

## Commonality and safety stocks

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# COMMONALITY AND SAFETY STOCKS\*

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## ABSTRACT

Introducing or increasing commonality in product structures is advocated frequently. However, introducing commonality gives rise to two questions: "Is it advantageous (in terms of service level) to keep common items at stock?" and "How should stock norms be determined for divergent systems?" It will be shown that in the cases of stationary demand, no lot-sizing and

unlimited capacity, the main deterioration of the service level is due to retaining common items in a depot. The impact of imbalance between inventories on the service level will appear to be negligible. This observation will lead to a rule to determine stock norms for divergent systems which is as simple as the classical rule for the one-stockpoint case.

## 1. INTRODUCTION

Commonality is a concept which is of interest in several areas of inventory control. In this paper two of those areas are considered: distribution and production. Commonality occurs if one type of product is shipped to several locations or if one type of product (the "common" component) is used to produce different types of products.

Both kinds of commonality are encountered in the Consumer Electronics Factory treated in [1]. Figure 1 sketches an example of the type of production process that may occur. The process starts with the procurement of two types of raw materials: specific components like a teletext-module called TXT and non-specific components called COMMONS. These components are used to produce two types of television sets: one with (TV1) and the other

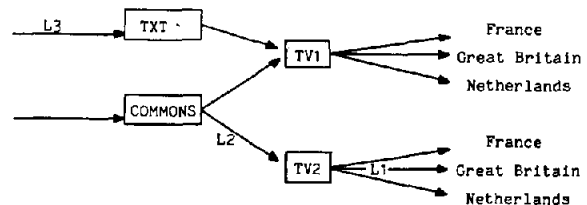


Fig. 1. The production process in the Consumer Electronics Factory.

without a teletext-module (TV2). These television sets are transported to several National Sales Organizations. Procurement, production and transportation require, respectively,  $L_3$ ,  $L_2$  and  $L_1$  periods.

The Consumer Electronics Factory aims at (increased) commonality. One of the main advantages of commonality is the well-known reduction of uncertainty: the forecast error in the total demand for television sets is relatively smaller than the forecast error in the demand for a specific type of television set. This reduction in uncertainty yields lower safety stocks.

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Implementing this commonality gives rise to the following two questions:

- (1) Is it advantageous (in terms of service level) to keep common items at stock?
- (2) How should stocknorms be determined?

Both questions will be dealt with in this paper. In doing so the attention will be focussed on two-level inventory systems. The multilevel case will be briefly discussed in Section 5.

As far as the first question is concerned, to keep common items at stock, the system service level is influenced in two ways. On the one hand, the service level is increased, because there is more inventory left in the depot to allocate to the different products (or locations). So if the order which arrives at the stockpoint for common items isn't large enough to bring the inventories of all products to an equivalent level, the commons in stock can be used for this. In that way it is possible to improve the balance of the inventories of the different products. This has a positive effect on the service level. On the other hand, the service level will decrease, since some inventory is retained at the depot.

The second question has been studied by several authors, see e.g. refs. [2-6]. In all these papers (as well as in the present one) it is assumed that the inventories are perfectly balanced. It is not obvious however, what the consequences of this assumption are for the service level. These consequences will be investigated by means of simulation. Results will be presented in the subsequent sections. Related results are those of Eppen and Schrage [4] and Federgruen and Zipkin [7]. Eppen and Schrage indicate the probability that the inventories are balanced. Federgruen and Zipkin report extensive simulation results for the system without a stockpoint for common items. Their results indicate the influence of imbalance on expected cost.

In Sections 2 and 3 the influence of unbalanced inventories on the service level is studied. Systems without and with a stockpoint for common items (called a depot) will be consid-

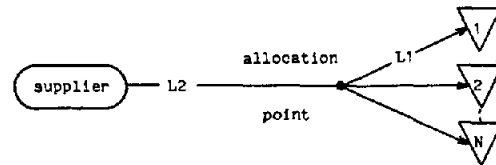


Fig. 2. A divergent system without depot.

ered. Whereas in those sections the demand of the lower level products are assumed to be distributed identically, Section 4 deals with non-identical distributions.

It will be shown that the effect of unbalanced inventories is limited if demand is stationary and the ordering policy is lot-for-lot. This result will lead to two conclusions:

- (1) Keeping common items at stock will yield little improvement (if any at all) in the service level.
- (2) To determine stocknorms for common items, there is little harm in assuming that the inventories are perfectly balanced.

This last result will be worked out in Section 5 and result in a method to determine stocknorms which is as simple as determining a norm for one stockpoint. Finally, some suggestions for further research will be given.

## 2. A DIVERGENT SYSTEM WITHOUT A CENTRAL DEPOT: THE IMPACT OF UNBALANCED INVENTORIES ON SERVICE LEVEL

Consider a divergent system as depicted in Fig. 2. This system can be interpreted as representing the production and transportation process of TV2 in the Consumers Electronics Factory. In this system a product (TV2) is produced and then allocated and transported to  $N$  identical locations (National Sales Organizations). These locations experience demand from customers out of the system. The demand at location  $j$  is assumed to be distributed normally with mean  $\mu_j = \mu$  and standard deviation  $\sigma_j = \sigma$ . The leadtimes for production and transportation are  $L2$  and  $L1$  periods.

TABLE 1

The probability that the inventories in the system without depot are balanced for various values of  $N$  and  $\mu/\sigma$

$N$	$\mu/\sigma$				
	0.5	1.0	1.5	2.0	2.5
2	40.5	67.5	86.3	95.3	98.8
3	23.6	55.0	79.7	93.4	98.0
4	14.2	46.7	74.3	91.0	96.9
5	8.1	37.1	70.0	88.5	96.9
6	5.1	31.0	65.1	87.4	96.4
7	3.3	25.6	60.1	85.1	96.0
8	1.7	21.5	54.5	82.4	95.2

The ordering policy for the system is an "order every period up to" policy. The order-up-to level (see ref. [4]) is denoted by  $S$  and equals:

$$S = (L1 + L2) \sum_{j=1}^N \mu_j + k \left[ L1 \left( \sum_{j=1}^N \sigma_j \right)^2 + L2 \sum_{j=1}^N \sigma_j^2 \right]^{1/2}$$

The safety factor  $k$  is determined by  $\Phi(k) = \gamma$ , where  $\gamma$  is the service level of the system, which could be achieved if the inventories were perfectly balanced and  $\Phi(\cdot)$  is the standard normal distribution function. The goods which arrive at the allocation point are sent to the locations with the lowest expected service level in order to keep the inventories as balanced as possible. In case of identical products these are just the locations with the lowest inventories.

Eppen and Schrage [4] showed that the possibility that the inventories are balanced is approximately zero if there are many locations and if the variance of the demand is relatively high. Simulation results corresponding to those of Eppen and Schrage are shown in Table 1. In the simulation,  $L1$  and  $L2$  are both chosen to be 3. The safety factor  $k$  is set equal to 1.645. All tables in this paper are in percentages.

Table 1 might suggest that the performance of the divergent system can be very poor. To check this, the system has been simulated for 5000 periods. The service level for each of the  $N$  locations was defined as the probability that demand can be met. The system service level,

TABLE 2

Service level for the system without depot for various values of  $N$  and  $\mu/\sigma$

$N$	$\mu/\sigma$				
	0.5	1.0	1.5	2.0	2.5
2	94.0	94.5	94.9	94.8	95.0
3	93.5	94.6	94.9	94.9	95.0
4	93.3	94.5	94.8	94.9	94.9
5	93.2	94.6	94.8	94.9	94.9
6	93.2	94.6	94.8	94.9	95.0
7	93.2	94.6	94.9	94.9	94.9
8	93.1	94.5	95.0	94.9	95.0
25	93.2	94.6	94.9	94.9	94.9

equal to the average of these  $N$  service levels, has been used as a performance indicator. The system service level, which could be achieved if the inventories were perfectly balanced, always is set equal to 95%. This corresponds with a safety factor  $k$  equal to 1.645. The results are summarized in Table 2.

The results are clear: even if the probability that the inventories are balanced is low, the impact on service level is limited. This is due to the fact that the service level not only depends on the probability that the system is unbalanced, but also depends on the size of imbalance. If there are many products, the probability that one of them gets out of balance will be large. However, with many products the size of imbalance will always be limited, since the amount of inventory available to be allocated is more regular. This results in the fact that the service level hardly depends on the number of products  $N$ .

### 3. A DIVERGENT SYSTEM WITH A CENTRAL DEPOT: THE IMPACT OF THIS DEPOT ON IMBALANCE AND SERVICE LEVEL

Again the system in Fig. 2 is considered, but with one distinction. Now there is also a central depot, which may hold back inventory. This system is depicted in Fig. 3.

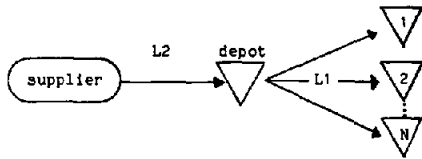


Fig. 3. A divergent system with central depot.

All ordering policies are "order every period up to" policies. The order-up-to level for the  $j$ -th location,  $S1(j)$  equals  $L1 \mu_j + k1 \sqrt{L1} \sigma_j$  with  $k1$  any safety factor. The order-up-to level for the system as a whole is the same as  $S$  in Section 2.

The impact of holding back inventory in the central depot is twofold: (1) the probability that the system is balanced will increase, and consequently the service level will also increase; (2) the service level will decrease, since some inventory is held back and not yet made available for the locations.

The system was simulated to get an indication of the probability that the inventories are balanced (see Table 3). Both safety factors were set equal to 1.645. All other parameters were set as in the previous section.

Tables 1 and 3 show that holding back inventory substantially increases the probability that the inventories are balanced, particularly if  $N$  is large and  $\mu/\sigma$  is small. This has a positive effect on the service level. As mentioned earlier, retaining inventory in the depot has

TABLE 3

The probability that the inventories in the system with depot are balanced for various values of  $N$  and  $\mu/\sigma$

$N$	$\mu/\sigma$				
	0.5	1.0	1.5	2.0	2.5
2	86.9	94.8	98.5	99.6	99.9
3	79.8	89.9	96.6	99.0	99.9
4	73.7	85.1	94.4	98.4	99.5
5	70.0	81.4	91.5	97.4	99.5
6	65.0	78.5	90.1	96.7	99.3
7	64.3	75.4	87.8	95.8	99.0
8	62.6	71.7	85.0	94.7	98.9
25	55.0	57.5	63.4	82.0	94.3

also a negative effect on the service level. The interesting question now is what the total effect will be.

The first column in Table 4 shows the impact on the service level of retaining inventory in the depot. These figures were obtained numerically by means of the formula in Section 4.2 of ref. [3]. They are based on the assumption that the inventories are perfectly balanced. The only influence which remains then is the influence of retaining inventory in the depot. The service level would have been 95% if no inventory were held back at the depot and if the inventories were perfectly balanced. The remaining columns in Table 4 show the total effect of holding back inventory in the depot. These figures were obtained by simulation.

Comparing the first column with the other ones makes it clear that the main deterioration of the service level is due to retaining inventory in the depot. The fact that inventories are not perfectly balanced deteriorates the service level only slightly.

From Tables 2 and 4, two conclusions can be drawn: (1) it appears that holding back inventory in a depot yields a lower service level than passing through all inventory; (2) since the increased ability to keep the inventories balanced has limited influence on the service level, there is little harm in assuming that the system is perfectly balanced.

It should be noted, that in the system above an "order every period up to" policy was used. So, implicitly, it was assumed that lot sizes were small. In addition, the demand distribution was stationary and the safety-factors were equal. These factors may influence the conclusions just drawn.

#### 4. NON-IDENTICAL DEMAND DISTRIBUTIONS FOR THE LOCATIONS

In the previous sections it was assumed that all locations had identical demand distributions. In practice this is never the case. In most cases there are many items with a low average

TABLE 4

Service level for the system with depot for various values of  $N$  and  $\mu/\sigma$ 

	$N$	Balanced inventories	$\mu/\sigma$				
			0.5	1.0	1.5	2.0	2.5
	2	92.6	92.4	92.4	92.5	92.5	92.6
	3	92.9	92.7	92.8	92.9	92.9	92.7
	4	93.1	92.8	93.0	93.1	92.9	93.1
	5	93.3	92.9	93.2	93.1	93.3	93.3
	6	93.4	93.0	93.4	93.3	93.5	93.3
	7	93.5	93.1	93.4	93.5	93.4	93.6
	8	93.6	93.1	93.4	93.6	93.5	93.7
	25	94.2	93.7	94.1	94.1	94.1	94.2

TABLE 5

Service level, derived by assuming that inventories are perfectly balanced or by simulating the system

	No depot	Depot
Balanced inventories	95.0	93.5
Simulation results	92.4	92.7

demand and a relatively high variation in demand (called slowmovers here) and only a few items with high average demand and a relatively small variation (called fastmovers here).

These considerations have led to the following model to investigate the impact of non-identical products on system service level:

- the number of products;  $N = 10$ ,
- the number of slowmovers;  $N_{\text{slow}} = 8$ ,
- mean and standard deviation for the demand of the slowmovers;  
 $\mu_j = 10$ ,  $\sigma_j = 20$ ,
- mean and standard deviation for the demand of the fastmovers;  
 $\mu_j = 160$ ,  $\sigma_j = 80$ .

The service level of the lower level products are weighted with  $\mu_j/\sum\mu_j$  to yield the system service level. The analytical and simulation results for the system service level are summarized in Table 5.

As opposed to the system with identical products considered in the previous Sections, the model used here yields a higher service level

for the system *with* a depot. The difference however is marginal.

Another interesting result from the simulation is the fact that the average service level for slowmovers and fastmovers is above and respectively, below the average service level for the whole system. For the system without depot the service levels for the slowmovers and the fastmovers are respectively, 95.4% and 91.6%. For the system with depot these figures are 93.7% and 92.5%.

Of course it is possible to define the system service level by means of any other weight factors than those mentioned above ( $\mu_j/\sum\mu_j$ ). It should be noted that the weight factors used here give a relatively high weight to the service level of the fastmovers. So the system service level may be relatively low compared to the one which would result from other weight factors.

## 5. DETERMINING STOCKNORMS

According to Sections 2 to 4 it may be better to have no central depot if the attention is restricted to the effects on service level. In practice there may be more reasons to retain inventory. One of these reasons is lower holding cost due to a lower added value at the intermediate level or a less voluminous size. Below, it will be indicated how stocknorms can be determined for systems with and without a depot.

For a system without a depot only one stocknorm is needed and this stocknorm can be determined in the way suggested by Epen and Schrage [4]. That means that the stocknorm  $S$  may be of the form  $S = \mu_{\text{div}} + k\sigma_{\text{div}}$ , where  $\mu_{\text{div}}$  (= average demand during the leadtime) and  $\sigma_{\text{div}}$  are determined by  $\mu_{\text{div}} = (L1 + L2) \sum \mu_j$  and  $\sigma_{\text{div}}^2 = L1 (\sum \sigma_j)^2 + L2 \sum \sigma_j^2$ . The safety factor  $k$  is derived directly from the equation  $\Phi(k) = \gamma$ , where  $\gamma$  is the desired service level. Their method assumes that the inventories are perfectly balanced and it has been shown in Section 2, that even if this assumption is violated, the impact on service level remains limited.

However if there is a depot, stocknorms have to be determined for each of the consecutive stockpoints. Clark and Scarf [2] use dynamic programming to determine these stocknorms. Their objective is to minimize the expected holding and penalty cost. What is needed however is a simple rule for determining stocknorms for consecutive stockpoints, such that the corresponding service level is equal to a prespecified level.

Using the results in Sections 2 through 4 and in ref. [3] it is possible to construct such a rule, which is of the same form and therefore as simple as the rule for the one-stockpoint case. It can be used for convergent, linear and divergent systems (for mixed systems: see remark number 3 below).

A convergent system can be treated as a linear one (see ref. [8]). A linear system is a special case of a divergent system ( $N=1$ ). So all that is needed, is a rule for a divergent system. Consider the divergent system in Fig. 3. To determine stocknorms of the form  $S = \mu_{\text{div}} + k\sigma_{\text{div}}$  for the  $N$  lower level products as well as for the whole system, it is necessary to indicate how  $\mu_{\text{div}}$ ,  $k$  and  $\sigma_{\text{div}}$  should be calculated to yield a prespecified service level. The stocknorms for the  $N$  lower level products are determined as in the one-stockpoint case. So  $S1(j) = L1 \mu_j + k1 \sqrt{L1} \sigma_j$ . For the system

stocknorm it is obvious that  $\mu_{\text{div}}$  should equal  $(L1 + L2) \sum_{j=1}^N \mu_j$ . As Eppen and Schrage already suggested  $\sigma_{\text{div}}^2$  should be equal to:  $L2 (\sum \sigma_j^2) + L1 (\sum \sigma_j)^2$ , that is:  $L2$  periods centralized demand and  $L1$  periods non-centralized demand.

According to Sections 2 to 4 there is little harm in assuming that inventories are balanced every period. This assumption was used in [3] to demonstrate, that the resulting system service level for safety factors  $k1$  and  $k$  equals  $\psi(k1, k; \rho)$ , where  $\psi(\dots; \rho)$  is the standard bivariate normal distribution function with correlation coefficient  $\rho$ . In this case  $\rho$  equals  $[\sqrt{(L1 (\sum \sigma_j)^2)}] / \sigma_{\text{div}}$ . If both safety factors are equal ( $k1 = k$ ), the system service level can be approximated by  $t\alpha^2 + (1-t)\alpha$ , where  $\alpha = \Phi(k)$  and  $t = \sqrt{(1-\rho^2)}$ . This means that for any desired service level  $\gamma$ , the simple quadratic equation  $\gamma = t\alpha^2 + (1-t)\alpha$  has to be solved for  $\alpha$ . This yields

$$\alpha = \frac{(t-1) + \sqrt{[(1-t)^2 + 4t\gamma]}}{2t}$$

if  $t \neq 0$  and  $\alpha = \gamma$  if  $t = 0$ . The resulting  $\alpha$  determines the safety factor  $k$ , as in the one-stockpoint case, by the relation  $\Phi(k) = \alpha$ . Some remarks on this method should be made:

(1) Until now only two systems were considered: system *with* a depot and equal safety factors and the system *without* a depot. The last system can be seen as a system with depot and safety factors  $k1 = \infty$  and  $k$  such that  $\Phi(k) = \gamma$ . It may be desirable to have a system with depot and different safety factors for different product-levels. In that case the approximation for the system service level in ref. [3] is no longer valid. An alternative seems to be:

System service level =  $t\alpha_1\alpha_2 + (1-t) \min(\alpha_1, \alpha_2)$ , where  $\alpha_1$  and  $\alpha_2$  are used to determine the safety factors for the lower level products, and for the whole system.  $\text{Min}(\alpha_1, \alpha_2)$  stands for the minimum of  $\alpha_1$  and  $\alpha_2$ .

This approximation has not yet been fully tested,

but the first tests indicate an absolute error for the service level, which is smaller than  $(1 - \gamma)^2$ .

(2) In practice demand is never stationary. In case of non-stationary demand  $\mu_{\text{div}}$  should be

$$\sum_{j=1}^N \sum_{i=1}^{L1+L2} D_j(t, t+i),$$

where  $D_j(t, t+i)$  is the forecast for demand in period  $t+i$  made in period  $t$  for product  $j$ . Here  $\sigma_{\text{div}}$  should be measured as follows:

$$\sigma_{\text{div}}^2 = \sigma_{L2}^2 + \sum_{j=1}^N \sigma_{L1}^2(j),$$

where  $\sigma_{L2}^2$  is the variance of the following forecast error:

$$\sum_{i=1}^{L2} \sum_{j=1}^N [D_j(t, t+i) - D_j(t+i)]$$

$$\sum_{i=1}^{L1} \sum_{j=1}^N [D_j(t, t+L2+i) - D_j(t+L2, t+L2+i)]$$

and where  $\sigma_{L1}^2(j)$  is the variance of the forecast error:

$$\sum_{i=1}^{L1} [D_j(t+L2, t+L2+i) - D_j(t+L2+i)].$$

In these formulas  $D_j(t)$  stands for the actual demand for product  $j$  in period  $t$ . Assume next that the inventories before ordering will never exceed their stocknorm for the next period. This assumption is valid if demand is not too dynamic. Then it can be proven that the system service level will be the same as in the stationary case. If demand becomes very dynamic, stocknorms will also be very dynamic and the content of the system before ordering may incidentally exceed the stocknorm. As a consequence the average inventory and service level will be too high. These effects remain to be studied.

(3) Figure 1 depicts part of a real-life production system. It shows that a product structure is usually mixed: TV1 in Fig. 1, for example, is part of a convergent structure (both COMMONS and TXT are components) as well as of a divergent structure (COMMONS is also used for TV2). In case of a mixed struc-

ture, stocknorms cannot be determined by simply combining the results for divergent and convergent systems. Further investigation is required here.

(4) The general method mentioned above is developed for two consecutive stockpoints (e.g. one depot and  $N$  locations or products). Systems with more consecutive stockpoints can be approximated by subsequently taking the last two consecutive stockpoints, approximating their service level with this method and then replacing them by one stockpoint. It is not known yet how well such an approximation works.

## 6. CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

Unbalanced inventories, due to variation in demand, have little impact on system service level. Thanks to this observation, a rule to determine stocknorms for consecutive stockpoints could be suggested for convergent, linear and divergent systems. This rule is as easy to understand and implement as in the one-stockpoint case.

However, just as in the one-stockpoint case, there are more factors which influence the service level. Perhaps the most important ones are lot-sizes and limited capacities. These factors, together with those mentioned earlier, constitute a wide field of research.

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## REFERENCES

- 1 Wijngaard, J. (1987). Structuring a production control system: a case study. *Engineering Costs and Production Economics*, 12:
- 2 Clark, A. and Scarf, H. (1960). Optimal policies for a multi-echelon inventory problem. *Manage. Sci.*, 6(4): 475-490.



- 3 Van Donselaar, K. and Wijngaard, J. (1986). Practical Application of the Echelon Approach for a System with Divergent Product Structures. In: S. Axsäter, Ch. Schneeweiss and E. Silver (Eds.), *Multi-Stage Production Planning and Inventory Control*. Springer Verlag, Berlin-Heidelberg, pp. 182-196.
- 4 Eppen, G. and Schrage, L. (1981). Centralized ordering policies in a multi-warehouse system with lead times and random demand. In: L.B. Schwarz (Ed.), *Multi-level Production/Inventory Control Systems: Theory and Practice*. North-Holland, Amsterdam, pp. 51-67.
- 5 Federgruen, A. and Zipkin, P. (1984). Approximations of dynamic, multilocation production and inventory problems. *Manage. Sci.*, 30(1): 69-84.
- 6 Federgruen, A. and Zipkin, P. (1984). Computational issues in an infinite-horizon, multi-echelon inventory model. *Oper. Res.*, 32(4): 818-836.
- 7 Federgruen, A. and Zipkin, P. (1984). Allocation policies and cost approximations for multilocation inventory systems. *Nav. Res. Logistics Q.*, 31: 97.
- 8 Wijngaard, J. and Wortmann, J.C. (1985). MRP and Inventories. *Eur. J. Oper. Res.*, 20: 281-293.