t,i}^{TD}]$$
(6.11)

The objective function in (6.11) is the same as the one in (6.5). The only difference is in  $\lambda_t^{DA}$  representing the actual wholesale market price. The corresponding constraints for (6.11) are similar to the ones for (6.5) which are explained in detail in chapter 4.

## 6.4 Input data and case studies

The case study consists of the same transmission and distribution grids as used in chapter 5 and shown in Appendix C. The data for the loads and DERs in the distribution system and the generators connected to the transmission system are also the same as the ones in chapter 5.

The wholesale market price  $(\lambda_{t,s})$  is dependent on scenarios which are generated based on historical data from [52] using an Artificial Neural Network approach and are shown in Appendix C.

To generate a distribution of the results, more than 10 scenarios need to be evaluated. The probability density function maintains its shape starting from around a hundred scenarios. So in the simulations 10 sets with 10 scenarios for the wholesale market price are used:  $\{\lambda_{t,s_1}, ..., \lambda_{t,s_{10}}\}_1, ..., \{\lambda_{t,s_1}, ..., \lambda_{t,s_{10}}\}_{10}$ . The demand in the transmission system is also dependent on scenarios and is obtained as follows.

- 1. First the bidding ladder of the generators in the transmission system is created, as shown in Figure 6.3. Note that the demand is assumed to be inelastic.
- 2. The corresponding demands for each wholesale market price scenario can be obtained as shown in Figure 6.3.
- 3. Finally, the scenarios for the demands are obtained as 10 sets with 10 scenarios each:  $\{D_{s_1}, ..., D_{s_{10}}\}_{1}, ..., \{D_{s_1}, ..., D_{s_{10}}\}_{10}$ .



Figure 6.3: The bidding ladder

How to proceed with simulations in min-max regret and scenario-based approaches when having different sets of scenarios for the demands and market prices in the TMO market, is explained as follows:

- Step 0: Generating 10 sets of scenarios for the demands and market prices in the TMO market, so that each set includes  $(\{D_{s1}, ..., D_{s10}\})$  and  $(\{\lambda_{t,s_1}, ..., \lambda_{t,s_{10}}\})$ , respectively.
- Step 1: Preliminary scheduling (step I in Figure 6.2): for each set of TMO market price  $\{\lambda_{t,s_1},...,\lambda_{t,s_{10}}\}$ , the DMO's preliminary scheduling is solved to calculate the DMO bidding in terms of price  $(\lambda_t)$  and quantity  $(P_t^{TD})$  for each time point. Therefore, there are 10 sets of DMO bidding prices and quantities:  $\{(\lambda_t, P_t^{TD})_{1,...,}, (\lambda_t, P_t^{TD})_{10}\}$ .
- Step 2: TMO day-ahead market (step II in Figure 6.2): for each set of scenario of demands and its corresponding DMO bidding price and quantity, the TMO market is solved to calculate the actual TMO market price and scheduled DMO energy:  $\{(\lambda_t, P_t^{TD})_1, ..., (\lambda_t, P_t^{TD})_{10}\}_1, ..., \{(\lambda_t, P_t^{TD})_{10}\}_{10}$ .
- Step 3: Local day-ahead market (step III in Figure 6.2): in this step, the set-points for each of DERs are disaggregated based on the overall results from step 2. For each set of TMO market price and DMO scheduled energy (i.e. power flow over the interface transformer), the local market is solved to obtain the total cost of the DMO and local day-ahead market results:  $\{(\lambda_t, P_t^{DER}, obj)_1, ..., (\lambda_t, P_t^{DER}, obj)_{10}\}_1, ..., \{(\lambda_t, P_t^{DER}, obj)_{10}\}_1, ..., \{(\lambda_t, P_t^{DER}, obj)_{10}\}_{10}\}$ . Note that  $obj_n$  depicts the value of the objective function calculated in (6.3) in Step III.



Figure 6.4: Average cost of the DMO in four different strategies

## 6.5 Results and discussions

In this section, the results of case study simulations when the DMO chooses different bidding approaches are discussed. The bar chart in Figure 6.4 shows the average total cost of the DMO ( $k \in$ ) for four different approaches. The green column belonging to the case perfect-knowledge has the lowest cost, while the blue column with the highest cost belongs to the deterministic approach. As the figure shows, the difference between the scenario-based case, shown in the dark red bar, and the min-max regret case shown in the orange bar, is not significant. However, the system cost in the regret case is slightly higher than in the scenario-based case.

To have a better comparison of the cost when the DMO chooses different approaches, a probability density function (PDF) of the system cost for regret, scenario-based and deterministic approaches with respect to the system cost in the perfect-knowledge case is shown in Figure 6.5. The PDFs are results of running simulations for the 10 sets, each set containing 10 scenarios. As Figure 6.5 shows, the deterministic approach has a ratio of 1.6-1.8 higher than the cost of perfect-knowledge. The PDFs belonging to the scenario-based and min-max regret cases are shifted to the left compared to the deterministic case, showing that they have a lower cost ratio with respect to the perfect-knowledge ideal situation. There are points in the scenario-based approach in which the costs are equal to that of perfect-knowledge (showing that some scenarios were lucky to capture the true realization). As already noted, the regret and scenario-based cases are cost-comparable for the most part, however, there are points in which the scenario-based case has a slightly lower cost than the regret case. However, the PDF of the cost in the regret case has the lowest variance compared with



Figure 6.5: Probability density functions for the system cost

the one in the scenario-based and deterministic cases. This corresponds with the risk-averse preference for the min-max regret decision-maker.

Now, the question is where do the higher system costs in the deterministic, regret and scenario-based approaches come from. To answer this question, the average DMO bidding prices in the wholesale market in regret, scenario-based, and deterministic approaches are shown in Figure 6.6 for a period of 24 hours in the day-ahead market. The green curve belongs to the perfect-knowledge situation, which is equivalent to the true wholesale market price. As the figure shows, the bidding prices of the DMO in the other three cases have different values from the one in the perfect-knowledge case. These differences increase the system cost in



Figure 6.6: DMO bidding price in the TMO market

Table 6.1: system costs in buying and selling modes			
	DMO sells to TMO	DMO buys from TMO	
Overestimate	Case 1: DMO stays out of	Case 3: DMO buys less	
the TMO mar-	the market	than the optimum hence	
ket price		activates more expensive	
		DERs	
Underestimate	Case 2: DMO stays in the	Case 4: DMO buys	
the TMO mar-	market but could have of-	more energy from whole-	
ket price	fered higher amount of en-	sale market than the op-	
	ergy	timum	

m 1 1

all three cases with respect to the perfect-knowledge case. The reason is summarized in Table 6.1. There are two modes of operation for the DMO: the DMO selling or buying energy to/from the TMO market. The selling mode means that there is excess energy from DERs after fulfilling the local demand, hence the DMO can sell this excess energy in the TMO market. In contrast, the buying mode means that there is deficit energy in fulfilling the demand by DERs and this must be compensated by importing energy from the TMO market. For each selling and buying mode, there are two situations: over and underestimating the TMO market price by the DMO. Consequently, four cases for the DMO bidding in the TMO market can be formed, as shown in Table 6.1. These four cases are explained below.

Starting with the selling mode, case 1 is related to a situation in which the DMO wants to sell energy to the TMO market while overestimating the TMO market price. In this case, the DMO stays out of the market as its bidding price is higher than the wholesale market price. Therefore, the DMO cannot sell the excess energy to the TMO market and it loses the revenue which otherwise could have been gained. Figure 6.7 illustrates the DMO's over and underestimating the TMO market price in selling mode. The thick red line belongs to the DMO bidding price. The dashed line and the brown line belong to the actual TMO



Figure 6.7: Over/underestimating the wholesale market price in the selling mode


Figure 6.8: Over/underestimating the wholesale market price in the buying mode

market price situations. The dashed line shows an overestimation that happens in case 1 and shows that the bidding price is higher than the market price which means the DMO will drop out of the market. In case 2, the DMO still wants to sell energy to the TMO market, however, it underestimates the TMO market price. In this case, the DMO stays in the market, however, as the brown line in Figure 6.7 shows, it could have offered higher amounts of energy to the TMO market. As a result, the DMO loses some revenue which otherwise could have been gained in the TMO market.

Figure 6.8 shows the over/underestimating of the TMO market price in the DMO buying mode. Figure 6.8.(a) shows the merit order based on DER's bidding and estimated TMO market price by the DMO (i.e DMO bidding price in the TMO



Figure 6.9: The total revenue and energy from DERs for various DMO bidding approaches

market). This figure assumes a large amount of energy from the TMO market. Figure 6.8.(b) and Figure 6.8.(c) depict the actual TMO market price. Therefore, Figure 6.8.(b) belongs to case 3 in Table 6.1 in which the DMO buys energy from the TMO market while it overestimates the TMO market price. In this case, as the bidding price of the DMO is higher than the actual TMO market price, the DMO buys less energy from the TMO market than otherwise, it could optimally buy from the TMO. Therefore, more expensive DERs must be activated to fulfil the demands in the local market. As a result, the system cost will increase. Figure 6.8.(c) belongs to case 4 in Table 6.1. In case 4, the DMO still wants to buy energy from the TMO market while it underestimates the TMO market price. In this case, as the DMO bids with a lower price than the actual TMO market price, it buys more energy from the TMO market than it could otherwise buy. In this case, the DMO imports more expensive energy from the TMO market, instead of activating cheaper DERs to fulfil the demand in the local market.

Using the information in Table 6.1 and Figures 6.7 and 6.8, one can compare the revenues and amounts of energy from DERs in different DMO bidding approaches. The bar-chart in Figure 6.9 shows the revenue and total energy from DERs in different approaches. The revenue of DERs is calculated based on average over scenarios and sum up over time and is shown by the hatched columns. The energy and revenue in the perfect-knowledge situation are obviously the highest, while in the deterministic, they are the lowest. In the scenario-based approach, energy and revenue are slightly higher than the ones in the min-max regret-based approach.

To explain these results, the PDFs of the difference between the DMO bidding price and the actual TMO market price are shown in Figures 6.10 and 6.11. According to the explanations in Table 6.1, in cases 1, 2, and 4, DERs are activated less than in the perfect-knowledge situation. Only in case 4, DERs are producing



Figure 6.10: Over/underestimating the market price in the DMO selling mode



Figure 6.11: Over/underestimating the market price in the DMO buying mode

energy higher than the optimal. Figure 6.10 shows the PDFs of DMO's overestimating and underestimating the TMO market price in its selling mode. The positive values in the horizontal axis indicate that the bidding market price minus the TMO market price is positive which means that the DMO overestimates the TMO market price. In contrast, negative values in the horizontal axis belong to DMO's underestimating the TMO market price. As this figure shows, the PDF in the scenario-based approach is placed toward the negative side of the x-axis, showing that underestimation in the selling mode is more likely to happen. In contrast, the deterministic approach has a PDF more toward the positive side, which means that overestimation has a higher chance to occur in selling mode. According to previous explanations in Table 6.1, overestimation in the selling mode leads to a case in which the DMO stays out of the market and cannot sell energy to the TMO market which means DERs activation is at the lowest. By contrast, Figure 6.11 shows the PDFs of DMO's overestimating and underestimating the TMO market price in its buying mode. As the figure shows, the PDF in the scenario-based approach is more shifted toward the positive values in the x-axis, thus showing that overestimation is more likely to happen in the buying mode. In the deterministic approach, however, the PDF is more shifted to the negative values, which means that underestimation in the buying mode is more likely to occur for this approach. According to Table 6.1, in the buying mode, overestimation leads to case 3 in which DERs are active with the highest rate. However, underestimation in the buying mode leads to case 4 in which the DMO imports more energy from the TMO market rather than activating DERs, therefore leading to lower revenue solutions. Finally, it should be mention that that in both Figures 6.10 and 6.11 the min-max regret approach is seen as having the tightest variance. So even though the revenue is slightly lower than when using a scenario-based approach, the min-max regret approach remains applicable as a less risk-averse strategy.

# 6.6 Conclusions

This chapter focused on studying the coupled market from the DMO's perspective. The roles and responsibilities of the DMO in the local market and its interaction with the main market participants and other entities – distributed energy resources, energy suppliers, the DSO, and the TMO – are explained. The interaction with the TMO market and the DMO bidding strategy in this market is the main focus of the rest of the chapter. The different DMO's bidding approaches in the TMO market and their impact on the performance of the local market are studied. In the preliminary scheduling of the coupled market model, the DMO must estimate the TMO market prices, as this step happens before the TMO market clearing. The DMO must deal with the uncertainty in the central market prices in such a way that it can minimize the total cost in its area. To handle this uncertainty, either a scenario- or regret-based approach can be considered. The perfect-knowledge and deterministic approaches are considered as references for the best and worst performance of the DMO in the local market, respectively.

Results show that the scenario-based approach is preferable for DERs as they can be activated in the local market at a higher rate than in the regret-based approach. However, the regret-based approach is most beneficial from the viewpoint of the social welfare, as the total system cost for different price scenarios has the lowest variance. Besides, the DSO can also have a more accurate estimation of how much power flows over the interface transformer between the distribution and transmission systems. Moreover, the DSO would prefer an approach in which more energy is produced from DERs rather than importing from the TMO market, to have fewer grid losses. Therefore, from this perspective, the scenariobased approach is more preferable for the DSO. Besides, DERs themselves prefer an approach in which they can earn higher revenue, and results show that in the scenario-based approach DERs have the highest revenues, however with a larger variance. In the end, which approach the DMO will choose is a trade-off between being risk-averse and having a higher amount of energy activated from DERs.

# 7

# The coupled market: DER's perspective

What the bidding strategy of DERs would be to maximise their revenue within the local market is still an open question. This chapter<sup>1</sup> tries to answer this question by studying DER's revenue maximization problem in the proposed coupled market. Since in the coupled market design, DERs participate in the local markets which has a relatively small size compared with the wholesale market, there can be a chance for exercising market power by DERs. This chapter focuses on investigating the revenue maximization problem for DERs with market power in the coupled market. Their revenue in the coupled market is compared with the one in the centralized market model in which DERs do not have market power. This chapter is organized as follows. Section 7.1 gives a short introduction to the revenue maximisation problem of DERs in electricity markets. Section 7.2 describes in detail the revenue maximization problem for DERs in the new coupled market. The corresponding mathematical formulations are presented subsequently in section 7.3. The non-strategic revenue maximization problem of DERs in the centralized market model is explained in section 7.4. Section 7.5 explains the input data and case studies. Section 7.6 discusses the numerical results of simulations. Finally, this chapter ends with conclusions derived in section 7.7.

<sup>&</sup>lt;sup>1</sup>This chapter is based on: Farrokhseresht, M., Slootweg, J.G., and Gibescu, M. (2020), Strategic bidding of distributed energy resources in the coupled local and central markets, *Sustainable energy, grid and network*, 24, 100390.

# 7.1 DERs behaviour in electricity markets

As discussed in chapter 2, the bidding behaviour of market players in electricity markets can be classified into strategic bidding and non-strategic bidding. Non-strategic bidding means market players can only solve a self-scheduling problem to determine their most beneficial actions for a given (external) price signal. In other words, being non-strategic means market participants are *price-takers*. In contrast, strategic bidding is behaviour by which a market player can affect market prices and as a result, increase its revenue. In other words, being strategic means market participants are *price-makers*. It is also mentioned in chapter 2 that exercising market power can be done in two ways; economic withholding and capacity withholding. In capacity withholding, a strategic market participant can influence the market price by withdrawing its cheaper units. In the economic withholding, a strategic market player can maximize its revenue mainly by taking advantage of system constraints.

In chapter 3, literature in which DER's market behaviour in central markets as being price-taker and price-maker are reviewed. Regardless of lack of research works in literature on performing market power by DERs in local markets, it is briefly discussed that DERs participating in local electricity markets can also have market power despite their small size, as the market size is still smaller and location matters for economic withholding. Therefore, local electricity markets can create opportunities for DERs to exercise market power. Moreover, it is important to understand strategic bidding behaviour by DERs in local markets, as market power leads to unwanted consequences for social welfare and unfair income distribution [190].

In the coupled market model, DERs participate in a relatively small local market and this can increase the chance of exercising market power by DERs. Moreover, in the coupled market, the local network constraints are taken into account during market-clearing, which can also increase the chance of market power through economic withholding. Therefore, In this chapter, a bi-level optimization approach is applied to the local electricity market to study the possibility of market power for a DER represented by a combination of wind farm and electrical storage system (WF-ESS).

The reason for implementing a storage system is due to possible benefits brought by the storage systems to the renewable energy resources among others: improving the dispatchability of renewable energy resources, shifting energy production closer to consumption (time arbitrage), forecast error correction to reduce disruptions due to unpredictable weather changes, and ramping to maintain constant power output during fast production fluctuations. Furthermore, in some cases, grid services (e.g. congestion management) can be stacked on top of the previous applications [59], but this is out of the scope of this thesis. Nevertheless, this should not be forgotten that storage systems have also significance drawbacks. First, they are expensive, and even though the cost has been decreasing drastically during recent years, it is still quite high, making many of the possible use-cases in large-scale energy applications non-viable. Second, it can be quite complex to build a safe storage systems, for example for battery cells that contain lithium - additional measures are required to protect from thermal runaway incidents. More thorough discussion for cost and benefits of storage systems can be found in [59].

In recent years, there has been a growing interest in bi-level approaches to model many operational and planning problems in power systems. In [191], the problem of optimal offering strategies by electricity producers in day-ahead energy auctions with step-wise energy offers is formulated as a bi-level optimization problem. Reference [192] presents a mathematical formulation to solve a Stackelberg game for a network-constrained energy market using integer programming. More information about the bi-level optimization and its application in power systems can be found in [193].

In short, this chapter formulates a revenue maximization problem for a wind farm with a storage system in a local day-ahead and balancing market set-up. To show that a DER such as a wind farm with a storage system can exercise its market power in a local market to generate higher revenue, a comparison between the WF-ESS's revenue in the coupled versus the centralized market is performed. Finally, the effect of the distribution system parameters on the ability to exercise market power by DERs in the local market is studied.

# 7.2 Revenue maximization of the WF-ESS in the coupled market

In this section, the revenue maximization problem for DERs in local day-ahead and balancing markets in the coupled market model is described. The grey flowchart in Figure 7.1 shows the timing and the steps for the day-ahead and balancing market in the coupled market model which was introduced earlier in chapter 4. The red and orange bars in Figure 7.1 – underneath the grey flowchart– show the steps taken by DERs to bid into the day-ahead and balancing markets, respectively. These steps are explained in the next two sections.

## 7.2.1 Day-ahead market

As it has been mentioned in previous chapters, the day-ahead market is a joint energy and reserve capacity market in which the wind farm with a storage system



Figure 7.1: DER's revenue maximization in the coupled market

(WF-ESS) actively bids.

In the day-ahead market in the coupled market model, the WF-ESS can behave strategically. The reason is that in the coupled market, DERs participate in the local market which is relatively small in size. Therefore, the chance for DERs to act strategically will increase, depending on their size and location. Moreover, in the coupled market model, the distribution system constraints are taken into account in the market-clearing. The DERs knowing that they might be activated due to power flow limits are provoked to exercise market power. In the coupled market due to the network constraints in the market clearing, the market power can be exercised through economic withholding. Capacity withholding is an unsafe strategy for market players. This is because, in the coupled market, the local market is not independent, it exchanges power with the wholesale market. Therefore, there is always a chance that cheaper energy from outside the local market becomes available, resulting in an import situation. In this situation, if DERs apply the capacity withholding strategy, their chance of being dropped out of the market will rise. Note that the other generators in the distribution system except the strategic WF-ESS are price-takers.

The DER's revenue maximization in the day-ahead market contains three steps, as shown by the red line underneath the grey flow chart in Figure 7.1. In step 1, the WF-ESS generates its strategic bids in terms of price and quantities in the day-ahead market. This strategic bidding of the WF-ESS in the day-ahead market is modelled through a bi-level optimization shown in Figure 7.2. The bi-level optimization contains two levels in which the upper-level problem is from the WF-ESS's perspective and the lower-level problem is from the DMO's market



Figure 7.2: Step 1: Bi-level optimization showing the link between day-ahead strategic bidding by the WF-ESS and the DMO market clearing problem

clearing's perspective. Through this bi-level optimization, the WF-ESS makes its offering decisions in the upper-level while anticipating the market behaviour of other market players which is modelled in the lower-level within the day-ahead market clearing problem of the DMO.

In step 2, the day-ahead market is cleared by the DMO. The reason for having this step – even though the day-ahead market clearing by the DMO is taken into account in the lower-level problem in step 1– is that there is uncertainty in how much power flow over the interface transformer between distribution and transmission systems will be. Therefore, the lower-level problem in step 1 cannot reflect the real market, and therefore, step 2 is needed. Step 2, the (deterministic) day-ahead market-clearing, contains all three steps (I-III) in the day-ahead market shown in the grey flowchart in Figure 7.1 which are explained earlier in chapter 4. The output of step 2 is the cleared local market prices and the scheduled energy and reserve capacities for DERs including the WF-ESS.

Finally, step 3 is the remuneration phase in which the WF-ESS calculates its day-ahead revenue based on the cleared market prices and its scheduled energy and reserve capacity obtained in step 2. The corresponding mathematical formulations are presented in section 7.3.1.

# 7.2.2 Balancing market

Unlike the day-ahead market, in the balancing market, it is more difficult to exercise market power by DERs. The reason is that the amount of MW energy traded in the balancing market is significantly smaller than in the day-ahead market. For this, the revenue maximization problem for the WF-ESS in the

Table 7.1: Dual-pricing mechanism		
	Short system	Long system
Positive imbalance	Profit $(\lambda_t^{DA})$	Cost $(\lambda_t^{BL})$
Negative imbalance	Cost $(\lambda_t^{BL})$	Profit $(\lambda_t^{DA})$

balancing market is the same as when the WF-ESS behaves non-strategically, i.e. as a price-taker. Moreover, the balancing market clearing is modelled through a shrinking rolling horizon due to having a storage system, since the energy or the state of charge of the storage system at time t depends on its energy at the previous time t-1. The starting point of the time horizon is reset each time the day-ahead market is cleared. This period includes 96 intervals in 24 hours. At the start of each interval, a new forecast becomes available. At each step inside the rolling horizon, the horizon is shrunk by one time-step and the balancing market is solved over the remaining horizon. Each of these shrinking horizon problems gives the bid for the current time. The last assumption is regarding the pricing mechanism in imbalance settlements. Dual pricing is applied. Dualpricing is more complicated than single-pricing, therefore, to have a more generic formulation, the dual-pricing mechanism has been chosen. However, the approach can easily be adapted for the single-pricing mechanism.

Table 7.1 explains how the dual pricing mechanism works. The short and long systems have been defined in chapter 2. The positive imbalance occurs when actual production of a generator is higher than its scheduled energy in the dayahead market while the negative imbalance occurs when actual production of a generator is lower than what he promises in the day-ahead market. In a situation in which the system's imbalance is short and the generator has a positive imbalance, or when the system's imbalance is long and the generator has a negative imbalance, the generator's imbalance status is in an opposite direction of the system's. Therefore, the generator is paid for improving the system's imbalance a price equal to the day-ahead market price  $(\lambda_t^{DA})$ . In contrast, if the generator's imbalance is in line with the system's, the generator has to pay a price equal to the balancing market prices  $(\lambda_t^{BL})$ .

The storage system of the WF-ESS can actively bid into the balancing market. However, the wind farm alone cannot participate in the balancing market. Instead, it can only pay or be paid in the imbalance settlement depending on its imbalance direction with respect to the total system imbalance. The DER's revenue maximization in the balancing market contains three steps shown in the orange line in Figure 7.1. Step 1 is generating non-strategic bids in terms of price and quantities. Therefore, the WF-ESS solves a self-scheduling problem to determine its most beneficial actions (bids) for given prices. Step 2 is the balancing market-clearing which contains all three steps (IV-VI) in the balancing market. The results of this step are local balancing market prices and quantities and imbalance volumes. Finally, step 3 is the remuneration in the imbalance settlement phase based on the dual-pricing mechanism. If the WF-ESS's imbalance is in the opposite direction of the system imbalance, he receives a price equal to the day-ahead market price. But if its imbalance is in the same direction with the total system imbalance, it has to pay a price based on the marginal cost of the last balancing unit deployed, which is usually higher than the day-ahead market price. The corresponding mathematical formulations are presented in section 7.3.2.

# 7.3 Mathematical formulations for WF-ESS's revenue maximization in the coupled market

In this section, the mathematical formulations for the revenue maximization problem of the WF-ESS are presented. Section 7.3.1 shows the mathematical formulation regarding the strategic optimization of the WF-ESS behaviour in the day-ahead market and section 7.3.2 describes the mathematical formulation for the non-strategic optimization of the WF-ESS behaviour in the balancing market.

## 7.3.1 Revenue maximization in the day-ahead market

As it has been explained, the revenue maximization problem of the WF-ESS in the day-ahead market consists of three steps. The mathematical formulation for steps 1-3 is described below.

#### • Step 1. Generate day-ahead strategic bids

In step 1, through a bi-level optimization shown in Figure 7.2, the WF-ESS tries to generate its strategic bidding in terms of price and quantities. The upper and lower levels of the bi-level optimization are formulated as follow:

- Upper-level problem:

The upper-level is from the WF-ESS's perspective which is maximizing the revenue of the WF-ESS and its objective function is shown in (7.1):

$$\min \sum_{t} \left[\sum_{c} \lambda_{c\in i,t}^{DA} P_{c,t}^{DA} + \sum_{e} \lambda_{t}^{UP} R_{e,t}^{dis} + \lambda_{t}^{DN} R_{e,t}^{ch}\right]$$
(7.1)

The objective function in (7.1) consists of several parts. The first part is the day-ahead energy bidding revenue of the WF-ESS. The expected day-ahead market price,  $\lambda_{cei,t}^{DA}$ , is the Lagrangian multiplier of the power balance equation in the lower-level problem. The second part is the revenue from selling upward and downward regulations by the storage in the reserve market. The expected reserve prices,  $\lambda_t^{UP}$  and  $\lambda_t^{DN}$ , are the Lagrangian multipliers belonging to constraints (7.17) and (7.18) in the lower-level problem. The following constraints need to be enforced:

$$\widehat{P}_{c,t}^{DA} = \sum_{w \in c} P_{w,t}^{DA} + \sum_{e \in c} (P_{e,t}^{dis} - P_{e,t}^{ch}) \quad c \in C, t \in T$$
(7.2)

$$0 \le P_{w,t}^{DA} \le P_w^{Wmax} \quad w \in W, t \in T$$
(7.3)

$$-\sum_{e \in c} P_e^{ch,max} \le \widehat{P}_{c,t}^{DA} \le \sum_{e,w \in c} (P_w^{Wmax} + P_e^{dis,max}) \quad c \in C, t \in T$$

$$(7.4)$$

$$0 \le \widehat{R}_{e,t}^{ch} \le P_e^{ch,max} \quad e \in E, t \in T$$

$$(7.5)$$

$$0 \le \widehat{R}_{e,t}^{dis} \le P_e^{dis,max} \quad e \in E, t \in T$$
(7.6)

$$0 \le P_{e,t}^{ch} \le u_{e,t} P_e^{ch,max} \quad e \in E, t \in T$$

$$(7.7)$$

$$0 \le P_{e,t}^{dis} \le (1 - u_{e,t}) P_e^{dis,max} \quad e \in E, t \in T$$
(7.8)

$$0 \le P_{e,t}^{ch} + \widehat{R}_{e,t}^{ch} \le P_e^{ch,max} \quad e \in E, t \in T$$

$$(7.9)$$

$$0 \le P_{e,t}^{dis} + \widehat{R}_{e,t}^{dis} \le P_e^{dis,max} \quad e \in E, t \in T$$

$$(7.10)$$

$$\widehat{o}_{e,t}^{Redis}, \widehat{o}_{e,t}^{Rech}, \widehat{o}_{c,t}^{E} \ge 0 \quad c \in c, e \in E, t \in T$$

$$(7.11)$$

$$E_e^{min} \le E_{e,t} \le E_e^{max} \quad e \in E, t \in T$$
(7.12)

$$E_{e,1} = E_e^{ini} \quad e \in E \tag{7.13}$$

$$E_{e,t} = E_{e,t-1} + (P_{e,t}^{ch} + \widehat{R}_{e,t}^{ch})\eta^{ch} - \frac{(P_{e,t}^{dis} + \widehat{R}_{e,t}^{dis})}{\eta^{dis}} \quad e \in E, 1 < t \in T$$
(7.14)

Constraint (7.2) defines the total energy bid by the WF-ESS in the day-ahead market which is a combination of the energy from the wind farm and the storage system. Constraint (7.3) limits the bidding by the wind farm in the day-ahead market to its installed capacity and (7.4) limits the total energy bids by the WF-ESS to the summation of the installed capacity of the wind farm and the storage system. Note that the total amount bid by the wind farm is limited to its installed capacity and not to a forecasted value of its output for each time interval. The reason is that the day-ahead market takes place far earlier than the delivery moment. Hence a realistic prediction by the wind farm output is not possible. However, later in the balancing market which closes to the real-time and delivery moment, a bid by

the wind farm is limited to the forecast of its output. Moreover, the storage system is assumed to be large enough to make up for any deficit between the actual production and the installed capacity.

Constraints (7.5) and (7.6) enforce limits for the downward and upward reserves by the storage system,  $R_{e,t}^{ch}$  and  $R_{e,t}^{cdis}$ , respectively. Similarly, (7.7) and (7.8) limit the bidding by the storage system in the day-ahead energy market. Constraints (7.9) and (7.10) show that the total charging and discharging of the storage in the reserve and energy markets should be less than the charging and discharging capacity of the storage system, respectively. Constraint (7.11) shows the offer prices of the WF-ESS in the day-ahead energy and reserve markets should be more than zero. Finally, (7.12)-(7.14) enforce limits to the state of charge for the storage system.

Note that in the above constraints, all offering and bidding decisions of the WF-ESS, which are all symbolized with a hat (e.g.  $\hat{P}_{c,t}^{DA}$ ), are variables in the upper-level problem, but are treated as parameters in the lower-level problems. This enables the WF-ESS to gain insight into the market-clearing outcomes as a function of its offering and bidding decisions, and to adjust them in the upper-level problem pursuing expected profit maximization. In the objective function in (7.1), however, the variables like  $P_{c,t}^{DA}$  and  $R_{e,t}^{dis/ch}$  are shown without hat. These are bidding of the WF-ESS for energy and upward and downward reserves in the day-ahead market hence are decision variables in the lower-level problem.

- Lower-level problem:

The lower-level is from the perspective of the day-ahead market clearing by the DMO. This local day-ahead market is cleared by solving the optimization problem defined by constraints (7.15)-(7.21). The primal variable is set  $\{P_{g,t}, Q_{g,t}, R_{g,t}^{UP}, R_{g,t}^{DN}, P_{c,t}^{DA}, R_{e,t}^{ch}, R_{e,t}^{dis}, V_{i,t}, I_{l,t}, f_{l,t}^{p}, f_{l,t}^{q}\}$ . All dual variables are given in a parentheses in front of constraints.

$$\min \sum_{t \in T} \sum_{g \in G_D} (O_{g,t}^E P_{g,t} + O_{g,t}^{RUP} R_{g,t}^{UP} + O_{g,t}^{RDN} R_{g,t}^{DN}) + \sum_{t \in T} [\sum_c \hat{o}_{c,t}^E P_{c,t}^{DA} + \sum_{e \in c} (\hat{o}_{e,t}^{Rech} R_{e,t}^{ch} + \hat{o}_{e,t}^{Redis} R_{e,t}^{dis})] + \sum_t \sum_{i \in N_{D-T}} \sum_s \pi_s \lambda_{t,s}^{TD} P_{t,i}^{TD}$$
(7.15)

The objective function in (7.15) minimizes the total energy and reserve capacity costs from other DERs (shown in the first line), and the energy and the reserve capacity cost from the WF-ESS (shown in the second line) in the joint day-ahead market. The objective function is subjected to the constraints which are mainly related to the distribution system, and are similar to the ones explained in chapter 4 including (4.2)-(4.18). The power balance equation, the upward and downward reserves and the constraints belonging to the WF-ESS are written below:

$$(\lambda_{i,t}^{DA}) : \sum_{l=(j,i)} (f_{l,t}^p - I_{l,t}r_l) + \sum_{g \in G_D} P_{g,t} + \sum_{w \in i} P_{c,t}^{DA} + P_{i,t}^{TD}$$
  
=  $P_{i,t}^{load} + \sum_{l=(i,j)} f_{l,t}^p + G_i V_{i,t} \quad i \in N_D, t \in T$  (7.16)

$$(\lambda_t^{UP}): \sum_{g \in G_D} R_{g,t}^{UP} + \sum_{e \in i} R_{e,t}^{dis} \ge \alpha_D (\sum_e P_e^{dis,max} + \sum_{g \in G_D} P_g^{max})$$

$$(7.17)$$

$$(\lambda_t^{DN}): \sum_{g \in G_D} R_{g,t}^{DN} + \sum_{e \in i} R_{e,t}^{ch} \ge \alpha_D (\sum_e P_e^{dis,max} + \sum_{g \in G_D} P_g^{max})$$

$$(7.18)$$

$$(\gamma_{c,t}^+, \gamma_{c,t}^-): \quad 0 \le P_{c,t}^{DA} \le \widehat{P}_{c,t}^{DA} \quad c \in C, t \in T$$
 (7.19)

$$(\psi_{e,t}^+, \psi_{e,t}^-): \quad 0 \le R_{e,t}^{dis} \le \widehat{R}_{e,t}^{dis} \quad e \in E, t \in T$$
 (7.20)

$$(\vartheta_{e,t}^+, \vartheta_{e,t}^-): \quad 0 \le R_{e,t}^{ch} \le \widehat{R}_{e,t}^{ch} \quad e \in E, t \in T$$
(7.21)

Equation (7.16) is the power balance equation and includes the energy from the generators connected to the distribution system, the WF-ESS, and the TMO market. Constraints (7.17) and (7.18) are limits for the upward and downward reserves in the distribution system. Constraints (7.19)-(7.21) impose limits for the energy and upward and downward reserves from the WF-ESS. The rest of the constraints are similar to the ones shown for the distribution network constraints in chapter 4 including (4.2)-(4.18).

#### - Solving the bi-level optimization problem:

To solve the bi-level problem, first, the lower-level problem of DMOmarket-clearing including equations (7.15) and its corresponding constraints are replaced by their Karush-Kuhn-Tucker (KKT) conditions. Note that these KKT conditions provide the optimality conditions since the lower-level problem is convex. Then, the KKT equations of the lower level problem will be added to the upper-level problem including equations (7.1)-(7.14). The resulting single-level optimization model is a mathematical problem with equivalent constraints (MPEC). This problem, however, is non-linear. There are two sources of nonlinearity that can be linearised as described below:

- \* The first source of non-linearity is the set of complementarity conditions that are within the KKT conditions. Each complementarity condition can be linearised using a "Big-M" approach [194].
- \* The second source of non-linearity comes from the bilinear terms in the objective function (7.1). Inspired from [195], we linearise those bilinear terms which are shown in Appendix D.

After solving the aforementioned non-linearity in the MPEC model, it turns into a mixed-integer linear problem with its output the strategic bidding prices  $(\hat{o}_{c,t}^{E}, \hat{o}_{e,t}^{Redis}, \hat{o}_{e,t}^{Rech})$  and quantities  $(\hat{P}_{c,t}^{DA}, \hat{R}_{e,t}^{ch}, \hat{R}_{e,t}^{dis})$  by which WF-ESS participates in the day-ahead market in step 2.

#### • Step 2. Day-ahead market clearing

In this step, the day-ahead market is cleared. As it is explained in chapter 4, the day-ahead market in the coupled market model consists of three steps. Sub-step I is the day-ahead preliminary scheduling by the DMO, sub-step II is the central market clearing and sub-step III is the local day-ahead market clearing. To clarify the inputs and outputs of this step, a short explanation together with a simplified formulation is presented below.

- Sub-step I. Day-ahead preliminary scheduling by the DMO:

The DMO first solves a preliminary scheduling problem where the objective function is minimizing the total cost of energy and reserve capacity from the bids and offers from DERs plus the TMO market. The objective function is shown in (7.22).

$$\min \sum_{t \in T} \sum_{g \in G_D} (O_{g,t}^E P_{g,t} + O_{g,t}^{RUP} R_{g,t}^{UP} + O_{g,t}^{RDN} R_{g,t}^{DN}) \\ + \sum_{t \in T} [\sum_c \hat{o}_{c,t}^E P_{c,t}^{DA} + \sum_{e \in c} (\hat{o}_{e,t}^{Rech} R_{e,t}^{ch} + \hat{o}_{e,t}^{Redis} R_{e,t}^{dis})] \\ + \sum_t \sum_{i \in N_{D-T}} \sum_s \pi_s \lambda_{t,s}^{TD} P_{t,i}^{TD}$$

$$(7.22)$$

Equation (7.22) minimizes the total cost of generation and reserve capacity of DERs in the local market plus the expected cost of buying/selling energy and reserve from/to the TMO market. The cost of this energy is the day-ahead price in the wholesale market. Note that  $\hat{o}_{c,t}^{E}$ ,  $\hat{o}_{e,t}^{Rech}$  and  $\hat{o}_{e,t}^{Redis}$  in (7.22) are no longer decision variables, but parameters which are the output of the bi-level optimization in step 1. Constraints of the objective function in (7.22) are similar to the ones in (7.16)-(7.21) plus the ones belong to distribution network constraints shown step I in chapter 4.

Through this preliminary scheduling, the local market price for energy and reserve will be determined. The energy price  $(\lambda_{i,t}^{DA})$  is the Lagrange multiplier of the power balance equation at the interface node between the TSO and DSO, which can be determined by deriving the KKT conditions of the above convex optimization problem. Moreover, the Lagrangian multipliers of (7.17) and (7.18) symbolized with  $\lambda_t^{UP}$ and  $\lambda_t^{DN}$ , are the price for the upward and downward reserve capacity in the distribution system. The power injected at the interface node  $(\widetilde{P_{i,t}^{TD}})$ , total reserve capacity of DERs  $(R_{g,t}^{UP/DN})$ , energy price  $(\lambda_{i,t}^{DA})$ and reserve price  $(\lambda_t^{UP/DN})$  are outputs of this step, by which the DMO participates in the wholesale market as an aggregator acting on behalf of DERs.

- Sub-step II. Central day-ahead market clearing:

In this step, the wholesale day-ahead joint energy and reserve capacity market is cleared by the TMO. The DMO and transmission-connected generators participate in this market. As explained in chapter 4, the objective of this market is maximizing the social welfare, however, the demand is considered to be inelastic, so social welfare is equivalent to minimizing the total generation cost.

$$min \quad \sum_{t \in T} \sum_{g \in G_T} (O_{g,t}^E P_{g,t} + O_{g,t}^{RUP} R_{g,t}^{UP} + O_{g,t}^{RDN} R_{g,t}^{DN}) \\ + \sum_{t \in T} \sum_{i \in N_{T-D}} (\lambda_{i,t}^E P_{i,t}^{DMO} + \lambda_t^{UP} R_{i,t}^{DMO/UP} - \lambda_t^{DN} R_{i,t}^{DMO/DN})$$
(7.23)

The first line in (7.23) consists of the cost of energy and reserve capacity procured by the transmission generators. The second line accounts for the total costs of energy and reserve capacity procured by DERs. The objective function is subjected to the transmission system constraints. The constraints are similar to the ones shown for the sub-step II of the coupled market in chapter 4. After clearing this market, the DMO is informed about the allocated power flow over the interface transformer  $(\widehat{P_{i,t}^{DMO}})$  and the required reserve capacity from DERs  $(R_{i,t}^{\widetilde{DMO}/UP} \text{ and } R_{i,t}^{\widetilde{DMO}/DN})$ .

- Sub-step III. Local day-ahead market clearing:

In this step, the DMO clears the day-ahead joint energy and reserve capacity market based on the updated information from sub-step II. This local day-ahead market is cleared with the objective function shown by (7.24). Same as in the TMO market, the demand considered as being inelastic.

$$\min \sum_{t \in T} \sum_{g \in G_D} (O_{g,t}^E P_{g,t} + O_{g,t}^{RUP} R_{g,t}^{UP} + O_{g,t}^{RDN} R_{g,t}^{DN}) + \sum_{t \in T} [\sum_c \hat{o}_{c,t}^E P_{c,t}^{DA} + \sum_{e \in c} (\hat{o}_{e,t}^{Rech} R_{e,t}^{ch} + \hat{o}_{e,t}^{Redis} R_{e,t}^{dis})]$$
(7.24)

The objective function in (7.24) is minimizing the total generation cost of DERs in the distribution system. The constraints in this step are mostly similar to the preliminary scheduling. The differences are in the power balance equation in (7.25) and the required upward and downward reserves in (7.26) and (7.27) which are written as follows:

$$\sum_{l=(j,i)} (f_{l,t}^p - I_{l,t}r_l) + \sum_{g \in G_D} P_{g,t} + \sum_{c \in i} P_{c,t}^{DA} - \widetilde{P_{i,t}^{DMO}}$$
  
=  $P_{i,t}^{load} + \sum_{l=(i,j)} f_{l,t}^p + G_i V_{i,t} \quad i \in N_D, t \in T$  (7.25)

$$\sum_{g \in G_D} R_{g,t}^{UP} + \sum_{e} R_{e,t}^{dis} \ge R_{i,t}^{\widetilde{DMO}/UP} \quad i \in N_{D-T}, t \in T$$
(7.26)

$$\sum_{g \in G_D} R_{g,t}^{DN} + \sum_e R_{e,t}^{ch} \ge R_{i,t}^{\widetilde{DMO/DN}} \quad i \in N_{D-T}, t \in T$$
(7.27)

In (7.25),  $\widetilde{P_{i,t}^{DMO}}$  is a parameter symbolizing the power flow injected at the interface node between the distribution and transmission systems. In (7.26) and (7.27),  $\widetilde{R_{i,t}^{DMO/UP}}$  and  $\widetilde{R_{i,t}^{DMO/DN}}$  are parameters symbolizing the required upward and downward reserves in the distribution system, respectively, which are generated in sub-step II. The rest of the constraints are similar with the ones explained earlier in chapter 4. The outputs of this step, regarding the upward and downward reserve capacity ( $\widetilde{R_{g,t}^{UP/DN}}$  and  $\widetilde{R_{e,t}^{dis/ch}}$ ) and the scheduled energy of dispatchable generators ( $\widetilde{P_{g,t}}$  and  $\widetilde{P_{c,t}^{DA}}$ ), are inputs for the balancing market which is explained further in section 7.3.2.

#### • Step 3. Remuneration

In this step, according to the cleared day-ahead market prices for energy and reserve  $(\lambda_{i,t}^{DA} \text{ and } \lambda_t^{UP/DN})$  and quantities  $(\widetilde{P_{c,t}^{DA}} \text{ and } \widetilde{R_{e,t}^{dis/ch}})$  obtained in step 2, the day-ahead revenue of the WF-ESS is calculated as shown in (7.28):

$$Revenue = \sum_{t} \left[\sum_{c} \lambda_{c\in i,t}^{DA} \widetilde{P_{c,t}^{DA}} + \sum_{e} \lambda_{t}^{UP} \widetilde{R_{e,t}^{dis}} + \lambda_{t}^{DN} \widetilde{R_{e,t}^{ch}}\right]$$
(7.28)

#### 7.3.2 Revenue maximization in the balancing market

As explained earlier in section 7.2.2, the bidding of the WF-ESS in the balancing market is non-strategic. The revenue calculations of the WF-ESS in the balancing market has three steps. Step 1 is to generate the bids through a selfoptimization problem. Step 2 is the balancing market clearing process including three step (IV-VI) of the coupled market model explained in chapter 4. Lastly, step 3 is the remuneration phase in which the WF-ESS calculates its balancing revenue based on a dual-pricing mechanism.

These three steps need to be performed through a shrinking rolling horizon approach, for reasons explained in section 7.2.2. The balancing market is cleared every 15 minutes in a time window of 24 hours. At each time interval inside the rolling horizon, the horizon is shrunk by a one time step and the optimization is solved over the remaining horizon with the new forecasts available for the current day. The forecast is for the uncertain parameters which are dependent on scenarios. Wind output  $(P_{w,t,s}^{Wact})$ , the imbalance prices  $(\lambda_{t,s}^{+/-})$ , and the total system imbalance  $(SI_{t,s})$  are the uncertain, scenario-based parameters in step 1. By contrast, there is no uncertainty in steps 2 and 3 which happen in real-time. Each of these shrinking horizon solutions gives the current variable outputs and at the current time. Mathematical formulations belonging to steps 1-3 are described as follows.

#### • Step 1: Generate balancing non-strategic bids

In step 1, the WF-ESS solves a self-scheduling problem to determine its most beneficial actions in the balancing market in terms of bidding volume for a given price and for the time-horizon of 24 hours.

$$\max \sum_{t} \sum_{e \in c} \sum_{s} \pi_{s} (\lambda_{t,s}^{+} P_{e,t,s}^{dis/BL} - \lambda_{t,s}^{-} P_{e,t,s}^{ch/BL}) + \sum_{c} (\lambda_{c \in i,t}^{DA} (1 - z_{t,s}) \Delta_{c,t,s}) + \sum_{c} (\lambda_{t,s}^{+} z_{t,s} y_{t,s} \Delta_{c,t,s}^{+} + \lambda_{t,s}^{-} z_{t,s} (1 - y_{t,s}) \Delta_{c,t,s}^{-}))]$$

$$(7.29)$$

The objective function in (7.29) consists of three parts. The first term is the revenue obtained by the storage system due to actively bidding in the balancing market. The second and the third terms belong to the imbalance settlement in which the storage system pays or is being paid, depending on whether or not its imbalance is in the opposite or same direction with the total system imbalance. As the dual-pricing mechanism is applied, in the case in which the deviation is in the same direction with the total system imbalance, the WF-ESS has to pay with a price equal to the  $\lambda_{c\in i,t}^{DA}$ , otherwise, it is being paid by a price equal to  $\lambda_{t,s}^+$  or  $\lambda_{t,s}^-$  depending on having a short or long imbalance, respectively. As mentioned earlier, this step happens before the real-time, hence the wind power and imbalance prices and the total imbalance of the system are scenario-dependent. Constraints (7.30)-(7.45) need to be enforced.

$$\Delta_{c,t,s} = P_{c,t,s}^{Total real time} - \widetilde{P_{c,t}^{DA}} \quad c,t,s$$
(7.30)

$$\Delta_{c,t,s}SI_{t,s} \le z_{t,s}M \quad c,t,s \tag{7.31}$$

$$\Delta_{c,t,s} SI_{t,s} \ge -(1 - z_{t,s})M \quad c, t, s$$
(7.32)

$$P_{c,t,s}^{Total real time} = \sum_{w \in c} P_{w,t,s}^{Wact} + \sum_{e \in c} (\widehat{P}_{e,t,s}^{dis/BL} - \widehat{P}_{e,t,s}^{ch/BL}) \quad c,t,s$$
(7.33)

$$\Delta_{c,t,s} = \Delta_{c,t,s}^+ - \Delta_{c,t,s}^- \quad c,t,s \tag{7.34}$$

$$0 \le \Delta_{c,t,s}^+ \le y_{t,s} (\sum_{w \in c} P_{w,t,s}^{Wact} + \sum_{e \in c} P_e^{dis,max}), \forall c, t, s$$

$$(7.35)$$

$$0 \le \Delta_{c,t,s}^{-} \le (1 - y_{t,s}) \widetilde{P_{c,t}^{DA}} \quad c, t, s$$

$$(7.36)$$

$$\widehat{P}_{e,t,s}^{ch/BL} + \widetilde{P}_{e,t}^{ch} \le P_e^{ch,max} \quad e,t,s \tag{7.37}$$

$$\widehat{P}_{e,t,s}^{dis/BL} + \widetilde{P_{e,t}^{dis}} \le P_e^{dia,max} \quad e,t,s$$
(7.38)

$$0 \le \widehat{P}_{e,t,s}^{ch/BL} \le u_{e,t,s} P_e^{ch,max} \quad e,t,s \tag{7.39}$$

$$0 \le \hat{P}_{e,t,s}^{dis/BL} \le (1 - u_{e,t,s}) P_e^{dis,max} \quad e, t, s \tag{7.40}$$

$$\widehat{P}_{e,t,s}^{ch/BL} \le \widetilde{R_{e,t}^{ch}} \quad e, t, s \tag{7.41}$$

$$\widehat{P}_{e,t,s}^{dis/BL} \le \widetilde{R_{e,t}^{dis}} \quad e, t, s \tag{7.42}$$

$$E_e^{min} \le E_{e,t} \le E_e^{max} \quad e, t, s \tag{7.43}$$

$$E_{e,1} = E_e^{ini} \quad e \tag{7.44}$$

$$E_{e,t} = E_{e,t-1} + \sum_{s} \pi_s \left( P_{e,t,s}^{ch/BL} \eta^{ch} - \frac{P_{e,t,s}^{dis/BL}}{\eta^{dis}} \right) \quad e, t > 1$$
(7.45)

Constraints (7.30) and (7.33) are related to the amount of the imbalances caused by the WF-ESS. Constraints (7.31) and (7.32) define the direction of the imbalance of the WF-ESS with respect to the total system imbalance.  $SI_{t,s}$  indicates the total system imbalance and is scenario-based. Constraints (7.34)-(7.36) define the positive and negative imbalance by WF-ESS. Constraints (7.37) and (7.38) limit the upward and downward

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energy of the storage system in the balancing market. Constraints (7.39)-(7.42) limit the charging and discharging of the storage system with respect to the scheduled energy of the storage in the day-ahead market ( $\widetilde{P_{e,t}^{dis}}$  and  $\widetilde{P_{e,t}^{ch}}$ ). Finally, (7.43)-(7.45) depict the state of the charging of the storage systems. The output of this optimization is an estimated bidding energy of the WF-ESS by which it will participate in the balancing market.

#### • Step 2: Balancing market clearing

This step includes sub-steps IV-VI in the coupled market model shown in Figure 7.1 and explained in chapter 4.

For clarity, each step is formulated briefly here as well. Note that,  $\tau$  in the equations below is the time unit of the balancing market in the shrinking rolling horizon.

- Sub-step IV. Balancing preliminary scheduling by the DMO:

In this step, the DMO estimates the local balancing market price by which it participates in the central real-time balancing market. The objective function is minimizing the expected cost of balancing services at the distribution system shown in (7.46).

$$\min \sum_{g \in G_D} (O_{g,\tau}^{EUP} P_{g,\tau}^{UP} - O_{g,\tau}^{EDN} P_{g,\tau}^{DN}) + \sum_e (O_{e,\tau}^{Edis} P_{e,\tau}^{dis/BL} - O_{e,\tau}^{Ech} P_{e,\tau}^{ch/BL}) + \sum_{i \in N_{D-T}} \sum_s \pi_s (\lambda_{\tau,s}^+ P_{i,\tau}^{DMO/UP} - \lambda_{\tau,s}^- P_{i,\tau}^{DMO/DN})$$
(7.46)

The first and the second term in (7.46) is the cost of the balancing services procured from DERs and the third term belongs to the cost of balancing services procured from the transmission system. The following constraints need to be imposed:

$$\begin{aligned} (\lambda_{i,\tau}^{EBL}) : \quad & \sum_{l=(j,i)} (f_{l,\tau}^p - I_{l,\tau}.r_l) + \sum_{g \in i} (\widetilde{P_{g,\tau}} + P_{g,\tau}^{UP} - P_{g,\tau}^{DN}) \\ & + (\widetilde{P_{c,\tau}^{DA}} + P_{e,\tau}^{dis/BL} - P_{e,\tau}^{ch/BL}) + \sum_{i \in N_{D-T}} (P_{i,\tau}^{DMO/UP} \\ & - P_{i,\tau}^{DMO/DN}) = \alpha_{Imb} SI_{\tau,s} + P_{i,\tau}^{load} \\ & + \sum_{l=(i,j)} f_{l,t}^p + G_i.V_{i,t} \quad i \in N_D \\ & \widetilde{P_{g,\tau}} + P_{g,\tau}^{UP} \leq P_g^{gmax} \quad g \in G_D \end{aligned}$$
(7.48)

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$$\widetilde{P_{g,\tau}} - P_{g,\tau}^{DN} \ge P_g^{gmin} \quad g \in G_D \tag{7.49}$$

$$-\widetilde{R_{g,\tau}^{UP/DN}} \le P_{g,\tau}^{UP/DN} \le \widetilde{R_{g,\tau}^{UP/DN}} \quad g \in G_D$$
(7.50)

Constraint (7.47) is the power balance equation.  $\alpha_{Imb}$  is a parameter shows the fraction of the total system imbalance which belongs to the distribution system.

Constraints (7.48)-(7.50) limit the upward and downward balancing regulations. The rest of the constraints are related to the system constraints which are similar with the ones shown for sub-step IV in chapter 4. The output of this step is the local balancing market price ( $\lambda_{i,\tau}^{EBL}$ : the Lagrangian multiplier of (7.47)) and quantities  $(P_{i,\tau}^{DMO/UP}$  and  $P_{i,\tau}^{DMO/DN})$ , by which the DMO participates in the TMO-balancing market in step V.

- Sub-step V. Central balancing market clearing:

In this step, the TMO clears the real-time central balancing market. Generators connected to the transmission system and the DMO participate in this market. This is the objective function:

$$\min \sum_{g \in G_{T}} (O_{g,\tau}^{EUP} P_{g,\tau}^{UP} - O_{g,\tau}^{EDN} P_{g,\tau}^{DN}) + \sum_{i \in N_{T-D}} \lambda_{i,\tau}^{EBL} (P_{i,\tau}^{DMO/UP} - P_{i,\tau}^{DMO/DN})$$
(7.51)

The first term is related to the cost of balancing services from the transmission-connected generators. In the second term,  $\lambda_{i,\tau}^{EBL}$  is the price of balancing services form the DMO. The network constraints of the transmission system are enforced. The results of this step, which will be passed on to the DMO, is indicating the deployed energy from transmission to the distribution system.

#### - Sub-step VI. Local balancing market clearing:

In this step, the DMO clears the local balancing market. The objective function is minimizing the balancing service costs from the deployed DERs:

$$\min \{ \sum_{g \in G_D} (O_{g,\tau}^{EUP} P_{g,\tau}^{UP} - O_{g,\tau}^{EDN} P_{g,\tau}^{DN}) + \sum_{e} (O_{e,\tau}^{Edis} P_{e,\tau}^{dis/BL} - O_{e,\tau}^{Ech} P_{e,\tau}^{ch/BL}) \}$$
(7.52)

The power balance equation is as follows:

$$\begin{aligned} (\lambda_{i,\tau}^{EBL}) : & \sum_{l=(j,i)} (f_{l,\tau}^p - I_{l,\tau}.r_l) + \sum_{g \in i} (\widetilde{P_{g,\tau}} + P_{g,\tau}^{UP} - P_{g,\tau}^{DN}) \\ & + (\widetilde{P_{c,\tau}^{DA}} + P_{e,\tau}^{dis/BL} - P_{e,\tau}^{ch/BL}) + \sum_{i \in N_{D-T}} (\widetilde{P_{i,\tau}^{DMO/UP}} \\ & - \widetilde{P_{i,\tau}^{DMO/DN}}) = P_{i,\tau}^{load} + \sum_{l=(i,j)} f_{l,\tau}^p + G_i V_{i,\tau} \quad i \in N_D \end{aligned}$$
(7.53)

In the power balance equation in (7.53),  $\Delta P_{i,\tau}^{DMO}$  is the scheduled adjustment from transmission level to the distribution level which has been calculated in sub-step V. The rest of the constraints are the system constraints which are shown in (4.55)-(4.59) of chapter 4.

The output of this step is the cleared balancing market price  $(\lambda_{i,\tau}^{EBL})$ and quantities  $(\widetilde{P_{e,\tau}^{dis/BL}}, \widetilde{P_{e,\tau}^{ch/BL}})$  by which the WF-ESS calculates its revenue in step 3.

#### • Step 3: Remuneration

This step, which happens at the imbalance settlement phase, calculates the revenue of the WF-ESS by the cleared balancing market price and quantities obtained in sub-step VI of step 2. As explained through Table 7.1 in section 7.2.2, in the imbalance settlement of the balancing market, the dual pricing mechanism is applied. Therefore, the revenue calculation is as follows:

$$\sum_{e \in c} (\lambda_{\tau}^{EBL} \widetilde{P_{e,\tau}^{dis/BL}} - \lambda_{\tau}^{EBL} \widetilde{P_{e,\tau}^{ch/BL}}) - \sum_{c} (a\lambda_{\tau}^{DA} \widetilde{\Delta_{c,\tau,s_r}^{+}} + b\lambda_{\tau}^{DA} \widetilde{\Delta_{c,\tau,s_r}^{-}}) + \sum_{c} (c\lambda_{\tau}^{EBL} \widetilde{\Delta_{c,\tau,s_r}^{+}} + d\lambda_{\tau}^{EBL} \widetilde{\Delta_{c,\tau,s_r}^{-}})$$
(7.54)

where a, b, c, d are binary parameters, out of which, at each moment, only one is equal to 1 and the rest are zero. For example, a = 1 means that in real-time the imbalance caused by the WF-ESS is positive  $(\Delta_{c,\tau,s_r}^+)$  and is in-line with the direction of the total system imbalance. Therefore, the WF-ESS should pay for causing this imbalance at the rate of the day-ahead market price.

The definitions of  $\Delta_{c,\tau,s_r}^+$ ,  $\overline{\Delta_{c,\tau,s_r}^-}$  are based on the (7.30)-(7.36). However, the difference is that in (7.54),  $\overline{\Delta_{c,\tau,s_r}^+}$  and  $\overline{\Delta_{c,\tau,s_r}^-}$  don't depend on the scenario, since this step is after the scenario realizations. Note that  $s_r$ is one realized scenario.  $P_{e,\tau}^{dis/BL}$  and  $P_{e,\tau}^{ch/BL}$  are cleared quantities for



Figure 7.3: Centralized market model

charging and discharging of the storage system in the balancing market obtained in sub-step VI of step 2.

# 7.4 Revenue maximization of the WF-ESS in the centralized market model

In this chapter, a design consisting of a centralized day-ahead and balancing market, which has been introduced in chapter 5, is considered as the benchmark. The centralized market model is shown in the grey flowchart in Figure 7.3 and the steps and their sequence in the WF-ESS's bidding in centralized day-ahead and balancing markets are shown in the red and orange lines. Relatively speaking, the volumes traded in the day-ahead and balancing markets in the centralized market model are much bigger than the capacity of the WF-ESS, hence the WF-ESS cannot behave strategically and exercise market power in neither the day-ahead market nor the balancing market. The balancing market is also modelled through the rolling shrinking horizon, the same as the one introduced in the coupled market model. The dual pricing mechanism is also applied in the imbalance settlement phase.

In both day-ahead and balancing markets, there are three steps. In step 1, generating the non-strategic bids, the WF-ESS solves a self-scheduling problem for a number of scenario-based market prices to determine its most beneficial actions in terms of bidding volumes. Thereafter, in step 2, the unit participates in

the day-ahead or balancing markets and the centralized market model is cleared. Finally, in step 3, according to the cleared price and quantities in step 2, the day-ahead or balancing revenue of the WF-ESS is calculated. The corresponding mathematical formulations of WF-ESS revenue maximization in day-ahead and balancing markets are presented below.

# 7.4.1 Mathematical formulations: WF-ESS's revenue in the day-ahead market

In this section, the mathematical formulation for the revenue maximization of the WF-ESS in the day-ahead market is presented. As shown in Figure 7.3, there are three steps in day-ahead bidding which are as follows:

#### • Step 1: Generate day-ahead non-strategic bids

In this step, the WF-ESS solves the following optimization problem to determine its most optimum bidding volume in the day-ahead market.

$$max \quad \sum_{t} \left[ \sum_{c} \sum_{s} \pi_{s} \lambda_{t,s}^{TD} P_{c,t}^{DA} + \sum_{e} \sum_{s} \pi_{s} (\lambda_{t,s}^{UP} R_{e,t}^{dis} + \lambda_{t,s}^{DN} R_{e,t}^{ch}) \right]$$
(7.55)

The objective function in eq. (7.55) consists of the revenue of the WF-ESS in the day-ahead energy market and the in the reserve market. The constraints for the objective (7.55) are the same as the ones in constraints (7.2)-(7.14). The outputs of this step are the energy  $(\widetilde{P_{c,t}^{DA}})$  and reserve  $(\widetilde{R_{e,t}^{dis/ch}})$  bidding volumes of the WF-ESS in the day-ahead market.

#### • Step 2: Day-ahead market clearing

The day-ahead joint market of the centralized model is quite similar to the day-ahead market clearing by the TMO in the coupled market model. The difference is that, for the objective function in the centralized model,  $\lambda_{i,t}^{DA}$  and  $\lambda_t^{UP/DN}$  in (7.23) are equal to zero. Moreover,  $P_{g,t}$  and  $R_{g,t}^{UP/DN}$  represent energy and reserve for all generators including DERs and generators connected to the transmission system. Therefore, the objective function and constraints are as follows:

$$max \quad \sum_{t \in T} \left[ \sum_{g \in (G_T \cup G_D)} O_{g,t}^E P_{g,t} + O_{c,t}^E P_{c,t}^{DA} + O_{g,t}^{RUP} R_{g,t}^{UP} + O_{g,t}^{RDN} R_{g,t}^{DN} + O_{e,t}^{RUP} R_{e,t}^{dis} + O_{e,t}^{RDN} R_{e,t}^{ch} \right]$$
(7.56)

The constraints of this optimization problem are similar to the constraints in the wholesale market clearing by the TMO in the coupled market model shown in (4.20)-(4.30) of chapter 4. The only difference is in the power balance equation in which the load refers to both the loads in the transmission and distribution systems. The output of this step is cleared market prices  $(\lambda_{i,t}^{DA}, \lambda_t^{UP/DN})$  and dispatching of energy and reserve from generators and storage  $(\widetilde{P_{g,t}}, \widetilde{P_{c,t}^{DA}}, \widetilde{R_{g,t}^{UP/DN}}, \widetilde{R_{e,t}^{dis/ch}})$ .

#### • Step 3: Remuneration

In this step, the revenue of the WF-ESS is calculated based on the cleared day-ahead market price and quantities obtained in step 2:

$$Revenue = \sum_{t} \left[\sum_{c} \lambda_{c\in i,t}^{DA} \widetilde{P_{c,t}^{DA}} + \sum_{e} (\lambda_{t}^{UP} \widetilde{R_{e,t}^{dis}} + \lambda_{t}^{DN} \widetilde{R_{e,t}^{ch}})\right]$$
(7.57)

# 7.4.2 Mathematical formulations: WF-ESS's revenue in the balancing market

As the orange line in Figure 7.3 shows, the revenue maximization problem of the WF-ESS in the balancing market consists of three steps, similar to the ones in the day-ahead market. These steps are explained below.

#### • Step 1: Generate balancing non-strategic bids

In this step, same as for in the coupled market, the WF-ESS tries to calculate its bidding position in the balancing market based on scenario-based positive and negative imbalance prices and the cleared day-ahead market prices:

$$max \qquad \sum_{t} \sum_{e \in c} \sum_{s} \pi_{s} (\lambda_{t,s}^{+} P_{e,t}^{dis} - \lambda_{t,s}^{-} P_{e,t}^{ch}) + \sum_{c} (\lambda_{t}^{DA} (1 - z_{t}) \Delta_{c,t} + \sum_{s} \pi_{s} . (\lambda_{t,s}^{+} z_{t} y_{t} \Delta_{c,t}^{+} + \lambda_{t,s}^{-} z_{t} (1 - y_{t}) \Delta_{c,t}^{-}))]$$
(7.58)

The formulation of the objective function in (7.58) is similar to the one in (7.29) hence its constraints are also the same as in (7.30)-(7.45). The output of this step is the bidding volume  $(\widetilde{P_{e,t}^{dis/ch}})$  of the WF-ESS unit in the balancing market.

#### • Step 2: Balancing market clearing

In this step, the balancing market is cleared. The shrinking rolling horizon approach is applied here as well to clear the balancing market. The objective function is minimizing the total costs of balancing service provision by the TMO as shown in (7.59):

$$\min \sum_{g \in G_T \cup G_D} (O_{g,\tau}^{EUP} . P_{g,\tau}^{UP} - O_{g,\tau}^{EDN} . P_{g,\tau}^{DN}) + \sum_{e} (O_{e,\tau}^{Edis} P_{e,\tau}^{dis/BL} - O_{e,\tau}^{Ech} P_{e,\tau}^{ch/BL})$$
(7.59)

The following constraints need to be enforced:

$$(\lambda_{i,\tau}^{BL}): \sum_{g \in G_T \cup G_D} (\widetilde{P_{g,\tau}} + (P_{g,\tau}^{UP} - P_{g,\tau}^{DN})) + \sum_{c,e \in c} (\widetilde{P_{c,\tau}^{DA}} + P_{e,\tau}^{dis/BL} - P_{e,\tau}^{ch/BL}) + \sum_{(j,i) \in l} f_{l,\tau}^p = SI + P_{i,\tau}^{load} + \sum_{(i,j) \in l} f_{l,\tau}^p$$

$$i \in N_T/N_{T-D}, l \in L_T, \tau$$

$$(7.60)$$

$$P_{e,\tau}^{ch/BL} \le \widetilde{P_{e,\tau}^{ch/BL}} \quad e \in E, \tau \tag{7.61}$$

$$P_{e,\tau}^{dis/BL} \le P_{e,\tau}^{dis/BL} \quad e \in E, \tau$$
(7.62)

The rest of the constraints are similar to the ones in the TMO balancing market clearing shown in chapter 4. The output of this step is the cleared balancing market price  $(\lambda_{i,\tau}^{BL})$  and the cleared upward and downward energy  $(P_{e,\tau}^{dis/BL}, P_{e,\tau}^{ch/BL})$  from the WF-ESS in the balancing market.

#### • Step 3: Remuneration

Finally, during the imbalance settlement phase, the WF-ESS calculates its revenue based on the dual pricing mechanism and cleared balancing market prices  $(\lambda_{i,\tau}^{BL})$  and the activated upward and downward energy  $(P_{e,\tau}^{dis/BL}, P_{e,\tau}^{ch/BL})$ , obtained in step 2.

# 7.5 Input data and Case studies

In this section, the input data and main case studies which have been used for the simulations of DERs behaviour in electricity markets are described.

## 7.5.1 Input data

The proposed revenue maximization of DERs in the coupled market model is tested using a radial 30-bus medium voltage Dutch distribution system and the IEEE-24 bus transmission system, the same as in chapters 5 and 6. The data for the offer prices of distributed generators are shown in Appendix C. The WF-ESS is located at the end of the feeder of the distribution system. The wind farm has an installed capacity of 6 MW. The storage system has 5 MW charging and discharging capacity with an efficiency of 80% and energy content of 20 MWh.

The wind speed data are from the Royal Netherlands Meteorological Institute (KNMI) [196]. The day-ahead and imbalance market prices and total system imbalances are for the Netherlands and obtained from the ENTSO-e transparency platform [52]. The residential loads in the distribution system are the same as the ones in chapter 5 and 6. Like in chapter 5, a set of scenarios for wind power generation, day-ahead, and imbalance market prices are generated which are shown in Appendix C. The time resolution of the day-ahead market is one hour with a time horizon of 24-hours and the balancing market is 15 minutes. The  $\alpha_T$  is considered as 30% of the total installed DERs to the total load of the system.

The mathematical models are formulated in the General Algebraic Modelling System (GAMS) and solved with the solvers CPLEX and MOSEK on a computer with CPU E5-2697 v3@2.6GHz. The computational time for the participation of the WF-ESS in one time-step of the day-ahead market (i.e. 1 hour) and the balancing market (i.e. 15 minutes) of the coupled market model is 34 seconds and 16 seconds, respectively.

## 7.5.2 Case studies

In this section, the case studies which are going to be analysed in the results section are introduced. The first one covers the market model and the second one is about different types of DERs.

#### Market model case studies

In addition to the coupled market model which is explained earlier, the revenue maximization of DERs in the centralized market model is also considered as the benchmark. This market model is more compatible with current electricity markets. More detailed information about the revenue maximization of DERs in the centralized market model and its mathematical formulation can be found in section 7.4.

As the WF-ESS is relatively small compared with the size of the market, the WF-ESS cannot behave strategically in the centralized market model and therefore its behaviour does not affect the market price. Consequently, the WF-ESS is a price-taker in both day-ahead and balancing markets in the case of the centralized market model. Hence, it solves a self-scheduling problem to determine its most beneficial actions for given prices in day-ahead and/or balancing markets.

#### DER case studies

In the WF-ESS case, the storage system participates in the energy and reserve capacity market and actively bids into the balancing markets. The wind farm alone, however, is limited in how it can participate in the market. Due to the stochastic nature of wind power, the wind farm alone is considered unable to participate in the reserve capacity market and/or actively bid into the balancing market, so it can only actively bid into the day-ahead energy market. However, in the balancing market, the wind farm may have to be paid or pay the market imbalance price (based on the assumed dual pricing scheme), depending on its real-time deviation with respect to the total system imbalance.

Because of higher complexity in the WF-ESS compared with the wind farm case, the mathematical formulations in section 7.3 have been presented for the WF-ESS case. However, these formulations can be easily adapted for the wind farm alone, if one sets the capacity of the storage system equal to zero.

# 7.6 Results and discussion

In this section, the numerical results of the simulations are shown. In section 7.6.1, the results of the wind farm's revenue in the coupled versus the centralized market model are presented. In section 7.6.2, the results of a sensitivity analysis on factors that may affect the ability to exercise market power are presented. This is done by changing the distribution system parameters, e.g. branch resistances and connected loads, and looking at their effects on the dayahead revenue and bidding behaviour by the wind farm. In section 7.6.3, the wind generation cleared in the coupled market is compared with the amount cleared in the (current) centralized market model. This is done for various higher/lower values of cable resistances as compared to the base case. Finally, in section 7.6.4, the revenue of the wind farm alone is compared with the case in which the wind farm is equipped with a storage system, in order to highlight its risk-hedging and profitability effects. The performance of the WF-ESS is analyzed for both the coupled and centralized market designs.



Figure 7.4: The revenue of the wind farm in the coupled versus the centralized market

# 7.6.1 Wind farm's revenue in the coupled versus centralized market models

Figure 7.4 shows the revenues of the wind farm in day-ahead and balancing markets for different market models. As this figure shows, the day-ahead revenue is significantly higher in the coupled market model, as compared to the one in the centralized market model. The reason is indeed the strategic behaviour of the wind farm in the day-ahead local market which leads to higher market prices. For the sake of comparison, Figure 7.5 shows the day-ahead market prices in the coupled versus centralized market models, where a relatively higher value for day-ahead market prices in the coupled market prices in the coupled market can be observed.

As expected, the revenue in balancing markets is lower than the revenue in the day-ahead market for both market models. Moreover, for both market models, the balancing market revenue is negative, which means that the wind farm has to pay imbalance penalty costs to the system operator. Compared to the day-ahead market, there is not much difference between the balancing revenue of coupled and centralized market models. The reason is that, as explained in section 7.2.2, the wind farm cannot exercise market power in the balancing market of the coupled market model. However, the difference is significantly higher in the day-ahead revenue of the wind farm when it participates strategically in the coupled market model behaviour. Note that, the increase in the revenue of any market player leads to increasing the cost of whole system. The burden of this cost can be carried by the end-users and/or distribution system operators.



Figure 7.5: The DA market price in the coupled versus centralized market models

# 7.6.2 The effect of distribution system parameters on exercising market power by the strategic wind farm

To see the effect of distribution system parameters on the revenue of strategic market players, two input parameters are being changed: loads of connected customers, and the resistance of branches. In distribution systems in the Netherlands, underground cables are usually being used, and their reactance compared to the resistance is relatively small. Therefore, changing the resistance of cables will be sufficient for studying the effect of branch parameters on the strategic biding of DERs. Performing this sensitivity analysis helps to understand whether or not changing loads and resistances affects the bidding volume and the revenue of the strategic wind farm. Before answering this question, one needs to study the effect of varying the resistance or loads on distribution system voltages, which is an indicator of network security. To better understand this effect, an example in Figure 7.6 has been developed. This figure shows a feeder where at its end, there is a generator, and in the middle, there are some loads. The generator's situation is similar to that of a wind farm. In the diagram in Figure 7.6, there are three curves showing voltage magnitude along the feeder in three different cases. The green curve in the middle is related to the normal situation where there are no large loads or high cable resistances, hence the voltage along the feeder is always in the secure range (assumed in this figure to be 0.9-1.1 p.u.). The red curve is related to the case where the cable resistance is increased. As shown, the voltage along the feeder is increasing too, so that at the end of the feeder, there is an over-voltage. In contrast, by increasing the loads, the voltage along the feeder is decreasing in such a way that at the end of the feeder an under-voltage appears. This is shown by the blue curve in the diagram.

Therefore, in both cases, i.e. either increasing the loads or increasing the cable resistance, the voltage at the end of the feeder, where the generator is located,



Figure 7.6: The effect of increasing resistance and loads on the voltage along the feeder

can be higher or lower than the security limits. Hence, the generator reacts differently to each of the two cases. In the case where the resistance is increased, to counteract the over-voltage, the generator has to reduce its production. This prevents the generator to exercise market power because the generator knows that its power is not required by the system. In contrast, in the case where the loads are increased, to counteract the under-voltages, the generator should inject more power to raise the voltage. Therefore, in this case, by knowing that its power is being required by the system operator, the generator might exercise market power. This market power is performed through an economic withholding which leads to a higher bidding price and a lower bidding quantity.

Now, returning to the case study for the wind farm, the effect of increased loads and cable resistances will be investigated. The effect of increasing the loads on the bidding behaviour of the wind farm is presented in Figure 7.7. In the horizontal axis, a different percentage of the load is shown. When the load in the distribution system is decreasing with respect to the base-case (labeled at 100%), the system is indicated as strong and when the load is increasing, the system is indicated as weak.

The red curve in Figure 7.7 shows the day-ahead revenue of the wind farm in the coupled market model. As shown in the figure, by increasing the loads, the day-ahead revenue has an overall increase. However, the revenue stays the same up to the point where the load is 80% of the base-case. After this point, the revenue starts rising, since the wind farm realizes that it is required by the system operator thanks to its geographical location and the under-voltage situation which is beginning to happen. Therefore, the wind farm raises the offer prices.

On the other hand, at the point where the revenue is increasing, the energy bid by the wind farm is decreasing as it is shown by the orange curve in Figure 7.7.



Figure 7.7: The effect of increasing loads on the day-ahead energy and revenue of the strategic wind farm

In short, Figure 7.7 depicts an exercise of market power by the wind farm when the loads are increasing. The exercise of market power by the wind farm is shown through a higher revenue for a lower amount of energy bid into the day-ahead market, which means a higher day-ahead price. As it has been mentioned earlier, this phenomenon is the basis of the economic withholding by which a strategic market player increases its revenue.

Figure 7.8 shows an increase in the resistance and its effect on the amount of wind energy bid in the day-ahead market and the resulting day-ahead market prices. The horizontal axis is the percentage difference of the cable resistance with respect to the base-case. When the resistance of the cables is decreasing with respect to the base-case, the system is indicated as strong, and when the resistance of the cables is increasing , the system is indicated as weak. As it is explained by Figure 7.6, increasing the resistance will cause an over-voltage at the end of the feeder and therefore, the wind farm has to decrease its power. This is shown by the orange curve in Figure 7.8, which has a downward trend. On the other hand, the day-ahead market prices, indicated by the red curve in Figure 7.8, are also decreasing as the resistance is increasing. This leads to a downward trend in the revenue of the winds farm as well. Therefore, it can be seen that by increasing the resistance, the wind farm cannot perform market power.

## 7.6.3 Renewable generation in the coupled versus centralized market models

In this part, the difference between the bidding energy by the wind farm in the coupled versus centralized market models with different resistance values is



Figure 7.8: The effect of increasing cable resistance on the day-ahead prices and energy offers by the wind farm

studied. In the centralized market model, as explained in chapter 5, distribution system constraints are not taken into account during market clearing. This means that in the centralized market model, distribution system constraints when DERs are getting dispatched is not considered, and therefore, there might be a chance that they cause network violations. To avoid this, a power flow for the distribution system with different resistance values is performed, to determine the maximum energy allowed by the wind farm which does not cause overload in the distribution system. Then, the maximum energy allowed by the wind farm is compared with its bidding energy in the centralized market model. If the bidding energy in the centralized market model for a certain resistance value, is lower than the maximum allowed energy, there will not be any wind curtailment, otherwise, there will be wind curtailment to reduce the bidding energy to the amount of the maximum allowed energy.

Figure 7.9 shows the wind farm bidding in the day-ahead market of the coupled versus the centralized market model. The red curve shows the energy offered by the wind farm in the centralized market model and the orange curve shows the one in the coupled market. As expected, the wind farm bidding in the centralized market model has a downward trend when increasing the cable resistance values, the same as the trend in the coupled market. However, at any resistance rate, the wind generation in the coupled market is higher than the wind generation in the centralized market model. In other words, in the coupled market the distribution system is dynamically checked at each moment while in the centralized market model, the distribution system is taken into account after the market-clearing. Therefore, in a weak system where the resistance is higher and the distribution system is more often in danger of network violations, the renewable-based DERs such as wind farms are more likely to be curtailed. In the coupled market,



Figure 7.9: Wind generation offers in the coupled versus the centralized market models

however, dynamically checking the distribution system constraints as part of the market clearing mechanism, allows the wind farm to inject energy at a higher rate. This can be seen in Figure 7.9 where for example at 140% resistance, the wind generation in the coupled market is almost 50% higher than the one in the centralized market model.

#### 7.6.4 Effect of storage system on the wind farm's revenue

In this section, the results for the comparison between the wind farm alone and the WF-ESS, in terms of their revenues, are presented. More results investigate whether or not being equipped with a storage system is profitable for the wind farm. This comparison is performed for both proposed and existing market models.

Figure 7.10 shows the total revenues of the WF-ESS over one day in day-ahead and balancing markets for different market models. To make the figure more easily interpretable, the results in Figure 7.4 are added to Figure 7.10 as well. As the latter figure shows, in the coupled market model, the day-ahead revenue either for the wind farm alone or the WF-ESS case is significantly higher compared to the ones in the centralized market model. The reason is indeed the strategic behaviour of the wind farm and the WF-ESS in the day-ahead local market, which leads to higher prices for the coupled market design.

As expected, Figure 7.10 shows that the revenue from the balancing markets, for both cases and both market models, are lower than the revenues from day-ahead markets. However, in the case where the wind farm acts alone, for both market


Figure 7.10: Revenue of the wind farm and the WF-ESS in the coupled versus the centralized market models

models, the balancing market revenue is negative which means that the wind farm has to pay imbalance penalty costs to the system operator. By contrast, as the storage system can actively bid into the balancing market, the revenue in the balancing market for the WF-ESS either in the coupled or centralized market models has positive values, which means the WF-ESS can earn some revenue in the balancing market, by acting to not only hedge for the wind farm's forecast errors, but also to compensate the system's imbalance. There is a slightly higher balancing market revenue in the coupled compared to the centralized market model but this difference is not very significant due to the non-strategic behaviour of market players in the balancing market.

To compare the balancing market revenues for different scenario realizations in the two configurations of wind farm versus WF-ESS, the probability density function (PDF) of balancing market revenues has been computed. Figure 7.11 shows the PDFs for the wind farm versus the WF-ESS case in the coupled market which is the result of 960 data points consisting of 10 scenarios realizations for each of the 96-time intervals in the balancing market. The PDF belonging to the WF-ESS case is shifted to the right in comparison with the one for the wind farm case and shows an increase in the positive side of the balancing market revenue for the WF-ESS. This means that when the wind farm is provided with a storage system, for different scenario realizations, there is a higher revenue compared with the case where the wind farm solely bids into the day-ahead market and consequently has to pay a penalty cost due to imbalances caused by the real-time wind power deviations from the energy bid into the day-ahead market.



Figure 7.11: The probability density function of the revenue in the balancing market for the wind farm and the WF-ESS

It should be mentioned here that to have a better comparison in terms of revenues between the wind farm case and the WF-ESS case, it is important to take into account the cost of the storage system as well. If a Li-ion battery is being considered, the Levelized Cost of Storage (LCOS) may be equal to  $388 \in MWh$ . In this case, for the WF-ESS in the coupled market, with the deducted LCOS from its total revenue, results in a 227  $\in$ /MWh net revenue which is equal to the  $227 \in MWh$  revenue of the case where the wind farm is alone. This means any cheaper storage system than  $388 \in MWh$  will result in a profitable business case for the WF-ESS. On the other hand, in the centralized market, deducting the LCOS from the total revenue requires a storage system of  $217 \in MWh$  to have an equal profit to a wind farm without a storage system. According to the literature, for the Li-ion battery, the LCOS is considered to be equal to  $235 \in /MWh$  based on [59]. Considering the case of WF-ESS in the coupled market, by deducting this LCOS from the DA revenue, it results in a  $335 \notin MWh$  net revenue which is slightly higher than the  $275 \in MWh$  revenue of the case where the wind farm is alone. On the other hand, in the case of the centralized market model, deducting the LCOS from the DA revenue leads to a lower profit for the WF-ESS compared to the case when the wind farm acts alone. Therefore, depending on the market model, price volatility and whether or not there is market power, a combined wind and storage unit can be an affordable option in comparison with a case where the wind farm is alone and cannot act strategically. Next to that, there is a decreasing trend in prices of battery storage systems, so the business case of the WF-ESS is expected to improve [197][198].

# 7.7 Conclusions

This chapter aimed to study the proposed coupled market design from the perspective of DERs by formulating the revenue maximization problem for DERs acting in the coupled market. First, the revenue maximization of DERs in electricity markets was explained, together with reviewing the literature about the possibilities for strategic and non-strategic biding behaviour of DERs. Noting that the strategic behaviour of DERs in a local market has not yet thoroughly been studied, this chapter proposed a strategic bidding method for the revenue maximization of DERs in the coupled market. The size of the local market is relatively small, and this increases the chance of some DERs to act strategically. The revenue maximization problem has been modelled through a bi-level shrinking rolling horizon optimization where the upper-level problem is from the strategic DER's perspective and the lower-level problem is from the market operator's (DMO's) perspective. As an illustration, a wind farm is considered as the strategic DER, showing that under certain assumptions, also intermittent resources can exercise market power by economic withholding.

In order to investigate how to quantify the strategic revenue maximization of the wind farm in the coupled market model, the results for the coupled market model were compared with the ones for a state-of-art centralized market model where DERs cannot employ strategic behaviour. The results confirm the applicability of the proposed revenue maximization problem and they show that, in general, the wind farm earns higher revenues in the coupled market where it can exercise market power, as compared with the centralized market model, where it cannot.

Next, it was tried to show whether or not changing the distribution system parameters will affect the revenue of the wind farm and its bidding strategy in the coupled market. Results show that a weak system, with longer feeders and thus higher cable resistances, leads to higher revenues for the wind farm, and lower amounts of energy cleared in the day-ahead market, while a stronger system has a reverse effect. In other words, a strategic market player in a weak (local) system can increase its market power and therefore earn a higher income. By contrast, a strong system prevents exercising market power by market players. Note that these results have to do with the presence of the wind farm at the end of a feeder, therefore having a positive effect on the voltage profile. Moreover, it was seen that in a weak system, the amount of cleared wind generation is significantly higher in the coupled market compared with the amounts cleared in the centralized market model. This means that the coupled market can better unlock the potential of renewable-based DERs which want to participate in the market, however the system must be strong enough to prevent the exercise of market power.

The last effort was to explore the financial benefits of adding a storage system to the wind farm. Results show that in both coupled and centralized market models, the combined wind and storage system (WF-ESS) has a higher income compared with the case of the wind farm alone. However, taking into account the Levelized Cost of the Storage and deducting it from the revenue can lead to different net revenues for the wind farm with the storage system in coupled and centralized market models. Clearly, the coupled market situation results in a more attractive business case for the WF-ESS combination.

Finally, it is important to recognize that exercising market power by market players leads to a higher end-user electricity price and consequently a higher social welfare cost. Since in the coupled market design, this market power exists due to the presence of system constraints, the distribution system operator should investigate the cost of upgrading the system to avoid the occurrence of market power.

8

# Conclusions & Recommendations

The transition in distribution and transmission systems due to increasing the number of distributed energy resources (DERs) connected to distribution systems and decreasing centralized generators connected to transmission systems is challenging for current electricity markets as well as for system operators. Therefore, the question emerges as to how to develop an electricity market to facilitate the participation of DERs into the market while the distribution network can still work within its secure operational limits. The previous chapters try to answer this question by designing a coupled market model for DER's market participation and by studying this market model from different perspectives. In this chapter, the conclusions which can be drawn regarding the coupled market are summarized. Thereafter, some recommendations for future research are given.

# 8.1 Conclusions

To address the requirement for a new market design which can cope with the transition in distribution systems due to increasing DERs, the following research question was raised in chapter 1:

## How should the current market designs be adapted to enable full participation of DERs in an economically efficient and system-secure way?

Based on the critical literature review presented in chapter 3 and the original research and modelling presented in chapters 4-7, the sub-questions posed in chapter 1 can now be answered.

## 1) What are the potential limitations for DERs to participate in a network-secure and economically efficient way in the current electricity markets?

In chapter 3, the main barriers for the participation of DERs in current electricity markets are introduced as: 1) minimum bid size requirement which is difficult for DERs to meet individually; and 2) the location of DERs (deep) in the distribution system, which leads to neglecting the distribution network constraints in the market-clearing process. To overcome these barriers, it was discussed in this chapter that the aggregator concept is a solution for the participation of DERs in electricity markets. However, aggregators do not have any incentive to take into account distribution network constraints. Also, aggregators do not usually have information about the geographical location of DERs in the distribution network. The control actions that impose on the connected units may influence the load flows as well as transformer and line loadings of one or several distribution systems. Consequently, dispatching of DERs can lead to over/under voltages or congestion problems in the distribution grid. Thus, extra care has to be taken if relevant distribution-grid constraints occur in the area of operation of an aggregator.

To deal with the aggregator related challenges, furthermore in this chapter it was shown that a local market is a promising alternative for the participation of DERs in electricity markets. The local market is defined as an institutional framework that allows the purchase and selling of local energy and ancillary services. Since the distribution grid covers a limited area, the contracting of energy and ancillary services in this context can be referred to as a "local market". In the context of participation of DERs in local markets, it is important to look at the coordination between the DSO and the TSO too as different TSO-DSO coordination can result in different market structures. The coordination has two aspects; the technical and the market perspective. The technical perspective includes congestion management, balancing, use of energy and ancillary services from DERs, real-time control and supervision, and network planning. The market perspective refers to different market structures with TSO-DSO coordination. Lastly, this chapter discussed that a correct pricing mechanism in local markets is mechanisms necessary to unlock more utilization of energy and ancillary services from DERs.

All in all, chapter 3 showed that the current electricity market design has limitations when it comes to the full participation of DERs in electricity markets. The existing literature has tried to address those limitations. However, some limitations remain, among others, the absence of considering distribution network constraints and neglecting the technical aspects of the TSO-DSO coordination during the market-clearing process. Therefore, an alternative option is to implement new market-based concepts to deal with these limitations to unlock the full potential of DERs in electricity markets.

### 2) How could the electricity market design be changed in order to overcome the discovered limitations?

To overcome the existing limitations in current electricity markets and to further improve the new designs found in the literature, a coupled market model was proposed in chapter 4. In this coupled market model, there is a local electricity market in which DERs can participate while distribution system constraints are taken into account. The coupled market model is a new market structure that allows full participation of DERs in day-ahead and balancing markets. The local market is operated by the distribution market operator (DMO), who can also participate in the central market that is operated by the transmission market operator (TMO).

The DMO can sell the excess energy from the local market to the TMO or buy the deficit energy from the TMO to fulfil the demand in the local market in the most economical way. This coordination between the central market operator and the local market operator is essential to unlock the full potential of DERs for the provision of energy and ancillary services to the benefit of the entire power system. Also, a coordinated approach can better balance supply and demand systemwide while resolving congestion issues locally. Moreover, the DMO enforces the distribution network constraints in the market clearing algorithm and keeping the distribution system within safe operating limits.

# 3) Do the changes in the market design result in economic and secure operation of the power system?

Chapter 5 answered this question by comparing the coupled market with the state-of-art centralized market model. The centralized market model represents the current electricity market design in which there is no local market for DERs. When proposing the coupled market model as an alternative to overcome the current limitations regarding DERs market participation, one needs to study whether the coupled market model results in the economical and secure operation of the power system when a large amount of DERs are present. This question was answered by comparing the total system cost for the coupled market and the currently applied centralized market model while including or ignoring the distribution network constraints.

The results showed that the coupled market model without the distribution network constraints is cost-comparable with the centralized market model even in the situations where the share of DERs in the system has been increased. This study concluded that although, the coupled market seems to be a more complicated model – e.g. DERs have to submit their bids earlier than in the centralized market model and there are more steps involved in the market clearing process – however, there are advantages in the coupled market over the centralized market model. First, the coupled market is more scalable and it can enable the participation of a large amount of DERs in the market without adding too much complexity in the overall system operation. Moreover, the coupled market model brings the opportunity to take into account the security of the distribution network during the market clearing process to avoid congestion, i.e. overloading or over/under voltages. Finally, from the system point of view, the coupled market can be cost-comparable with the centralized market model. Adding the distribution network constraints to the coupled market model will alleviate congestion while raising the system cost. However, to have a more accurate overview of the costs, the increase in the system cost due to adding the distribution network constraints in the market clearing process, should be balanced against the cost of a system blackout or a partial electricity cut-off due to the congestion that may happen when neglecting the distribution network constraints.

## 4) Do the changes in the market design result in an economic and secure operation from the perspective of stakeholders within the market?

Further analysis of the coupled market was needed to see whether it can also be economically and financially secure from the points of view of main stakeholders in the local market. The main stakeholders in the local markets are the DMO and DERs which their perspectives were investigated in chapter 6 and chapter 7, respectively.

Chapter 6 showed that the analysis of the coupled market from the DMO's perspective requires studying different DMO approaches in the preliminary scheduling in which the DMO determines its bidding strategy in terms of price and quantity for the TMO market. Moreover, in the preliminary scheduling, the DMO has to take into account the uncertainty in the TMO market prices. Different ways for addressing this uncertainty by the DMO were considered, including scenariobased, min-max regret and deterministic (with perfect knowledge as an idealized situation). The perfect knowledge and deterministic approaches are considered as limits for the best and worst performance of the DMO, respectively. Thereafter, the effect of the DMO's different bidding strategies is studied in terms of the cost in the local market and the energy output from DERs.

The results showed that the scenario-based approach is preferable for DERs as the total produced energy from DERs in the local market is at a higher rate than in the regret-based approach. However, the regret-based approach is most beneficial from the viewpoint of the system cost, as the total system cost in different price scenarios has the lowest variance. Therefore, this approach supports more accurate assessment of the system cost. Besides, the DSO and TSO can also have a better estimate of the power flow over the interface transformer between the distribution and transmission systems. Moreover, the DSO would prefer an approach in which more energy is produced from DERs rather than importing from the TMO market, to have less grid utilization and losses. On the other hand, DERs prefer an approach in which they can earn higher revenue, and results showed that in the scenario-based approach DERs has the highest revenues. In the end, which approach the DMO will choose is a trade-off between being risk-averse or having a higher amount of energy output from DERs.

Chapter 7 performed more in-depth research to analyse the proposed coupled market from DER's perspective in the local market. For this, the bidding strategy of DERs and their profit maximization problem within the coupled market were addressed. When DERs participate in the coupled market, they can have market power despite their size, as the DMO market size is much smaller than the centralized market model, and the location of DERs can make a difference. The strategic operation of DERs was investigated through the case of a wind farm which is equipped with a battery energy storage system.

Several questions about the exercising of market power in a local electricity market were answered in this chapter. Can the wind farm raise its revenue in the day-ahead market compared to a centralized market model? How do the distribution system parameters, e.g. resistance and loads, affect the use of market power of the wind farm? How would the inclusion of storage system affect the revenue of the wind farm? To answer the first question, the results for coupled market were compared with those for a centralized market model where DERs cannot employ strategic behaviour. The results confirmed the applicability of the proposed revenue maximization problem and showed that, in general, the wind farm earns higher revenues in the coupled market where it can exercise market power, compared to the centralized market model.

To answer the second question, the effect of changing the distribution system parameters on the revenue of the wind farm and its bidding strategy in the coupled market was studied. Results showed that a weak system, with longer feeders and thus higher branch resistances, leads to higher revenues for the wind farm, and lower amounts of energy cleared in the day-ahead market, while a stronger system has the reverse effect. In other words, a strategic market player in a weak system can increase its market power and therefore earn a higher income. In contrast, a strong system prevents exercising market power by market players. Moreover, it was seen that in a weak system, wind generation is significantly higher in the coupled market compared with the amounts cleared in the centralized market model. This means that the coupled market can better unlock the potential of the renewable-based DERs which want to participate in the market.

For the last question, the affordability of a storage system for the wind farm was investigated. The results showed that in both coupled and centralized market models, the combined wind and storage system has a higher income compared with the case of the wind farm alone. However, taking into account the Levelized Cost of the Storage and deducting it from the revenue can lead to different net revenues for the wind farm with the storage system in the coupled and centralized market models. In the end, it is important to mention that exercising market power by market players leads to a higher end-user electricity price and consequently a higher social cost. Since in the coupled market design, this market power exists due to the presence of network constraints, the distribution system operator must also investigate the cost of upgrading the system to avoid the occurrence of market power.

# 8.2 Recommendations

The research in this thesis gives an overview of how current electricity markets can be adjusted through the proposed coupled market design in order to enable market participation of DERs economically and securely. There are, however, other aspects of designing a local market in the transition toward sustainable power systems which require more elaborate investigation. In this section, some of the possible future research directions are presented.

• Policy changes to unlock coupled markets

Policies for supporting the implementation of renewable energy resources

(RES) need to be adjusted to support the local concentration of RES, rather than blanket policies targeting the rise of RES nationwide. As enabling the concentration of RES in a small area, allows for the preconditions of the coupled market to be met in more areas. This enables unlocking the full potential of renewable energy resources through their participation in the coupled market.

#### • Regulatory adjustments

Regulatory frameworks also need to be adjusted to allow for the introduction of a coupled market. The legal framework should provide market rules for the regulation of the local market and the interaction between the local and central market. These regulations are essential for the correct functioning of the coupled market. Moreover, the DMO should be established as a new legal entity within the coupled market. An entity as DMO, however, has not yet been defined in electricity markets and therefore, the DMO needs regulation concerning its role and responsibility. How DMO's roles and responsibilities fit with other market players in the market needs to be further investigated.

Similar entities as the DMO in the coupled market, are emerging in the real world. A good example is a Neutral Market Facilitator introduced in the Local Energy Oxfordshire project recently launched by DNO Scottish & Southern Electricity Networks [199]. The realization of these projects demonstrates the necessity of regulatory adjustments for a new entity as the DMO.

#### • Extending the coupled market to include other services

In addition to balancing and reserve capacity which have been implemented in this research work, more services can be implemented by the DSO at the distribution network level. For instance, there is an incentive for the DSO to limit the power exchange between the distribution and transmission networks to reduce the tariffs the DSO pays to the TSO. DSOs also need to keep the voltage under control which in the coupled market is done through an optimal power flow. However, voltage control can also be done through a market-based approach, such as reactive power/voltage control markets. However, these other services are not yet addressed in the presented coupled market design. Therefore, it should be investigated which other services can be included in the coupled market and how they can be implemented.

#### • Effect of the coupled market on grid reinforcements

Currently, the DSO is required by the regulator to solve capacity limitations by reinforcing the network within a reasonable time frame. In the coupled market, if congestion happens, the market price goes up. This increase in the market price can give an incentive to the DSO to alleviate the congestion through the local market rather than the regulation. In other words, the DSO can receive an incentive to perform network reinforcements through a local market. Moreover, the grid reinforcement can also affect the business case of generators. Therefore, how the coupled market affects the reinforcement of the network by the DSO needs to be determined to test the efficacy of leaving the incentive to reinforce the network to the local market rather than to the regulator.

#### • Interaction between several DMOs

Participation of several DMOs on the TMO market can affect the performance of the coupled market. The interaction between the DMOs can also affect the market results. For example, how market prices or DERs would change if all the DMOs use the same bidding strategy. Therefore, for future research, it is important to look at the effect of several DMOs on the TMO market, both on the level of the performance of the central market as well as the individual local markets.

# Bibliography

- [1] "Short Assessment of Renewable Energy Sources," eurostat, Tech. Rep., 2020. [Online]. Available: https://ec.europa.eu/eurostat/data/database
- [2] A. D. Vita, N. Kouvaritakis, P. Capros, L. Mantzos, and N. Tasios, EU energy trends to 2030. European Commission, 2009.
- [3] Sweco, Ecofys, Tractbel Engineering, and PWC, "Study on the effective integration of demand energy recourses for providing flexibility to the electricity system," *Final report to The European Commission*, no. April, p. 179, 2015.
- [4] "Active Distribution System Management: A Key Tool for the Smooth Integration," Eurelectric, Technical Report, 2013.
- [5] D. Kirschen and G. Strbac, Fundamentals of Power System Economics, 2nd Edition. John Wiley & Sons, Ltd, 2018.
- [6] M. Shahidehpour and M. Alomoush, *Restructured Electrical Power Systems*. CRC Press, dec 2017.
- [7] J. Karas and P. Sulamaa, "The Increasing Scope and Authority for Power Exchanges," Tech. Rep., 2013.
- [8] H. Bontius and J. Hodemaekers, "An introduction to EU Market-based congestion management models," Universal Smart Energy Framework (USEF), Tech. Rep., 2018.
- [9] European Energy Regulators' White Paper # 2, "The Role of the DSO, Relevant to European Commission's Clean Energy Proposals," Tech. Rep., 2017.
- [10] M. Mulder, "Energy transition and the electricity market: an exploration of an electrifying relationship," Centre for Energy Economics Research (CEER), University of Groningen, Tech. Rep.
- [11] A. J. Conejo, M. Carrión, and J. M. Morales, Decision Making Under Uncertainty in Electricity Markets, ser. International Series in Operations

Research & Management Science. Boston, MA: Springer US, 2010, vol. 153.

- [12] V. Bianco, Analysis of Energy Systems: Management, Planning and Policy, illustrate ed., V. Bianco, Ed. CRC Press, 2017.
- [13] "How the ICE market works," International Exchange, Tech. Rep., 2018. [Online]. Available: https://www.theice.com/publicdocs/How{\_} the{\_}ICE{\_}Market{\_}Works.pdf
- [14] P. Mäntysaari, EU Electricity Trade Law: The Legal Tools of Electricity Producers in the Internal Electricity Market. Springer, 2015.
- [15] N. van Bracht, A. Maaz, and A. Moser, "Simulating electricity market bidding and price caps in the European power markets," European Commission, Tech. Rep., 2019.
- [16] Winter Energy Package, "Regulation of the European Parliament and of the Council on the internal market for electricity," Commission Regulation (EU), Tech. Rep., 2016.
- [17] TenneT, "Imbalance Pricing System," Tech. Rep., 2019.
- [18] R. A. der Veen and R. A.Hakvoort, "The electricity balancing market: Exploring the design challenge," *Utilities Policy*, vol. 43, pp. 186–194, 2016.
- [19] A. Motte-Cortes and M. Eising, "Assessment of balancing market designs in the context of European coordination," *International Conference on the European Energy Market, EEM*, vol. 2019-Septe, 2019.
- [20] S. Burger, J. P. Chaves-Ávila, C. Batlle, and I. J. Pérez-Arriaga, "The value of aggregators in electricity systems," MIT Center for Energy and Environmental Policy Research, Technical Report.
- [21] A. van der Welle, "Required adjustments of electricity market design for a more flexible energy system in the short term," ECN, Tech. Rep., 2016.
- [22] "Electricity Balancing in Europe," European Network of Transmission System Operators for Electricity (ENTSO-E), Tech. Rep., 2018.
- [23] Reishus Consulting LLC, "Electricity Ancillary Services Primer," New England States Committee on Electricity (NESCOE), Tech. Rep., 2017.
- [24] Eurelectric, "Ancillary Services. Unbundling Electricity Products an Emerging Market," Eurelectric Thermal Working Group, Tech. Rep., 2004.
- [25] "Concerning common rules for the internal market in electricity and repealing Directive 2003/54/EC," Tech. Rep., 2003. [Online]. Available: https://eur-lex.europa.eu/legal-content/en/ALL/?uri=CELEX{%} 3A32009L0072

- [26] S. Just, "The German market for system reserve capacity and balancing energy," 2015.
- [27] P. Cramton, "Electricity market design," Oxford Review of Economic Policy, vol. 33, no. 4, pp. 589–612, nov 2017.
- [28] M. Ilic, F. Galiana, and L. Fink, Power Systems Restructuring: Engineering and Economics. Springer, 2013.
- [29] S. Vaheesan, "Market power in power markets: The filed-rate doctrine and competition in electricity," *Journal of Law Reform*, vol. 46, 2013.
- [30] C. A. Holt, Markets, Games, & Strategic Behavior. Princeton University Press, 2019.
- [31] D. R. Biggar and M. R. Hesamzadeh, "Market Power in Electricity Markets," in *The Economics of Electricity Markets*. John Wiley & Sons, may 2014, ch. 15, pp. 171–182.
- [32] European Commission, "The economic impact of enforcement of competition policies on the functioning of EU energy markets," European Commission, Tech. Rep., 2016. [Online]. Available: https: //ec.europa.eu/competition/publications/reports/kd0216007enn.pdf
- [33] "Market Power in Electricity Markets : Do Electricity Markets Require Special Regulatory Rules?" Market Analysis Ltd, Tech. Rep., 2000.
- [34] W. Strielkowski, "Traditional power markets and an evolution to smart grids," in Social Impacts of Smart Grids. Elsevier, 2020, pp. 9–54.
- [35] J. Dutton, "Market Power in Electricity Markets," in *Economics of Elec*tricity. Cambridge University Press, 2019, pp. 171–182.
- [36] J. Bergler, S. Heim, and K. Hüschelrath, "Strategic capacity withholding through failures in the German-Austrian electricity market," *Energy Policy*, vol. 102, no. July 2016, pp. 210–221, mar 2017.
- [37] H. Zhou, B. Chen, Z. Han, and F. Zhang, "Study on economic withholding in electricity market of Zhejiang Province, China," *Communications in Nonlinear Science and Numerical Simulation*, vol. 14, no. 5, pp. 2495–2501, may 2009.
- [38] "Europex Association of European Energy Exchange." [Online]. Available: https://www.europex.org/members/epex-spot/
- [39] GlobalData, "The Netherlands Power Market Outlook to 2030, Update 2019- Market Trends, Regulations, and Competitive Landscape," Tech. Rep., 2019.

- [40] "APX Group." [Online]. Available: https://apx.com/
- [41] TenneT, "Annual Market Update 2018," Tech. Rep., 2018.
   [Online]. Available: https://www.tennet.eu/fileadmin/user{\_}upload/ Company/Publications/Technical{\_}Publications/Dutch/Annual{\_} Market{\_}Update{\_}2018{\_}-{\_}Final.pdf
- [42] "Quarterly Report on European Electricity Markets 2011," EU Open data Portal, Tech. Rep., 2011. [Online]. Available: https://data.europa.eu/ euodp/en/data/dataset/european-electricity-market-reports
- "Market [43] TenneT, Review 2016.Electricity market insights," Tech. Rep., 2016.[Online]. Available: https: //www.tennet.eu/fileadmin/user{\_}upload/Company/Publications/ Technical{\_}Publications/Dutch/2016{\_}Market{\_}Review{\_}TenneT.pdf
- [44] —, "Market Review 2017 Electricity market insights," Tech. Rep. [Online]. Available: https://www.tennet.eu/fileadmin/user{\_} upload/Company/Publications/Technical{\_}Publications/Dutch/2017{\_} TenneT{\_}Market{\_}Review.pdf
- [45] A. Adair, "Market review," Journal of Property Research, vol. 18, no. 1, pp. 89–97, 2001.
- [46] TenneT, GEN, and E-Bridge, "Imbalance Management TenneT Analysis report," pp. 1–39, 2011.
- [47] J.-B. Spitzley, E. B. Zane, D. Bauknecht, R. Frank, and M. C. Sieber, "Integration of electricity from renewables to the electricity grid and to the electricity market RES integration," Tech. Rep., 2011.
- [48] Energypedia, "Feed-in Premiums (FIP)." [Online]. Available: https: //energypedia.info/wiki/Feed-in{\_}Premiums{\_}(FIP)
- [49] "Premium tariff (SDE+)," 2019. [Online]. Available: http://www.res-legal.eu/search-by-country/netherlands/single/s/ res-e/t/promotion/aid/premium-tariff-sde/lastp/171
- [50] "Over 72 percent of Germany's power needs to be met by nonhydro renewable energy by 2030," 2019. [Online]. Available: https: //www.globaldata.com
- [51] N. Sönnichsen, "Energy mix in Germany 2016-2018 Published by N. Sönnichsen, Jan 31, 2020 Despite renewables increasing during this period to a share of 35 percent of electricity generation, Germany is still heavily reliant on fossil fuels. In 2018, approximately 35 perce," 2020. [Online]. Available: https://www.statista.com/statistics/736640/ energy-mix-germany/

- [52] ENTSO-E, "ENTSO-e Transparancy Platform," 2020. [Online]. Available: https://transparency.entsoe.eu/
- [53] K. De Vos, "Negative Wholesale Electricity Prices in the German, French and Belgian Day-Ahead, Intra-Day and Real-Time Markets," *The Electricity Journal*, vol. 28, no. 4, pp. 36–50, 2015.
- [54] "What is market liquidity and why is it important?" 2019. [Online]. Available: https://www.ig.com/en/trading-strategies/ what-is-market-liquidity-and-why-is-it-important--190214
- [55] "Renewable Energy Sources Act (EEG 2017)." [Online]. Available: https://www.bmwi.de/Redaktion/EN/ Downloads/renewable-energy-sources-act-2017.pdf{%}3F{\_}blob{%} 3DpublicationFile{%}26v{%}3D3
- [56] ENTSO-E, "ENTSO-e Transparancy Platform," 2019. [Online]. Available: https://transparency.entsoe.eu/
- [57] M. Farrokhseresht, N. G. Paterakis, M. Gibescu, Y. Tohidi, and J. G. Slootweg, "A Survey on the Participation of Distributed Energy Resources in Balancing Markets," in 2018 15th International Conference on the European Energy Market (EEM), vol. 2018-June. IEEE, jun 2018, pp. 1–5.
- [58] Y. Tohidi, M. Farrokhseresht, and M. Gibescu, "A Review on Coordination Schemes Between Local and Central Electricity Markets," in 2018 15th International Conference on the European Energy Market (EEM), vol. 2018-June. IEEE, jun 2018, pp. 1–5.
- [59] M. Killer, M. Farrokhseresht, and N. G. Paterakis, "Implementation of large-scale Li-ion battery energy storage systems within the EMEA region," *Applied Energy*, vol. 260, no. June 2019, p. 114166, feb 2020.
- [60] S. Chowdhury, S. P. Chowdhury, and P. Crossley, "Distributed energy resources," in *Microgrids and Active Distribution Networks*. Institution of Engineering and Technology, jan 2009, pp. 15–34.
- [61] European Commission, "Winter Energy Package," 2016. [Online]. Available: https://ec.europa.eu/energy/news/ commission-proposes-new-rules-consumer-centred-clean-energy-transition{\_ }en?redir=1
- [62] "ENTSO-E." [Online]. Available: https://www.entsoe.eu/
- [63] Tenne, "Ancillary Services." [Online]. Available: https://www.tennet.eu/ electricity-market/ancillary-services/

- [64] S. Mischinger, Η. Seidl, E.-L. Limbacher, S. Fasben-D. "Ancillary and F. Stalleicken, Services," Deutsche Energieder. Tech. Agentur GmbH (dena), Rep., 2008.[Online]. Available: https://www.dena.de/fileadmin/dena/Publikationen/PDFs/2019/ 2018{\_}Innovation{\_}report{\_}ancillary{\_}services.pdf
- [65] S. Minniti, N. Haque, P. Nguyen, and G. Pemen, "Local Markets for Flexibility Trading: Key Stages and Enablers," *Energies*, vol. 11, no. 11, p. 3074, nov 2018.
- [66] ENTSO-E, "Supporting Paper For The Load-Frequency and Reserves Network Code," 2013.
- [67] O. Ma, N. Alkadi, P. Cappers, P. Denholm, J. Dudley, S. Goli, M. Hummon, S. Kiliccote, J. MacDonald, N. Matson, D. Olsen, C. Rose, M. D. Sohn, M. Starke, B. Kirby, and M. O'Malley, "Demand Response for Ancillary Services," *IEEE Transactions on Smart Grid*, vol. 4, no. 4, pp. 1988–1995, dec 2013.
- [68] "Qualitative assessment of balancing market design for the Netherlands," Frontier Economics, Tech. Rep. April, 2011. [Online]. Available: https://www.acm.nl/download/documenten/nma/103542Rapport.pdf
- [69] A. Pinto-Bello, "The smartEn Map European Balancing Markets Edition," p. 60, 2018.
- [70] F. P. Sioshansi, Behind and Beyond the Meter: Digitalization, Aggregation, Optimization, Monetization. Academic Press, 2020.
- [71] A. Roos, S. Ø. Ottesen, and T. F. Bolkesjø, "Modeling Consumer Flexibility of an Aggregator Participating in the Wholesale Power Market and the Regulation Capacity Market," *Energy Proceedia*, vol. 58, pp. 79–86, 2014.
- [72] S. Burger, J. P. Chaves-Ávila, C. Batlle, and I. J. Pérez-Arriaga, "A review of the value of aggregators in electricity systems," *Renewable and Sustainable Energy Reviews*, vol. 77, no. February 2016, pp. 395–405, sep 2017.
- [73] S. Koch, Assessment of Revenue Potentials of Ancillary Service Provision by Flexible Unit Portfolios. Elsevier Inc., 2015. [Online]. Available: http://dx.doi.org/10.1016/B978-0-12-410491-4.00002-6
- [74] G. Lipari, G. Del Rosario, C. Corchero, F. Ponci, and A. Monti, "A realtime commercial aggregator for distributed energy resources flexibility management," *Sustainable Energy, Grids and Networks*, vol. 15, pp. 63–75, sep 2018.
- [75] D. Della Giustina, F. Ponci, and S. Repo, "Automation for smart grids in Europe," in Application of Smart Grid Technologies. Elsevier, 2018, pp. 231–274.

- [76] Nordic Council of Ministers, *Flexible demand for electricity and power*, 2017.
- [77] W. Yang, M. Wilkinson, and P. Tavner, "Condition monitoring and fault diagnosis of a wind turbine synchronous generator drive train," *IET Renewable Power Generation*, vol. 3, no. 1, pp. 1–11, 2003.
- [78] N. Ruiz, I. Cobelo, and J. Oyarzabal, "A direct load control model for virtual power plant management," *IEEE Transactions on Power Systems*, vol. 24, no. 2, pp. 959–966, 2009.
- [79] H. PandZić, J. M. Morales, A. J. Conejo, and I. Kuzle, "Offering model for a virtual power plant based on stochastic programming," *Applied Energy*, vol. 105, pp. 282–292, may 2013.
- [80] H. Pandžić, I. Kuzle, and T. Capuder, "Virtual power plant mid-term dispatch optimization," Applied Energy, vol. 101, pp. 134–141, 2013.
- [81] M. Peik-Herfeh, H. Seifi, and M. K. Sheikh-El-Eslami, "Decision making of a virtual power plant under uncertainties for bidding in a day-ahead market using point estimate method," *International Journal of Electrical Power and Energy Systems*, vol. 44, no. 1, pp. 88–98, 2013.
- [82] M. Giuntoli and D. Poli, "Optimized thermal and electrical scheduling of a large scale virtual power plant in the presence of energy storages," *IEEE Transactions on Smart Grid*, vol. 4, no. 2, pp. 942–955, 2013.
- [83] F. Kamyab and S. Bahrami, "Efficient operation of energy hubs in time-ofuse and dynamic pricing electricity markets," *Energy*, vol. 106, pp. 343–355, 2016.
- [84] D. B. Nguyen, J. M. Scherpen, B. Ter Haar, and F. Bliek, "Modeling and optimization in USEF-compliant hierarchical energy markets," in *IEEE PES Innovative Smart Grid Technologies Conference Europe*. IEEE, 2016, pp. 1–6.
- [85] I. Lampropoulos, T. Alskaif, J. Blom, and W. van Sark, "A framework for the provision of flexibility services at the transmission and distribution levels through aggregator companies," *Sustainable Energy, Grids and Networks*, vol. 17, p. 100187, mar 2019.
- [86] J. Rivera, C. Goebel, and H.-A. Jacobsen, "Distributed Convex Optimization for Electric Vehicle Aggregators," *IEEE Transactions on Smart Grid*, vol. 8, no. 4, pp. 1852–1863, jul 2017.
- [87] A. Esmat, J. Usaola, and M. Moreno, "Distribution-Level Flexibility Market for Congestion Management," *Energies*, vol. 11, no. 5, p. 1056, apr 2018.

- [88] P. Olivella-Rosell, E. Bullich-Massagué, M. Aragüés-Peñalba, A. Sumper, S. Ø. Ottesen, J.-A. Vidal-Clos, and R. Villafáfila-Robles, "Optimization problem for meeting distribution system operator requests in local flexibility markets with distributed energy resources," *Applied Energy*, vol. 210, no. September 2017, pp. 881–895, jan 2018.
- [89] M. Zipf and D. Möst, "Cooperation of TSO and DSO to provide ancillary services," in 13th International Conference on the European Energy Market (EEM). Porto: IEEE, 2016.
- [90] M. Gancheva, S. O'Brien, and N. Crook, Models of Local Energy Ownership and the Role of Local Energy Communities in Energy Transition in Europe, 2018. [Online]. Available: http://www.cor.europa.eu
- [91] P. Olivella-Rosell, P. Lloret-Gallego, I. Munné-Collado, R. Villafafila-Robles, A. Sumper, S. Ottessen, J. Rajasekharan, and B. Bremdal, "Local Flexibility Market Design for Aggregators Providing Multiple Flexibility Services at Distribution Network Level," *Energies*, vol. 11, no. 4, p. 822, apr 2018.
- [92] R. Belmans, A. Ramos, C. D. Jonghe, and G. Virginia, "Realizing the smart grid 's potential: Defining local markets for flexibility," *Utilities Policy*, vol. 40, pp. 26–35, 2016.
- [93] "GridWise Transactive Energy Framework," The GridWise Architecture Council, Tech. Rep., 2015.
- [94] "Flexibility and Aggregation Requirements for their interaction in the market," EURELECTRIC, Tech. Rep.
- [95] C. Eid, L. A. Bollinger, B. Koirala, D. Scholten, E. Facchinetti, J. Lilliestam, and R. Hakvoort, "Market integration of local energy systems: Is local energy management compatible with European regulation for retail competition?" *Energy*, vol. 114, pp. 913–922, nov 2016.
- [96] "Decentralized Flexibility Market 2.0, A market based solution for organizing access to flexibility on the local distribution network," Bundesverband Neue Energiewirtschaft e.V. (bne) / Association of Energy Market Innovators, Tech. Rep.
- [97] C. Zhang, Y. Ding, N. C. Nordentoft, P. Pinson, and J. Ostergaard, "A Danish market solution for DSO congestion management through DER flexibility services," *Power Systems Clean Energy*, vol. 2, pp. 126–133, 2014.
- [98] M. Ampatzis, P. H. Nguyen, and W. Kling, "Local electricity market design for the coordination of distributed energy resources at district level," *IEEE PES Innovative Smart Grid Technologies Conference Europe*, vol. 2015-Janua, no. January, pp. 1–6, 2015.

- [99] A. Lüth, J. M. Zepter, P. Crespo del Granado, and R. Egging, "Local electricity market designs for peer-to-peer trading: The role of battery flexibility," *Applied Energy*, vol. 229, no. May, pp. 1233–1243, nov 2018.
- [100] S. S. Torbaghan, N. Blaauwbroek, P. Nguyen, and M. Gibescu, "Local market framework for exploiting flexibility from the end users," *International Conference on the European Energy Market, EEM*, vol. 2016-July, pp. 1–6, 2016.
- [101] T. Esterl, R. Schwalbe, D. Burnier De Castro, F. Kupzog, S. Kadam, and M. Kolenc, "Impact of market-based flexibility on distribution grids," *IET Conference Publications*, vol. 2016, no. CP686, pp. 1–4, 2016.
- [102] A. Esmat and J. Usaola, "DSO congestion management using demand side flexibility," in *CIRED Workshop 2016*, no. 0343. Institution of Engineering and Technology, 2016.
- [103] A. Esmat, J. Usaola, and M. Á. Moreno, "Congestion management in smart grids with flexible demand considering the payback effect," in *IEEE PES Innovative Smart Grid Technologies Conference Europe*. Ljubljana: IEEE, 2016.
- [104] K. Spiliotis, A. I. Ramos Gutierrez, and R. Belmans, "Demand flexibility versus physical network expansions in distribution grids," *Applied Energy*, vol. 182, pp. 613–624, nov 2016.
- [105] S. Huang and Q. Wu, "Real-time congestion management in distribution networks by flexible demand swap," *IEEE Transactions on Smart Grid*, vol. 9, no. 5, pp. 4346–4355, 2018.
- [106] J. Hu, G. Yang, H. W. Bindner, and Y. Xue, "Application of Network-Constrained Transactive Control to Electric Vehicle Charging for Secure Grid Operation," *IEEE Transactions on Sustainable Energy*, vol. 8, no. 2, pp. 505–515, 2017.
- [107] J. Hu, G. Yang, and Y. Xue, "Economic assessment of network-constrained transactive energy for managing flexible demand in distribution systems," *Energies*, vol. 10, no. 5, 2017.
- [108] A. Ramos, C. De Jonghe, V. Gómez, and R. Belmans, "Realizing the smart grid's potential: Defining local markets for flexibility," *Utilities Policy*, vol. 40, pp. 26–35, 2016.
- [109] P. Olivella-Rosell, G. Vinals-Canal, A. Sumper, R. Villafafila-Robles, B. A. Bremdal, I. Ilieva, and S. O. Ottesen, "Day-ahead micro-market design for distributed energy resources," 2016 IEEE International Energy Conference, ENERGYCON 2016, 2016.

- [110] F. Teotia and R. Bhakar, "Local energy markets: Concept, design and operation," in 2016 National Power Systems Conference, NPSC 2016. IEEE, 2017, pp. 1–6.
- [111] A. G. Madureira and J. A. Peças Lopes, "Ancillary services market framework for voltage control in distribution networks with microgrids," *Electric Power Systems Research*, vol. 86, pp. 1–7, 2012.
- [112] T. Jenkin, P. Beiter, R. Margolis, T. Jenkin, P. Beiter, and R. Margolis, "Capacity Payments in Restructured Markets under Low and High Penetration Levels of Renewable Energy," Tech. Rep. February, 2016.
- [113] G. Munoz-Delgado, J. Contreras, and J. M. Arroyo, "Distribution System Expansion Planning Considering Non-Utility-Owned DG and an Independent Distribution System Operator," *IEEE Transactions on Power Systems*, vol. 34, no. 4, pp. 2588–2597, jul 2019.
- [114] D. Bao Nguyen, J. M. Scherpen, and F. Bliek, "Distributed Optimal Control of Smart Electricity Grids with Congestion Management," *IEEE Transactions on Automation Science and Engineering*, vol. 14, no. 2, pp. 494–504, 2017.
- [115] J. Meese, T. Kornrumpf, B. Dahlmann, A. Volschow, T. Marquardt, and M. Zdrallek, "Multi-market optimization of industrial flexibility: market comparison and field test results," in *CIRED Workshop 2016*, no. 0035. Institution of Engineering and Technology, 2016.
- [116] R. A. Verzijlbergh, L. J. De Vries, Z. Lukszo, and L. J. D. Vries, "Renewable Energy Sources and Responsive Demand. Do We Need Congestion Management in the Distribution Grid?" *IEEE Transactions on Power Systems*, vol. 29, no. 5, pp. 2119–2128, 2014.
- [117] "Distributed Connection Energy **Resources:** Modeling and Reliability Considerations," American Electric North Reliabilitv Corporation (NERC), Tech. Rep., 2017.[Online]. Available: https://www.nerc.com/comm/Other/essntlrlbltysrvcstskfrcDL/ Distributed [\_] Energy [\_] Resources [\_] Report.pdf
- [118] A. Zegers and H. Brunner, "TSO-DSO interaction : An Overview of current interaction between transmission and distribution system operators and an assessment of their cooperation in Smart Grids," ISGAN, Tech. Rep., 2014.
- [119] G. CEDEC, EDSO, ENTSOE, Eurelectric, "TsoDso Data Management Report," Tech. Rep., 2016.
- The European Commission, "System Operations Guideline 2017," Official Journal of the European Union, vol. L 220, no. 25 August 2017, 2017. [Online]. Available: http://eur-lex.europa.eu/legal-content/EN/ TXT/PDF/?uri=CELEX:32017R1485{&}from=EN

- [121] European "Comission Regulation (EU)2017/2195Commission, electricity balancing," establishing a guideline on Official Journal oftheEuropean Union, vol. 2017,no. November, 312/6312/53,2017.Available: pp. [Online]. https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=uriserv:OJ.L{\_} .2017.312.01.0006.01.ENG{&}toc=OJ:L:2017:312:TOC{#}d1e2376-6-1
- [122] ENTSO-E, "Supporting Document for the Network Code on Electricity Balancing," no. September, pp. 1–178, 2013.
- [123] C. Hansen and E. Syriani, "An MDE solution for the development of robotic device drivers," in *Proceedings of the Annual Southeast Conference*, 2013.
- [124] "Smart Grid Traffic Light Concept," 2015. [Online]. Available: https://www.bdew.de/media/documents/Stn{\_}20150310{\_} Smart-Grids-Traffic-Light-Concept{\_}english.pdf
- [125] S. Mathieu, D. Ernst, and B. Corneiusse, "Agent-based analysis of dynamic access ranges to the distribution network," in 2016 IEEE PES Innovative Smart Grid Technologies Conference Europe (ISGT-Europe). IEEE, oct 2016, pp. 1–6. [Online]. Available: http: //ieeexplore.ieee.org/document/7856191/
- [126] J. Villar, R. Bessa, and M. Matos, "Flexibility products and markets: Literature review," *Electric Power Systems Research*, vol. 154, pp. 329–340, 2018.
- [127] H. Gerard, R. Puente, E. Israel, and D. Six, "Coordination between transmission and distribution system operators in the electricity sector: A conceptual framework," *Utilities Policy*, 2017.
- [128] H. Gerard, E. Rivero, and D. Six, "Basic Schemes for TSO-DSO coordination and ancillary services provision," Tech. Rep. D1.3, 2016. [Online]. Available: http://smartnet-project.eu/wp-content/ uploads/2016/12/D1.3{\_}20161202{\_}V1.0.pdf
- [129] S. Ahn, S. Nam, J. Choi, and S. Moon, "Power scheduling of distributed generators for economic and stable operation of a microgrid," *Utilities Policy*, vol. 4, no. 1, pp. 398–405.
- [130] M. Caramanis, E. Ntakou, W. W. Hogan, A. Chakrabortty, and J. Schoene, "Co-Optimization of Power and Reserves in Dynamic T&D Power Markets With Nondispatchable Renewable Generation and Distributed Energy Resources," *Proceedings of the IEEE*, vol. 104, no. 4, pp. 807–836, apr 2016.
- [131] A. Papavasiliou and I. Mezghani, "Coordination Schemes for the Integration of Transmission and Distribution System Operations," in 2018 Power Systems Computation Conference (PSCC), Dublin, 2018.

- [132] D. Ilic, P. G. D. Silva, S. Karnouskos, and M. Griesemer, "An energy market for trading electricity in smart grid neighbourhoods," in 6 th International Conference on Digital Ecosystems Technologies, Campione d'Italia, 2012.
- [133] E. Buchmann, S. Kessler, P. Jochem, and K. Bohm, "The costs of privacy in local energy markets," in *IEEE 15th Conference on Business Informatics*. Vienna: IEEE, 2013.
- [134] C. Zhang, Q. Wang, J. Wang, M. Korpaas, and M. E. Khodayar, "Strategymaking for a proactive distribution company in the real-time market with demand response," *Applied Energy*, vol. 181, pp. 540–548, 2016.
- [135] Q. Wang, C. Zhang, J. Wang, P. Pinson, and J. Ostergaard, "Real-Time Trading Strategies of Proactive DISCO with Heterogeneous DG Owners," *IEEE Transactions on Smart Grid*, vol. 9, no. 3, pp. 1688–1697.
- [136] D. Biggar and A. Reeves, "Network Pricing for the Prosumer Future: Demand-Based Tariffs or Locational Marginal Pricing?" in *Future of Utilities-Utilities of the Future*. Elsevier, 2016, no. 1, pp. 247–265.
- [137] Y. Ding, S. Pineda, P. Nyeng, J. Ostergaard, E. M. Larsen, and Q. Wu, "Real-Time Market Concept Architecture for EcoGrid EUA Prototype for European Smart Grids," *IEEE Transactions on Smart Grid*, vol. 4, no. 4, pp. 2006–2016, dec 2013.
- [138] S. Shmuel, "Distribution Locational Marginal Pricing for Optimal Electric Vehicle Charging," *IEEE Transactions on Power Systems*, vol. 29, no. 1, pp. 203 – 211, 2014.
- [139] S. Huang, Q. Wu, S. S. Oren, and R. Li, "Distribution Locational Marginal Pricing through Quadratic Programming for Congestion Management in Distribution Networks," *IEEE Transactions on Power Systems*, vol. 30, no. 4, pp. 2170 – 2178, 2015.
- [140] A. Papavasiliou, "Analysis of Distribution Locational Marginal Prices," IEEE Transactions on Smart Grid, vol. 9, no. 5, pp. 4872–4882, sep 2018.
- [141] S. Minniti, A. Haque, N. Paterakis, and P. Nguyen, "A Hybrid Robust-Stochastic Approach for the Day-Ahead Scheduling of an EV Aggregator," in 2019 IEEE Milan PowerTech. IEEE, jun 2019, pp. 1–6.
- [142] D. T. Nguyen and L. B. Le, "Risk-Constrained Profit Maximization for Microgrid Aggregators With Demand Response," *IEEE Transactions on Smart Grid*, vol. 6, no. 1, pp. 135–146, jan 2015.
- [143] A. Bagherian and S. M. Tafreshi, "A developed energy management system for a microgrid in the competitive electricity market," in 2009 IEEE Bucharest PowerTech. IEEE, jun 2009, pp. 1–6.

- [144] B. Alinia, M. H. Hajiesmaili, and N. Crespi, "Microgrid Revenue Maximization by Charging Scheduling of EVs in Multiple Parking Stations," arXivLabs, p. 8, 2016.
- [145] K. Dietrich, J. M. Latorre, L. Olmos, and A. Ramos, "Modelling and assessing the impacts of self supply and market-revenue driven Virtual Power Plants," *Electric Power Systems Research*, vol. 119, no. 2015, pp. 462–470, feb 2015.
- [146] M. A. Tajeddini, A. Rahimi-Kian, and A. Soroudi, "Risk averse optimal operation of a virtual power plant using two stage stochastic programming," *Energy*, vol. 73, pp. 958–967, aug 2014.
- [147] A. G. Zamani, A. Zakariazadeh, S. Jadid, and A. Kazemi, "Stochastic operational scheduling of distributed energy resources in a large scale virtual power plant," *International Journal of Electrical Power & Energy Systems*, vol. 82, pp. 608–620, nov 2016.
- [148] L. Ju, H. Li, J. Zhao, K. Chen, Q. Tan, and Z. Tan, "Multi-objective stochastic scheduling optimization model for connecting a virtual power plant to wind-photovoltaic-electric vehicles considering uncertainties and demand response," *Energy Conversion and Management*, vol. 128, pp. 160– 177, nov 2016.
- [149] D. Koraki and K. Strunz, "Wind and Solar Power Integration in Electricity Markets and Distribution Networks Through Service-Centric Virtual Power Plants," *IEEE Transactions on Power Systems*, vol. 33, no. 1, pp. 473–485, jan 2018.
- [150] S. Borenstein, J. Bushnell, and C. R. Knittel, "Market power in electricity markets: Beyond concentration measures," *Energy Journal*, vol. 20, no. 4, pp. 65–88, 1999.
- [151] E. Nasrolahpour, J. Kazempour, H. Zareipour, and W. D. Rosehart, "A Bilevel Model for Participation of a Storage System in Energy and Reserve Markets," *IEEE Transactions on Sustainable Energy*, vol. 9, no. 2, pp. 582–598, 2018.
- [152] Y. Ye, D. Papadaskalopoulos, R. Moreira, and G. Strbac, "Strategic capacity withholding by energy storage in electricity markets," 2017 IEEE Manchester PowerTech, Powertech 2017, pp. 1–6, 2017.
- [153] N.-H. M. von der Fehr and D. Harbord, "Spot Market Competition in the UK Electricity Industry," *The Economic Journal*, vol. 103, no. 418, p. 531, 1993.
- [154] M. Vilim and A. Botterud, "Wind power bidding in electricity markets with high wind penetration," *Applied Energy*, vol. 118, pp. 141–155, apr 2014.

- [155] S. S. Oren, "Economic inefficiency of passive transmission rights in congested electricity systems with competitive generation," *Energy Journal*, vol. 18, no. 1, pp. 63–83, 1997.
- [156] R. W. Ferrero and V. C. Ramesh, "Transaction Analysis in Deregulated Power Systems," *Transactions on Power Systems*, vol. 12, no. 3, pp. 1340– 1347, 1997.
- [157] B. F. Hobbs, C. B. Metzler, and J. S. Pang, "Strategic gaming analysis for electric power systems: An MPEC approach," *IEEE Transactions on Power Systems*, vol. 15, no. 2, pp. 638–645, 2000.
- [158] E. G. Tsimopoulos and M. C. Georgiadis, "Optimal strategic offerings for a conventional producer in jointly cleared energy and balancing markets under high penetration of wind power production," *Applied Energy*, vol. 244, no. March, pp. 16–35, jun 2019.
- [159] E. G. Kardakos, C. K. Simoglou, and A. G. Bakirtzis, "Optimal Offering Strategy of a Virtual Power Plant: A Stochastic Bi-Level Approach," *IEEE Transactions on Smart Grid*, vol. 7, no. 2, pp. 1–1, 2015.
- [160] E. Martinot, L. Kristov, and J. D. Erickson, "Distribution System Planning and Innovation for Distributed Energy Futures," *Current Sustainable/Renewable Energy Reports*, vol. 2, no. 2, pp. 47–54, jun 2015.
- [161] "Developing the REV Market in New York: DPS Staff Straw Proposal on Track One Issues," New York State Department of Public Service, Technical Report.
- [162] D. R. Biggar and M. R. Hesamzadeh, Eds., The Economics of Electricity Markets. Chichester, United Kingdom: John Wiley & Sons Ltd, aug 2014.
- [163] J. A. Momoh, Yan Xia, and G. D. Boswell, "Locational Marginal Pricing for real and reactive power," in 2008 IEEE Power and Energy Society General Meeting - Conversion and Delivery of Electrical Energy in the 21st Century, jul 2008, pp. 1–6.
- [164] L. Gan, N. Li, U. Topcu, and S. H. Low, "Exact Convex Relaxation of Optimal Power Flow in Radial Networks," *IEEE Transactions on Automatic Control*, vol. 60, no. 1, pp. 72–87, jan 2015.
- [165] M. Farivar and S. H. Low, "Branch Flow Model: Relaxations and ConvexificationPart I," *IEEE Transactions on Power Systems*, vol. 28, no. 3, pp. 2554–2564, aug 2013.
- [166] M. Baradar and M. R. Hesamzadeh, "AC Power Flow Representation in Conic Format," *IEEE Transactions on Power Systems*, vol. 30, no. 1, pp. 546–547, jan 2015.

- [167] N. Papazacharopoulos, "Voltage control in MV distribution networks with a large share of distributed renewable generation," Master thesis, 2014.
- [168] A. van der Welle, "Required adjustments of electricity market design for a more flexible energy system in the short term," ECN, Tech. Rep.
- [169] P. Gonzalez, J. Villar, C. A.Diaz, and F. A. Campos, "Joint energy and reserve markets: Current implementations and modeling trends," *Electric Power Systems Research*, vol. 109, pp. 101–111.
- [170] M. Farrokhseresht, N. G. Paterakis, and M. Gibescu, "Joint day-ahead clearing of energy, active and reactive power reserves for systems with significant RES-related uncertainty," in 2016 13th International Conference on the European Energy Market (EEM), vol. 2016-July, no. 2016. IEEE, jun 2016, pp. 1–5.
- [171] N. Lauritzen, "Convex optimization," in Undergraduate Convexity. WORLD SCIENTIFIC, may 2013, pp. 223–251.
- [172] A. Gomez-Exposito, A. J. Conejo, and C. Canizares, *Electric Energy Systems Analysis and Operation*, 2008.
- [173] C. Grigg, P. Wong, P. Albrecht, R. Allan, M. Bhavaraju, R. Billinton, Q. Chen, C. Fong, S. Haddad, S. Kuruganty, W. Li, R. Mukerji, D. Patton, N. Rau, D. Reppen, A. Schneider, M. Shahidehpour, and C. Singh, "The IEEE Reliability Test System-1996. A report prepared by the Reliability Test System Task Force of the Application of Probability Methods Subcommittee," *IEEE Transactions on Power Systems*, vol. 14, no. 3, pp. 1010–1020, aug 1999.
- [174] M. Xie, X. Ji, X. Hu, P. Cheng, Y. Du, and M. Liu, "Autonomous optimized economic dispatch of active distribution system with multi-microgrids," *Energy*, vol. 153, pp. 479–489, jun 2018.
- [175] M. Nijhuis, M. Gibescu, and J. F. Cobben, "Bottom-up Markov Chain Monte Carlo approach for scenario based residential load modelling with publicly available data," *Energy and Buildings*, vol. 112, no. 2016, pp. 121– 129, 2016.
- [176] "NEDU Verbruiksprofielen," 2020. [Online]. Available: https://www.nedu.nl/documenten/verbruiksprofielen/https://www.nedu.nl
- [177] "Elia Belgium's transmission system operator," 2017. [Online]. Available: http://www.elia.be
- [178] B. S. Mccafferty, "It's time to Spotify the grid with DSO functions," 2020. [Online]. Available: https://www.utilitydive.com/news/ its-time-to-spotify-the-grid-with-dso-functions/550321/

- [179] IRENA, "Future Role of Distribution System Operators," International Renewable Energy Agency, Abu Dhabi, Tech. Rep., 2019.
- [180] S. Parhizi, A. Khodaei, and M. Shahidehpour, "Market-Based Versus Price-Based Microgrid Optimal Scheduling," *IEEE Transactions on Smart Grid*, vol. 9, no. 2, pp. 615–623, mar 2018.
- [181] J. D. Kueck, R. Staunton, S. D. Labinov, and B. Kirby, "Microgrid Energy Management System," Tech. Rep., 2003. [Online]. Available: https://building-microgrid.lbl.gov/about-microgrids
- [182] Z. Wang, B. Chen, J. Wang, M. M. Begovic, and C. Chen, "Coordinated Energy Management of Networked Microgrids in Distribution Systems," *IEEE Transactions on Smart Grid*, vol. 6, no. 1, pp. 45–53, jan 2015.
- [183] I. Ilieva, B. Bremdal, and P. Olivella, "local Electricity retail Markets for Prosumer smart grid pOWER services (EMPOWER)," 2015. [Online]. Available: http://empowerh2020.eu/wp-content/uploads/2016/ 05/D6.1{\_}Market-design.pdf
- [184] "Deterministic and probabilistic risk," 2015. [Online]. Available: https: //www.preventionweb.net/risk/deterministic-probabilistic-risk
- [185] K.-H. Chang, "Reliability Analysis," in e-Design. Elsevier, 2015, pp. 523– 595.
- [186] L. Wu, M. Shahidehpour, and T. Li, "Stochastic Security-Constrained Unit Commitment," *IEEE Transactions on Power Systems*, vol. 22, no. 2, pp. 800–811, may 2007.
- [187] J. Aghaei and H. Barati, "Demand/supply-side reserves in electricity markets: A scenario-based approach," Australian Journal of Basic and Applied Sciences, vol. 5, no. 12, pp. 1386–1397, 2011.
- [188] L. Fan, J. Wang, R. Jiang, and Y. Guan, "Min-max regret bidding strategy for thermal generator considering price uncertainty," *IEEE Transactions on Power Systems*, vol. 29, no. 5, pp. 2169–2179, 2014.
- [189] H. Aissi, C. Bazgan, and D. Vanderpooten, "Minmax and minmax regret versions of combinatorial optimization problems: A survey," *European Journal of Operational Research*, vol. 197, no. 2, pp. 427–438, sep 2009.
- [190] J. F. De Guevara and J. Maudos \*, "Measuring welfare loss of market power: an application to European banks," *Applied Economics Letters*, vol. 11, no. 13, pp. 833–836, oct 2004.

- [191] G. A. Bakirtzis, A. G. Bakirtzis, C. K. Simoglou, N. P. Ziogos, and A. C. Tellidou, "Electricity producer offering strategies in day-ahead energy markets," in 2007 iREP Symposium Bulk Power System Dynamics and Control VII. Revitalizing Operational Reliability, vol. 22, no. 4. IEEE, aug 2007, pp. 1–18.
- [192] S. A. Gabriel and F. U. Leuthold, "Solving discretely-constrained MPEC problems with applications in electric power markets," *Energy Economics*, vol. 32, no. 1, pp. 3–14, jan 2010.
- [193] D. Pozo, E. Sauma, and J. Contreras, "Basic theoretical foundations and insights on bilevel models and their applications to power systems," *Annals* of Operations Research, vol. 254, no. 1-2, pp. 303–334, jul 2017.
- [194] A. Baillo, M. Ventosa, M. Rivier, and A. Ramos, "Optimal offering strategies for generation companies operating in electricity spot markets," *IEEE Transactions on Power Systems*, vol. 19, no. 2, pp. 745–753, 2004.
- [195] M. J. Rider, J. M. López-Lezama, J. Contreras, and A. Padilha-Feltrin, "Bilevel approach for optimal location and contract pricing of distributed generation in radial distribution systems using mixed-integer linear programming," *IET Generation, Transmission and Distribution*, vol. 7, no. 7, pp. 724–734, 2013.
- [196] KNMI Koninklijk Nederlands Meteorologisch Instituut, "KNMI -Koninklijk Nederlands Meteorologisch Instituut." [Online]. Available: www.knmi.nl
- [197] Prof Research, "Global Energy Storage Market Report 2019 Market Size, Share, Price, Trend and Forecast," Tech. Rep., 2019.
- [198] Bloomberg, "Energy storage market overview," 2019. [Online]. Available: https://www.bloomberg.com/press-releases/2019-11-07
- [199] J. S. JOHN, "How UK the Is Building Grid Flexible Markets to Reward Distributed Energy." [Onhttps://www.greentechmedia.com/articles/read/ line]. Available: how-the-uk-is-building-grid-markets-to-reward-flexible-distributed-energy
- [200] M. Farivar and S. H. Low, "Branch flow model: Relaxations and convexification-part I," *IEEE Tran. on Power Syst.*, pp. 2554–2564, 2013.



This appendix includes the KKT conditions belonged to constraints (4.1)-(4.18) shown in chapter 4.

In order to have the KKT conditions, first the Lagrangian needs to be derived. The Lagrangian multipliers are mentioned in parentheses next to each constraint in (4.1)-(4.18).

$$\begin{split} & L = \sum_{t \in T} \sum_{g \in G_D} \left( O_{g,t}^{g} P_{g,t} + O_{g,t}^{RUP} R_{g,t}^{UP} + O_{g,t}^{RDN} R_{g,t}^{DN} \right) \\ & + \sum_{t} \sum_{i \in N_D - T} \sum_{s} \pi_s \lambda_{t,s}^{TD} P_{t,i}^{TD} \\ & + \sum_{t} \sum_{i \in (i,j)} \theta_{i,l,t}(V_{i,t} - V_{A_{i,t}} - 2(r_l f_{l,t}^p - x_l f_{l,t}^q) + I_{l,t}(r_l^2 + x_l^2)) \\ & + \sum_{t} \sum_{i \in (i,j)} \lambda_{i,t}^{ED} \left( \sum_{l = (A_{i,i})} (f_{l,t}^p - I_{l,t} r_l) + \sum_{g \in G_D \in i} P_{g,t} - P_{i,t}^{load} - \sum_{l = (i,j)} f_{l,t}^p - G_i V_{i,t} \right) \\ & + \sum_{t} \sum_{i \in (i,j)} \lambda_{i,t}^{ED} \left( \sum_{l = (A_{i,i})} (f_{l,t}^p - I_{l,t} r_l) + P_{i,t}^{TD} - \sum_{l = (i,j)} f_{l,t}^p - G_i V_{i,t} \right) \\ & + \sum_{t} \sum_{i \in (i,j)} \lambda_{i,t}^{RUP} \left( \sum_{l = (A_{i,i})} (f_{l,t}^q - I_{l,t} x_l) + Q_{i,t}^{TD} - \sum_{l = (i,j)} f_{l,t}^q + b_i V_{i,t} \right) \\ & + \sum_{t} \sum_{i \in (i,j)} \lambda_{i}^{RUP} \left( \sum_{g \in G_D} R_{g,t}^{UP} - \alpha_D \sum_{g \in G_D} P_g^{gmax} \right) \\ & + \sum_{t} \sum_{g \in G_D} \lambda_{t}^{RUP} \left( \sum_{g \in G_D} R_{g,t}^{RD} - \alpha_D \sum_{g \in G_D} P_g^{gmax} \right) \\ & + \sum_{t} \sum_{g \in G_D} \lambda_{t}^{RDN} \left( \sum_{g \in G_D} R_{g,t}^{RD} - \alpha_D \sum_{g \in G_D} P_g^{gmax} \right) \\ & + \sum_{t} \sum_{g \in G_D} \lambda_{t}^{RUP} \left( \sum_{g \in G_D} R_{g,t}^{RD} - \alpha_D \sum_{g \in G_D} P_g^{gmax} \right) \\ & + \sum_{t} \sum_{g \in G_D} \lambda_{t}^{RUP} \left( \sum_{g \in G_D} R_{g,t}^{DD} - \alpha_D \sum_{g \in G_D} P_g^{gmax} \right) \\ & + \sum_{t} \sum_{g \in G_D} \lambda_{t}^{RUP} \left( \sum_{g \in G_D} R_{g,t}^{DD} - \alpha_D \sum_{g \in G_D} P_g^{gmax} \right) \\ & + \sum_{t} \sum_{g \in G_D} \lambda_{t}^{RUP} \left( \sum_{g \in G_D} R_{g,t}^{DD} - \alpha_D \sum_{g \in G_D} P_g^{gmax} \right) \\ & + \sum_{t} \sum_{g \in G_D} \lambda_{t}^{RUP} \left( \sum_{g \in G_D} R_{g,t}^{DD} - \alpha_D \sum_{g \in G_D} P_g^{gmax} \right) \\ & + \sum_{t} \sum_{g \in G_D} \lambda_{t}^{RUP} \left( \sum_{g \in G_D} N_{g,t}^{TD} - \sum_{i \in (i,j)} V_{i,t} I_{i,t} \right) \\ & + \sum_{t} \sum_{g \in G_D} \lambda_{t}^{RUP} \left( \sum_{g \in G_D} N_{g,t}^{TD} - \sum_{i \in (i,j)} V_{i,t} I_{i,t} \right) \\ & + \sum_{t} \sum_{g \in G_{g,t}} \left( P_{g,t}^2 + Q_{g,t}^2 - S_g^2 \right) + \sum_{t} \sum_{g \in G_D} \lambda_{i,t}^{TU} \left( V_{i,t}^{TD} - V_{i,t} I_{i,t} \right) \\ & + \sum_{t} \sum_{g \in G_D} \lambda_{g,t}^{TU} \left( P_{g,t}^2 - Q_{g,t}^2 - S_g^2 \right) + \sum_{t} \sum_{g \in G_D} \lambda_{i,t}^{TU} \left( V_{i,t}^{TD} - V_{i,t} \right) \\ & + \sum_{t} \sum_{g \in G_D} \lambda_{i,t}^{TU} \left( P_{g,t}^2$$

Beside the primary conditions which are constraints (4.2)-(4.18), the stationarity condition of the Lagrangian function with respect to variables are:

$$(P_{g,t}): O_{g,t}^{ED} + \lambda_{i,t}^{ED} + \varphi_{g,t}^{+} - \varphi_{g,t}^{-} + \beta_{g,t}^{+} - \beta_{g,t}^{-} = 0$$
(A.2)

$$(P_{i,t}^{TD}): \sum_{s} (\pi \lambda_{t,s}^{TD}) + \lambda_{i,t}^{E} + 2\vartheta_{i,t} P_{i,t}^{TD} = 0$$
(A.3)

$$(R_{g,t}^{UP}): O_{g,t}^{RUP} - \lambda_t^{RUP} + \varphi_{g,t}^+ + \varphi_{g,t}^- = 0$$
(A.4)

$$(R_{g,t}^{DN}): O_{g,t}^{RDN} - \lambda_t^{RDN} + \varphi_{g,t}^+ + \varphi_{g,t}^- = 0$$
(A.5)

$$(Q_{g,t}): \mu_{i,t} + \delta_{g,t}^+ - \delta_{g,t}^- = 0$$
(A.6)

$$(Q_{i,t}^{TD}): \mu_{i,t}^0 + 2\vartheta_{i,t}Q_{i,t}^{TD} = 0$$
(A.7)

$$(f_{l,t}^{p}): -2r_{l}\theta_{i,l,t} + \lambda_{i,t}^{ED} - \lambda_{j,t}^{ED} + \lambda_{i,t}^{E} - \lambda_{j,t}^{E} + 2\xi f_{l,t}^{p} + 2\zeta f_{l,t}^{p} = 0$$
(A.8)

$$(f_{l,t}^q) : -2x_l\theta_{i,l,t} + \mu_{i,t} - \mu_{j,t} + \mu_{i,t}^0 - \mu_{j,t}^0 + 2\xi f_{l,t}^q + 2\zeta f_{l,t}^q = 0$$
(A.9)

$$(V_{i,t}) : \theta_{i,l,t} - \theta_{j,l,t} - G_i \lambda_{i,t}^{ED} - G_i \lambda_{i,t}^E - \mu_{i,t} b_i - \mu_{i,t}^0 b_i - \xi_{l,t} I_{l,t} - \vartheta_{i,t} I_{l,t} + \sigma_{i,t}^+ - \sigma_{i,t}^- = 0$$
 (A.10)

$$(I_{l,t}):(r_l^2 + x_l^2)\theta_{i,l,t} - r_l\lambda_{i,t}^{ED} - r_l\lambda_{i,t}^E - \mu_{i,t} - \mu_{i,t}^0 - \xi V_{i,t} - \vartheta_{i,t}V_{i,t} = 0$$
(A.11)

Finally, the complementary conditions are as follows:

$$0 \le \lambda_t^{RUP} \perp \left(-\sum_g R_{g,t}^{UP} + \alpha_D \sum_g P_g^{gmax}\right) \tag{A.12}$$

$$0 \le \lambda_t^{RDN} \perp \left(-\sum_g R_{g,t}^{DN} + \alpha_D \sum_g P_g^{gmax}\right) \tag{A.13}$$

$$0 \le \varphi_{g,t}^+ \perp (P_{g,t} + R_{g,t}^{UP} - P_g^{gmax})$$
 (A.14)

$$0 \le \varphi_{g,t}^{-} \perp (P_{g,t} - R_{g,t}^{DN} - P_{g}^{gmin})$$
(A.15)

$$0 \le \xi_{l,t} \perp (f_{l,t}^{p^2} + f_{l,t}^{q^2} - V_{i,t}I_{l,t})$$
(A.16)

$$0 \le \zeta_{l,t} \perp (f_{l,t}^{p^2} + f_{l,t}^{q^2} - S_l^2)$$
(A.17)

$$0 \le \vartheta_{i,t} \perp (P_{i,t}^{TD^2} + Q_{i,t}^{TD^2} - V_{i,t}I_{l,t})$$
(A.18)

$$0 \le \phi_{g,t} \perp (P_{g,t}^2 + Q_{g,t}^2 - S_g^2)$$
(A.19)

$$0 \le \sigma_{i,t}^+ \perp (V_{i,t} - V_i^{max}) \tag{A.20}$$

$$0 \le \sigma_{i,t}^- \perp \left( V_i^{min} - V_{i,t} \right) \tag{A.21}$$

$$0 \le \delta_{g,t}^+ \perp (Q_{g,t} - Q_i^{gmax}) \tag{A.22}$$

$$0 \le \delta_{q,t}^- \perp (Q_i^{gmin} - Q_{g,t}) \tag{A.23}$$

$$0 \le \beta_{g,t}^+ \perp \left( P_{g,t} - P_g^{gmax} \right) \tag{A.24}$$

$$0 \le \beta_{q,t}^- \perp \left( P_q^{gmin} - P_{q,t} \right) \tag{A.25}$$

The KKT conditions (D.10) and (D.11) connects the real and reactive power price at the node *i* to the price of its ancestor. In order to replace  $\theta_{i,l,t}$  and  $\xi_{l,t}$ in (D.10), an additional KKT condition (D.13) is derived. Therefore, it can be concluded that the distribution locational marginal price (DLMP) at node *i* is a function of the active power price at the ancestor node *j*, the reactive power price at the current and ancestor node, and the price of congestion over the line attached to *i* and *j*.

# Distribution network

In this appendix, the mathematical formulation applied in modelling the distribution network in section 4.2 in chapter 4 is explained. First, the branch flow model of the radial distribution network is described in section B.1. Next, in section B.2 the second-order conic AC power flow to convexify the branch flow model and finally, calculating the distribution locational marginal price (DLMP) is presented.

# **B.1** Branch flow model

A radial distribution network is modelled by a tree graph T = (N, L) where  $N = \{0, ..., n\}$  represents the set of nodes and l represent the set of distribution lines connecting the nodes in N. The root of the tree is shown by  $N = \{0\}$  and  $N - \{0\}$  indicates the other nodes. For each node i, it has a unique ancestor  $A_i$  and a set of children nodes, denoted by  $C_i$ . An example for the branch flow is shown in fig. B.1. Each directed line connects a node i and its unique ancestor  $A_i$ . The lines l are from the set of  $L = \{1, ..., n\}$  and each  $i \in l(A_i, i)$  indicates a line from i to  $A_i$ . The root of the tree T is a substation node that is connected to the



Figure B.1: A branch flow model in the tree graph T

transmission network and has a fixed voltage and redistributes the power received from the transmission network to other nodes in the distribution network and/or transfers the power from the distribution network to the transmission network. Each node i is associated with the following decision variables  $(\chi)$ : total real power at node  $i P_i$ , total reactive power at node  $i Q_i$ , real power generated by g generation  $P_g$ , reactive power generated by generation  $g Q_g$ , real power consumption  $P_i^c$ , reactive power consumption  $Q_i^c$ , square of voltage magnitude  $V_i$ , square of current magnitude  $I_l$  and following parameters ( $\gamma$ ): susceptance  $B_i$ , conductance  $G_i$ , cost of real power generation  $O_g$ , maximum and minimum real power capacity for generators  $P_g^{max}$ ,  $P_g^{min}$ , maximum and minimum reactive for generation  $Q_g^{max}$ ,  $Q_g^{min}$ , maximum and minimum voltage limits  $V_i^{max}$ ,  $V_i^{imin}$ , and maximum and minimum current limits  $I_l^{max}, I_l^{min}$ . In addition, each  $i \in l(i, A_i)$ is associated with the following decision variables ( $\chi$ ): real power flow  $f_l^p$  from node i to  $A_i$  (with flow measured on the side of i), reactive power flow  $f_i^q$  from node i to  $A_i$  (with flow measured on the side of i), current magnitude squared  $I_i$ ; and the following parameters  $(\gamma)$ : line resistance  $R_l$ , reactance  $X_l$ , and complex power flow limit  $S_l$ . A general form of optimal power flow (OPF) for the branch flow model shown in fig. B.1 is as follows:
$$OPF: min \quad f(\chi, \gamma)$$
subject to:  
1)  $V_{A_i} = V_i - 2(R_l f_l^p + X_l f_l^q) + I_l(R_l^2 + X_l^2) \quad i \in N, l(A_i, i) \in L$   
2)  $\sum_{j \in C_i} (f_l^p - I_l R_l) + P_i = 0 \quad i \in N, l(A_i, i) \in L$   
3)  $\sum_{j \in C_i} (f_l^q - I_l X_l) + Q_i = 0 \quad i \in N, l(A_i, i) \in L$   
4)  $P_i^2 + Q_i^2 = V_i I_l \quad i \in N, l(A_i, i) \in L$   
5)  $P_i^2 + Q_i^2 = S_i \quad i \in N$   
6)  $V_i^{min} \leq V_i \leq V_i^{max} \quad i \in N$   
(B.1)

The relaxed branch flow model in (B.1) is adopted in a way that it ignores the phase angles of voltages and currents and uses only the aforementioned decision variables  $(\chi)$ . The branch flow model compared with the e.g. bus injection model is more numerically stable and has broad application in distribution networks [200]. Given a vector of decision variables that satisfies (B.1), the phase angles of the voltages and currents can be uniquely determined if the network is a tree. Hence the set of constraints (1-6) in (B.1) is equivalent to a full AC power flow model. The OPF problem in (B.1) is, however, non-linear non-convex due to equation number (4) and with few hundreds of variables it can already be intractable. To convexify this OPF, one can change the equality (4) into the inequality shown in (B.2):

$$P_i^2 + Q_i^2 \le V_i I_l \quad i \in N, l(A_i, i) \in L \tag{B.2}$$

The inequality in (B.2) is showing a second-order cone constraint and needs to be satisfied as equality for the problem to admit physical interpretation. Depending on the objective function in (B.1), the SOCP relaxation is exact under several conditions. If the objective function in (B.1) equals to  $f(\chi, \gamma) = \sum_{i \in N} (O_i^c P_i^c - O_i^g P_i^g)$ , which is equivalent of maximizing the social welfare, the condition in which the SOCP relaxation is exact, is when there are no upper bounds on the loads [165]. By exact, what is meant is that there exists an optimal solution of (B.1) such that the inequality constraint (B.2) is satisfied as equality. Through the next subsections, it is explained how to derive the duality problem of a secondorder conic optimization and consequently calculate the distribution locational marginal price (DLMP) in the second-order conic AC power flow.

# B.2 DLMP in the second-order conic AC power flow

In this section, the second-order conic AC power flow is presented. First, the general form of the second-order conic optimization problem and its duality problem is explained. Next, the second-order conic AC power flow is explained. How to calculate the DLMP is subsequently explained.

#### • Dual problem of second-order conic optimization

The optimization problem shown in (B.3) is a general form of the secondorder conic optimization for a minimizing objective. The objective function and corresponding constraints are:

$$\begin{aligned} \min_{x,y,t} g^T x \\ subject \quad to \\ \|y_i\| \le t_j \quad j = 1, ..., m \\ y_j = A_j^x x + b_j \quad j = 1, ..., m \\ t_j = w_j^T x + z_j \quad j = 1, ..., m \end{aligned}$$
(B.3)

 $A_i$  is an  $a \times n$  matrix, and x,g, and  $w_j$  are  $n \times 1$  vectors.  $y_j$  and  $b_j$  are  $a \times 1$  vectors.  $z_j$  and  $t_j$  are scalars. In order to find the dual problem of (B.3), first, the Lagrangian should be formed as:

$$L = g^{T}x + \sum_{j} + \sum_{j} (\|y_{j}\| - t_{j}) + \sum_{j} \nu_{j}^{T}(y_{j} - A_{j}x - b_{j}) + \sum_{j} \mu_{j}(t_{j} - w_{j}^{T}x - z_{j}) = (g - \sum_{j} A_{j}^{T}\nu_{j} - \sum_{j} \mu_{j}\omega_{j})^{T}x + \sum_{j} (\lambda_{j}\|y_{j}\| + \nu_{j}^{T}y_{j}) + \sum_{j} (-\lambda_{j} + \mu_{j})t_{j} - \sum_{j} (\nu_{j}b_{j}\mu_{j}z_{j})$$
(B.4)

The dual is the infimum of the Lagrangian:

$$g(\lambda,\nu,\mu) = inf_{x,y,t}(L) \tag{B.5}$$

The derivatives of the infimum with respect to x, y, and t are:

$$inf_x(L) = 0, \quad if: \sum_j A_j^T \nu_j + \sum_j \mu_j \omega_j = g \tag{B.6}$$

$$inf_x(L) = 0, \quad if: \|\nu_j\| \le \lambda_j \quad j = 1, ..., m$$
 (B.7)

$$inf_x(L) = 0, \quad if: \lambda_j = \mu_j \quad j = 1, ..., m$$
 (B.8)

Equations (B.6) and (B.8) are derived from the derivative of L with respect to x and t, respectively. As proven in [171], the infimum of L with respect to y in (B.7) is either zero (if the form is positive semidefinite) or  $-\infty$  (if the form is not positive semidefinite).

Finally, the dual problem in (B.3) is formulated as:

$$Max_{\lambda} - \sum_{j} (\nu_{j}b_{j} + \mu_{j}z_{j})$$
  
subject to  
$$\sum_{j} A_{j}^{T}\nu_{j} + \sum_{j} \mu_{j}\omega_{j} = g$$
(B.9)  
$$\|\nu_{j}\| \leq \lambda_{j} \quad j = 1, ..., m$$
$$\lambda_{j} = \mu_{j} \quad j = 1, ..., m$$

Applying the strong duality theorem, the optimization model in (B.9) can be replaced with the primal feasibility, dual feasibility and strong duality conditions presented below:

$$y_j = A_j x + b_j \quad j = 1, ..., m$$
 (B.10)

$$t_j = \omega_j^T + z_j \quad j = 1, ..., m$$
 (B.11)

$$\sum_{j} A_j^T \nu_j + \sum_{j} \mu_j \omega_j = g \tag{B.12}$$

$$\|\nu_j\| \le \lambda_j \quad j = 1, ..., m \tag{B.13}$$

$$\lambda_j = \mu_j \quad j = 1, ..., m \tag{B.14}$$

$$\sum_{j} (\nu_j b_j + \mu_j z_j) = g^T x \tag{B.15}$$

The term  $\mu_j z_j$  is a non-linear, non-quadratic term. In order to linearise these bilinear terms, in the strong duality condition, new variables  $\tau'_{jrl}$ ,  $v'_{jrh}$ and  $\kappa'_{jrm}$  are introduced together with the following disjunctive equations:

$$-M(1-\tau_{rl}) \le \tau'_{jrl} - \mu_j \le M(1-\tau_{rl})$$
(B.16)

$$-M\tau_{rl} \le \tau'_{jrl} - \mu_j \le M\tau_{rl} \tag{B.17}$$

$$-M(1 - v_{rl}) \le v'_{jrl} - \mu_j \le M(1 - v_{rl})$$
(B.18)

$$-Mv_{rl} \le v'_{jrl} - \mu_j \le Mv_{rl} \tag{B.19}$$

$$-M(1 - \kappa rl) \le \kappa j rl - \mu_j \le M(1 - \kappa rl)$$
(B.20)

$$-M\kappa rl \le \kappa jrl - \mu_j \le M\kappa rl \tag{B.21}$$

The results of the optimization in (B.10)-(B.21) is the mixed-integer secondorder conic problem.

#### • Second-order conic AC power flow and its duality

The following OPF shown in (B.22)-(B.31) is adopting the OPF in (B.1) based on the second-order conic optimization belonging to (4.1)-(4.18) in section 4.2. The objective function and corresponding constraints are:

$$min \quad \sum_{g} O_g P_g \tag{B.22}$$

Subject to:

$$(\lambda_i): \sum_{l(i,j)} f_l^p - \sum_{l(A_i,i)} (f_l^p - I_l R_l) + \sum_{g \in i} P_g - P_i^{load} + G_i v_i = 0 \quad (B.23)$$

$$(\mu_i): \sum_{l(i,j)} f_l^q - \sum_{l(A_i,i)} (f_l^q - I_l X_l) + \sum_{g \in i} Q_g - Q_i^{load} + B_i v_i = 0 \quad (B.24)$$

$$(\theta_{i,l(A_i,i)}): \quad V_i = V_{A_i} + 2(R_l f_l^p + X_l f_l^q) - I_l(R_l^2 + X_l^2)$$
(B.25)

$$(\xi_l): (f_l^p)^2 + (f_l^q)^2 \le V_{i \in l(A_i,i)} I_l$$
 (B.26)

$$(\zeta_l): (f_l^p)^2 + (f_l^q)^2 \le S_l^2$$
 (B.27)

$$(\phi_g): (P_g)^2 + (Q_g)^2 \le S_g^2$$
 (B.28)

$$(\sigma_i^+, \sigma_i^-): \quad V_i^{min} \le V_i \le V_i^{max}$$
(B.29)

$$(\delta_g^+, \delta_g^-): \quad Q_i^{gmin} \le Q_g \le Q_i^{gmax} \tag{B.30}$$

$$(\beta_g^+, \beta_g^-): \quad P_g^{gmin} \le P_g \le P_g^{gmax} \tag{B.31}$$

Below, a recursive formula of the corresponding dual problem for this SOC-ACPF optimization which leads to the DLMPs is derived. The formula expresses the DLMP at a certain node as a function of the active and reactive power price at that same node, the price of real and reactive power at the ancestor node, and the contribution of the capacity constraint of the distribution line that connects a node to its ancestor. This formula is derived by resorting to the KKT conditions of the SOCP relaxation. The Lagrangian of the above optimization is:

$$\begin{split} L &= \sum_{g} (O_{g}P_{g}) + \sum_{i} \sum_{l=(i,A_{i})} \theta_{i,l}(V_{i} - V_{A_{i}} - 2(r_{l}f_{l}^{p} - x_{l}f_{l}^{q}) \\ &+ I_{l}(r_{l}^{2} + x_{l}^{2})) + \sum_{i} \lambda_{i} (\sum_{l=(A_{i},i)} (f_{l}^{p} - I_{l}r_{l}) + \sum_{g \in i} P_{g} \\ &- P_{i}^{load} - \sum_{l=(i,A_{i})} f_{l}^{p} - G_{i}V_{i}) + \sum_{i} \mu_{i} (\sum_{l=(A_{i},i)} (f_{l}^{q} - I_{l}x_{l}) \\ &+ \sum_{g \in i} Q_{g} - Q_{i}^{load} - \sum_{l=(i,A_{i})} f_{l}^{q} - b_{i}V_{i}) \\ &+ \sum_{l=(i,A_{i})} \xi_{l}((f_{l}^{p})^{2} + (f_{l}^{q})^{2} - \sum_{i} V_{i}I_{l}) \\ &+ \sum_{l} \zeta_{l}(f_{l}^{p})^{2} + ((f_{l}^{q})^{2} - S_{l}^{2}) + \sum_{g} \phi_{g}(P_{g}^{2} + Q_{g}^{2} - S_{g}^{2}) \\ &+ \sum_{i} \sigma_{i}^{+}(V_{i} - V_{i}^{max}) + \sum_{i} \sigma_{i}^{-}(V_{i}^{min} - V_{i}) \\ &+ \sum_{g} \delta_{g}^{+}(Q_{g} - Q_{i}^{gmax}) + \sum_{g} \delta_{g}^{-}(Q_{g}^{gmin} - Q_{g}) \\ &+ \sum_{g} \beta_{g}^{+}(P_{g} - P_{g}^{gmax}) + \sum_{g} \beta_{g}^{-}(P_{g}^{gmin} - P_{g}) \end{split}$$

Beside the primary conditions which are constraints (B.23)-(B.31), the stationary conditions of the Lagrangian function with respect to the optimization variables are:

$$(P_g): O_g + \lambda_i + \varphi_g^+ - \varphi_g^- + \beta_g^+ - \beta_g^- = 0$$
 (B.33)

$$(Q_g): \mu_i + \delta_g^+ - \delta_g^- = 0$$
 (B.34)

$$(f_l^p) :- 2r_l \theta_{i,l} + \lambda_i - \lambda_{A_i} + 2\xi f_l^p + 2\zeta f_l^p = 0$$
 (B.35)

$$(f_l^q) :- 2x_l \theta_{i,l} + \mu_i - \mu_{A_i} + \mu_i^0 - \mu_{A_i}^0 + 2\xi f_l^q + 2\zeta f_l^q = 0$$
(B.36)

$$(V_i): \theta_{i,l} - \theta_{A_i,l} - G_i \lambda_i - \mu_i b_i - \xi_l I_l - \vartheta_i I_{l,t} + \sigma_i^+ - \sigma_i^- = 0$$
(B.37)

$$(I_l): (r_l^2 + x_l^2)\theta_{i,l} - r_l\lambda_i - \mu_i - \mu_i^0 - \xi V_i - \vartheta_i V_i = 0$$
(B.38)

$$0 \le \xi_l \perp (f_l^{p^2} + f_l^{q^2} - V_i I_l)$$
(B.39)

$$0 \le \zeta_l \perp (f_l^{p^2} + f_l^{q^2} - S_l^2) \tag{B.40}$$

$$0 \le \phi_g \perp (P_g^2 + Q_g^2 - S_g^2) \tag{B.41}$$

$$0 \le \sigma_i^+ \perp (V_i - V_i^{max}) \tag{B.42}$$

$$0 \le \sigma_i^- \perp (V_i^{min} - V_i) \tag{B.43}$$

$$0 \le \delta_g^+ \perp (Q_g - Q_i^{gmax}) \tag{B.44}$$

$$0 \le \delta_q^- \perp (Q_i^{gmin} - Q_g) \tag{B.45}$$

$$0 \le \beta_g^+ \perp (P_g - P_g^{gmax}) \tag{B.46}$$

$$0 \le \beta_g^- \perp (P_g^{gmin} - P_g) \tag{B.47}$$

The KKT conditions (D.10) and (D.11) connect the real and reactive power price at the node *i* to the price of its ancestor. In order to replace  $\theta_{i,l,t}$  and  $\xi_{l,t}$  in (D.10), an additional KKT condition (D.13) is derived. Therefore, it can be concluded that the DLMP ( $\lambda_i$ )at node *i* is a function of the active power price at the ancestor node  $A_i$ , the reactive power price at the current and ancestor node, and the price of congestion over the line attached to *i* and  $A_i$ , hence the DLMP at a distribution node *i* can be expressed as the sum of the following terms:

$$\lambda_i = \Gamma_1(\chi) \cdot \lambda_{A_i} + \Gamma_2(\chi) \cdot \mu_i + \Gamma_3(\chi) \cdot \mu_{A_i} + \Gamma_4(\chi) \cdot \zeta_l \tag{B.48}$$

where,  $\Gamma_n(\chi)$  is a function of decision variables,  $\chi$ , including active power flow  $(f_l^p)$ , reactive power flow  $(f_l^q)$  and current magnitude  $(I_l)$ .  $\Gamma_n(\chi)$  are calculated as:

$$\Gamma_{1} = \frac{((f_{l}^{p})^{2} + (f_{l}^{p})^{2})X_{l} + I_{l}f_{l}^{q}(R_{l}^{2} - X_{l}^{2}) - 2I_{l}f_{l}^{p}R_{l}X_{l}}{((f_{l}^{p})^{2} + (f_{l}^{p})^{2})X_{l} - I_{l}f_{l}^{q}(R_{l}^{2} + X_{l}^{2})}$$
(B.49)

$$\Gamma_2 = \frac{((f_l^p)^2 + (f_l^p)^2)R_l - I_l f_l^p (R_l^2 + X_l^2)}{((f_l^p)^2 + (f_l^p)^2)X_l - I_l f_l^q (R_l^2 + X_l^2)}$$
(B.50)

$$\Gamma_3 = \frac{-((f_l^p)^2 + (f_l^p)^2)R_l + I_l f_l^p (R_l^2 - X_l^2) + 2I_l f_l^q R_l X_l}{((f_l^p)^2 + (f_l^p)^2)X_l - I_l f_l^q (R_l^2 + X_l^2)}$$
(B.51)

$$\Gamma_4 = \frac{2((f_l^q)^3 R_l - (f_l^p)^3 X_l) + 2f_l^p f_l^q (f_l^p R_l - f_l^q X_l)}{((f_l^p)^2 + (f_l^p)^2) X_l - I_l f_l^q (R_l^2 + X_l^2)}$$
(B.52)

# C Input data

This appendix explains the input data have been used for the research work in this thesis. Section C.1 shows the data belonged to the transmission grid. Section C.2 presents the input data belonged to the distribution grid.



Figure C.1: Transmission and distribution systems case studies

#### C.1 Transmission grid data

Figure C.1 shows the IEEE-24-bus test system used as the transmission grid. Data related to the branch susceptance (as used in modelling DC power flow) are provided in the test system [173]. The data related to generators and loads connected to the transmission grid are presented as follow.

#### C.1.1 Generators

Table C.1 summarizes the data for generators at transmission grid including their location, maximum capacity, their energy and revere costs.

#### C.1.2 Loads

The loads connected to the transmission grid are forecasted through Neural Network scenario generation techniques. The inputs of 29 neurons in three layers with hyper parameter tuning, is preprocessed through minimax mapping. For the training of the network, Levenberg-Marquardt back-propagation has been applied. Sigmoid Symmetric has been used as the activation Function. The data for day-ahead market price, imbalance rices and MW total imbalances are based on 2019 from ENTSO-e transparency platform. Table C.2 shows the load data produced for 10 scenarios in day-ahead market.

Gens. bus no.	$P_g^{gmax}$	$P_g^{gmax}$	$O_{g,t}^{ET}$	$O_{g,t}^{RT}$
	$\mathbf{M}\mathbf{W}$	$\mathbf{M}\mathbf{W}$	€/MWh	€/MW
1	152	$_{30,4}$	90.58	50
2	152	$_{30,4}$	90.58	50
7	300	75	130.63	70
13	591	$206,\!85$	130.27	70
15	60	12	210	120
15	155	$54,\!25$	60.75	40
16	155	$54,\!25$	60.75	40
18	400	400	30.39	20
21	400	400	30.39	20
23	310	108,5	60.75	40
23	350	140	70.03	50

Table C.1: Data for transmission generators

		Tab	le $C.2$	: Data	a for t	ransm	ission	loads		
	1	<b>2</b>	3	4	<b>5</b>	6	7	8	9	10
1	116	107	126	122	121	123	123	115	104	127
2	109	105	121	118	115	117	116	109	103	122
3	106	106	118	116	113	114	113	106	104	120
4	104	106	117	114	111	112	111	105	105	120
5	103	108	117	115	112	113	111	104	107	120
6	103	116	121	120	117	118	111	103	115	125
$\overline{7}$	101	126	131	130	129	128	112	102	129	136
8	100	142	144	144	142	140	113	101	146	150
9	100	149	116	150	146	146	114	101	153	124
10	100	150	152	150	146	142	114	100	154	159
11	100	148	149	150	145	138	110	99	151	157
12	99	148	147	149	146	136	108	98	151	154
13	97	146	142	147	144	132	109	96	149	151
14	95	146	145	151	144	131	109	95	149	153
15	93	146	146	150	142	131	107	94	148	153
16	96	148	148	152	143	131	108	96	150	155
17	106	148	152	153	144	133	111	104	151	159
18	118	153	155	155	149	139	120	116	154	162
19	121	148	151	147	146	138	123	120	149	156
20	123	148	152	149	147	138	125	121	149	156
21	125	150	156	149	150	140	130	125	154	160
22	128	150	156	148	148	137	133	125	151	156
23	123	143	147	139	140	131	129	120	144	148
24	116	136	138	132	132	126	123	113	137	139

#### C.1.3 Day-ahead and balancing market price and transmission imbalances

The day-ahead and balancing market prices are forecasted through Neural Network scenario generation techniques from 2019 ENTSO-e transparency platform.



Figure C.2: Scenarios for day-ahead market prices over 24 hours

	Table	e C.3	Da	ta ior	· day	-anea	ia ma	arket	price	)
	1	<b>2</b>	3	4	<b>5</b>	6	7	8	9	10
1	25	23	23	23	23	25	27	22	23	23
2	25	23	23	23	23	25	27	22	23	23
3	28	25	25	25	22	24	26	23	25	25
4	29	25	36	30	21	26	26	23	26	19
5	30	27	25	24	23	17	29	30	29	18
6	28	26	26	25	24	28	31	15	27	26
7	25	23	23	22	22	26	25	22	28	35
8	31	29	42	23	28	40	25	40	31	42
9	49	48	48	48	49	51	35	39	46	61
10	49	53	52	56	56	54	47	41	48	50
11	49	50	50	50	52	58	39	48	51	50
12	43	50	51	39	51	52	40	62	51	51
13	47	48	49	48	52	48	41	52	49	48
14	45	49	47	47	51	47	37	37	45	49
15	47	46	42	50	50	47	34	42	45	46
16	44	44	42	45	49	43	33	40	46	45
17	44	46	44	44	47	46	35	41	40	45
18	47	47	48	48	46	45	35	46	47	48
19	43	46	47	36	53	40	35	46	45	46
20	42	41	41	41	42	40	32	27	26	53
21	41	23	34	35	41	27	28	34	30	34
22	38	25	31	26	36	31	24	23	38	25
23	27	27	28	27	29	43	26	21	37	34
24	28	24	28	22	30	38	26	24	33	28

	1	<b>2</b>	3	4	5	6	7	8	9	10
1	82	86	31	22	48	58	24	23	74	67
2	33	86	78	31	1	80	10	41	47	88
3	68	58	56	54	47	43	4	102	51	97
4	82	70	12	4	66	33	44	29	47	22
5	47	50	37	65	28	65	27	32	7	84
6	65	69	56	72	1	53	83	14	23	112
7	65	35	75	41	62	54	64	28	144	77
8	39	33	48	45	57	64	94	7	30	51
9	53	86	54	82	54	54	137	11	46	102
10	47	76	107	69	29	77	56	50	43	113
11	51	60	113	81	78	87	31	23	21	93
12	33	51	76	57	41	67	21	21	58	111
13	50	77	78	17	74	61	1	53	81	123
14	68	78	118	4	$\overline{7}$	36	20	3	102	132
15	68	97	102	56	2	38	33	1	54	78
16	68	71	23	49	16	38	16	7	14	56
17	33	52	61	12	92	34	110	37	48	76
18	65	48	100	14	22	20	33	48	58	92
19	65	55	97	56	28	25	91	48	44	115
20	64	56	53	66	41	25	28	27	8	58
21	51	61	42	2	24	61	38	14	29	48
22	65	87	34	40	19	58	8	80	33	80
23	65	74	46	15	43	43	16	46	70	106
24	65	42	36	20	62	4	26	42	78	95

Table C.4: Data for positive imbalance price

#### C.2 Distribution grid data

Figure C.1 shows the radial 30-bus medium voltage Dutch distribution grid connected to the transmission grid. The corresponding data for branches, generators and loads are given as follow.

#### C.2.1 Branches

Table C.7 shows the data for distribution branches including their resistance (R), reactants (X) and susceptance (B) and admittance (G).

#### C.2.2 Generators

Table C.8 summarizes the data for generators at distribution grids including their location, maximum capacity, their energy and revere costs. A wind turbine

	T	2	3	4	5	6	1	8	9	10
1	15	21	26	49	51	16	57	55	13	34
2	19	27	16	25	12	13	48	20	24	47
3	17	20	10	29	39	15	24	72	6	24
4	15	23	26	11	27	15	9	54	18	33
5	17	20	6	7	32	14	5	15	17	14
6	16	17	19	26	32	14	4	11	10	11
7	16	15	17	51	53	16	20	33	17	27
8	20	26	20	17	43	14	40	14	23	42
9	17	17	19	11	49	16	18	2	15	22
10	17	2	10	20	23	14	39	32	13	13
11	18	10	3	10	10	12	3	15	$\overline{7}$	2
12	19	17	5	25	17	17	8	23	23	4
13	18	16	2	30	2	10	27	30	23	16
14	17	$\overline{7}$	6	43	13	17	47	16	1	4
15	17	19	8	46	64	17	17	16	20	16
16	17	21	26	36	45	17	16	38	19	5
17	19	19	13	52	59	17	21	21	17	3
18	16	19	9	27	48	15	42	3	18	24
19	16	13	20	10	20	19	19	20	12	1
20	16	19	8	9	26	19	39	1	11	28
21	18	19	22	45	31	19	63	35	25	16
22	16	10	23	4	32	16	73	37	24	2
23	16	14	17	28	$\overline{7}$	15	57	6	13	5
24	16	27	21	49	38	22	56	4	9	3

Table C.5: Data for negative imbalance price for 10 scenarios and over 24-hours

is located at bus number 18 (at the end of the feeder) of the distribution grid.

#### C.2.3 Loads

Table C.9 shows the data for loads connected to each bus (rows of the table denotes the bus number) at the distribution grid for 24 hours day-ahead market.

#### C.2.4 Wind power

Figure C.3 shows the data for wind turbine connected to the distribution grid.

	1	<b>2</b>	3	4	5	6	7	8	9	10
1	11	7	7	8	6	7	11	4	8	11
2	8	8	9	6	10	9	5	-1	7	12
3	10	12	9	3	2	9	9	7	8	9
4	11	10	6	8	13	$\overline{7}$	5	-3	7	7
5	9	7	5	7	7	8	7	11	6	13
6	10	10	6	9	2	$\overline{7}$	7	-4	7	14
7	10	10	10	9	7	8	7	-2	9	10
8	8	11	8	11	18	9	7	-2	5	5
9	9	12	8	9	6	8	7	-5	8	11
10	9	8	11	10	9	9	13	14	9	12
11	10	10	12	8	6	11	12	1	7	12
12	8	12	10	6	8	11	10	5	$\overline{7}$	13
13	9	11	9	5	0	10	10	-5	8	14
14	10	11	11	1	6	$\overline{7}$	4	-8	13	13
15	10	11	12	7	-3	$\overline{7}$	3	0	10	11
16	10	11	8	8	8	$\overline{7}$	8	1	4	12
17	8	9	9	7	-10	8	5	2	7	13
18	10	10	9	10	-2	9	6	10	8	12
19	10	7	10	10	2	5	9	6	6	14
20	9	9	10	9	12	5	4	-7	5	9
21	10	8	7	3	2	9	3	-9	5	9
22	10	12	6	8	8	$\overline{7}$	4	-6	8	9
23	10	10	6	7	6	9	6	-8	11	15
24	10	10	9	11	10	6	7	-14	13	13

Table C.6: Data for imbalance at the transmission system



Figure C.3: Scenarios for wind power over 24 hours

From	<u>To</u>			<u>nanches</u>
$\mathbf{bus}$	$\mathbf{bus}$	R	Х	В
1	2	1.30E-02	5.21E-03	6.58E-04
1	16	6.65E-03	2.66E-03	3.37E-04
1	31	2.06E-02	8.24E-03	1.04E-03
2	3	4.00E-03	9.21E-04	1.16E-04
3	4	2.00E-03	4.60E-04	5.78E-05
4	5	2.08E-02	4.78E-03	6.00E-04
5	6	3.59E-03	8.25E-04	1.04E-04
6	7	1.20E-02	2.76E-03	3.47E-04
7	8	3.90E-02	$8.97 \text{E}{-} 03$	1.13E-03
8	9	8.34E-03	1.92E-03	2.41E-04
10	11	2.30E-02	9.21E-03	1.17E-03
11	12	5.95E-03	2.38E-03	3.01E-04
12	13	8.11E-03	3.25E-03	4.11E-04
13	14	7.68E-03	3.07E-03	3.89E-04
14	15	1.29E-02	5.18E-03	6.56E-04
15	16	2.29E-02	9.16E-03	1.16E-03
10	17	3.07E-02	7.06E-03	8.87E-04
17	18	4.21E-03	9.69E-04	1.22E-04
18	19	6.42E-03	1.48E-03	1.86E-04
20	21	1.28E-02	2.95E-03	3.71E-04
21	22	5.55E-03	1.28E-03	1.60E-04
22	23	5.51E-03	1.27E-03	1.59E-04
23	24	5.76E-03	1.32E-03	1.66E-04
24	25	5.84E-03	1.34E-03	1.69E-04
25	26	4.30E-03	9.88E-04	1.24E-04
26	27	8.97E-03	2.06E-03	2.59E-04
27	28	5.80E-03	1.33E-03	1.67E-04
28	29	9.09E-03	2.09E-03	2.63E-04
29	30	5.67 E-03	1.30E-03	1.64E-04
30	31	4.33E-03	1.73E-03	2.19E-04

Table C.7: Data for distribution branches

 Table C.8: Data for distribution generators

Gens. bus no.	$P_g^{gmax}$ <b>MW</b>	$O_{g,t}^{ED}$ ${ { f / MWh }}$	$O_{g,t}^{RD}$ ${ { {                                $
3	5	25	12
4	5	20	10
5	5	15	7.5
17	5	30	15
19	5	22	12
26	5	22	12
29	5	18	9
31	5	18	9

			Lai	<u>ле с.:</u>	<u>). Dat</u>	<u>a 101 (</u>	nsum	ution .	luaus			
	1	2	3	4	5	6	7	8	9	10	11	12
1	0.10	0.09	0.09	0.08	0.09	0.09	0.12	0.18	0.19	0.23	0.33	0.24
0	0.10	0.00	0.00	0.00	0.00	0.00	1.04	1.94	1.00	1.00	0.00	0.21
2	0.92	0.75	0.67	0.61	0.62	0.74	1.04	1.34	1.68	1.83	2.04	2.14
3	0.30	0.27	0.26	0.26	0.27	0.31	0.41	0.54	0.62	0.68	0.70	0.69
4	0.09	0.08	0.08	0.08	0.08	0.10	0.13	0.17	0.19	0.21	0.22	0.22
-	0.94	0.91	0.20	0.20	0.91	0.94	0.20	0.42	0.40	0.52	0.55	0.54
Э	0.24	0.21	0.20	0.20	0.21	0.24	0.32	0.43	0.49	0.53	0.55	0.54
6	0.32	0.28	0.25	0.25	0.26	0.33	0.45	0.58	0.66	0.64	0.74	0.67
7	0.52	0.42	0.37	0.34	0.32	0.39	0.48	0.76	0.86	0.80	0.95	1.09
	0.05	0.04	0.04	0.04	0.07	0.07	0.07	0.10	0.10	0.10	0.00	0.07
0	0.05	0.04	0.04	0.04	0.07	0.07	0.07	0.12	0.12	0.10	0.09	0.07
9	0.62	0.54	0.46	0.43	0.42	0.49	0.69	0.88	1.10	1.18	1.26	1.22
10	0.33	0.29	0.23	0.22	0.23	0.26	0.42	0.49	0.54	0.53	0.56	0.76
11	0.22	0.10	0.19	0.17	0.16	0.22	0.42	0.47	0.42	0.42	0.52	0.57
11	0.23	0.19	0.18	0.17	0.10	0.22	0.42	0.47	0.43	0.45	0.55	0.57
12	0.90	0.73	0.64	0.60	0.56	0.69	0.92	1.26	1.43	1.56	1.82	1.75
13	0.63	0.52	0.45	0.41	0.45	0.60	0.80	0.95	1.29	1.26	1.35	1.31
14	0.01	0.76	0.65	0.64	0.65	0.70	0.02	1 49	1.61	1 72	1.07	1 02
14	0.91	0.70	0.05	0.04	0.05	0.19	0.92	1.42	1.01	1.12	1.97	1.92
15	0.51	0.40	0.38	0.35	0.35	0.38	0.57	0.73	0.90	0.97	1.01	1.04
16	0.05	0.05	0.05	0.05	0.05	0.05	0.12	0.14	0.12	0.12	0.09	0.09
17	0.03	0.03	0.03	0.03	0.03	0.04	0.05	0.06	0.07	0.08	0.08	0.08
11	0.03	0.05	0.03	0.03	0.03	0.04	0.05	0.00	0.07	0.08	0.08	0.08
18	0.94	0.72	0.66	0.62	0.62	0.77	1.15	1.50	1.67	1.73	1.66	1.96
19	0.24	0.22	0.21	0.21	0.22	0.25	0.33	0.44	0.50	0.55	0.57	0.56
20	0.41	0.32	0.27	0.27	0.26	0.29	0.38	0.59	0.65	0.73	0.92	0.70
01	0.11	0.02	0.21	0.21	0.20	0.20	0.00	1.10	1.10	1.00	1.05	1.40
21	0.67	0.54	0.51	0.45	0.48	0.59	0.88	1.16	1.19	1.26	1.35	1.43
22	0.26	0.20	0.19	0.17	0.18	0.19	0.27	0.53	0.51	0.44	0.52	0.56
23	0.32	0.27	0.24	0.26	0.23	0.27	0.50	0.59	0.62	0.70	0.72	0.73
24	0.54	0.45	0.40	0.20	0.40	0.46	0.69	0.00	1.06	1 10	1 1 1	1 10
24	0.34	0.45	0.40	0.39	0.40	0.40	0.08	0.90	1.00	1.10	1.11	1.10
25	0.73	0.60	0.53	0.52	0.51	0.62	0.89	1.32	1.46	1.54	1.61	1.64
26	0.15	0.14	0.13	0.13	0.14	0.16	0.21	0.28	0.32	0.34	0.36	0.35
27	0.21	0.26	0.24	0.21	0.24	0.28	0.42	0.51	0.59	0.67	0.56	0.56
41	0.31	0.20	0.24	0.21	0.24	0.28	0.42	0.51	0.38	0.07	0.50	0.50
28	0.03	0.03	0.03	0.03	0.03	0.04	0.05	0.06	0.07	0.08	0.08	0.08
29	0.10	0.09	0.08	0.08	0.09	0.10	0.13	0.18	0.20	0.22	0.23	0.22
30	0.74	0.61	0.54	0.51	0.52	0.62	0.86	1 1 1	1 28	1.20	1 / 8	1.45
0	0.14	0.01	0.04	0.01	0.02	0.02	0.00	1.11	1.20	1.20	1.40	1.40
	13	14	15	16	17	18	19	20	91	22	23	24
	13	14	15	16	17	18	19	20	21	22	23	24
1	13 0.18	14 0.24	15 0.21	16 0.22	17 0.27	18 0.23	19 0.27	20 0.28	21 0.24	22 0.22	23 0.17	24 0.12
1 2	$13 \\ 0.18 \\ 2.15$	$     \begin{array}{r}       14 \\       0.24 \\       2.13     \end{array} $	$15 \\ 0.21 \\ 2.24$	$16 \\ 0.22 \\ 2.19$	$17 \\ 0.27 \\ 2.46$	$     \begin{array}{r}       18 \\       0.23 \\       2.47     \end{array} $	$19 \\ 0.27 \\ 2.43$	20 0.28 2.23	21 0.24 2.14	22 0.22 2.07	$23 \\ 0.17 \\ 1.71$	24 0.12 1.21
1 2 3	$13 \\ 0.18 \\ 2.15 \\ 0.68$	14 0.24 2.13 0.68	$15 \\ 0.21 \\ 2.24 \\ 0.66$	16 0.22 2.19 0.65	17 0.27 2.46 0.64	18 0.23 2.47 0.62	$     19 \\     0.27 \\     2.43 \\     0.61 $	20 0.28 2.23 0.63	21 0.24 2.14 0.59	22 0.22 2.07 0.54	23 0.17 1.71 0.45	24 0.12 1.21 0.37
$\frac{1}{2}$	$     \begin{array}{r}       13 \\       0.18 \\       2.15 \\       0.68 \\       0.21     \end{array} $	14 0.24 2.13 0.68 0.21	$     \begin{array}{r}       15 \\       0.21 \\       2.24 \\       0.66 \\       0.21     \end{array} $	$     \begin{array}{r}       16 \\       0.22 \\       2.19 \\       0.65 \\       0.20 \\     \end{array} $	$     \begin{array}{r}       17 \\       0.27 \\       2.46 \\       0.64 \\       0.20 \\     \end{array} $	$     18 \\     0.23 \\     2.47 \\     0.62 \\     0.10 $	$     19 \\     0.27 \\     2.43 \\     0.61 \\     0.10 $	20 0.28 2.23 0.63 0.20	$   \begin{array}{r}     21 \\     0.24 \\     2.14 \\     0.59 \\     0.18 \\   \end{array} $	$\begin{array}{r} 22 \\ 0.22 \\ 2.07 \\ 0.54 \\ 0.17 \end{array}$	$\begin{array}{r} 23 \\ 0.17 \\ 1.71 \\ 0.45 \\ 0.14 \end{array}$	24 0.12 1.21 0.37
$\begin{array}{c} 1\\ 2\\ 3\\ 4 \end{array}$	$     \begin{array}{r}       13 \\       0.18 \\       2.15 \\       0.68 \\       0.21 \\     \end{array} $	$     \begin{array}{r}       14 \\       0.24 \\       2.13 \\       0.68 \\       0.21 \\     \end{array} $	$     \begin{array}{r}       15 \\       0.21 \\       2.24 \\       0.66 \\       0.21 \\     \end{array} $	$     \begin{array}{r}       16 \\       0.22 \\       2.19 \\       0.65 \\       0.20 \\     \end{array} $	$     \begin{array}{r}       17 \\       0.27 \\       2.46 \\       0.64 \\       0.20 \\     \end{array} $	$     \begin{array}{r}       18 \\       0.23 \\       2.47 \\       0.62 \\       0.19 \\     \end{array} $	$     \begin{array}{r}       19 \\       0.27 \\       2.43 \\       0.61 \\       0.19 \\     \end{array} $	20 0.28 2.23 0.63 0.20	$\begin{array}{r} 21 \\ 0.24 \\ 2.14 \\ 0.59 \\ 0.18 \end{array}$	22 0.22 2.07 0.54 0.17	$\begin{array}{r} 23 \\ 0.17 \\ 1.71 \\ 0.45 \\ 0.14 \end{array}$	24 0.12 1.21 0.37 0.12
$\begin{array}{c} 1\\ 2\\ 3\\ 4\\ 5 \end{array}$	$     \begin{array}{r}       13 \\       0.18 \\       2.15 \\       0.68 \\       0.21 \\       0.53 \\     \end{array} $	$     \begin{array}{r}       14 \\       0.24 \\       2.13 \\       0.68 \\       0.21 \\       0.53 \\     \end{array} $	$     \begin{array}{r}       15 \\       0.21 \\       2.24 \\       0.66 \\       0.21 \\       0.52     \end{array} $	$     \begin{array}{r}       16 \\       0.22 \\       2.19 \\       0.65 \\       0.20 \\       0.51 \\     \end{array} $	$     \begin{array}{r}       17 \\       0.27 \\       2.46 \\       0.64 \\       0.20 \\       0.50 \\     \end{array} $	$     18 \\     0.23 \\     2.47 \\     0.62 \\     0.19 \\     0.49   $	$     \begin{array}{r}       19 \\       0.27 \\       2.43 \\       0.61 \\       0.19 \\       0.47     \end{array} $	20 0.28 2.23 0.63 0.20 0.49	$\begin{array}{r} 21 \\ 0.24 \\ 2.14 \\ 0.59 \\ 0.18 \\ 0.46 \end{array}$	$\begin{array}{r} 22\\ 0.22\\ 2.07\\ 0.54\\ 0.17\\ 0.42 \end{array}$	$\begin{array}{r} 23 \\ 0.17 \\ 1.71 \\ 0.45 \\ 0.14 \\ 0.35 \end{array}$	24 0.12 1.21 0.37 0.12 0.29
$     \begin{array}{c}       1 \\       2 \\       3 \\       4 \\       5 \\       6     \end{array} $	$     \begin{array}{r}       13 \\       0.18 \\       2.15 \\       0.68 \\       0.21 \\       0.53 \\       0.85 \\     \end{array} $	$ \begin{array}{r} 14\\ 0.24\\ 2.13\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ \end{array} $	$     \begin{array}{r}       15 \\       0.21 \\       2.24 \\       0.66 \\       0.21 \\       0.52 \\       0.82 \\     \end{array} $	$     \begin{array}{r}       16 \\       0.22 \\       2.19 \\       0.65 \\       0.20 \\       0.51 \\       0.74 \\     \end{array} $	$     \begin{array}{r}       17 \\       0.27 \\       2.46 \\       0.64 \\       0.20 \\       0.50 \\       0.87 \\     \end{array} $	$     18 \\     0.23 \\     2.47 \\     0.62 \\     0.19 \\     0.49 \\     0.95   $	$     \begin{array}{r}       19 \\       0.27 \\       2.43 \\       0.61 \\       0.19 \\       0.47 \\       0.90 \\     \end{array} $	20 0.28 2.23 0.63 0.20 0.49 0.83	$\begin{array}{r} 21 \\ 0.24 \\ 2.14 \\ 0.59 \\ 0.18 \\ 0.46 \\ 0.89 \end{array}$	$\begin{array}{r} 22\\ 0.22\\ 2.07\\ 0.54\\ 0.17\\ 0.42\\ 0.83 \end{array}$	$\begin{array}{r} 23 \\ 0.17 \\ 1.71 \\ 0.45 \\ 0.14 \\ 0.35 \\ 0.61 \end{array}$	$\begin{array}{r} 24 \\ 0.12 \\ 1.21 \\ 0.37 \\ 0.12 \\ 0.29 \\ 0.48 \end{array}$
$     \begin{array}{c}       1 \\       2 \\       3 \\       4 \\       5 \\       6 \\       7     \end{array} $	$ \begin{array}{r} 13\\ 0.18\\ 2.15\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 1.18 \end{array} $	$ \begin{array}{r} 14\\ 0.24\\ 2.13\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 0.87\\ \end{array} $	$     \begin{array}{r}       15 \\       0.21 \\       2.24 \\       0.66 \\       0.21 \\       0.52 \\       0.82 \\       0.91 \\     \end{array} $	$ \begin{array}{r} 16\\ 0.22\\ 2.19\\ 0.65\\ 0.20\\ 0.51\\ 0.74\\ 0.07 \end{array} $	$     \begin{array}{r}       17 \\       0.27 \\       2.46 \\       0.64 \\       0.20 \\       0.50 \\       0.87 \\       1.20 \\     \end{array} $	$     18 \\     0.23 \\     2.47 \\     0.62 \\     0.19 \\     0.49 \\     0.95 \\     1.20 $	$     \begin{array}{r}       19 \\       0.27 \\       2.43 \\       0.61 \\       0.19 \\       0.47 \\       0.90 \\       1.27 \\     \end{array} $	20 0.28 2.23 0.63 0.20 0.49 0.83 1.22	$\begin{array}{r} 21 \\ 0.24 \\ 2.14 \\ 0.59 \\ 0.18 \\ 0.46 \\ 0.89 \\ 1.02 \end{array}$	$\begin{array}{r} 22\\ 0.22\\ 2.07\\ 0.54\\ 0.17\\ 0.42\\ 0.83\\ 0.97 \end{array}$	$\begin{array}{c} 23 \\ 0.17 \\ 1.71 \\ 0.45 \\ 0.14 \\ 0.35 \\ 0.61 \\ 0.82 \end{array}$	24 0.12 1.21 0.37 0.12 0.29 0.48 0.64
$     \begin{array}{c}       1 \\       2 \\       3 \\       4 \\       5 \\       6 \\       7 \\       7     \end{array} $	$\begin{array}{c} 13 \\ 0.18 \\ 2.15 \\ 0.68 \\ 0.21 \\ 0.53 \\ 0.85 \\ 1.18 \\ 0.68 \end{array}$	14 0.24 2.13 0.68 0.21 0.53 0.85 0.85	$     \begin{array}{r}       15 \\       0.21 \\       2.24 \\       0.66 \\       0.21 \\       0.52 \\       0.82 \\       0.91 \\       0.51 \\       0.52 \\       0.82 \\       0.91 \\       0.51 \\       0.52 \\       0.82 \\       0.91 \\       0.52 \\       0.82 \\       0.91 \\       0.52 \\       0.82 \\       0.91 \\       0.52 \\       0.82 \\       0.91 \\       0.52 \\       0.82 \\       0.91 \\       0.52 \\       0.82 \\       0.91 \\       0.52 \\       0.82 \\       0.91 \\       0.52 \\       0.82 \\       0.91 \\       0.52 \\       0.82 \\       0.91 \\       0.52 \\       0.82 \\       0.91 \\       0.52 \\       0.82 \\       0.91 \\       0.52 \\       0.82 \\       0.91 \\       0.52 \\       0.82 \\       0.91 \\       0.52 \\       0.91 \\       0.52 \\       0.91 \\       0.52 \\       0.82 \\       0.91 \\       0.52 \\       0.82 \\       0.91 \\       0.52 \\       0.82 \\       0.91 \\       0.52 \\       0.82 \\       0.91 \\       0.52 \\       0.82 \\       0.91 \\       0.52 \\       0.82 \\       0.91 \\       0.52 \\       0.82 \\       0.91 \\       0.52 \\       0.82 \\       0.91 \\       0.52 \\       0.82 \\       0.91 \\       0.52 \\       0.82 \\       0.91 \\       0.52 \\       0.82 \\       0.91 \\       0.52 \\       0.82 \\       0.91 \\       0.52 \\       0.91 \\       0.52 \\       0.$	$ \begin{array}{r} 16\\ 0.22\\ 2.19\\ 0.65\\ 0.20\\ 0.51\\ 0.74\\ 0.97\\ 0.57\\ 0.74\\ 0.97\\ $	$     \begin{array}{r}       17 \\       0.27 \\       2.46 \\       0.64 \\       0.20 \\       0.50 \\       0.87 \\       1.20 \\       1.20 \\       \hline       1.20 \\        1.20 \\      1.20 \\ $	18     0.23     2.47     0.62     0.19     0.49     0.95     1.30     1.30     1	$     \begin{array}{r}       19 \\       0.27 \\       2.43 \\       0.61 \\       0.19 \\       0.47 \\       0.90 \\       1.27 \\       1.27 \\       1.2     $	20 0.28 2.23 0.63 0.20 0.49 0.83 1.23	$\begin{array}{c} 21 \\ 0.24 \\ 2.14 \\ 0.59 \\ 0.18 \\ 0.46 \\ 0.89 \\ 1.03 \\ 1.03 \end{array}$	22 0.22 2.07 0.54 0.17 0.42 0.83 0.97	$\begin{array}{r} 23 \\ 0.17 \\ 1.71 \\ 0.45 \\ 0.14 \\ 0.35 \\ 0.61 \\ 0.83 \end{array}$	$\begin{array}{r} 24 \\ 0.12 \\ 1.21 \\ 0.37 \\ 0.12 \\ 0.29 \\ 0.48 \\ 0.64 \\ 0.64 \end{array}$
$     \begin{array}{c}       1 \\       2 \\       3 \\       4 \\       5 \\       6 \\       7 \\       8     \end{array} $	$\begin{array}{c} 13 \\ 0.18 \\ 2.15 \\ 0.68 \\ 0.21 \\ 0.53 \\ 0.85 \\ 1.18 \\ 0.08 \end{array}$	$\begin{array}{r} 14\\ 0.24\\ 2.13\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 0.87\\ 0.10\\ \end{array}$	$\begin{array}{c} 15 \\ 0.21 \\ 2.24 \\ 0.66 \\ 0.21 \\ 0.52 \\ 0.82 \\ 0.91 \\ 0.19 \end{array}$	$\begin{array}{c} 16\\ 0.22\\ 2.19\\ 0.65\\ 0.20\\ 0.51\\ 0.74\\ 0.97\\ 0.17\\ \end{array}$	$\begin{array}{r} 17\\ 0.27\\ 2.46\\ 0.64\\ 0.20\\ 0.50\\ 0.87\\ 1.20\\ 0.14 \end{array}$	$\begin{array}{r} 18 \\ 0.23 \\ 2.47 \\ 0.62 \\ 0.19 \\ 0.49 \\ 0.95 \\ 1.30 \\ 0.17 \end{array}$	$     \begin{array}{r}       19 \\       0.27 \\       2.43 \\       0.61 \\       0.19 \\       0.47 \\       0.90 \\       1.27 \\       0.13 \\     \end{array} $	$\begin{array}{r} 20\\ 0.28\\ 2.23\\ 0.63\\ 0.20\\ 0.49\\ 0.83\\ 1.23\\ 0.15 \end{array}$	$\begin{array}{c} 21 \\ 0.24 \\ 2.14 \\ 0.59 \\ 0.18 \\ 0.46 \\ 0.89 \\ 1.03 \\ 0.13 \end{array}$	$\begin{array}{c} 22\\ 0.22\\ 2.07\\ 0.54\\ 0.17\\ 0.42\\ 0.83\\ 0.97\\ 0.11 \end{array}$	$\begin{array}{c} 23 \\ 0.17 \\ 1.71 \\ 0.45 \\ 0.14 \\ 0.35 \\ 0.61 \\ 0.83 \\ 0.10 \end{array}$	$\begin{array}{r} 24\\ 0.12\\ 1.21\\ 0.37\\ 0.12\\ 0.29\\ 0.48\\ 0.64\\ 0.08 \end{array}$
$     \begin{array}{c}       1 \\       2 \\       3 \\       4 \\       5 \\       6 \\       7 \\       8 \\       9     \end{array} $	$\begin{array}{c} 13\\ 0.18\\ 2.15\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 1.18\\ 0.08\\ 1.19\end{array}$	$\begin{array}{r} 14\\ 0.24\\ 2.13\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 0.87\\ 0.10\\ 1.32\\ \end{array}$	$     \begin{array}{r}       15 \\       0.21 \\       2.24 \\       0.66 \\       0.21 \\       0.52 \\       0.82 \\       0.91 \\       0.19 \\       1.28 \\     \end{array} $	$\begin{array}{r} 16\\ 0.22\\ 2.19\\ 0.65\\ 0.20\\ 0.51\\ 0.74\\ 0.97\\ 0.17\\ 1.25\\ \end{array}$	$\begin{array}{r} 17\\ 0.27\\ 2.46\\ 0.64\\ 0.20\\ 0.50\\ 0.87\\ 1.20\\ 0.14\\ 1.64 \end{array}$	$     \begin{array}{r}       18 \\       0.23 \\       2.47 \\       0.62 \\       0.19 \\       0.49 \\       0.95 \\       1.30 \\       0.17 \\       1.80 \\     \end{array} $	$ \begin{array}{r} 19\\ 0.27\\ 2.43\\ 0.61\\ 0.19\\ 0.47\\ 0.90\\ 1.27\\ 0.13\\ 1.50\\ \end{array} $	$\begin{array}{r} 20\\ 0.28\\ 2.23\\ 0.63\\ 0.20\\ 0.49\\ 0.83\\ 1.23\\ 0.15\\ 1.44 \end{array}$	$\begin{array}{r} 21 \\ 0.24 \\ 2.14 \\ 0.59 \\ 0.18 \\ 0.46 \\ 0.89 \\ 1.03 \\ 0.13 \\ 1.29 \end{array}$	$\begin{array}{c} 22\\ 0.22\\ 2.07\\ 0.54\\ 0.17\\ 0.42\\ 0.83\\ 0.97\\ 0.11\\ 1.18 \end{array}$	$\begin{array}{c} 23\\ 0.17\\ 1.71\\ 0.45\\ 0.14\\ 0.35\\ 0.61\\ 0.83\\ 0.10\\ 0.98 \end{array}$	$\begin{array}{c} 24 \\ 0.12 \\ 1.21 \\ 0.37 \\ 0.12 \\ 0.29 \\ 0.48 \\ 0.64 \\ 0.08 \\ 0.74 \end{array}$
$     \begin{array}{c}       1 \\       2 \\       3 \\       4 \\       5 \\       6 \\       7 \\       8 \\       9 \\       10 \\       \end{array} $	13 0.18 2.15 0.68 0.21 0.53 0.85 1.18 0.08 1.19 0.71	14 0.24 2.13 0.68 0.21 0.53 0.85 0.85 0.87 0.10 1.32 0.72	$\begin{array}{c} 15\\ 0.21\\ 2.24\\ 0.66\\ 0.21\\ 0.52\\ 0.82\\ 0.91\\ 0.19\\ 1.28\\ 0.72\end{array}$	$\begin{array}{c} 16\\ 0.22\\ 2.19\\ 0.65\\ 0.20\\ 0.51\\ 0.74\\ 0.97\\ 0.17\\ 1.25\\ 0.77\\ \end{array}$	$\begin{array}{r} 17\\ 0.27\\ 2.46\\ 0.64\\ 0.20\\ 0.50\\ 0.87\\ 1.20\\ 0.14\\ 1.64\\ 0.85\end{array}$	$\begin{array}{r} 18\\ 0.23\\ 2.47\\ 0.62\\ 0.19\\ 0.49\\ 0.95\\ 1.30\\ 0.17\\ 1.80\\ 0.80\end{array}$	$\begin{array}{r} 19\\ 0.27\\ 2.43\\ 0.61\\ 0.19\\ 0.47\\ 0.90\\ 1.27\\ 0.13\\ 1.50\\ 0.88\end{array}$	$\begin{array}{c} 20\\ 0.28\\ 2.23\\ 0.63\\ 0.20\\ 0.49\\ 0.83\\ 1.23\\ 0.15\\ 1.44\\ 0.75\end{array}$	$\begin{array}{c} 21 \\ 0.24 \\ 2.14 \\ 0.59 \\ 0.18 \\ 0.46 \\ 0.89 \\ 1.03 \\ 0.13 \\ 1.29 \\ 0.72 \end{array}$	22 0.22 2.07 0.54 0.17 0.42 0.83 0.97 0.11 1.18 0.62	$\begin{array}{c} 23\\ 0.17\\ 1.71\\ 0.45\\ 0.14\\ 0.35\\ 0.61\\ 0.83\\ 0.10\\ 0.98\\ 0.57\end{array}$	$\begin{array}{c} 24 \\ 0.12 \\ 1.21 \\ 0.37 \\ 0.12 \\ 0.29 \\ 0.48 \\ 0.64 \\ 0.08 \\ 0.74 \\ 0.42 \end{array}$
$   \begin{array}{c}     1 \\     2 \\     3 \\     4 \\     5 \\     6 \\     7 \\     8 \\     9 \\     10 \\     10 \\   \end{array} $	$\begin{array}{c} 13\\ 0.18\\ 2.15\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 1.18\\ 0.08\\ 1.19\\ 0.71\\ 0.71\\ 0.61\\ 0$	$\begin{array}{c} 14\\ 0.24\\ 2.13\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 0.87\\ 0.10\\ 1.32\\ 0.73\\ 0.73\end{array}$	$\begin{array}{c} 15\\ 0.21\\ 2.24\\ 0.66\\ 0.21\\ 0.52\\ 0.82\\ 0.91\\ 0.19\\ 1.28\\ 0.72\\ 0.72\\ 0.46\\ 0.72\\ 0.62\\ 0$	$\begin{array}{c} 16 \\ 0.22 \\ 2.19 \\ 0.65 \\ 0.20 \\ 0.51 \\ 0.74 \\ 0.97 \\ 0.17 \\ 1.25 \\ 0.77 \\ 0.77 \\ \end{array}$	$\begin{array}{c} 17\\ 0.27\\ 2.46\\ 0.64\\ 0.20\\ 0.50\\ 0.87\\ 1.20\\ 0.14\\ 1.64\\ 0.85\\ 0.57\\ \end{array}$	$\begin{array}{c} 18 \\ 0.23 \\ 2.47 \\ 0.62 \\ 0.19 \\ 0.49 \\ 0.95 \\ 1.30 \\ 0.17 \\ 1.80 \\ 0.89 \\ 0.87 \end{array}$	$ \begin{array}{r} 19\\ 0.27\\ 2.43\\ 0.61\\ 0.19\\ 0.47\\ 0.90\\ 1.27\\ 0.13\\ 1.50\\ 0.88\\ 0.88\\ \end{array} $	$\begin{array}{c} 20\\ 0.28\\ 2.23\\ 0.63\\ 0.20\\ 0.49\\ 0.83\\ 1.23\\ 0.15\\ 1.44\\ 0.75\\ \end{array}$	$\begin{array}{c} 21 \\ 0.24 \\ 2.14 \\ 0.59 \\ 0.18 \\ 0.46 \\ 0.89 \\ 1.03 \\ 0.13 \\ 1.29 \\ 0.72 \\ 0.72 \end{array}$	$\begin{array}{c} 22\\ 0.22\\ 2.07\\ 0.54\\ 0.17\\ 0.42\\ 0.83\\ 0.97\\ 0.11\\ 1.18\\ 0.63\\ 0.45\end{array}$	$\begin{array}{c} 23 \\ 0.17 \\ 1.71 \\ 0.45 \\ 0.14 \\ 0.35 \\ 0.61 \\ 0.83 \\ 0.10 \\ 0.98 \\ 0.57 \\ 0.57 \\ \end{array}$	$\begin{array}{c} 24 \\ 0.12 \\ 1.21 \\ 0.37 \\ 0.12 \\ 0.29 \\ 0.48 \\ 0.64 \\ 0.08 \\ 0.74 \\ 0.43 \\ 0.43 \\ 0.20 \end{array}$
$   \begin{array}{c}     1 \\     2 \\     3 \\     4 \\     5 \\     6 \\     7 \\     8 \\     9 \\     10 \\     11 \\   \end{array} $	$\begin{array}{c} 13\\ 0.18\\ 2.15\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 1.18\\ 0.08\\ 1.19\\ 0.71\\ 0.49\\ \end{array}$	$\begin{array}{c} 14\\ 0.24\\ 2.13\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 0.87\\ 0.10\\ 1.32\\ 0.73\\ 0.44 \end{array}$	$\begin{array}{c} 15\\ 0.21\\ 2.24\\ 0.66\\ 0.21\\ 0.52\\ 0.82\\ 0.91\\ 0.19\\ 1.28\\ 0.72\\ 0.48\\ \end{array}$	$\begin{array}{c} 16\\ 0.22\\ 2.19\\ 0.65\\ 0.20\\ 0.51\\ 0.74\\ 0.97\\ 0.17\\ 1.25\\ 0.77\\ 0.55\\ \end{array}$	$\begin{array}{c} 17\\ 0.27\\ 2.46\\ 0.64\\ 0.20\\ 0.50\\ 0.87\\ 1.20\\ 0.14\\ 1.64\\ 0.85\\ 0.65\\ \end{array}$	$\begin{array}{c} 18\\ 0.23\\ 2.47\\ 0.62\\ 0.19\\ 0.95\\ 1.30\\ 0.17\\ 1.80\\ 0.89\\ 0.75\\ \end{array}$	$\begin{array}{c} 19\\ 0.27\\ 2.43\\ 0.61\\ 0.19\\ 0.47\\ 0.90\\ 1.27\\ 0.13\\ 1.50\\ 0.88\\ 0.67\\ \end{array}$	$\begin{array}{c} 20\\ 0.28\\ 2.23\\ 0.63\\ 0.20\\ 0.49\\ 0.83\\ 1.23\\ 0.15\\ 1.44\\ 0.75\\ 0.57\\ \end{array}$	$\begin{array}{c} 21 \\ 0.24 \\ 2.14 \\ 0.59 \\ 0.18 \\ 0.46 \\ 0.89 \\ 1.03 \\ 0.13 \\ 1.29 \\ 0.72 \\ 0.59 \end{array}$	$\begin{array}{c} 22\\ 0.22\\ 2.07\\ 0.54\\ 0.17\\ 0.42\\ 0.83\\ 0.97\\ 0.11\\ 1.18\\ 0.63\\ 0.45\\ \end{array}$	$\begin{array}{c} 23\\ 0.17\\ 1.71\\ 0.45\\ 0.14\\ 0.35\\ 0.61\\ 0.83\\ 0.10\\ 0.98\\ 0.57\\ 0.39\\ \end{array}$	$\begin{array}{c} 24 \\ 0.12 \\ 1.21 \\ 0.37 \\ 0.12 \\ 0.29 \\ 0.48 \\ 0.64 \\ 0.08 \\ 0.74 \\ 0.43 \\ 0.29 \end{array}$
$   \begin{array}{c}     1 \\     2 \\     3 \\     4 \\     5 \\     6 \\     7 \\     8 \\     9 \\     10 \\     11 \\     12 \\   \end{array} $	$\begin{array}{c} 13 \\ 0.18 \\ 2.15 \\ 0.68 \\ 0.21 \\ 0.53 \\ 0.85 \\ 1.18 \\ 0.08 \\ 1.19 \\ 0.71 \\ 0.49 \\ 1.68 \end{array}$	$\begin{array}{c} 14\\ 0.24\\ 2.13\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 0.87\\ 0.10\\ 1.32\\ 0.73\\ 0.44\\ 1.74\end{array}$	$\begin{array}{c} 15\\ 0.21\\ 2.24\\ 0.66\\ 0.21\\ 0.52\\ 0.82\\ 0.91\\ 0.19\\ 1.28\\ 0.72\\ 0.48\\ 1.65\\ \end{array}$	$\begin{array}{c} 16 \\ 0.22 \\ 2.19 \\ 0.65 \\ 0.20 \\ 0.51 \\ 0.74 \\ 0.97 \\ 0.17 \\ 1.25 \\ 0.77 \\ 0.55 \\ 1.90 \end{array}$	$\begin{array}{c} 17\\ 0.27\\ 2.46\\ 0.64\\ 0.20\\ 0.50\\ 0.87\\ 1.20\\ 0.14\\ 1.64\\ 0.85\\ 0.65\\ 2.03\end{array}$	$\begin{array}{c} 18\\ 0.23\\ 2.47\\ 0.62\\ 0.19\\ 0.95\\ 1.30\\ 0.17\\ 1.80\\ 0.89\\ 0.75\\ 2.18\\ \end{array}$	$\begin{array}{c} 19\\ 0.27\\ 2.43\\ 0.61\\ 0.19\\ 0.47\\ 0.90\\ 1.27\\ 0.13\\ 1.50\\ 0.88\\ 0.67\\ 2.25\\ \end{array}$	$\begin{array}{c} 20\\ 0.28\\ 2.23\\ 0.63\\ 0.20\\ 0.49\\ 0.83\\ 1.23\\ 0.15\\ 1.44\\ 0.75\\ 0.57\\ 2.16\end{array}$	$\begin{array}{c} 21 \\ 0.24 \\ 2.14 \\ 0.59 \\ 0.18 \\ 0.46 \\ 0.89 \\ 1.03 \\ 0.13 \\ 1.29 \\ 0.72 \\ 0.59 \\ 1.96 \end{array}$	$\begin{array}{c} 22\\ 0.22\\ 2.07\\ 0.54\\ 0.17\\ 0.42\\ 0.83\\ 0.97\\ 0.11\\ 1.18\\ 0.63\\ 0.45\\ 1.79\\ \end{array}$	$\begin{array}{c} 23 \\ 0.17 \\ 1.71 \\ 0.45 \\ 0.14 \\ 0.35 \\ 0.61 \\ 0.83 \\ 0.10 \\ 0.98 \\ 0.57 \\ 0.39 \\ 1.52 \end{array}$	$\begin{array}{c} 24 \\ 0.12 \\ 1.21 \\ 0.37 \\ 0.12 \\ 0.29 \\ 0.48 \\ 0.64 \\ 0.08 \\ 0.74 \\ 0.43 \\ 0.29 \\ 1.20 \end{array}$
$ \begin{array}{c} 1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11\\12\\13\end{array}$	$\begin{array}{c} 13 \\ 0.18 \\ 2.15 \\ 0.68 \\ 0.21 \\ 0.53 \\ 0.85 \\ 1.18 \\ 0.08 \\ 1.19 \\ 0.71 \\ 0.49 \\ 1.68 \\ 1.37 \end{array}$	$\begin{array}{c} 14\\ 0.24\\ 2.13\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 0.87\\ 0.10\\ 1.32\\ 0.73\\ 0.44\\ 1.74\\ 1.28\end{array}$	$\begin{array}{c} 15\\ 0.21\\ 2.24\\ 0.66\\ 0.21\\ 0.52\\ 0.82\\ 0.91\\ 0.19\\ 1.28\\ 0.72\\ 0.48\\ 1.65\\ 1.31\\ \end{array}$	$\begin{array}{c} 16 \\ 0.22 \\ 2.19 \\ 0.65 \\ 0.20 \\ 0.51 \\ 0.74 \\ 0.97 \\ 0.17 \\ 1.25 \\ 0.77 \\ 0.55 \\ 1.90 \\ 1.51 \end{array}$	$\begin{array}{c} 17\\ 0.27\\ 2.46\\ 0.64\\ 0.20\\ 0.50\\ 0.87\\ 1.20\\ 0.14\\ 1.64\\ 0.85\\ 2.03\\ 1.59\end{array}$	$\begin{array}{c} 18 \\ 0.23 \\ 2.47 \\ 0.62 \\ 0.19 \\ 0.95 \\ 1.30 \\ 0.17 \\ 1.80 \\ 0.89 \\ 0.75 \\ 2.18 \\ 1.67 \end{array}$	$\begin{array}{c} 19\\ 0.27\\ 2.43\\ 0.61\\ 0.19\\ 0.47\\ 0.90\\ 1.27\\ 0.13\\ 1.50\\ 0.88\\ 0.67\\ 2.25\\ 1.62\end{array}$	$\begin{array}{c} 20\\ 0.28\\ 2.23\\ 0.63\\ 0.20\\ 0.49\\ 0.83\\ 1.23\\ 0.15\\ 1.44\\ 0.75\\ 0.57\\ 2.16\\ 1.55\end{array}$	$\begin{array}{c} 21 \\ 0.24 \\ 2.14 \\ 0.59 \\ 0.18 \\ 0.46 \\ 0.89 \\ 1.03 \\ 0.13 \\ 1.29 \\ 0.72 \\ 0.59 \\ 1.96 \\ 1.51 \end{array}$	$\begin{array}{c} 22\\ 0.22\\ 2.07\\ 0.54\\ 0.17\\ 0.42\\ 0.83\\ 0.97\\ 0.11\\ 1.18\\ 0.63\\ 0.45\\ 1.79\\ 1.31\\ \end{array}$	$\begin{array}{c} 23 \\ 0.17 \\ 1.71 \\ 0.45 \\ 0.14 \\ 0.35 \\ 0.61 \\ 0.83 \\ 0.10 \\ 0.98 \\ 0.57 \\ 0.39 \\ 1.52 \\ 0.98 \end{array}$	$\begin{array}{c} 24 \\ 0.12 \\ 1.21 \\ 0.37 \\ 0.12 \\ 0.29 \\ 0.48 \\ 0.64 \\ 0.08 \\ 0.74 \\ 0.43 \\ 0.29 \\ 1.20 \\ 0.69 \end{array}$
$ \begin{array}{c} 1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11\\12\\13\\14\end{array} $	$\begin{array}{c} 13 \\ 0.18 \\ 2.15 \\ 0.68 \\ 0.21 \\ 0.53 \\ 1.18 \\ 0.08 \\ 1.19 \\ 0.71 \\ 0.49 \\ 1.68 \\ 1.37 \\ 1.88 \end{array}$	$\begin{array}{c} 14 \\ 0.24 \\ 2.13 \\ 0.68 \\ 0.21 \\ 0.53 \\ 0.85 \\ 0.87 \\ 0.10 \\ 1.32 \\ 0.73 \\ 0.44 \\ 1.74 \\ 1.28 \\ 1.85 \end{array}$	$\begin{array}{c} 15 \\ 0.21 \\ 2.24 \\ 0.66 \\ 0.21 \\ 0.52 \\ 0.91 \\ 0.19 \\ 1.28 \\ 0.72 \\ 0.48 \\ 1.65 \\ 1.31 \\ 1.90 \end{array}$	$\begin{array}{c} 16 \\ 0.22 \\ 2.19 \\ 0.65 \\ 0.20 \\ 0.51 \\ 0.74 \\ 0.97 \\ 0.17 \\ 1.25 \\ 0.77 \\ 0.55 \\ 1.90 \\ 1.51 \\ 1.90 \end{array}$	$\begin{array}{c} 17\\ 0.27\\ 2.46\\ 0.64\\ 0.20\\ 0.50\\ 0.87\\ 1.20\\ 0.14\\ 1.64\\ 0.85\\ 0.65\\ 2.03\\ 1.59\\ 2.26\end{array}$	$\begin{array}{c} 18 \\ 0.23 \\ 2.47 \\ 0.62 \\ 0.19 \\ 0.49 \\ 0.95 \\ 1.30 \\ 0.17 \\ 1.80 \\ 0.89 \\ 0.75 \\ 2.18 \\ 1.67 \\ 2.55 \end{array}$	$\begin{array}{c} 19\\ 0.27\\ 2.43\\ 0.61\\ 0.19\\ 0.47\\ 0.90\\ 1.27\\ 0.13\\ 1.50\\ 0.88\\ 0.67\\ 2.25\\ 1.62\\ 2.42\end{array}$	$\begin{array}{c} 20\\ 0.28\\ 2.23\\ 0.63\\ 0.20\\ 0.49\\ 0.83\\ 1.23\\ 0.15\\ 1.44\\ 0.75\\ 0.57\\ 2.16\\ 1.55\\ 2.26\end{array}$	$\begin{array}{c} 21 \\ 0.24 \\ 2.14 \\ 0.59 \\ 0.18 \\ 0.46 \\ 0.89 \\ 1.03 \\ 0.13 \\ 1.29 \\ 0.72 \\ 0.59 \\ 1.96 \\ 1.51 \\ 2.08 \end{array}$	$\begin{array}{c} 22\\ 0.22\\ 2.07\\ 0.54\\ 0.17\\ 0.42\\ 0.83\\ 0.97\\ 0.11\\ 1.18\\ 0.63\\ 0.45\\ 1.79\\ 1.31\\ 1.00\end{array}$	$\begin{array}{c} 23 \\ 0.17 \\ 1.71 \\ 0.45 \\ 0.14 \\ 0.35 \\ 0.61 \\ 0.83 \\ 0.10 \\ 0.98 \\ 0.57 \\ 0.39 \\ 1.52 \\ 0.99 \\ 1.52 \\ 0.98 \\ 1.5c \end{array}$	$\begin{array}{c} 24 \\ 0.12 \\ 1.21 \\ 0.37 \\ 0.12 \\ 0.29 \\ 0.48 \\ 0.64 \\ 0.08 \\ 0.74 \\ 0.43 \\ 0.29 \\ 1.20 \\ 0.69 \\ 1.24 \end{array}$
$ \begin{array}{c} 1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11\\12\\13\\14\end{array}$	$\begin{array}{c} 13 \\ 0.18 \\ 2.15 \\ 0.68 \\ 0.21 \\ 0.53 \\ 0.85 \\ 1.18 \\ 0.08 \\ 1.19 \\ 0.71 \\ 0.49 \\ 1.68 \\ 1.37 \\ 1.88 \end{array}$	$\begin{array}{c} 14\\ 0.24\\ 2.13\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 0.87\\ 0.10\\ 1.32\\ 0.73\\ 0.44\\ 1.74\\ 1.28\\ 1.85\end{array}$	$\begin{array}{c} 15 \\ 0.21 \\ 2.24 \\ 0.66 \\ 0.21 \\ 0.52 \\ 0.91 \\ 0.19 \\ 1.28 \\ 0.72 \\ 0.48 \\ 1.65 \\ 1.31 \\ 1.90 \end{array}$	$\begin{array}{c} 16\\ 0.22\\ 2.19\\ 0.65\\ 0.20\\ 0.51\\ 0.74\\ 0.97\\ 0.17\\ 1.25\\ 0.77\\ 0.55\\ 1.90\\ 1.51\\ 1.99\\ \end{array}$	$\begin{array}{c} 17\\ 0.27\\ 2.46\\ 0.64\\ 0.20\\ 0.50\\ 0.87\\ 1.20\\ 0.14\\ 1.64\\ 0.85\\ 0.65\\ 2.03\\ 1.59\\ 2.36\end{array}$	$\begin{array}{c} 18 \\ 0.23 \\ 2.47 \\ 0.62 \\ 0.19 \\ 0.49 \\ 0.95 \\ 1.30 \\ 0.17 \\ 1.80 \\ 0.89 \\ 0.75 \\ 2.18 \\ 1.67 \\ 2.55 \end{array}$	$\begin{array}{c} 19\\ 0.27\\ 2.43\\ 0.61\\ 0.19\\ 0.47\\ 0.90\\ 1.27\\ 0.13\\ 1.50\\ 0.88\\ 0.67\\ 2.25\\ 1.62\\ 2.43\end{array}$	$\begin{array}{c} 20\\ 0.28\\ 2.23\\ 0.63\\ 0.20\\ 0.49\\ 0.83\\ 1.23\\ 0.15\\ 1.44\\ 0.75\\ 0.57\\ 2.16\\ 1.55\\ 2.26\end{array}$	$\begin{array}{c} 21 \\ 0.24 \\ 2.14 \\ 0.59 \\ 0.18 \\ 0.46 \\ 0.89 \\ 1.03 \\ 0.13 \\ 1.29 \\ 0.72 \\ 0.59 \\ 1.96 \\ 1.51 \\ 2.08 \end{array}$	$\begin{array}{c} 22\\ 0.22\\ 2.07\\ 0.54\\ 0.17\\ 0.42\\ 0.83\\ 0.97\\ 0.11\\ 1.18\\ 0.63\\ 0.45\\ 1.79\\ 1.31\\ 1.90\\ \end{array}$	$\begin{array}{c} 23 \\ 0.17 \\ 1.71 \\ 0.45 \\ 0.14 \\ 0.35 \\ 0.61 \\ 0.83 \\ 0.10 \\ 0.98 \\ 0.57 \\ 0.39 \\ 1.52 \\ 0.98 \\ 1.56 \end{array}$	$\begin{array}{c} 24 \\ 0.12 \\ 1.21 \\ 0.37 \\ 0.12 \\ 0.29 \\ 0.48 \\ 0.64 \\ 0.08 \\ 0.74 \\ 0.43 \\ 0.29 \\ 1.20 \\ 0.69 \\ 1.24 \end{array}$
$ \begin{array}{c} 1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11\\12\\13\\14\\15\end{array} $	$\begin{array}{c} 13\\ 0.18\\ 2.15\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 1.18\\ 0.08\\ 1.19\\ 0.71\\ 0.49\\ 1.68\\ 1.37\\ 1.88\\ 1.16\end{array}$	$\begin{array}{c} 14\\ 0.24\\ 2.13\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 0.87\\ 0.10\\ 1.32\\ 0.73\\ 0.44\\ 1.74\\ 1.28\\ 1.85\\ 1.13\\ \end{array}$	$\begin{array}{c} 15\\ 0.21\\ 2.24\\ 0.66\\ 0.21\\ 0.52\\ 0.82\\ 0.91\\ 0.19\\ 1.28\\ 0.72\\ 0.48\\ 1.65\\ 1.31\\ 1.90\\ 1.10\\ \end{array}$	$\begin{array}{c} 16\\ 0.22\\ 2.19\\ 0.65\\ 0.20\\ 0.51\\ 0.74\\ 0.97\\ 1.25\\ 0.77\\ 0.55\\ 1.90\\ 1.51\\ 1.99\\ 1.16\\ \end{array}$	$\begin{array}{c} 17\\ 0.27\\ 2.46\\ 0.64\\ 0.20\\ 0.50\\ 0.87\\ 1.20\\ 0.14\\ 1.64\\ 0.85\\ 0.65\\ 2.03\\ 1.59\\ 2.36\\ 1.28\\ \end{array}$	$\begin{array}{c} 18\\ 0.23\\ 2.47\\ 0.62\\ 0.19\\ 0.95\\ 1.30\\ 0.17\\ 1.80\\ 0.75\\ 2.18\\ 1.67\\ 2.55\\ 1.31\\ \end{array}$	$\begin{array}{c} 19\\ 0.27\\ 2.43\\ 0.61\\ 0.19\\ 0.47\\ 0.90\\ 1.27\\ 0.13\\ 1.50\\ 0.88\\ 0.67\\ 2.25\\ 1.62\\ 2.43\\ 1.37\\ \end{array}$	$\begin{array}{c} 20\\ 0.28\\ 2.23\\ 0.63\\ 0.20\\ 0.49\\ 0.83\\ 1.23\\ 0.15\\ 1.44\\ 0.75\\ 0.57\\ 2.16\\ 1.55\\ 2.26\\ 1.33\\ \end{array}$	$\begin{array}{c} 21 \\ 0.24 \\ 2.14 \\ 0.59 \\ 0.18 \\ 0.46 \\ 0.89 \\ 1.03 \\ 0.13 \\ 1.29 \\ 0.72 \\ 0.59 \\ 1.96 \\ 1.51 \\ 2.08 \\ 1.18 \end{array}$	$\begin{array}{c} 22\\ 0.22\\ 2.07\\ 0.54\\ 0.17\\ 0.42\\ 0.83\\ 0.97\\ 0.11\\ 1.18\\ 0.63\\ 0.45\\ 1.79\\ 1.31\\ 1.90\\ 1.15\\ \end{array}$	$\begin{array}{c} 23 \\ 0.17 \\ 1.71 \\ 0.45 \\ 0.14 \\ 0.35 \\ 0.61 \\ 0.83 \\ 0.10 \\ 0.98 \\ 0.57 \\ 0.39 \\ 1.52 \\ 0.98 \\ 1.56 \\ 0.95 \end{array}$	$\begin{array}{c} 24\\ 0.12\\ 1.21\\ 0.37\\ 0.12\\ 0.29\\ 0.48\\ 0.64\\ 0.08\\ 0.74\\ 0.43\\ 0.29\\ 1.20\\ 0.69\\ 1.20\\ 0.69\\ 1.24\\ 0.70\\ \end{array}$
$ \begin{array}{c} 1\\ 2\\ 3\\ 4\\ 5\\ 6\\ 7\\ 8\\ 9\\ 10\\ 11\\ 12\\ 13\\ 14\\ 15\\ 16\\ \end{array} $	$\begin{array}{c} 13 \\ 0.18 \\ 2.15 \\ 0.68 \\ 0.21 \\ 0.53 \\ 0.85 \\ 1.18 \\ 0.08 \\ 1.19 \\ 0.71 \\ 0.49 \\ 1.68 \\ 1.37 \\ 1.88 \\ 1.16 \\ 0.10 \end{array}$	$\begin{array}{c} 14\\ 0.24\\ 2.13\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 0.87\\ 0.10\\ 1.32\\ 0.73\\ 0.44\\ 1.74\\ 1.28\\ 1.85\\ 1.13\\ 0.15\\ \end{array}$	$\begin{array}{c} 15\\ 0.21\\ 2.24\\ 0.66\\ 0.21\\ 0.52\\ 0.82\\ 0.91\\ 0.19\\ 1.28\\ 0.72\\ 0.48\\ 1.65\\ 1.31\\ 1.90\\ 1.10\\ 0.09\end{array}$	$\begin{array}{c} 16\\ 0.22\\ 2.19\\ 0.65\\ 0.20\\ 0.51\\ 0.74\\ 0.97\\ 0.17\\ 1.25\\ 0.77\\ 0.55\\ 1.90\\ 1.51\\ 1.99\\ 1.16\\ 0.13\\ \end{array}$	$\begin{array}{c} 17\\ 0.27\\ 2.46\\ 0.60\\ 0.50\\ 0.50\\ 0.87\\ 1.20\\ 0.14\\ 1.64\\ 0.85\\ 0.65\\ 2.03\\ 1.59\\ 2.36\\ 1.28\\ 0.11\\ \end{array}$	$\begin{array}{c} 18 \\ 0.23 \\ 2.47 \\ 0.62 \\ 0.19 \\ 0.95 \\ 1.30 \\ 0.17 \\ 1.80 \\ 0.89 \\ 0.75 \\ 2.18 \\ 1.67 \\ 2.55 \\ 1.31 \\ 0.11 \\ \end{array}$	$\begin{array}{c} 19\\ 0.27\\ 2.43\\ 0.61\\ 0.19\\ 0.47\\ 0.9\\ 1.27\\ 0.13\\ 1.50\\ 0.88\\ 0.67\\ 2.25\\ 1.62\\ 2.43\\ 1.37\\ 0.16\end{array}$	$\begin{array}{c} 20\\ 0.28\\ 2.23\\ 0.63\\ 0.20\\ 0.49\\ 0.83\\ 1.23\\ 0.15\\ 1.44\\ 0.75\\ 0.57\\ 2.16\\ 1.55\\ 2.26\\ 1.33\\ 0.14 \end{array}$	$\begin{array}{c} 21 \\ 0.24 \\ 2.14 \\ 0.59 \\ 0.18 \\ 0.46 \\ 0.89 \\ 1.03 \\ 0.13 \\ 1.29 \\ 0.72 \\ 0.59 \\ 1.96 \\ 1.51 \\ 2.08 \\ 1.18 \\ 0.12 \end{array}$	$\begin{array}{c} 22\\ 0.22\\ 2.07\\ 0.54\\ 0.17\\ 0.42\\ 0.83\\ 0.97\\ 0.11\\ 1.18\\ 0.63\\ 0.45\\ 1.79\\ 1.31\\ 1.90\\ 1.15\\ 0.15 \end{array}$	$\begin{array}{c} 23 \\ 0.17 \\ 1.71 \\ 0.45 \\ 0.14 \\ 0.35 \\ 0.61 \\ 0.83 \\ 0.10 \\ 0.98 \\ 0.57 \\ 0.39 \\ 1.52 \\ 0.98 \\ 1.56 \\ 0.95 \\ 0.09 \end{array}$	$\begin{array}{c} 24\\ 0.12\\ 1.21\\ 0.37\\ 0.12\\ 0.29\\ 0.48\\ 0.64\\ 0.08\\ 0.74\\ 0.43\\ 0.29\\ 1.20\\ 0.69\\ 1.24\\ 0.70\\ 0.08\\ \end{array}$
$ \begin{array}{c} 1\\ 2\\ 3\\ 4\\ 5\\ 6\\ 7\\ 8\\ 9\\ 10\\ 11\\ 12\\ 13\\ 14\\ 15\\ 16\\ 17\\ \end{array} $	$\begin{array}{c} 13\\ 0.18\\ 2.15\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 1.18\\ 0.08\\ 1.19\\ 0.71\\ 0.49\\ 1.68\\ 1.37\\ 1.88\\ 1.16\\ 0.10\\ 0.28\end{array}$	$\begin{array}{c} 14\\ 0.24\\ 2.13\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 0.87\\ 0.10\\ 1.32\\ 0.73\\ 0.44\\ 1.28\\ 1.85\\ 1.13\\ 0.15\\ 1.13\\ 0.28\end{array}$	$\begin{array}{c} 15\\ 0.21\\ 2.24\\ 0.66\\ 0.21\\ 0.52\\ 0.82\\ 0.91\\ 0.19\\ 1.28\\ 0.72\\ 0.48\\ 1.65\\ 1.31\\ 1.90\\ 1.10\\ 0.09\\ 0.98\end{array}$	$\begin{array}{c} 16 \\ 0.22 \\ 2.19 \\ 0.65 \\ 0.20 \\ 0.51 \\ 0.74 \\ 0.97 \\ 0.17 \\ 1.25 \\ 0.77 \\ 0.55 \\ 1.90 \\ 1.51 \\ 1.99 \\ 1.16 \\ 0.13 \\ 0.97 \end{array}$	$\begin{array}{c} 17\\ 0.27\\ 2.46\\ 0.64\\ 0.20\\ 0.50\\ 0.87\\ 1.20\\ 0.14\\ 1.64\\ 0.85\\ 0.65\\ 2.03\\ 1.59\\ 2.36\\ 1.28\\ 0.11\\ 0.27\\ \end{array}$	$\begin{array}{c} 18 \\ 0.23 \\ 2.47 \\ 0.62 \\ 0.19 \\ 0.95 \\ 1.30 \\ 0.17 \\ 1.80 \\ 0.89 \\ 0.75 \\ 2.18 \\ 1.67 \\ 2.55 \\ 1.31 \\ 0.11 \\ 0.27 \end{array}$	$\begin{array}{c} 19\\ 0.27\\ 2.43\\ 0.61\\ 0.19\\ 0.47\\ 0.90\\ 1.27\\ 0.13\\ 1.50\\ 0.88\\ 0.67\\ 2.25\\ 1.62\\ 2.43\\ 1.37\\ 0.16\\ 0.07\\ \end{array}$	$\begin{array}{c} 20\\ 0.28\\ 2.23\\ 0.63\\ 0.20\\ 0.49\\ 0.83\\ 1.23\\ 0.15\\ 1.44\\ 0.75\\ 0.57\\ 2.16\\ 1.55\\ 2.26\\ 1.33\\ 0.14\\ 0.07\\ \end{array}$	$\begin{array}{c} 21 \\ 0.24 \\ 2.14 \\ 0.59 \\ 0.18 \\ 0.46 \\ 0.89 \\ 1.03 \\ 0.13 \\ 1.29 \\ 0.72 \\ 0.59 \\ 1.96 \\ 1.51 \\ 2.08 \\ 1.18 \\ 0.12 \\ 0.07 \end{array}$	$\begin{array}{c} 22\\ 0.22\\ 2.07\\ 0.54\\ 0.17\\ 0.42\\ 0.83\\ 0.97\\ 0.11\\ 1.18\\ 0.63\\ 0.45\\ 1.79\\ 1.31\\ 1.90\\ 1.15\\ 0.15\\ 0.26\end{array}$	$\begin{array}{c} 23 \\ 0.17 \\ 1.71 \\ 0.45 \\ 0.14 \\ 0.35 \\ 0.61 \\ 0.98 \\ 0.57 \\ 0.39 \\ 1.52 \\ 0.98 \\ 1.56 \\ 0.95 \\ 0.95 \\ 0.05 \end{array}$	$\begin{array}{c} 24 \\ 0.12 \\ 1.21 \\ 0.37 \\ 0.12 \\ 0.29 \\ 0.48 \\ 0.64 \\ 0.08 \\ 0.74 \\ 0.43 \\ 0.29 \\ 1.20 \\ 0.69 \\ 1.24 \\ 0.70 \\ 0.69 \\ 0.24 \\ \end{array}$
$ \begin{array}{c} 1\\ 2\\ 3\\ 4\\ 5\\ 6\\ 7\\ 8\\ 9\\ 10\\ 11\\ 12\\ 13\\ 14\\ 15\\ 16\\ 17\\ 16\\ 16\\ 17\\ 16\\ 16\\ 17\\ 16\\ 16\\ 17\\ 16\\ 16\\ 17\\ 16\\ 16\\ 17\\ 16\\ 16\\ 17\\ 16\\ 16\\ 17\\ 16\\ 16\\ 17\\ 16\\ 16\\ 16\\ 17\\ 16\\ 16\\ 16\\ 16\\ 16\\ 16\\ 16\\ 16\\ 16\\ 16$	$\begin{array}{c} 13\\ 0.18\\ 2.15\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 1.18\\ 0.08\\ 1.19\\ 0.71\\ 0.49\\ 1.68\\ 1.37\\ 1.88\\ 1.16\\ 0.10\\ 0.08\\ 0.5\\ \end{array}$	$\begin{array}{c} 14\\ 0.24\\ 2.13\\ 0.68\\ 0.21\\ 0.53\\ 0.87\\ 0.10\\ 1.32\\ 0.73\\ 0.44\\ 1.28\\ 1.85\\ 1.13\\ 0.15\\ 0.08\\ 0.56\\ 0$	$\begin{array}{c} 15\\ 0.21\\ 2.24\\ 0.66\\ 0.21\\ 0.52\\ 0.91\\ 0.19\\ 1.28\\ 0.72\\ 0.48\\ 1.65\\ 1.31\\ 1.90\\ 1.10\\ 0.09\\ 0.08\\ 0.55\\ 0$	$\begin{array}{c} 16\\ 0.22\\ 2.19\\ 0.65\\ 0.20\\ 0.51\\ 0.71\\ 0.51\\ 0.77\\ 0.55\\ 1.90\\ 1.51\\ 1.99\\ 1.16\\ 0.13\\ 0.07\\ \end{array}$	$\begin{array}{c} 17\\ 0.27\\ 2.46\\ 0.64\\ 0.20\\ 0.50\\ 0.87\\ 1.20\\ 0.14\\ 1.64\\ 0.85\\ 2.03\\ 1.59\\ 2.36\\ 1.59\\ 2.36\\ 1.28\\ 0.11\\ 0.07\\ 0.16\end{array}$	$\begin{array}{c} 18 \\ 0.23 \\ 2.47 \\ 0.62 \\ 0.19 \\ 0.49 \\ 1.30 \\ 0.17 \\ 1.80 \\ 0.75 \\ 2.18 \\ 1.67 \\ 2.55 \\ 1.31 \\ 0.11 \\ 0.71 \\ 0.75$	$\begin{array}{c} 19\\ 0.27\\ 2.43\\ 0.61\\ 0.19\\ 0.47\\ 0.90\\ 1.27\\ 0.13\\ 1.50\\ 0.8\\ 0.67\\ 2.25\\ 1.62\\ 2.43\\ 1.37\\ 0.16\\ 0.07\\ \end{array}$	$\begin{array}{c} 20\\ 0.28\\ 2.23\\ 0.63\\ 0.20\\ 0.49\\ 0.83\\ 1.23\\ 0.15\\ 1.44\\ 0.75\\ 2.16\\ 1.55\\ 2.26\\ 1.33\\ 0.14\\ 0.07\\ \end{array}$	$\begin{array}{c} 21 \\ 0.24 \\ 2.14 \\ 0.59 \\ 0.18 \\ 0.46 \\ 0.89 \\ 1.03 \\ 0.13 \\ 1.29 \\ 0.72 \\ 0.59 \\ 1.96 \\ 1.51 \\ 2.08 \\ 1.18 \\ 0.12 \\ 0.07 \\ \end{array}$	$\begin{array}{c} 22\\ 0.22\\ 2.07\\ 0.54\\ 0.17\\ 0.42\\ 0.83\\ 0.97\\ 0.11\\ 1.18\\ 0.63\\ 0.45\\ 1.79\\ 1.31\\ 1.90\\ 1.15\\ 0.15\\ 0.15\\ 0.15\\ 0.65\\ 0.75\\ 0$	$\begin{array}{c} 23\\ 0.17\\ 1.71\\ 0.45\\ 0.14\\ 0.35\\ 0.68\\ 0.10\\ 0.98\\ 0.57\\ 0.39\\ 1.52\\ 0.98\\ 1.56\\ 0.95\\ 0.09\\ 0.05\\ 0.09\\ 0.05\\ 0.09\\ 0.05\\ 0.09\\ 0.05\\ 0.09\\ 0.05\\ 0.09\\ 0.05\\ 0.09\\ 0.05\\ 0$	$\begin{array}{c} 24 \\ 0.12 \\ 1.21 \\ 0.37 \\ 0.12 \\ 0.29 \\ 0.48 \\ 0.64 \\ 0.08 \\ 0.74 \\ 0.43 \\ 0.29 \\ 1.20 \\ 0.69 \\ 1.24 \\ 0.70 \\ 0.08 \\ 0.04 \\ 0.64 \end{array}$
$\begin{array}{c} 1\\ 2\\ 3\\ 4\\ 5\\ 6\\ 7\\ 8\\ 9\\ 10\\ 11\\ 12\\ 13\\ 14\\ 15\\ 16\\ 17\\ 18\\ \end{array}$	$\begin{array}{c} 13\\ 0.18\\ 2.15\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 1.18\\ 0.08\\ 1.19\\ 0.71\\ 0.49\\ 1.68\\ 1.37\\ 1.88\\ 1.16\\ 0.10\\ 0.08\\ 1.94\\ \end{array}$	$\begin{array}{c} 14\\ 0.24\\ 2.13\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 0.87\\ 0.10\\ 1.32\\ 0.73\\ 0.44\\ 1.74\\ 1.28\\ 1.85\\ 1.13\\ 0.15\\ 0.08\\ 1.87\\ \end{array}$	$\begin{array}{c} 15\\ 0.21\\ 2.24\\ 0.66\\ 0.21\\ 0.52\\ 0.82\\ 0.91\\ 0.19\\ 1.28\\ 0.72\\ 0.48\\ 1.65\\ 1.31\\ 1.90\\ 1.10\\ 0.09\\ 0.08\\ 1.85\\ \end{array}$	$\begin{array}{c} 16\\ 0.22\\ 2.19\\ 0.65\\ 0.20\\ 0.51\\ 0.74\\ 0.97\\ 0.17\\ 1.25\\ 0.77\\ 0.55\\ 1.90\\ 1.51\\ 1.99\\ 1.16\\ 0.13\\ 0.07\\ 2.00\\ \end{array}$	$\begin{array}{c} 17\\ 0.27\\ 2.46\\ 0.64\\ 0.20\\ 0.50\\ 0.87\\ 1.20\\ 0.14\\ 1.64\\ 0.85\\ 2.03\\ 1.59\\ 2.36\\ 1.28\\ 0.11\\ 0.07\\ 2.13\\ \end{array}$	$\begin{array}{c} 18\\ 0.23\\ 2.47\\ 0.62\\ 0.19\\ 0.95\\ 1.30\\ 0.17\\ 1.80\\ 0.89\\ 0.75\\ 2.18\\ 1.67\\ 2.55\\ 1.31\\ 0.11\\ 0.07\\ 2.41\\ \end{array}$	$\begin{array}{c} 19\\ 0.27\\ 2.43\\ 0.61\\ 0.19\\ 0.47\\ 0.90\\ 1.27\\ 0.13\\ 1.50\\ 0.88\\ 0.67\\ 2.25\\ 1.62\\ 2.43\\ 1.37\\ 0.16\\ 0.07\\ 2.23\\ \end{array}$	$\begin{array}{c} 20\\ 0.28\\ 2.23\\ 0.63\\ 0.20\\ 0.49\\ 0.83\\ 1.23\\ 0.15\\ 1.44\\ 0.75\\ 2.16\\ 1.55\\ 2.26\\ 1.33\\ 0.14\\ 0.07\\ 2.08\\ \end{array}$	$\begin{array}{c} 21 \\ 0.24 \\ 2.14 \\ 0.59 \\ 0.18 \\ 0.46 \\ 0.89 \\ 1.03 \\ 0.13 \\ 1.29 \\ 0.72 \\ 0.59 \\ 1.96 \\ 1.51 \\ 2.08 \\ 1.18 \\ 0.12 \\ 0.07 \\ 2.06 \end{array}$	$\begin{array}{c} 22\\ 0.22\\ 2.07\\ 0.54\\ 0.17\\ 0.42\\ 0.83\\ 0.97\\ 0.11\\ 1.18\\ 0.63\\ 0.45\\ 1.79\\ 1.31\\ 1.90\\ 1.15\\ 0.15\\ 0.06\\ 1.78\\ \end{array}$	$\begin{array}{c} 23 \\ 0.17 \\ 1.71 \\ 0.45 \\ 0.14 \\ 0.35 \\ 0.61 \\ 0.98 \\ 0.57 \\ 0.99 \\ 1.52 \\ 0.98 \\ 1.56 \\ 0.95 \\ 0.95 \\ 0.05 \\ 1.52 \end{array}$	$\begin{array}{c} 24\\ 0.12\\ 1.21\\ 0.37\\ 0.12\\ 0.29\\ 0.48\\ 0.64\\ 0.08\\ 0.74\\ 0.43\\ 0.29\\ 1.20\\ 0.69\\ 1.24\\ 0.70\\ 0.69\\ 1.24\\ 0.70\\ 0.04\\ 1.14 \end{array}$
$ \begin{array}{c} 1\\ 2\\ 3\\ 4\\ 5\\ 6\\ 7\\ 8\\ 9\\ 10\\ 11\\ 12\\ 13\\ 14\\ 15\\ 16\\ 17\\ 18\\ 19\\ \end{array} $	$\begin{array}{c} 13\\ 0.18\\ 2.15\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 1.18\\ 0.08\\ 1.19\\ 0.71\\ 0.49\\ 1.68\\ 1.37\\ 1.88\\ 1.16\\ 0.10\\ 0.08\\ 1.94\\ 0.54\end{array}$	$\begin{array}{c} 14\\ 0.24\\ 2.13\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 0.87\\ 0.10\\ 1.32\\ 0.73\\ 0.44\\ 1.74\\ 1.28\\ 1.85\\ 1.13\\ 0.15\\ 0.08\\ 1.87\\ 0.54\\ \end{array}$	$\begin{array}{c} 15\\ 0.21\\ 2.24\\ 0.66\\ 0.21\\ 0.52\\ 0.82\\ 0.91\\ 0.19\\ 1.28\\ 0.72\\ 0.48\\ 1.65\\ 1.31\\ 1.90\\ 1.10\\ 0.09\\ 0.08\\ 1.85\\ 0.53\end{array}$	$\begin{array}{c} 16\\ 0.22\\ 2.19\\ 0.65\\ 0.20\\ 0.51\\ 0.74\\ 0.97\\ 0.17\\ 1.25\\ 0.77\\ 0.55\\ 1.90\\ 1.51\\ 1.99\\ 1.16\\ 0.13\\ 0.07\\ 2.00\\ 0.53\\ \end{array}$	$\begin{array}{c} 17\\ 0.27\\ 2.46\\ 0.64\\ 0.20\\ 0.50\\ 0.87\\ 1.20\\ 0.14\\ 1.64\\ 0.85\\ 0.65\\ 2.03\\ 1.59\\ 2.36\\ 1.28\\ 0.11\\ 0.07\\ 2.13\\ 0.51\\ \end{array}$	$\begin{array}{c} 18\\ 0.23\\ 2.47\\ 0.62\\ 0.19\\ 0.95\\ 1.30\\ 0.17\\ 1.80\\ 0.75\\ 2.18\\ 1.67\\ 2.55\\ 1.31\\ 0.11\\ 0.01\\ 2.55\\ 1.31\\ 0.11\\ 0.07\\ 2.41\\ 0.50\\ \end{array}$	$\begin{array}{c} 19\\ 0.27\\ 2.43\\ 0.61\\ 0.19\\ 0.47\\ 0.90\\ 1.27\\ 0.13\\ 1.50\\ 0.88\\ 0.67\\ 2.25\\ 1.62\\ 2.43\\ 1.37\\ 0.16\\ 0.07\\ 2.23\\ 0.49\\ \end{array}$	$\begin{array}{c} 20\\ 0.28\\ 2.23\\ 0.63\\ 0.20\\ 0.49\\ 0.83\\ 1.23\\ 0.15\\ 1.44\\ 0.75\\ 0.57\\ 2.16\\ 1.55\\ 2.26\\ 1.33\\ 0.14\\ 0.07\\ 2.08\\ 0.51\\ \end{array}$	$\begin{array}{c} 21 \\ 0.24 \\ 2.14 \\ 0.59 \\ 0.18 \\ 0.46 \\ 0.89 \\ 1.03 \\ 0.13 \\ 1.29 \\ 0.72 \\ 0.59 \\ 1.96 \\ 1.51 \\ 2.08 \\ 1.18 \\ 0.12 \\ 0.07 \\ 2.06 \\ 0.47 \end{array}$	$\begin{array}{c} 22\\ 0.22\\ 2.07\\ 0.54\\ 0.17\\ 0.42\\ 0.83\\ 0.97\\ 0.11\\ 1.18\\ 0.63\\ 0.45\\ 1.79\\ 1.31\\ 1.90\\ 1.15\\ 0.15\\ 0.15\\ 0.15\\ 0.43\\ \end{array}$	$\begin{array}{c} 23\\ 0.17\\ 1.71\\ 0.45\\ 0.14\\ 0.35\\ 0.61\\ 0.83\\ 0.10\\ 0.98\\ 0.57\\ 0.39\\ 1.52\\ 0.98\\ 1.56\\ 0.95\\ 0.09\\ 0.05\\ 1.52\\ 0.36\end{array}$	$\begin{array}{c} 24\\ 0.12\\ 1.21\\ 0.37\\ 0.12\\ 0.29\\ 0.48\\ 0.64\\ 0.08\\ 0.74\\ 0.43\\ 0.29\\ 1.20\\ 0.69\\ 1.24\\ 0.70\\ 0.08\\ 0.04\\ 1.14\\ 0.30\\ \end{array}$
$\begin{array}{c} 1\\ 1\\ 2\\ 3\\ 4\\ 5\\ 6\\ 7\\ 8\\ 9\\ 10\\ 11\\ 12\\ 13\\ 14\\ 15\\ 16\\ 17\\ 18\\ 19\\ 20\\ \end{array}$	$\begin{array}{c} 13\\ 0.18\\ 2.15\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 1.18\\ 0.08\\ 1.19\\ 0.71\\ 0.49\\ 1.68\\ 1.37\\ 1.88\\ 1.16\\ 0.10\\ 0.08\\ 1.94\\ 0.54\\ 0.71\\ \end{array}$	$\begin{array}{c} 14\\ 0.24\\ 2.13\\ 0.68\\ 0.21\\ 0.53\\ 0.87\\ 0.10\\ 1.32\\ 0.73\\ 0.44\\ 1.74\\ 1.28\\ 1.85\\ 1.13\\ 0.15\\ 0.08\\ 1.87\\ 0.54\\ 0.72\\ \end{array}$	$\begin{array}{c} 15\\ 0.21\\ 2.24\\ 0.66\\ 0.21\\ 0.52\\ 0.82\\ 0.91\\ 0.19\\ 1.28\\ 0.72\\ 0.48\\ 1.65\\ 1.31\\ 1.90\\ 1.10\\ 0.09\\ 0.08\\ 1.85\\ 0.53\\ 0.71\\ \end{array}$	$\begin{array}{c} 16\\ 0.22\\ 2.19\\ 0.65\\ 0.20\\ 0.51\\ 0.74\\ 0.97\\ 0.17\\ 1.25\\ 0.77\\ 0.55\\ 1.90\\ 1.51\\ 1.99\\ 1.16\\ 0.13\\ 0.07\\ 2.00\\ 0.53\\ 0.87\\ \end{array}$	$\begin{array}{c} 17\\ 0.27\\ 2.46\\ 0.64\\ 0.20\\ 0.50\\ 0.87\\ 1.20\\ 0.14\\ 1.64\\ 0.85\\ 2.03\\ 1.59\\ 2.36\\ 1.28\\ 0.11\\ 0.07\\ 2.13\\ 0.51\\ 1.05\\ \end{array}$	$\begin{array}{c} 18 \\ 0.23 \\ 2.47 \\ 0.62 \\ 0.19 \\ 0.49 \\ 0.95 \\ 1.30 \\ 0.17 \\ 1.80 \\ 0.89 \\ 0.75 \\ 2.18 \\ 1.67 \\ 2.55 \\ 1.31 \\ 0.11 \\ 0.07 \\ 2.41 \\ 0.07 \\ 1.4 \end{array}$	$\begin{array}{c} 19\\ 0.27\\ 2.43\\ 0.61\\ 0.19\\ 0.47\\ 0.9\\ 1.27\\ 0.13\\ 1.50\\ 0.88\\ 0.67\\ 2.25\\ 1.62\\ 2.43\\ 1.37\\ 0.16\\ 0.07\\ 2.23\\ 0.49\\ 1.08\end{array}$	$\begin{array}{c} 20\\ 0.28\\ 2.23\\ 0.63\\ 0.20\\ 0.49\\ 0.83\\ 1.23\\ 0.15\\ 1.44\\ 0.75\\ 0.57\\ 2.16\\ 1.55\\ 2.26\\ 1.33\\ 0.14\\ 0.07\\ 2.08\\ 0.51\\ 0.93\\ \end{array}$	$\begin{array}{c} 21 \\ 0.24 \\ 2.14 \\ 0.59 \\ 0.18 \\ 0.46 \\ 0.89 \\ 1.03 \\ 0.13 \\ 1.29 \\ 0.72 \\ 0.59 \\ 1.96 \\ 1.51 \\ 2.08 \\ 1.18 \\ 0.12 \\ 0.07 \\ 2.06 \\ 0.47 \\ 0.87 \end{array}$	$\begin{array}{c} 22\\ 0.22\\ 2.07\\ 0.54\\ 0.17\\ 0.42\\ 0.83\\ 0.97\\ 0.11\\ 1.18\\ 0.63\\ 0.45\\ 1.79\\ 1.31\\ 1.90\\ 1.15\\ 0.15\\ 0.06\\ 1.78\\ 0.45\\ 0.80\\ 0$	$\begin{array}{c} 23\\ 0.17\\ 1.71\\ 0.45\\ 0.14\\ 0.35\\ 0.61\\ 0.83\\ 0.10\\ 0.98\\ 0.57\\ 0.39\\ 1.52\\ 0.98\\ 1.56\\ 0.98\\ 1.56\\ 0.98\\ 1.56\\ 0.09\\ 0.05\\ 1.52\\ 0.09\\ 0.05\\ 1.52\\ 0.36\\ 0.71\\ 0$	$\begin{array}{c} 24\\ 0.12\\ 1.21\\ 0.37\\ 0.12\\ 0.29\\ 0.48\\ 0.64\\ 0.08\\ 0.74\\ 0.43\\ 0.29\\ 1.20\\ 0.69\\ 1.24\\ 0.70\\ 0.08\\ 0.04\\ 1.14\\ 0.30\\ 0.53\end{array}$
$ \begin{array}{c} 1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11\\12\\13\\14\\15\\16\\17\\18\\19\\20\\\end{array} $	$\begin{array}{c} 13\\ 0.18\\ 2.15\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 1.18\\ 0.08\\ 1.19\\ 0.71\\ 0.49\\ 1.68\\ 1.37\\ 1.88\\ 1.16\\ 0.10\\ 0.08\\ 1.94\\ 0.54\\ 0.54\\ 0.54\\ 0.71\\ 0.54\\ 0$	$\begin{array}{c} 14\\ 0.24\\ 2.13\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 0.87\\ 0.10\\ 1.32\\ 0.73\\ 0.44\\ 1.28\\ 1.85\\ 1.13\\ 0.15\\ 1.13\\ 0.15\\ 0.08\\ 1.87\\ 0.54\\ 0.72\\ 0.54\\ 0$	$\begin{array}{c} 15\\ 0.21\\ 2.24\\ 0.66\\ 0.21\\ 0.52\\ 0.82\\ 0.91\\ 0.19\\ 1.28\\ 0.72\\ 0.48\\ 1.65\\ 1.31\\ 1.90\\ 1.10\\ 0.09\\ 1.85\\ 0.53\\ 0.71\\ 0.75\\ 0$	$\begin{array}{c} 16\\ 0.22\\ 2.19\\ 0.65\\ 0.20\\ 0.51\\ 0.74\\ 0.9\\ 0.17\\ 1.25\\ 0.77\\ 0.55\\ 1.90\\ 1.51\\ 1.99\\ 1.16\\ 0.13\\ 0.07\\ 2.00\\ 0.53\\ 0.87\\ 0.53\\ 0.87\\ 0.55\\ 0.87\\ 0.53\\ 0.87\\ 0.53\\ 0.87\\ 0.55\\ 0.87\\ 0.53\\ 0.87\\ 0.55\\ 0.87\\ 0.53\\ 0.87\\ 0.55\\ 0.87\\ 0.53\\ 0.87\\ 0.53\\ 0.87\\ 0.55\\ 0.87\\ 0.53\\ 0.87\\ 0.53\\ 0.87\\ 0.55\\ 0.87\\ 0.53\\ 0.87\\ 0.53\\ 0.87\\ 0.55\\ 0.87\\ 0.$	$\begin{array}{c} 17\\ 0.27\\ 2.46\\ 0.64\\ 0.20\\ 0.50\\ 0.87\\ 1.20\\ 0.14\\ 1.64\\ 0.85\\ 0.65\\ 2.03\\ 1.59\\ 2.36\\ 1.28\\ 0.11\\ 0.07\\ 2.13\\ 0.51\\ 1.05\\ 1.23\\ 0.51\\ 1.05\\ 1.23\\ 0.51\\ 1.05\\ 0.51\\ 1.05\\ 0.51\\ 0$	$\begin{array}{c} 18 \\ 0.23 \\ 2.47 \\ 0.62 \\ 0.19 \\ 0.95 \\ 1.30 \\ 0.17 \\ 1.80 \\ 0.75 \\ 2.18 \\ 1.67 \\ 2.55 \\ 1.31 \\ 0.11 \\ 0.07 \\ 2.41 \\ 0.50 \\ 1.14 \\ 0.50 \\ 0.14$	$\begin{array}{c} 19\\ 0.27\\ 2.43\\ 0.61\\ 0.19\\ 0.47\\ 0.90\\ 1.27\\ 0.13\\ 1.50\\ 0.88\\ 0.67\\ 2.25\\ 1.62\\ 2.43\\ 1.37\\ 0.16\\ 0.07\\ 2.23\\ 0.49\\ 1.08\\ 0.49\\ 0$	$\begin{array}{c} 20\\ 0.28\\ 2.23\\ 0.63\\ 0.20\\ 0.49\\ 0.83\\ 1.23\\ 0.15\\ 1.44\\ 0.75\\ 0.57\\ 2.16\\ 1.55\\ 2.26\\ 1.33\\ 0.14\\ 0.07\\ 2.08\\ 0.51\\ 0.93\\ 0.93\\ 0.51\\ 0.93\\ 0$	$\begin{array}{c} 21 \\ 0.24 \\ 2.14 \\ 0.59 \\ 0.18 \\ 0.46 \\ 0.89 \\ 1.03 \\ 0.13 \\ 1.29 \\ 0.72 \\ 0.59 \\ 1.51 \\ 2.08 \\ 1.18 \\ 0.12 \\ 0.07 \\ 2.06 \\ 0.47 \\ 0.87 \\ 0.87 \\ 0.47 \\ 0.87 \\ 0.47 \\ 0.87 \\ 0.47 \\ 0.87$	$\begin{array}{c} 22\\ 0.22\\ 2.07\\ 0.54\\ 0.17\\ 0.42\\ 0.83\\ 0.97\\ 0.11\\ 1.18\\ 0.63\\ 0.45\\ 1.79\\ 1.31\\ 1.90\\ 1.15\\ 0.15\\ 0.15\\ 0.16\\ 0.43\\ 0.43\\ 0.43\\ 0.80\\ 0.55\\ 0$	$\begin{array}{c} 23 \\ 0.17 \\ 1.71 \\ 0.45 \\ 0.14 \\ 0.35 \\ 0.61 \\ 0.98 \\ 0.57 \\ 0.39 \\ 1.52 \\ 0.98 \\ 1.56 \\ 0.95 \\ 0.95 \\ 0.05 \\ 1.52 \\ 0.36 \\ 0.71 \\ 1.52 \\ 0.36 \\ 0.71 \\ 1.52 \\ 0.36 \\ 0.71 \\ 1.52 \\ 0.36 \\ 0.71 \\ 1.52 \\ 0.36 \\ 0.71 \\ 1.52 \\ 0.36 \\ 0.71 \\ 1.52 \\ 0.36 \\ 0.71 \\ 1.52 \\ 0.36 \\ 0.71 \\ 1.52 \\ 0.36 \\ 0.71 \\ 1.52 \\ 0.36 \\ 0.71 \\ 1.52 \\ 0.36 \\ 0.71 \\ 1.52 \\ 0.36 \\ 0.71 \\ 1.52 \\ 0.36 \\ 0.71 \\ 1.52 \\ 0.36 \\ 0.71 \\ 1.52 \\ 0.36 \\ 0.71 \\ 1.52 \\ 0.36 \\ 0.71 \\ 1.52 \\ 0.36 \\ 0.71 \\ 1.52 \\ 0.36 \\ 0.71 \\ 1.52 \\ 0.52$	$\begin{array}{c} 24\\ 0.12\\ 1.21\\ 0.37\\ 0.12\\ 0.29\\ 0.48\\ 0.64\\ 0.08\\ 0.74\\ 0.43\\ 0.29\\ 1.20\\ 0.69\\ 1.24\\ 0.70\\ 0.69\\ 1.24\\ 0.70\\ 0.08\\ 0.04\\ 1.14\\ 0.30\\ 0.53\\ 0.04\\ 1.14\\ 0.30\\ 0.53\\ 0.54\\ 0.55\\ 0$
$\begin{array}{c} \hline 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 19 \\ 20 \\ 21 \\ \end{array}$	$\begin{array}{c} 13\\ 0.18\\ 2.15\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 1.18\\ 0.08\\ 1.19\\ 0.71\\ 0.49\\ 1.68\\ 1.37\\ 1.88\\ 1.16\\ 0.10\\ 0.08\\ 1.94\\ 0.51\\ 1.48\\ \end{array}$	$\begin{array}{c} 14\\ 0.24\\ 2.13\\ 0.68\\ 0.21\\ 0.53\\ 0.87\\ 0.10\\ 1.32\\ 0.73\\ 0.44\\ 1.74\\ 1.28\\ 1.85\\ 1.13\\ 0.15\\ 0.08\\ 1.87\\ 0.54\\ 1.87\\ 0.72\\ 1.28\\ \end{array}$	$\begin{array}{c} 15\\ 0.21\\ 2.24\\ 0.66\\ 0.21\\ 0.52\\ 0.91\\ 0.19\\ 1.28\\ 0.72\\ 0.48\\ 1.65\\ 1.31\\ 1.90\\ 1.10\\ 0.09\\ 0.08\\ 1.85\\ 0.53\\ 0.71\\ 1.40\\ \end{array}$	$\begin{array}{c} 16\\ 0.22\\ 2.19\\ 0.65\\ 0.20\\ 0.51\\ 0.71\\ 0.51\\ 0.77\\ 1.25\\ 0.77\\ 0.55\\ 1.90\\ 1.51\\ 1.99\\ 1.16\\ 0.13\\ 0.07\\ 2.00\\ 0.53\\ 0.87\\ 1.38\\ \end{array}$	$\begin{array}{c} 17\\ 0.27\\ 2.46\\ 0.64\\ 0.20\\ 0.50\\ 0.87\\ 1.20\\ 0.14\\ 1.64\\ 0.85\\ 0.65\\ 2.03\\ 1.59\\ 2.36\\ 1.28\\ 0.11\\ 0.07\\ 2.13\\ 0.51\\ 1.05\\ 1.90\\ \end{array}$	$\begin{array}{c} 18\\ 0.23\\ 2.47\\ 0.62\\ 0.19\\ 0.49\\ 0.95\\ 1.30\\ 0.17\\ 1.80\\ 0.75\\ 2.18\\ 1.67\\ 2.55\\ 1.31\\ 0.67\\ 2.41\\ 0.07\\ 2.41\\ 0.07\\ 1.14\\ 1.82 \end{array}$	$\begin{array}{c} 19\\ 0.27\\ 2.43\\ 0.61\\ 0.19\\ 0.47\\ 0.90\\ 1.27\\ 0.13\\ 1.50\\ 0.88\\ 0.67\\ 2.25\\ 1.62\\ 2.43\\ 1.37\\ 0.16\\ 0.07\\ 2.23\\ 0.49\\ 1.08\\ 1.92 \end{array}$	$\begin{array}{c} 20\\ 0.28\\ 2.23\\ 0.63\\ 0.20\\ 0.49\\ 0.83\\ 1.23\\ 0.15\\ 1.44\\ 0.75\\ 0.57\\ 2.16\\ 1.55\\ 2.26\\ 1.33\\ 0.14\\ 0.07\\ 2.08\\ 0.51\\ 0.93\\ 1.68\\ \end{array}$	$\begin{array}{c} 21 \\ 0.24 \\ 2.14 \\ 0.59 \\ 0.18 \\ 0.46 \\ 0.89 \\ 1.03 \\ 0.13 \\ 1.29 \\ 0.72 \\ 0.59 \\ 1.96 \\ 1.51 \\ 2.08 \\ 1.18 \\ 0.12 \\ 0.07 \\ 2.06 \\ 0.47 \\ 0.87 \\ 1.48 \end{array}$	$\begin{array}{c} 22\\ 0.22\\ 2.07\\ 0.54\\ 0.17\\ 0.42\\ 0.83\\ 0.97\\ 0.11\\ 1.18\\ 0.63\\ 0.45\\ 1.79\\ 1.31\\ 1.90\\ 1.15\\ 0.15\\ 0.15\\ 0.06\\ 1.78\\ 0.48\\ 0.43\\ 0.80\\ 1.36\\ \end{array}$	$\begin{array}{c} 23\\ 0.17\\ 1.71\\ 0.45\\ 0.14\\ 0.35\\ 0.61\\ 0.98\\ 0.57\\ 0.39\\ 1.52\\ 0.98\\ 1.56\\ 0.95\\ 0.09\\ 0.05\\ 1.52\\ 0.09\\ 0.05\\ 1.52\\ 0.09\\ 0.05\\ 1.52\\ 0.36\\ 0.71\\ 1.11\\ \end{array}$	$\begin{array}{c} 24\\ 0.12\\ 1.21\\ 0.37\\ 0.12\\ 0.29\\ 0.48\\ 0.64\\ 0.08\\ 0.74\\ 0.43\\ 0.29\\ 1.20\\ 0.69\\ 1.24\\ 0.70\\ 0.08\\ 0.04\\ 1.14\\ 0.30\\ 0.53\\ 0.88\\ \end{array}$
$\begin{array}{c} 1\\ 2\\ 3\\ 4\\ 5\\ 6\\ 7\\ 8\\ 9\\ 10\\ 11\\ 12\\ 13\\ 14\\ 15\\ 16\\ 17\\ 18\\ 19\\ 20\\ 21\\ 22\\ \end{array}$	$\begin{array}{c} 13\\ 0.18\\ 2.15\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 1.18\\ 0.08\\ 1.19\\ 0.71\\ 0.49\\ 1.68\\ 1.37\\ 1.88\\ 1.16\\ 0.10\\ 0.08\\ 1.94\\ 0.54\\ 0.71\\ 1.48\\ 0.50\\ \end{array}$	$\begin{array}{c} 14\\ 0.24\\ 2.13\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 0.87\\ 0.10\\ 1.32\\ 0.73\\ 0.44\\ 1.74\\ 1.28\\ 1.85\\ 1.13\\ 0.15\\ 0.08\\ 1.87\\ 0.54\\ 0.72\\ 1.28\\ 0.49\\ \end{array}$	$\begin{array}{c} 15\\ 0.21\\ 2.24\\ 0.66\\ 0.21\\ 0.52\\ 0.82\\ 0.91\\ 1.28\\ 0.72\\ 0.48\\ 1.65\\ 1.31\\ 1.90\\ 1.10\\ 0.09\\ 0.08\\ 1.85\\ 0.53\\ 0.71\\ 1.40\\ 0.43\\ \end{array}$	$\begin{array}{c} 16\\ 0.22\\ 2.19\\ 0.65\\ 0.20\\ 0.51\\ 0.74\\ 0.97\\ 0.17\\ 1.25\\ 0.77\\ 0.55\\ 1.90\\ 1.51\\ 1.99\\ 1.16\\ 0.13\\ 0.07\\ 2.00\\ 0.53\\ 0.87\\ 1.38\\ 0.66\end{array}$	$\begin{array}{c} 17\\ 0.27\\ 2.46\\ 0.64\\ 0.20\\ 0.50\\ 0.87\\ 1.20\\ 0.14\\ 1.64\\ 0.85\\ 2.03\\ 1.59\\ 2.36\\ 1.28\\ 0.11\\ 0.07\\ 2.13\\ 0.51\\ 1.05\\ 1.90\\ 0.63\\ \end{array}$	$\begin{array}{c} 18\\ 0.23\\ 2.47\\ 0.62\\ 0.19\\ 0.95\\ 1.30\\ 0.95\\ 1.30\\ 0.75\\ 1.80\\ 0.89\\ 0.75\\ 2.18\\ 1.67\\ 2.55\\ 1.31\\ 0.11\\ 0.07\\ 2.41\\ 0.50\\ 1.14\\ 1.82\\ 0.70\\ \end{array}$	$\begin{array}{c} 19\\ 0.27\\ 2.43\\ 0.61\\ 0.19\\ 0.47\\ 0.90\\ 1.27\\ 0.13\\ 1.50\\ 0.88\\ 0.67\\ 2.25\\ 1.62\\ 2.43\\ 1.37\\ 0.16\\ 0.07\\ 2.23\\ 0.49\\ 1.08\\ 1.92\\ 0.60\\ \end{array}$	$\begin{array}{c} 20\\ 0.28\\ 2.23\\ 0.63\\ 0.20\\ 0.49\\ 0.83\\ 1.23\\ 0.15\\ 1.44\\ 0.75\\ 0.57\\ 2.16\\ 1.55\\ 2.26\\ 1.33\\ 0.14\\ 0.07\\ 2.08\\ 0.51\\ 0.93\\ 1.68\\ 0.53\end{array}$	$\begin{array}{c} 21 \\ 0.24 \\ 2.14 \\ 0.59 \\ 0.18 \\ 0.46 \\ 0.89 \\ 1.03 \\ 0.13 \\ 1.29 \\ 0.72 \\ 0.59 \\ 1.96 \\ 1.51 \\ 2.08 \\ 1.18 \\ 0.12 \\ 0.07 \\ 2.06 \\ 0.47 \\ 0.87 \\ 1.48 \\ 0.49 \end{array}$	$\begin{array}{c} 22\\ 0.22\\ 2.07\\ 0.54\\ 0.17\\ 0.42\\ 0.83\\ 0.97\\ 0.11\\ 1.18\\ 0.63\\ 0.45\\ 1.79\\ 1.31\\ 1.90\\ 1.15\\ 0.16\\ 0.66\\ 1.78\\ 0.43\\ 0.80\\ 1.36\\ 0.51\\ \end{array}$	$\begin{array}{c} 23\\ 0.17\\ 1.71\\ 0.45\\ 0.14\\ 0.35\\ 0.61\\ 0.98\\ 0.57\\ 0.98\\ 1.52\\ 0.98\\ 1.56\\ 0.95\\ 0.95\\ 0.05\\ 1.52\\ 0.36\\ 0.71\\ 1.11\\ 0.44 \end{array}$	$\begin{array}{c} 24\\ 0.12\\ 1.21\\ 0.37\\ 0.12\\ 0.29\\ 0.48\\ 0.64\\ 0.08\\ 0.74\\ 0.43\\ 0.29\\ 1.20\\ 0.69\\ 1.24\\ 0.70\\ 0.69\\ 1.24\\ 0.70\\ 0.08\\ 0.04\\ 1.14\\ 0.30\\ 0.58\\ 0.33\\ 0.88\\ 0.33\\ \end{array}$
$\begin{array}{c} \hline 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 19 \\ 20 \\ 21 \\ 22 \\ 23 \\ \end{array}$	$\begin{array}{c} 13\\ 0.18\\ 2.15\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 1.18\\ 0.08\\ 1.19\\ 0.71\\ 0.49\\ 1.68\\ 1.37\\ 1.88\\ 1.16\\ 0.10\\ 0.08\\ 1.94\\ 0.54\\ 0.71\\ 1.48\\ 0.54\\ 0.71\\ 1.48\\ 0.54\\ 0.67\\ \end{array}$	$\begin{array}{c} 14\\ 0.24\\ 2.13\\ 0.68\\ 0.21\\ 0.53\\ 0.87\\ 0.10\\ 1.32\\ 0.73\\ 0.44\\ 1.28\\ 1.85\\ 1.13\\ 0.08\\ 1.87\\ 0.51\\ 0.08\\ 1.87\\ 0.54\\ 0.72\\ 1.28\\ 0.49\\ 0.72\\ 1.28\\ 0.49\\ 0.74\\ \end{array}$	$\begin{array}{c} 15\\ 0.21\\ 2.24\\ 0.66\\ 0.21\\ 0.52\\ 0.91\\ 0.19\\ 1.28\\ 0.72\\ 0.48\\ 1.65\\ 1.31\\ 1.90\\ 1.10\\ 0.09\\ 0.08\\ 1.85\\ 0.53\\ 0.71\\ 1.40\\ 0.43\\ 0.80\\ \end{array}$	$\begin{array}{c} 16\\ 0.22\\ 2.19\\ 0.65\\ 0.20\\ 0.51\\ 0.71\\ 0.51\\ 0.77\\ 0.55\\ 1.90\\ 1.51\\ 1.99\\ 1.16\\ 0.13\\ 0.07\\ 2.00\\ 0.53\\ 0.87\\ 1.38\\ 0.66\\ 0.74 \end{array}$	$\begin{array}{c} 17\\ 0.27\\ 2.46\\ 0.64\\ 0.20\\ 0.50\\ 0.87\\ 1.20\\ 0.14\\ 1.64\\ 0.85\\ 2.03\\ 1.59\\ 2.36\\ 1.59\\ 2.36\\ 1.28\\ 0.11\\ 0.07\\ 2.13\\ 0.51\\ 1.05\\ 1.90\\ 0.63\\ 0.77\\ \end{array}$	$\begin{array}{c} 18 \\ 0.23 \\ 2.47 \\ 0.62 \\ 0.19 \\ 0.95 \\ 1.30 \\ 0.17 \\ 1.80 \\ 0.87 \\ 2.18 \\ 1.67 \\ 2.55 \\ 1.31 \\ 0.11 \\ 0.07 \\ 2.41 \\ 0.50 \\ 1.14 \\ 1.82 \\ 0.70 \\ 0.90 \\ \end{array}$	$\begin{array}{c} 19\\ 0.27\\ 2.43\\ 0.61\\ 0.19\\ 0.47\\ 0.90\\ 1.27\\ 0.13\\ 1.50\\ 0.8\\ 0.67\\ 2.25\\ 1.62\\ 2.43\\ 1.37\\ 0.16\\ 0.07\\ 2.23\\ 1.37\\ 0.16\\ 0.07\\ 2.23\\ 0.49\\ 1.08\\ 1.92\\ 0.60\\ 0.89\end{array}$	$\begin{array}{c} 20\\ 0.28\\ 2.23\\ 0.63\\ 0.20\\ 0.49\\ 0.83\\ 1.23\\ 0.15\\ 1.44\\ 0.75\\ 2.16\\ 1.55\\ 2.26\\ 1.55\\ 2.26\\ 1.33\\ 0.14\\ 0.07\\ 2.08\\ 0.51\\ 0.93\\ 1.68\\ 0.53\\ 0.81\\ \end{array}$	$\begin{array}{c} 21 \\ 0.24 \\ 2.14 \\ 0.59 \\ 0.18 \\ 0.46 \\ 0.89 \\ 1.03 \\ 0.13 \\ 1.29 \\ 0.72 \\ 0.59 \\ 1.96 \\ 1.51 \\ 2.08 \\ 1.18 \\ 0.12 \\ 0.07 \\ 2.06 \\ 0.47 \\ 0.87 \\ 1.48 \\ 0.49 \\ 0.77 \end{array}$	$\begin{array}{c} 22\\ 0.22\\ 2.07\\ 0.54\\ 0.17\\ 0.42\\ 0.83\\ 0.97\\ 0.11\\ 1.18\\ 0.63\\ 0.45\\ 1.79\\ 1.31\\ 1.90\\ 1.15\\ 0.06\\ 1.78\\ 0.43\\ 0.80\\ 1.36\\ 0.51\\ 0.69\end{array}$	$\begin{array}{c} 23\\ 0.17\\ 1.71\\ 0.45\\ 0.14\\ 0.35\\ 0.61\\ 0.83\\ 0.10\\ 0.98\\ 0.57\\ 0.39\\ 1.52\\ 0.98\\ 1.56\\ 0.95\\ 0.09\\ 0.05\\ 1.52\\ 0.39\\ 1.52\\ 0.95\\ 0.09\\ 0.05\\ 1.52\\ 0.36\\ 0.71\\ 1.11\\ 0.44\\ 0.50\\ 0.51\\ 0$	$\begin{array}{c} 24\\ 0.12\\ 1.21\\ 0.37\\ 0.12\\ 0.29\\ 0.48\\ 0.64\\ 0.08\\ 0.74\\ 0.43\\ 0.29\\ 1.20\\ 0.69\\ 1.24\\ 0.70\\ 0.08\\ 0.04\\ 1.14\\ 0.30\\ 0.53\\ 0.88\\ 0.33\\ 0.40\end{array}$
$\begin{array}{c} 1\\ 2\\ 3\\ 4\\ 5\\ 6\\ 7\\ 8\\ 9\\ 10\\ 11\\ 12\\ 13\\ 14\\ 15\\ 16\\ 17\\ 18\\ 19\\ 20\\ 21\\ 22\\ 23\\ 34 \end{array}$	$\begin{array}{c} 13\\ 0.18\\ 2.15\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 1.18\\ 0.08\\ 1.19\\ 0.71\\ 0.49\\ 1.68\\ 1.37\\ 1.88\\ 1.16\\ 0.10\\ 0.08\\ 1.94\\ 0.54\\ 0.71\\ 1.48\\ 0.50\\ 0.67\\ 1.66\\ 0.10\\ 0.08\\ 1.94\\ 0.54\\ 0.74\\ 0.54\\ 0.51\\ 0.67\\ 0.67\\ 1.66\\ 0.67\\ 0$	$\begin{array}{c} 14\\ 0.24\\ 2.13\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 0.87\\ 0.10\\ 1.32\\ 0.73\\ 0.44\\ 1.74\\ 1.28\\ 1.85\\ 1.13\\ 0.15\\ 0.08\\ 1.87\\ 0.54\\ 0.72\\ 1.28\\ 0.49\\ 0.74\\ 1.28\\ 0.49\\ 0.74\\ 1.28\\ 0.49\\ 0.74\\ 1.28\\ 0.49\\ 0.74\\ 1.28\\ 0.49\\ 0.74\\ 1.28\\ 0.49\\ 0.74\\ 1.28\\ 0.49\\ 0.74\\ 1.28\\ 0.49\\ 0.74\\ 1.28\\ 0.49\\ 0.74\\ 1.28\\ 0.49\\ 0.74\\ 1.28\\ 0.49\\ 0.74\\ 1.28\\ 0.49\\ 0.74\\ 1.28\\ 0.49\\ 0.74\\ 1.28\\ 0.49\\ 0.74\\ 1.28\\ 0.49\\ 0.74\\ 1.28\\ 0.49\\ 0.74\\ 0$	$\begin{array}{c} 15\\ 0.21\\ 2.24\\ 0.66\\ 0.21\\ 0.52\\ 0.82\\ 0.91\\ 0.19\\ 1.28\\ 0.72\\ 0.48\\ 1.65\\ 1.31\\ 1.90\\ 1.10\\ 0.09\\ 0.08\\ 1.85\\ 0.53\\ 0.71\\ 1.40\\ 0.43\\ 0.80\\ 1.85\\ 0.43\\ 0.80\\ 1.85\\ 0.53\\ 0.71\\ 0.43\\ 0.80\\ 1.85\\ 0.53\\ 0.71\\ 0.43\\ 0.80\\ 1.85\\ 0.53\\ 0.71\\ 0.43\\ 0.80\\ 1.85\\ 0.53\\ 0.71\\ 0.43\\ 0.80\\ 1.85\\ 0.53\\ 0.71\\ 0.43\\ 0.80\\ 1.85\\ 0.53\\ 0.71\\ 0.43\\ 0.80\\ 1.85\\ 0.53\\ 0.71\\ 0.85\\ 0.53\\ 0.71\\ 0.85\\ 0.53\\ 0.71\\ 0.85\\ 0.53\\ 0.71\\ 0.85\\ 0.53\\ 0.71\\ 0.85\\ 0$	$\begin{array}{c} 16\\ 0.22\\ 2.19\\ 0.65\\ 0.20\\ 0.51\\ 0.74\\ 0.97\\ 0.17\\ 1.25\\ 0.77\\ 0.55\\ 1.90\\ 1.51\\ 1.99\\ 1.16\\ 0.13\\ 0.07\\ 2.00\\ 0.53\\ 0.87\\ 1.38\\ 0.66\\ 0.74\\ .25\\ 0.74\\ 0.75\\ 0.$	$\begin{array}{c} 17\\ 0.27\\ 2.46\\ 0.64\\ 0.20\\ 0.50\\ 0.87\\ 1.20\\ 0.14\\ 1.64\\ 0.85\\ 0.65\\ 2.03\\ 1.59\\ 2.36\\ 1.28\\ 0.11\\ 0.07\\ 2.13\\ 0.51\\ 1.05\\ 1.90\\ 0.63\\ 0.77\\ .\end{array}$	$\begin{array}{c} 18 \\ 0.23 \\ 2.47 \\ 0.62 \\ 0.19 \\ 0.49 \\ 0.95 \\ 1.30 \\ 0.17 \\ 1.80 \\ 0.89 \\ 0.75 \\ 2.18 \\ 1.67 \\ 2.55 \\ 1.31 \\ 0.17 \\ 0.50 \\ 1.14 \\ 1.82 \\ 0.70 \\ 0.90 \\ 1.62 \\ 1.92 \\ 1.82 \\ 0.70 \\ 0.90 \\ 0.90 \\ 1.92 \\ 0.90 \\ 1.92 \\ 0.90$	$\begin{array}{c} 19\\ 0.27\\ 2.43\\ 0.61\\ 0.19\\ 0.47\\ 0.90\\ 1.27\\ 0.13\\ 1.50\\ 0.88\\ 0.67\\ 2.25\\ 1.62\\ 2.43\\ 1.37\\ 0.16\\ 0.07\\ 2.23\\ 0.49\\ 1.08\\ 1.92\\ 0.60\\ 0.89\\ 1.92\\ 0.60\\ 0.89\\ 1.92\\ 0.60\\ 0.89\\ 1.92\\ 0.60\\ 0.89\\ 1.92\\ 0.60\\ 0.89\\ 1.92\\ 0.60\\ 0.89\\ 1.92\\ 0.60\\ 0.89\\ 1.92\\ 0.60\\ 0.89\\ 1.92\\ 0.60\\ 0.89\\ 1.92\\ 0.60\\ 0.89\\ 1.92\\ 0.85\\ 0$	$\begin{array}{c} 20\\ 0.28\\ 2.23\\ 0.63\\ 0.20\\ 0.49\\ 0.83\\ 1.23\\ 0.15\\ 1.44\\ 0.75\\ 0.57\\ 2.16\\ 1.55\\ 2.26\\ 1.33\\ 0.14\\ 0.07\\ 2.08\\ 0.51\\ 0.93\\ 1.68\\ 0.53\\ 0.81\\ 1.25\\ 0.81\\ 1.25\\ 0.81\\ 1.25\\ 0.81\\ 1.25\\ 0.81\\ 1.25\\ 0.81\\ 1.25\\ 0.81\\ 1.25\\ 0.81\\ 1.25\\ 0.81\\ 1.25\\ 0.81\\ 1.25\\ 0.81\\ 1.25\\ 0.81\\ 1.25\\ 0.81\\ 1.25\\ 0.81\\ 1.25\\ 0.81\\ 1.25\\ 0.81\\ 0$	$\begin{array}{c} 21 \\ 0.24 \\ 2.14 \\ 0.59 \\ 0.18 \\ 0.46 \\ 0.89 \\ 1.03 \\ 0.13 \\ 1.29 \\ 0.72 \\ 0.59 \\ 1.51 \\ 2.08 \\ 1.51 \\ 2.08 \\ 1.51 \\ 2.08 \\ 1.51 \\ 2.06 \\ 0.47 \\ 0.47 \\ 1.48 \\ 0.49 \\ 0.77 \\ 1.48 \\ 0.49 \\ 0.77 \\ 1.28 \\ 0.49 \\ 0.77 \\ 1.48 \\ 0.49 \\ 0.77 \\ 0.87$	$\begin{array}{c} 22\\ 0.22\\ 2.07\\ 0.54\\ 0.17\\ 0.42\\ 0.83\\ 0.97\\ 0.11\\ 1.18\\ 0.63\\ 0.45\\ 1.79\\ 1.31\\ 1.90\\ 1.15\\ 0.15\\ 0.06\\ 1.78\\ 0.43\\ 0.80\\ 1.36\\ 0.51\\ 0.69\\ 1.46\\ \end{array}$	$\begin{array}{c} 23\\ 0.17\\ 1.71\\ 0.45\\ 0.14\\ 0.35\\ 0.61\\ 0.98\\ 0.57\\ 0.39\\ 1.52\\ 0.98\\ 1.56\\ 0.98\\ 1.56\\ 0.98\\ 1.56\\ 0.09\\ 0.05\\ 1.52\\ 0.36\\ 0.71\\ 1.11\\ 0.44\\ 0.50\\ 0.95\\ 0.95\\ 0.95\\ 0.05\\ 0$	$\begin{array}{c} 24\\ 0.12\\ 1.21\\ 0.37\\ 0.12\\ 0.29\\ 0.48\\ 0.64\\ 0.08\\ 0.74\\ 0.43\\ 0.29\\ 1.20\\ 0.69\\ 1.24\\ 0.70\\ 0.08\\ 0.04\\ 1.14\\ 0.30\\ 0.04\\ 1.14\\ 0.30\\ 0.88\\ 0.33\\ 0.88\\ 0.33\\ 0.40\\ 0.56\\ 0$
$\begin{array}{c} 1\\ 2\\ 3\\ 4\\ 5\\ 6\\ 7\\ 8\\ 9\\ 10\\ 11\\ 12\\ 13\\ 14\\ 15\\ 16\\ 17\\ 18\\ 19\\ 20\\ 21\\ 22\\ 23\\ 24\\ \end{array}$	$\begin{array}{c} 13\\ 0.18\\ 2.15\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 1.18\\ 0.08\\ 1.19\\ 0.71\\ 0.49\\ 1.68\\ 1.37\\ 1.88\\ 1.16\\ 0.10\\ 0.08\\ 1.94\\ 0.54\\ 0.71\\ 1.48\\ 0.54\\ 0.71\\ 1.48\\ 0.50\\ 0.67\\ 1.10\\ \end{array}$	$\begin{array}{c} 14\\ 0.24\\ 2.13\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 0.87\\ 0.10\\ 1.32\\ 0.73\\ 0.44\\ 1.74\\ 1.28\\ 1.85\\ 1.13\\ 0.15\\ 1.28\\ 1.85\\ 1.13\\ 0.15\\ 1.28\\ 1.85\\ 1.13\\ 0.54\\ 0.72\\ 1.28\\ 0.49\\ 0.74\\ 1.11\\ \end{array}$	$\begin{array}{c} 15\\ 0.21\\ 2.24\\ 0.66\\ 0.21\\ 0.52\\ 0.82\\ 0.91\\ 0.19\\ 1.28\\ 0.72\\ 0.48\\ 1.65\\ 1.31\\ 1.90\\ 1.10\\ 0.09\\ 1.85\\ 0.53\\ 0.71\\ 1.40\\ 0.43\\ 0.80\\ 1.13\\ \end{array}$	$\begin{array}{c} 16\\ 0.22\\ 2.19\\ 0.65\\ 0.20\\ 0.51\\ 0.74\\ 0.9\\ 0.17\\ 1.25\\ 0.77\\ 0.55\\ 1.90\\ 1.51\\ 1.99\\ 1.16\\ 0.13\\ 0.07\\ 2.00\\ 0.53\\ 0.87\\ 1.38\\ 0.66\\ 0.74\\ 1.25\\ \end{array}$	$\begin{array}{c} 17\\ 0.27\\ 2.46\\ 0.64\\ 0.20\\ 0.50\\ 0.87\\ 1.20\\ 0.14\\ 1.64\\ 0.85\\ 0.65\\ 2.03\\ 1.59\\ 2.36\\ 1.28\\ 0.11\\ 0.07\\ 2.13\\ 0.51\\ 1.05\\ 1.90\\ 0.63\\ 0.77\\ 1.54 \end{array}$	$\begin{array}{c} 18\\ 0.23\\ 2.47\\ 0.62\\ 0.19\\ 0.95\\ 1.30\\ 0.95\\ 1.30\\ 0.75\\ 2.18\\ 1.67\\ 2.55\\ 1.31\\ 0.11\\ 0.17\\ 2.41\\ 0.50\\ 1.14\\ 1.82\\ 0.70\\ 1.14\\ 1.82\\ 0.90\\ 1.56\\ \end{array}$	$\begin{array}{c} 19\\ 0.27\\ 2.43\\ 0.61\\ 0.19\\ 0.47\\ 0.90\\ 1.27\\ 0.13\\ 1.50\\ 0.88\\ 0.67\\ 2.25\\ 1.62\\ 2.43\\ 1.37\\ 0.16\\ 0.07\\ 2.23\\ 0.49\\ 1.08\\ 1.92\\ 0.60\\ 0.89\\ 1.35\end{array}$	$\begin{array}{c} 20\\ 0.28\\ 2.23\\ 0.63\\ 0.20\\ 0.49\\ 0.83\\ 1.23\\ 0.15\\ 1.44\\ 0.75\\ 0.57\\ 2.16\\ 1.55\\ 2.26\\ 1.33\\ 0.14\\ 0.07\\ 2.08\\ 0.51\\ 0.93\\ 1.68\\ 0.53\\ 0.81\\ 1.36\\ \end{array}$	$\begin{array}{c} 21 \\ 0.24 \\ 2.14 \\ 0.59 \\ 0.18 \\ 0.46 \\ 0.89 \\ 1.03 \\ 0.13 \\ 1.29 \\ 0.72 \\ 0.59 \\ 1.51 \\ 2.08 \\ 1.51 \\ 2.08 \\ 1.18 \\ 0.12 \\ 0.07 \\ 2.06 \\ 0.47 \\ 0.87 \\ 1.48 \\ 0.49 \\ 0.77 \\ 1.24 \end{array}$	$\begin{array}{c} 22\\ 0.22\\ 2.07\\ 0.54\\ 0.17\\ 0.42\\ 0.83\\ 0.97\\ 0.11\\ 1.18\\ 0.63\\ 0.45\\ 1.79\\ 1.31\\ 1.90\\ 1.15\\ 0.15\\ 0.06\\ 1.78\\ 0.43\\ 0.80\\ 1.36\\ 0.51\\ 0.69\\ 1.18\\ \end{array}$	$\begin{array}{c} 23 \\ 0.17 \\ 1.71 \\ 0.45 \\ 0.14 \\ 0.35 \\ 0.61 \\ 0.98 \\ 0.57 \\ 0.39 \\ 1.52 \\ 0.98 \\ 1.56 \\ 0.95 \\ 0.95 \\ 1.52 \\ 0.05 \\ 1.52 \\ 0.36 \\ 0.71 \\ 1.11 \\ 0.44 \\ 0.50 \\ 0.92 \end{array}$	$\begin{array}{c} 24\\ 0.12\\ 1.21\\ 0.37\\ 0.12\\ 0.29\\ 0.48\\ 0.64\\ 0.08\\ 0.74\\ 0.43\\ 0.29\\ 1.20\\ 0.69\\ 1.24\\ 0.70\\ 0.69\\ 1.24\\ 0.70\\ 0.69\\ 1.24\\ 0.70\\ 0.69\\ 1.24\\ 0.70\\ 0.69\\ 1.24\\ 0.70\\ 0.69\\ 1.24\\ 0.70\\ 0.69\\ 1.24\\ 0.70\\ 0.69\\ 0.69\\ 1.24\\ 0.70\\ 0$
$\begin{array}{c} \hline 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 19 \\ 20 \\ 21 \\ 22 \\ 23 \\ 24 \\ 25 \\ \end{array}$	$\begin{array}{c} 13\\ 0.18\\ 2.15\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 1.18\\ 0.08\\ 1.19\\ 0.71\\ 0.49\\ 1.68\\ 1.37\\ 1.88\\ 1.16\\ 0.10\\ 0.08\\ 1.94\\ 0.71\\ 1.48\\ 0.50\\ 0.67\\ 1.10\\ 1.77\\ \end{array}$	$\begin{array}{c} 14\\ 0.24\\ 2.13\\ 0.68\\ 0.21\\ 0.53\\ 0.87\\ 0.10\\ 1.32\\ 0.73\\ 0.44\\ 1.74\\ 1.28\\ 1.85\\ 1.13\\ 0.15\\ 0.08\\ 1.87\\ 0.54\\ 0.72\\ 1.28\\ 0.49\\ 0.74\\ 1.11\\ 1.68\end{array}$	$\begin{array}{c} 15\\ 0.21\\ 2.24\\ 0.66\\ 0.21\\ 0.52\\ 0.91\\ 0.19\\ 1.28\\ 0.72\\ 0.48\\ 1.65\\ 1.31\\ 1.90\\ 1.10\\ 0.09\\ 0.08\\ 1.85\\ 0.53\\ 0.71\\ 1.40\\ 0.43\\ 0.80\\ 1.13\\ 1.77\end{array}$	$\begin{array}{c} 16\\ 0.22\\ 2.19\\ 0.65\\ 0.20\\ 0.51\\ 0.74\\ 0.97\\ 0.17\\ 1.25\\ 0.77\\ 0.55\\ 1.90\\ 1.51\\ 1.99\\ 1.16\\ 0.13\\ 0.07\\ 2.00\\ 0.53\\ 0.87\\ 1.38\\ 0.66\\ 0.74\\ 1.25\\ 1.82\\ \end{array}$	$\begin{array}{c} 17\\ 0.27\\ 2.46\\ 0.64\\ 0.20\\ 0.50\\ 0.87\\ 1.20\\ 0.14\\ 1.64\\ 0.85\\ 2.03\\ 1.59\\ 2.36\\ 1.28\\ 0.11\\ 0.07\\ 2.13\\ 0.51\\ 1.05\\ 1.90\\ 0.63\\ 0.77\\ 1.54\\ 1.96\end{array}$	$\begin{array}{c} 18\\ 0.23\\ 2.47\\ 0.62\\ 0.19\\ 0.49\\ 0.95\\ 1.30\\ 0.17\\ 1.80\\ 0.75\\ 2.18\\ 1.67\\ 2.55\\ 1.31\\ 0.11\\ 0.07\\ 2.41\\ 0.07\\ 1.14\\ 1.82\\ 0.70\\ 0.90\\ 1.56\\ 1.82\end{array}$	$\begin{array}{c} 19\\ 0.27\\ 2.43\\ 0.61\\ 0.19\\ 0.47\\ 0.90\\ 1.27\\ 0.13\\ 1.50\\ 0.88\\ 0.67\\ 2.25\\ 1.62\\ 2.43\\ 1.37\\ 0.16\\ 0.07\\ 2.23\\ 0.49\\ 1.08\\ 1.92\\ 0.60\\ 0.89\\ 1.35\\ 1.80\\ \end{array}$	$\begin{array}{c} 20\\ 0.28\\ 2.23\\ 0.63\\ 0.20\\ 0.49\\ 0.83\\ 1.23\\ 0.15\\ 1.44\\ 0.75\\ 2.16\\ 1.55\\ 2.26\\ 1.35\\ 0.14\\ 0.07\\ 2.08\\ 0.51\\ 0.93\\ 1.68\\ 0.53\\ 0.81\\ 1.36\\ 1.80\\ \end{array}$	$\begin{array}{c} 21 \\ 0.24 \\ 2.14 \\ 0.59 \\ 0.18 \\ 0.46 \\ 0.89 \\ 1.03 \\ 0.13 \\ 1.29 \\ 0.72 \\ 0.59 \\ 1.96 \\ 1.51 \\ 2.08 \\ 1.18 \\ 0.12 \\ 0.07 \\ 2.06 \\ 0.47 \\ 0.87 \\ 1.48 \\ 0.49 \\ 0.77 \\ 1.24 \\ 1.60 \end{array}$	$\begin{array}{c} 22\\ 0.22\\ 2.07\\ 0.54\\ 0.17\\ 0.42\\ 0.83\\ 0.97\\ 0.11\\ 1.18\\ 0.63\\ 0.45\\ 1.79\\ 1.31\\ 1.90\\ 1.15\\ 0.15\\ 0.15\\ 0.15\\ 0.15\\ 0.15\\ 0.15\\ 0.15\\ 0.15\\ 0.15\\ 0.15\\ 0.15\\ 1.78\\ 0.43\\ 0.80\\ 1.36\\ 0.51\\ 0.69\\ 1.18\\ 1.41\\ \end{array}$	$\begin{array}{c} 23\\ 0.17\\ 1.71\\ 0.45\\ 0.14\\ 0.35\\ 0.61\\ 0.83\\ 0.10\\ 0.98\\ 0.57\\ 0.39\\ 1.52\\ 0.98\\ 1.56\\ 0.98\\ 1.56\\ 0.98\\ 1.52\\ 0.09\\ 0.05\\ 0.00\\ 0.05\\ 0.00\\ 0.05\\ 0.00\\ 0.05\\ 0.00\\ 0.05\\ 0.00\\ 0.05\\ 0.00\\ 0.05\\ 0.00\\ 0$	$\begin{array}{c} 24\\ 0.12\\ 1.21\\ 0.37\\ 0.12\\ 0.29\\ 0.48\\ 0.64\\ 0.74\\ 0.43\\ 0.29\\ 1.20\\ 0.69\\ 1.24\\ 0.70\\ 0.69\\ 1.24\\ 0.70\\ 0.08\\ 0.04\\ 1.14\\ 0.33\\ 0.88\\ 0.33\\ 0.40\\ 0.70\\ 0.93\\ \end{array}$
$\begin{array}{c} 1\\ 2\\ 3\\ 4\\ 5\\ 6\\ 7\\ 8\\ 9\\ 10\\ 11\\ 12\\ 13\\ 14\\ 15\\ 16\\ 17\\ 18\\ 19\\ 20\\ 21\\ 22\\ 23\\ 24\\ 25\\ 26\end{array}$	$\begin{array}{c} 13\\ 0.18\\ 2.15\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 1.18\\ 0.08\\ 1.19\\ 0.71\\ 0.49\\ 1.68\\ 1.37\\ 1.88\\ 1.16\\ 0.10\\ 0.08\\ 1.94\\ 0.54\\ 0.71\\ 1.48\\ 0.54\\ 0.71\\ 1.48\\ 0.50\\ 0.67\\ 1.10\\ 1.77\\ 0.34 \end{array}$	$\begin{array}{c} 14\\ 0.24\\ 2.13\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 0.87\\ 0.10\\ 1.32\\ 0.73\\ 0.44\\ 1.74\\ 1.28\\ 1.85\\ 1.13\\ 0.15\\ 1.85\\ 1.13\\ 0.08\\ 1.87\\ 0.54\\ 0.72\\ 1.28\\ 0.49\\ 0.74\\ 1.11\\ 1.68\\ 0.34\end{array}$	$\begin{array}{c} 15\\ 0.21\\ 2.24\\ 0.66\\ 0.21\\ 0.52\\ 0.82\\ 0.91\\ 1.28\\ 0.72\\ 0.48\\ 1.65\\ 1.31\\ 1.90\\ 1.10\\ 0.09\\ 0.08\\ 1.85\\ 0.53\\ 0.71\\ 1.40\\ 0.43\\ 0.80\\ 1.13\\ 1.77\\ 0.34 \end{array}$	$\begin{array}{c} 16\\ 0.22\\ 2.19\\ 0.65\\ 0.20\\ 0.51\\ 0.74\\ 0.97\\ 0.17\\ 1.25\\ 0.77\\ 0.55\\ 1.90\\ 1.51\\ 1.99\\ 1.16\\ 0.13\\ 0.07\\ 2.00\\ 0.53\\ 0.87\\ 1.38\\ 0.66\\ 0.74\\ 1.25\\ 1.82\\ 0.33\\ \end{array}$	$\begin{array}{c} 17\\ 0.27\\ 2.46\\ 0.64\\ 0.20\\ 0.50\\ 0.87\\ 1.20\\ 0.14\\ 1.64\\ 0.85\\ 2.03\\ 1.59\\ 2.36\\ 1.28\\ 0.11\\ 1.05\\ 1.90\\ 2.13\\ 0.51\\ 1.05\\ 1.90\\ 0.63\\ 0.77\\ 1.54\\ 1.96\\ 0.32\end{array}$	$\begin{array}{c} 18\\ 0.23\\ 2.47\\ 0.62\\ 0.19\\ 0.95\\ 1.30\\ 0.95\\ 1.30\\ 0.75\\ 1.80\\ 0.89\\ 0.75\\ 2.18\\ 1.67\\ 2.55\\ 1.31\\ 0.17\\ 0.55\\ 1.31\\ 0.07\\ 2.41\\ 0.50\\ 1.14\\ 1.82\\ 0.70\\ 0.90\\ 1.56\\ 1.82\\ 0.70\\ 0.90\\ 1.56\\ 1.82\\ 0.32\\ 0$	$\begin{array}{c} 19\\ 0.27\\ 2.43\\ 0.61\\ 0.19\\ 0.47\\ 0.90\\ 1.27\\ 0.13\\ 1.50\\ 0.88\\ 0.67\\ 2.25\\ 1.62\\ 2.43\\ 1.37\\ 0.16\\ 0.07\\ 2.23\\ 0.49\\ 1.08\\ 1.92\\ 0.60\\ 0.89\\ 1.35\\ 1.80\\ 0.31\\ \end{array}$	$\begin{array}{c} 20\\ 0.28\\ 2.23\\ 0.63\\ 0.20\\ 0.49\\ 0.83\\ 1.23\\ 0.15\\ 1.44\\ 0.75\\ 0.57\\ 2.16\\ 1.55\\ 2.26\\ 1.33\\ 0.14\\ 1.55\\ 2.26\\ 1.33\\ 0.16\\ 1.55\\ 2.26\\ 1.33\\ 0.16\\ 1.55\\ 0.51\\ 0.93\\ 1.68\\ 0.53\\ 0.81\\ 1.36\\ 1.86\\ 0.32\end{array}$	$\begin{array}{c} 21 \\ 0.24 \\ 2.14 \\ 0.59 \\ 0.18 \\ 0.46 \\ 0.89 \\ 1.03 \\ 0.13 \\ 1.29 \\ 0.72 \\ 0.59 \\ 1.96 \\ 1.51 \\ 2.08 \\ 1.18 \\ 0.12 \\ 0.07 \\ 2.06 \\ 0.47 \\ 0.87 \\ 1.48 \\ 0.49 \\ 0.77 \\ 1.24 \\ 1.60 \\ 0.30 \\ \end{array}$	$\begin{array}{c} 22\\ 0.22\\ 2.07\\ 0.54\\ 0.17\\ 0.42\\ 0.83\\ 0.97\\ 0.11\\ 1.18\\ 0.63\\ 0.45\\ 1.79\\ 1.31\\ 1.90\\ 1.15\\ 0.16\\ 0.63\\ 0.43\\ 0.43\\ 0.80\\ 1.36\\ 0.51\\ 0.69\\ 1.18\\ 1.41\\ 0.27\end{array}$	$\begin{array}{c} 23 \\ 0.17 \\ 1.71 \\ 0.45 \\ 0.14 \\ 0.35 \\ 0.61 \\ 0.98 \\ 0.57 \\ 0.98 \\ 1.56 \\ 0.98 \\ 1.56 \\ 0.95 \\ 0.95 \\ 0.05 \\ 1.52 \\ 0.36 \\ 0.71 \\ 1.11 \\ 0.44 \\ 0.50 \\ 0.92 \\ 1.22 \\ 0.23 \end{array}$	$\begin{array}{c} 24\\ 0.12\\ 1.21\\ 0.37\\ 0.12\\ 0.29\\ 0.48\\ 0.64\\ 0.08\\ 0.74\\ 0.43\\ 0.29\\ 1.20\\ 0.69\\ 1.24\\ 0.70\\ 0.69\\ 1.24\\ 0.70\\ 0.08\\ 0.04\\ 1.14\\ 0.30\\ 0.53\\ 0.88\\ 0.33\\ 0.40\\ 0.70\\ 0.93\\ 0.19\end{array}$
$\begin{array}{c} 1\\ 2\\ 3\\ 4\\ 5\\ 6\\ 7\\ 8\\ 9\\ 10\\ 11\\ 12\\ 13\\ 14\\ 15\\ 16\\ 17\\ 18\\ 19\\ 20\\ 21\\ 22\\ 23\\ 24\\ 25\\ 26\\ 24\\ 25\\ 26\\ 26\\ 7\\ 7\end{array}$	$\begin{array}{c} 13 \\ 0.18 \\ 2.15 \\ 0.68 \\ 0.21 \\ 0.53 \\ 0.85 \\ 1.18 \\ 0.08 \\ 1.19 \\ 0.71 \\ 0.49 \\ 1.68 \\ 1.37 \\ 1.88 \\ 1.16 \\ 0.10 \\ 0.08 \\ 1.94 \\ 0.54 \\ 0.71 \\ 1.48 \\ 0.50 \\ 0.67 \\ 1.10 \\ 1.77 \\ 0.34 \\ 0.57 \\ 1.10 \\ 1.77 \\ 0.35 \\ 0.57$	$\begin{array}{c} 14\\ 0.24\\ 2.13\\ 0.68\\ 0.21\\ 0.53\\ 0.87\\ 0.10\\ 1.32\\ 0.73\\ 0.44\\ 1.74\\ 1.28\\ 1.85\\ 1.13\\ 0.15\\ 0.08\\ 1.87\\ 0.54\\ 0.72\\ 1.28\\ 0.49\\ 0.74\\ 1.11\\ 1.68\\ 0.49\\ 0.72\\ 1.28\\ 0.49\\ 0.72\\ 1.28\\ 0.49\\ 0.72\\ 1.28\\ 0.49\\ 0.72\\ 1.28\\ 0.49\\ 0.72\\ 1.28\\ 0.49\\ 0.72\\ 1.28\\ 0.49\\ 0.72\\ 1.28\\ 0.49\\ 0.72\\ 1.28\\ 0.49\\ 0.72\\ 1.28\\ 0.49\\ 0.72\\ 1.28\\ 0.49\\ 0.72\\ 1.28\\ 0.49\\ 0.72\\ 1.28\\ 0.49\\ 0.72\\ 1.28\\ 0.49\\ 0.72\\ 1.28\\ 0.49\\ 0.72\\ 1.28\\ 0.49\\ 0.72\\ 0.58\\ 0.49\\ 0.72\\ 0.58\\ 0.49\\ 0.72\\ 0.58\\ 0.49\\ 0.72\\ 0.58\\ 0.49\\ 0.72\\ 0.58\\ 0.49\\ 0.72\\ 0.58\\ 0.49\\ 0.72\\ 0.58\\ 0.49\\ 0.72\\ 0.58\\ 0.49\\ 0.72\\ 0.58\\ 0$	$\begin{array}{c} 15\\ 0.21\\ 2.24\\ 0.66\\ 0.21\\ 0.52\\ 0.91\\ 0.19\\ 1.28\\ 0.72\\ 0.48\\ 1.65\\ 1.31\\ 1.90\\ 1.10\\ 0.09\\ 0.08\\ 1.85\\ 0.53\\ 0.71\\ 1.40\\ 0.43\\ 0.80\\ 1.13\\ 1.77\\ 0.34\\ 0.64\\ \end{array}$	$\begin{array}{c} 16\\ 0.22\\ 2.19\\ 0.65\\ 0.20\\ 0.51\\ 0.71\\ 0.51\\ 0.77\\ 1.25\\ 0.77\\ 0.55\\ 1.90\\ 1.51\\ 1.99\\ 1.16\\ 0.13\\ 0.07\\ 2.00\\ 0.53\\ 0.87\\ 1.38\\ 0.66\\ 0.74\\ 1.25\\ 1.82\\ 0.33\\ 0.86\end{array}$	$\begin{array}{c} 17\\ 0.27\\ 2.46\\ 0.64\\ 0.20\\ 0.50\\ 0.87\\ 1.20\\ 0.14\\ 1.64\\ 0.85\\ 0.65\\ 2.03\\ 1.59\\ 2.36\\ 1.28\\ 0.11\\ 0.07\\ 2.13\\ 0.51\\ 1.90\\ 0.63\\ 0.77\\ 1.54\\ 1.96\\ 0.32\\ 0.87\\ \end{array}$	$\begin{array}{c} 18 \\ 0.23 \\ 2.47 \\ 0.62 \\ 0.19 \\ 0.49 \\ 0.95 \\ 1.30 \\ 0.17 \\ 1.80 \\ 0.75 \\ 2.18 \\ 1.67 \\ 2.55 \\ 1.31 \\ 0.11 \\ 0.07 \\ 2.41 \\ 0.07 \\ 2.41 \\ 0.114 \\ 1.82 \\ 0.70 \\ 0.90 \\ 1.56 \\ 1.82 \\ 0.32 \\ 0.32 \\ 0.34 \\ 0.56 \\ 1.82 \\ 0.32 \\ 0.34 \\ 0.56 \\ 0.56 \\ 1.82 \\ 0.32 \\ 0.34 \\ 0.56 \\ 0.5$	$\begin{array}{c} 19\\ 0.27\\ 2.43\\ 0.61\\ 0.19\\ 0.47\\ 0.90\\ 1.27\\ 0.13\\ 1.50\\ 0.88\\ 0.67\\ 2.25\\ 1.62\\ 2.43\\ 1.37\\ 0.16\\ 0.07\\ 2.23\\ 0.49\\ 1.92\\ 0.60\\ 0.89\\ 1.35\\ 1.80\\ 0.31\\ 0.92\end{array}$	$\begin{array}{c} 20\\ 0.28\\ 2.23\\ 0.63\\ 0.20\\ 0.49\\ 0.83\\ 1.23\\ 0.15\\ 1.44\\ 0.75\\ 2.16\\ 1.55\\ 2.26\\ 1.55\\ 2.26\\ 1.33\\ 0.14\\ 0.07\\ 2.08\\ 0.51\\ 0.93\\ 1.68\\ 0.53\\ 0.81\\ 1.36\\ 1.80\\ 0.32\\ 0.85\\ 0$	$\begin{array}{c} 21 \\ 0.24 \\ 2.14 \\ 0.59 \\ 0.18 \\ 0.46 \\ 0.89 \\ 1.03 \\ 0.13 \\ 1.29 \\ 0.72 \\ 0.59 \\ 1.96 \\ 1.51 \\ 2.08 \\ 1.18 \\ 0.12 \\ 0.07 \\ 2.06 \\ 0.47 \\ 0.87 \\ 1.48 \\ 0.49 \\ 0.77 \\ 1.24 \\ 1.60 \\ 0.30 \\ 0.77 \end{array}$	$\begin{array}{c} 22\\ 0.22\\ 2.07\\ 0.54\\ 0.17\\ 0.42\\ 0.83\\ 0.97\\ 0.11\\ 1.18\\ 0.63\\ 0.45\\ 1.79\\ 1.31\\ 1.90\\ 1.15\\ 0.06\\ 1.78\\ 0.43\\ 0.80\\ 1.36\\ 0.51\\ 0.69\\ 1.18\\ 1.41\\ 0.27\\ 0.02\end{array}$	$\begin{array}{c} 23 \\ 0.17 \\ 1.71 \\ 0.45 \\ 0.14 \\ 0.35 \\ 0.61 \\ 0.83 \\ 0.10 \\ 0.98 \\ 0.57 \\ 0.39 \\ 1.52 \\ 0.98 \\ 1.56 \\ 0.95 \\ 0.09 \\ 0.05 \\ 1.52 \\ 0.09 \\ 0.05 \\ 1.52 \\ 0.09 \\ 0.05 \\ 1.52 \\ 0.09 \\ 0.05 \\ 1.52 \\ 0.09 \\ 0.05 \\ 1.52 \\ 0.23$	$\begin{array}{c} 24\\ 0.12\\ 1.21\\ 0.37\\ 0.12\\ 0.29\\ 0.48\\ 0.64\\ 0.08\\ 0.74\\ 0.43\\ 0.29\\ 1.20\\ 0.69\\ 1.24\\ 0.70\\ 0.08\\ 0.04\\ 1.14\\ 0.30\\ 0.53\\ 0.88\\ 0.33\\ 0.40\\ 0.53\\ 0.88\\ 0.33\\ 0.40\\ 0.70\\ 0.93\\ 0.19\\ 0.93\\ 0.19\\ 0.93\\ 0.19\\ 0.46\\ 0.56\\ 0.93\\ 0.93\\ 0.93\\ 0.94\\ 0.93\\ 0.93\\ 0.93\\ 0.94\\ 0.93\\ 0$
$\begin{array}{c} 1\\ 2\\ 3\\ 4\\ 5\\ 6\\ 7\\ 8\\ 9\\ 10\\ 11\\ 12\\ 13\\ 14\\ 15\\ 16\\ 17\\ 18\\ 19\\ 20\\ 21\\ 22\\ 23\\ 24\\ 25\\ 26\\ 27\\ \end{array}$	$\begin{array}{c} 13\\ 0.18\\ 2.15\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 1.18\\ 0.08\\ 1.19\\ 0.71\\ 0.49\\ 1.68\\ 1.37\\ 1.88\\ 1.16\\ 0.10\\ 0.08\\ 1.94\\ 0.54\\ 0.54\\ 0.54\\ 0.54\\ 0.54\\ 0.54\\ 0.54\\ 0.54\\ 0.54\\ 0.51\\ 1.48\\ 0.50\\ 0.67\\ 1.10\\ 1.77\\ 0.34\\ 0.55\\ \end{array}$	$\begin{array}{c} 14\\ 0.24\\ 2.13\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 0.87\\ 0.10\\ 1.32\\ 0.73\\ 0.44\\ 1.74\\ 1.28\\ 1.85\\ 1.13\\ 0.15\\ 1.13\\ 0.08\\ 1.87\\ 0.54\\ 0.72\\ 1.28\\ 0.49\\ 0.74\\ 1.11\\ 1.68\\ 0.34\\ 0.63\\ \end{array}$	$\begin{array}{c} 15\\ 0.21\\ 2.24\\ 0.66\\ 0.21\\ 0.52\\ 0.82\\ 0.91\\ 1.28\\ 0.72\\ 0.48\\ 1.65\\ 1.31\\ 1.90\\ 1.10\\ 0.09\\ 0.08\\ 1.85\\ 0.53\\ 0.71\\ 1.40\\ 0.43\\ 0.80\\ 1.13\\ 1.77\\ 0.34\\ 0.64\\ \end{array}$	$\begin{array}{c} 16\\ 0.22\\ 2.19\\ 0.65\\ 0.20\\ 0.51\\ 0.74\\ 0.97\\ 0.17\\ 1.25\\ 0.77\\ 0.55\\ 1.90\\ 1.51\\ 1.99\\ 1.16\\ 0.13\\ 0.07\\ 2.00\\ 0.53\\ 0.87\\ 1.38\\ 0.66\\ 0.74\\ 1.25\\ 1.82\\ 0.33\\ 0.82\\ \end{array}$	$\begin{array}{c} 17\\ 0.27\\ 2.46\\ 0.64\\ 0.20\\ 0.50\\ 0.87\\ 1.20\\ 0.14\\ 1.64\\ 0.85\\ 0.65\\ 2.03\\ 1.59\\ 2.36\\ 1.28\\ 0.11\\ 1.05\\ 1.99\\ 0.63\\ 0.77\\ 1.54\\ 1.96\\ 0.32\\ 0.85\\ \end{array}$	$\begin{array}{c} 18\\ 0.23\\ 2.47\\ 0.62\\ 0.19\\ 0.49\\ 0.95\\ 1.30\\ 0.17\\ 1.80\\ 0.89\\ 0.75\\ 2.18\\ 1.67\\ 2.55\\ 1.31\\ 0.17\\ 0.50\\ 1.14\\ 1.82\\ 0.70\\ 0.90\\ 1.56\\ 1.82\\ 0.32\\ 0.94 \end{array}$	$\begin{array}{c} 19\\ 0.27\\ 2.43\\ 0.61\\ 0.19\\ 0.47\\ 0.90\\ 1.27\\ 0.13\\ 1.50\\ 0.88\\ 0.67\\ 2.25\\ 1.62\\ 2.43\\ 1.37\\ 0.16\\ 2.23\\ 0.49\\ 1.08\\ 1.92\\ 0.60\\ 0.89\\ 1.35\\ 1.80\\ 0.31\\ 0.92 \end{array}$	$\begin{array}{c} 20\\ 0.28\\ 2.23\\ 0.63\\ 0.20\\ 0.49\\ 0.83\\ 1.23\\ 0.15\\ 1.44\\ 0.75\\ 0.57\\ 2.16\\ 1.55\\ 2.26\\ 1.33\\ 0.14\\ 1.55\\ 2.26\\ 1.33\\ 0.16\\ 1.85\\ 0.51\\ 0.93\\ 1.68\\ 0.53\\ 0.81\\ 1.36\\ 1.80\\ 0.32\\ 0.85\\ \end{array}$	$\begin{array}{c} 21 \\ 0.24 \\ 2.14 \\ 0.59 \\ 0.18 \\ 0.46 \\ 0.89 \\ 1.03 \\ 0.13 \\ 1.29 \\ 0.72 \\ 0.59 \\ 1.51 \\ 2.08 \\ 1.51 \\ 2.08 \\ 1.51 \\ 2.08 \\ 1.51 \\ 2.06 \\ 0.47 \\ 0.77 \\ 1.48 \\ 0.49 \\ 0.77 \\ 1.24 \\ 1.60 \\ 0.30 \\ 0.77 \end{array}$	$\begin{array}{c} 22\\ 0.22\\ 2.07\\ 0.54\\ 0.17\\ 0.42\\ 0.83\\ 0.97\\ 0.11\\ 1.18\\ 0.63\\ 0.45\\ 1.79\\ 1.31\\ 1.90\\ 1.15\\ 0.16\\ 0.66\\ 1.78\\ 0.43\\ 0.80\\ 1.36\\ 0.51\\ 0.69\\ 1.18\\ 1.41\\ 0.27\\ 0.69\end{array}$	$\begin{array}{c} 23\\ 0.17\\ 1.71\\ 0.45\\ 0.14\\ 0.35\\ 0.61\\ 0.98\\ 0.57\\ 0.39\\ 1.52\\ 0.98\\ 1.56\\ 0.98\\ 1.56\\ 0.98\\ 1.52\\ 0.05\\ 1.52\\ 0.36\\ 0.71\\ 1.11\\ 0.44\\ 0.50\\ 0.92\\ 1.22\\ 0.23\\ 0.52\\ \end{array}$	$\begin{array}{c} 24\\ 0.12\\ 1.21\\ 0.37\\ 0.12\\ 0.29\\ 0.48\\ 0.64\\ 0.08\\ 0.74\\ 0.43\\ 0.29\\ 1.20\\ 0.69\\ 1.24\\ 0.70\\ 0.08\\ 0.04\\ 1.14\\ 0.30\\ 0.04\\ 1.14\\ 0.30\\ 0.58\\ 0.04\\ 1.14\\ 0.30\\ 0.58\\ 0.33\\ 0.40\\ 0.70\\ 0.93\\ 0.19\\ 0.40\\ \end{array}$
$\begin{array}{c} 1\\ 2\\ 3\\ 4\\ 5\\ 6\\ 7\\ 8\\ 9\\ 10\\ 11\\ 12\\ 13\\ 14\\ 15\\ 16\\ 17\\ 18\\ 19\\ 20\\ 21\\ 22\\ 23\\ 24\\ 25\\ 26\\ 27\\ 28 \end{array}$	$\begin{array}{c} 13\\ 0.18\\ 2.15\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 1.18\\ 0.08\\ 1.19\\ 0.71\\ 0.49\\ 1.68\\ 1.37\\ 1.88\\ 1.16\\ 0.10\\ 0.08\\ 1.94\\ 0.54\\ 0.71\\ 1.48\\ 0.54\\ 0.71\\ 1.48\\ 0.54\\ 0.71\\ 1.48\\ 0.54\\ 0.71\\ 1.48\\ 0.54\\ 0.54\\ 0.54\\ 0.54\\ 0.54\\ 0.54\\ 0.54\\ 0.54\\ 0.54\\ 0.54\\ 0.54\\ 0.54\\ 0.54\\ 0.54\\ 0.55\\ 0.08\\ 0$	$\begin{array}{c} 14\\ 0.24\\ 2.13\\ 0.68\\ 0.21\\ 0.53\\ 0.87\\ 0.10\\ 1.32\\ 0.73\\ 0.44\\ 1.28\\ 1.85\\ 1.13\\ 0.08\\ 1.87\\ 0.54\\ 0.72\\ 1.28\\ 0.49\\ 0.74\\ 1.11\\ 1.68\\ 0.34\\ 0.63\\ 0.08\\ \end{array}$	$\begin{array}{c} 15\\ 0.21\\ 2.24\\ 0.66\\ 0.21\\ 0.52\\ 0.91\\ 0.19\\ 1.28\\ 0.72\\ 0.48\\ 1.65\\ 1.31\\ 1.90\\ 1.10\\ 0.09\\ 0.08\\ 1.85\\ 0.53\\ 0.71\\ 1.40\\ 0.09\\ 1.13\\ 1.77\\ 0.34\\ 0.64\\ 0.07\\ \end{array}$	$\begin{array}{c} 16\\ 0.22\\ 2.19\\ 0.65\\ 0.20\\ 0.51\\ 0.71\\ 0.51\\ 0.77\\ 0.55\\ 1.90\\ 1.51\\ 1.99\\ 1.16\\ 0.13\\ 0.07\\ 2.00\\ 0.53\\ 0.87\\ 1.38\\ 0.66\\ 0.74\\ 1.25\\ 1.82\\ 0.33\\ 0.82\\ 0.07\\ \end{array}$	$\begin{array}{c} 17\\ 0.27\\ 2.46\\ 0.64\\ 0.20\\ 0.50\\ 0.87\\ 1.20\\ 0.14\\ 1.64\\ 0.85\\ 2.03\\ 1.59\\ 2.36\\ 1.28\\ 0.11\\ 0.07\\ 2.13\\ 0.51\\ 1.05\\ 1.90\\ 0.63\\ 0.77\\ 1.54\\ 1.96\\ 0.32\\ 0.85\\ 0.07\\ \end{array}$	$\begin{array}{c} 18 \\ 0.23 \\ 2.47 \\ 0.62 \\ 0.19 \\ 0.95 \\ 1.30 \\ 0.17 \\ 1.80 \\ 0.75 \\ 2.18 \\ 1.67 \\ 2.55 \\ 1.31 \\ 0.07 \\ 2.41 \\ 0.75 \\ 1.14 \\ 1.82 \\ 0.70 \\ 0.90 \\ 1.14 \\ 1.82 \\ 0.70 \\ 0.90 \\ 1.56 \\ 1.82 \\ 0.32 \\ 0.94 \\ 0.07 \end{array}$	$\begin{array}{c} 19\\ 0.27\\ 2.43\\ 0.61\\ 0.19\\ 0.47\\ 0.90\\ 1.27\\ 0.13\\ 1.50\\ 0.8\\ 0.67\\ 2.25\\ 1.62\\ 2.43\\ 1.37\\ 0.16\\ 0.07\\ 2.23\\ 1.37\\ 0.16\\ 0.07\\ 2.23\\ 1.37\\ 0.16\\ 0.07\\ 1.92\\ 1.08\\ 1.92\\ 1.08\\ 1.92\\ 0.60\\ 0.89\\ 1.35\\ 1.80\\ 0.31\\ 0.92\\ 0.07\\ \end{array}$	$\begin{array}{c} 20\\ 0.28\\ 2.23\\ 0.63\\ 0.20\\ 0.49\\ 0.83\\ 1.23\\ 0.15\\ 1.44\\ 0.75\\ 2.16\\ 1.55\\ 2.26\\ 1.33\\ 0.57\\ 2.16\\ 1.55\\ 2.26\\ 1.33\\ 0.57\\ 2.16\\ 1.55\\ 2.26\\ 1.33\\ 0.57\\ 2.16\\ 1.33\\ 0.14\\ 0.07\\ 2.08\\ 0.51\\ 0.93\\ 1.68\\ 0.51\\ 0.93\\ 1.68\\ 0.51\\ 0.93\\ 1.68\\ 0.51\\ 0.93\\ 1.68\\ 0.51\\ 0.93\\ 0.81\\ 1.36\\ 1.80\\ 0.32\\ 0.85\\ 0.07\\ \end{array}$	$\begin{array}{c} 21\\ 0.24\\ 2.14\\ 0.59\\ 0.18\\ 0.46\\ 0.89\\ 1.03\\ 0.13\\ 1.29\\ 0.72\\ 0.59\\ 1.96\\ 1.51\\ 2.08\\ 1.18\\ 0.12\\ 0.07\\ 2.06\\ 0.47\\ 0.87\\ 1.48\\ 0.49\\ 0.77\\ 1.24\\ 1.60\\ 0.30\\ 0.77\\ 0.07\end{array}$	$\begin{array}{c} 22\\ 0.22\\ 2.07\\ 0.54\\ 0.17\\ 0.42\\ 0.83\\ 0.97\\ 0.11\\ 1.18\\ 0.63\\ 0.45\\ 1.79\\ 1.31\\ 1.90\\ 1.15\\ 0.06\\ 1.78\\ 0.43\\ 0.80\\ 1.36\\ 0.51\\ 0.69\\ 1.18\\ 1.41\\ 0.27\\ 0.69\\ 0.06\end{array}$	$\begin{array}{c} 23\\ 0.17\\ 1.71\\ 0.45\\ 0.14\\ 0.35\\ 0.61\\ 0.83\\ 0.10\\ 0.98\\ 0.57\\ 0.39\\ 1.52\\ 0.98\\ 1.56\\ 0.95\\ 0.09\\ 0.05\\ 1.52\\ 0.39\\ 1.52\\ 0.98\\ 1.52\\ 0.98\\ 1.52\\ 0.98\\ 1.52\\ 0.95\\ 0.09\\ 1.22\\ 0.23\\ 0.52\\ 0.05\\ \end{array}$	$\begin{array}{c} 24\\ 0.12\\ 1.21\\ 0.37\\ 0.12\\ 0.29\\ 0.48\\ 0.64\\ 0.08\\ 0.74\\ 0.43\\ 0.29\\ 1.20\\ 0.69\\ 1.24\\ 0.70\\ 0.08\\ 0.04\\ 1.14\\ 0.30\\ 0.53\\ 0.88\\ 0.33\\ 0.40\\ 0.70\\ 0.93\\ 0.19\\ 0.40\\ 0.04\\ \end{array}$
$\begin{array}{c} 1\\ 2\\ 3\\ 4\\ 5\\ 6\\ 7\\ 8\\ 9\\ 10\\ 11\\ 12\\ 13\\ 14\\ 15\\ 16\\ 17\\ 18\\ 19\\ 20\\ 21\\ 22\\ 23\\ 24\\ 25\\ 26\\ 27\\ 28\\ 29\\ \end{array}$	$\begin{array}{c} 13\\ 0.18\\ 2.15\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 1.18\\ 0.08\\ 1.19\\ 0.71\\ 0.49\\ 1.68\\ 1.37\\ 1.88\\ 1.16\\ 0.10\\ 0.08\\ 1.94\\ 0.71\\ 1.48\\ 0.50\\ 0.67\\ 1.10\\ 0.67\\ 1.148\\ 0.50\\ 0.67\\ 1.00\\ 1.77\\ 0.34\\ 0.55\\ 0.08\\ 0.22\\ \end{array}$	$\begin{array}{c} 14\\ 0.24\\ 2.13\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 0.87\\ 0.10\\ 1.32\\ 0.73\\ 0.44\\ 1.74\\ 1.28\\ 1.85\\ 1.13\\ 0.15\\ 0.08\\ 1.87\\ 0.54\\ 0.72\\ 1.28\\ 0.49\\ 0.74\\ 1.11\\ 1.68\\ 0.34\\ 0.63\\ 0.08\\ 0.22\\ \end{array}$	$\begin{array}{c} 15\\ 0.21\\ 2.24\\ 0.66\\ 0.21\\ 0.52\\ 0.82\\ 0.91\\ 0.19\\ 1.28\\ 0.72\\ 0.48\\ 1.65\\ 1.31\\ 1.90\\ 1.10\\ 0.09\\ 0.08\\ 1.85\\ 0.53\\ 0.71\\ 1.40\\ 0.43\\ 0.80\\ 1.13\\ 0.80\\ 1.13\\ 1.77\\ 0.34\\ 0.64\\ 0.07\\ 0.21\\ \end{array}$	$\begin{array}{c} 16\\ 0.22\\ 2.19\\ 0.65\\ 0.20\\ 0.51\\ 0.74\\ 0.97\\ 0.17\\ 1.25\\ 0.77\\ 0.55\\ 1.90\\ 1.51\\ 1.99\\ 1.16\\ 0.13\\ 0.07\\ 2.00\\ 0.53\\ 0.87\\ 1.38\\ 0.66\\ 0.74\\ 1.25\\ 1.82\\ 0.33\\ 0.82\\ 0.07\\ 2.1\\ 0.21\\ 0.$	$\begin{array}{c} 17\\ 0.27\\ 2.46\\ 0.64\\ 0.50\\ 0.50\\ 0.50\\ 0.7\\ 1.20\\ 0.14\\ 1.64\\ 0.85\\ 2.03\\ 1.59\\ 2.36\\ 1.28\\ 0.11\\ 0.07\\ 2.13\\ 0.51\\ 1.05\\ 1.90\\ 0.63\\ 0.77\\ 1.54\\ 1.96\\ 0.32\\ 0.85\\ 0.07\\ 0.21\\ \end{array}$	$\begin{array}{c} 18\\ 0.23\\ 2.47\\ 0.62\\ 0.19\\ 0.49\\ 0.95\\ 1.30\\ 0.17\\ 1.80\\ 0.89\\ 0.75\\ 2.18\\ 1.67\\ 2.55\\ 1.31\\ 0.17\\ 2.55\\ 1.31\\ 0.17\\ 2.41\\ 0.07\\ 2.41\\ 0.07\\ 2.41\\ 0.07\\ 2.41\\ 0.07\\ 2.41\\ 0.07\\ 2.41\\ 0.07\\ 2.41\\ 0.07\\ 2.41\\ 0.02\\ 0.90\\ 1.56\\ 1.82\\ 0.90\\ 1.56\\ 0.90\\ 1.82\\ 0.90\\ 1.82\\ 0.94\\ 0.07\\ 0.20\\ 0.90\\ 0$	$\begin{array}{c} 19\\ 0.27\\ 2.43\\ 0.61\\ 0.19\\ 0.47\\ 0.90\\ 1.27\\ 0.13\\ 1.50\\ 0.88\\ 0.67\\ 2.25\\ 1.62\\ 2.43\\ 1.37\\ 0.16\\ 0.07\\ 2.23\\ 0.49\\ 1.08\\ 1.92\\ 0.60\\ 0.89\\ 1.35\\ 1.80\\ 0.31\\ 0.92\\ 0.07\\ 0.20\\ \end{array}$	$\begin{array}{c} 20\\ 0.28\\ 2.23\\ 0.63\\ 0.20\\ 0.49\\ 0.83\\ 1.23\\ 0.15\\ 1.44\\ 0.75\\ 0.57\\ 2.16\\ 1.55\\ 2.26\\ 1.33\\ 0.14\\ 0.07\\ 2.08\\ 0.51\\ 0.93\\ 1.68\\ 0.53\\ 0.81\\ 1.36\\ 1.80\\ 0.32\\ 0.85\\ 0.07\\ 0.20\\ \end{array}$	$\begin{array}{c} 21 \\ 0.24 \\ 2.14 \\ 0.59 \\ 0.18 \\ 0.46 \\ 0.89 \\ 1.03 \\ 0.13 \\ 1.29 \\ 0.72 \\ 0.59 \\ 1.96 \\ 1.51 \\ 2.08 \\ 1.18 \\ 0.12 \\ 0.07 \\ 2.06 \\ 0.47 \\ 1.24 \\ 0.07 \\ 1.24 \\ 0.67 \\ 1.48 \\ 0.49 \\ 0.77 \\ 1.24 \\ 1.60 \\ 0.30 \\ 0.77 \\ 0.07 \\ 0.19 \\ \end{array}$	$\begin{array}{c} 22\\ 0.22\\ 2.07\\ 0.54\\ 0.17\\ 0.42\\ 0.83\\ 0.97\\ 0.11\\ 1.18\\ 0.63\\ 0.45\\ 1.79\\ 1.31\\ 1.90\\ 1.15\\ 0.15\\ 0.15\\ 0.15\\ 0.15\\ 0.66\\ 1.78\\ 0.43\\ 0.80\\ 1.36\\ 0.51\\ 0.69\\ 1.18\\ 1.41\\ 0.27\\ 0.69\\ 0.06\\ 0.17\\ \end{array}$	$\begin{array}{c} 23\\ 0.17\\ 1.71\\ 0.45\\ 0.14\\ 0.35\\ 0.14\\ 0.35\\ 0.14\\ 0.83\\ 0.10\\ 0.98\\ 0.57\\ 0.39\\ 1.52\\ 0.98\\ 1.56\\ 0.98\\ 1.56\\ 0.98\\ 1.52\\ 0.09\\ 0.05\\ 1.52\\ 0.09\\ 0.05\\ 1.52\\ 0.09\\ 1.52\\ 0.05\\ 1.52\\ 0.22\\ 0.23\\ 0.52\\ 0.14\\ 0$	$\begin{array}{c} 24\\ 0.12\\ 1.21\\ 0.37\\ 0.12\\ 0.29\\ 0.48\\ 0.64\\ 0.08\\ 0.74\\ 0.43\\ 0.29\\ 1.20\\ 0.69\\ 1.24\\ 0.70\\ 0.69\\ 1.24\\ 0.70\\ 0.08\\ 0.04\\ 1.14\\ 0.30\\ 0.53\\ 0.88\\ 0.33\\ 0.40\\ 0.70\\ 0.93\\ 0.19\\ 0.40\\ 0.12\\ \end{array}$
$\begin{array}{c} 1\\ 2\\ 3\\ 4\\ 5\\ 6\\ 7\\ 8\\ 9\\ 10\\ 11\\ 12\\ 13\\ 14\\ 15\\ 16\\ 17\\ 18\\ 19\\ 20\\ 21\\ 223\\ 24\\ 25\\ 26\\ 27\\ 28\\ 29\\ 20\\ \end{array}$	$\begin{array}{c} 13\\ 0.18\\ 2.15\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 1.18\\ 0.08\\ 1.19\\ 0.71\\ 0.49\\ 1.68\\ 1.37\\ 1.88\\ 1.16\\ 0.10\\ 0.08\\ 1.94\\ 0.54\\ 0.71\\ 1.48\\ 0.50\\ 0.67\\ 1.10\\ 1.77\\ 0.34\\ 0.55\\ 0.08\\ 0.22\\ 1.44\end{array}$	$\begin{array}{c} 14\\ 0.24\\ 2.13\\ 0.68\\ 0.21\\ 0.53\\ 0.85\\ 0.87\\ 0.10\\ 1.32\\ 0.73\\ 0.44\\ 1.74\\ 1.28\\ 1.85\\ 1.13\\ 0.15\\ 0.08\\ 1.87\\ 0.54\\ 0.72\\ 1.28\\ 0.49\\ 0.74\\ 1.11\\ 1.68\\ 0.34\\ 0.63\\ 0.08\\ 0.22\\ 1.26\\ 0.74\\ 0.63\\ 0.08\\ 0.22\\ 1.26\\ 0.82\\ 0$	$\begin{array}{c} 15\\ 0.21\\ 2.24\\ 0.66\\ 0.21\\ 0.52\\ 0.82\\ 0.91\\ 0.19\\ 1.28\\ 0.72\\ 0.48\\ 1.65\\ 1.31\\ 1.90\\ 1.10\\ 0.09\\ 0.08\\ 1.85\\ 0.53\\ 0.71\\ 1.40\\ 0.43\\ 0.80\\ 1.13\\ 1.77\\ 0.34\\ 0.64\\ 0.07\\ 0.21\\ 1.40\end{array}$	$\begin{array}{c} 16\\ 0.22\\ 2.19\\ 0.65\\ 0.20\\ 0.51\\ 0.74\\ 0.97\\ 0.17\\ 1.25\\ 0.77\\ 0.55\\ 1.90\\ 1.51\\ 1.99\\ 1.16\\ 0.13\\ 0.07\\ 2.00\\ 0.53\\ 0.87\\ 1.38\\ 0.66\\ 0.74\\ 1.25\\ 1.82\\ 0.33\\ 0.82\\ 0.07\\ 0.21\\ 1.25\\ 1.82\\ 0.33\\ 0.82\\ 0.07\\ 0.21\\ 1.25\\ 0.33\\ 0.82\\ 0.07\\ 0.21\\ 0.22\\ 0.32\\ 0.07\\ 0.21\\ 0.22\\ 0.32\\ 0.07\\ 0.21\\ 0.22\\ 0.32\\ 0.07\\ 0.21\\ 0.22\\ 0.32\\ 0.07\\ 0.21\\ 0.22\\ 0.32\\ 0.07\\ 0.21\\ 0.22\\ 0.32\\ 0.07\\ 0.21\\ 0.22\\ 0.32\\ 0.07\\ 0.21\\ 0.22\\ 0.32\\ 0.07\\ 0.21\\ 0.22\\ 0.32\\ 0.07\\ 0.21\\ 0.22\\ 0.32\\ 0.07\\ 0.21\\ 0.22\\ 0.07\\ 0.07\\ 0$	$\begin{array}{c} 17\\ 0.27\\ 2.46\\ 0.64\\ 0.20\\ 0.50\\ 0.87\\ 1.20\\ 0.14\\ 1.64\\ 0.85\\ 2.03\\ 1.59\\ 2.36\\ 1.28\\ 0.11\\ 0.07\\ 2.13\\ 0.51\\ 1.05\\ 1.90\\ 0.63\\ 0.77\\ 1.54\\ 1.96\\ 0.32\\ 0.85\\ 0.07\\ 0.21\\ \end{array}$	$\begin{array}{c} 18\\ 0.23\\ 2.47\\ 0.62\\ 0.19\\ 0.95\\ 1.30\\ 0.95\\ 1.30\\ 0.71\\ 1.80\\ 0.89\\ 0.75\\ 2.18\\ 1.67\\ 2.55\\ 1.31\\ 0.11\\ 0.07\\ 2.41\\ 0.50\\ 1.14\\ 1.82\\ 0.70\\ 0.90\\ 1.56\\ 1.82\\ 0.32\\ 0.94\\ 0.07\\ 0.20\\ \end{array}$	$\begin{array}{c} 19\\ 0.27\\ 2.43\\ 0.61\\ 0.19\\ 0.47\\ 0.90\\ 1.27\\ 0.13\\ 1.50\\ 0.88\\ 0.67\\ 2.25\\ 1.62\\ 2.43\\ 1.37\\ 0.16\\ 2.23\\ 0.49\\ 1.08\\ 1.92\\ 0.60\\ 0.89\\ 1.35\\ 1.80\\ 0.89\\ 1.35\\ 1.80\\ 0.31\\ 0.92\\ 0.07\\ 0.20\\ 0.7\\ 0.20\\ 0.7\\ 0.20\\ 0.7\\ 0.20\\ 0.92\\ 0.07\\ 0.20\\ 0.07\\ 0.02\\ 0.07\\ 0.02\\ 0.07\\ 0.02\\ 0.07\\ 0.02\\ 0.07\\ 0.02\\ 0.07\\ 0.02\\ 0.07\\ 0.02\\ 0.07\\ 0.02\\ 0.07\\ 0.02\\ 0.07\\ 0.02\\ 0.07\\ 0.02$	$\begin{array}{c} 20\\ 0.28\\ 2.23\\ 0.63\\ 0.20\\ 0.49\\ 0.83\\ 1.23\\ 0.15\\ 1.44\\ 0.75\\ 0.57\\ 2.16\\ 1.55\\ 2.26\\ 1.33\\ 0.14\\ 0.07\\ 2.08\\ 0.51\\ 0.93\\ 1.68\\ 0.53\\ 0.81\\ 1.36\\ 1.80\\ 0.32\\ 0.85\\ 0.07\\ 0.20\\ \end{array}$	$\begin{array}{c} 21 \\ 0.24 \\ 2.14 \\ 0.59 \\ 0.18 \\ 0.46 \\ 0.89 \\ 1.03 \\ 0.13 \\ 1.29 \\ 0.72 \\ 0.59 \\ 1.96 \\ 1.51 \\ 2.08 \\ 1.18 \\ 0.12 \\ 0.07 \\ 2.06 \\ 0.47 \\ 0.87 \\ 1.48 \\ 0.49 \\ 0.77 \\ 1.24 \\ 1.60 \\ 0.30 \\ 0.77 \\ 0.07 \\ 0.07 \\ 0.07 \\ 0.97 \\ 1.72 \\ 0.07 \\ 0.97$	$\begin{array}{c} 22\\ 0.22\\ 2.07\\ 0.54\\ 0.17\\ 0.42\\ 0.83\\ 0.97\\ 0.11\\ 1.18\\ 0.63\\ 0.45\\ 1.79\\ 1.31\\ 1.90\\ 1.15\\ 0.15\\ 0.06\\ 1.78\\ 0.43\\ 0.80\\ 1.36\\ 0.51\\ 0.69\\ 1.18\\ 1.41\\ 0.27\\ 0.69\\ 0.06\\ 0.17\\ 0.69\\ 0.06\\ 0.17\\ 0.69\\ 0.06\\ 0.17\\ 0.69\\ 0.06\\ 0.17\\ 0.69\\ 0.06\\ 0.17\\ 0.69\\ 0.06\\ 0.17\\ 0.69\\ 0.06\\ 0.17\\ 0.69\\ 0.06\\ 0.17\\ 0.69\\ 0.06\\ 0.17\\ 0.69\\ 0.06\\ 0.17\\ 0.69\\ 0.06\\ 0.17\\ 0.69\\ 0.06\\ 0.17\\ 0.69\\ 0.06\\ 0.17\\ 0.69\\ 0.06\\ 0.17\\ 0.69\\ 0.06\\ 0.17\\ 0.69\\ 0.06\\ 0.17\\ 0.69\\ 0.06\\ 0.17\\ 0.69\\ 0.06\\ 0.17\\ 0.69\\ 0.06\\ 0.17\\ 0.5\\ 0.06\\ 0.17\\ 0.5\\ 0.06\\ 0.17\\ 0.5\\ 0.06$	$\begin{array}{c} 23 \\ 0.17 \\ 1.71 \\ 0.45 \\ 0.14 \\ 0.35 \\ 0.61 \\ 0.98 \\ 0.57 \\ 0.98 \\ 1.56 \\ 0.99 \\ 1.52 \\ 0.98 \\ 1.56 \\ 0.95 \\ 0.09 \\ 1.52 \\ 0.36 \\ 0.71 \\ 1.11 \\ 0.44 \\ 0.50 \\ 0.92 \\ 1.22 \\ 0.23 \\ 0.52 \\ 0.05 \\ 0.14 \\ 1.1 \\ 0.51 \\ 0.52 \\ 0.52 \\ 0.52 \\ 0.52 \\ 0.14 \\ 0.52 \\ 0.52 \\ 0.52 \\ 0.52 \\ 0.52 \\ 0.52 \\ 0.52 \\ 0.52 \\ 0.52 \\ 0.52 \\ 0.52 \\ 0.51 \\ 0.52 $	$\begin{array}{c} 24\\ 0.12\\ 1.21\\ 0.37\\ 0.12\\ 0.29\\ 0.48\\ 0.64\\ 0.08\\ 0.74\\ 0.43\\ 0.29\\ 1.20\\ 0.69\\ 1.24\\ 0.70\\ 0.69\\ 1.24\\ 0.70\\ 0.08\\ 0.04\\ 1.14\\ 0.30\\ 0.53\\ 0.88\\ 0.33\\ 0.40\\ 0.70\\ 0.93\\ 0.19\\ 0.40\\ 0.04\\ 0$

Table C.9: Data for distribution loads

# D

This appendix explains the linearisation technique applied in chapter 7. The source of non-linearity in the objective function (7.1) comes from the bilinear terms in  $\{\lambda_{c\in i,t}^{DA}P_{c,t}^{DA} + \lambda_{t}^{UP}R_{e,t}^{dis} + \lambda_{t}^{DN}R_{e,t}^{ch}\}$ . Based on the duality-theorem, those bilinear terms are linearised in this appendix. For this, the strong duality condition for the lower-level problem, belonging to the optimization shown in (7.15)-(7.21), should be formed which will be explained in this appendix.

First, one needs to derive the KKT conditions of the constraints belonging to the objective function (7.15). The constraints include (7.16)-(7.21) and the rest belonged to (4.2)-(4.18) which are shown in chapter 4. The Lagrangian multipliers are mentioned in parentheses next to each constraint. The Lagrangian is driven as below.

$$\begin{split} L &= \sum_{t \in T} \sum_{g \in G_D} (O_{g,t}^E P_{g,t} + O_{g,t}^{RUP} R_{g,t}^{UP} + O_{g,t}^{RDN} R_{g,t}^{DN}) + \sum_{t \in T} [\sum_c \widehat{\partial}_{c,t}^E P_{c,t}^{DA} \\ &+ \sum_{e \in c} (\widehat{\partial}_{e,t}^{Rech} R_{e,t}^{ch} + \widehat{\partial}_{e,t}^{Redis} R_{e,t}^{dis})] + \sum_t \sum_{i \in N_{D-T}} \sum_s \pi_s \lambda_{t,s}^{TD} P_{t,i}^{TD} \\ &+ \sum_t \sum_i \sum_{l = (i,j)} \theta_{i,l,t} (V_{i,t} - V_{j,t} - 2(r_l f_{l,t}^P - x_l f_{l,t}^q) - I_{l,t} (r_l^2 + x_l^2)) \\ &+ \sum_t \sum_i \lambda_{i,t}^{DA} (\sum_{l = (j,i)} (f_{l,t}^P - I_{l,t} r_l) + \sum_{g \in i} P_{g,t} + \sum_{c \in i} P_{c,t}^{DA} + P_{t,i}^{TD} - P_{i,t}^{load} \\ &- \sum_{l = (i,j)} f_{l,t}^P - G_i V_{i,t}) + \sum_t \sum_i \mu_{i,t} (\sum_{l = (j,i)} (f_{l,t}^q - I_{l,t} x_l) + \sum_{g \in i} Q_{g,t} \\ &+ Q_{t,i}^{TD} - Q_{i,t}^{load} - \sum_{l = (i,j)} f_{l,t}^q - b_i V_{i,t}) \\ &+ \sum_t \sum_g \lambda_t^{UP} (-R_{g,t}^{UP} - R_{e,t}^{dis}) + \sum_t \sum_g \lambda_t^{DN} (-R_{g,t}^{DN} - R_{e,t}^{ch}) \\ &+ \sum_t \sum_g \varphi_{g,t}^+ (P_{g,t} + R_{g,t}^{UP} - P_g^{gmax}) + \sum_t \sum_g \varphi_{g,t}^- (P_g^{gmin} - P_{g,t} + R_{g,t}^{DN}) \\ &+ \sum_t \sum_g \varphi_{g,t}^+ (P_{g,t} + R_{g,t}^{UP} - P_g^{gmax}) + \sum_t \sum_g \varphi_{g,t}^- (P_g^{gmin} - P_{g,t} + R_{g,t}^{DN}) \\ &+ ((f_{l,t}^q)^2 - S_l^2) + \sum_t \sum_g \varphi_{g,t} (P_{g,t}^2 + Q_{g,t}^2 - S_g^2) \\ &+ \sum_t \sum_g \delta_{g,t}^+ (Q_{g,t} - Q_g^{gmax}) + \sum_t \sum_g \delta_{g,t}^- (Q_g^{gmin} - Q_{g,t}) \\ &+ \sum_t \sum_g \beta_{g,t}^+ (P_{g,t} - P_g^{gmax}) + \sum_t \sum_g \beta_{g,t}^{-1} (Q_g^{gmin} - P_{g,t}) \\ &+ \sum_t \sum_g \beta_{g,t}^+ (P_{c,t}^2 - \hat{P}_{c,t}^D) + \sum_t \sum_c \gamma_{c,t}^{-1} (0 - R_{c,t}^{dis}) \\ &+ \sum_t \sum_e \psi_{e,t}^+ (R_{e,t}^{dis} - \hat{R}_{e,t}^{dis}) + \sum_t \sum_e \psi_{e,t}^{-1} (0 - R_{e,t}^{dis}) \\ &+ \sum_t \sum_e \psi_{e,t}^+ (R_{e,t}^{ch} - \hat{R}_{e,t}^C) + \sum_t \sum_e \psi_{e,t}^{-1} (0 - R_{e,t}^{dis}) \\ &+ \sum_t \sum_e \psi_{e,t}^{-1} (R_{e,t}^{ch} - \hat{R}_{e,t}^{ch}) + \sum_t \sum_e \psi_{e,t}^{-1} (0 - R_{e,t}^{ch}) \\ \end{aligned}$$

Besides the primary conditions which include constraints in (7.16)-(7.21) and the (4.2)-(4.18), the stationarity conditions of the Lagrangian function are:

$$(P_{g,t}): O_{g,t}^E + \lambda_{i,t}^{DA} + \varphi_{g,t}^+ - \varphi_{g,t}^- + \beta_{g,t}^+ - \beta_{g,t}^- + 2\phi_{g,t}P_{g,t} = 0$$
(D.1)

$$(P_{i,t}^{TD}): \sum_{s} (\pi \lambda_{t,s}^{TD}) + \lambda_{i,t}^{DA} + 2\phi_{i \in g,t} P_{i,t}^{TD} = 0$$
(D.2)

$$(R_{g,t}^{UP}): O_{g,t}^{RUP} - \lambda_t^{UP} + \varphi_{g,t}^+ = 0$$
 (D.3)

$$(R_{g,t}^{DN}): O_{g,t}^{RDN} - \lambda_t^{DN} + \varphi_{g,t}^- = 0$$
 (D.4)

$$(P_{c,t}^{DA}): \hat{o}_{c,t}^{E} - \lambda_{i,t}^{DA} + \gamma_{c,t}^{+} - \gamma_{c,t}^{-} = 0$$
(D.5)

$$(R_{e,t}^{dis}): \hat{o}_{e,t}^{Redis} - \lambda_t^{UP} + \psi_{e,t}^+ - \psi_{e,t}^- = 0$$
(D.6)

$$(R_{e,t}^{ch}):\hat{o}_{e,t}^{Rech} - \lambda_t^{DN} + \vartheta_{e,t}^+ - \vartheta_{e,t}^- = 0$$
(D.7)

$$(Q_{g,t}): \mu_{i,t} + \delta_{g,t}^+ - \delta_{g,t}^- + 2\phi_{g,t}Q_{g,t} = 0$$
(D.8)

$$(Q_{i,t}^{TD}): \mu_{i,t} + 2\phi_{i \in g,t} Q_{i,t}^{TD} + = 0$$
(D.9)

$$(f_{l,t}^p) :- 2r_l\theta_{i,l,t} + \lambda_{i,t}^{DA} - \lambda_{j,t}^{DA} + 2\xi f_{l,t}^p + 2\zeta f_{l,t}^p = 0$$
(D.10)  
$$(f_{l,t}^q) :- 2x_l\theta_{i,l,t} + \mu_{i,t} - \mu_{j,t} + 2\xi f_{l,t}^q + 2\zeta f_{l,t}^q = 0$$
(D.11)

$$f_{l,t}^q) :- 2x_l \theta_{i,l,t} + \mu_{i,t} - \mu_{j,t} + 2\xi f_{l,t}^q + 2\zeta f_{l,t}^q = 0$$
(D.11)

$$(V_{i,t}):\theta_{i,l,t} - \theta_{j,l,t} - G_i \lambda_{i,t}^{DA} - \mu_{i,t} b_i - \xi_{l,t} I_{l,t} - \vartheta_{i,t} I_{l,t} + \sigma_{i,t}^+ - \sigma_{i,t}^- = 0$$
(D.12)

$$(I_{l,t}): (r_l^2 + x_l^2)\theta_{i,l,t} - r_l\lambda_{i,t}^{DA} - \mu_{i,t} - \xi V_{i,t} - \vartheta_{i,t}V_{i,t} = 0$$
(D.13)

Finally, the complementarity conditions are as follows:

$$0 \le \lambda_t^{UP} \perp \left(-\sum_g R_{g,t}^{UP} - \sum_e R_{e,t}^{dis}\right) \tag{D.14}$$

$$0 \le \lambda_t^{DN} \perp (-\sum_g R_{g,t}^{DN} - \sum_e R_{e,t}^{ch})$$
(D.15)

$$0 \le \varphi_{g,t}^+ \perp (P_{g,t} + R_{g,t}^{UP} - P_g^{gmax})$$
(D.16)

$$0 \le \varphi_{g,t}^{-} \perp (P_{g,t} - R_{g,t}^{DN} - P_g^{gmin})$$
(D.17)

$$0 \le \xi_{l,t} \perp (f_{l,t}^{p^2} + f_{l,t}^{q^2} - V_{i,t}I_{l,t})$$
(D.18)

$$0 \le \zeta_{l,t} \perp (f_{l,t}^{p^{-2}} + f_{l,t}^{q^{-2}} - S_l^2)$$
(D.19)

$$0 \le \phi_{g,t} \perp (P_{g,t}^2 + Q_{g,t}^2 - S_g^2)$$
 (D.20)

$$0 \le \sigma_{i,t}^+ \perp (V_{i,t} - V_i^{max}) \tag{D.21}$$

$$0 \le \sigma_{i,t}^- \perp (V_i^{min} - V_{i,t}) \tag{D.22}$$

$$0 \le \delta_{g,t}^+ \perp (Q_{g,t} - Q_i^{gmax}) \tag{D.23}$$

$$0 \le \delta_{g,t}^- \perp (Q_i^{gmin} - Q_{g,t}) \tag{D.24}$$

$$0 \le \beta_{g,t}^+ \perp (P_{g,t} - P_g^{gmax}) \tag{D.25}$$

 $0 \le \beta_{g,t}^- \perp \left( P_g^{gmin} - P_{g,t} \right) \tag{D.26}$ 

$$0 \le \gamma_{c,t}^+ \perp (P_{c,t}^{DA} - \hat{P}_{c,t}^{DA})$$
(D.27)

$$0 \le \gamma_{c,t}^- \perp (0 - P_{c,t}^{DA})$$
 (D.28)

$$0 \le \psi_{e,t}^+ \perp (R_{e,t}^{ch} - \hat{R}_{e,t}^{ch})$$
 (D.29)

$$0 \le \psi_{e,t}^{-} \perp (0 - R_{e,t}^{ch}) \tag{D.30}$$

$$0 \le \vartheta_{e,t}^+ \perp (R_{e,t}^{dis} - \hat{R}_{e,t}^{dis})$$
 (D.31)

$$0 \le \vartheta_{e,t}^- \perp (0 - R_{e,t}^{dis}) \tag{D.32}$$

The following equation shows the condition for the strong duality:

$$- \lambda_{i,t}^{DA} P_{i,t}^{load} - \mu_{i,t} Q_{i,t}^{load} - \phi_{g,t}^{+} P_{g}^{max} + \phi_{g,t}^{-} P_{g}^{min} - \sigma_{i,t}^{+} V_{i}^{max} + \sigma_{i,t}^{-} V_{i}^{min} \\ - \delta_{g,t}^{+} Q_{g}^{max} + \delta_{g,t}^{-} Q_{g}^{min} - \beta_{g,t}^{+} P_{g}^{max} + \beta_{g,t}^{-} P_{g}^{min} - s_{g,t} X - \gamma_{c,t}^{+} \hat{P}_{c,t}^{DA} - \psi_{e,t}^{+} \hat{R}_{e,t}^{ch} \\ - \vartheta_{e,t}^{+} \hat{R}_{e,t}^{dis} = O_{g,t}^{E} P_{g,t} + O_{g,t}^{RUP} R_{g,t}^{UP} + O_{g,t}^{RDN} R_{g,t}^{DN} + \hat{o}_{c,t}^{E} P_{c,t}^{DA} \\ + \hat{o}_{e,t}^{Rech} R_{e,t}^{ch} + \hat{o}_{e,t}^{Redis} R_{e,t}^{dis} + \sum_{s} \pi_{s} \lambda_{t,s}^{TD} P_{t,i}^{TD}$$

$$(D.33)$$

in which the term  $s_{g,t}X$  belongs to the constraint in (4.14) of chapter 4 is nonlinear and its linearisation leads to a set of linear constraints shown below. The linearisation is based on [170].

$$\begin{aligned} (\phi_{g,t}^{1}) &: \quad Q_{g,t} \leq (\sqrt{3} - 2)P_{g,t} + S_{g,t} \\ (\phi_{g,t}^{2}) &: \quad Q_{g,t} \geq -(\sqrt{3} - 2)P_{g,t} - S_{g,t} \\ (\phi_{g,t}^{3}) &: \quad Q_{g,t} \leq -P_{g,t} + \frac{S_{g,t}}{2}(\sqrt{3} + 1) \\ (\phi_{g,t}^{4}) &: \quad Q_{g,t} \geq P_{g,t} - \frac{S_{g,t}}{2}(\sqrt{3} + 1) \\ (\phi_{g,t}^{5}, \phi_{g,t}^{6}) &: \quad -\frac{P_{g,t} - S_{g,t}}{\sqrt{3} - 2} \leq Q_{g,t} \leq \frac{P_{g,t} - S_{g,t}}{\sqrt{3} - 2} \end{aligned}$$
(D.34)

and the term  $s_{g,t}X$  is equal to:

$$s_{g,t}X = s_{g,t}((\sqrt{3}-2)\phi_{g,t}^1 + (\sqrt{3}-2)\phi_{g,t}^2 - \phi_{g,t}^3 + \phi_{g,t}^4 - \frac{1}{\sqrt{3}-2}\phi_{g,t}^5 + \frac{1}{\sqrt{3}-2}\phi_{g,t}^6)$$
(D.35)

Therefore, the equation in (D.33) can be rewritten as:

$$\begin{split} \Gamma &- \gamma_{c,t}^{+} \hat{P}_{c,t}^{DA} - \psi_{e,t}^{+} \hat{R}_{e,t}^{ch} - \vartheta_{e,t}^{+} \hat{R}_{e,t}^{dis} = O_{g,t}^{E} P_{g,t} + O_{g,t}^{RUP} R_{g,t}^{UP} + O_{g,t}^{RDN} R_{g,t}^{DN} + \hat{o}_{c,t}^{E} P_{c,t}^{DA} \\ &+ \hat{o}_{e,t}^{Rech} R_{e,t}^{ch} + \hat{o}_{e,t}^{Redis} R_{e,t}^{dis} + \sum_{s} \pi_{s} \lambda_{t,s}^{TD} P_{t,i}^{TD} \end{split}$$
(D.36)

in which  $\Gamma$  equals to:

$$\Gamma = -\lambda_{i,t}^{DA} P_{i,t}^{load} - \mu_{i,t} Q_{i,t}^{load} - \phi_{g,t}^{+} P_g^{max} + \phi_{g,t}^{-} P_g^{min} - \sigma_{i,t}^{+} V_i^{max} + \sigma_{i,t}^{-} V_i^{min} \\
- \delta_{g,t}^{+} Q_g^{max} + \delta_{g,t}^{-} Q_g^{min} - \beta_{g,t}^{+} P_g^{max} + \beta_{g,t}^{-} P_g^{min} - s_{g,t} X$$
(D.37)

Therefore:

$$\Gamma - O_{g,t}^{E} P_{g,t} - O_{g,t}^{RUP} R_{g,t}^{UP} - O_{g,t}^{RDN} R_{g,t}^{DN} - \sum_{s} \pi_{s} \lambda_{t,s}^{TD} P_{t,i}^{TD} 
= (\hat{o}_{c,t}^{E} + \gamma_{c,t}^{+}) \hat{P}_{c,t}^{DA} + (\hat{o}_{e,t}^{Rech} + {}^{+}_{e,t}) \hat{R}_{e,t}^{ch} + (\hat{o}_{e,t}^{Redis} + \vartheta_{e,t}^{+}) \hat{R}_{e,t}^{dis}$$
(D.38)

From the above-mentioned stationarity equations, the following equations can be obtained:

$$\lambda_{i,t}^{DA} = \hat{o}_{c,t}^E + \gamma_{c,t}^+ - \gamma_{c,t}^- \tag{D.39}$$

$$\lambda_{i,t}^{DA} P_{c,t}^{DA} = \hat{o}_{c,t}^{E} P_{c,t}^{DA} + \gamma_{c,t}^{+} P_{c,t}^{DA} - \gamma_{c,t}^{-} P_{c,t}^{DA}$$
(D.40)

in which, from the complementarity conditions we have  $\gamma_{c,t}^{-}P_{c,t}^{DA} = 0$ . A similar approach can be applied to find an equilibrium for  $\lambda_t^{UP} R_{e,t}^{dis}$  and  $\lambda_t^{DN} R_{e,t}^{ch}$  which are shown as follows:

$$\lambda_t^{UP} = \hat{o}_{e,t}^{Redis} + \psi_{e,t}^+ - \stackrel{-}{}_{e,t} \tag{D.41}$$

$$\lambda_t^{UP} R_{e,t}^{dis} = \widehat{o}_{e,t}^{Redis} R_{e,t}^{dis} + \stackrel{+}{}_{e,t} R_{e,t}^{dis} - \stackrel{-}{}_{e,t} R_{e,t}^{dis}$$
(D.42)

in which, from the complementarity conditions,  $-\frac{1}{e,t}R_{e,t}^{dis} = 0$ .

$$\lambda_t^{DN} = \hat{o}_{e,t}^{Rech} + \vartheta_{e,t}^+ - \vartheta_{e,t}^- \tag{D.43}$$

$$\lambda_t^{DN} R_{e,t}^{ch} = \hat{o}_{e,t}^{Rech} R_{e,t}^{ch} + \vartheta_{e,t}^+ R_{e,t}^{ch} - \vartheta_{e,t}^- R_{e,t}^{ch}$$
(D.44)

in which, from the complementarity conditions:  $\vartheta_{e,t}^{-}R_{e,t}^{ch} = 0$ .

Finally, based on equations in (D.38), (D.40), (D.42), and (D.44), the equivalent linearised form of the non-linear term in  $\{\lambda_{i,t}^{DA}P_{c,t}^{DA} + \lambda_t^{UP}R_{e,t}^{dis} + \lambda_t^{DN}R_{e,t}^{ch}\}$  is:

$$\lambda_{i,t}^{DA} P_{c,t}^{DA} + \lambda_{t}^{UP} R_{e,t}^{dis} + \lambda_{t}^{DN} R_{e,t}^{ch} = \Gamma - O_{g,t}^{E} P_{g,t} - O_{g,t}^{RUP} R_{g,t}^{UP} - O_{g,t}^{RDN} R_{g,t}^{DN} - \sum_{s} \pi_{s} \lambda_{t,s}^{TD} P_{t,i}^{TD}$$
(D.45)

Chapter D: Linearisation

## List of Publications

#### Journal papers

Farrokhseresht, M., Slootweg, J.G., and Gibescu, M. (2020), Day-ahead bidding strategies of a distribution market operator in a coupled local and central market, *submitted*.

Farrokhseresht, M., Slootweg, J.G., and Gibescu, M. (2020), Strategic bidding of distributed energy resources in the coupled local and central markets, *Sustainable energy, grid and network*, 24, 100390.

Farrokhseresht, M., Paterakis, N., Gibescu, M., and Slootweg, J.G. (2020), Enabling market participation of distributed energy resources through a coupled market design, *IET Renewable Power Generation*, 14, 4.

Killer, M., Farrokhseresht, M., and Paterakis, N. (2020), Implementation of largescale Li-ion battery energy storage systems within the EMEA region, *Applied Energy*, 260, 114166.

#### **Conference** papers

Farrokhseresht, M., Paterakis, N., Gibescu, M., and Slootweg, J.G. (2018), A survey on the participation of distributed energy resources in balancing markets, In *IEEE 15th International Conference on the European Energy Market*, ŁÓdŹ, Poland.

Farrokhseresht, M., Paterakis, N., Gibescu, M., and Slootweg, J.G. (2018), Participation of a combined wind and storage unit in the day-ahead and local balancing markets, In *IEEE 15th International Conference on the European Energy Market*, LÓdŹ, Poland. Tohidi, Y., Farrokhseresht, M. and Gibescu, M. (2018), A review on coordination schemes between local and central electricity markets, In *IEEE 15th International Conference on the European Energy Market*, LÓdŹ, Poland.

Farrokhseresht, M., Paterakis, N., Gibescu, M., and Slootweg, J.G. (2017), Minimization of distribution system losses by exploiting storage and anticipating market-driven behaviour of wind power producers, In *IEEE 14th International Conference on the European Energy Market*, Dresden, Germany.

Farrokhseresht, M., Paterakis, N., and Gibescu, M. (2016), Joint day-ahead clearing of energy, active and reactive power reserves for systems with significant RES-related uncertainty, In *IEEE 13th International Conference on the European Energy Market*, Porto, Portugal.

Farrokhseresht, M., Farrokhseresht, N., and Hesamzadeh, M. R. (2016), Optimizing the net transfer capacity of a cross-border interconnector by reactive power planning, In *IEEE PES General Meeting*, Boston, US.

Farrokhseresht, M., Rahimi, S., Hesamzadeh, M. R., and Lin, J. (2016), Optimal power flow for reactive power compensation increasing the cross-border transmission capacity, In *IEEE International Conference on the Electrical and Environmental Engineering*, Rome, Italy.

Farrokhseresht, M., Rahimi, S., and Hesamzadeh, M. R. (2014), Evaluation of reactive power compensation on voltage stability and NTC values in the Swedish transmission network, In *IEEE 11th International Conference on the European Energy Market*, Krakow, Poland.

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# Curriculum Vitae



Mana Farrokhseresht was born on 29<sup>th</sup> of January 1986, in Hamedan, Iran. She received her BSc degree in Industrial Engineering from Iran University of Science and Technology, Tehran, Iran (2010) and received her double degree MSc in Energy Engineering by EIT-KIC program from KU Leuven, Belgium and KTH Royal Institute of Technology, Sweden (2014). In her master thesis, she developed a multi-objective optimization model to reduce the overall cost of the system, improve the security elements of the power systems, and increasing the net transfer capacity between neighbouring countries. After completing her MSc, she started working as a researcher with Electricity Market Research Group at KTH. In 2015, she started working as a PhD candidate with Electrical Energy Systems Group at Eindhoven University of Technology, the Netherlands. Her research is associated with Smart Energy Systems in the Built Environment (SES-BE) program. The main focus of her research is on market frameworks that enable increased penetration of renewable energy sources in the distribution grid.



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