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Carrier Inversion Noise Has Important Influence on the Dynamics of a Semiconductor Laser

Mirvais Yousefi, *Member, IEEE*, Daan Lenstra, and Gautam Vemuri

Abstract—We find that, although inversion noise has only a marginal effect on the linewidth of a semiconductor laser in continuous wave operation, in the presence of dynamics, it may play an important role in determining the final dynamical state. It is, therefore, essential to include realistic carrier noise when analysing semiconductor laser dynamics.

Index Terms—Noise, nonlinear dynamics, rate equations, semiconductor lasers.

I N SEMICONDUCTOR lasers, spontaneous recombination of electrons and holes results in two types of noise contribution to the laser output field, i.e., spontaneous emission noise due to a fraction of spontaneously emitted photons ending up in the lasing mode, referred to as field noise, and carrier or inversion noise due to the discrete, random and instantaneous character of each recombination event, often also referred to as shot noise. Usually, for edge-emitting lasers, it is only a small portion of the total spontaneous radiative recombination events that lead to a photon ending up in the lasing mode. These photons have random phases and, hence, lead to random fluctuations of the power and the phase of the field in the lasing mode. Another, much larger portion, of the randomly recombining carriers, decay from the positions in their respective energy states by nonradiative means, that is, without contributing a photon to the laser field. However, this does considerably alter the amount of carriers available for lasing, i.e., the inversion, and it is, therefore, quite surprising, as Henry showed [1] and others confirmed later, [2], [14], that the linewidth of the optical field emitted by a continuous wave (CW)-emitting semiconductor laser is to a very good approximation not affected by the carrier noise at all. This has led, quite unintentionally, to the general belief that carrier noise can be ignored in numerical simulations of semiconductor laser operation.

It is the purpose of this paper to show that, in the presence of dynamics, it is of crucial importance to include in any numerical analysis, apart from the field noise, the effect of the carrier noise as well. We will give an explanation for this and illustrate our point with the example of a semiconductor laser with delayed optical feedback. This example was chosen not only because of the large attention it receives as an object of study for generating chaotic or other complicated type of dynamics that can be used, for instance, in chaotic encryption applications [3], but also because of its paradigmatic status as a nonlinear delay system [4], [5].

The single-longitudinal-mode description of a semiconductor laser in terms of rate equations has proven to be a fruitful approach for analysing the dynamics that occur in perturbed semiconductor lasers [5]–[8]. In this description, the spontaneous recombination noise is included as Langevin forces [1], [2], [14], [9], thus transforming the rate equations into a set of stochastic nonlinear differential equations. As they stem from the same spontaneous recombination events, these forces are correlated, but in view of the spontaneous emission rate into the lasing mode roughly being a factor 10^{-5} smaller than the total spontaneous carrier decay rate, this correlation is irrelevant. The reason why carrier noise has a negligible effect on the CW-laser linewidth is the following: the optical linewidth is primarily caused by low-frequency phase fluctuations, notably in the megahertz regime, and the inversion makes minimal contribution to the linewidth due to its clamping and restoring property on the microsecond time scale. On the other hand, the inversion is very susceptible to fluctuations in the gigahertz regime and since this is the regime where dynamics normally occur (because they are often induced by undamping of the relaxation oscillations), we may readily expect that carrier noise should not be neglected when the laser operates in a non-CW dynamical state.

In this paper, we will present numerical results that emphasize the dynamics that appear when a single-mode semiconductor laser is subjected to delayed optical feedback with delay time of $\tau = 2.4$ ns. We will consider both spontaneous carrier-recombination noise (carrier noise) and spontaneous photon emission into the laser mode (field noise). In previous work, we confirmed via numerical simulations that the carrier noise has no detectable influence on the linewidth of the solitary semiconductor laser in CW-operation [10], consistent with Henry [1] and Petermann [2], [14]. However, if the semiconductor laser is performing *dynamics* (for instance, as a result of the external optical feedback), we find circumstantial evidence for the carrier noise strength becoming a factor in codetermining the resulting dynamics. We will identify attractors that are stable in the presence of field noise alone, but become unstable as carrier noise of realistic strength is included in the analysis. In fact, we will demonstrate that the dynamics of semiconductor lasers are more sensitive to the strength of carrier noise than of field noise,

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which leads us to conclude that omission of carrier noise may result in an unphysical description of the dynamics, leading to incorrect predictions.

The simulations we will present here are for a semiconductor laser subject to external optical feedback. They are based on the equations of Lang and Kobayashi [6], extended with Langevin noise terms. These equations are widely used but, in most cases, only field noise is taken into account [11], [15], [16]. The rate equations for the (complex) slowly varying envelope of the electrical field $E(t)$, and the inversion $n(t)$ for the Lang-Kobayashi system read

$$\dot{E}(t) = (1 + i\alpha)\xi n(t)E(t) + \gamma E(t - \tau)e^{i\omega_0\tau} + F_E(t) \quad (1)$$

$$\dot{n}(t) = \Delta J - \frac{n(t)}{T_1} - (\Gamma_0 + \xi n(t))|E(t)|^2 + F_n(t). \quad (2)$$

Here, ω_0 is the optical free-running laser angular frequency; and the inversion $n(t)$ is taken with respect to its value at threshold. The gain is described by the differential gain coefficient ξ and α is the linewidth enhancement factor. Γ_0 is the photon decay rate, T_1 is the carrier lifetime, and ΔJ is the pump rate with respect to the threshold value. The feedback strength is represented by γ and τ is the external delay time of the light in the feedback circuit. Finally, the two random forces, $F_E(t)$ and $F_n(t)$, account for the spontaneous emission and the shot noise. They represent Gaussian white noise with zero average and their autocorrelation functions read

$$\langle F_E(t)F_E^*(t')\rangle = R\delta(t - t')$$
$$\langle F_n(t)F_n(t')\rangle = D\delta(t - t'). \quad (3)$$

As stated earlier, the correlation between $F_E(t)$ and $F_n(t)$ will be neglected. The noise strength is set to $R = 10^{12}$ s^{-1} and D will be varied between $\sim 10^{16}$ and 10^{18} s^{-1}. In [10] a calibration curve for the solitary laser and $D \approx 10^{16}$ is presented, using the same approach as ours. As first noted by Henry [1], [9], the linewidth decreases with the pump level; the low-frequency relative intensity noise (RIN) was used to calibrate the carrier noise level, in accordance with Petermann [2], [14].

We will present the analysis of the dynamics in the (η, P)-plane, where P is the photon number, $P = |E|^2$, and η is the instantaneous phase difference between the actual and the delayed laser field, (see [10] for precise definitions). The CW-modes of operation are represented in this space by points, often also denoted fixed points. These correspond to the external cavity modes (ECM). The fixed points will be labeled with a number $1, 2, 3, \ldots$ according to their corresponding η value, such that $\eta_1 < \eta_2 < \eta_3 < \ldots$. The parameter values used in (1) and (2) are given in Table I and correspond to the coherence collapse regime [4].

Equations (1) and (2) were integrated using a modified fourth-order Runge–Kutta method with a time step of 0.1 ps [8]. In the phase portraits, such as in Fig. 1(a), the projection of the phase space trajectory is plotted every 10 ps, after an initial $100\,\tau$ long transient. This enables a satisfactory visualization of the attractor in the presence of noise. In all cases presented, the total integration time was sufficiently long to yield representative phase space results. Since in numerical simulations of delay equations it is always possible to miss a certain attractor, we applied many different initial conditions in a systematic manner,

TABLE I
PARAMETER VALUES USED IN THE SIMULATIONS

Quantity	Symbol	Value
Linewidth enhancement factor	α	3
Feedback rate	γ	$5.27 \cdot 10^9$s^{-1}
External Cavity roundtrip time	τ	2.4 ns
Differential gain coefficient	ξ	$2.8 \cdot 10^4$s^{-1}
Photon decay rate	Γ_0	10^{11}s^{-1}
Carrier decay rate	T_1	0.7 ns
Threshold pump rate	J_{thr}	$1.23\ 10^{17}$s^{-1}
Pump rate	J	$\sim 1.3\ J_{\text{thr}}$
Average carrier pair number	$<N(t)>$	$\sim 10^{17}$
Spontaneous emission rate	R	10^{12} s^{-1}

as described in [8], [10]. In this method, we use all the fixed points as initial conditions and "shoot" in eight different directions for each fixed point and this gives us confidence, through earlier experience of comparison with analytical theory, that all stable attractors are indeed found.

For the purpose of demonstration, a special case has been selected in Fig. 1, where in absence of noise, six different attractors coexist: two attracting fixed points (CW-operation); two limit cycles, one of which, the "fast" one, has the relaxation oscillation period (218 ps), while the other, the "slow" one, has the roundtrip period (2.4 ns); and finally two chaotic attractors. We will use the CW-states to investigate the influence of carrier noise on the linewidth properties of the laser in absence of dynamics. The "fast" limit cycle with relaxation oscillation period [case c in Fig. 1(b)] will be used to investigate how carrier noise influences the relaxation oscillation limit cycle, while the "slow" limit cycle with period τ [case a in Fig. 1(b)] will be used to probe the effects of noise on the coherence properties of the feedback system. Finally, the robustness of the chaotic attractors (b and d in Fig. 1) will be investigated in presence of noise. It should be noted that the topological structure of phase space for our example system was numerically verified to be stable against parameter variations of up to $\pm 5\%$. This is important because it is a good indicator of the fact that we are not too close to a bifurcation point in the parameter space of the deterministic system.

In Fig. 1(a), the fixed point solutions of (1) and (2) are indicated by circles. Only fixed points 1 and 3 are stable, while all other fixed points are unstable. Earlier studies have identified that the fixed points are born pair-wise in saddle-node bifurcations, where a potentially stable fixed point (node) is born together with an unstable one (saddle) [5]. In Fig. 1(b), power time series for the attractors **a, b**, and **c** in Fig. 1(a) are depicted. The oscillation frequency of the fast limit cycle, **c**, is ω_{RO}, while the slow limit cycle, **a**, has angular frequency $2\pi/\tau$. In the projection of Fig. 1(a), limit cycle **a** follows a complicated trajectory, including several relaxation oscillation during one complete τ-oscillation. Although, the power time series of the chaotic attractor **b** suggests rather regular behavior, the phase
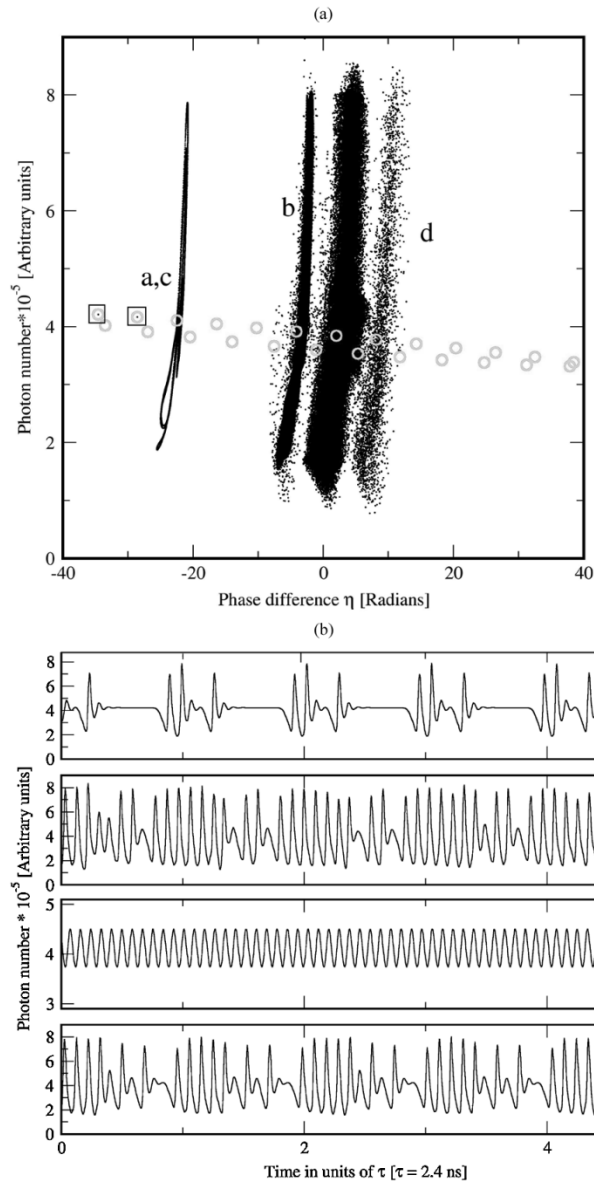
Fig. 1. (a) Phase plane projection of (1) and (2) in absence of noise in the (η, P) plane. The circles represent the fixed points (steady states) and the two states of CW-operation have been highlighted with boxes. The "blobs" show the four remaining attractors in a stroboscopic manner. (b) Time series of the photon number of the four attractors, as indicated in (a).

Fig. 2. (a) Phase plane projection of (1) and (2) in presence of field noise alone, in the (η, P) plane ($R = 10^{12}$ s^{-1}). The symbols are similar to Fig. 1(a). (b) Time series of the photon number of the attractors as indicated in Fig. 2(a).

difference moves irregularly around fixed points 11–13 (case **b**) and 13–16 (case **d**).

We now include field noise (no carrier noise) of realistic strength and the results are shown in Fig. 2. We note several changes. The fixed points 1 and 3 remain stable but are broadened by intensity and phase fluctuations, indicative of the line broadening of the corresponding CW-states. The slow attractor, **a**, has moved in phase space from its position near fixed point number 5 in Fig. 1 to fixed point 9, and its time series reveals a stronger relaxation oscillation component than in the deterministic case. It is well known that the inclusion of noise may change the nature of a dynamical system, and as such it is not surprising that the time series of the slow limit cycle has changed. Moreover, the diode laser is very susceptible to picking up noise around the relaxation frequency. Noise may also move bifurcation points of the deterministic system in
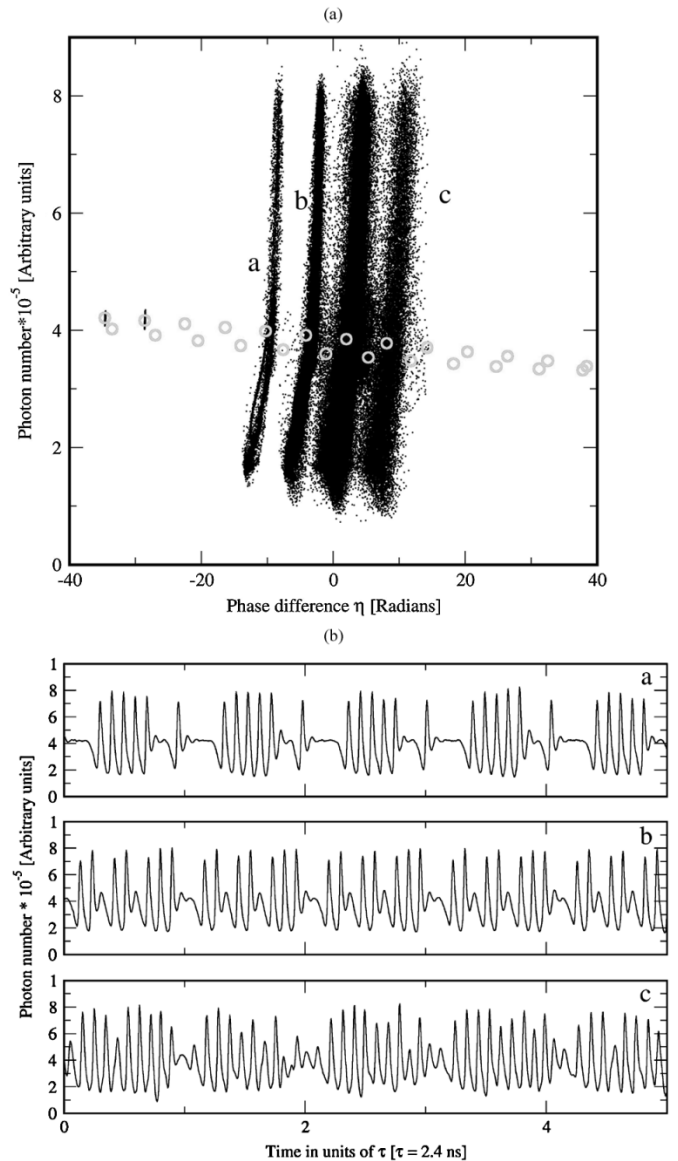
parameter space [12], and this may explain the displacement of the slow limit cycle. The fast limit cycle (**c** in Fig. 1) has completely disappeared, and lastly, the two chaotic attractors have not moved in the phase plane projection, although their time traces have changed slightly.

Fig. 3 shows what happens if we include carrier noise of realistic strength in the simulations, in addition to the field noise. The stable fixed points 1 and 3 remain stable and their broadening is not affected by the inclusion of carrier noise. The slow limit cycle **a** has moved "backward" to fixed point 3 where it coexists with the stable fixed point. Also, its time series has significantly changed compared to Figs. 1 and 2, containing fewer relaxation oscillations and longer periods of time-independence. This may be explained by the fact that the carrier noise is resonantly enhanced near the relaxation frequency while insignificant at the roundtrip frequency. Therefore, this attractor manages to reproduce its motion after each roundtrip by minimizing its relaxation oscillation content. The chaotic attractors maintain
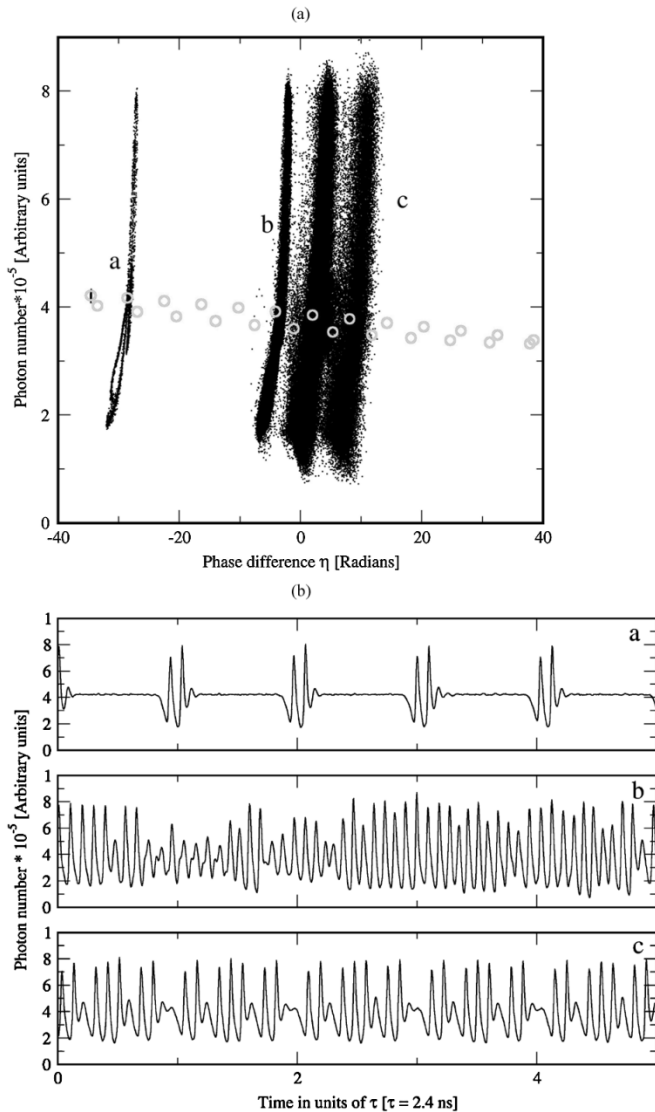
Fig. 3. (a) Phase plane projection of (1) and (2) in presence of field noise and carrier noise in the (η, P) plane ($R = 10^{12}$ s^{-1} and $D = 10^{17}$ s^{-1}). The symbols are similar to Fig. 1(a). (b) Time series of the photon number of the attractors as indicated in Fig. 3(a).

Fig. 4. Phase plane projection of (1) and (2) in presence of field noise alone ($R = 10^{13}$ s^{-1}).

their position in phase space, while their time series are again different from those in Fig. 2.

It has now been made clear that inclusion of carrier noise changes the dynamics and this is most evidently seen in the slow limit cycle **a**. In general, limit cycles can survive in the presence of noise only if the noise does not affect their periodicities significantly. Given the enhanced susceptibility to noise of the laser near the relaxation frequency, this explains why the relaxation oscillation limit cycle disappears when only field noise is added, while a roundtrip oscillation survives even with carrier noise, although it has moved so as to minimize the relaxation oscillation content.

Further evidence for the important influence of carrier noise on the dynamical properties was obtained by comparing the dynamics when the noise sources were varied in strength. For this purpose, we first neglect the carrier noise and increase the field noise strength by a factor of 10, relative to the noise strength in Fig. 2(a). The resulting phase portrait shows a broadening in the η-direction of all attract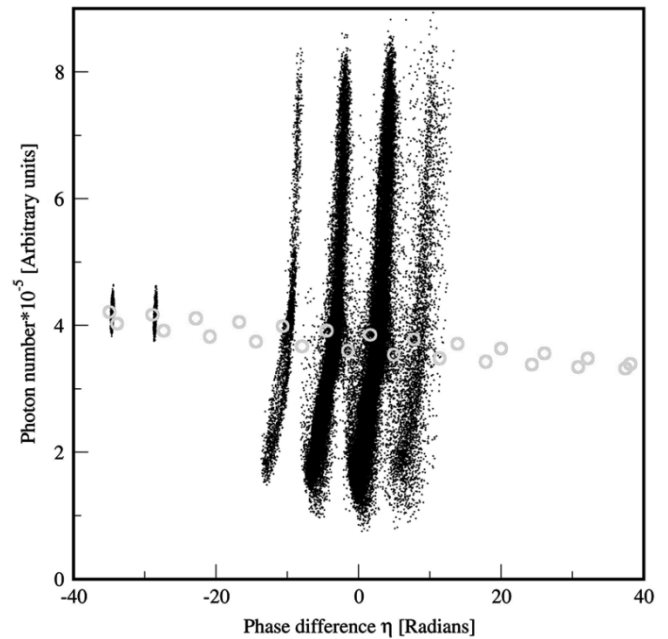ors (Fig. 4), but apart from that the portrait is not different from Fig. 2(a). Hence, the influence of realistic carrier noise could not be simulated by simply increasing the strength of the field noise to unrealistic values.

Next, we decreased the carrier noise diffusion strength by a factor of 10 compared to Fig. 3(a), and observed that the phase portrait [Fig. 5(a)] is qualitatively similar to the case without carrier noise of Fig. 2(a). On the other hand, increasing the carrier noise diffusion strength by a factor of 10 compared to Fig. 3(a), makes the slow limit cycle disappear [Fig. 5(b)]. Also, the CW-states (fixed points 1 and 3) show larger excursions in the P-direction as compared to Fig. 3(a). These results indicate that the regular attractors, such as limit cycles or tori, tend to be more sensitive to carrier noise than to field noise. The CW-states (fixed points 1 and 3) remain almost unaffected, apart from some broadening, and the same holds for the chaotic attractors.

In conclusion, we have presented circumstantial evidence that the type of noise that is included in the analysis profoundly affects the nonlinear dynamics of a semiconductor laser, modeled by rate equations. This point is illustrated using the Lang–Kobayashi equations with added Langevin noise terms, which describe a semiconductor laser with delayed external optical feedback—a representative example of a semiconductor laser system that exhibits nontrivial dynamics. We compared the numerically predicted dynamics in the presence of field noise only, with those where the carrier noise was also present, and concluded that there is no justification for ignoring the carrier inversion noise in simulations of semiconductor laser dynamics. Neglecting carrier noise may be justified though for CW-operation of the laser, and this confirms the findings of Henry [1], [9] and Petermann [2], [14] for linewidth broadening. However, in the cases with nonlinear dynamics that we investigated, the dynamics tend to be equally sensitive to changes in the strength of the carrier noise as to changes in the field noise. Moreover, the two noise sources, i.e., inversion and field noise, give rise to clearly distinguishable types of stochastic dynamics.
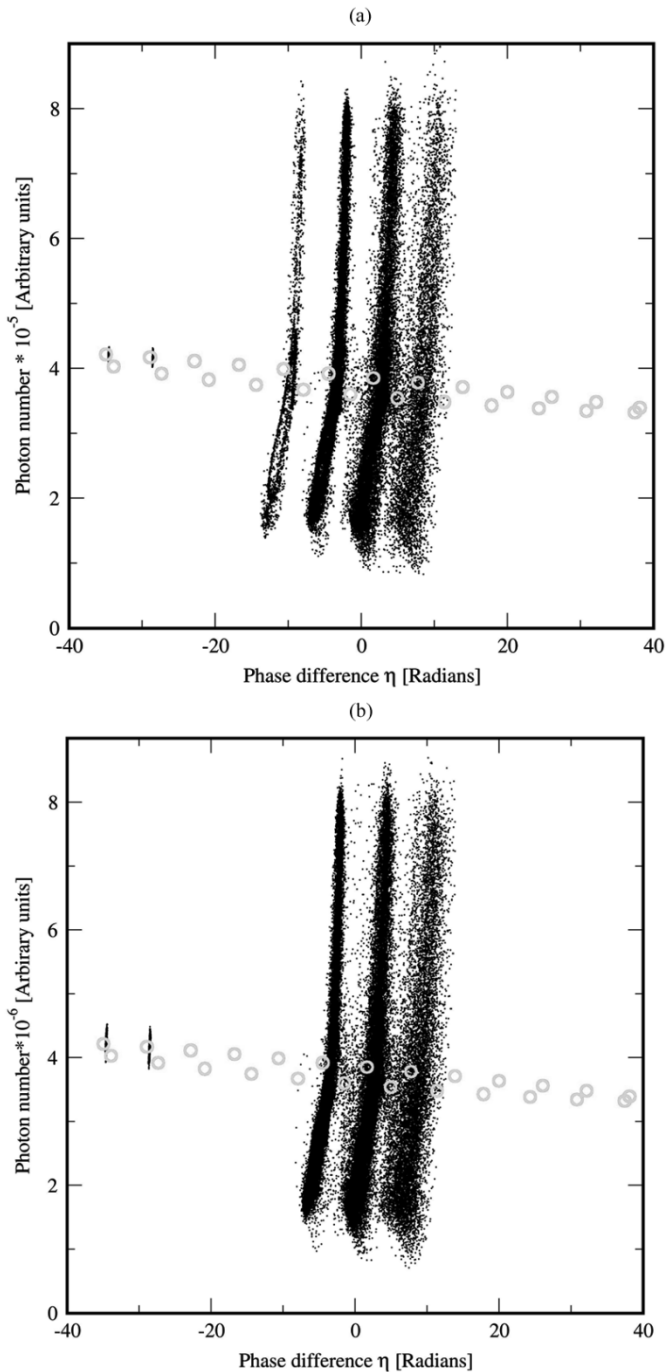
(a)

(b)

Fig. 5. Phase plane projection of (1) and (2) in presence of field noise and carrier noise. (a) $R = 10^{12}$ s^{-1} and $D = 10^{16}$ s^{-1}. (b) $R = 10^{12}$ s^{-1} and $D = 10^{18}$ s^{-1}.

It is well known that the presence of noise in a deterministic dynamical system will change the location of the bifurcation points in parameter space. It may also change the phase space structure, resulting in different attractors and basins of attraction. Therefore, if one would attempt to predict the dynamical behavior of a semiconductor laser system on the basis of a rate equations model, one should take into account the correct field and carrier inversion noise. Although these conclusions were drawn from results obtained for a semiconductor laser with feedback, they should hold quite generally and in particular for an optically injected semiconductor laser. In this latter system, a

variety of attractors, such as fixed points, limit cycles, tori, and strange attractors are known to occur [7]. A noise analysis for this system was reported by Liu [13], but unfortunately without taking the carrier noise into account.

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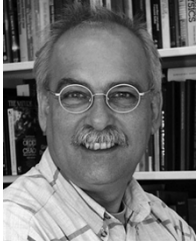
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