E-textbooks and connectivity: proposing an analytical framework

E-textbooks and Connectivity: Proposing an Analytical Framework

Abstract

This paper is concerned with the development of e-textbooks. We claim that analysis (and design) of e-textbooks require the development of a specific frame. Digital affordances provide particular opportunities (e.g. in terms of interactions between users) that require specific considerations for their analysis, as teachers and students use them for their individual and collective purposes. In this study we develop a framework for mathematics e-textbook analysis, based on the notion of "connectivity". We introduce criteria to assess the different aspects of connectivity and build an analysis grid for e-textbooks. We illustrate the framework proposed by analyzing two commonly used French grade 10 mathematics e-textbooks. The results of the analyses show that there are major differences between the two e-textbooks in terms of connectivity, which can be related to differences in their design. Beyond these two examples, we claim that focusing on connectivity is a useful and relevant way of analyzing e-textbooks, as it can provide a window into issues of interactivity, both practically, and cognitively.

Keywords

Connectivity; Design; Digital resources; E-textbooks; Mathematics textbooks

Introduction

Mathematics teachers around the world use a variety of resources, inside and outside the classroom when planning and enacting classroom activities. One of the crucial resources for teaching, it is said, is the textbook, and the textbook is considered to be the most important component of a reformed curriculum (Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002; Authors, 2001). Numerous studies have shown that curriculum materials, including textbooks, play a central role in mathematics teachers' instruction (e.g. Remillard, 2000; Grave & Pepin, 2015; Lepik, Grevholm, & Viholainen, 2015). Moreover, textbooks are said to offer learning opportunities for students (Pepin & Haggarty, 2001). For example, Tőrnroos (2005) measured the mathematical topics included in Trends in Mathematics and Science Study (TIMSS 1999, see Mullis & IAEEA, 2000) using three categories: learning opportunities in the content of each chapter; learning opportunities offered by teachers; and the content of the textbooks in terms of material presented. He concluded that textbooks are the best source for providing learning opportunities, and furthermore the best indicator for measuring them. According to Schmidt *et al*. (2001), learning opportunities are an important factor in explaining differences in student performance, and Schmidt (2012) claims that: "how textbooks are designed provides a window into the nature of the mathematics that students are expected to learn […] They mediate between instruction and the actual behaviors that the students undertake as a part of their learning" (p. 143).

In the current digital age we are surrounded by technology, and this applies to textbooks too, which have become "digital textbooks"; we term them *e-textbook* here (for a definition, see section 2 of this paper): they are said to be interactive and to include digital resources associated with particular learning sequences. These developments and changes, due to technology, are likely to lead to a re-examination of our perceptions on teaching and learning of mathematics, of our understandings of what knowledge is and the associated teaching methods, and of key-characteristics for technology/digital resources with regard to teaching and learning mathematics.

Hence, researchers investigating e-textbooks need new analytical dimensions and frames. We refer here to the theoretical perspective of the *documentational approach* (Authors 2009, 2012a). According to this perspective, the possibilities of use and adaptation for the teacher are especially relevant for the analysis of a digital teaching resource: we consider indeed that the work of the teacher with resources, which we term *teacher* *documentation work*, is central in her professional activity. The teacher chooses resources, modifies them, combines them and develops his/her own *resource system* (meaning the whole set of teacher's resources, appropriated, designed and structured by her along her teaching activity, Authors, 2012b). The possibilities of adaptation of an e-textbook: downloading parts of it, inserting in it resources designed by the teacher, and more generally the possibility of establishing links between the teacher's resource system and the e-textbook, are central factors shaping the teacher's use of the e-textbook.

In this study we have chosen to introduce and use the notion of "connectivity" as a critical feature of e-textbooks, and beyond the case of textbooks, as a critical feature of teachers' working environments in time of digitalization (Authors, 2016b). This choice allows us to develop a frame for analyzing e-textbooks, and to see what technology/digital resources might have to offer in terms of support for quality teaching in a digital age.

Our paper is divided into eight sections: after this introductory section (1), we present our theoretical frame (2), in particular the concept of *connectivity*. We describe our methods, in particular the choice of the two e-textbooks (3); and subsequently present the development of an analysis grid drawing on the concept of connectivity (4). We apply this method to analyze the two e-textbooks chosen (sections 5 and 6), and to compare these two books concerning their connectivity (7). Finally, we present our conclusion (8).

Our research is part of the French ReVEA[1] project, which concerns the interactions between secondary school teachers and their resources in four disciplines, including mathematics.

Theoretical Frame

Definition of an e-textbook and its different types. Over the past decade, mathematics textbooks have changed due to an abundance of digital resources available on the web: hard copy textbooks have changed to e-textbooks, more or less interactive, and the variety of resources at teachers' disposal (e.g. on the web; as accompaniment to their textbooks) when planning and enacting their lessons has increased enormously. In (Authors, 2016a), we define *e-textbooks* as:

"an evolving structured set of digital resources, dedicated to teaching, initially designed by different types of authors, but open for re-design by teachers, both individually and collectively." (p.644)

We have also proposed to distinguish between three kinds of e-textbooks:

"1- the *integrative e-textbook* refers to an 'adds-on' type model where the digital version of a (traditional) textbook is connected to other learning objects [..];

2- the *evolving or 'living' e-textbook* refers to a accumulative/developing type model, authored where a core community (e.g. of teachers, IT specialists) has authored a digital textbook, which is permanently developing due to the input of other practicing members/teachers [..];

3- the *interactive e-textbook* refers to a 'toolkit' model where the e-textbook (authored to function only as an interactive textbook) is based upon a set of learning objects: tasks and interactives (diagrams and tools) that can be linked and combined." (Authors 2016a, p. 640).

Connectivity: a theoretical construct to analyze e-textbooks. Traditionally, textbooks have been analyzed with respect to different criteria (see e.g. Fan, 2013), often using quantitative measures (e.g. in TIMSS), by policy makers and by researchers. In Brazil, the Ministry of education has appointed a national commission, composed of teachers, researchers and inspectors, in charge of assessing all the textbooks on the basis of a grid[2], considering issues of language ergonomics, citizenship, and of interdisciplinary aspects (Bellemain et al., 2016). Furthermore, mathematics education researchers have considered textbook analyzes at (a) macro level; and at (b) micro level:

(a) A macro level analysis would include what Charalambous *et al*. (2010) call the 'horizontal' analysis of the textbook, where the textbook "is examined as a whole, as part of technology in the educational system" (p. 119). Studies on this level would include those that focus on general textbook characteristics (e. g. Schmidt *et al*., 1997);

(b) At micro level, different studies have identified ways of analyzing textbooks for 'quality' according to their particular aims (e.g. Österholm & Bergqvist, 2013). Often, in these studies, a single mathematical concept is examined (e.g. Li, 2000; Mesa, 2010), viewing the textbook as an "environment for construction of knowledge" (Herbst, 1995, p. 3). Charalambous *et al*. (2010) would call this a 'vertical analysis'. Other examples of vertical textbook analyzes are investigations of reasoning and proof in school mathematics textbooks (e.g. Stylianides, 2009); or international comparisons, for example, of how textbooks teach mathematical problem solving in different countries (e.g. Mayer, Sims, & Tajika, 1995). In these studies, the mathematical tasks proposed by the textbook typically provide the basis for analysis, in particular when opportunities for learning mathematics are the focus (Valverde *et al*., 2002), or indeed 'learning mathematics with understanding' (e.g. Hiebert & Carpenter, 1992). Questions such as the following may be asked: Do these exercises only require the application of procedures? Do they propose to combine different representations of mathematical objects? Do they foster a richer mathematical activity for the students (Newton & Newton, 2006)?

Going back to what we regard as the nature and construction of knowledge, and the implications for mathematics teaching, students' conceptual understanding and their expertise in terms of mathematical thinking, reasoning and problem-solving has often been linked to "making connections" and "connectivity". Hiebert and Carpenter (1992), for example, propose that it is essential to make connections in mathematics, if one intends to develop mathematical understanding and knowledge. According to them, understanding can be defined as:

"…the way information is represented and structured. A mathematical idea or fact is understood if its mental representation is part of a network of representations. The degree of understanding is determined by the number and strength of the connections." (p. 67).

Moreover, in recent years an epistemological position, *connectivism*, has emerged, and this is particularly relevant considering digital technology tools, and digital curriculum materials such as e-textbooks. Siemens (2005) has introduced this epistemological position as a new theory of learning, taking into account the deep modifications of learning processes in the digital age. He argues that:

"*connectivism* presents a model of learning that acknowledges the tectonic shifts in society where learning is no longer an internal, individualistics activity. … Learning (defined as actionable knowledge) can reside outside of ourselves (within an organization or a database)." (p. 56)

This explanation has motivated us to look

- for connections in, between, and across individuals' cognitive/learning tasks and activities, and how e-textbooks may support those (*micro* level*)*; as well as
- for 'connected learning' between and across groups of individuals, teachers or students (*macro* level).

It is said that the principles of *connectivism* include in particular the "ability to see connections between fields, ideas, and concepts" (as a 'core skill"; and "Nurturing and maintaining connections is needed to facilitate continual learning" (p. 56). Downes (2008) states that:

"at its heart, *connectivism* is the thesis that knowledge is distributed across a network of connections, and therefore that learning consists of the ability to construct and traverse those networks. … [connectivism] implies a pedagogy that:

(a) seeks to describe 'successful' networks (as identified by their properties, which I have characterised as diversity, autonomy, openness, and connectivity) and

(b) seeks to describe the practices that lead to such networks, both in the individual and in society- which I have characterised as modeling and demonstration (on the part of the teacher) – and practice and reflection (on the part of the learner."(p. 57).

Interestingly, the works of Confrey and her team (e.g. Confrey, Maloney, & Corley, 2014) provide an example of "connecting standards with curriculum" through learning trajectories: the team has developed connections, and they explain the process of linking standards (e.g. the Common Core Standards for Mathematics in the United States) to learning trajectories, illustrating the "rich connections possible among standards, [their] descriptors, and elaborated learning trajectory, and related curriculum materials." (p. 719)

Even if comparing e-textbooks and traditional textbooks on paper is not our focus here, the possibilities of connectivity offered by e-textbooks compared with traditional textbooks motivates our interest for these questions. We distinguish here between two levels, as evoked above: (a) *macro*; and (b) *micro* (see Authors, 2016b for details).

(a) Connectivity at macro level refers to the potential of linking to and between subjects/users and resources/tools outside the textbook. It includes the potential to create virtual communities, connecting users with users (both teachers and students), as well as users and designers, and the textbook's interaction with other resources, via web links, or on

platforms, for example. More generally, we have argued (ibid.) that this macro level connectivity could include the following criteria:

- Connections across grades; connections to the national curriculum; connections with other disciplines;

- Connection to an assessment system;

- Connections to other resources (files to download or websites of different kinds);

- Connections between the textbook and teacher *resource systems* (for synergetic effects);

- Connections in terms of teacher collective work; connections between teacher/s and students; connections between teachers using the textbook and the authors of the textbook.

Connectivity at micro level refers to connections made inside the e-textbook. It concerns the specific mathematical content, i.e. that the e-textbook offers different kinds of combined materials (which can be definitions, properties, exercises but also, in the case of an e-textbook, software files, videos etc.). At micro level connectivity could include the following criteria:

- Connections between different topic areas;

- Connections between different semiotic representations (e.g. text, figures, static and dynamic); connections between different software/s for carrying out a particular task;

- Connecting different concepts; connecting different strategies for problem solving - this is linked to the issue of procedural vs problem-solving tasks as proposed by the textbook; connecting different moments of appropriating a given concept (e.g. spiral progression, progressively deepening a concept instead of proposing a complete presentation of it in the same chapter);

- Connection to assessments (concerning the particular mathematical content studied);

- Connection to different students' needs.

In summary, we have outlined the different ways of "making connections" within and from/to an e-textbook. The connectivity of a mathematics digital resource, in our case an e-textbook, could be defined as its "*connecting potential for a given user (student or teacher) both practically as well as cognitively*".

The aim of the work presented here is to propose an analytical framework for e-textbooks, drawing on the concept of *connectivity* as introduced above, and to investigate if differences in terms of connectivity appear for e-textbooks of different types. Hence our research questions can be formulated as:

(1) What are the factors/dimensions that should be considered for developing a framework for analyzing mathematics e-textbooks in terms of *connectivity*?

(2) What are, considering these factors/dimensions, the similarities and differences between e-textbooks of different types, in terms of *connectivity*?

Drawing on the criteria presented above, we developed an analysis grid for e-textbooks. We also chose two different types of e-textbooks (one "integrative", and one "evolving"; there are no interactive e-textbooks in France). An initial version of the grid had been tested on the first e-textbook chosen, refined, and tested on the second one. The design of this grid can be regarded as a cyclic (design) process, which is likely to evolve after further e-textbook analyses. Nevertheless, we consider that the present state of the grid/analysis schedule is rich enough to be discussed with the readers of this article.

Methodology

This section is divided into three parts, relating to (1) the sampling of the two textbooks; (2) the choice of the mathematical topic for applying the grid; and (3) issues of validity and reliability, in particular in terms of inter-researcher reliability.

(1) The first step relates to the sampling of our e-textbooks: we have purposely chosen "contrasting" e-textbooks (relating to the three types: *integrative, evolving, interactive/toolkit type*), in order to be able to develop a deeper understanding of their potential to connect, at the levels described. Ideally, we would have liked to analyze and compare three e-textbooks: one for each category we identified. However, at the moment there does not exist a *toolkit* e-textbook in France – publishers seem to be keen to develop such an e-book (personal communications), but at the time of our study it did not exist. Thus, we decided to choose one *integrative* e-textbook; and one *evolving* e-textbook.

Our study took place in France; it started in 2014. In 2014 the commercial publishers proposed new textbooks and e-textbooks for grade 10 (whilst the official curriculum had not changed): these e-textbooks were of the *integrative* type. Seven integrative e-textbooks were published in 2014; the official selling figures were not available. We retained the "Barbazo e-textbook", which has been designed by a team of six authors coordinated by Eric Barbazo, who was a former president of the French mathematics teacher association (APMEP[3]), hence well-known by many mathematics teachers, likely to trust him as author.

As a representative of the *evolving* type, we chose an e-textbook published by a French association, Sesamath[4], gathering teachers for collaboratively designing teaching resources, particularly textbooks and associated e-textbooks, freely available online (Authors, 2011). Hence, we included the Sesamath grade 10 e-textbook as the only one of its type.

(2) Secondly, we have chosen a mathematical theme for our analysis at the micro level: the introduction of *functions*. According to the French National Curriculum[5], the notion of function is firstly introduced in grade 9. At grade 10, the official curriculum includes, firstly, general concepts associated with functions (e.g. image, pre-image, graphical representations), variation of a function, and notions of minimum and maximum. Secondly, specific "reference functions" are studied: linear functions (already met in grade 9); second

degree polynomials; and rational functions. Moreover, equations and inequalities (linked to the variations of functions) are also studied.

The research literature (Akkoç & Tall, 2005) asserts that the theme of functions is rich in terms of didactical choices associated with potentially useful digital resources: different representations can be connected, in a first approach which can be intuitive or formal. The relationship between the definition of functions and the mental image students may develop has been investigated (Vinner, 1983): there are various symbolizations of functions, and the representations experienced by students influence their concept images. In order to develop a rich meaning for and use of the concept, students should encounter functions in different representations and make connections between these (Thompson, 1994). The following are important to include: symbolic; graphic; diagrammatic; verbal; tabular; implicit (e.g. functions emerging from parametrizing solution sets of equations).

Research on the use of technology supporting the introduction of functions has evidenced a positive impact of dynamic visualizations on concept development (e.g. Falcade, Laborde & Mariotti, 2007) with specific software or with the calculator, under certain conditions of implementation in the classroom (Authors, 1999). Some potential opportunities include the following: the use of multiple representations of functions on computers; facilitation of the use of functions for modeling purposes; software for explorations of properties of functions; software for consolidating functions as objects (e.g. families of functions). In summary, it can be said that the area of functions offers many opportunities for connectivity with digital resources, in particular in e-textbooks.

(3) Concerning inter-researcher reliability, we took four steps: (a) Two researchers developed an initial version of the analysis grid from the literature and previous research; (b) this initial grid was trialed and tested by a team of two researchers (one of them who developed the grid) on the Sesamath e-textbook; (c) two other researchers subsequently

trialed the grid on the Barbazo e-textbook; (d) amendments and explanations were added to the different categories, in back-and-forth discussions, until inter-rater reliability was established, and both e-textbooks were then analyzed on the basis of these established understandings by the five authors of this paper.

Developing and using the analysis grid

We built the initial version of the analysis grid drawing on our definition of connectivity. Thus we firstly followed the distinction between macro and micro levels, which led to a grid in two parts; we subsequently considered all the criteria, and "translated" them in terms of possible actual features of an e-textbook. For a given category, we firstly noted whether a connection existed or not; then, if there was a connection, we described the nature of this connection (web link, quote, file to download etc.).

Table 1

Macro level grid (resources network)

Connections to/in terms of
Official text of the National Curriculum
Teacher guide
Across grades to: textbooks of the same family (resp. making links to notion before and notion follow up of a given notion of the given textbook)
Across disciplines to: textbooks of the same family (resp. making links to related notions linked, in the other disciplines, to the math. notion at stake)
Exam texts, assessments
Website or other resources authored by the editor of the textbook
Other websites or resources (not authored by the editor of the textbook), in particular open resources, collaborative websites
Software, Calculators. Different from web links to a free software or calculator: mentioning the use of software, giving a tutorial in the textbook etc.
Possibility of annotations
Teachers' own resources: possibility for the teacher to incorporate his/her own resources in the textbook.

Possibility for the teacher to download parts of the textbook, to incorporate elements of the textbook at stake in her own resources (in her progression, in her lesson plan, in her repertoire of exercises...
Teachers' collective work: e.g. forum, possibility to build resources collectively, to share resources etc.
Teachers' and students' joint work: e.g. space for discussion, possibility for the teacher to propose homework, for the students to upload homework etc.
Teachers and textbook authors or editors
Other joint work or communication: between students and students: e.g. space for discussion, for working collectively; between teachers and parents etc.

At macro level the connections mostly happen between the e-textbook and external resources. The connections with the national curriculum, with other grades or other disciplines are taken into account by describing if connections exist with the official text of the National Curriculum, with textbooks of other grades or other disciplines, and under which form (a web link, an extract that can be downloaded etc.). The teacher's guide, when it exists, is also an important resource that can take different forms and thus be connected in different ways with the e-textbook for students (e.g. via windows opening when browsing a special part of the e-textbook).

Concerning the connections with websites, we distinguish between websites authored by the publisher of the e-textbook (which have been designed to complement the e-textbook; and following these links, the teacher remains in the resources system of the publisher), and websites with other authors. We observe in particular the potential connections to collaborative websites: following such connections, the teacher can him/herself become involved in a new community, outside of the publisher's resource system (and this could be deliberately avoided by a commercial publisher).

For assessing the potential connections between the e-textbook and teachers' resource systems, we distinguish between two categories:

- The possibility for the teacher to incorporate his/her own resources in the e-textbook, e.g. uploading a file, or annotating the text;

- The possibility for the teacher to download selected parts of the e-textbook: specific sections, or any part of a section; as a static (pdf) file, or under a format which permits modifications.

Table 2

Micro level grid (for a given set of concepts)

Connections to/in terms of
Previous knowledge (learned in previous chapters in the same textbook)
Further knowledge (role of the chapter considered in further chapters).
Different concepts
Different moments of appropriation of the same concept
Different topic areas within mathematics
Authentic situations, real-life problems, problems referring to other disciplines
Different semiotic representations
Different software and calculator
Different strategies for the same exercise
Variations of the same exercise: same exercise with different values, different exercises around the same question etc.
Different students' needs: presence of exercises for high-achieving students, for students who have difficulties etc.
Assessment procedures and storage of results

At the micro level, we consider a part of the e-textbook corresponding to a given mathematical theme: e.g. probabilities, statistics, functions (this part often constitutes one or two chapters in the e-textbook). The connections observed can happen within this part of the e-textbook, or between this part and other parts within the e-textbook. Some of these connections concern the "progression" chosen by the authors: connection with previous knowledge, with further knowledge, between different moments of appropriation of the same concept. Others concern networking different concepts within the same mathematical topic, or across different topics, or even across different disciplines, for example through the use of the mathematical topic learned to solve problems in economy, in physics, biology etc.

Analyzing Connectivity: First Case, the Sesamath E-textbook

We start this section with a description of the grade 10 Sesamath e-textbook structure; of a network of resources associated with it; and of the function chapter content. We analyzed it in terms of macro and micro level connectivity using all the categories of the grids; we present the main conclusions of this analysis (a complete summary is presented in the comparison section).

The Sesamath e-textbook in a complex resource system with associated services. The Sesamath e-textbook links different resources to different users:

- for the student: the e-textbook integrates various digital resources (figure 1), and has links to interactive exercises (Mathenpoche)*;* a corresponding paper textbook can be bought;

- for the teacher: the same e-textbook and exercises are accompanied by a teacher's guide and different platforms (e.g. Sesaprof) where teachers can discuss in forums (together and with the authors). It is integrated in LaboMEP (see below).

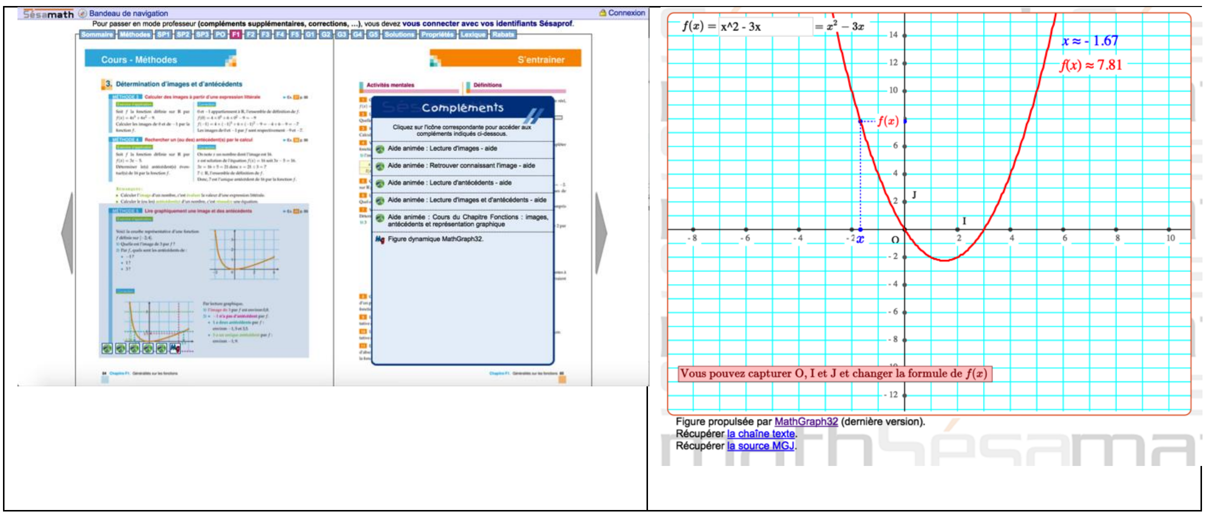

Figure *1*. The Sesamath e-textbook, a double page on functions

The teachers (with their login and password, freely offered to all teachers) have access to many Sesamath resources, outside of the e-textbook. The most important is LaboMEP, a platform where the teachers have access to all Sesamath resources, can upload their own resources and can manage their communication with students. Teachers can create different tasks for different groups of students; so the use of the e-textbook by students must always be considered according to what the teacher proposes in LaboMEP.

Introduction of functions in the Sesamath e-textbook. The theme of functions is divided into four chapters: "Functions, generalities", "Solve an (in)equation… (or not)!", "Variations and extrema" and "Factorisation and study of signs". For this analysis we focus on the first chapter: "Functions, generalities".

The chapter starts by recalling some knowledge learned at grade 9; if proposes then introductory activities (problems referring to geometry, physics, meteorology & arithmetic). These activities are followed by a section "course-methods", offering definitions, exposition of methods, and applications with correction concerning: functions; reference functions; different representations of a function; determination of images and pre-images. The following part of the chapter proposes exercises, organized into three categories: "practice"; "deepen"; student self-assessment and "multiple choice questions". In the last part, the authors expose four "Practical Work" activities (PW activities) linked with geometry, physics and algorithmics.

Analyzing Sesamath e-textbook connectivity, macro level. The Sesamath e-textbook does not offer a proper "teacher guide", but many complements for teachers: the solutions of the exercises; comments about the objective of each chapter, for example. The teacher has access to other platforms developed by the Sesamath team (e.g. Sesaprof or LaboMEP). Students and teachers can download the whole book, or each chapter, either in pdf or in Tex, make changes and adapt according to their objectives and teaching context.

We also note a connection to assessment: in each chapter two tests are proposed for the student. At the beginning of the chapter, to test what s/he knows already; and at the end of the chapter, what s/he has learnt. The connections with software or calculator are limited; but many connections exist to digital resources: e.g. animated helps, exercises or dynamic representations of functions.

Through the Sesamath website, connections are possible with the other level e-textbooks; but not with e-textbooks for other disciplines, since Sesamath naturally only produce resources for mathematics teaching.

Analyzing Sesamath e-textbook connectivity, micro level. In this introductory chapter, there is clearly a strong emphasis on the *connections between concepts,* which in this chapter are: functions, domain of a function, representative curve, table of values, image and pre-image. Links between the notions of functions, image, pre-image are presented in this chapter (both in the lesson and in the exercises). Concerning the representations many connections between the different semiotic representations are presented. We note a particular emphasis on the articulation between the graphical representation and the table of values.

The *connections with real-life situations* or other disciplines are present, but in special parts of the textbook: in the introductory activities, in the "deepening" exercises and in the Practical Work (PW) activities. The *connections between different methods* for solving the same exercise or problem are not explicitly mentioned. The authors seem to prefer focusing on a given method and to implement it by using variations of the same exercise (connections between different versions of the same exercise). There are *connections with dynamic representations*: dynamic representations that offer possibilities to articulate numerical and graphical representation. The possibility of using dynamic geometry, calculator and spreadsheet is mentioned for many exercises.

Concerning *connections to different students' needs*, there are "step-by-step" solutions of the exercises and the "animated help" for students with difficulties, and PW activities for high achieving students.

Analyzing Connectivity: Second Case, the Barbazo E-textbook

The "Barbazo e-textbook" for grade 10 has been published by one of the most famous (in terms of economic figures) French publishers: Hachette. Our analysis of the "Barbazo e-textbook" is organized like the analysis of the Sesamath e-textbook in the previous section.

The Barbazo e-textbook in a complex resource system with associated services. The publisher's website presents a resource system, called the "Barbazo collection", composed of materials for students, and materials for the teacher:

- for the student: a paper textbook (small or large size) and two digital counterparts, named *student e-textbook* (which is only a pdf version of the paper textbook) and *enriched student e-textbook* (we describe its features below);

- for the teacher: a teacher guide on paper, and two digital counterparts, named *teacher e-textbook* (the pdf version of the teacher guide), and *enriched teacher e-textbook* (see below).

In the enriched e-textbook, students and teachers can find the same text as in the paper textbook, but here they can annotate it; select parts of it to keep in his/her own space; add to this space personal resources (files of different kinds, weblinks). The e-textbook also offers additional resources: animated presentations of the main points of the chapter; a tutorial for using his/her calculator; a quiz for self-testing (see figure 2 below).

A teacher is offered different possibilities:

- each teacher may have a free *working space* and a *personal library* on the publisher's website;

- once identified, the teacher can freely download a set of resources: the teacher guide; slides associated with the chapter; software files and associated exercises; slides for mental computation, in particular;
- The publisher's website also proposes a service for managing a class online: "the virtual class 2.0", allowing teachers to propose specific tasks to their students and follow their work.

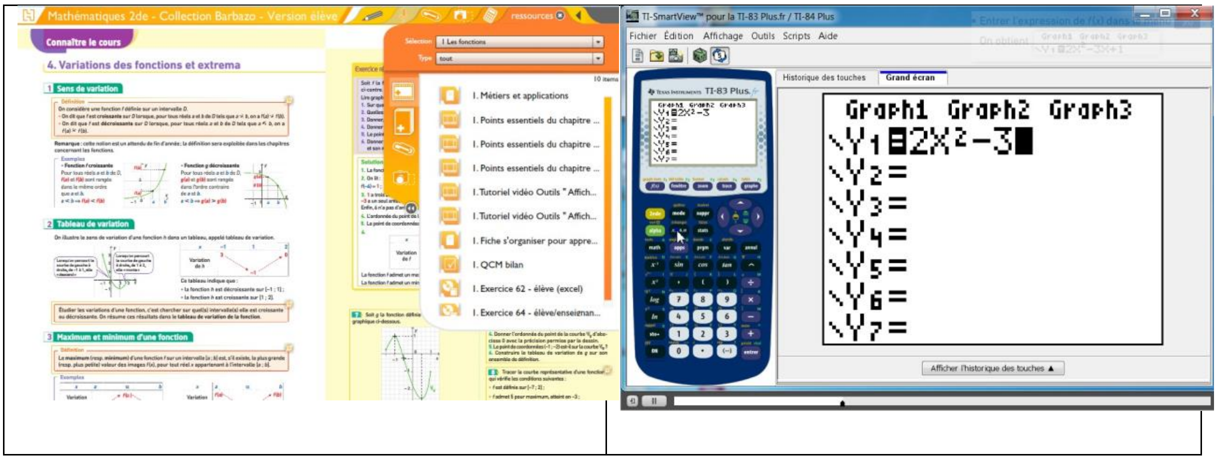

Figure 2. The Barbazo students' e-textbook enriched version.

Introduction of functions in the Barbazo e-textbook. In the Barbazo e-textbook, a chapter is devoted to the introduction of functions. It comprises five subsections: Notion of function; Three ways to represent a function; Graphs and Functions; Variations of functions and extrema; Solving graphically equations and inequalities.

The chapter starts with revisions from grade 9; then introductory activities linked with geometry, physic, economy, programming, biology, technology, medicine, human sciences. Then the course is presented, each page of course is associated with one page of worked examples and simple application exercises. It is followed by "tools and methods", with four examples: two on the calculator, one on algorithms, one on solving second degree equations algebraically. One page called "Organisation for learning" comprises a simple test, and a synthesis on functions with holes to be complemented. Then the book proposes exercises, organised as follows: firstly exercises linked with the "students' capacities" (coming from the official curriculum); then *mental* activities; simple exercises; medium-level exercises; *complex* exercises, *synthesis* exercises; one page for « personalised support », and finally exercises in English, for bilingual classes.

In addition, the e-textbook offers one pdf page about professions (professions using functions: medicine, statistician, economist); three videos presenting a summary of the course on functions; two videos about the use of the calculator ; one multiple choice test, and two excel files corresponding to two exercises of the textbook.

Analyzing Barbazo e-textbook connectivity, macro level. The e-textbook offers connections to the official curriculum (as a pdf file which can be downloaded); to calculators or software. Many connections are possible through the publisher's website: connections to textbooks of the same publisher for other grades or other disciplines; connection to the teacher guide. The user may *personalize* the e-textbook, and tools are given for annotating the e-textbook; or for integrating user resources in the e-textbook. Hence, these are means to connect the e-textbook with the user's resource system, mainly by bringing the user's resource system *inside* the e-textbook. The web 2.0 virtual class offers a working space, connecting the teacher and his/her students outside of the e-textbook, but still within the publisher's website. The working space of the user is not open, it remains within the publishers' spaces. Some exercises are mentioned as authored by the APMEP association but without a link towards the APMEP website[3].

Analyzing Barbazo e-textbook connectivity, micro level. For introducing functions, in this e-textbook there is clearly a strong emphasis on the *connections between concepts*, and *connections between representations*. Concerning the concepts, the links between the notions of functions, variations of functions, equations and inequalities are presented in this chapter (both in the lesson and in the exercises). They are linked again in further chapters concerning special kinds of functions. The same can be observed for the connections between various representations.

The *connections with real-life situations* or other disciplines are present, albeit in special parts of the textbook: in the introductory activities; and in complex exercises. The simple exercises remain within mathematics. The *connections between different methods*, for solving the same exercise or problem, are not explicitly mentioned in this textbook; nevertheless some problems can be solved with different methods.

There are *no connections with dynamic representations*; the possible use of dynamic geometry is only mentioned in one introductory activity, and in one exercise. Amongst the files offered on the textbook's website under the title "software exercises", there are spreadsheet files (actually Excel files); Algobox files; but no GeoGebra files, since GeoGebra does not allow links from commercial publishers. In the e-textbook, the links made with digital files are directed towards slideshows; video tutorials; but not to dynamic representations.

We notice that the Barbazo e-textbook offers rich connections between mathematics and other subjects, through problems/tasks which include other disciplines. It also offers rich connections between concepts, and between different representations of the concept of functions, albeit static ones, as the authors do not use dynamic representations.

Comparing the Connectivity of Two E-textbooks

Concerning connectivity at the macro level. Table 3 presents a summary comparing the different aspects of connectivity respectively at the macro level between the two e-textbooks.

Table 3

Connectivity comparison, Macro level

Connections to/in terms of	*Sesamath*	*Barbazo*
National Curriculum	Yes, link	Yes, link
Teachers' guide	Comments	Yes, pdf guide
Across grades	Yes	Yes
Across disciplines	No	Yes
Exam texts, assessments	Self assessment	Self assessment
Editor's website	Yes, with LaboMEP	Yes
Other websites or resources	Yes, with LaboMEP	No
Software, Calculators	Limited	Yes
Possibility of annotations	No	Yes
Teachers' own resources	Yes, with LaboMEP	Yes
Possibility to download	Yes	No
Teachers' collective work	Yes, with Sesaprof	No
Teachers and students	Yes, with LaboMEP	Yes, Hachette website
Teachers and textbook authors	Yes, with Sesaprof	No
Other collaboration	No	No

We observe that in both cases the e-textbook is inserted in a rich network of resources developed by the publishers. For Sesamath, LaboMEP offers to the teacher the possibility to build his/her own sessions (and potentially different sessions for different groups of students) incorporating extracts of the e-textbook, interactive exercises, software files and his/her own resources; the teacher can also follow the work of individual students. For Hachette-Barbazo, the virtual class 2.0 offers similar possibilities. A major difference here is that the Sesamath resources network is freely accessible; while the availability of the Hachette resources depends on how much is paid for, and it can be quite expensive in the case of the virtual classes; different systems of subscription exist for the schools.

We also note that, in both cases, an important connectivity is possible between the e-textbook's content and the teachers' and students' own resources. For the Sesamath's case, the teacher can incorporate his/her own resources in LaboMEP sessions, can upload parts of the e-textbook and modify them. For the Barbazo e-textbook, teachers and students have a personal space within the e-textbook to store selected parts of the e-textbook, but also their own resources. They can make annotations in the e-texbook. In the case of Sesamath, the student has access to his/her own e-textbook, to LaboMEP sessions prepared by his/her teacher, but cannot store his/her own resources in the e-textbook, nor make annotations. The possibilities of annotations for students offered by the Barbazo e-textbook are typically a feature of an *integrative* e-textbook: the original content, proposed by the authors, remains the same. But it can be complemented by the user who can add her own content, to build a personalised e-textbook. For Sesamath teachers can modify the content of the e-textbook to build their own sessions; they can also communicate with the authors, and suggest evolutions of the e-textbook content: we recognize here a typical feature of an *evolving* e-textbook.

Concerning connectivity at the micro level. Table 4 presents a summary of the comparison between the two e-textbooks in terms of micro level connectivity.

Table 4

Connectivity comparison, Micro level

Connections to/in terms of	*Sesamath*	*Barbazo*
Previous knowledge	Yes, grade 9	Yes, grade 9
Further knowledge	Yes	Yes
Different concepts	Yes	Yes
Moments of appropriation	Yes	Yes
Topic areas within mathematics	Yes, geometry	Yes, geometry
Other disciplines	Yes, but limited	Yes

Semiotic representations	Yes	Yes
Software and calculator	Limited	Yes
Different strategies	No	Yes, implicit
Variations of the same exercise	Yes, for dynamic exercises	No
Different students' needs	Yes	Yes
Assessment	Yes	Yes

In terms of introduction of functions in grade 10, we note that both e-textbooks offer many connections between the different concepts (function, graph, image etc.) and between the different representations: algebraic, graphic, table of values in particular. These connections are actually already present in the paper versions of the textbooks. The Sesamath e-textbook draws more on the digital possibilities, by offering connections with many dynamic representations. The Barbazo e-textbook offers more connections with other disciplines, and problem that can be solved with different methods. But once again, this could concern the paper textbook: there are no web links or digital resources to support the links between mathematics and other disciplines. Both e-textbooks offer connections with calculators or software. Sesamath offers moreover connections with dynamic exercises, and thus connections between different versions of the same exercise, through the interactive Mathenpoche exercises. In the Sesamath textbook, the authors have chosen to emphasize the connection on one hand between concepts introduced (table of values, image and pre-image) and on the other hand between algebraic, graphic and numerical representation. While the authors of Barbazo e-textbook have chosen to introduce the functions insisting on a different kind of connectivity: connection between different mathematics subjects and between mathematics and other disciplines. This can be interpreted as a consequence of the different design modes: in the large Sesamath authors' team, there are teachers who developed important programming competencies and where able to design the interactive digital resources. For the Barbazo e-textbook, the publisher did not choose to hire a specialist for designing dynamic resources.

Conclusions

In this paper we have introduced the concept of *connectivity* and rationalized our interest for this concept. We have defined the connectivity of an e-textbook as its "*connecting potential for a given user (student or teacher) both practically as well as cognitively*". We have used it as a theoretical tool to develop a methodology for analyzing e-textbooks. Since we distinguished in previous research different types of e-textbook (*integrative*, *evolving*, and *interactive*), our research questions where:

(1) What are the factors/dimensions that should be considered for developing a framework for analyzing mathematics e-textbooks in terms of *connectivity*?

(2) What are the similarities and differences between e-textbooks of different types, in terms of *connectivity*, as evidenced by the use of the analytical framework developed?

We actually focused on two French e-textbooks, chosen as representative of the *integrative* and of the *evolving* types. We distinguished between a *macro* and a *micro* level for analyzing connectivity.

The macro level examines the potential insertion of the e-textbook in networks of other resources: resources offered by the publisher, other external resources, or resources of the user, student or teacher. This macro level also takes into account the possibilities of modifications of the e-textbook (by the student or by the teacher), and the possibilities of networking between teachers, students or even parents offered by the e-textbook. These networking and adaptation issues are directly linked to the possibilities offered by digital resources.

The comparison of the two French e-textbooks we studied evidences commonalities in terms of macro-level connectivity: a number of connections with other resources, possibilities of collective work (but these possibilities could be more developed in both cases). It also evidences major differences:

- In the Barbazo's case, no changes of the initial e-textbook were possible for the teachers or the students. However, there were possibilities for creating a personal space within the e-textbook, including selected parts of this e-textbook and personal resources. But the teacher cannot import parts of the e-textbook in his/her resource system;

- In the Sesamath case, the teacher can download all the parts of the e-textbook, modify them and integrate them in his/her resource system; s/he can also create his/her own sessions on LaboMEP and share them with colleagues.

From these examples we hypothesize that *evolving* e-textbooks are likely to foster more connections between the users' resource systems and a rich network of resources than *integrative* e-textbooks. This hypothesis must be tested on other representatives of these two types; nevertheless, we consider that it is coherent with the differences between the two types: an *integrative* e-textbook is organized around a stable kernel, which the students and teachers cannot modify, whilst an *evolving* e-textbook is a more open network of resources. For the two e-textbooks we studied, this difference is also linked with an important difference in the authorship and economical model. The commercial e-textbook has a protected content, and the publisher tries to attract the users within his collection. If a teacher inserts her own resources within the e-textbook, she will most probably renew her subscription in due course.

The micro level examines, for each learning purpose – and we chose here the theme of functions – connections between concepts, between different representations; connections between the mathematical theme studied and other themes, or with other disciplines. These connections are likely to support a rich mathematical activity for students with the e-textbook and foster their development of cognitive connections.

Analyzing and comparing the two e-textbooks with respect to this dimension, we observed that while both e-textbooks offer connections between concepts, differences appear concerning the connections with other disciplines or between different methods (more developed in the Barbazo e-textbook), and concerning connections with dynamic representations (more developed in the Sesamath e-textbook).

We claim that these differences are linked to the type of authorship. For the Barbazo e-textbook, members the team of authors are aware of mathematics education research results, in particular concerning the interest of connections to other disciplines and between different methods; for Sesamath, the team includes ICT specialists who are able to produce dynamic representations. An evolution in the authors teams could lead to a more developed connectivity at the micro level for both types of e-textbooks.

Finally, for both levels, we note that the kind of authorship plays a major role. At macro level, the commercial e-textbook has a connectivity directed towards the publisher's productions, while the associative e-textbook has a more networking kind of connectivity. At the micro level, the authors who are more experts in math education make more connections with other fields and between methods, while the authors experts in ICT make connections with dynamic representations. At the end of this first exploratory study, we hypothesize that differences in the authorship lead both to different types of e-textbooks and to different kinds of connectivity (as a consequence, different types of e-textbooks have different connectivity).

We studied here a particular context: two French e-textbooks, with a focus on the teaching of functions for grade 10. For further studies, it would be beneficial to analyze more e-textbooks (including *interactive* e-textbooks), and other topic areas.

However, we consider that the analytic framework developed here can be used for a didactical analysis of any mathematics e-textbook, independently of the mathematical theme, classroom level or country. We claim that the concept of *connectivity* is particularly suitable in the context of teacher/student interaction with digital resources, which e-textbooks will increasingly provide. It could also be used by e-textbooks designers: our analysis evidences that the two e-textbooks do not fully exploit the connectivity potential offered by digital resources. We contend that exploiting this potential is still a major challenge for future mathematics education.

Endnotes

1. ReVEA is a project funded by the French National Agency for Research http://anr-revea.fr/
2. http://www.fnde.gov.br/pnld-2016/
3. http://www.apmep.fr/
4. http://www.sesamath.net/
5. http://www.education.gouv.fr/cid22120/mene0817023a.html

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