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I. Abstract

Enumerative sphere shaping (ESS) is a technique to index amplitude sequences that satisfy a maximum energy constraint, i.e., the ones inside a sphere [1], [2]. ESS assumes that these sequences can be ordered lexicographically, and defines the index of a sequence as the number of sequences which are lexicographically 'smaller'. To realize this ordering, an energy-bounded enumerative amplitude trellis is constructed.

In practice, for binary transmission, the number of actually transmitted sequences is an integer power of two. Therefore some of the sequences at the end of the lexicographical list, which are not necessarily located on the surface of the sphere, are never used. This phenomenon degrades the operational energy efficiency of ESS, especially for short block lengths. Furthermore, other sphere shaping algorithms which sort sequences based on their energy, e.g., shell mapping (SM) [3], do not suffer from this loss and always operate with a set of sequences having the minimum possible average energy.

In this work, we first evaluate the maximum possible difference in average energies of lexicographical and energy-based ordering approaches. The simulation results show that this difference is less than a few tenths of a dB for block lengths more than a hundred. Then we consider a heuristic to optimize the enumerative trellis such that the same is achieved for shorter block lengths. By removing connections from the enumerative trellis, we deliberately take out sequences from the output set of the shaper. These connections are selected such that the removed sequences are from the surface of the sphere, i.e., they have the largest possible energy. In this way, for a fixed rate, we obtain a set of sequences that approach the least possible average energy.

References

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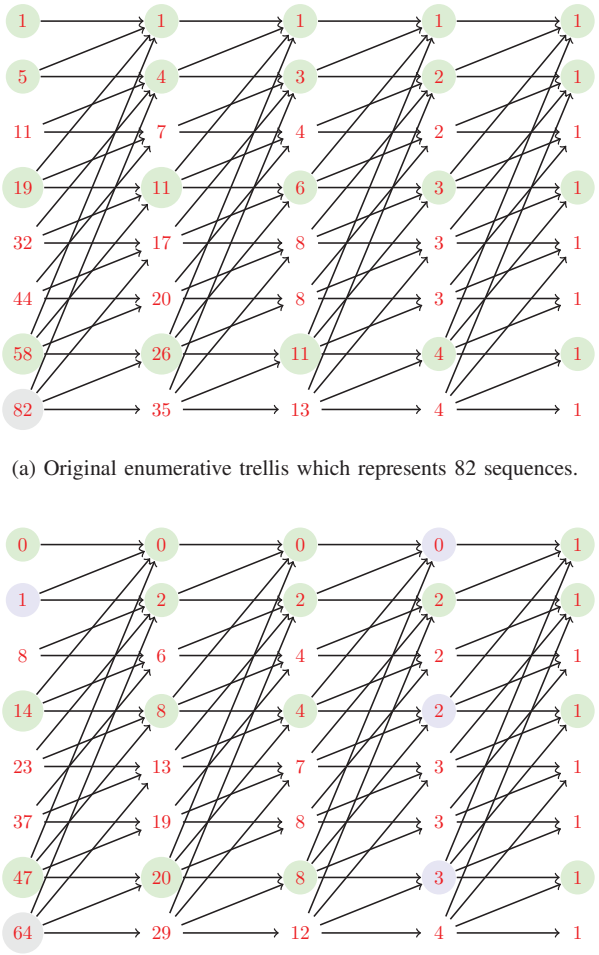

(a) Original enumerative trellis which represents 82 sequences.

(b) Modified enumerative trellis which represents 64 sequences.

Figure 1: Original (top) and modified (bottom) trellises. Green nodes have a connection to the surface. By decreasing these nodes by one, we can remove sequences from the surface. Blue nodes are the particular ones we decreased in this example. With this modification, number of sequences is decreased from 82 to 64. All removed sequences have the highest energy.