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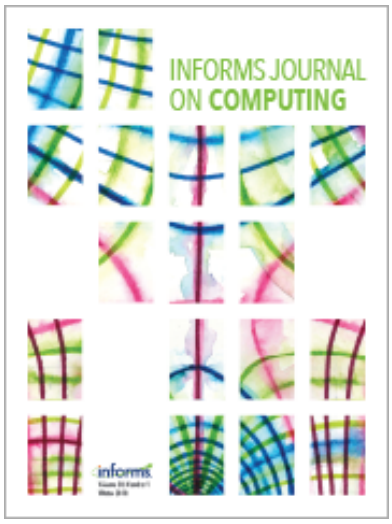


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# No-Wait Scheduling for Locks

**Ward Passchyn,[a,b] Dirk Briskorn,[c] Frits C. R. Spieksma[a,d]**

[a] Faculty of Economics and Business, ORSTAT, KU Leuven, 3000 Leuven, Belgium; [b] OM Partners, 2160 Wommelgem, Belgium; [c] Lehrstuhl für Produktion und Logistik, Bergische Universität Wuppertal, 42119 Wuppertal, Germany; [d] Department of Mathematics and Computer Science, Eindhoven University of Technology, 5600 Eindhoven, Netherlands

**Contact:** ward.passchyn@kuleuven.be, https://orcid.org/0000-0002-5422-5526 (WP); briskorn@uni-wuppertal.de, https://orcid.org/0000-0003-1829-8100 (DB); f.c.r.spieksma@tue.nl, https://orcid.org/0000-0002-2547-3782 (FCRS)

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**Abstract.** We introduce and investigate the problem of scheduling a single lock with parallel chambers. Special cases of this problem are related to interval scheduling. We focus on the existence of no-wait schedules and characterize their feasibility for a lock consisting of two chambers using new graph-theoretical concepts. We obtain a linear time algorithm for this special case. We also provide an efficient algorithm for the case where all chambers of the lock are identical. Furthermore, we describe a dynamic programming algorithm for the general case with arbitrary chambers. Finally, we indicate how our methods for the no-wait case can be applied to practical settings where waiting time is unavoidable.

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**Keywords:** lock scheduling • algorithms • interval scheduling

## 1. Introduction

Locks are a necessity on many inland waterways; they maintain the water level while allowing ships traversing these waterways to overcome the resulting water-level differences. We consider the following scheduling problem that deals with ships passing through a lock: consider a single lock that consists of $m$ parallel chambers. The chambers operate independent of each other and are each characterized by two numbers: their lockage time (denoted by $T_j, j = 1, \ldots, m$) and their capacity (denoted by $C_j, j = 1, \ldots, m$). A *lockage* or *lock movement* refers to a single operation of a chamber of the lock (i.e., allowing a ship to enter and subsequently change the water level in the lock from the downstream water level to the upstream water level or vice versa, which allows the ship to exit the lock and continue its journey). The term *lockage time* refers to the time needed to complete this operation. The *capacity* of a chamber refers to the number of ships that can be simultaneously served during a single lockage. Ships arrive at the lock at given times $t_1, t_2, \ldots, t_n$. We use $\mathcal{T}$ to denote the set of arrival times (i.e., $\mathcal{T} = \{t_i \mid 1 \le i \le n\}$). A ship can arrive from either the upstream side of the lock or the downstream side of the lock. We will refer to ships by their direction of travel (i.e., ships arriving on the upstream (downstream) side of the lock are referred to as downstream-traveling (upstream-traveling) ships). Our interest in this paper is primarily on the existence of so-called *no-wait* schedules. A no-wait schedule is a schedule where each ship, on its arrival, can enter a chamber of the lock immediately. Thus, in a no-wait schedule, each ship $i$ $(1 \le i \le n)$ leaves the lock at $t_i + T_1$, $t_i + T_2$, …, or $t_i + T_m$ depending on the particular chamber to which the ship is assigned. The question that we address is thus: given the arrival time and the direction of travel for each ship, does there exist an assignment of each ship to a chamber such that no ship has to wait; more compactly, does a no-wait schedule exist?

The existence of no-wait schedules is relevant in ports that connect inland waterways with seas and oceans. In ports such as Antwerp and Rotterdam locks are used to protect inland waterways from the influence of the tide. In these ports, large vessels coming from the sea can only reach the port by using the high tide. Hence, these vessels are subject to a "tidal window" (i.e., a limited time period when they are able to enter a port). The length of such a tidal window depends on specific circumstances (wind, waves, and ship dimensions); in extreme situations, no tidal window is available, or its length is reduced to a very short period. In such cases, a policy is enforced where such vessels do not incur any waiting time at the locks, allowing entry into the port area (see Savenije 1997 and Vantorre et al. 2014 for a detailed description).

Another setting where no-wait schedules are important is in the transport of hazardous cargo. To minimize the risk of potential incidents, the scheduling of locks on inland waterways that serve certain classes of ships carrying dangerous cargo is constrained [see, e.g., Cornell Law School (Legal Information Institute) 2006]. Depending on precise local regulations, it can be the case that lockages are forbidden to contain both regular

ships as well as ships carrying hazardous cargo, or it can be the case that ships carrying hazardous cargo should immediately be served by the lock at their time of arrival.

In this work, we provide a complete classification of the complexity of this no-wait scheduling problem (see Section 1.3). In some cases, algorithms that are linear in the number of ships (i.e., $O(n)$) are obtained. In addition to deciding the existence of a no-wait solution, we describe algorithms that actually find such solutions, provided that they exist. We also describe in Section 6 how our algorithms can be expanded to find heuristic solutions for the problem of minimizing the waiting time, in particular for those instances that do not admit a no-wait solution.

### 1.1. Literature

Scheduling locks is a problem that is receiving an increasing amount of attention. In particular, when confronted with a series of locks (e.g., along a canal), the problem of operating the locks jointly to minimize total waiting time or emissions is dealt with in Prandtstetter et al. (2015) and Passchyn et al. (2016a) (see also Disser et al. 2015 for a related more abstract setting).

Furthermore, the problem of scheduling a lock consisting of one chamber is treated by Hermans (2014), who presents a $O(n^4 \log n)$ dynamic programming (DP) algorithm that asserts feasibility with respect to given ship deadlines when the single chamber has unit capacity. Passchyn et al. (2016c) deal with minimizing total waiting time for a single lock chamber. They give an $O(n^4)$ algorithm for the bidirectional single-chamber setting and discuss results for such practical features as ship handling times, drought, etc. Smith et al. (2011) view the single-lock, single-chamber setting as a two-stage queue and describe a mixed integer programming model as well as a heuristic solution procedure.

The research mentioned above concentrates on single-chamber locks. In practice, however, many locks consist of more than one chamber. On the Panama Canal, for example, each lock consists of two identical parallel chambers. Another example is the Wijnegem lock, situated in Belgium, which connects the Albert Canal to the Port of Antwerp. Like all other locks on the Albert Canal, this lock consists of three nonidentical chambers. Furthermore, the construction of a fourth lock chamber in Wijnegem is currently under consideration (see also Waterwegen en Zeekanaal NV and nv De Scheepvaart 2014).

Ting and Schonfeld (2001) mention a heuristic for a lock consisting of two chambers. Other works that deal with scheduling a lock with multiple chambers are Verstichel et al. (2011, 2014) and Verstichel (2013). In Verstichel et al. (2014), they formulate a mixed integer program (MIP) that models, among other things, the physical placement of the ships within the chamber. The objective of this MIP contains terms that correspond to the number of lockages as well as the tardiness of ships. Solutions are found by solving the MIP.

### 1.2. Interval Scheduling

An important special case of our problem arises when all arrival times are distinct. Observe that in this case the capacities of the chambers lose their meaning because then, in a no-wait schedule, no two ships can be in the same lockage. This resulting special case can be phrased as an interval scheduling problem as follows. Let a chamber be a *machine*, and let a ship be a *job*. Furthermore, let the direction of travel for each ship correspond to the *job type* for each job. Multiple intervals are associated to each job: one for each machine in the instance. The starting time of each of the intervals corresponding to a particular job $i$ is equal to $t_i$; the ending times of these intervals are given by $t_i + T_j$, $j \in \{1, \ldots, m\}$. Notice that, when considering a particular interval, it is associated with a job and a machine. A feasible solution consists of a selection of intervals such that (i) one interval corresponding to each job is selected and (ii) the selected intervals that correspond to a machine are disjoint; even more, when two consecutive intervals of a machine correspond to jobs with the same job type, there must be a difference of $T_j$ between the ending point of the earlier interval and the starting point of the later interval. The requirement involving this difference is needed, because a chamber transporting a ship needs $T_j$ time units to return before transporting another ship that travels in the same direction. Notice that, in the special case where all ships travel in the same direction, this difference requirement vanishes, because it can be modeled by assuming that the length of all intervals equals $2T_j$. We refer to this setting, which corresponds to a well-known interval scheduling problem, as the *unidirectional* case.

Interval scheduling is a well-studied subject (see Kolen et al. 2007 for an overview). Krumke et al. (2011) deal with interval scheduling on related machines: given are $m$ machines, each with a certain speed $s_j$ ($1 \leq j \leq m$), and $n$ intervals specified by a starting point $r_i$ and a processing time $p_i$ ($1 \leq i \leq n$). They show that even deciding the existence of a schedule is NP complete. This setting is related to the unidirectional variant of our problem with distinct arrival times. Indeed, a chamber corresponds to a machine (so that there are $m$ machines), and the speed of machine $j$ (the $s_j$) is set equal to $1/T_j$. Furthermore, there is an interval for each ship (so that there are $n$ intervals); we set the starting points of the $n$ intervals (the $r_i$) equal to the arrival times of the corresponding ships. The processing time of each interval (the $p_i$) equals one. Notice that the resulting interval scheduling problem has the property that the lengths of the intervals processed by the same machine are equal.

The unidirectional case with distinct arrival times is also related to a problem studied in Böhmová et al. (2013). In their setting, a job corresponds to a set of intervals: one for each machine. To schedule a job, exactly one of its intervals must be selected, and a selection of intervals is called feasible if no two intervals corresponding to the same machine overlap. In particular, they consider the problem with so-called cores, where all intervals corresponding to the same job have a point in time in common. They prove that deciding the existence of a feasible selection that schedules all jobs is NP complete, thereby solving an open problem from Sung and Vlach (2005). However, they allow arbitrary lengths of the intervals corresponding to a job, which differs from this special case of our problem (unidirectional with distinct arrival times) where all interval lengths are identical. They also mention a DP given in Sung and Vlach (2005), which, translated into our terms, solves the unidirectional case with distinct arrival times in $O(mn^{m+1})$ time. In addition, they mention without specifying additional details that the special case with two machines is polynomially solvable by a reduction to 2-SAT.

### 1.3. Summary of Results

We formulate our results in terms of locks and ships; for example, the phrase "assigning ship $i$ to chamber $j$" can, in machine scheduling terms, be read as "executing job $i$ on machine $j$."

As mentioned before, the input of our problem consists of lockage times $T_j$ and capacities $C_j$ for $j \in \{1,\dots,m\}$ and arrival times $t_i$ and direction $d_i \in \{\text{upstream}, \text{downstream}\}$ for $i \in \{1,\dots,n\}$. We consider the following question: does a no-wait solution exist? We refer to this problem as "no-wait lock scheduling" (NLS). In addition to this problem, we consider different special cases in what follows, deciding on whether a no-wait solution exists subject to additional restrictions imposed on the input. For convenience, we introduce a notation to refer to these special cases. We refer to the problem settings as NLS{-uni, ∅}{-2, $-m$, ∅}{-id, ∅}{-distinct, ∅}, where uni refers to the unidirectional case (omitting this implies bidirectional traffic), 2 refers to the setting with two lock chambers, $m$ refers to the setting where the number of chambers $m$ is fixed (omitting these implies that the number of chambers is part of the input), id refers to the setting with identical chambers (omitting this implies that values for lockage time and capacity are arbitrary), and distinct refers to the setting where all arrival times are distinct (omitting this implies that multiple ships may arrive simultaneously). We point out that specifying additional parameters in the problem name increasingly yields a more specific case of the problem. NLS, which describes the setting with an arbitrary number of nonidentical chambers and bidirectional traffic, thus describes the most general problem covered by this notation. Table 1 lists the notation as well as the results discussed in this paper for the different special cases of the problem that we consider. Based on the chamber characteristics, we consider the following settings.

1. The setting where $m = 2$ with two arbitrary chambers. For the unidirectional setting, called NLS-uni-2, we give necessary and sufficient conditions for deciding the existence of a no-wait schedule (Section 2). This result is obtained by introducing the concept of a "bad path," and this characterization allows us to establish the existence of such a schedule in $O(n)$ time, provided that the arrival times are sorted (see Section 1.4). Because reading the input takes $O(n)$ time, this is an optimal algorithm. Furthermore, for the bidirectional case NLS-2, we give a reduction to 2-SAT that leads to an $O(n^2)$ algorithm (Section 3).
2. The setting where $C_j = C$ and $T_j = T$ for all $j \in \{1,\dots,m\}$ (i.e., the setting with $m$ identical chambers). The resulting problems are called NLS-uni-id and NLS-id, and we show that we can solve these problems in $O(n)$ time for sorted arrival times (Section 4).
3. The setting where the number of chambers $m$ is fixed. We give a DP for our problem that runs in polynomial time (Section 5). For the problem of finding a feasible no-wait schedule described here, this DP strengthens the result in Sung and Vlach (2005) that can only be applied to the unidirectional case with distinct arrival times.
4. The setting with an arbitrary number of chambers. We state that the unidirectional case of this variant

**Table 1.** Summary of Results

| | Two arbitrary chambers | $m$ Identical chambers | $m$ Arbitrary chambers | $m$ Arbitrary chambers ($m$ part of the input) |
|---|---|---|---|---|
| Unidirectional | NLS-uni-2 | NLS-uni-id | NLS-uni-$m$ | NLS-uni |
| | $O(n)$[a] | $O(n)$[a] | $O(mn^m)$ | strongly NP complete |
| | (Section 2) | (implied) | (implied) | (Section 5) |
| Bidirectional | NLS-2 | NLS-id | NLS-$m$ | NLS |
| | $O(n^2)$ | $O(n)$[a] | $O(mn^m)$ | strongly NP complete |
| | (Section 3) | (Section 4.3) | (Section 5) | (implied) |

[a]These results apply to input with arrival times given in sorted order (see Section 1.4).

is NP complete (Section 5). This result strengthens both the result given in Krumke et al. (2011) and a result in Böhmová et al. (2013).

Finally, not all instances will admit a feasible solution where no ship incurs any waiting time. Thus, an important issue concerns the question of whether our algorithms can be used in a situation where no-wait solutions do not exist. In Section 6, we sketch two directions in which algorithms for no-wait solutions can be modified so as to deal with instances that do not admit a no-wait solution.

### 1.4. A Note on Sorted Arrival Times

Notice that some of the results stated in Table 1 apply exclusively to sorted input (i.e., these results require the assumption that arrival times are given in nondecreasing order). Because of the well-known $\Omega(n \log n)$ lower bound on comparison-based sorting algorithms, it may not be possible to improve beyond a complexity of $O(n \log n)$ in the general case of unsorted arrival times. However, assuming that the arrival times are known in sorted order may be justified. For instance, when iteratively applying the presented methods, a large part of the input may have remained unchanged since earlier iterations so that sorting is no longer required for a large subset of the given arrival times. Furthermore, the input may be unsorted, but it may satisfy additional assumptions that allow the use of a non–comparison-based sorting algorithm, such as radix sort, which runs in linear time. Therefore, we choose to report the complexity assuming that the arrival times are sorted.

## 2. Two Arbitrary Chambers, Unidirectional Case

In this section, we deal with the case of two chambers: more specifically, with the unidirectional setting. To serve a ship with chamber $j \in \{1, 2\}$, a lockage time $T_j$ is incurred. We assume that $T_1 \leq T_2$. We refer to the two chambers as the short and long chambers, respectively, and their lockages as the short and long lockages.

In Section 2.1, we first look at problem NLS-uni-2-distinct (i.e., the special case where (i) all ships travel in the same direction and (ii) all arrival times are distinct). Notice that the numbers $C_1, C_2$ are then irrelevant, because a chamber contains at most one ship. In Sections 2.2 and 2.3, we extend our approach to a more general setting where some ships are preassigned to a chamber, and we describe how this result can be used to model the setting NLS-uni-2, where the numbers $C_1$ and $C_2$ become relevant.

### 2.1. Unidirectional Setting with Distinct Arrival Times

As argued in Section 1.4, we assume throughout this section that the arrival times are given in sorted order. We describe an $O(n)$ algorithm under this assumption. We organize this section as follows. In Section 2.1.1, we describe the construction of a graph corresponding to an instance of NLS-uni-2-distinct and discuss some basic observations. In Section 2.1.2, we prove a theorem characterizing feasibility, and Section 2.1.3 describes an $O(n)$ algorithm.

**2.1.1. Graph and Concepts.** An instance $\mathcal{I}$ is given by specifying distinct ordered arrival times $t_1 < t_2 < \ldots < t_n$ and lockage times $T_1 < T_2$. We say that an instance is feasible if there exists a no-wait solution; otherwise, it is not feasible. Given an instance $\mathcal{I}$, we create a graph $G(\mathcal{I})$; we will, in the sequel, simply write $G$. Notice that we allow multiple edges between a pair of nodes in $G$; more precisely, the edge set of $G$ consists of a set $E^S$ and a set $E^L$. The graph is constructed as follows. There is a node for each arrival time; we say that two nodes $i < j$ are connected via an *s-edge* $(i, j) \in E^S$ if and only if $t_j - t_i < 2T_1$; two nodes $i < j$ are connected via an *l-edge* $(i, j) \in E^L$ if and only if $t_j - t_i < 2T_2$. Observe that the resulting graphs $(V, E^S)$ and $(V, E^L)$ are undirected, and they are, in fact, so-called unit interval graphs.

Because the arrival times are assumed to be strictly ordered, this same ordering applies to the nodes of $V$. Node $i$ then refers to the arrival of a ship at time $t_i$, with $1 \leq i \leq n$. We may thus refer to "first" and "last" or "earlier" and "later" nodes without ambiguity. We call a pair of nodes $(i, j)$ *consecutive* when $j = i + 1$. In all figures, we represent an s-edge by a straight line segment (—), whereas an l-edge is represented by a segment in the form of an arc (⌒).

A no-wait solution to the given instance can exist only if each ship can be assigned to one of the two lock chambers so that a lockage starts at the ship's time of arrival. We can immediately observe the following: for a solution to be no-wait, two arrivals connected by an s-edge cannot both be served by the short chamber; two arrivals connected by an l-edge cannot both be served by the long chamber. Furthermore, we can observe a number of interesting properties in graph $G$. In addition to highlighting some structural characteristics of an instance and its graph, we will refer back to these observations in the Proof of Theorem 1, where a more general structure is described that characterizes infeasible instances.

**Observation 2.1.** If there exists a node $i \in V \setminus \{n\}$ such that $(i, i + 1) \notin E^L$ (i.e., if there exists a pair of consecutive nodes that are not connected by an l-edge), the instance splits into two independent subproblems. This is easily seen, because both chambers are then available at time $t_{i+1}$ for any no-wait solution, regardless of the assignment of nodes $1, \ldots, i$. A no-wait solution thus exists if and only if there is a no-wait solution for each of the two subproblems.

**Observation 2.2.** If there exists a node $i \in V$ such that $(i, i+2) \in E^S$ (i.e., if there exists a triangle of s-edges in $G$), then the instance is not feasible. This readily follows from the fact that there does not exist a proper two-coloring in a triangle graph.

**Observation 2.3.** If an l-edge contains two s-edges that are node disjoint (i.e., if there exist nodes $i, j \in V$ with $j > i+1$ so that $(i, i+1) \in E^S$, $(j, j+1) \in E^S$, and $(i, j+1) \in E^L$), then the instance is not feasible. This is easily verified by enumerating all possible assignments for nodes $i$, $i+1$, $j$, and $j+1$. An example is shown in Figure 1.

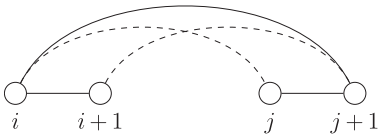
**Observation 2.4.** If there exists a node $i \in V$ such that there exist s-edges $(i, i+1), (i+1, i+2), (i+2, i+3) \in E^S$ and l-edges $(i, i+2), (i+1, i+3) \in E^L$, the instance is not feasible. Figure 2 shows the graph for an instance consisting of four such nodes. It is easily verified (see also the next observation) that both nodes $i+1$ and $i+2$ must be served by the short chamber in all feasible solutions, which is impossible because of the presence of s-edge $(i+1, i+2)$. Hence, an instance containing this structure is not feasible.

For convenience, we restrict ourselves in the remainder of Section 2 to instances $\mathcal{I}$ with corresponding graphs $G(\mathcal{I})$ that do not contain any of the structures described in Observations 2.1–2.4. This can be done without loss of generality, because we can recognize these structures efficiently (see Section 2.1.3). We also point out that the existence of an l-edge $(i, j) \in E^L$ implies that each edge of the form $(p, q)$ with $i \le p < q \le j$ is also in $E^L$. For clarity, our figures contain only the maximal l-edges.

Furthermore, an additional observation allows us to recognize certain nodes that must be assigned to the short chamber.

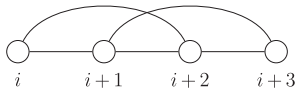
**Observation 2.5.** If there exist nodes $i, j \in V$ with $j > i+1$ such that $(i, i+1) \in E^S$ and $(i, j) \in E^L$, a feasible solution can exist only when node $j$ is assigned to the short chamber. This follows readily from the fact that either $i$ or $i+1$ must be assigned to the long chamber, and an l-edge to $j$ starts at both $i$ and $i+1$. From symmetry, it immediately follows that the same holds for a node $i$, where $(i, j) \in E^L$ and $(j, j-1) \in E^S$. The associated graphs are shown in Figure 3.

**Figure 1.** Structure Described in Observation 2.3




*Note.* The existence of $(i, j+1) \in E^L$ implies that $(i, j) \in E^L$ and $(i+1, j+1) \in E^L$, because $t_i < t_{i+1} < t_j < t_{j+1}$.

**Figure 2.** Structure Described in Observation 2.4




We proceed by describing paths and nodes in the graph that have a specific structure. We show later that checking for the presence of such paths suffices to decide the feasibility of a given instance.

**Definition 1.** Given an instance $\mathcal{I}$ and graph $G(\mathcal{I})$, a *bad path* is any sequence of distinct nodes $(i_1, i_2, \ldots, i_k)$ with $k \ge 4$ that satisfies the following.

1. The nodes in the sequence appear in the order defined on $V$, with the exceptions of $i_1$ and $i_k$, which satisfy $i_2 < i_1 < i_3$ and $i_{k-2} < i_k < i_{k-1}$. More formally, $i_x < i_{x+1}$ for all $x \in \{2, \ldots, k-2\}$, $i_2 < i_1 < i_3$, and $i_{k-2} < i_k < i_{k-1}$.
2. The pairs of consecutive nodes in the sequence are alternatingly connected by an s-edge and an l-edge, with the first and last edges in the sequence being both s-edges. More formally, $(i_x, i_{x+1}) \in E^S$ for all odd $x \in \{1, \ldots, k-1\}$, $(i_x, i_{x+1}) \in E^L$ for all even $x \in \{1, \ldots, k-1\}$, and $k$ is even.

Note that a bad path necessarily contains an even number of nodes. An example is shown in Figure 4. Observe that a bad path with $k = 4$ is present in Figure 1.

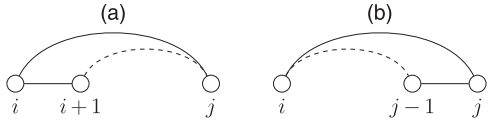
We also describe specific nodes that are closely related to the definition of a bad path. Intuitively, we define a *potentially bad node* as the latest node in a path $(i_1, i_2, \ldots, i_k)$ that contains at least five nodes and that could be extended to a bad path if there would exist a node $j$ with $i_{k-1} < j < i_k$ and an s-edge $(j, i_k)$. Note that the existence of such a node $j$ is not required for $i_k$ to be a potentially bad node. More formally, we define a potentially bad node as follows.

**Definition 2.** A node $i_k$ is a potentially bad node if there exists a path $(i_1, i_2, \ldots,$ and $i_k)$ that satisfies the following.

- $i_x < i_{x+1}$ for all $x \in \{2, \ldots, k-1\}$, and $i_2 < i_1$.
- $(i_x, i_{x+1}) \in E^S$ for all odd $x \in \{1, \ldots, k-1\}$ and $(i_x, i_{x+1}) \in E^L$ for all even $x \in \{1, \ldots, k-1\}$.
- $k \ge 5$, and $k$ is odd.

Observe that, in a bad path, all nodes $i_j$ in the path with $j \ge 5$ and $j$ odd are potentially bad nodes (e.g., in Figure 4, nodes $i_5$ and $i_7$ are potentially bad nodes, whereas $i_3$ is not).

**Figure 3.** Structures Described in Observation 2.5




*Note.* The existence of the dashed edges is implied.

**Figure 4.** Example of a Bad Path


*Note.* The structure described in Definition 1 remains satisfied if the dashed edges are replaced with a longer sequence (with odd numbers of nodes) consisting alternatingly of s-edges and l-edges.

**2.1.2. Characterizing Feasible Instances.** We present the following theorem to characterize when an instance is feasible.

**Theorem 1.** *An instance $\mathcal{I}$ of NLS-uni-2 distinct is feasible if and only if its corresponding graph $G(\mathcal{I})$ does not contain a bad path.*

**Proof.** $\Rightarrow$ We argue that, if $G(\mathcal{I})$ contains a bad path, the instance is not feasible. Consider a bad path $(i_1, \ldots, i_k)$ in $G$. It follows from Observation 2.5 and $i_2 < i_1 < i_3$ that node $i_3$ must be assigned to the short chamber. We now trace the path from $i_3$ to $i_{k-1}$. Note that, because l-edges and s-edges alternate, any solution must assign nodes $i_3, \ldots, i_{k-1}$ to the short and long chambers in alternating order. This implies that node $i_{k-2}$ must be assigned to the long chamber, whereas node $i_{k-1}$ must be assigned to the short chamber. However, the bad path implies that $(i_{k-1}, i_k) \in E^S$ and $(i_{k-2}, i_k) \in E^L$ so that no chamber is available for $i_k$; hence, no feasible solution exists.

$\Leftarrow$ We now argue by contradiction that a feasible solution exists whenever the graph does not contain a bad path. For this, let us assume that there exists an instance that is not feasible while its corresponding graph does not contain a bad path. Assuming that such instances exist, consider one with a minimum number of arrivals. Let $G^* = (V^*, E^*)$ be the graph corresponding to this instance, with $E^* = E^{S*} \cup E^{L*}$, where $E^{S*}$ and $E^{L*}$ denote the sets of s-edges and l-edges, respectively. The proof is organized as follows. We first show that $G^*$ must contain at least one potentially bad node. We then argue that there cannot be a latest potentially bad node in $G^*$: a contradiction.

We claim that graph $G^*$ must satisfy the following properties.

**Property 2.1.** *For each $i \in V^*$, there exists a feasible solution in $G^* \setminus \{i\}$ with distinct nodes $j_s, j_l \in V^* \setminus \{i\}$ such that $(i, j_s) \in E^{S*}$ and $(i, j_l) \in E^{L*}$, while $j_s$ ($j_l$) is assigned to the short (long) chamber in that feasible solution.*

**Proof.** Let $i$ be an arbitrary node in $G^*$. Because $G^*$ is a counterexample with a minimum number of nodes and because no bad paths are introduced by removing a node and its incident edges, it follows immediately that a feasible assignment of chambers is possible in $G^* \setminus \{i\}$. Consider an arbitrary feasible assignment in $G^* \setminus \{i\}$, and let $S$ and $L$ be the set of nodes that are assigned to the short and long chambers in this solution, respectively; clearly, $S \cap L = \emptyset$. If, in $G^*$, none of the nodes in $S$ are connected to $i$ by an s-edge, it is easily seen that node $i$ can be assigned to the short chamber, thus extending the solution in $G^* \setminus \{i\}$ to a feasible solution in $G^*$. Because no feasible solution is possible in $G^*$, it follows that a node $j_s \in S$ must exist such that $(i, j_s) \in E^{S*}$. Repeating this reasoning for the long chamber immediately implies that there exists a node $j_l \in L$ such that $(i, j_l) \in E^{L*}$. □

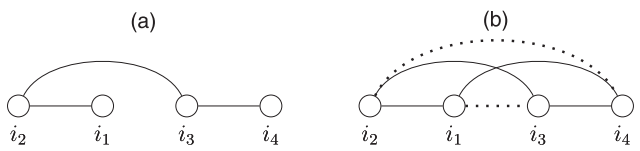
**Property 2.2.** *$G^*$ contains at least one potentially bad node.*

**Proof.** Consider the earliest node, say $i_2$, in $G^*$. Applying Property 2.1 to this node, it follows that there must exist two additional nodes $i_1, i_3$ such that $(i_2, i_1) \in E^{S*}$ and $(i_2, i_3) \in E^{L*}$. We select $i_1$, $i_3$ to be minimal (i.e., $i_1$ is the successor of $i_2$, and $i_3$ is the successor of $i_1$). We obtain the structure shown in Figure 3, implying that node $i_3$ cannot be assigned to the long chamber (Observation 2.5). Applying Property 2.1 to node $i_3$ implies that there is an s-edge incident to node $i_3$. We can distinguish two possible cases.

1. The s-edge is incident to a node $i_4 > i_3$. Observe that $i_4$ must be the successor of $i_3$. We obtain the structure shown in Figure 5(a). If $G^*$ consists of four nodes, applying Property 2.1 to $i_1$ implies that $(i_1, i_4) \in E^{L*}$. Recall that $i_3$ cannot be assigned to the long chamber. Furthermore, observe that $(i_1, i_3) \notin E^{S*}$ (by our assumption that the structure from Observation 2.4 is not present in an instance) and that $(i_2, i_4) \notin E^{L*}$ (by our assumption that the structure from Observation 2.3 is not present). Then, however, $G^*$ corresponds to an instance that is feasible, which is not the case. This is illustrated in Figure 5(b). Thus, $G^*$ consists of at least five nodes. Observation 2.1 implies that $(i_4, i_5) \in E^{L*}$, and thus, $i_5$ is a potentially bad node.

2. The s-edge is incident to node $i_1$. Observe that, if $G^*$ consists of only three nodes, a feasible solution is easily found. Thus, there exists a fourth node $i_4$ in $G^*$. Because $i_2$, $i_1$, and $i_3$ are the earliest nodes in the instance, $i_4$ is the successor of $i_3$. Applying Property 2.1 to node $i_1$, it follows that $(i_1, i_4) \in E^{L*}$. Observe that nodes $i_2$ and $i_3$ cannot be selected as node $j_l$ in the property,

**Figure 5.** Structures Described in Case 1 in the Proof




*Note.* In (b), the dotted edges are not present in $G^*$, and a feasible solution assigns $i_1$ and $i_3$ to the short chamber and $i_2$ and $i_4$ to the long chamber.

because they must not be assigned to the long chamber (Observation 2.5). $G^*$ then contains the structure shown in Figure 6(a). We then apply Property 2.1 to node $i_4$. Because we may assume that $(i_3, i_4) \notin E^{S*}$ (otherwise, the case described above applies), it follows that there exists a node $i_5 > i_4$ with $(i_4, i_5) \in E^{S*}$. The resulting structure is shown in Figure 6(b). Finally, we apply Property 2.1 again, now to $i_5$. It follows that there is an l-edge incident to $i_5$. As before, note that nodes $i_3$ and $i_4$ cannot be chosen as node $j_l$ in the property, because they must not be assigned to the long chamber (Observation 2.5). Because the existence of the l-edge $(i_1, i_5)$, dotted in Figure 6(b), implies a bad path, this l-edge cannot be present, and there thus exists a node $i_6 > i_5$ with $(i_5, i_6) \in E^{L*}$. Node $i_6$ is then a potentially bad node, with path $(i_3, i_1, i_4, i_5, i_6)$ satisfying the definition above.

Thus, $G^*$ must contain at least one potentially bad node. □

Property 2.2 implies that $G^*$ contains a latest potentially bad node. We now show that this leads to a contradiction. Figure 7 illustrates the reasoning below. Let $i_k$ be the latest potentially bad node in $G^*$. Applying Property 2.1 to $i_k$ implies that there exists an s-edge $(i_k, i_{k+1})$. Because $i_k$ is a potentially bad node, $i_{k+1} < i_k$ would imply the existence of a bad path. Node $i_{k+1}$ is thus the successor of $i_k$. Applying Property 2.1 again, now to $i_{k+1}$, implies the existence of an additional l-edge. Observe that the l-edge $(i_{k-1}, i_{k+1})$, illustrated by the dotted l-edge in Figure 7, cannot be present, because it implies the existence of a bad path. Thus, there exists a node $i_{k+2} > i_{k+1}$ with $(i_{k+1}, i_{k+2}) \in E^{L*}$. Note that $i_{k+2}$ is a potentially bad node, and $i_{k+2} > i_k$. This contradicts our choice of $i_k$ as the latest potentially bad node.

Thus, the existence of $G^*$ leads to a contradiction, proving the theorem. □

**2.1.3. An $O(n)$ Algorithm for Deciding Feasibility and Constructing a Solution.** Notice that Theorem 1 characterizes feasibility of an instance. We now present an $O(n)$ algorithm that actually recognizes whether $G$ contains a bad path. We start with the arrival times $t_i$ for $i \in \{1, \ldots, n\}$ in sorted order and avoid explicitly constructing $G$, which may contain up to $O(n^2)$ l-edges. Because $(V, E^L)$ is a unit interval graph, we can avoid checking each of the l-edges explicitly as follows. Instead of constructing all edges in graph $G$, we will check for the existence of edges, using their definition, only when needed. For example, when we write "if $(v_i, v_j) \in E^L$" for nodes $v_i, v_j \in V$ in what follows, this equals the expression "if $t_j - t_i < 2T_2$," corresponding to the definition of an l-edge. For any given pair of nodes, this is easily checked in constant time. In addition to recognizing feasibility, the algorithm assigns each node to a chamber such that the corresponding solution is a no-wait solution, provided that no bad path exists.

Recall that, in Figure 8, each of the nodes labeled $s$ must be assigned to the short chamber in any feasible solution as argued in Observation 2.5.

The outline of the procedure is as follows (see Algorithm 1). We first identify all nodes that must be assigned to the short chamber because of the structures shown in Figure 8 (lines 4–17 of Algorithm 1). We then use implications from these assignments to assign other nodes to the chambers (lines 18–33 of Algorithm 1). In this way, all "forced" assignments are handled. Finally, we apply a simple greedy procedure to assign the remaining nodes to chambers (lines 35–39 of Algorithm 1). In the remainder of this section, we will show that, if no bad paths are present, the greedy procedure always yields a feasible assignment of lock chambers (i.e., correctness of the algorithm) and that each of these steps can be performed in linear time (i.e., complexity of the algorithm).

We will call any node that must be assigned to a specific chamber in all feasible solutions a *fixed node*. We distinguish *s-fixed* and *l-fixed* nodes for nodes that must be assigned to the short and long chambers, respectively. As argued in Observation 2.5, the nodes labeled $s$ in Figure 8 are s-fixed nodes. We start by identifying all occurrences of these structures in $G$. Observe that these structures correspond to the "beginning" (i.e., the first three nodes) and the "end" (i.e., the last three nodes) of a bad path. Finding these structures is easily done in linear time by considering each node once and checking for the presence of the edges shown in the figures. We obtain an initial set of s-fixed nodes. Initially, no nodes are l-fixed.

**Figure 6.** Structures Described in Case 2 in the Proof

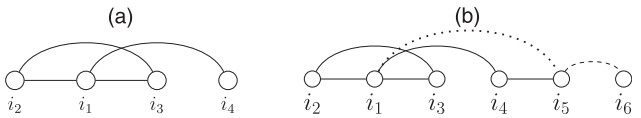
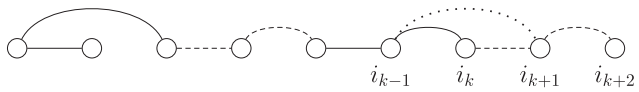
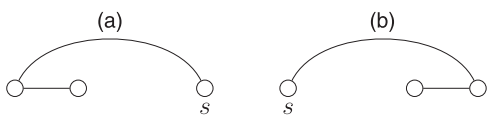



**Figure 7.** Structure Describing the Latest Potentially Bad Node in $G$




*Note.* Observe that the dotted l-edge cannot be present.

**Figure 8.** The Nodes Labeled $s$ Must Be Assigned to the Short Chamber

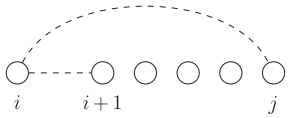


Observe that any node connected to an s-fixed node by an s-edge is necessarily l-fixed and that any node connected to an l-fixed node by an l-edge is necessarily s-fixed. The following step is to identify all remaining nodes that can be fixed using these observations. We first consider all nodes in order; let $i$ be the current node. If $i$ is s-fixed and $(i, i+1) \in E^S$, add $i+1$ to the set of l-fixed nodes. If $i$ is l-fixed, add all $j > i$ for which $(i, j) \in E^L$ to the set of s-fixed nodes. Next, we repeat this for all nodes in reverse order; again, let $i$ be the current node. If $i$ is s-fixed and $(i, i-1) \in E^S$, add $i-1$ to the set of l-fixed nodes. If $i$ is l-fixed, add all $j < i$ for which $(i, j) \in E^L$ to the set of s-fixed nodes. Clearly, when a node is both s-fixed and l-fixed, no feasible solution exists. Observe that such a conflict is only possible if a path starting from the beginning of a bad path is extended to a node where it meets the end of a bad path.

We claim that, after considering all nodes in increasing order (the increasing round) and once in decreasing order (the decreasing round), no new nodes can be fixed in a feasible instance. Indeed, assuming the opposite implies the existence of a bad path: consider a node $i$, which is not fixed after the decreasing round but becomes fixed in a second increasing round. If $i$ becomes s-fixed, there also exists a node that becomes l-fixed in the second increasing round. Thus, we can assume that node $i$ becomes l-fixed in the second increasing round, implying that node $i-1$ is s-fixed. Moreover, if node $i-1$ is s-fixed in the first increasing round, node $i$ is l-fixed in the first increasing round. Thus, node $i-1$ only becomes s-fixed in the decreasing round, and there exists a node $j > i$ with $(i-1, j) \in E^L$, which is l-fixed. This implies that node $i$ is s-fixed and that the instance is thus infeasible. From symmetry, the same also holds for nodes fixed in a second decreasing round.

As noted above, if a bad path exists, at least one node must be both s-fixed and l-fixed. If no such contradiction is found, the instance is thus feasible, and no additional nodes can be fixed. In the corresponding solution, all s-fixed nodes are assigned to the short chamber, and all l-fixed nodes are assigned to the long chamber. On completing this step, there may remain nodes with no chamber assignment. We describe a straightforward greedy rule that assigns the remaining nodes to the chambers and show that applying this rule to a node does not prevent us from applying it to any later node, thus yielding a feasible solution. The greedy procedure considers all nodes in order. Let $i$ be the current node to be labeled: if $(i-1, i) \in E^S$ and $i-1$ is assigned to the short chamber, assign $i$ to the long chamber; otherwise, assign $i$ to the short chamber. To see that applying this rule does not prevent us from applying it to any later node, consider node $i$ in Figure 9. If $(i, i+1) \in E^S$, then node $i+1$ must be assigned to the long chamber only if $i$ is assigned to the short chamber; this is consistent with the rule. If there exists a $j > i+1$ for which $(i, j) \in E^L$, no s-edges may be incident to any nodes $k$ with $i < k < j$, because this would imply that either $i$ or $j$ is a fixed node and hence, already assigned to a chamber. All such nodes $k$ as well as node $j$ can thus be assigned to the short chamber, which is consistent with the rule. Note that it cannot be the case that both $(i, i+1) \in E^S$ and there exists a $j > i+1$ with $(i, j) \in E^L$, because this would imply that $j$ is a fixed node.

**Figure 9.** Illustration of the Assignment Rule for Remaining Nodes




*Note.* Only one of the dashed edges may be present for any $j > i+1$.

After applying the greedy rule, all nodes have been assigned to a chamber; if no bad path was identified, we have thus obtained a feasible solution. A pseudocode of this procedure is presented in Algorithm 1. Note that the algorithm assumes that $n \geq 3$; instances with $n < 3$ can be solved trivially. The discussion above establishes correctness of the algorithm.

**Algorithm 1** (Pseudocode for the Unidirectional Problem with Two Arbitrary Chambers and Distinct Arrival Times)

**Input:** arrival times $t_1 < t_2 < \ldots < t_n$ and lockage durations $T_1 < T_2$

$v_i \leftarrow$ node corresponding to the arrival at time $t_i$ for all $i \in \{1, \ldots, n\}$
s-fixed $\leftarrow \emptyset$
l-fixed $\leftarrow \emptyset$
// identify initial s-fixed nodes:
$j \leftarrow 1$
**for** $i = 1$ **to** $|V|$**, do**
  $j \leftarrow \max(i+2, j)$
  **if** $(v_i, v_{i+1}) \in E^S$**, then**
    **while** $(v_i, v_j) \in E^L$**, do**
      s-fixed $\leftarrow$ s-fixed $\cup \{v_j\}$
      $j \leftarrow j+1$
$j \leftarrow |V|$
**for** $i = |V|$ **to** 1**, do**
  $j \leftarrow \min(i-2, j)$
  **if** $(v_{i-1}, v_i) \in E^S$**, then**
    **while** $(v_j, v_i) \in E^L$**, do**
      s-fixed $\leftarrow$ s-fixed $\cup \{v_j\}$
      $j \leftarrow j-1$
// extend implications of fixed nodes:
$j \leftarrow 1$
**for** $i = 1$ **to** $|V|$**, do**

**if** $v_i \in$ s-fixed **and** $(v_i, v_{i+1}) \in E^S$, **then** l-fixed $\leftarrow$ l-fixed $\cup \{v_{i+1}\}$
$j \leftarrow \max(i+1, j)$
**if** $v_i \in$ l-fixed, **then**
**while** $(v_i, v_j) \in E^L$, **do**
s-fixed $\leftarrow$ s-fixed $\cup \{v_j\}$
$j \leftarrow j + 1$
$j \leftarrow |V|$
**for** $i = |V|$ **to** 1, **do**
**if** $v_i \in$ s-fixed **and** $(v_{i-1}, v_i) \in E^S$, **then** l-fixed $\leftarrow$ l-fixed $\cup \{v_{i-1}\}$
$j \leftarrow \min(i-1, j)$
**if** $v_i \in$ l-fixed, **then**
**while** $(v_j, v_i) \in E^L$, **do**
s-fixed $\leftarrow$ s-fixed $\cup \{v_j\}$
$j \leftarrow j - 1$
// check for conflicts:
**if** s-fixed $\cap$ l-fixed $\neq \emptyset$, **then return** "not feasible"
// assign nodes to chambers using a greedy rule for nonfixed nodes:
**for** $i = 1$ **to** $|V|$, **do**
**if** $v_i \in$ s-fixed, **then** chambers$_i \leftarrow$ "short"
**else if** $v_i \in$ l-fixed, **then** chambers$_i \leftarrow$ "long"
**else if** $(v_{i-1}, v_i) \in E^S$ **and** chambers$_{i-1}$ = short, **then** chambers$_i \leftarrow$ long
**else** chambers$_i \leftarrow$ short
**return** chambers

*Note.* $n \geq 3$ is assumed for the input.

It remains to show that the procedure described above runs in linear time. As argued above, finding the initial set of s-fixed nodes takes at most $O(n)$ time. Finding all additional s-fixed nodes is also possible in linear time. Indeed, each node has at most two s-edges incident to it, and each node is considered only once in order and once in reverse order. Graph $G$ contains up to $O(n^2)$ l-edges; to see that finding all additional l-fixed nodes takes only $O(n)$ time, we make use of the fact that $(V, E^L)$ is an interval graph. By definition, it follows that, if l-edge $(u, v)$ exists, all l-edges $(u, w)$ with $u < w < v$ must also exist. Thus, when traversing the nodes in order, if a node was labeled as s-fixed because of the presence of an l-edge, no nodes earlier than this fixed node need to be checked at a later time, because such nodes are already s-fixed. Thus, it suffices to remember the latest node that was s-fixed to avoid checking all possible l-edges for each of the nodes. In Algorithm 1, this is achieved by keeping track of $j$, which represents the next node to be checked for the existence of an l-edge. The same argument applies when considering nodes in reverse order, where it suffices to start from the earliest s-fixed node. Because in each iteration, either $i$ or $j$ increases and no nodes earlier than $j$ are checked, finding the s-fixed nodes completes in $O(n)$ time.

Finally, we note that recognizing the structures described in Observations 2.1–2.4, which were assumed not to be present in the graph, can be achieved in linear time. This is easily seen for Observations 2.1, 2.2, and 2.4, which deal only with adjacent nodes. To recognize these structures, it suffices to traverse each of the nodes and check for the existence of the incident edges described in the observations. For Observation 2.3, we use the interval graph structure in the same way as when recognizing the s-fixed nodes in Algorithm 1. That is, we remember the latest node that was s-fixed to avoid checking earlier nodes that are already known to be s-fixed. If any of these structures is identified, it immediately follows that no feasible solution exists.

### 2.2. Fixed Chamber Assignments

We describe here how the algorithm can be extended to the case where some nodes are preassigned to a specific chamber. Note that, when specific chamber assignments are imposed, an instance may not have a feasible solution while its corresponding graph does not contain a bad path. Our algorithm, however, is able to take these given assignments into account.

If a subset of the nodes, say $S \subseteq V$, is preassigned to the short chamber, we initialize, in Algorithm 1, the set of s-fixed nodes to this set $S$. Similarly, if a given subset of nodes, say $L \subseteq V$, is preassigned to the long chamber, we initialize, in Algorithm 1, the set of l-fixed nodes to this set $L$. Clearly, when a node is both s-fixed and l-fixed, the instance is not feasible. After the sets of fixed nodes are initialized, the initial sets of fixed nodes are extended as in Algorithm 1. The analysis of the assignment rule for all nodes that are not fixed remains valid. Note that feasibility is still identified by verifying for each node whether it is both s-fixed and l-fixed. These adjustments suffice to solve the generalization with fixed chamber assignments. The computational complexity remains unchanged.

### 2.3. Simultaneous Arrivals

In this section, we will consider the generalization where simultaneous arrivals can occur (i.e., where arrival times need not be distinct). Because a chamber may then simultaneously serve more than one ship in a no-wait solution, we need to take the capacity of the chambers into account. Let $C_{\text{small}} = \min(C_1, C_2)$ and $C_{\text{large}} = \max(C_1, C_2)$. Note that $C_{\text{small}}$ does not necessarily correspond to the chamber with the shortest lockage duration. We now show how to modify graph $G$ and Algorithm 1 to determine whether a feasible solution exists in this setting with simultaneous arrivals.

For each time $t \in \mathcal{T}$, let $k_t$ be the number of ships arriving at time $t$. Clearly, if there exists a $t$ with $k_t > C_{\text{small}} + C_{\text{large}}$, the instance is not feasible, because the number of arriving ships at time $t$ exceeds the combined capacity of both chambers. Let us thus assume that the instance has $k_t \leq C_{\text{small}} + C_{\text{large}}$ for all $t \in \mathcal{T}$. For each $t \in \mathcal{T}$, we distinguish three cases.

1. Case 1: $2 \le k_t \le C_{\text{small}}$. We modify graph $G$ as follows: we let a single node represent all of the simultaneous arrivals at time $t$. Either chamber is a valid assignment to simultaneously serve all $k_t$ ships; we may thus treat these simultaneous arrivals as a single ship.

2. Case 2: $C_{\text{small}} < k_t \le C_{\text{large}}$. As in the case above, let a single node represent all simultaneous arrivals at time $t$. In addition, we impose that this node must be assigned to the large chamber. Recall that the large chamber may be the chamber with either the short or the long lockage duration depending on the values of $T_1$, $T_2$, $C_1$, and $C_2$. It is easily seen that it is never required to use both chambers to serve these $k_t$ ships, because the large chamber must be used regardless, and this single chamber can serve all ships simultaneously.

3. Case 3: $C_{\text{large}} < k_t \le C_{\text{small}} + C_{\text{large}}$. Again, let a single node represent all simultaneous arrivals at time $t$. It follows that, because $k_t > C_{\text{large}}$, both chambers must be used simultaneously to transfer all ships without introducing waiting time. We add this node to the set $B$ designated to identify all nodes that must be assigned to both chambers. We argue below that, after modifying the graph for all $t \in \mathcal{T}$, the algorithm is easily adjusted to enforce this assignment for each node in $B$.

After graph $G$ has been modified, we take $B$ into account by initializing the algorithm as follows. For each node $i \in B$, we add all implications that follow from imposing that $i$ is assigned to both chambers: for all $j \in V$ with $(i,j) \in E^S$, add $j$ to the set l-fixed; for all $j \in V$ with $(i,j) \in E^L$, add $j$ to the set s-fixed. It is easily argued that each of the added implications must hold in any feasible solution. When assigning the nodes to chambers, we set $\text{chambers}_i = \text{short} + \text{long}$ for all $i \in B$. Furthermore, for a feasible solution to exist, it must hold that none of the nodes in $B$ are fixed when running the algorithm. Indeed, if a node $k \in B$ with corresponding time $t_k$ would be fixed, this implies that at least one chamber is unavailable at time $t_k$ so that there is no feasible assignment for $k$. In addition to modifying the initialization of sets s-fixed and l-fixed, we thus extend the check for conflicts in the algorithm to "if s-fixed $\cap$ l-fixed $\neq \emptyset$, s-fixed $\cap$ $B \neq \emptyset$, or l-fixed $\cap$ $B \neq \emptyset$."

Using the result for fixed chamber assignments described in Section 2.2 and by modifying Algorithm 1 as outlined above, it follows that we can solve the resulting instance in $O(n)$ time. The following theorem concludes this discussion.

**Theorem 2.** *If a no-wait solution exists for the unidirectional lock scheduling problem with two chambers, it can be found (i.e., problem NLS-uni-2 can be solved) in $O(n)$ time.*

We also point out the following remark. We will come back to this issue in Section 6.

**Remark 1.** In case a no-wait solution does not exist, it is not difficult to adapt Algorithm 1 to identify the ship with the latest arrival $t_k$ such that the instance formed by arrival times $\{t_1, t_2, \ldots, t_k\}$ allows a no-wait solution.

## 3. Two Arbitrary Chambers

We now focus on the more general setting with two lock chambers, where ships may travel in both directions. We show how to model this problem as a 2-SAT problem. The 2-SAT problem is well known to be solvable in polynomial time.

**Lemma 1.** *An instance of NLS-2 can be modeled as an instance of 2-SAT using $O(n)$ variables and $O(n^2)$ clauses.*

**Proof.** In the NLS-2 setting, ships may arrive simultaneously. To take this into account, we first describe how each instance of NLS-2 can be transformed into an equivalent instance, where $C_1 = C_2 = 1$ and where some ships are preassigned to chambers. We then provide a reduction to 2-SAT for the setting with two unit-capacity chambers and preassigned ships.

Similar to the approach discussed in Section 2.3, we distinguish multiple cases when constructing the instance with unit capacity. For each time $t \in \mathcal{T}$, let $k_{t,d}$ be the number of ships arriving at time $t$ and traveling in direction $d$. Clearly, the instance is not feasible if there exist $t$ and $d$ such that $k_{t,d} > C_1 + C_2$. Because this can be easily verified in linear time, we assume that $k_{t,d} \le C_1 + C_2$ for all $t \in \mathcal{T}$, $d \in \{\text{upstream}, \text{downstream}\}$.

Consider a given instance $\mathcal{I}$ of NLS-2. As in Section 2.3, let $C_{\text{small}} = \min(C_1, C_2)$ and $C_{\text{large}} = \max(C_1, C_2)$. We construct an instance $\mathcal{I}_{\text{unit}}$, where $C_1^{\mathcal{I}_{\text{unit}}} = C_2^{\mathcal{I}_{\text{unit}}} = 1$, leaving the lockage times and the set of arrival times unaltered. The set of arriving ships is constructed as follows for each $t \in \mathcal{T}$ and $d \in \{\text{upstream}, \text{downstream}\}$.

1. If $k_{t,d} \le C_{\text{small}}$, replace these $k_{t,d}$ ships by a single ship traveling in direction $d$ arriving at time $t$. It is immediately clear that either lock chamber suffices to handle all $k_{t,d}$ ships so that we effectively ignore these simultaneous arrivals.

2. If $C_{\text{small}} < k_{t,d} \le C_{\text{large}}$, replace all of these ships by a single ship traveling in direction $d$ arriving at time $t$ and additionally impose that this ship must be assigned to the large chamber (i.e., the second chamber if $C_1 \le C_2$; the first chamber otherwise). To serve all ships without introducing waiting time, the large chamber must be used to serve at least one ship arriving at time $t$ and traveling in direction $d$. By preassigning the ship to the large chamber, it follows that this chamber must be available at time $t$ to serve ships traveling in direction $d$. In fact, the capacity of the large chamber suffices to serve all ships arriving at time $t$ and traveling in direction $d$. Consequently, we can ignore the simultaneously arriving ships in what follows.

3. If $C_{\text{large}} < k_{t,d}$, replace all of these ships with two ships traveling in direction $d$ arriving at time $t$. Furthermore, preassign the first ship to the first chamber and the second ship to the second chamber. It follows that both chambers must be available at time $t$ to serve ships traveling in direction $d$.

The resulting instance is called $\mathcal{I}_{\text{unit}}$. Finding a no-wait schedule for $\mathcal{I}_{\text{unit}}$ with preassigned ships thus yields a no-wait solution for the original instance $\mathcal{I}$ of NLS-2. We now describe a reduction to 2-SAT for the problem of finding a no-wait solution for instances with two unit-capacity chambers and preassigned ships.

Recall that a no-wait schedule exists if and only if each ship can be assigned to either the short or the long chamber such that, for each chamber, lockages do not overlap. For lock chamber $j \in \{1, 2\}$, it is easily seen that there is no overlap if and only if $|t_i - t_{i'}| \geq T_j$ for each pair of ships $i$ and $i'$ that are assigned to chamber $j$, and in addition, $|t_i - t_{i'}| \geq 2T_j$ if ships $i$ and $i'$ travel in the same direction, as argued in Section 1. We create the following instance of 2-SAT: for each ship $i \in \{1, \ldots, n\}$, define a literal $x_i$. We will argue later that $x_i$ = false corresponds to assigning ship $i$ to the short chamber and that $x_i$ = true corresponds to assigning ship $i$ to the long chamber. Let the Boolean expression in conjunctive normal form consist of clauses described as follows for each pair of ships $i, i' \in \{1, \ldots, n\}$.

1. If the ships travel in opposite directions and $|t_i - t_{i'}| < T_1$, add the clause $(x_i \vee x_{i'})$.
2. If the ships travel in the same direction and $|t_i - t_{i'}| < 2T_1$, add the clause $(x_i \vee x_{i'})$.
3. If the ships travel in opposite direction and $|t_i - t_{i'}| < T_2$, add the clause $(\neg x_i \vee \neg x_{i'})$.
4. If the ships travel in the same direction and $|t_i - t_{i'}| < 2T_2$, add the clause $(\neg x_i \vee \neg x_{i'})$.

Observe that, if $x_i$ = false where ship $i$ is assigned to the short chamber, and $x_i$ = true where it is assigned to the long chamber, $(\neg x_i \vee \neg x_{i'})$ suffices to prevent overlapping lockages for the long chamber, whereas $(x_i \vee x_{i'})$ ensures that there is no overlap for the short chamber.

In addition, to enforce that all preassignments are respected, we add a clause containing only the literal $x_i$ for all ships $i$ that are preassigned to the long chamber and a clause consisting of $\neg x_i$ for all ships $i$ that are preassigned to the short chamber.

We claim that the existence of a truth assignment satisfying the Boolean formula is equivalent to the existence of a no-wait schedule. Indeed, given a truth assignment, we assign ship $i$ to the short chamber if $x_i$ = false and to the long chamber if $x_i$ = true. The definition of the clauses implies that no overlapping lockages exist for either chamber while each ship is assigned to a chamber, and hence, we found a no-wait schedule. Also, the existence of a no-wait schedule immediately translates into a satisfying truth assignment. □

Note that the number of clauses in the 2-SAT instance described above, as well as the time needed to construct this instance, is quadratic in the number of ships. To find a no-wait solution for the NLS-2 problem, it follows that we can construct an instance of 2-SAT as described above and use any algorithm for 2-SAT with a running time linear in the number of clauses. We can summarize this in the following theorem.

**Theorem 3.** *The bidirectional case for two arbitrary chambers (i.e., problem NLS-2) can be solved in $O(n^2)$ time.*

For an explicit description of methods deciding the feasibility of 2-SAT, we refer to Even et al. (1976) and Aspvall et al. (1979).

## 4. Identical Chambers

We now focus on problem NLS-id (i.e., $C_j = C$ and $T_j = T$ for each $j \in \{1, \ldots, m\}$). Again, we assume that arrival times are given in sorted order (Section 1.4). In fact, we consider a more general optimization version of NLS-id, where we aim at finding the minimum number of chambers allowing a no-wait solution. We first show that this problem is a special case of coloring trapezoid graphs. In Section 4.2, we provide a description of a greedy procedure and argue that it always finds a no-wait schedule while using a minimum number of chambers. We then prove in Section 4.3 that this procedure can be implemented with an $O(n)$ running time for both the unidirectional and the bidirectional cases.

### 4.1. Coloring Trapezoid Graphs

For the definition of trapezoid graphs, we consider a pair of parallel lines labeled *up* and *down*. A trapezoid between these lines is defined by two points per line. Let this construction of lines and trapezoids be called the trapezoid instance. A graph $G = (V, E)$ is called a trapezoid graph if there exists a trapezoid instance with $|V|$ trapezoids, each corresponding to a node in $V$, such that there is an edge in $E$ connecting nodes $u$ and $v$ if and only if the trapezoids corresponding to $u$ and $v$ intersect (see Figures 10 and 11 for an example illustrating this definition). Felsner et al. (1997) discuss the coloring of trapezoid graphs and show that a proper coloring with a minimum number of colors can be found in $O(n \log n)$ time.

**Figure 10.** Example Instance Illustrating the Definition of the Corresponding Trapezoids

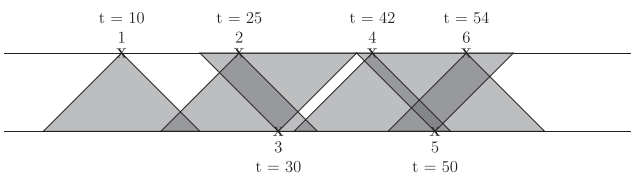
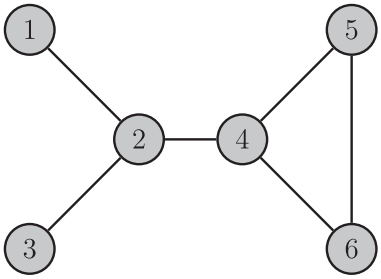


**Figure 11.** Trapezoid Graph Corresponding to the Example Instance of Figure 10




The special case of this trapezoid graph coloring problem that we consider is the following: given a trapezoid instance where all trapezoids are identical isosceles triangles, find a proper coloring with a minimum number of colors. We denote this problem by TC. Notice that, although the triangles are identical, their orientation may differ depending on which of the parallel lines contains a single point. In a trapezoid coloring instance, we say that a triangle is *up oriented* if this triangle has a single point on the up line and two points on the down line; a triangle is *down oriented* if it has a single point on the down line and two points on the up line.

We argue that we can reduce NLS-id to TC and vice versa. As in Section 3, let $k_{t,d}$ denote the number of ships arriving at time $t$ and traveling in direction $d$; we first argue that we can deal with simultaneous arrivals by transforming each instance $\mathcal{I}$ of NLS-id into an equivalent instance $\mathcal{I}_{\text{unit}}$ with unit capacity. In the remainder of Section 4, we can then restrict ourselves to instances where $C = 1$. Given $\mathcal{I}$, we construct $\mathcal{I}_{\text{unit}}$ as follows. For each $t \in \mathcal{T}$, replace the $k_{t,d}$ arrivals by $\lceil k_{t,d}/C \rceil$ arrivals at time $t$ and traveling in direction $d$. Clearly, in $\mathcal{I}$, at least $\lceil k_{t,d}/C \rceil$ chambers are needed to serve these $k_{t,d}$ ships. It is also clear that any remaining capacity in the chosen lock chambers cannot be used to serve other ships without introducing waiting time. Consequently, a solution in $\mathcal{I}_{\text{unit}}$ corresponds directly to a solution in $\mathcal{I}$. Observe that constructing $\mathcal{I}_{\text{unit}}$ can be done in $O(n)$ time.

We thus turn our attention to the case with unit capacity. Given an instance of NLS-id where $C = 1$, we specify a set of identical isosceles triangles between parallel lines up and down as follows. For each downstream-traveling ship $i$, we construct a triangle with a point on the up line at $t_i$ and points on the down line at $t_i - T$ and $t_i + T$; for each upstream-traveling ship $i$, we construct a triangle with a point on the down line at $t_i$ and points on the up line at $t_i - T$ and $t_i + T$. From this construction, we can derive two fundamental properties.

1. The triangles corresponding to two ships $i$ and $i'$ traveling in the same direction intersect if and only if $|t_i - t_{i'}| < 2T$.

2. The triangles corresponding to two ships $i$ and $i'$ traveling in opposite directions intersect if and only if $|t_i - t_{i'}| < T$.

Note that, in either case, ships $i$ and $i'$ cannot be served by the same chamber. In conclusion, ships can be assigned to the same chamber if and only if the corresponding triangles do not intersect. Hence, a proper coloring of the corresponding graph represents a no-wait schedule, where a color refers to a chamber. The chromatic number of the trapezoid graph then represents the minimum number of chambers allowing a no-wait schedule. Each instance of NLS-id can thus be modeled as an instance of TC. Similarly, it is easily seen that each instance of TC can be modeled as an instance of NLS-id.

To illustrate this reduction, we provide an example instance in Figure 10. We have downstream-traveling ships 1, 2, 4, and 6 arriving at times 10, 25, 42, and 54 and upstream-traveling ships 3 and 5 arriving at times 30 and 50. The lockage time is $T = 10$. Consequently, the pairs of ships that cannot both be served by a single chamber are (1,2), (2,3), (2,4), (4,5), (4,6), and (5,6). This is represented by intersections of triangles accurately. The graph corresponding to this instance is shown in Figure 11. It is immediately seen that at least three colors are required to color the graph, because it contains a clique on nodes 4, 5, and 6. One feasible solution could consist of assigning ships 1 and 4 to the first chamber, ships 2 and 5 to the second chamber, and ships 3 and 6 to the third chamber.

Because the chromatic number of a trapezoid graph can be found in $O(n \log n)$ time (Felsner et al. 1997), it immediately follows that an $O(n \log n)$ algorithm exists that solves NLS-id. In the remainder of this section, we improve this result to yield an $O(n)$ algorithm.

### 4.2. Correctness of a Greedy Procedure for NLS-id

We argued in Section 4.1 that we can reduce each instance of NLS-id to an equivalent instance with $C = 1$. We thus restrict ourselves to the setting with $C = 1$. We say that a chamber is available at time $t$ for direction $d$ if the last ship handled by this chamber (say ship $i$) traveled in direction $d$ and arrived at time $t_i \le t - 2T$ or traveled in the direction opposite to $d$ and arrived at time $t_i \le t - T$. In the former case, we say that the availability period of the chamber equals $t - t_i - 2T$; in the latter case, the availability period of the chamber equals $t - t_i - T$. The solution procedure is as follows. Initially, let the number of chambers be zero. Consider all arriving ships in order; let $t_i$ and $d_i$ be the arrival time and direction, respectively, of ship $i$ being considered. If no chambers are available at time $t_i$ for direction $d_i$, add an additional chamber, and assign ship $i$ to this chamber; otherwise, assign ship $i$ to the chamber with the largest availability period for direction $d_i$ at time $t_i$.

We argue that this greedy procedure yields a solution with a minimum number of chambers. Whenever a chamber is added to serve ship $i$, there is no possible

assignment of ships $1,\dots,i-1$ to chambers so that at least one chamber is available at time $t_i$. Indeed, because all chambers are identical, the choice of which ship is assigned to which chamber is, in fact, irrelevant. When considering a ship $i$, a chamber is added if and only if the current number of chambers is not sufficient to serve ships $1,\dots,i$ without waiting time. It follows that, after considering all ships, we have a no-wait solution that uses a minimum number of chambers.

### 4.3. An $O(n)$ Algorithm for NLS-id

To see that the procedure from Section 4.2 can be implemented to run in linear time, we first briefly discuss the unidirectional setting before extending the implementation to the bidirectional case. When all ships travel in the upstream direction, it is easily seen that there is an optimal solution where a chamber, after transferring a ship, immediately returns to the downstream side. A chamber that serves a ship is then always unavailable for $2T$ time units starting from the arrival time of that ship. Solving this problem corresponds to a basic interval scheduling problem, for which Ford and Fulkerson (1962) describe a "staircase rule" based on Dilworth's chain decomposition theorem. Gupta et al. (1979) provide a more efficient algorithm, which runs in $O(n)$ time when the intervals have equal length and are sorted by starting time. Applying this algorithm thus immediately yields a solution to NLS-uni-id.

Although this approach is straightforward for NLS-uni-id, the time at which a chamber becomes available depends on the direction of travel in the more general NLS-id. Indeed, a chamber that finishes an upward lockage is immediately available to serve a downstream-traveling ship; the next upstream-traveling ship, however, can only be served after an additional $T$ time units needed to return to the downstream side. As a result, for a given time $t$ and direction $d$, a chamber that started a lock movement at time $t_1$ may not be available, whereas a different chamber that started a lock movement at time $t_2 > t_1$ is available. The main challenge of implementing our greedy rule is thus to efficiently keep track of the different chambers and the moments in time at which they are available to serve ships depending on their direction.

To achieve this, we maintain the following lists throughout the solution procedure. Each of the entries that will be added to these lists consists of a pair $(t,i)$, where $t$ specifies the time at which the chamber that serves ship $i$ becomes available for a given direction.

1. List $A^{UU}$: availability for upstream, ship $i$ is upstream traveling.
2. List $A^{UD}$: availability for upstream, ship $i$ is downstream traveling.
3. List $A^{DU}$: availability for downstream, ship $i$ is upstream traveling.
4. List $A^{DD}$: availability for downstream, ship $i$ is downstream traveling.

As outlined in the description in Section 4.2, we keep track of the required number of chambers $m$. Additionally, we follow-up whether each chamber remains available as the algorithm runs. Throughout the algorithm, $R_i$ are Boolean values indicating whether, after serving ship $i$, a chamber has been used to serve another ship ($1 \le i \le n$).

A pseudocode for the algorithm is provided in Algorithm 2. In words, we consider each ship in order and first verify whether one of the existing chambers is available at the position where the ship arrives. Let $i$ be the ship under consideration. If a chamber $j$ is available, it contains an entry in two of the lists. On assigning a ship to $j$, we update $R_i$ so that the second entry corresponding to $j$ becomes invalid. We update the times at which $j$ becomes available for each direction. If no chamber is available, we update $m$ and proceed as above.

**Algorithm 2** (Pseudocode for Identical Chambers (i.e., NLS-id) with Unit Capacity)

**Input:** arrival times $t_1 \le t_2 \le \dots < t_n$, directions $d_1, d_2, \dots, d_n$, lockage duration $T$

$A^{UU} \leftarrow \emptyset$, $A^{UD} \leftarrow \emptyset$, $A^{DU} \leftarrow \emptyset$, $A^{DD} \leftarrow \emptyset$
$R_i \leftarrow$ false for all $i \in \{1,\dots,n\}$
$m \leftarrow 0$
**for** $i = 1$ **to** $n$**, do**
  reUsed = false
  **if** $d_i$ == downstream**, then**
    $(t^*, i^*) \leftarrow$ earliest entry in $A^{DU} \cup A^{DD}$
    **while** $t^* < t_i$ **and** reUsed == false**, do**
      **if** $R_i$ == false**, then**
        reUsed = true
        $R_{i^*} \leftarrow$ true
      delete entry $(t^*, i^*)$ from the list in which it is contained
      $(t^*, i^*) \leftarrow$ earliest entry in $A^{DU} \cup A^{DD}$
  **else**
    $(t^*, i^*) \leftarrow$ earliest entry in $A^{UU} \cup A^{UD}$
    **while** $t^* < t_i$ **and** reUsed == false**, do**
      **if** $R_i$ == false**, then**
        reUsed = true
        $R_{i^*} \leftarrow$ true
      delete entry $(t^*, i^*)$ from the list in which it is contained
      $(t^*, i^*) \leftarrow$ earliest entry in $A^{UU} \cup A^{UD}$
  **if** reUsed == false**, then**
    $m \leftarrow m + 1$
  **if** $d_i$ == downstream**, then**
    add entry $(t_i + T, i)$ to the back of list $A^{UD}$
    add entry $(t_i + 2T, i)$ to the back of list $A^{DD}$
  **else**
    add entry $(t_i + T, i)$ to the back of list $A^{DU}$
    add entry $(t_i + 2T, i)$ to the back of list $A^{UU}$
**return** $m$

To see that Algorithm 2 runs in linear time, note the following. Each ship is considered only once in the given input order. New entries in the lists $A^{UU}$, $A^{UD}$, $A^{DU}$, and $A^{DD}$ are always added to the end of the list. Furthermore, the time value of newly inserted entries is nondecreasing with $i$, because the increment when constructing the entry is the same for all entries within each of the lists; for example, all entries inserted in list $A^{UU}$ have a time value of $t_i + 2T$ for some $t_i$, and all entries inserted in list $A^{UD}$ have a time value of $t_i + T$ for some $t_i$. Each of the lists thus remains sorted by the time value of the contained entries at all times. Finding the earliest entry in two of the lists is then easily performed in constant time by comparing the first entry of each of the lists. Deleting the first entry as well as adding a new entry to the back of a list also require only constant time. Whenever an entry of one of the lists is iterated over, it is deleted. Because only $O(n)$ entries are added to lists throughout the procedure, iterating through the lists thus also takes $O(n)$ time in total. It follows that the entire procedure runs in linear time. We can summarize the discussion above as follows.

**Theorem 4.** *For the setting with identical chambers, a no-wait schedule can be found or shown not to exist (i.e., problem NLS-id can be solved) in $O(n)$ time.*

**Corollary 1.** *Finding the chromatic number of a trapezoid graph where the trapezoids are identical up- or down-oriented isosceles triangles can be done in $O(n)$ time.*

Similar to Remark 1, we point out the following. We will come back to this issue in Section 6.

**Remark 2.** In case a no-wait solution does not exist for a given number of chambers $m$, it is trivial to modify Algorithm 2 to identify the ship with the latest arrival $t_k$ such that the instance formed by arrival times $\{t_1, t_2, \ldots, t_k\}$ allows a no-wait solution.

## 5. Results for NLS

In this section, we propose a DP approach that solves the general problem NLS stated in Section 1. This algorithm runs in polynomial time when the number of chambers is fixed. In contrast, if the number of chambers is part of the input, we state that NLS is NP complete, even in the unidirectional case with distinct arrival times.

Recall that, in NLS, we consider an arbitrary number of chambers $m$ with nonidentical lockage times $T_j$ and capacities $C_j$ for $j = 1, \ldots, m$. We assume that ships are numbered in nondecreasing order of arrival times. Ties are broken by letting ships arriving downstream have lower numbers than ships arriving upstream. Remaining ties are broken arbitrarily. Our DP approach assigns ships to chambers in increasing order of these numbers. We consider states $(i_1, \ldots, i_m)$, where $i_j$ represents the last ship $1 \le i_j \le n$ that has been assigned to chamber $j$ $(1 \le j \le m)$. Furthermore, we restrict ourselves to states where a ship is assigned to a chamber only if all earlier ships have also been assigned. Thus, in a given state $(i_1, \ldots, i_m)$, the first $\max_{j \in \{1,\ldots,m\}} i_j$ ships have been assigned to chambers. We consider a transition from $(i_1, \ldots, i_m)$ to $(i'_1, \ldots, i'_m)$ if there is a $j^* \in \{1, \ldots, m\}$ such that:

1. $i'_{j^*} > i_{j^*}$ and $i'_j = i_j$ for each $j \in \{1, \ldots, m\}$ with $j \neq j^*$,
2. ships $(\max_j i_j) + 1, \ldots, i'_{j^*}$ travel in the same direction and arrive at the same time,
3. ship $i'_{j^*}$ arrives at least $T_{j^*}$ time units later than ship $i_{j^*}$ if both travel in opposite direction and at least $2T_{j^*}$ time units later than ship $i_{j^*}$ otherwise, and
4. $i'_{j^*} - (\max_j i_j) \le C_{j^*}$.

This transition represents assigning ships $(\max_j i_j) + 1, \ldots, i'_{j^*}$ to chamber $j^*$. This is allowed only if chamber $j^*$ is available after handling ship $i_{j^*}$. If more than one ship is assigned to chamber $j^*$, then these ships must arrive simultaneously and travel in the same direction, and the chamber's capacity must not be exceeded. We consider an initial state $(0, \ldots, 0)$ representing that no ships are assigned to any chambers yet. The question is whether we can reach a state $(i_1, \ldots, i_m)$ with $\max\{i_j \mid j = 1, \ldots, m\} = n$ by any sequence of transitions.

In this DP, we use $O(n^m)$ states and $O(mn^{m+1})$ transitions. However, we can further restrict the set of transitions by always assigning the maximum number of ships (up to the chamber's capacity) that travel in the same direction and arrive at the same time as ship $i_{j^*} + 1$. Each state then has $m$ transitions: one per chamber. This leaves us with $O(mn^m)$ transitions, which also constitutes the runtime complexity. Note that the complexity is polynomially bounded if the number of chambers is fixed. We can conclude with the following theorem.

**Theorem 5.** *The problem setting with a fixed number of arbitrary chambers (i.e., problem NLS-m) can be solved in $O(mn^m)$ time.*

Again, we point out the following. We will come back to this issue in Section 6.

**Remark 3.** In case a no-wait solution does not exist, it is not difficult to adapt the procedure above to identify the ship with the latest arrival $t_k$ such that the instance formed by arrival times $\{t_1, t_2, \ldots, t_k\}$ allows a no-wait solution.

We now state that problem NLS is NP complete.

**Theorem 6.** *Deciding whether a no-wait solution exists for an arbitrary number of chambers, even when all ships travel in the same direction and arrival times are distinct (i.e., problem NLS-uni-distinct), is strongly NP complete.*

**Proof.** We refer to Passchyn et al. (2016b) for the proof. □

**Remark 4.** As mentioned in Section 1, when viewing a chamber as a machine and an arrival as a job represented by $m$ intervals (one for each machine; each starting at the same moment $t_i$), the reduction referred to above shows that the problem considered by Böhmová et al. (2013) (called interval selection with cores) remains NP complete even when all intervals that correspond to the same machine have the same length.

## 6. Minimizing Waiting Time

An interesting extension of the work presented for no-wait schedules is to consider instances where some waiting time cannot be avoided. For such instances, it makes sense to look for a schedule that minimizes the waiting time. For example, two sensible objective functions to consider would be to minimize (i) the total sum of waiting times of all ships or (ii) the maximum waiting time that any ship incurs. In this section, we sketch how extensions of algorithms introduced in the previous sections can be used to find solutions for instances that do not have a no-wait solution. We discuss two distinct approaches. Of course, it may not be possible to make strong guarantees about the solution quality of the proposed heuristics for all possible problem instances.

### 6.1. Iterative Application

Essentially, we iteratively identify a maximal instance that allows a no-wait schedule, build a corresponding partial schedule, and postpone the arrival times of the ships that are incompatible with the partial schedule. A more precise description is as follows. Suppose that a given instance of NLS-$m$ does not admit a no-wait schedule. As indicated by Remark 3, the DP sketched in Section 5 will then identify the largest arrival time $t_k$ (with $t_k < t_n$) such that the partial instance $\{t_1, \ldots, t_k\}$ allows a no-wait schedule. Then, we schedule the first $k$ ships accordingly and refer to these assignments as a partial schedule. Clearly, all lock chambers are unavailable at time $t_{k+1}$; otherwise, the partial schedule was not maximal. Let $t^*$ be the earliest time at which a chamber becomes available after serving all ships in the partial schedule. Next, we update the arrival times $t_i$ that satisfy $t_i < t^*$ with $i \geq k+1$ as follows: $t_i \leftarrow t^*$. Updating these arrival times leads to a new partial instance $\{t_{k+1}, \ldots, t_n\}$ to which we apply the same procedure.

Notice that, in each iteration, we are sure that at least one ship is scheduled. This guarantees that this procedure ends with a feasible solution and in fact, runs in polynomial time.

Clearly, by Remarks 1 and 2, similar procedures can be applied for instances of NLS-uni-2 and NLS-id, respectively, making use of the efficient no-wait algorithms for these special cases discussed in Sections 2 and 4.

This approach can be used for minimizing the maximum waiting time, because in each iteration, the updated arrival times reflect the minimum possible delays that allow a feasible extension.

### 6.2. Rounding of Arrival Times

Instead of associating an arrival time with each ship that is a unique moment in time, one can also associate a predefined interval with each ship. Thus, let us assume that time is split up into five-minute intervals and that we know in which interval each ship arrives. In fact, we assume that each ship arrives at the end of its five-minute interval. Next, by treating the resulting instance as an instance of NLS, we verify whether there exists a schedule in which a ship does not wait for more than five minutes. If the answer is yes, we can implement this schedule with a bound on the maximum (and total) waiting time; otherwise, we might (incrementally) increase the length of the time interval and verify whether a feasible solution exists for this new length.

This approach allows for the clustering of ships that arrive close to each other and may find no-wait schedules for these rounded data where the original data do not admit such schedules. Such an approach is likely to work well when the interval is small compared with the lockage time and where there is enough capacity for the locks. Clearly, these issues may depend on the precise practical situation.

## 7. Conclusion and Future Research

In this paper, we have investigated the problem of scheduling a lock consisting of multiple chambers in parallel. We showed the connection of this problem to a known interval scheduling problem. We focused on the relevant case of finding so-called no-wait schedules, and we obtained algorithmic results for different special cases.

More specifically, for the problem setting where a lock consists of two (distinct) chambers and traffic is unidirectional, we showed how to characterize feasibility and how this result leads to a linear time algorithm that either identifies a no-wait solution or proves that no such solutions exist. For the problem setting where a lock consists of (many) identical chambers, we showed how a known result for the unidirectional setting can be extended to the bidirectional setting and how this result extends to a special case of the trapezoid coloring problem. Furthermore, we described a DP approach for the more general problem setting with arbitrary chambers. We also showed that the general problem of deciding whether a no-wait schedule exists is strongly NP complete, thereby strengthening a result from interval scheduling. Finally, we discussed two approaches that make use of the algorithms for the no-wait setting to obtain solutions for the problem of minimizing waiting time in case a no-wait solution does not exist.

One direction for future research would be to expand on Section 6 with a more in-depth study of the problem of minimizing waiting time. In addition to this extension, taking into account ship properties, such as individual deadlines and size or other restrictions, is worth studying. Finally, multiple multi-chambered locks in sequence (which are present in many inland waterways) are an interesting research area.

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