A local measure for perceived contrast

Institute for Perception Research
PO Box 513, 5600 MB Eindhoven

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W.M.C.J. van Overveld

Voor akkoord: Dr.ir. J.B. Martens

J.B. Martens

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Summary

In this report we investigate an objective measure which predicts the subjective, or perceived, contrast of an image. The measure, called the "local contrast index", makes use of the local grey-value distribution in different parts of a grey-value image. This local information is then combined into a global measure for contrast, This contrast measure is an extension of the contrast index as introduced by Kayargadde [Kay95].
We check the validity of the contrast measure against the results of three experiments. In one experiment, we varied the so-called "gamma" of three X-ray images whilst keeping the luminance constant. Subjects were asked to rate the perceived contrast of the images on a scale from 1 to 10. In a second experiment, we varied both the physical contrast (the grey-value range) and the amount of blur in three X-ray images. Again subjects had to rate the perceived contrast. The third experiment was similar to the second, but now the amount of noise was varied simultaneously with the contrast.
We found that the effect of gamma was predicted fairly well: both the contrast indices and the experiments showed that perceived contrast increases with gamma. The same holds for the effect of window width: both the perceived contrast and the contrast indices decrease with decreasing window width.
For the sharpness effect, the experiments showed a clear drop of perceived contrast when an image was blurred. The contrast index behaved differently for different images: it either increased or decreased with increasing blur, depending on the image. We discuss some reasons for this and possible ways to improve the performance of the contrast index in this case.
For the effect of noise, finally, we found that perceived contrast dropped only slightly when noise was added to an image. This was found in the experiments, but the contrast indices were hardly sensitive to the effect of noise. Only in the case of a subtraction image, we found that the contrast index dropped with increasing noise.

1. Introduction

It is known that brightness contrast plays an important role in the judgement of image quality, both for "natural" images and for medical images (cf. [RG88], [RKT94], [Ove94], [Ove95]). An objective measure of contrast (and preferably one of low computational complexity) can help a designer of an imaging system to optimize the image quality produced by the imaging system - or at least, to optimize the contrast.
A few of these objective contrast measures already exist. Lillesaeter [Lil93] describes a procedure for computing the contrast of an object separated from a background by a well-defined contour. Peli [Pel90] gives a definition of contrast for arbitrary images, using a description of the image in the frequency domain. The local contrast for a given frequency band can be computed by band-pass and low-pass filtering the image, and dividing the two filtered versions of the image. This method, which is quite complex, does not give a single contrast measure that predicts the overall perceived contrast in the image. Such a contrast measure is provided by the "global contrast index" as introduced by Kayargadde [Kay95]. Since the contrast measure proposed by us is an extension of the global contrast index, we will explain this index in some detail.
The computation of the global contrast index starts with a transformation from the grey value image to the luminance image; that is, the luminance produced by the monitor or the combination of hard copy unit and view box when the digital grey value image is used as input. The next step is the pixel-by-pixel translation from the luminance image to the (perceived) brightness image. For this, the psychometric lightness formula proposed by the CIE in 1976 (cf. [Hun78]) is used, which is basically a power function with a power of 1/3.
The histogram of the brightness values is used to estimate the perceived contrast. The width of the brightness histogram indicates how large the global contrast is. When the brightness distribution is uniform, the cumulative histogram is linear and the inverse of the slope of this cumulative histogram is proportional to the width of the original brightness histogram. For an arbitrary brightness distribution, linear regression is applied to find a linear part of the cumulative histogram and the inverse of the slope of this linear part is used as the global contrast index. To avoid instability of the linear regression due to isolated points at the lower and upper ends of the histogram, the algorithm only uses the part of the histogram between the cumulative fractions of 5% and 95%. Also, only the brightness values between 5 and 95 are used (where the brightness is computed on a scale from 0 to 100).The procedure is illustrated in Figure 1.

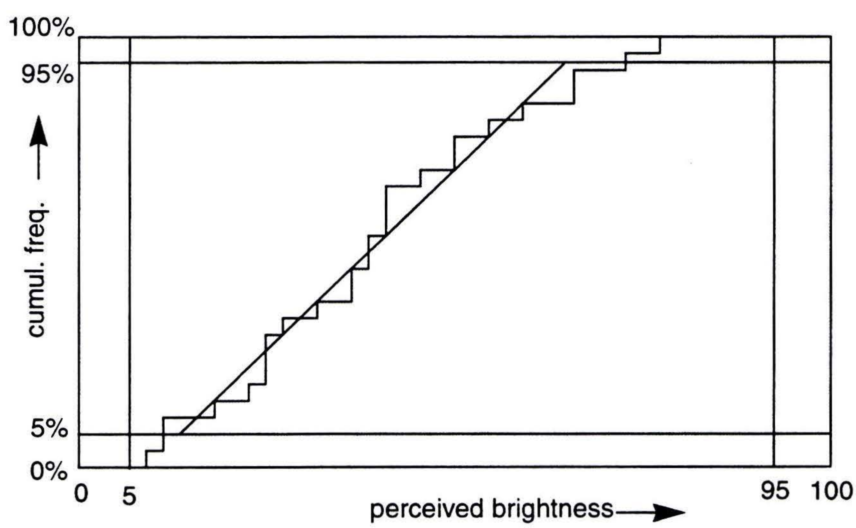

FIGURE 1. Hypothetical cumulative brightness histogram with indication of the part used for linear regression.

We have extended the notion of "contrast index" as described in Section 2. This new type of contrast index is verified by means of three experiments, described in Sections 3 through 5. Final discussion and conclusions are given in Section 6.

2. The local contrast index

In this report, we extend the concept of the global contrast index to a "local contrast index". The reason for this is that we expect perceived contrast not to be exclusively determined by the dynamic range of the grey values or brightness. Apart from such global aspects of contrast, it is likely that local aspects play a role as well. We can think of things like local dynamic range and "edge contrast" (which is probably closely related to sharpness). If local aspects of contrast play a role, we may try to capture these by computing the contrast index (as defined in Section 1) in some small region. The contrast in such a region will be called a "local contrast index". The local contrast indices in different regions can then be combined into one overall contrast index.
In order to keep the complexity of the computation of the overall contrast index fairly low, we divide a - square - image into square windows where horizontal or vertical neighbouring windows have 50% overlap: thus we use windows of size $2m$ (m still has to be specified) with a sampling distance of m. If we take just one window with size equal to the image size, then the local contrast index computed in this region is the same as the global contrast index defined before. By taking smaller and smaller windows, local effects of contrast will contribute more and more to the overall contrast index. However, if we make the windows too small, we run into problems with the linear regression. This will become very inaccurate when based on too few points. Also, if we consider a mondrian-like image with many small homogeneous regions, we would find a local contrast index of 0 in most of the windows, so that the overall contrast index is also likely to be small (although of course this depends on the combination rule, which we haven't specified yet). Yet the small homogeneous regions may have very different grey values so that the perceived contrast may be high.
As a compromise, we decided to follow the approach used for adaptive histogram equalization (cf. [PZS84]).Thus as a first guess, we take windows of width 1/8 times the width of the image. However, to study the effect of the window size we also consider windows of size 1/4 and 1/16 times the image width, respectively.
As for the combination of the various local indices into one overall index, we opt for a Minkowski sum:

$$C_o = \sqrt[p]{\frac{1}{n_w} \sum_w (C_w)^p}. \tag{1}$$

In the above formula, w is an arbitrary window, C_w is the local contrast index computed in window w, and n_w is the number of windows in which the local contrast index can be computed (see below). C_o is the overall contrast index. We do not use all windows in the computation, for reasons already indicated above: when the "useful" part of the cumulative brightness histogram (between 5 and 95, and 5% and 95%) consists of too few points, we discard the local contrast index computed for that window and we exclude it from the Minkowski sum. We take a limit of 10 pixels.
We also exclude windows for which the local contrast index is close to zero. These values correspond to nearly homogeneous regions. From the literature (e.g., the first three chapters of [Gil94]) it is known that the human visual system uses intensity gradients (luminance edges) as a first stage in the encoding of image information, so that brightness and contrast impressions are also based on regions in which the intensity varies.
The choice of a Minkowski sum is motivated by the literature (e.g. [Rid92]), where this combination rule is used to describe the combined effect of different perceptual attributes. The parameter p can be used to control the contribution of isolated regions with high contrast: for p=1, the contrast indices of all regions are weighted equally, and with increasing p, the regions with higher contrast index will get larger weights. As a limit case, for p infinitely large, the sum is equal to the largest one among the contributing terms. In fact it is easy to show that the Minkowski sum is a monotonously increasing function of p. To find out which value of p best fits the experimental data, we have used values of p equal to 1, 2 and 3 in our computations.

3. Experiment 1: gamma variation

Stimuli

The set-up of the experiment (method, stimuli, and viewing conditions) is the same as described in [Ove94], "Experiment 2". We used three angiographic images, hereafter called "scenes". These images had been digitally acquired in hospitals using an X-ray system with an image intensifier and a camera and consisted of 512 by 512 pixels. The scenes will be referred to as follows:

- "contr": a lateral image of the neck, jaw and part of the shoulder, showing the carotid artery in non-subtracted, positive contrast mode (dense regions are black). Some overexposure has occurred on the outside of the neck.
- "ksub": a subtracted image of the kidney and the vessels feeding the kidney. The background is bright, the kidney is medium grey and the vessels are dark. Very thin low-contrast vessels (around a tumour) are visible in the original image.
- "i32": a non-subtracted AP view of the upper part of the chest and the left shoulder, with contrast liquid in the subclavian artery. The brightest parts of this image are in the lung area, but there is no overexposure.

The contrast of the images was changed by changing the grey-value-to-luminance exponent gamma. We used hard copy images. The hard copy had a g-to-L (or actually g-to-D) characteristic of its own, but we changed this using look-up tables (LUTs), to a gamma curve with variable gamma γ_v:

$$L = C_v \cdot g^{\gamma_v} . \qquad (2)$$

Here g is the 8-bit digital grey value (the digitized video level at the output of an image intensifier-TV chain) and L is the luminance as measured on a patch of film with grey value g when this film is mounted on a view box with a given luminance L_{box}. The multiplicative constant C_v varied with g_v to keep the average luminance in an image constant with varying γ_v (without this, the image would get darker with increasing γ_v).The value of C_v was determined using the grey value histogram of an image, where the black backgrounds of the images were excluded from the histogram in these computations. We used γ_v = 1.5 through 6.0 with increments of 0.75.

Method

Images were viewed on a view box having a luminance of L_{box} = 2000 cd/m^2, which is a common value for view boxes encountered in hospitals. The viewing distance was 50 cm - a natural distance for a radiologist standing in front of a view box - and the radius of the images was 12 cm. Thus the viewing angle was 13 degrees. To mimic the low level of ambient light in the average hospital viewing room, we put a desk lamp behind the view box, which illuminated a white wall. The luminance reflecting from the front of the view box was 1 cd/m^2, corresponding to 70 cd/m^2 being reflected from the white wall.
All seven stimuli corresponding to one scene (the seven different gamma values) were mounted next to each other on a view box, in a random order. The remaining parts of the view box were completely covered. Subjects had to express the contrast of each image on a scale from 1 to 10. We used 16 subjects in this experiment: 8 non-experts and 8 radiologists. None of these had any previous experience with image quality experiments. All subjects had normal or corrected-to-normal visual acuity.

Results

We performed the following analysis of the contrast scores. To correct for the fact that one subject may only use scale values from 1 to 6 while another uses 4 to 10, we first computed z scores (by subtracting per subject the score averaged over the stimuli and dividing by the standard deviation). We averaged the z scores over the 16 subjects and transformed these back to the original scale values. Since we found no significant differences between the scores of non-experts and experts, we averaged the results over all subjects irrespective of their level of expertise.
The results are shown as the solid lines in Figures 2-4 below (after a linear transformation of the scores which will be described below).

Furthermore we computed the local contrast index for each of these images by taking all combinations of m and p with m = 16, 32 and 64; p = 1, 2 and 3. We also computed the global index by taking m = 256. The plots in Figures 2-4 show the 10 versions of the contrast index each time compared to the experimental data. The horizontal axis shows the seven different gamma values used, and the vertical axis shows the contrast scores and contrast index values. We linearly scaled the experimental data such that the square error between the data and the contrast index was minimum. The correlation coefficient r between the experimental data and the theoretical predictions is also given in each plot.

Discussion

We see that all contrast measures give a good prediction of the perceived contrast, but the local contrast indices work better than the global ones. Intermediate window sizes generally give the highest correlation with the experimental results, and p=1 gives the best result for most scenes and window sizes. Still the effect of p and m is extremely small.
Further discussion is postponed to the final section.

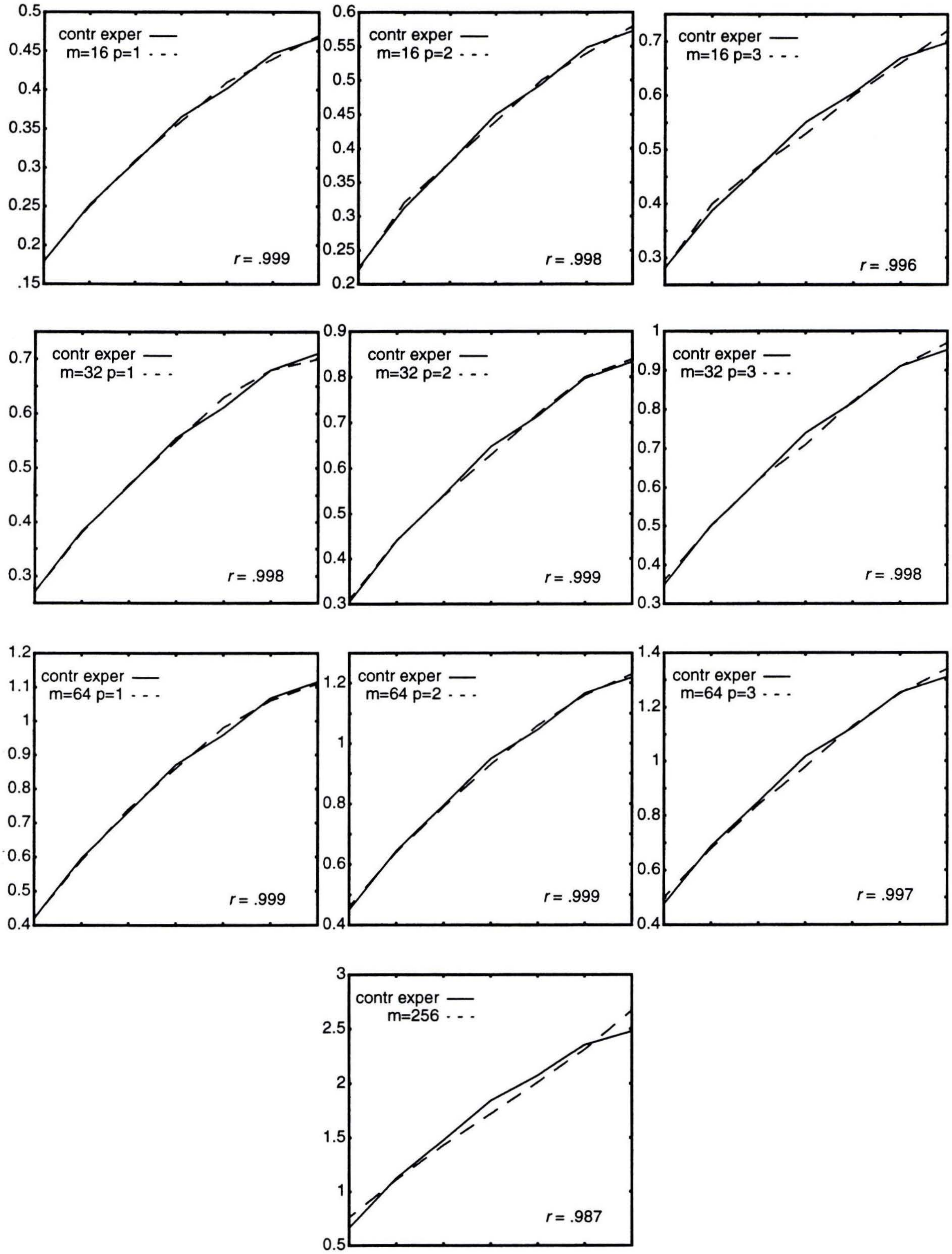

FIGURE 2. Experimental contrast scores compared to contrast indices for Experiment 1, scene: "contr".

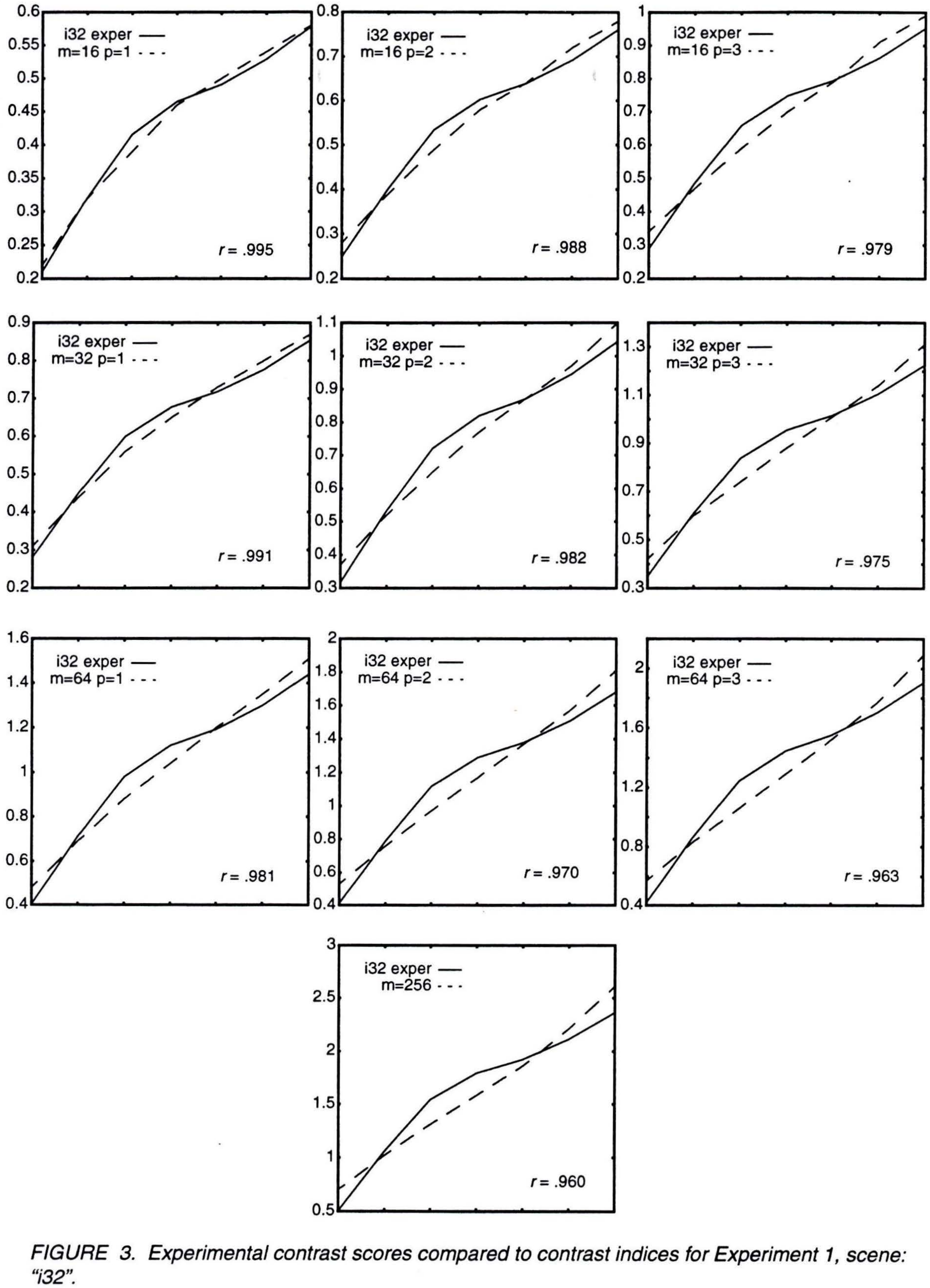

FIGURE 3. Experimental contrast scores compared to contrast indices for Experiment 1, scene: "i32".

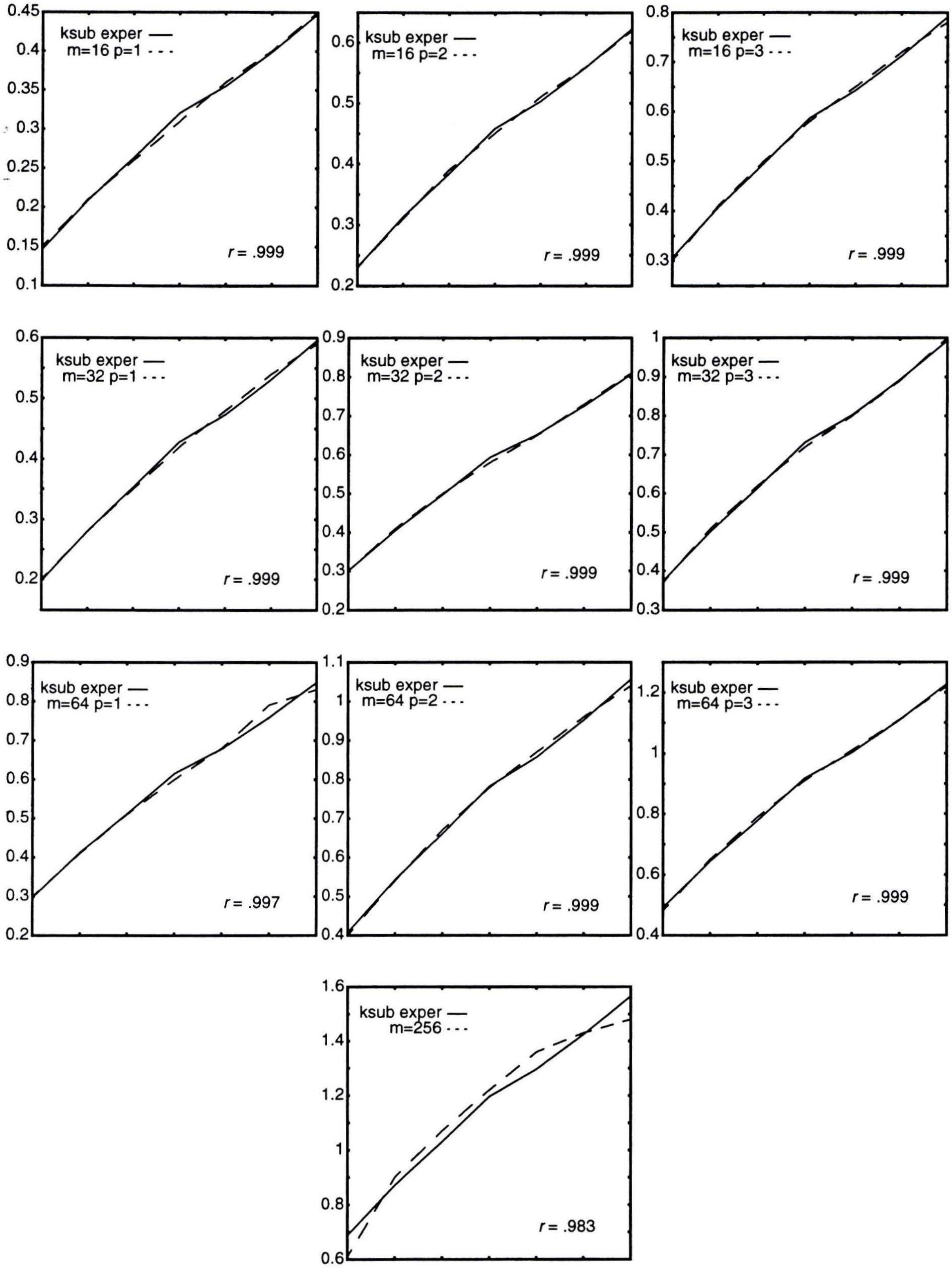

FIGURE 4. Experimental contrast scores compared to contrast indices for Experiment 1, scene: "ksub".

4. Experiment 2: effect of sharpness

Stimuli and method

This experiment was meant to study perceived contrast in the presence of unsharpness: to see how both the experimentally assessed contrast and the objective measures behave when physical contrast (the grey value range) is varied simultaneously with physical blur (the kernel size of a Gaussian blurring filter). For this experiment, we used three different medical images, as in [Ove95]: the subtraction angiogram "cer" and the contrast angiograms "kid" and "leg".
Variation of physical contrast was done through linear windowing of the grey value range, with different window widths (i.e., ranges), keeping the average brightness constant by adjusting the window level (i.e., offset). In this case, constant brightness means: a constant average grey value. This is true because the hard copy unit contains a perceptually linearizing LUT, cf. [Piz81].
Variation of sharpness was achieved through linear filtering with Gaussian filters of different kernel widths. The filters were applied to the images after the contrast variation had taken place. We used four levels of contrast: window widths of 100, 150, 200 and 255 in terms of grey values. We also used four levels of blur: Gaussian kernels with a radius of 0, 1, 2 and 3 pixels. We refer to these as blur levels 0 through 3 (note that blur level 0 means "no blur"). Thus we had 4 x 4 = 16 stimuli per scene. These were viewed on the view box in exactly the same set-up as in Experiment 1, except for the fact that stimuli were arranged in a 4x4 matrix instead of in one row. Eight subjects participated in the experiment, none of whom had any medical background. Each image was assessed four times, where in each of the four presentations, the images were shown in a spatially different random order.
After the experimental session, each subject was asked to point out the details which he or she had used for the contrast judgement. In this way we could find out to which extent global and local image aspects played a role.

Results

We averaged the four scores assigned to a stimulus per subject and, since the results of different subjects agreed well, we also averaged over the eight subjects. The results are show in Figures 5 to 7. Here we do not show the experimental data and the contrast indices in one graph as in the previous section, because we already have four curves in each plot due to the four blur levels. Again, the physical contrast parameter is shown on the horizontal axis (in this case, the window width in grey value units) and the perceived contrast is shown on the vertical axis. The length of the error bars indicates twice the standard error in the mean (over the 8x4 observations). In all plots in this section, the solid line depicts the images with the highest sharpness; and the shorter the dashes, the more blurred the images are.

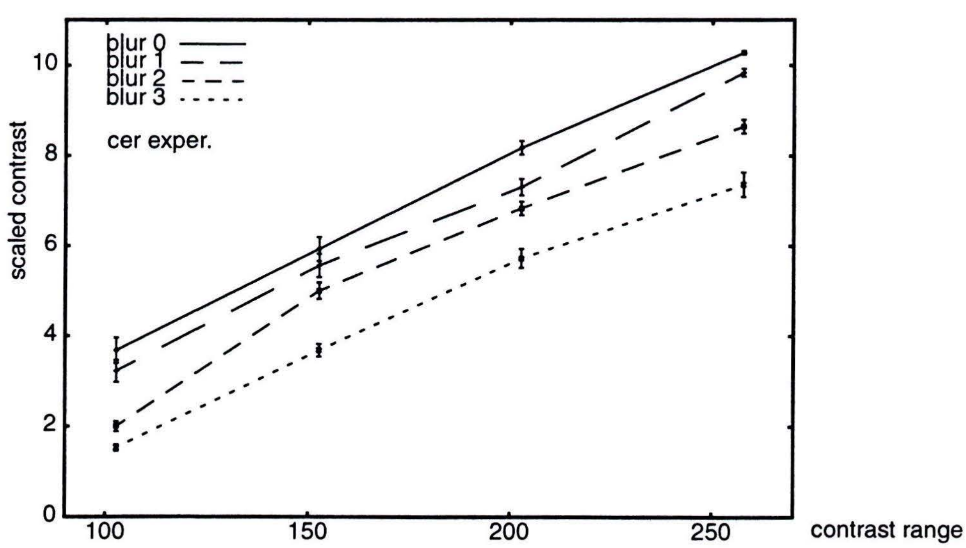

FIGURE 5. Experimentally determined perceived contrast for scene "cer" when physical contrast and blur are varied.

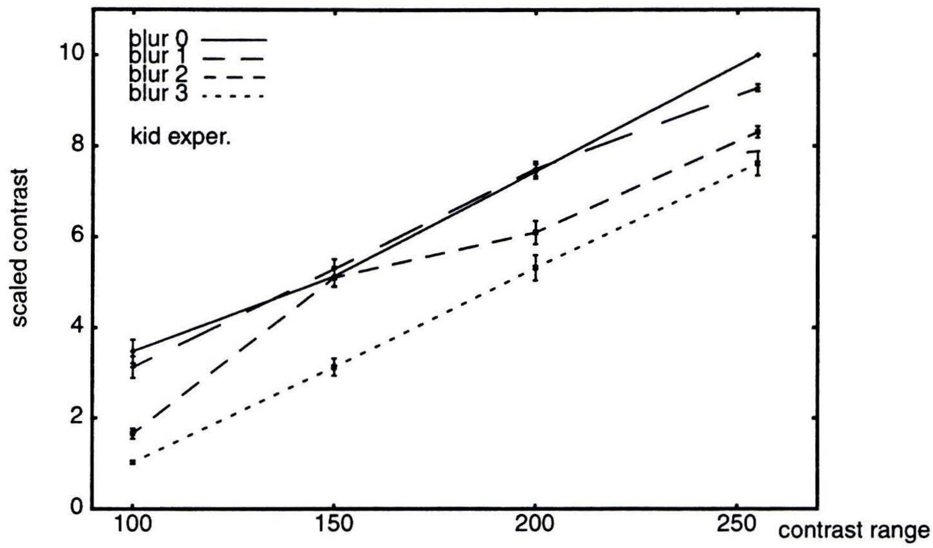

FIGURE 6. Experimentally determined perceived contrast for scene "kid" when physical contrast and blur are varied.

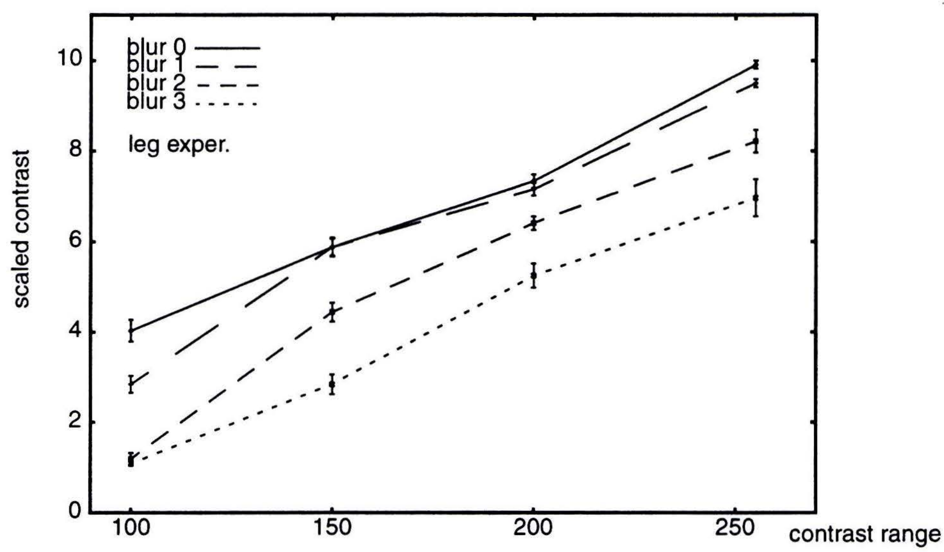

FIGURE 7. Experimentally determined perceived contrast for scene "leg" when physical contrast and blur are varied.

We also computed local contrast indices for different values of m and p, as described in Sections 2 and 3. Since these images consisted of 1024 x 1024 pixels instead of 512 x 512 as in Experiment 1, however, we doubled the window sizes (parameter m) compared to those of Experiment 1. The results are shown in Figures 8 to 10, where the ten different contrast indices are arranged into ten plots in a configuration similar to Figures 2 to 4.

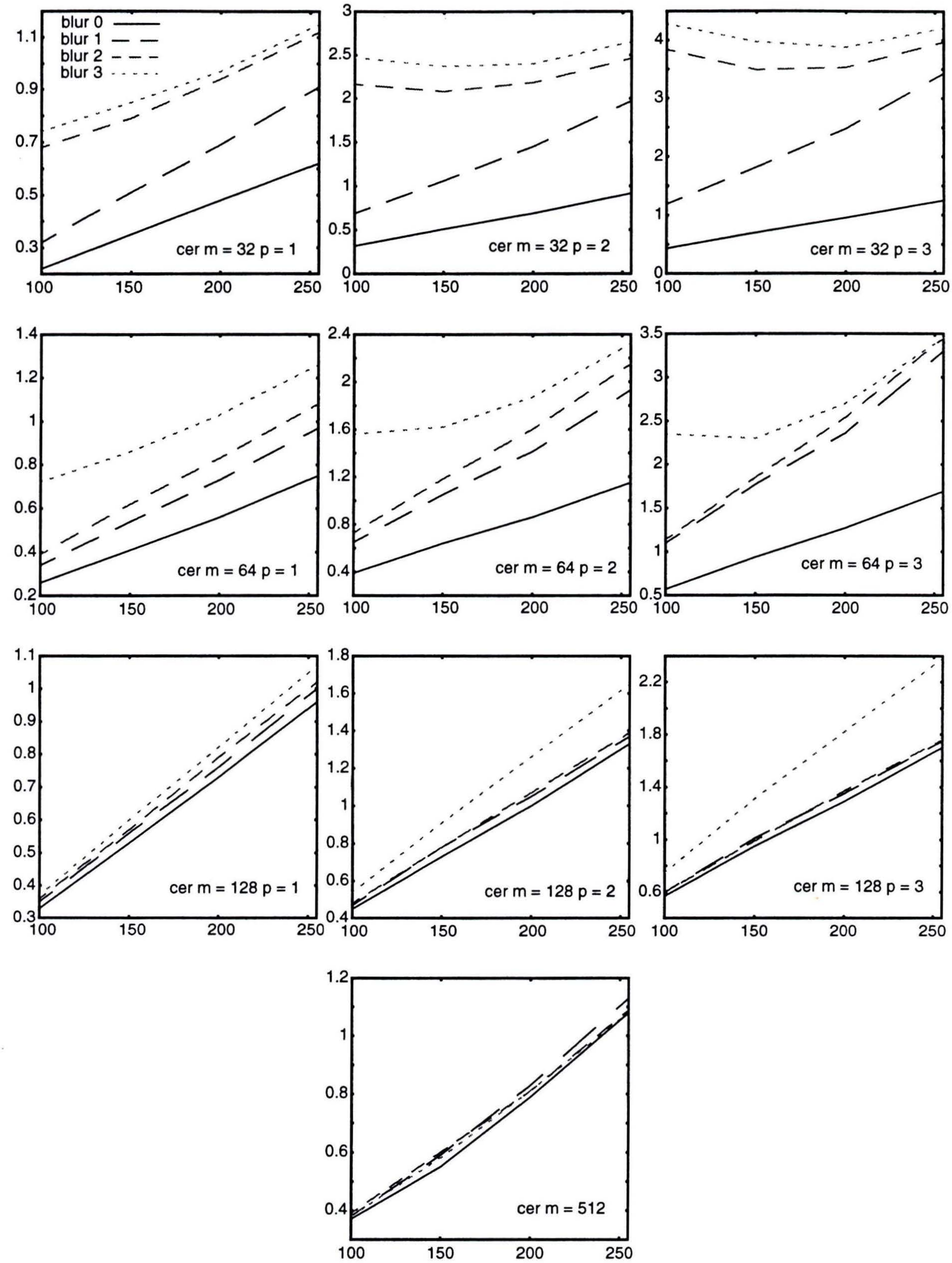

FIGURE 8. Contrast indices for scene "cer" with varying contrast range and blur level.

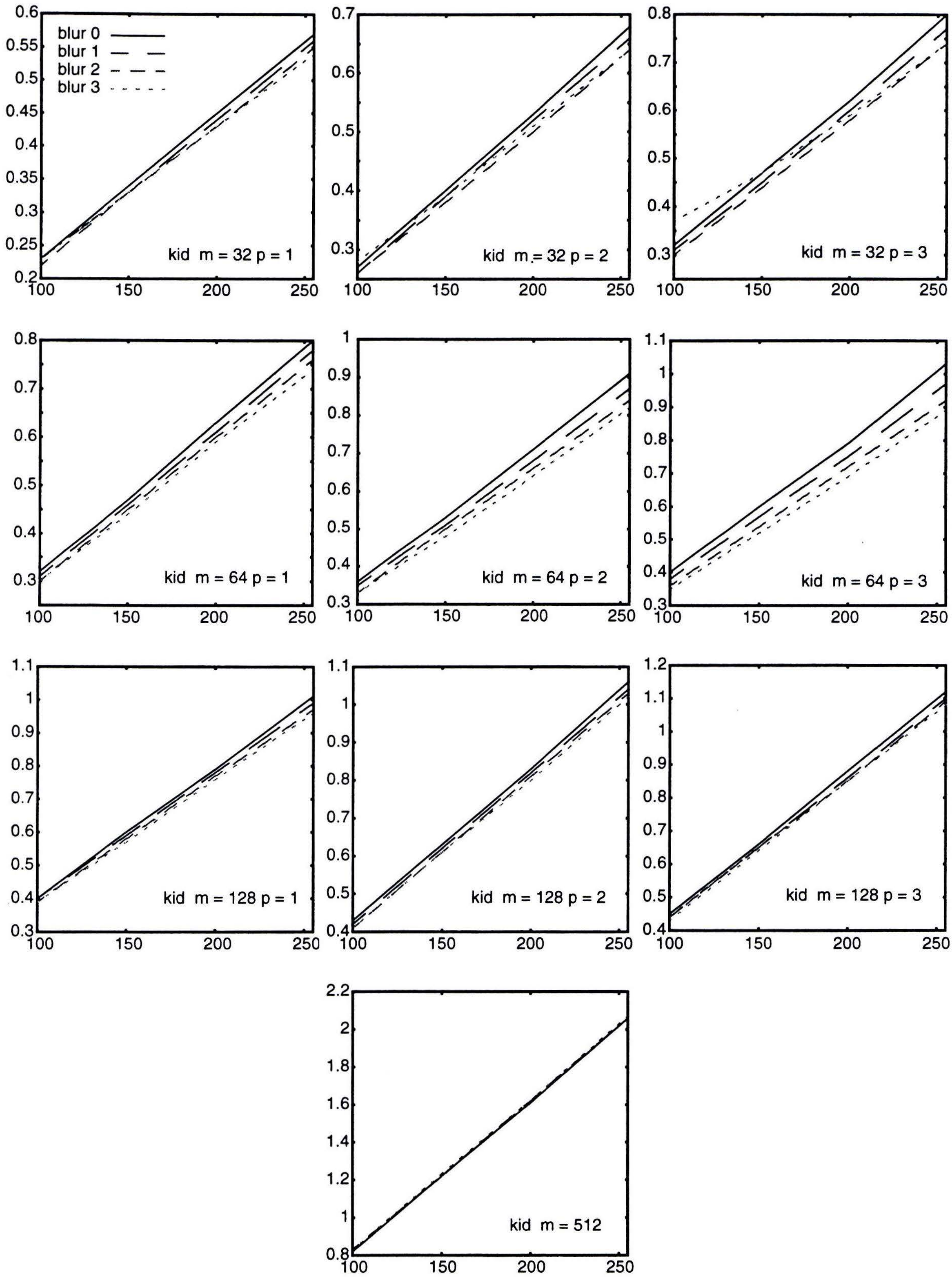

FIGURE 9. Contrast indices for scene "kid" with varying contrast range and blur level.

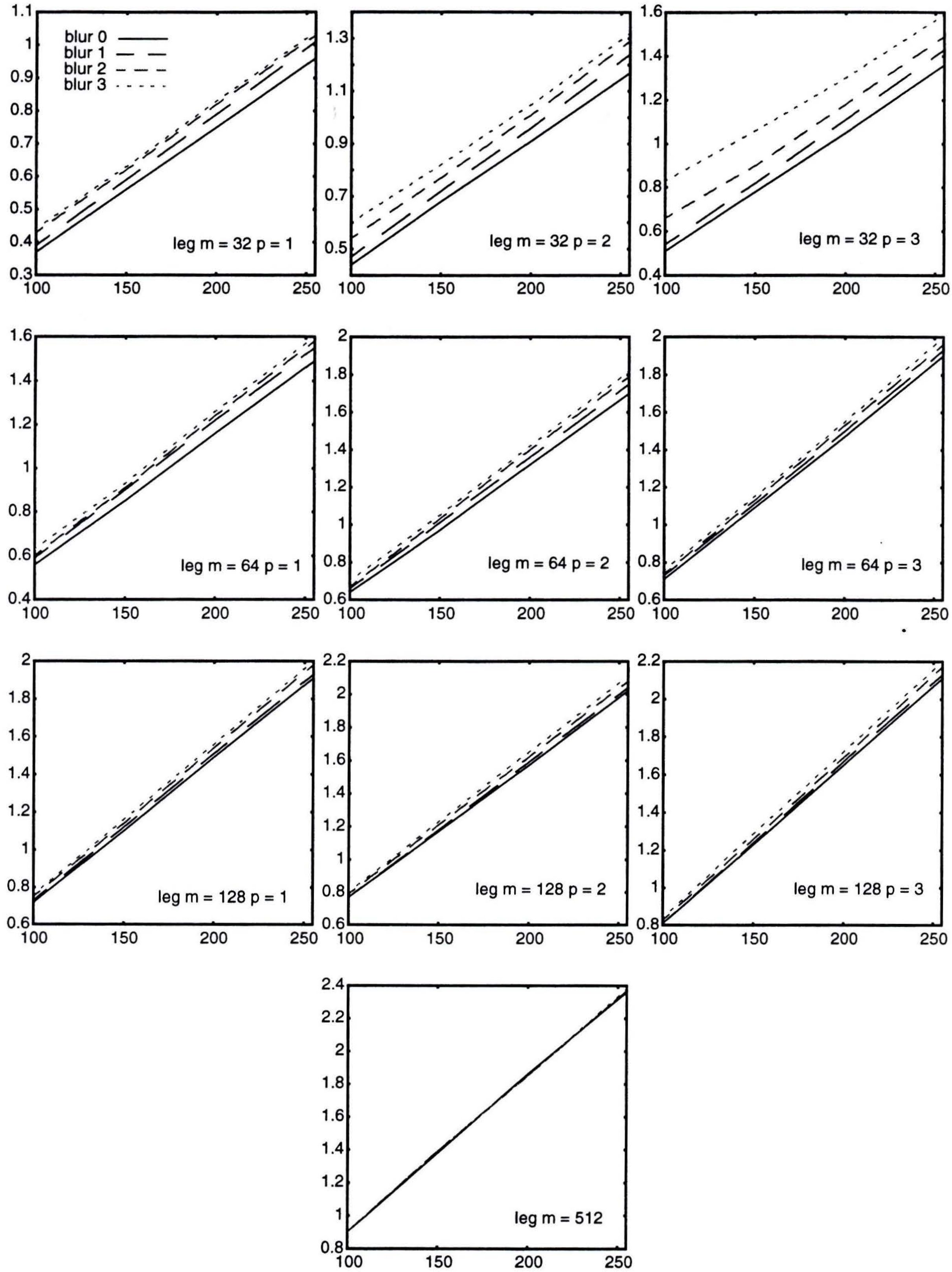

FIGURE 10. Contrast indices for scene "leg" with varying contrast range and blur level.

Discussion

First we discuss the experimental results. From Figures 5-7 it is observed that perceived contrast increases with window width and it decreases with the size of the Gaussian filter kernel. This is found for all three scenes, and also for all individual subjects (although a drop in sharpness has a stronger effect on perceived contrast for some subjects than for others).
This is confirmed by the subjects' remarks: most of them looked at the visibility of thin blood vessels and other fine details, which was clearly affected by blur. Thus the contrast judgements were lower for blurred images.
As for the objective contrast measures, we find that these behave very differently for the three scenes. The figures for scene "cer" show that the local contrast indices tend to increase with increasing blur levels, which is contrary to the experimental findings. In some extreme cases (small m and large p), blur has even a much larger effect than physical contrast: for the least sharp images, the contrast indices do not even increase with physical contrast. The fact that the objective measures do not correlate well with the scores found in the subjective experiment is supported by the correlation coefficients we find when we try to obtain a best fit for the experimental data to the objective measures, like we did in the case of Experiment 1. When we apply a linear scaling to the scores of the experiment, we have to apply the same scaling to all stimuli that have been compared to each other: thus to all stimuli of a single scene. The correlation coefficients of the experimental data with each of the ten different local contrast indices are given in the following tables (organized in the same configuration as the plots in Figures 8 to 10).

.323	-.068	-.126
.505	.404	.444
.879	.820	.741
	.919	

Table 1: Correlation coefficients of contrast indices and experimental scores, scene "cer"

.944	.940	.928
.954	.962	.969
.947	.944	.943
	.927	

Table 2: Correlation coefficients of contrast indices and experimental scores, scene "kid"

.841	.799	.722
.860	.863	.873
.874	.875	.877
	.898	

Table 3: Correlation coefficients of contrast indices and experimental scores, scene "leg"

An explanation for the bad fit of the "cer" data can be found in the fact that the brightness histogram of this scene is extremely peaked. For a local histogram, only very bright and very dark regions might occur, so that it is difficult to find a reliable estimate of the slope of the cumulative histogram.
Also, it can be seen that if we blur this image, the dark regions are spread over a larger area. Thus the black regions will contribute more to the brightness histogram (discarding the bottom

5%) after blurring, so that the dark parts get more weight in the linear regression and the slope will become smaller.
For the other two scenes, we find smaller effects of the blur, but the effects are contrary for the two scenes: for the "kid" scene we see that blur lowers the contrast index and for "leg" the opposite holds.
When taking all scenes and window sizes into account, the best fits are generally found for small p values. For "kid", intermediate m values give the best results, but for the other two scenes, the global contrast index is better than any of the local ones.

One might think that local contrast indices should be able to describe the effects of blurring better than a global index, because of the local nature of the measurements. However, this was not found in our data. This could be due to the fact that local aspects of contrast can be divided into two types of effects (as mentioned in the beginning of Section 2): the local dynamic range and edge effects. Local dynamic range effects may be explained using a local contrast index as proposed in this report, but edge effects probably play a role in a much smaller environment than the size of the windows we used. This latter type of contrast can only be described by some edge-finding procedure and a measurement of the "strength" of the edge, i.e. the luminance difference at either side of the edge.

5. Experiment 3: effect of noise

The experiment described in this section is similar to Experiment 2, but now the amount of noise was varied instead of the amount of blur. We first varied the noise in each of the scenes, using a computer package to simulate the effect of a varying X-ray dose. Thus noise with a Poisson distribution was computed for each image, this noise pattern was filtered by the MTF of the imaging chain, and this filtered noise was added to the original image. For more details about this procedure, the reader is referred to [Ove95]. We used four different noise levels, indicated by 1 to 4 where 1 is the lowest noise level (the highest X-ray dose). The contrast variation following the noise variation was the same as in Experiment 2.
The results were treated in the same way as in the previous experiment. Figures 11 through 13 show the result of the scaling experiments and Figures 14 through 16 show the corresponding results of contrast index computations.

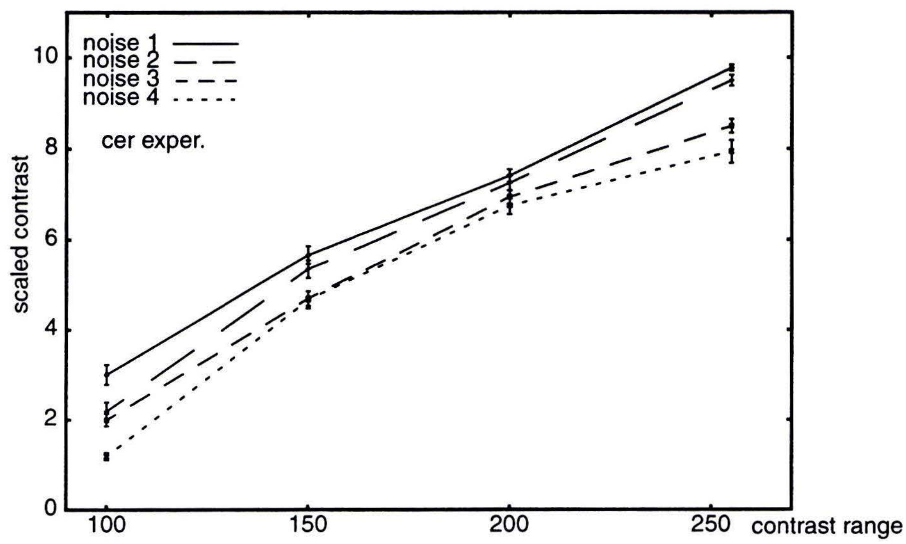

FIGURE 11. Experimentally determined perceived contrast for scene "cer" when physical contrast and noise are varied.

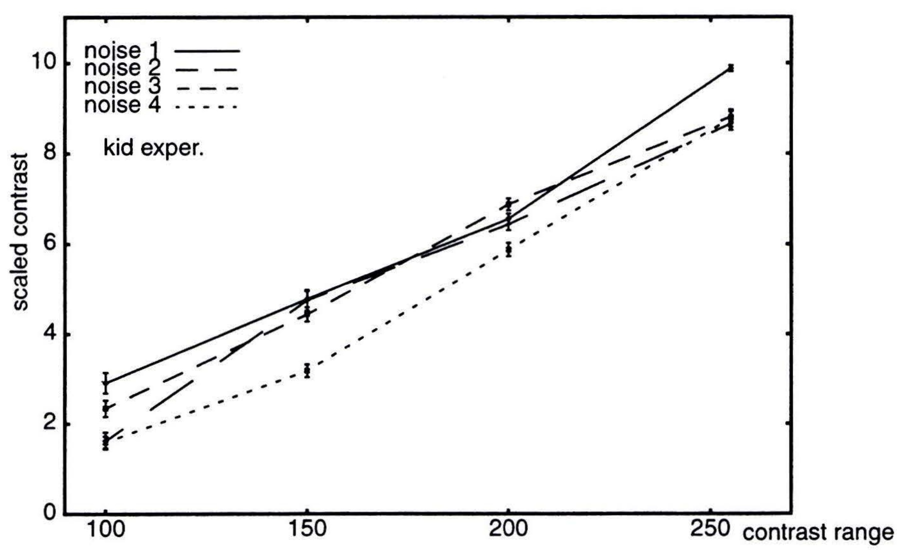

FIGURE 12. Experimentally determined perceived contrast for scene "kid" when physical contrast and noise are varied.

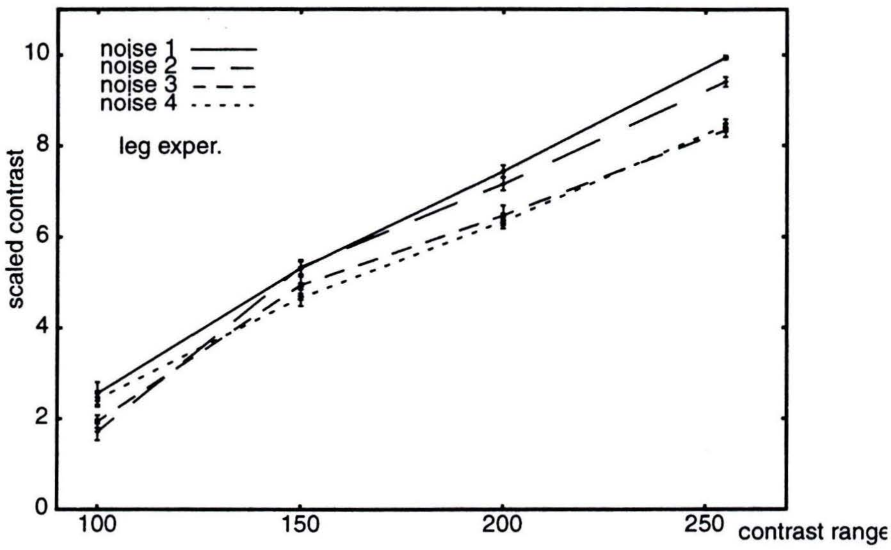

FIGURE 13. Experimentally determined perceived contrast for scene "leg" when physical contrast and noise are varied.

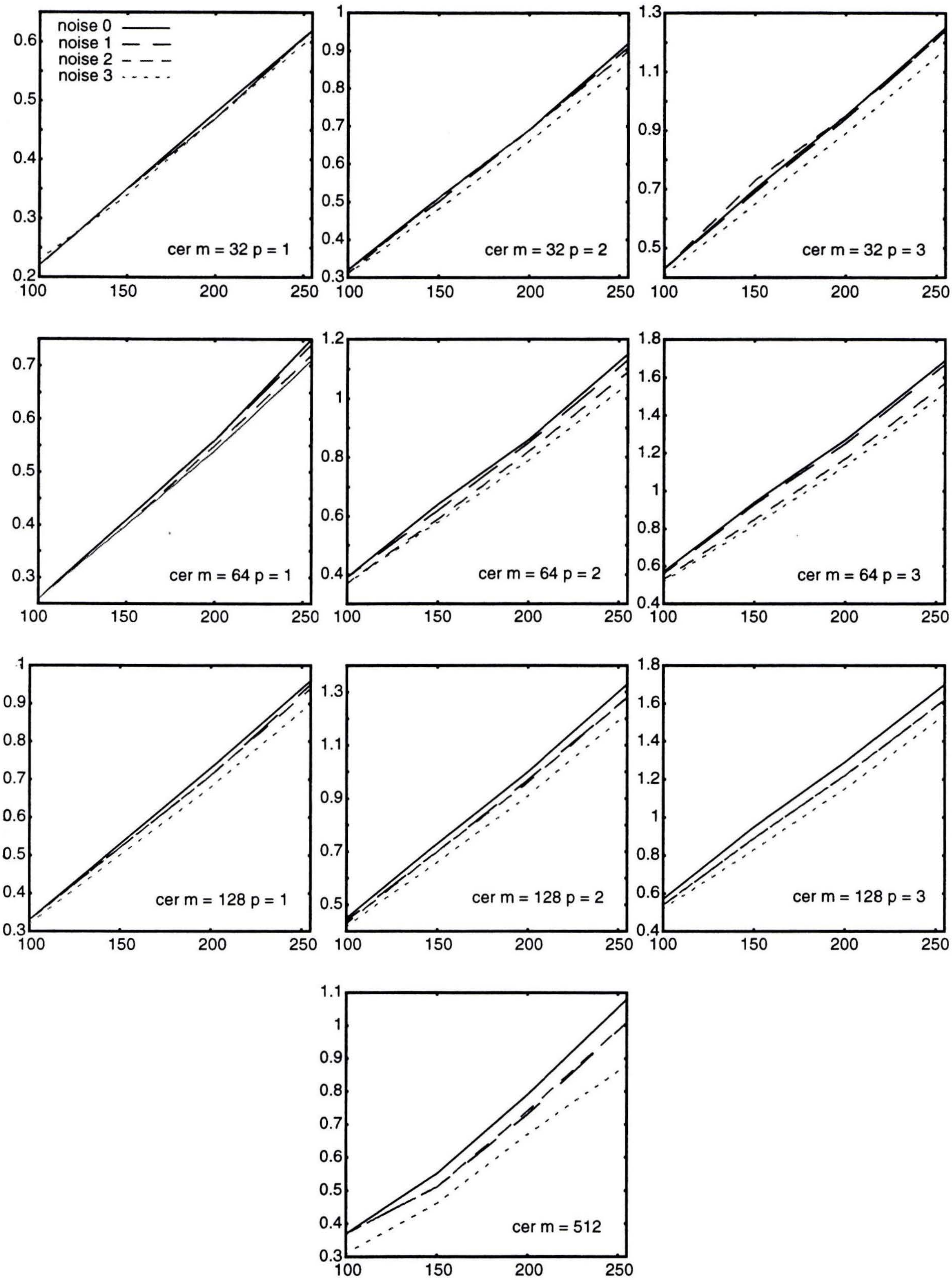

FIGURE 14. Contrast indices for scene "cer" with varying contrast range and noise level.

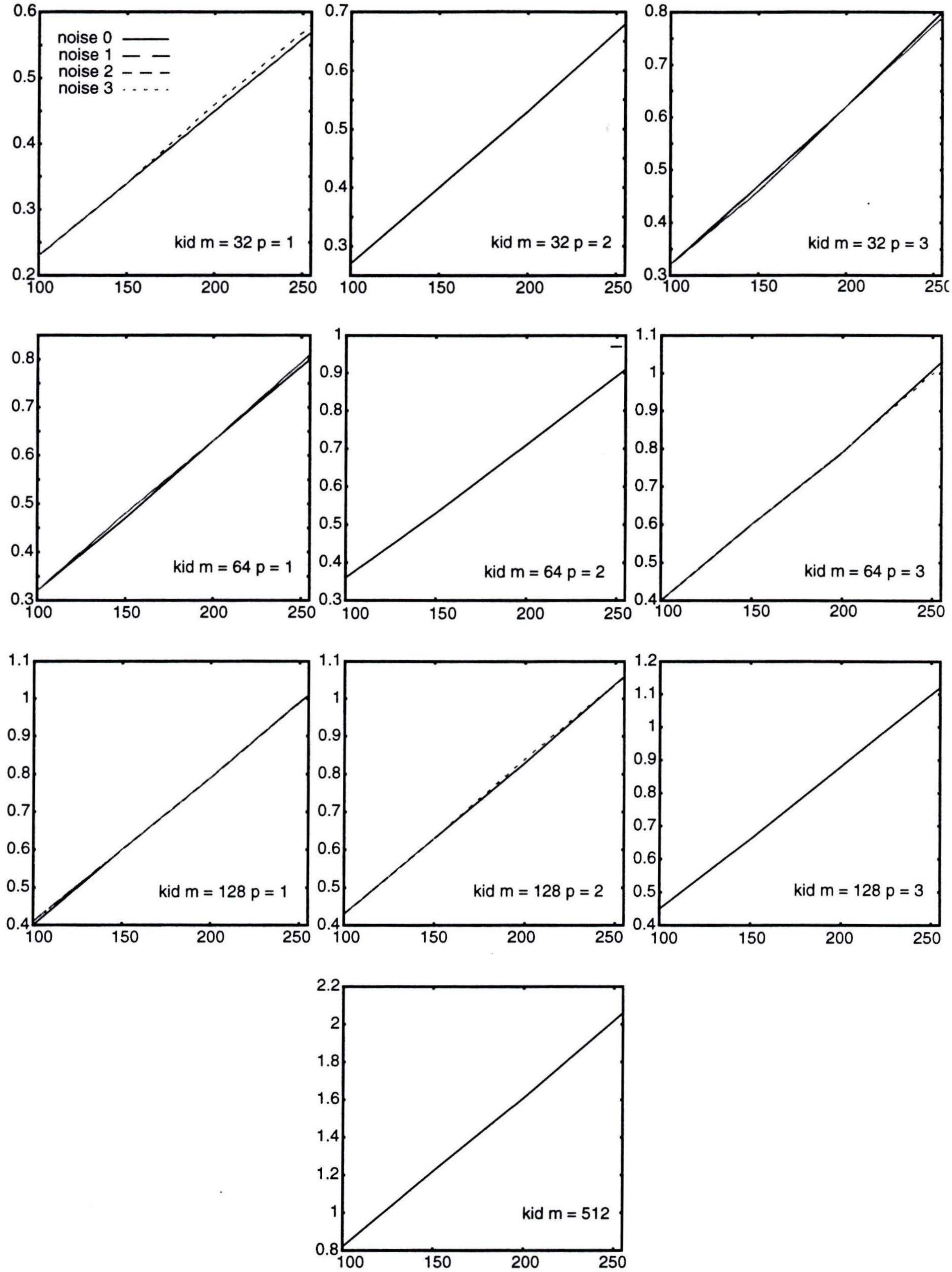

FIGURE 15. Contrast indices for scene "kid" with varying contrast range and noise level.

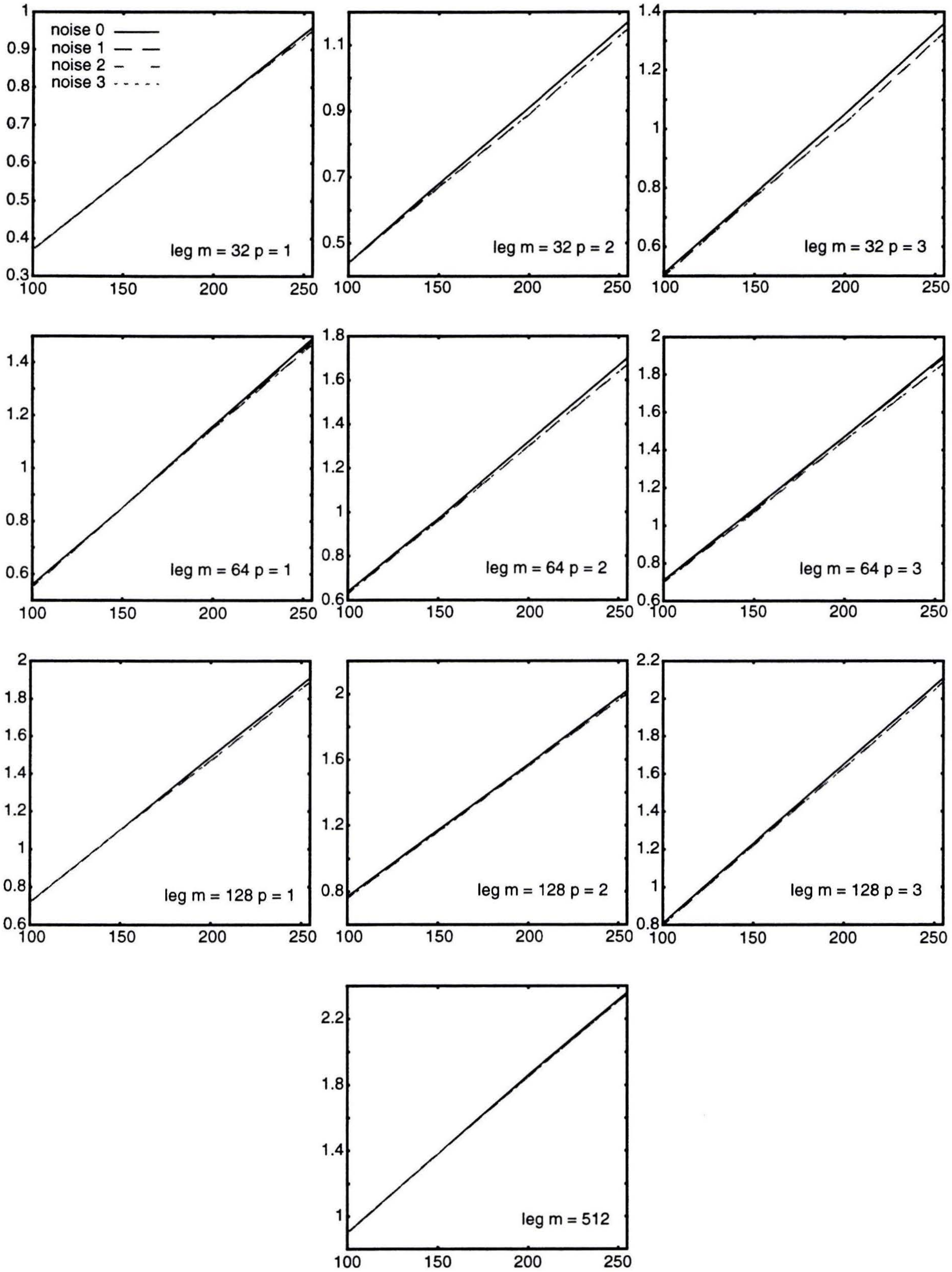

FIGURE 16. Contrast indices for scene "leg" with varying contrast range and noise level.

Discussion

In the results of the subjective experiments, we see a small but significant effect: perceived contrast slightly decreases with noise. This was found for all three scenes, and it also holds for all individual subjects. In the case of scene "cer", this was confirmed by the subjects' remarks: most of them mentioned that the background in the subtraction gets darker when noise is added, so that the (dark) vessels don't stand out from the background as well as in the case of the lowest noise level. For the other two scenes, we found the same effect, but there the subjects did not mention it explicitly.
As for the objective contrast measures, we find that these are not very sensitive to noise. Contrast indices drop a little when noise is added for scenes "cer" and "leg", but not for "kid". Note that the effect is strongest for "cer", for the largest window width. This indicates that it is actually a global effect of the darkening of the background, as mentioned by the subjects.
To find out how well the contrast indices correlate with the experimental data, we computed the correlation coefficients as described in the previous section. It is seen that all correlations are fairly high, and that the largest value of p seems - marginally - optimal. If we compare the graphs in Figures 11 to 16 visually, it would seem that the experimentally found interaction of contrast and noise is captured best by the objective measure for scene "cer", but this is not reflected in the correlation coefficients, which are actually lowest for this scene.

.967	.973	.974
.972	.978	.981
.973	.976	.979
	.965	

Table 4: Correlation coefficients of contrast indices and experimental scores, scene "cer".

.978	.980	.982
.978	.980	.980
.979	.979	.980
	.980	

Table 5: Correlation coefficients of contrast indices and experimental scores, scene "kid".

.979	.982	.982
.979	.980	.981
.980	.979	.980
	.979	

Table 6: Correlation coefficients of contrast indices and experimental scores, scene "leg".

6. Conclusions and discussion

A number of conclusions can be drawn from the experiments described in this report. First of all, we have found how changes in physical sharpness and noise affect the perceived contrast of an image: a decrease in sharpness induces a lower perceived contrast, and an increase in noisiness brings about a slightly lowered perceived contrast. This holds for all scenes and for all observers.
Secondly, we have studied how well perceived contrast can be described by an objective measure called the local contrast index. We have seen that fairly high correlations can be obtained for a wide range of parameter choices (p and m). Among the values we tried, the choice p=1 gives the best overall results (taken over Experiments 1-3). As for the window size m, we have not found one value giving optimum results for all cases. Generally, most window sizes give reasonable to good results. In particular, the global contrast index also does fairly well in general, although it is usually outperformed by a local contrast index with an intermediate window size (in Experiments 1 and 3). Only when the image is corrupted by blur (Experiment 2) and the brightness histogram is strongly non-uniform, the global contrast index gives the best prediction of the perceived contrast.
The fact that blur is not described well by the contrast index was found previously by Teunissen [Teu96]. He found this for the global contrast index; here we have shown that the localized version of this does even worse. In fact, the smaller the window size, the worse the prediction for two of the three scenes. This is quite counterintuitive, since one would expect the local effects of blurring to be reproduced best by the smallest window size. We see that smaller windows indeed have a large effect on the outcome of the computations, but the effect is contrary to the empirical observations. An explanation of this finding is given at the end of Section 4.
To better describe the effect of blur, we might use a separate measure of perceived sharpness (such as the one developed in [Kay95], [KM94]) and to base the final contrast measure both on the contrast index as computed here and on the sharpness measure.
Other possible extensions of the contrast measure presented here are: 1) a multiresolution approach (computing contrast indices over various window sizes, or scales), and 2) a feature-based approach (first finding features such as edges in an image and deriving the local contrast from differences in brightness in the neighbourhood of such a feature, more or less like [Lil93]). Both methods would require an extra combination rule by which to average the contrast indices computed at different scales and/or locations. Such approaches are however considerably more complex to implement than the one proposed here and for that reason they fall outside the scope of this report.

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