Efficient estimation of sensitivity and bias in detection tables

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Abstract

In this report a practical approach is outlined how the significance of sensory detection or discrimination can be statistically assessed when there is strong reponse bias in a 2-alternative forced choice (2AFC) task. Classical Signal Detection Theory cannot be employed without additional assumptions, therefore, use is made of the response strength theory of Luce (1959) in deriving efficient estimators.

Frequently used designs in sensory detection experiments by human observers are Yes-No detection and 2-Alternative Forced Choice (2AFC). Both designs lead to similar detection tables; the one for detection runs as follows:

		Observer response	
		stimulus	*no stimulus*
Stimulus presentation	stimulus	p	$1 - p$
	no stimulus	$1 - q$	q

(1)

A correct detection (probability p) is called *hit*, failure to detect the stimulus is called a *miss*; the detection of a stimulus when none was presented is a *falsealarm* and the response of no stimulus when none was presented is a *correct rejection* (q). In a 2AFC design the stimulus is contained in one of two observation intervals and the subject decides at each trial which one contained the stimulus. The corresponding detection table is:

		Observer response	
		interval 1	*interval 2*
Stimulus presentation	interval 1	p	$1 - p$
	interval 2	$1 - p$	p

(2)

in which p corresponds to a *hit* and its complement to a *false alarm*. In this table, however, it is assumed that the subject has no preference, or *bias*, to choose one specific interval more often than the other and in general this assumption is violated in practice.

If the subject displays a bias towards one of the response alternatives, the response proportions in one column of the detection table will be increased, which implies that the estimate of the detection probability p in one row will be increased, but decreased in the other row. On the level of the observed data, therefore, sensory sensitivity and bias are then completely confounded. This, by itself, need not at all impede proper estimation of sensitivity, as classical signal detection techniques (Swets, Tanner and Birdsall, 1961) and response strength models from Choice theory (Luce, 1959) are available for exaxctly this purpose.

In order to establish in the 2AFC task whether the alternatives can consistently be discriminated the hypothesis to be tested is that the response probability p departs significantly from 0.5, which is the guessing probability. Whenever there is response bias the confounding of bias and sensitivity prevents such an approach, as the null hypothesis that the guessing probability is 0.5 is obviously invalid.

What is needed is an estimate of the "real" guessing probability when the effect of sensitivity has been factored out. So, whereas in classical signal detection theory one tries to obtain a sensitivity measure, independent of bias (i.e. the point(s) lying on the equisensitivity curve) we now try to obtain a measure of bias independent of sensitivity, i.e. the point lying on the *equibias* curve.

One problem here is that equibias curves are not well defined in classical signal detection theory and can only be obtained at the expense of additional, sometimes rather arbitrary assumptions.

Response strength theory, however, leads in a very natural way to equibias curves and to the solution of the problem stated above. Response strength theory (Luce, 1959) is called that way because it relates hypothetical response tendencies to observable response probabilities, containing no assumptions about sensory processing *per se*.

For a short description of the theory suppose that an observer is unable to distinguish between stimuli S0 and S1, but that he or she has to respond with the corresponding R0 and R1. It is supposed that for each response alternative there is a response strength v_i on presentation of one of the stimuli. The response strength matrix is then:

		Response	
		R0	*R1*
Stimuli	S0	v_0	v_1
	S1	v_0	v_1

(3)

In this scheme v_i is a positive real constant lying on a ratio scale. This means that the matrix can be simplified by dividing all entries by v_1. Setting v_0/v_1 equal to β, this leads to the following matrix:

		Response	
		R0	*R1*
Stimulus	S0	1	β
	S1	1	β

(4)

from which the predicted response probabilities can be obtained:

$$\text{Stimulus} \quad \begin{array}{c|cc} & \multicolumn{2}{c}{\textbf{Response}} \\ & R0 & R1 \\ \hline S0 & \frac{1}{1+\beta} & \frac{\beta}{1+\beta} \\ S1 & \frac{1}{1+\beta} & \frac{\beta}{1+\beta} \end{array} \tag{5}$$

Whenever there is sensory evidence for either S0 or S1 this leads to an increment in the response tendencies by a multiplicative sensory parameter α. It is easy to verify that the corresponding response strength and detection probability matrices then read:

$$\text{Stimulus} \quad \begin{array}{c|cc} & \multicolumn{2}{c}{\textbf{Response}} \\ & R0 & R1 \\ \hline S0 & \alpha & \beta \\ S1 & 1 & \alpha\beta \end{array} \tag{6}$$

$$\text{Stimulus} \quad \begin{array}{c|cc} & \multicolumn{2}{c}{\textbf{Response}} \\ & R0 & R1 \\ \hline S0 & \frac{\alpha}{\alpha+\beta} & \frac{\beta}{\alpha+\beta} \\ S1 & \frac{1}{1+\alpha\beta} & \frac{\alpha\beta}{1+\alpha\beta} \end{array} . \tag{7}$$

This is the detection matrix for the 2AFC task, and for this matrix it has to be assessed whether detection performance is better than chance, in particular when response bias is large. For this purpose we assume that response bias β is independent of sensory detection in order to arrive at the bias matrix (5), which is equivalent to setting α to 1. Both α and β can easily be estimated from a data matrix D using the structure of (7). If the response fractions are denoted as follows:

		Response	
		interval 1	*interval 2*
Stimulus	interval 1	p	$1-p$
	interval 2	$1-r$	r

(8)

the parameters α and β can be solved directly from the equations below.

$$\alpha/(\alpha+\beta) = p \qquad (10)$$

$$1/(1+\alpha\beta) = 1 - r. \qquad (11)$$

These lead to the following estimates:

$$\hat{\beta} = \sqrt{\left(\frac{r}{1-r}\right)\left(\frac{1-p}{p}\right)} \qquad (12)$$

and

$$\hat{\alpha} = \sqrt{\left(\frac{r}{1-r}\right)\left(\frac{p}{1-p}\right)}\ . \qquad (13)$$

The statistical significance of a sensory effect can now be assessed by testing (preferably with the binomial test) whether the hit rate deviates sufficiently from the expected response fraction $\beta/(1+\beta)$ or $1/(1+\beta)$, whichever is appropriate; see (5). This is basically a test of whether the sensory effect α is significant.

Final comments

While the estimation method for the parameters α and β follows the procedures given in Luce (1959), it must be noted that the number of free parameters in the matrix is equal to the number of estimated parameters. Least squares optimization is therefore not possible; the theoretical fit will always be perfect. Also, the statistical properties of the estimators are unknown; in quite asymmetric cases there may be a strong covariance between them. Lastly, because of the occurrence of divisions involving potentially small numbers the estimates may in some cases show a lack of robustness.

References

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