

On the accessibility of rotating convection regimes in laboratory experimental studies

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On the Accessibility of Rotating Convection Regimes in Laboratory Experimental Studies

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Abstract

Many geophysical and astrophysical phenomena are driven by massively-turbulent, multiscale fluid dynamics. These fluid systems are often both too remote and too complex to fully grasp without employing forward models. While attempts to directly simulate geophysical systems have made important strides, such models still inhabit modest ranges of the governing parameters that cannot be extrapolated to extreme planetary settings with certainty. An alternate approach is to isolate the fundamental physics in a reduced setting. The canonical problem of rotating Rayleigh-Bénard convection in a plane layer provides such a reduced framework. Laboratory experiments are capable of resolving broad ranges of length and time scales and are thus well-suited for reaching the extreme conditions where asymptotic behaviors distinctly manifest. In this study, we discuss how to optimize laboratory experiments toward testing asymptotically-predicted rotating convection regimes. We also discuss the limitations that arise in designing these experiments. We apply these criteria to several of the most extreme rotating convection setups to date and predict their capabilities. The achievable parameter ranges of these current and upcoming devices demonstrate that laboratory studies likely still remain on the cusp of exploring geophysically-relevant flow behaviors in rotating convection.

I. INTRODUCTION

Turbulent flows underlie many geophysical and astrophysical phenomena in the universe, from the dynamics of the oceans and atmosphere on Earth to the fluid dynamos generating magnetic fields in planets and stars [e.g., 1–4]. These flows are inherently difficult to investigate because their settings are too remote to allow for direct measurements. Thus, the canonical method for examining many such flows is to develop forward models [e.g., 3, 5–10]. Forward models aim to capture the underlying dynamics of geophysical systems in a simplified setting. Two common methods for modeling planetary physics are to directly simulate the governing flow equations using numerical models, or to investigate fluid behaviors in a laboratory setting. While direct simulations more faithfully model the overall geometry and orientation of force vectors in a geophysical system [11, 12], laboratory experiments can approach more extreme, geophysically-relevant conditions [13–18].

Of the many forces involved in geophysical and astrophysical fluid processes, buoyant instabilities and rotational effects are broadly present and often dominant. A reduced problem with great relevance to these processes, then, is plane layer thermal convection under the influence of rotation. This canonical approach takes advantage of an expansive literature of Rayleigh-Bénard convection and rotating convection studies, including theory, direct numerical simulations (DNS), and laboratory experiments [e.g., 19–23]. Some recent numerical models take a unique approach to rotating convection by solving the governing equations in the limit of asymptotically rapid rotation [24–27]. Predictions from theory and from these ‘asymptotically-reduced’ models have established that many of the behavioral regimes which are likely relevant to planetary-scale flows cannot yet be accessed by direct models of geophysical systems [16, 28]. The simpler geometry of the rotating convection problem is better suited for reaching parameter ranges where these regimes are expected to manifest [e.g., 17, 29–31].

The Rayleigh number Ra describes the strength of the buoyancy forcing in convection. It is the ratio between the viscous diffusion and thermal diffusion time scales, τ_ν and τ_κ , and the square of the buoyancy forcing (free-fall) time scale, τ_{ff} :

$$Ra = \frac{\tau_\nu \cdot \tau_\kappa}{\tau_{ff}^2} = \frac{\gamma g \Delta T H^3}{\nu \kappa}, \quad (1)$$

where γ is the coefficient of thermal expansion, g is the gravitational acceleration, ΔT is the adverse superadiabatic temperature gradient, H is the height of the fluid layer, ν is the kinematic viscosity and κ is the thermal diffusivity. The characteristic time scales are defined as $\tau_\nu = H^2/\nu$, $\tau_\kappa = H^2/\kappa$, and $\tau_{ff} = H/U_{ff} = H^{1/2}(\gamma g \Delta T)^{-1/2}$, where the convective free-fall velocity is $U_{ff} = (\gamma g \Delta T H)^{1/2}$.

Convective heat transfer dynamics vary between fluids with different material properties [e.g., 20, 23]. The Prandtl number quantifies the relative strength of thermal and viscous diffusion as the ratio between timescales τ_κ and τ_ν :

$$Pr = \frac{\tau_\kappa}{\tau_\nu} = \frac{\nu}{\kappa}. \quad (2)$$

The Ekman number E describes the influence of rotation, and is given by the ratio between the characteristic rotational time scale, $\tau_\Omega = 1/(2\Omega)$ and the viscous time scale τ_ν :

$$E = \frac{\tau_\Omega}{\tau_\nu} = \frac{\nu}{2\Omega H^2}, \quad (3)$$

where Ω is the angular rotation rate of the body. The Rossby number Ro describes the influence of rotation by comparing the inertial time scale $\tau_i = H/U$ to the rotational time scale τ_Ω :

$$Ro = \frac{\tau_\Omega}{\tau_i} = \frac{U}{2\Omega H}. \quad (4)$$

where U is the characteristic flow velocity. In convectively driven flows, the buoyant free-fall time scale serves as a lower bound on the inertial time scale, where all the heating power goes toward fluid motions [e.g., 21, 32, 33]. Assuming $\tau_i \sim \tau_{ff}$, we can then define the convective Rossby number to be:

$$Ro_C = \frac{\tau_\Omega}{\tau_{ff}} = \left(\frac{\gamma g \Delta T}{(2\Omega)^2 H} \right)^{1/2} = \left(\frac{Ra E^2}{Pr} \right)^{1/2}. \quad (5)$$

In geophysical settings these parameters take on extreme values. For example, in the Earth's outer core, estimates give $Ra \sim 10^{20} - 10^{30}$, $E \sim 10^{-15}$ and $Ro \sim 10^{-6}$ [34–36]. A massive separation exists between the viscous and inertial time scales, as well as between the inertial and rotational time scales ($\tau_\nu \gg \tau_i \gg \tau_\Omega$). The majority of direct simulations of the outer core, in contrast, are confined to ranges of $Ra \lesssim 10^7$, $E \gtrsim 10^{-6}$, and $Ro \gtrsim 10^{-2}$ due to

numerical resolution constraints [e.g., 37, 38]. We therefore posit that many of the behaviors expected from theory and asymptotic models, which are likely relevant at planetary scales, do not yet manifest in these models.

While numerical models need to resolve the scale separation between different behaviors to simulate geophysically meaningful flows, laboratory experiments inherently ‘resolve’ all of the physics, even when behaviors occur that are too small to detect [4]. The laboratory approach is thus uniquely well-suited for investigating geophysical-style rotating convection at extreme values of the governing parameters. In this study, we will show that modern experiments are capable of achieving parameter ranges where asymptotically-predicted flow behaviors potentially appear, allowing us to elucidate their relevance to planetary flows. We will discuss the constraints that factor into optimizing the laboratory approach to rotating convection.

In Section II, we describe the behavioral regimes found in theoretical studies of rotating convection as well as the behaviors observed so far in laboratory experiments and DNS. In Section III, we outline the design considerations for laboratory experiments to access and characterize these regimes. In Section IV, we discuss the rotational and heat transfer constraints needed for ensuring that the physics remains within the bounds of classical Boussinesq rotating convection. To contextualize these design considerations and theoretical constraints, we will detail the achievable parameter ranges in several extreme rotating convection experiments, shown in Figure 1. Our findings and predictions are summarized in Section V.

II. FLOW REGIMES

Results from laboratory experiments, direct numerical simulations, and asymptotically-reduced studies indicate that a variety of rotating convection flow regimes occupy the range between rotationally-controlled and buoyancy-controlled convection [28]. To analyze the capabilities of a given experiment, we first outline these regimes and the parameter ranges over which they are expected to arise. One method for categorizing flow behavior is to track the strength of the nondimensional heat transfer: different behaviors likely lead to different modes of heat transport, and thus to differences in the scaling properties [e.g., 19, 21–23, 43, 44].

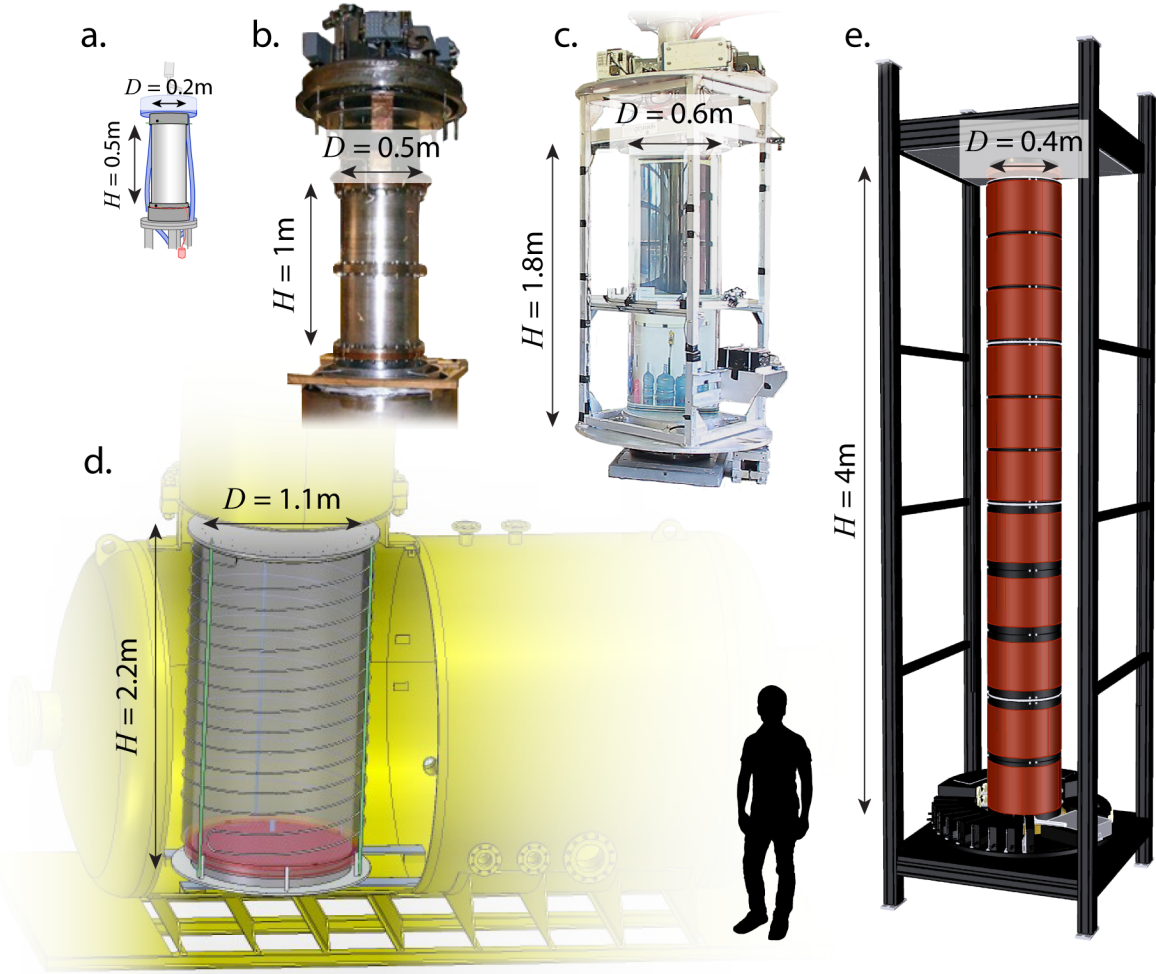


FIG. 1. Images of several extreme rotating convection setups. a) ‘RoMag’ at UCLA (liquid gallium, $Pr \approx 0.025$) [39]. b) Trieste experiment at ICTP (cryogenic liquid He, $Pr \approx 0.7$) [13, 40]. c) ‘NoMag’ at UCLA (water, $Pr \approx 4 - 7$). d) ‘U-Boot’ at the Max Planck Institute for Dynamics and Self-Organization (SF_6 , N_2 , He gas, $Pr \approx 0.8$) [41, 42]. e) ‘TROCONVEX’ at Eindhoven University of Technology (water, $Pr \approx 2 - 7$).

We parametrize the heat transfer using the Nusselt number:

$$Nu = \frac{\text{Total heat transfer}}{\text{Conductive heat transfer}} = \frac{qH}{\kappa\rho C_p \Delta T}, \quad (6)$$

where C_p is the specific heat capacity and q is the heat flux per unit area. The higher the Nusselt number, the more efficiently heat is being transported heat across the layer through fluid motions.

The governing parameters tend to be related by power law scalings [e.g., 22, 45]. Here,

we assume that the Nusselt number scales as:

$$Nu \sim Ra^\alpha E^\beta Pr^\delta, \quad (7)$$

where α , β and δ are constants in a given scaling regime. Behavioral transitions are associated with changes in the mode of heat transfer and therefore with changes in the scaling exponents between the governing parameters [e.g., 17, 28, 31, 40, 46].

Figure 2 is a schematic demonstrating how the Nusselt number scales with the Rayleigh number over multiple rotating convection regimes, assuming fixed E and Pr values. The ‘columnar’, ‘plumes’ and ‘geostrophic turbulence’ regimes are derived from the asymptotic results of Julien et al. [28] and Nieves et al. [47] while the ‘nonrotating heat transfer’ regime is established in classical experiments and theory [e.g., 19, 43–45]. We theorize that the ‘unbalanced boundary layer’ exists between the ‘geostrophic turbulence’ and ‘nonrotating heat transfer’ regimes. Transitions between different regimes are distinguished as differently-dashed vertical lines in Figure 2.

Since Julien et al. [28] find that the columnar convection regime only appears for $Pr > 3$, $Pr = 3$ is used as an approximate threshold between ‘large’ and ‘small’ Prandtl number fluids. Figure 2a shows the predicted regimes for ‘large’ Pr (> 3) while 2b shows the predicted regimes for ‘small’ Pr ($\lesssim 3$). In both cases, the Nu – Ra scaling slope α flattens as the convective forcing increases in strength relative to rotational effects. This predicted variation in α functions as a simple diagnostic for detecting transitions in the flow morphology. In contrast to the clearly-defined asymptotic regimes in Figure 2, though, most rotating convection studies find a smooth transition in α between the endpoints of ‘rotationally-dominated’ convection and ‘buoyancy-dominated’ convection [e.g., 17, 20, 48].

To understand existing rotating convection results in the context of the theoretically-predicted regimes, we first describe the flow physics of each regime in Section II A. Then, in Section II B, we present the heat transfer scalings (α values) that have been inferred from rotating convection studies. In Section II C, we discuss the possible regime transition locations (Ra_T values) based on theoretical arguments and laboratory and numerical results. These transition values are compiled in Table I. Convection behaviors in $Pr \ll 1$ fluids, such as liquid metals, differ greatly from the $Pr \gtrsim 0.5$ cases we address here, and are therefore discussed separately in the Appendix.

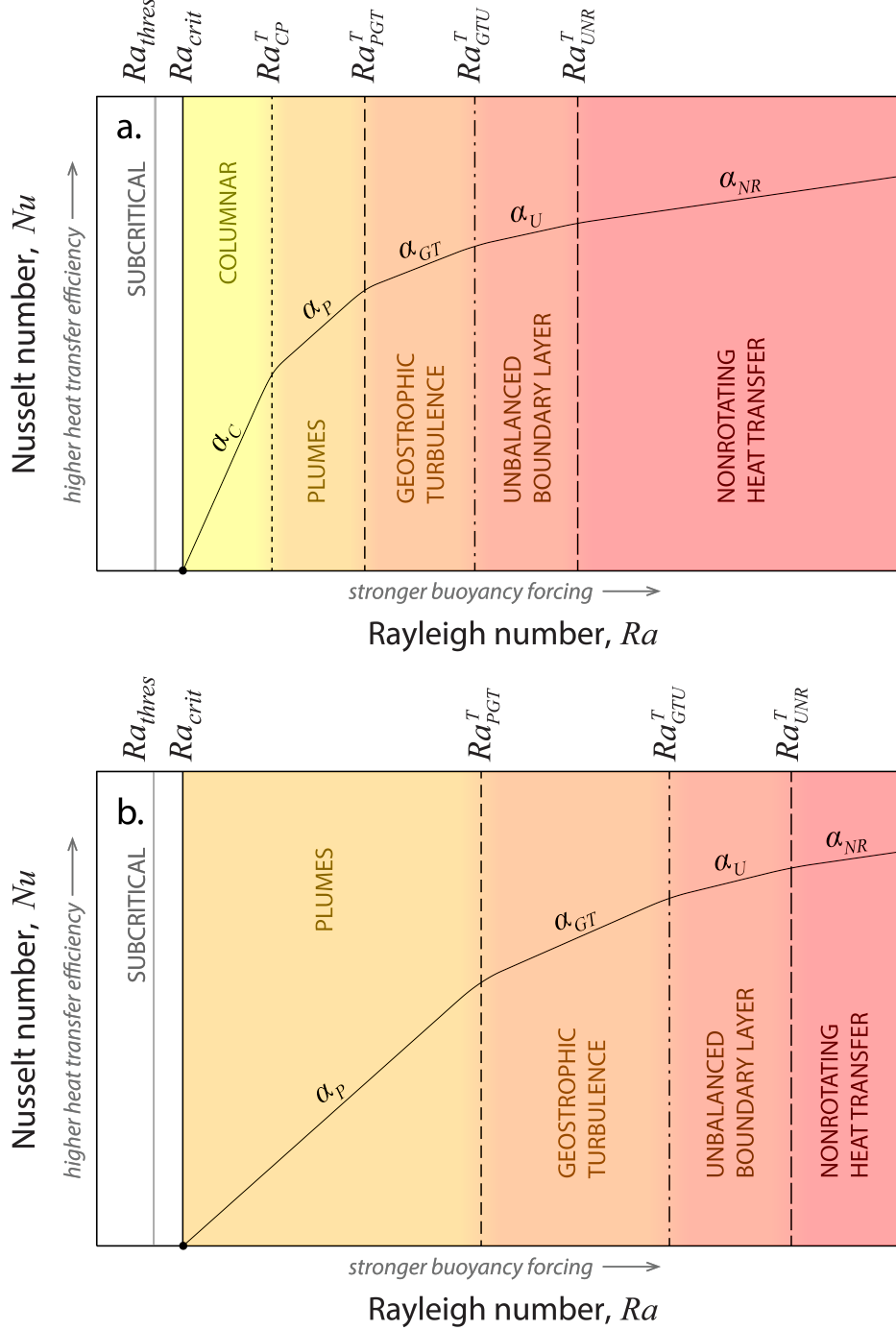


FIG. 2. (Color online) Schematic showing the distribution of rotating convection regimes in terms of Nusselt number (Nu) versus Rayleigh number (Ra) for a fixed Ekman number (E) and a) $Pr > 3$ and b) $Pr \lesssim 3$. The vertical lines indicate transition Rayleigh values: Ra_{crit} denotes convective onset, Ra_{CP}^T denotes the transition between columnar-style convection and plumes, Ra_{PGT}^T between plumes and geostrophic turbulence, Ra_{GTU}^T between geostrophic turbulence and unbalanced boundary layers, and Ra_{UNR}^T to nonrotating-style convection. Though the transitions are delimited by lines, each likely occurs gradually over a range of Ra values. Their locations are not yet well-determined, and Table I and Figure 8 list various existing predictions. For $Pr \lesssim 3$, steady columnar convection does not occur [e.g., 28, 46].

A. Regime predictions

In a finite container, convection first onsets via ‘wall modes’ - drifting waves attached to the sidewalls [49, 50]. At the critical Rayleigh number for stationary onset Ra_{crit} , shown in Figure 2 as a solid vertical black line, the bulk begins to convect in the form of overturning cells. For $Pr \gtrsim 0.68$ and $E \lesssim 10^{-3}$ [51],

$$Ra_{crit} = 8.7E^{-4/3}. \quad (8)$$

The cellular regime persists until $Ra/Ra_{crit} \sim 2$ [52], and is not marked separately on Figure 2, as the heat transfer scaling does not change appreciably between this regime and the next [28].

For $Pr > 3$, as Ra/Ra_{crit} increases, the ‘columnar’ regime manifests [24, 53]. The bulk flow in this regime is dominated by quasi-steady convective Taylor columns, created by synchronization of the plumes emitting from the top and bottom boundary layers, and consisting of vortex cores surrounded by a shield of oppositely-signed vorticity [28]. In both the cellular and convective Taylor column regimes, the geostrophic balance between the Coriolis force and the pressure is perturbed by viscous effects. This leads to narrow structures with a horizontal length scale of [e.g., 54]:

$$\ell = cE^{1/3}H = c \left(\frac{\nu H}{2\Omega} \right)^{1/3}, \quad (9)$$

where c is a prefactor. While Chandrasekhar [51] derives an asymptotic value of $c = 4.8$ for the infinite plane layer, we use $c = 2.4$ instead to account for the effects of Ekman pumping at $E > 10^{-7}$ [55]. For $Pr \lesssim 3$ the steady columnar regime is not expected to manifest [e.g., 28, 39].

At $Pr > 3$ and Rayleigh numbers in the vicinity of Ra_{CP}^T (marked by the short-dashed lines in Figure 2), the shields surrounding the vortex cores in the columnar regime deteriorate and the flow enters the ‘plumes’ regime. For $Pr \lesssim 3$, this regime develops directly out of cellular convection. Here, the rising and falling plumes ejected from the boundaries are exposed to strong vortex-vortex interactions due to the deterioration of their shields, preventing them from synchronizing. This leads to structures which share the same horizontal length scale

as columns and cells but which do not extend across the entire fluid layer [28]. The loss of direct heat conduits connecting the boundaries and increased lateral mixing results in enhanced lateral heat transport compared to the columnar regime [21, 24, 28]. Thus, a given increase in Ra boosts the heat transfer less efficiently in the plume regime than in the columnar regime, implying $\alpha_P < \alpha_C$.

Around Ra_{PGT}^T , shown as medium-dashed lines in Figure 2, the ‘geostrophic turbulence’ regime manifests. Here, the plumes become confined close to the thermal boundary layers and the bulk of the fluid becomes dominated by strong mixing and vortex-vortex interactions [28]. The increased lateral mixing causes Nu to scale even more weakly with Ra , such that $\alpha_{GT} < \alpha_P$. Though small-scale turbulence is present, geostrophy still persists as the primary force balance and still imparts an effective vertical stiffness to the flow field. Geostrophic turbulence now acts as the bottleneck to heat transport and throttles the heat flux to the more efficient laminar boundary layers.

Within the geostrophic turbulence regime, recent DNS and asymptotically-reduced studies have observed an inverse cascade of energy from small-scale turbulence into coherent, axially-aligned, barotropic ‘large-scale vortices’ (LSVs) [27–31, 46, 56]. These LSVs characterize the flow dynamics on the domain scale while geostrophic turbulent structures persist on the small scales [28]. LSVs have, thus far, only appeared in $Pr \leq 1$ simulations with free-slip boundary conditions. Laboratory experiments will help to elucidate whether LSVs can develop under non-slip boundary conditions in more extreme parameter ranges.

Around Ra_{GTU}^T , shown as dot-dashed lines in Figure 2, geostrophy in the thermal boundary layers breaks down, leading to the theorized ‘unbalanced boundary layer’ regime. This breakdown of rotational control causes the boundary layers to become less efficient in transporting heat and more restrictive in heat transfer compared to the bulk, indicating that $\alpha_U < \alpha_{GT}$. The flow morphology here remains virtually unexplored and should be the subject of future laboratory, DNS and asymptotic studies.

Finally, around Rayleigh number Ra_{UNR}^T , shown as long-dashed lines in Figure 2, the ‘nonrotating-style heat transfer’ regime is established as the flow field becomes effectively insensitive to Coriolis forces. For large enough Ra , the bulk of the fluid becomes nearly isothermal and the temperature gradients are almost entirely confined to the boundary layers.

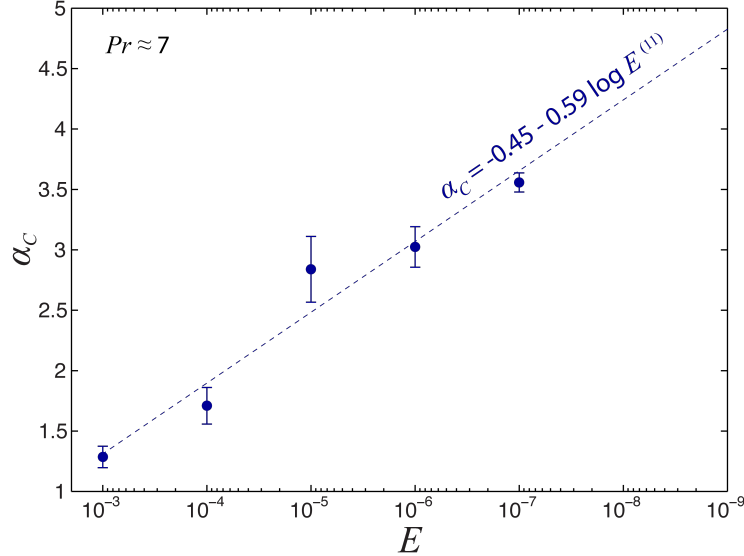


FIG. 3. Slope of Nusselt number, Nu , versus Rayleigh number, Ra , in the columnar regime plotted versus Ekman number, E , for $Pr \simeq 7$ laboratory-numerical rotating convection data from Cheng et al. (2015). A best-fit slope between α_c and E is estimated for $E \leq 10^{-3}$. Above $E = 10^{-3}$, the range of Nu - Ra space inhabited by columnar convection is too small for any meaningful slope to manifest.

B. Heat transfer scaling (α) predictions

In the columnar regime, the heat transfer follows steep $Nu \sim Ra^{\alpha_c}$ trends, with $\alpha_c \gtrsim 3$ for $E \lesssim 10^{-6}$ [17, 46, 48]. The steepness of this trend is due to Ekman pumping effects, which greatly boost the heat transfer for a given thermal forcing [31, 46]. Julien et al. [26] theorize that Ekman pumping effects kick in above a threshold Rayleigh number:

$$Ra_{thres} \sim E^{-13/9}, \quad (10)$$

plotted as the solid vertical grey line in Figure 2. This scaling implies that Ekman pumping should affect the flow immediately upon the onset of bulk convection for $E \gtrsim 10^{-9}$.

Figure 3 demonstrates that the α_c scaling exponent continues to steepen as E decreases in $Pr \simeq 7$ laboratory and numerical rotating convection data from [17]. The best-fit trend is given by:

$$\alpha_c = -0.45 - 0.59 \log E. \quad (11)$$

In asymptotically-reduced simulations with parametrized Ekman pumping, Plumley et al. [27] find steep α_c trends at $E = 10^{-7}$, in agreement with laboratory and numerical re-

sults. As experiments become capable of reaching more extreme E ranges, they will confirm whether the Nu - Ra scaling law in the columnar regime continues to steepen as projected.

In the geostrophic turbulence regime, Julien et al. [25] argue that the heat transport law should be independent of dissipation and predict that $Nu \sim Ra^{3/2} E^2 Pr^{-1/2}$ ($\alpha_{GT} = 3/2$) [cf. 57]. This is corroborated by Gastine et al. [12], whose $Pr = 1$ spherical shell rotating convection data follow a $Nu \sim Ra^{3/2}$ scaling for $Ra > 0.4E^{-8/5}$.

In the nonrotating-style heat transfer regime, a large variety of Ra and Pr dependent predictions exist for α_{NR} [22, 23]. At high enough Ra , the temperature gradient may be entirely confined to the boundary layers. The heat transfer then becomes independent of the total height, leading to $\alpha_{NR} = 1/3$ [19]. Water experiments have found evidence for this regime at $Ra \gtrsim 10^{10}$ [17, 58]. At even higher Ra values, Kraichnan [43] and Spiegel [45] predict that the boundary layers should become fully turbulent and the heat transfer should become independent of the diffusivities, leading to $\alpha_{NR} = 1/2$ (with logarithmic corrections). While some experiments find an increase in the heat transfer scaling at high Ra [59, 60], this result is not universally supported [13].

The majority of studies at $Ra < 10^{10}$ and $Pr \gtrsim 1$ instead find $\alpha_{NR} \approx 2/7$ [20, 61, 62], theorized to be a nonasymptotic modification to the $\alpha_{NR} = 1/3$ scaling [e.g., 44, 63]. However, estimates of Ra for planetary and stellar systems place them well beyond the expected parameter range for which the $\alpha_{NR} = 2/7$ scaling remains valid [e.g., 36].

In summary, a wealth of different heat transfer scalings exist for nonrotating heat transfer. The relevance of each scaling to asymptotic settings, as well as their applicability to geophysical systems, remain open questions.

C. Transition Rayleigh number (Ra_T) predictions

An empirical prediction for the columnar-to-plume transition is given by $Ra_{CP}^T \sim 5.4E^{-1.47}$, derived from laboratory and numerical $E = 10^{-4}$ to $E = 3 \times 10^{-8}$ rotating convection data in [17]. It was determined by finding the intersection between the best-fit trend for the rotationally-controlled, steep Nu - Ra scaling cases and the best-fit trend for nonrotating convection cases. Visualizations of the flow field in [17] and thermal measurements in [64] indicate that the breakdown of columnar structures into plume-like structures coincides with this intersection. For the Ra - E ranges explored, the transition corresponds

TABLE I. Table showing various predictions for the transitions between different flow regimes (shown schematically in Figures 2 and 4). In the ‘Type’ column, Ra_{CP}^T refers to the breakdown of well-organized convective columns into plumes, Ra_{PGT}^T refers to the breakdown of plumes into geostrophic turbulence, Ra_{GTU}^T refers to the local loss of rotational influence leading to unbalanced boundary layers and Ra_{UNR}^T refers to the global loss of rotational influence leading to nonrotating-style convection. The ‘ Pr ’ column refers to the approximate Prandtl number for which the transition is observed or is predicted to apply. The ‘Reference’ column gives the study from which each prediction originated. The ‘Figure abbreviation’ column gives the label assigned to each transition in Figures 8 and 9.

Transition prediction	Type	Pr	Reference	Figure abbr.
$Ra \sim 5.4E^{-1.47}$	Ra_{CP}^T	≈ 7	Cheng et al. [17]	$Ra_{T,Ch'15}$
$Ra/Ra_{crit} \sim 3$	Ra_{PGT}^T	< 3	Julien et al. [25]	$Ra/Ra_{crit} = 3$
$Ra \sim E^{-8/5} Pr^{3/5}$	Ra_{GTU}^T	any*	Julien et al. [25]	$Ra_{T,Ju'12}$
$Ra \sim 1.3E^{-1.65}$	Ra_{GTU}^T	≈ 6	Ecke and Niemela [40]	$Ra_{T,Ec'14(1)}$
$Ra \sim 0.25E^{-1.8}$	Ra_{GTU}^T	≈ 0.7	Ecke and Niemela [40]	$Ra_{T,Ec'14(2)}$
$Ro_C \sim 0.35$	Ra_{UNR}^T	≈ 0.7	Ecke and Niemela [40]	$Ro_C = 0.35$
$Ra \sim 100E^{-12/7}$	Ra_{UNR}^T	1	Gastine et al. [12]	$Ra_{T,Ga'16}$
$Ro_C \sim 1$	Ra_{UNR}^T	any	Gilman [32]	$Ro_C = 1$
$Ro_C \sim 2$	Ra_{UNR}^T	≈ 6	Ecke and Niemela [40]	$Ro_C = 2$

*While [25] did not reach the geostrophic turbulence regime for any $Pr > 3$ cases, the asymptotic argument for this transition is Pr -independent.

closely to the $Ra \sim 10E^{-3/2}$ argument from [48], where the $-3/2$ exponent describes the Ra – E relationship for which the thickness of the Ekman boundary layer and the thickness of the thermal boundary layer become comparable.

Though no specific predictions exist for Ra_{PGT}^T , in the asymptotically-reduced cases of [25], the heat transfer diverges from the geostrophic turbulence scaling $\alpha = 3/2$ when $Ra \lesssim 3Ra_{crit}$. Ecke and Niemela [40] use this lower bound on the GT-scaling heat transfer as a broad estimate for Ra_{PGT}^T in $Pr > 3$ fluids.

Julien et al. [25] predict that $Ra_{GTU}^T \sim E^{-8/5} Pr^{3/5}$, where geostrophic balance in the thermal boundary layer breaks down in the asymptotic equations. Ecke and Niemela [40] find separate predictions for Ra_{GTU}^T depending on Pr : for $Pr = 0.7$ they argue that $Ra_{GTU}^T \sim 1.3E^{-1.65}$ while for $Pr = 6$ they argue that $Ra_{GTU}^T \sim 0.25E^{-1.8}$. These empirical estimates assume that transitions take the form $Ra_T \sim E^\chi$ and are derived by determining the best-fit value of χ .

Gilman [32] predicts that the transition to nonrotating-style convection, Ra_{UNR}^T , occurs when the system-scale buoyancy and Coriolis time scales become similar, or when

$Ro_C \sim 1 \Rightarrow Ra \sim E^{-2}Pr$. This prediction has been found to adequately describe the breakdown of large-scale zonal flows in spherical shell rotating convection simulations with free-slip boundary conditions [65, 66]. Gastine et al. [12] empirically estimate $Ra_{UNR}^T = 100E^{-12/7}$ based on their spherical shell rotating convection simulations with non-slip boundary conditions. In the vicinity of this Ra value, they find that all measurable quantities become indistinguishable from the nonrotating cases. Finally, Ecke and Niemela [40] suggest that for $Pr \approx 6$, the transition to nonrotating-style convection occurs at $Ro_C \sim 2$ while for $Pr \approx 0.7$, this transition happens at $Ro_C \sim 0.35$.

These transition arguments have been compiled in Table I. Notably, no predictions have been made for Ra_{CTU}^T and Ra_{UNR}^T values. Larger datasets of more extreme rotating convection cases are needed, both to establish the validity of existing predictions and to develop new predictions for presently-unconstrained transitions.

III. EXPERIMENTAL DESIGN

Above, we have catalogued the expected rotating convection regimes in the limit of asymptotically rapid rotation. But while detailed asymptotic and theoretical analyses exist, present-day laboratory and DNS studies are only on the cusp of being able to capture the asymptotic physics [cf. 29, 30, 46]. To further our understanding of extreme rotating convection, it is essential to develop experiments that enable optimal comparison with theory by covering broad ranges of Ra/Ra_{crit} at extremely low values of E . Here, we will discuss the physical considerations for designing such experiments.

Figure 4 is a schematic showing the accessible E and Ra ranges in a rotating convection setup with fixed height and width. Assuming the fluid properties are also fixed, the bounds on E are determined solely by the minimum and maximum rotation rates of the system, Ω , and the bounds on Ra are determined solely by the minimum and maximum imposed temperature difference, ΔT .

A. Maximizing parameter coverage

Experimental bounds on Ω and ΔT are initially determined by the mechanical limitations of the setup and by the physical properties of the working fluid. For many laboratory

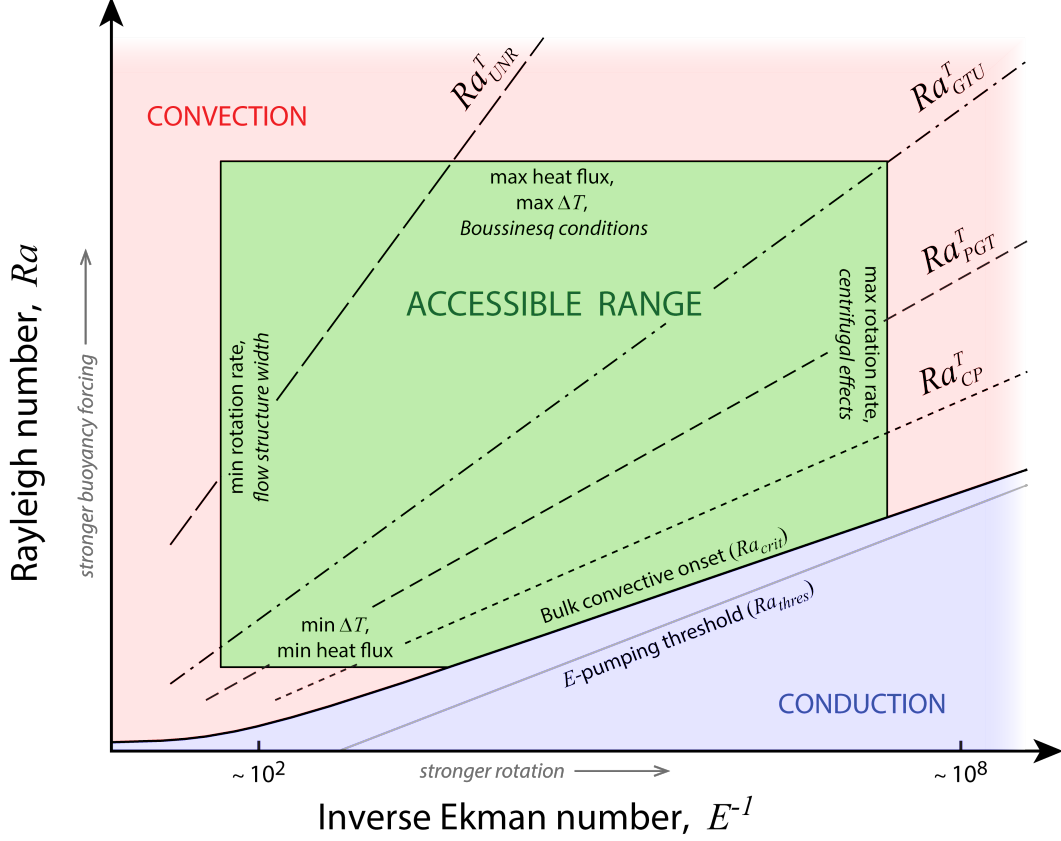


FIG. 4. (Color online) Schematic of accessible Ekman number, E , and Rayleigh number, Ra ranges for a given rotating convection experiment. Assuming the tank size and fluid properties are fixed, the absolute bounds on E are determined by the minimum and maximum rotation rates, Ω , and the absolute bounds on Ra are determined by the minimum and maximum temperature difference, ΔT . Minimizing sidewall effects and minimizing centrifugation effects require separate lower and upper bounds on Ω . Maintaining Boussinesq conditions requires a separate upper bound on ΔT . The onset of bulk convection, Ra_{crit} is indicated by a solid black line. The threshold for Ekman pumping effects on the heat transport is indicated by a solid grey line. Different flow regimes are separated by transition Rayleigh values Ra_{CP}^T , Ra_{PGT}^T , Ra_{GTU}^T and Ra_{UNR}^T (see Table I). These transitions and the regimes they separate are difficult to distinguish at moderate-to-high E values but become distinct as E decreases.

experiments, the temperature difference is imposed by applying a heat flux q to the bottom boundary [e.g., 17, 40, 41]. Assuming fixed fluid properties and tank size, the associated control parameter is the flux Rayleigh number:

$$Ra_F = Nu \cdot Ra = \frac{\gamma g H^4 q}{\nu \kappa k}, \quad (12)$$

where $k = \rho C_p \kappa$ is the thermal conductivity of the fluid. The dependence of Ra on q is therefore variable – for a shallow Nu – Ra scaling such as $\alpha_{NR} \simeq 1/3$, $Ra \propto q^{3/4}$, while for

a steep Nu - Ra scaling such as $\alpha_C \simeq 3$, $Ra \propto q^{1/4}$. In either case, varying q is relatively inefficient for accessing broad ranges of Ra .

In contrast to the linear dependence of ΔT on Ra and Ω on E , eqs. (1) and (3) demonstrate that Ra varies with H^3 and E varies with H^{-2} . Changing the height of the experiment is far more effective for reaching a broad range of Ra and E values than varying Ω or ΔT , and building a larger experiment is the most effective way to reach extreme Ra and E values.

Increasing the height, however, simultaneously hinders the ability to access low values of Ra/Ra_{crit} . From (1) and (8), the supercriticality is given by:

$$\frac{Ra}{Ra_{crit}} = \frac{Ra}{8.7E^{-4/3}} = \frac{\gamma g \nu^{1/3} \Delta T H^{1/3}}{21.9\kappa \Omega^{4/3}}. \quad (13)$$

Since $Ra/Ra_{crit} \propto H^{1/3}$, experiments in higher tanks will be less able to reach low supercriticalities, and thus less able to access near-onset flow regimes. To overcome this limitation, the RoMag, NoMag, U-Boot, and TROCONVEX experiments (Figure 1a, c, d and e) use interchangeable tanks of various heights. Table III in the Appendix contains more information about these experiments.

The U-Boot and Trieste experiments (Figure 1b and d) can access high Ra values by taking advantage of the large ratio between thermal expansivity and the thermal and viscous diffusivities ($\gamma/\nu\kappa$) in cryogenic helium and other compressed gases [13, 41, 42, 67]. For example, at a typical operating temperature for cryogenic helium of 4.7 K and pressure of 0.12 bar, $\gamma/\nu\kappa \approx 10^{11} \text{ s}^2\text{m}^{-4}\text{K}^{-1}$. At a typical operating temperature of 25 °C for water, $\gamma/\nu\kappa \approx 2 \times 10^9 \text{ s}^2\text{m}^{-4}\text{K}^{-1}$, a factor of 50 lower.

Furthermore, the ability to vary the pressure allows for a greater Ra -range in gas experiments. From (1), we see that for an ideal gas:

$$Ra = \frac{\gamma g \Delta T H^3 \rho^2 C_P}{k\eta} \propto \rho^2 \propto P^2 M^2, \quad (14)$$

where η is the dynamic viscosity, P is the pressure and M is the molecular weight [41]. The quadratic relation between Ra and pressure is especially useful since the pressure can be varied over several decades in these devices. The U-Boot device can also be filled with different gases of varying molecular weight in order to reach broader ranges of Ra and lower values of Ra/Ra_{crit} (see Table III).

B. Detecting flow regimes

In addition to accessing multiple flow regimes, experiments must accommodate the necessary diagnostics for characterizing and distinguishing them. In Figure 2 we express the regimes in terms of the heat transfer parameters Ra and Nu , for which only the top and bottom boundary temperatures need to be measured. However, heat transfer scalings alone may not suffice for determining the underlying physics: for example, transitions may not manifest clearly in the heat transfer, such as between cellular convection and convective Taylor columns [e.g., 16, 28]. Transitions may occur over a broad range of Ra values, making it difficult to distinguish between regimes even when they contain different heat transfer scalings. Measurements that directly assess the flow field are also necessary.

Beyond determining the overall heat transfer parameters, thermal measurements can impart knowledge of the flow field: temperature time series allow for statistical analyses of the flow [e.g., 47, 68, 69], and correlations between temperature time series at different locations can delineate the flow morphology [64, 70].

Direct visualizations of the flow can qualitatively reveal flow structures, useful for contextualizing the data from other diagnostics [e.g., 17, 47, 71–73]. Point velocity measurement techniques such as Laser Doppler Velocimetry (LDV) can produce characteristic velocities, flow statistics, velocity profiles and scaling laws [e.g., 70, 74, 75]. Techniques such as Particle Image Velocimetry (PIV) provide a snapshot of the velocity field on a plane, allowing for analyses such as spatial statistics measurements [e.g., 76–79]. Direct visualizations and velocity field measurements are particularly suited to confirming the presence of large-scale vortices, should they arise in laboratory experiments.

Visualizations and velocity measurement techniques often require transparent sidewalls and seeding of the fluid with neutrally-buoyant tracers. They may also require that the sensors, cameras and lighting components all be in corotation with the fluid layer. For opaque fluids such as liquid metals, or fluids that are difficult to seed with particles such as gases, ultrasonic doppler techniques can be used to estimate fluid velocities instead [e.g., 80].

Some features are difficult to detect in laboratory settings regardless of the diagnostics used. For example, in a rotating convection case, the thickness of the viscous boundary layers scales as $\delta_\nu \sim E^{1/2}H$. Even in the 4 m high tank of water in TROCONVEX, these

TABLE II. Table cataloging upper and lower bounds on the rotation rate (Ω) and Ekman number (E), and the upper bounds on the temperature gradient (ΔT) and Rayleigh number (Ra), for cylindrical rotating convection experiments. The constant prefactor $c = 2.4$. The constraints on the flow structure width ratio (m) and the Froude number (Fr) are described in (15) and (16), respectively.

Condition	Dimensional constraint	Nondimensional constraint
$m \geq 10$	$\Omega_{\min} = \frac{500c^3\nu H}{D^3}$	$E_{\max} = \left(\frac{D}{10cH}\right)^3$
$Fr < 0.1$	$\Omega_{\max} = \left(\frac{0.2g}{D}\right)^{1/2}$	$E_{\min} = \left(\frac{1.25\nu^2 D}{H^4 g}\right)^{1/2}$
$\gamma\Delta T < 0.1$	$\Delta T_{\max} = \frac{0.1}{\gamma}$	$Ra_{\max} = \frac{0.1\alpha g H^3}{\gamma\nu\kappa}$

layers are only ~ 0.4 mm thick for $E = 10^{-8}$. Comparison with high-resolution direct numerical simulations may be the only feasible way to diagnose the behaviors in such thin boundary layers.

IV. EXPERIMENTAL CONSTRAINTS

Along with maximizing the parameter coverage and incorporating the appropriate diagnostics, experimental designs must simultaneously ensure that the fluid physics remains consistent with the fundamental rotating Rayleigh-Bénard convection problem. This imposes a separate set of limitations on the heat transfer and rotation rate, indicated in italics in Figure 4. We address the rotational constraints in Section IV A and the heat transfer constraints in Section IV B. These arguments are compiled in Table II. For further context, we apply the resulting Ra and E limitations to the experimental devices shown in Figure 1.

A. Rotational constraints

In an experimental setup, the presence of sidewalls can affect the physics appreciably. In contrast, in many low- E numerical simulations, the horizontal conditions are doubly-periodic with effectively no walls. Limiting sidewall effects in experiments is therefore important for ensuring valid comparisons with DNS. To this end, we implement the criterion that a large number of flow structures must fit horizontally across the tank.

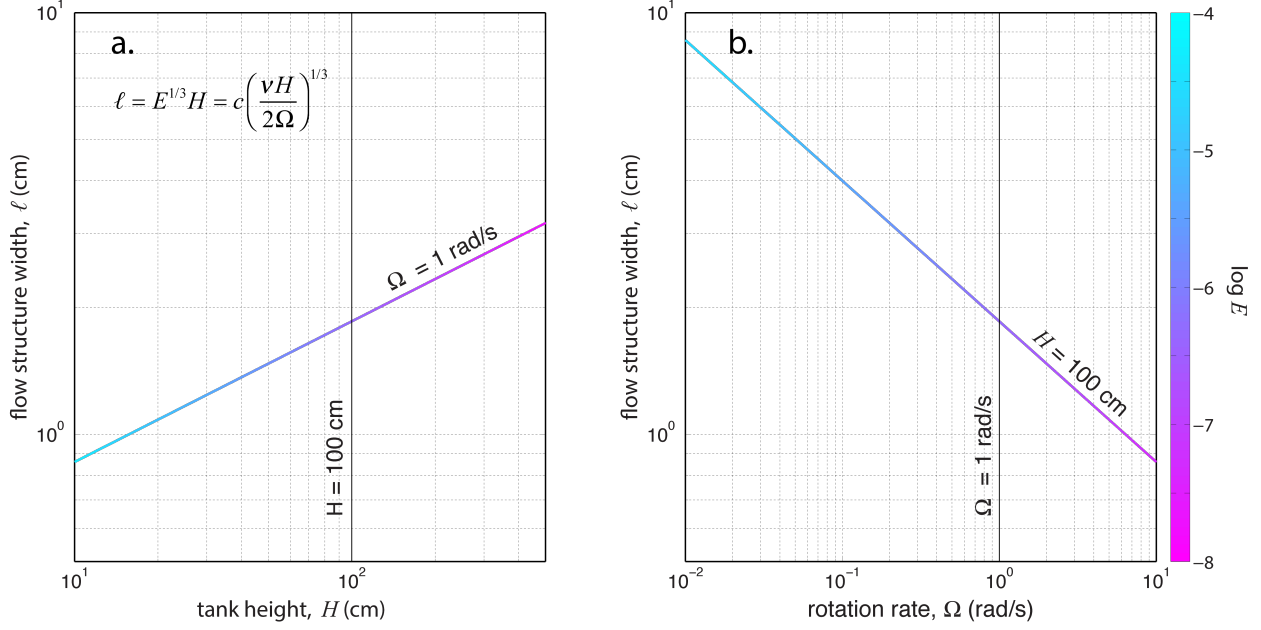


FIG. 5. (Color online) Flow structure width plotted versus: a) tank height H at a fixed rotation rate of $\Omega = 1$ rad/s, and b) rotation rate Ω at a fixed tank height of $H = 1$ m. The color scale shows the Ekman number, E . The flow structure width, ℓ , is given by the relation shown in panel a, with prefactor $c = 2.4$.

We define the flow structure width ratio m as:

$$m = D/\ell = c^{-1} E^{-1/3} \Gamma = c^{-1} \left(\frac{2\Omega D^3}{\nu H} \right)^{1/3}, \quad (15)$$

where D is the diameter of the tank and Γ is the aspect ratio D/H of the experiment. We contend that for $m \geq 10$, sidewall effects do not dominate the bulk flows in a given experiment. This choice is somewhat arbitrary: while the thickness of the sidewall boundary layers scales as $E^{-1/3}$ [e.g., 81, 82], the depth of sidewall effects on the bulk flow at low E is not well-known. However, numerical simulations often employ periodicity lengths of $D \simeq 10\ell$ [28, 31, 46].

Figure 5 demonstrates how the flow structure width scales with the height and rotation rate of an experiment based on (9). In Figure 5a, the flow structure width ℓ is plotted versus tank height H at a fixed rotation rate of $\Omega = 1$ rad/s. The color scale demonstrates that, while increasing the height of the tank lowers the Ekman number, it also increases the width of flow structures based on (9). If the height of the tank is increased by a factor of x , then the diameter must increase by a factor of $x^{1/3}$ to retain the same upper bound on E .

In Figure 5b, the flow structure width ℓ is instead plotted versus rotation rate Ω at a

fixed tank height of $H = 1$ m. As the rotation rate is increased, both flow structure width and E decrease.

Centrifugal effects contribute an upper bound on Ω , and, thus, a lower bound on E . Centrifugation is parametrized via the Froude number [83–85]:

$$Fr = \frac{\text{centrifugation}}{\text{gravity}} = \frac{\Omega^2 D}{2g}. \quad (16)$$

In the case of high Fr , the centrifugal acceleration becomes significant relative to gravitational acceleration, causing denser parcels of fluid to travel outward radially. This leads to circulation patterns that are not found in the canonical rotating convection problem [e.g., 84, 86]. To avoid the potential dynamical effects of centrifugation, we assign an upper limit of $Fr < 0.1$. Note that it is not entirely clear when Froude number effects actually become significant: different studies have found different minimum Fr values at which centrifugation first alters the flow [cf. 86, 87].

In Figures 6 and 7, we compare the mechanical limitations to the limitations imposed by the $Fr < 0.1$, $m \geq 10$ constraints for each of the experimental setups shown in Figure 1. Of these devices, TROCONVEX can access the lowest Ekman number at $\approx 5 \times 10^{-9}$ due to its 4 meter high tallest tank. However, the device also has the thinnest aspect ratio at $\Gamma = D/H = 1/10$. For a given experiment, the accessible E range based on (15) and (16) is:

$$\frac{E_{\max}}{E_{\min}} = \frac{\Omega_{\max}}{\Omega_{\min}} = \frac{(8gFr_{\max})^{1/2} D^{5/2}}{c^3 \nu m_{\min}^3 H}. \quad (17)$$

Thus, the large height and small diameter on the highest TROCONVEX tank cause its accessible E range to be relatively small ($E_{\max}/E_{\min} = 15$), while the wide diameter of the U-Boot tank ($D = 1.1$ m, $\Gamma = 1/2$) causes its accessible E range to be relatively large ($E_{\max}/E_{\min} = 780$).

B. Heat transfer constraints

Apart from the physical capabilities of the experiment, the maximum heat transfer is also restricted by the dependence of the fluid properties on the temperature. In the Boussinesq approximation, the material properties - ρ , k , γ , ν and κ - do not change appreciably with temperature. A flow is considered to follow Boussinesq theory when the density difference

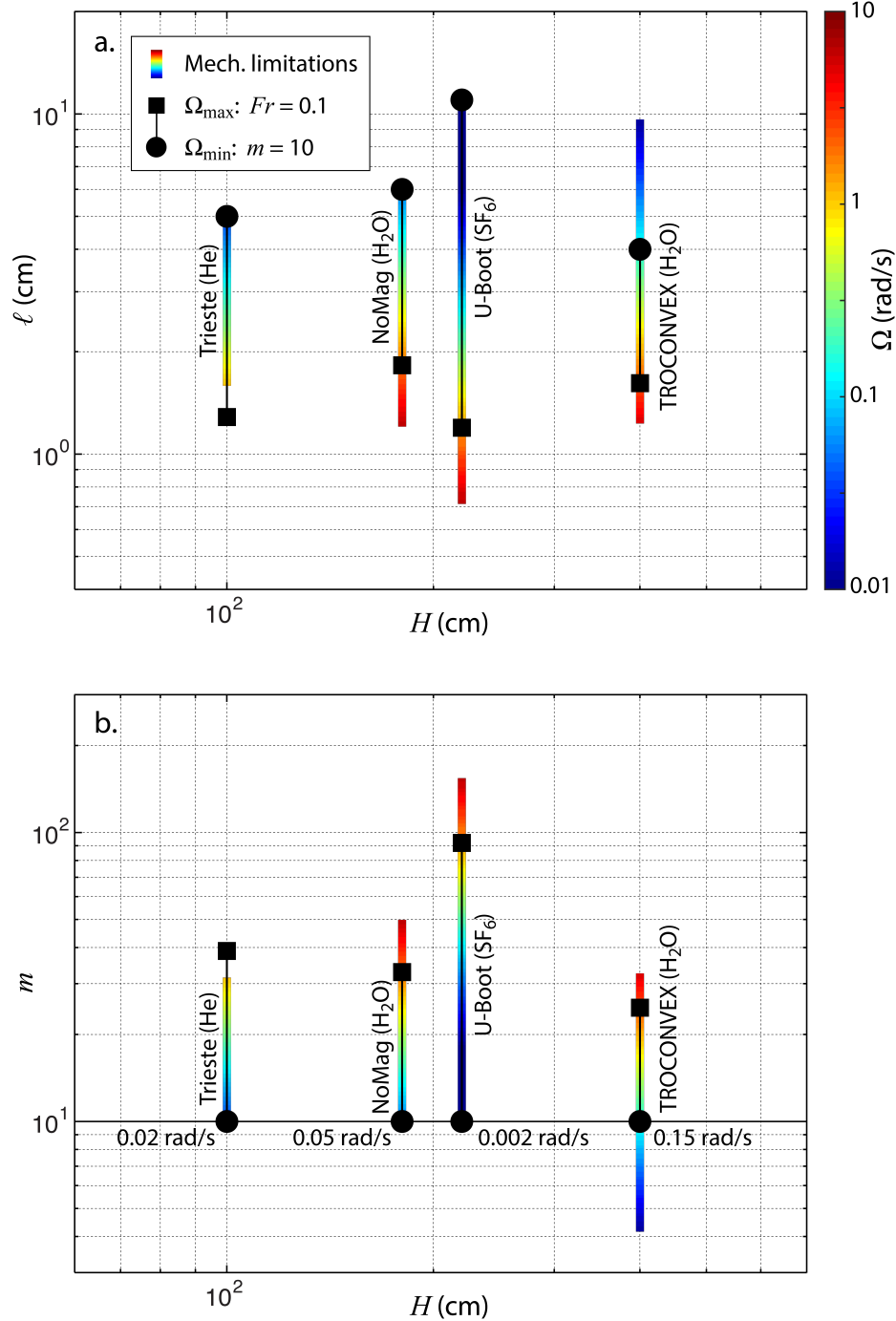


FIG. 6. (Color online) Flow structure width (panel a) and flow structure width ratio (panel b) plotted versus tank height H for four extreme rotation convection experiments. The tallest available tank in each experiment is used. The color bars show rotation rate and specify the minimum and maximum achievable rotation rates based on mechanical limitations of each device. The filled black square represents the maximum rotation rate Ω_{\max} for which the Froude number $Fr = 0.1$, while the filled black circle represents the minimum rotation rate Ω_{\min} for which the flow structure width ratio $m = 10$. The values for Ω_{\min} in each experiment are given at the solid horizontal line in panel b.

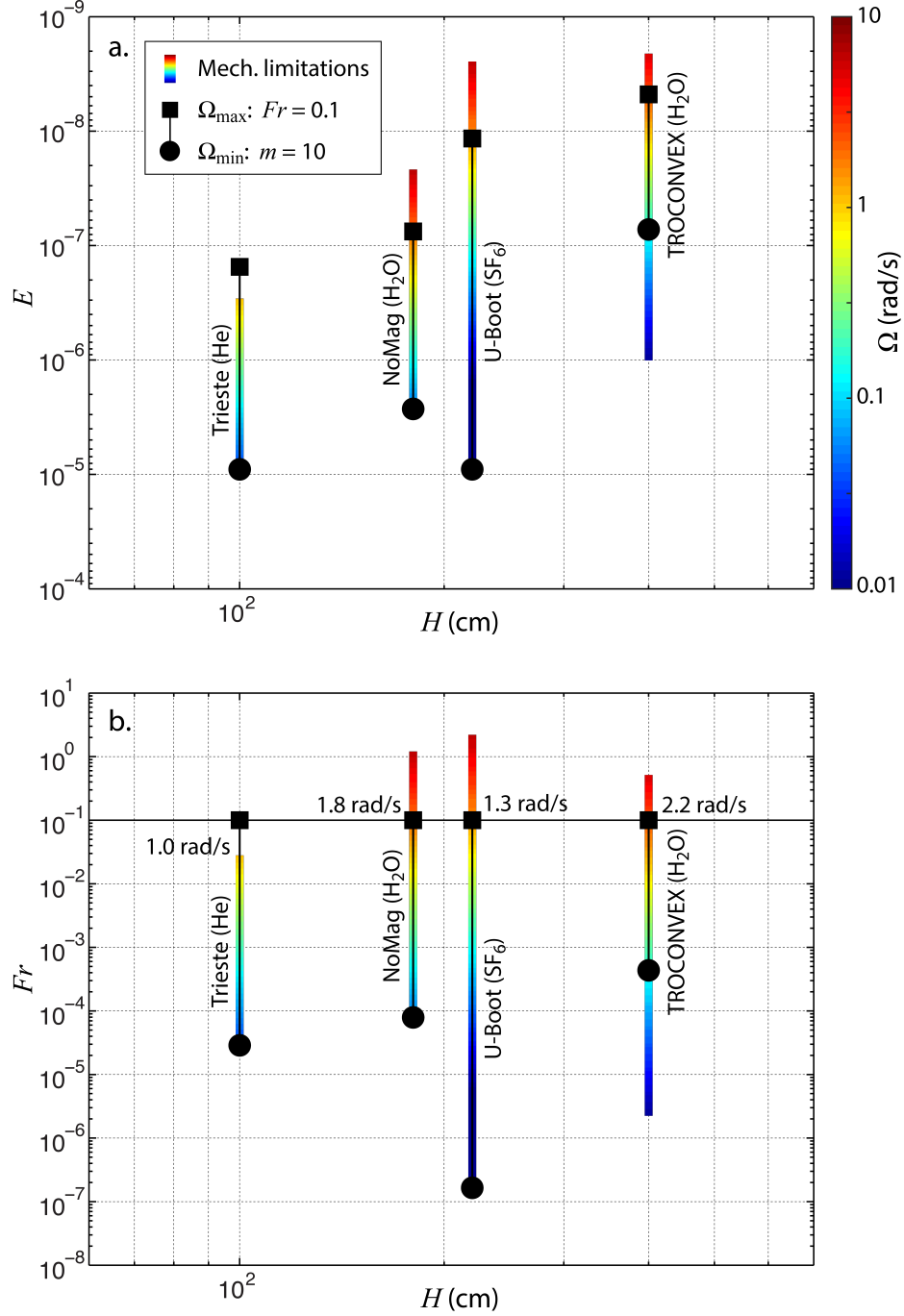


FIG. 7. (Color online) Ekman number E (panel a) and Froude number Fr (panel b) plotted versus tank height H for four extreme rotation convection experiments. Plot characteristics are the same as Figure 6. The values for Ω_{\max} in each experiment are given at the solid horizontal line in panel b.

that is driving the convection is small compared to the background fluid density [85, 88–90]:

$$\frac{\Delta\rho}{\rho_0} \ll 1 \rightarrow \gamma\Delta T \ll 1, \quad (18)$$

where ρ_0 is the background density of the fluid and $\Delta\rho$ is the density perturbation. This enforces a separate upper bound on ΔT . We have chosen the condition $\gamma\Delta T = 0.1$. However, some previous experimental studies have used more relaxed conditions such as $\gamma\Delta T = 0.2$ [e.g., 13], while others have suggested additional criteria for ensuring Boussinesq conditions [89, 91].

Figure 8 shows the accessible Ra versus E values for water rotating convection experiments a) TROCONVEX, and b) NoMag. The E range in each experiment is constrained by the bounds given in Figure 7a for maintaining $m \geq 10$, $Fr < 0.1$ flows. This range of accessible Ra values is plotted as a green box for each device. The $\gamma\Delta T \leq 0.1$ condition does not actually change the accessible Ra range in the water experiments - instead, the maximum applicable heat flux determines the maximum ΔT . The minimum ΔT in each case is determined by the precision of the temperature measurements.

Transition predictions are given as dashed lines. We were unable to find any formal predictions for the separation of plumes and geostrophic turbulence in $Pr > 3$ rotating convection; thus, investigating this transition should be an immediate priority when these devices become active. Furthermore, a broad gap at least two decades in Ra exists between the transitions into and out of the unbalanced boundary layer regime. Both the TROCONVEX and NoMag experiments should be well-suited for exploring this still-poorly-characterized regime.

Figure 8a shows that the highest TROCONVEX tank is best suited for studying the columnar, plume and geostrophic turbulence regimes at $5 \times 10^{-9} \lesssim E \lesssim 2 \times 10^{-8}$ and the unbalanced boundary layers regime for $2 \times 10^{-8} \lesssim E \lesssim 7 \times 10^{-8}$. Figure 8b shows that the NoMag tank is best suited for studying the columnar, plume, and geostrophic turbulence regimes for $7 \times 10^{-8} \lesssim E \lesssim 3 \times 10^{-7}$. For the entire range of $7 \times 10^{-8} \lesssim E \lesssim 3 \times 10^{-6}$, the NoMag device should be able to access the unbalanced boundary layer regime.

In Figure 9 we plot the accessible Ra versus E values for gas rotating convection experiments in a) the 1 m high tank on the Trieste device and b) the 2.2 m high tank on the U-Boot. The E ranges are constrained to the bounds given in Figure 7a. Gases tend to have greater thermal expansivities and are more likely to exceed Boussinesq limitations based on (18). For example, $\Delta T < 10.6$ K is the maximum allowable temperature gradient for SF_6 in the U-Boot.

The Trieste and U-Boot experiments are nevertheless capable of covering broad Ra ranges

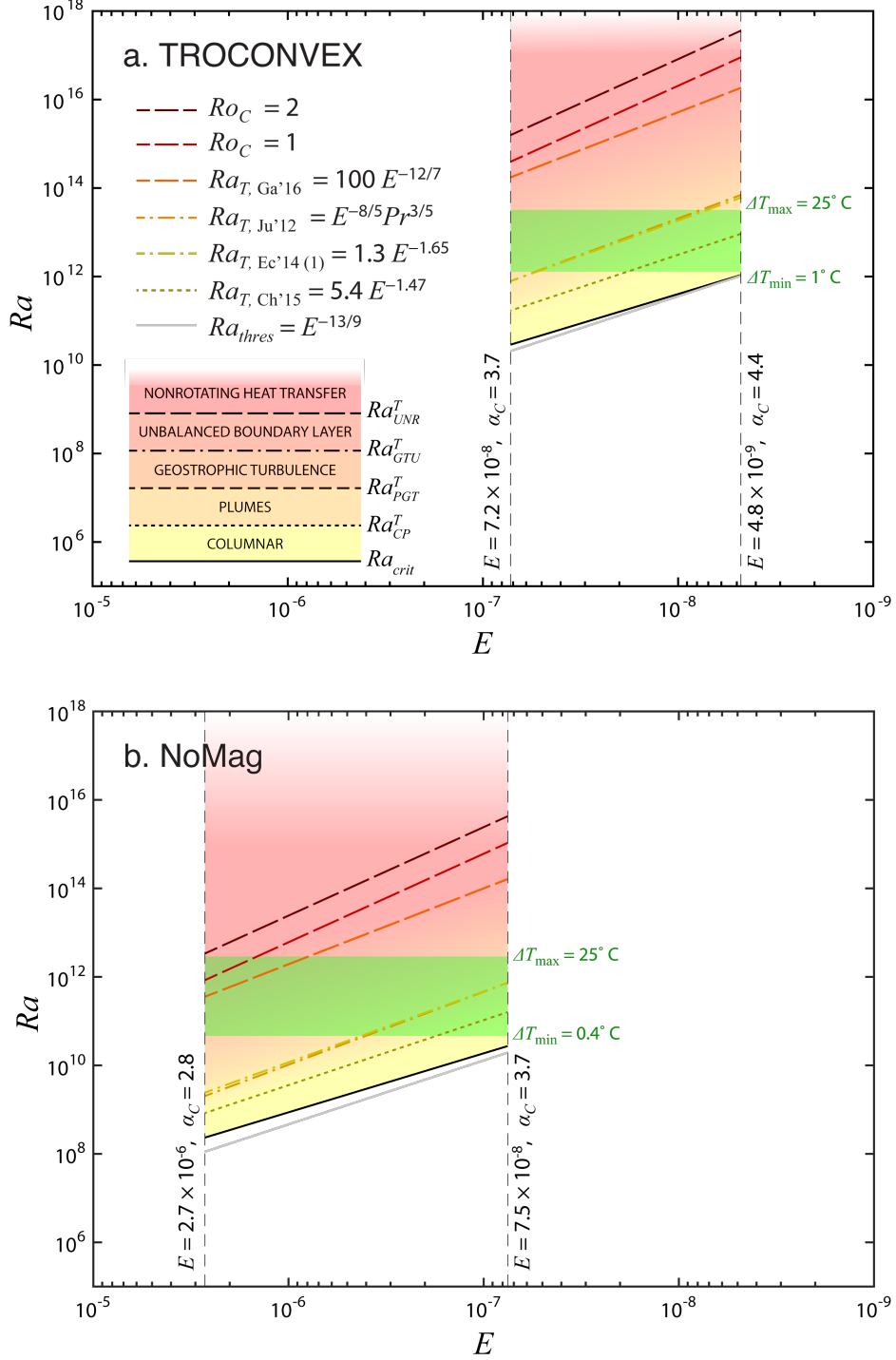


FIG. 8. (Color online) Rayleigh number Ra , plotted versus Ekman number, E , for the highest available tank size in the: a. TROCONVEX and b. NoMag rotating convection experiments at $Pr = 7$. The green box shows the range of Ra - E space accessible to each experiment, determined by the upper and lower ΔT limits and assuming fluid properties for a fixed temperature of $25^\circ C$. The predicted transitions between different flow regimes are plotted, with the line style indicating the type of transition and the line color indicating the specific prediction (following the legend; see Table I). The different background colors depict approximate locations in Ra - E space of different flow regimes. At the upper and lower E bounds, the slope of the Nu - Ra scaling expected near the onset of convection is indicated, based on the α_C trend from Figure 3.

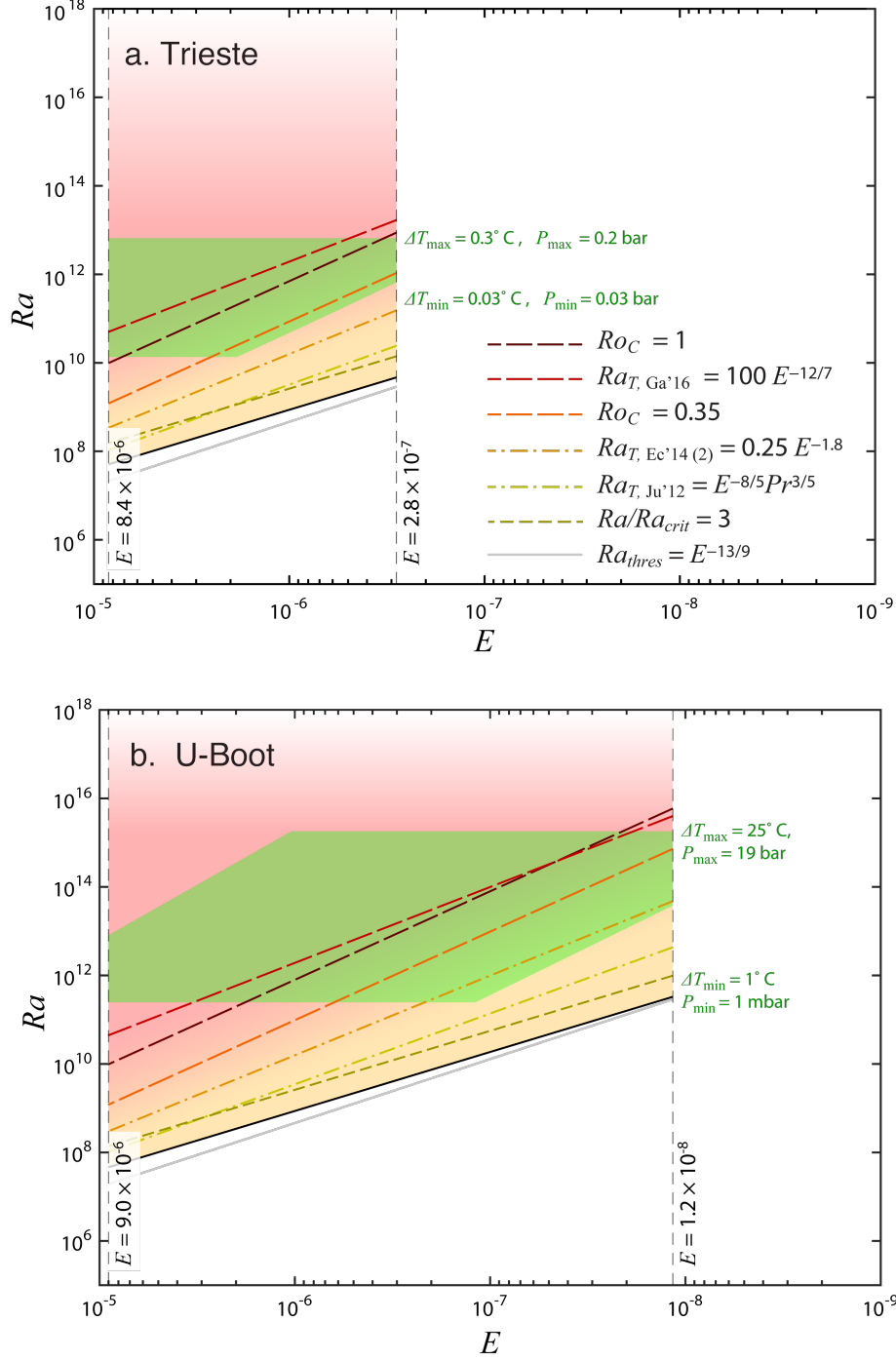


FIG. 9. (Color online) Rayleigh number Ra , plotted versus Ekman number, E , for the highest available tank size in the a. Trieste and b. U-Boot rotating convection experiments at $Pr \simeq 0.7$. The green box shows the range of Ra - E space accessible to each experiment, determined by the upper and lower ΔT limits and pressure limits. The lines depicting predicted transitions between flow regimes are plotted following the legend and Table I. Background colors depict the approximate locations of different flow regimes.

by varying the pressure in the tanks. The Trieste tank examined in Figure 9a is most suited for studying the nonrotating heat transfer regime over the $3 \times 10^{-7} \lesssim E \lesssim 8 \times 10^{-6}$ range, and the transition to the unbalanced boundary layers regime, Ra_{UNR}^T , for $E < 2 \times 10^{-6}$. The U-Boot tank examined in Figure 9b is most suited for studying the nonrotating heat transfer regime for $10^{-8} \lesssim E \lesssim 9 \times 10^{-6}$, the unbalanced boundary layers regime for $E < 10^{-6}$ and potentially the geostrophic turbulence regime for $E < 2 \times 10^{-7}$.

Extreme rotating convection experiments may be capable of reaching parameter regimes where large-scale vortices manifest. Under stress-free boundary conditions, simulations observe LSVs forming at E values as high as 10^{-4} , and for $Ra/Ra_{crit} \gtrsim 4$ [30] to $\lesssim 12$ [29]. These ranges might indicate that each of the experiments listed above can access LSV-containing regimes, but some important caveats apply: Under non-slip boundary conditions, simulations have observed an inverse energy cascade toward larger scales but no stable formation of coherent LSVs [27]. LSV formation is also a slow process – the upward cascade of energy is gradual and can take tens to hundreds of viscous diffusion times before manifesting LSVs [28]. In addition, the TROCONVEX and NoMag water experiments may have more difficulty reaching appropriate conditions for LSVs to develop: the higher relative momentum diffusivity in water ($Pr \simeq 7$) could hinder the ability of the flow to maintain broad-scale coherent structures. Finally, cylindrical experimental geometries may not be ideally shaped for containing the oppositely-signed dipole structure that LSVs take on in simulations.

The appearance of LSVs in laboratory experiments is therefore still an open question. However, experiments in forced turbulent flows have shown that inverse energy cascades can occur with sufficiently strong forcing and rapid enough rotation [92]. The possibility remains that the more extreme conditions accessible to experiments (for example, where Ekman friction is extremely weak) could be favorable to LSV formation in ways that overcome the issues listed above. Determining the circumstances in which LSVs occur provides another compelling goal for rotating convection experiments.

V. DISCUSSION

Laboratory experiments are a robust tool for studying rotating convection in extreme, turbulent settings. We have mapped out predictions for the fundamental rotating convection

regimes and possible locations for these transitions. Some of these predictions are corroborated by parallel studies: for example, agreement between flow regime morphologies have been observed between laboratory experiments, DNS, and asymptotically-reduced studies [46]. Asymptotically-predicted scaling behaviors have manifested in some extreme, present-day experiments and simulations [e.g., 12, 46, 58]. Laboratory observations are essential because these phenomena may be relevant to geophysical regimes but cannot yet be found at the parameter ranges that geophysical models occupy. In order to fully understand the relationship between asymptotic results and existing geophysical and astrophysical problems, we must access parameter ranges where more of the behaviors seen in asymptotic models are capable of manifesting in a real-world setting.

To this end, we have defined guidelines for conducting laboratory experiments that can be applied toward the study of extreme, turbulent rotating convection. The experiments discussed in Section IV represent some of the most extreme present-day rotating convection devices. By making use of different tank sizes, working fluids, pressures, and temperatures, these experiments are designed to cover a broad range of Ekman and Rayleigh numbers in order to access and describe the asymptotically-predicted regimes of rotating convection. Numerous aspects of these flow regimes require elucidation. For example, various, often conflicting predictions for transitions between different regimes exist in the literature. The precise heat transfer and velocity scalings in each regime are uncertain. There is a scarcity of data concerning the effects of non-slip boundary conditions and Ekman pumping at extreme E and Ra/Ra_{crit} [cf. 26, 27, 46]. The flow morphology in the unbalanced boundary layer regime remains to be determined. In each of these open questions, extreme rotating convection experiments should help to expand our understanding.

Another strength of laboratory experiments lies in their flexibility toward exploring other geophysically-relevant problems. Many planetary fluid layers, such as Jupiter’s atmosphere, do not fall within the bounds of the Boussinesq approximation. While the constraints on the rotation rate listed in Section IV maintain relevance to the canonical rotating RBC problem, the physics of high Fr rotating convection have not been thoroughly explored in turbulent systems [cf. 85, 86]. High Fr rotating convection data in Cheng et al. [17] indicate that a traditional overshoot in Nu at the transition to nonrotating-style convection, [e.g., 20, 33, 93], may not occur under conditions of strong centrifugation. Rough boundaries are pertinent to geophysical systems containing surface topography [e.g., 94]. Installing rough

top and bottom boundaries can both destabilize the viscous boundary layer, potentially leading to different heat transfer modes [e.g., 95, 96]. Apart from addressing the canonical rotating convection problem, several of the laboratory experiments discussed in Section IV are designed with the capacity to address these related problems.

Rotating convection provides a robust framework for approaching the turbulent physics that dominate many geophysical and astrophysical systems. By foraying into more extreme conditions than existing global-style models, studies have uncovered purely hydrodynamic behaviors that could be vital to explaining phenomena in more complex systems. For example, large-scale vortices develop under conditions of geostrophic turbulence for many rotating convection simulations [e.g., 29–31, 46]. These LSVs could provide a hydrodynamic explanation for the pattern of temperature anomalies on the surface of the Sun [97]. They may also be relevant to the Earth’s magnetic field, whose large-scale flux patches could be explained by similarly large-scale, vortical flow structures in outer core [e.g., 16, 98]. By providing predictions for the parameter ranges in where large-scale vortices develop, numerical results also provide a launching point for laboratory investigations of these phenomena. Laboratory studies can test whether these structures develop in real-world settings, and how the physics in moderate parameter ranges extrapolate to more extreme conditions. Developments in rotating convection are already bridging the gap between small-scale models and planetary-scale systems. Further advancing our understanding of rotating convection will require insights gained from both the current suite of experiments and from future experimental endeavors.

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Appendix A: Experiment information

TABLE III. Design properties and constraints for the experiments shown in Figure 1 and discussed in Figures 6, 7, 8 and 9. For entries with multiple values, the bolded quantity is the one used in our study. We include some references for devices used in previously published results. Minimum and maximum rotation rates Ω_{\min} and Ω_{\max} are determined by $m \geq 10$ and $Fr < 0.1$, respectively. Minimum and maximum temperature differences ΔT_{\min} and ΔT_{\max} are determined by the measurement sensitivity and the Boussinesq limitation $\gamma \Delta T < 0.1$, respectively. An asterisk indicates that a mechanical or diagnostic limitation is given instead because it is more restrictive than the corresponding theoretical constraint.

Experiment name	working fluid	H (m)	D (m)	Ω_{\min} (rad/s)	Ω_{\max} (rad/s)	ΔT_{\min} ($^{\circ}\text{C}$)	ΔT_{\max} ($^{\circ}\text{C}$)
RoMag [39]	liquid Ga	0.03, 0.05, 0.1, 0.2, 0.5	0.2	0.14	3.1	1*	40*
Trieste [13, 40]	cryogenic He	1.0	0.5	0.02	1.0*	0.03*	0.2*
NoMag	water , air	0.05, 0.1, 0.2, 0.4, 0.8, 1.8	0.2, 0.6	0.05	1.8	0.4*	25*
U-Boot [41, 42]	He, N ₂ , SF₆	1.1, 2.2	1.1	0.002	1.3	1*	10.6
TROCONVEX	water , air	0.8, 2.0, 3.0, 4.0	0.4	0.15	2.2	1*	25*

Appendix B: $Pr \ll 1$ fluids

Liquid metal rotating convection follows a different set of predictions than those given for water and gas due to thermal diffusion operating on a far shorter time scale than viscous diffusion ($Pr \simeq 0.025$) [e.g., 39]. Rotating convection onsets via oscillatory modes at [51, 99]:

$$Ra_{crit,osc} \simeq 17.4 (E/Pr)^{-4/3} . \quad (\text{B1})$$

The horizontal scale of these oscillatory structures are set by the thermal diffusivity [51, 99]:

$$\ell_{osc} \simeq c(E/Pr)^{1/3}H = c \left(\frac{\kappa H}{2\Omega} \right)^{1/3} , \quad (\text{B2})$$

where $c = 2.4$.

The flow structure width ratio is then given by:

$$m = D/\ell = c^{-1} \left(\frac{E}{Pr} \right)^{-1/3} \Gamma = c^{-1} \left(\frac{2\Omega D^3}{\kappa H} \right)^{1/3} , \quad (\text{B3})$$

and the $m \geq 10$ condition for experiments using liquid metals is:

$$\Omega_{\min} = \frac{500c^3\kappa H}{D^3} , \quad E_{\max} = \left(\frac{D}{10cH} \right)^3 Pr . \quad (\text{B4})$$

King and Aurnou [39] find that their liquid gallium rotating convection cases ($Pr = 0.025$) conform to a nonrotating Nu - Ra scaling for $Ra_{UNR}^T \simeq (E^2/Pr)^{-1}$. However, the scaling exponent varies smoothly at lower Ra values, such that other transitions cannot be clearly discerned.

Liquid metals also behave differently from moderate Pr fluids under nonrotating convection. At $Ra \gtrsim 2 \times 10^9$, Cioni et al. [100] find a $Nu \sim Ra^{2/7}$ scaling, consistent with $Pr \gtrsim 1$ results albeit with a different constant prefactor [101]. At lower Rayleigh numbers ($\lesssim 5 \times 10^8$), though, studies find an $\alpha_{NR} \simeq 1/4$ scaling where the heat transfer is controlled by inertially-driven, container-scale flows in the bulk rather than by viscous boundary layer processes [e.g., 39, 100, 102–104].

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