Are model and predictor filters equivalent?

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Studying identifiability in a dynamic network identification setting

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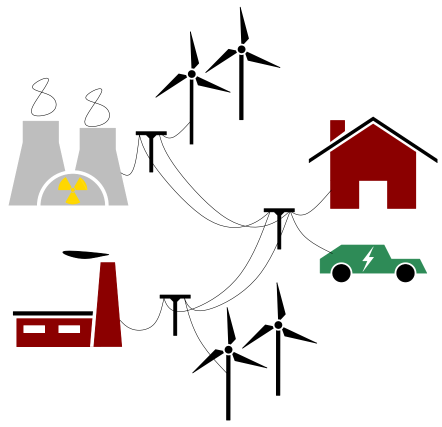

Main question

In open- and closed-loop identification there is a one-to-one relationship between (parameterized) predictor filters and model[1], is that the case for dynamic networks?

Example: Can we distinguish the two networks below on the basis of their transfers $r \to w$?

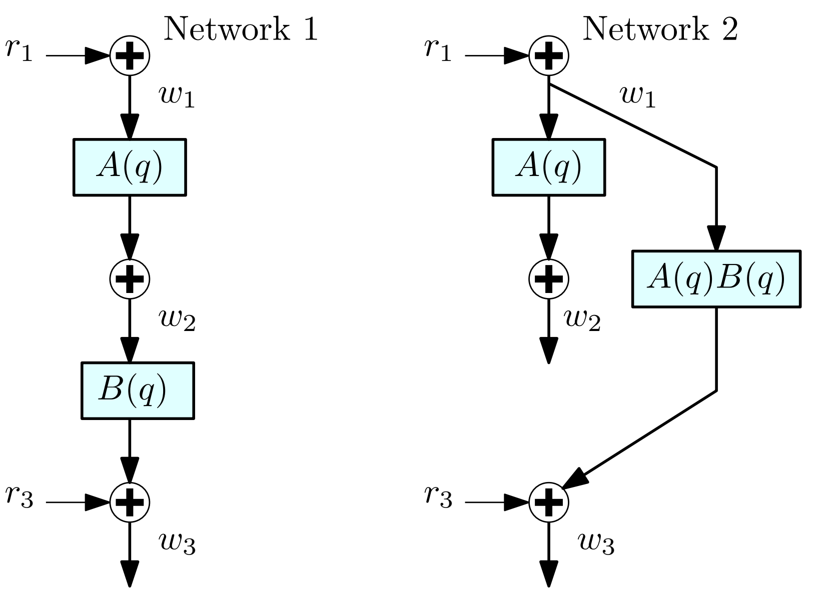

Figure 1: **OH NO!! Two models generate the same mapping from external variables r to internal variables w.**

Network, predictor, model structure

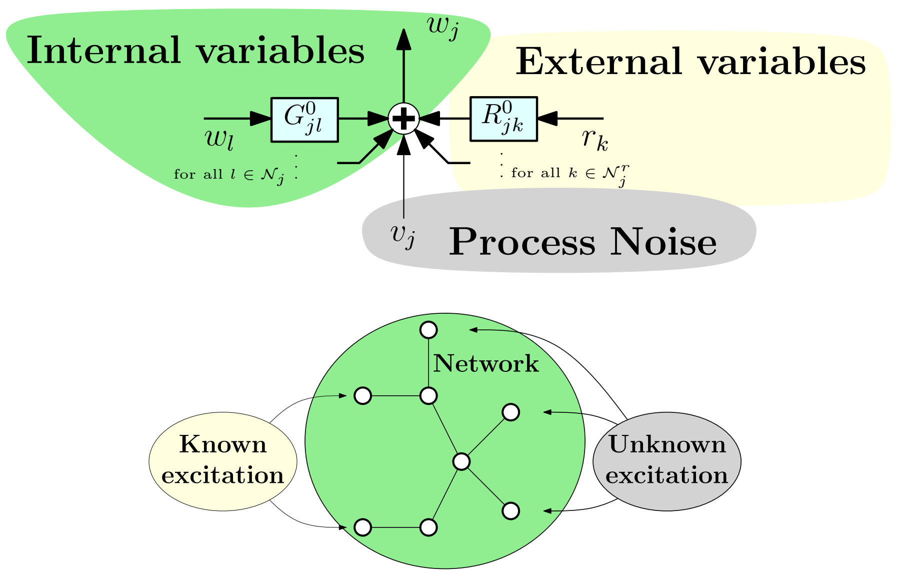

Figure 2: **The network (top) is constructed by connecting all the building blocks (bottom).**

All nodes w_j are stacked into columnvector w. Then we have

Network:	$w = G^0 w + R^0 r + H^0 e$
Predictor:	$\hat{w}(t\|t-1,\theta) = W_w(q,\theta)w(t) + W_r(q,\theta)r(t)$
Predictor filters:	$W_w = I - H^{-1}(I-G), \qquad W_r = H^{-1}R$
Model structure:	$\mathcal{M} = \{G(q,\theta),\ H(q,\theta),\ R(q,\theta)\}$.

Note that G and G^0 have zeros on the diagonal.

Approach of the problem

Predictor filters have a one-to-one relation to the filter

$$T(q,\theta) = \big(I - G(q,\theta)\big)^{-1}\Big[H(q,\theta) \quad R(q,\theta)\Big]$$

which is under conditions has a one-to-one relation to the filters in $\mathcal{M}(q,\theta)$.

$\hat{w}(\theta_1) = \hat{w}(\theta_2)$	*Predictor equality*
① $\Updownarrow$	Conditional: Data must be **informative.**
$W_w(\theta_1) = W_w(\theta_2)$ $W_r(\theta_1) = W_r(\theta_2)$	*Predictor filter equality*
② $\Updownarrow$	Unconditional [2]
$T(\theta_1) = T(\theta_2)$	*Mapping equality*
③ $\Updownarrow$	Conditional [2]: see conditions below,
$\mathcal{M}(\theta_1) = \mathcal{M}(\theta_2)$	*Model equality*
④ $\Updownarrow$	Conditional: classical **identifiability**.
$\theta_1 = \theta_2$	*Parameter equality*

Results

Definition: $\mathcal{M}$ is called a **globally network identifiable** model structure when ③ holds.

Theorem: When G is as flexible as possible then ③ holds when $\Big[H(q,\theta) \quad R(q,\theta)\Big] P(q)$ has a leading diagonal for some $P(q)$ [2].

example: Choose $H(q,\theta)$ diagonal. Plugging it into predictor filter W_w shows that $\mathcal{M}$ is globally network identifiable. Diagonal H corresponds to having uncorrelated noise on every node.

Conclusion

The one-to-one relation between (parameterized) predictor filters and model is conditional in case of dynamic networks. Combinations of H and R can lead to a network identifiable $\mathcal{M}$, it is not necessary that H is diagonal. When H is not diagonal networks where noises are correlated can be considered.

References

[1] L. Ljung. *System Identification: Theory for the User.* Prentice-Hall, 1999.

[2] H.H.M. Weerts, A.G. Dankers, and P.M.J. Van den Hof. Identifiability in dynamic network identification. *In 17th IFAC Symposium on System Identification*, 2015.