Experimental identification of model parameters for SMA wires

Document status and date:
Published: 01/01/2003

Document Version:
Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

• A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
• The final author version and the galley proof are versions of the publication after peer review.
• The final published version features the final layout of the paper including the volume, issue and page numbers.

Link to publication

Download date: 17. Nov. 2023

Experimental Identification of
Model Parameters for SMA wires

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DCT 2003.115

Traineeship report

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Eindhoven, February, 2004

Abstract

Shape Memory Alloys are extensive applied in Medical Robotics nowadays as active elements. They could be used as actuators, which are able to perform accurate movements in minimal invasive surgery (MIS). In MIS the precision of the incisions has to be guaranteed.

The need of precision leads to try to find a reliable approach that describes the behavior of SMA. Departing from the knowledge of the characteristics of SMA, a survey of Models available is performed. This models describe the response of the SMA under different conditions.

Finally the goal is to determine the most important features in the model and compare the simulations with the result of the experiments performed in the SMA.

Contents

Chapter 1

Introduction

1.1 Description of Smart Memory Alloys (SMA)

Smart Memory Alloys (SMA) are metallic alloys, which exhibits the Shape Memory Effect (SME) when they are deformed. SMA can recover their permanent strength Figure 1.1 when they are heated above a certain temperature while a Martensitic phase change is occurring.

The stress - temperature curve is shown in Figure 1.1. The parent Austenitic material is cooled and it is transformed from Austenite to twinned Martensite (1 to 2). When the stress is increasing, the SMA wire experiments a phase change into detwinned martensite (2 to 3). From 3 to 4, the temperature is raised and then the phase transformation is between detwinned Martensite to Austenite, arriving to the initial structure configuration. This phenomenon is called one-way shape memory effect, because the shape recovery is achieved only during heating.
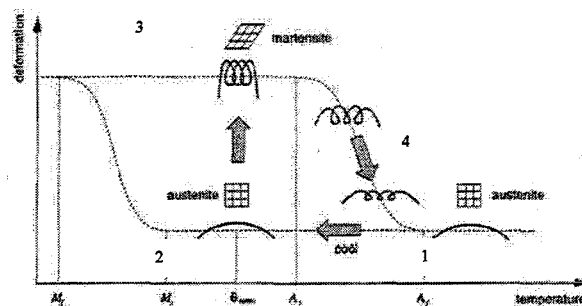

Figure 1.1: Thermomechanical loading path demonstrating the shape memory effect

1.1.1 Properties of SMA wires

The pseudoelasticity behavior of SMA wire is associated with the recovery of the transformation strain upon unloading and encompasses both super-elastic and rubber-like behavior.

Super-elastic behavior is observed during loading and unloading above the austenitic Star transformation temperature, A_{os}, and it is associated with stress-induced martensite and reversal to austenitic upon unloading. When the loading and unloading of the SMA wire occurs above A_{os}, partial transformation strain recovery takes place and when loading and unloading occurs above the Austenitic Final transformation temperature, A_{of}, full recovery happens. The rubber-like behavior occurs if the material is in the Martensite state and detwinning and twinning of the Martensite variants occur upon loading and unloading respectively, by reversible movement of the twin boundaries.

Hysteresis is present due to phase changes in the material; this phenomenon is given by the difference between Stress Martensite final transformation s_{Mf} and Stress Austenitic start transformation s_{As} and between Stress Martensite start transformation s_{Ms} and Stress Austenitic final transformation s_{Af}. This can be seen in the loading/unloading stress-strain diagram, see Figure 1.2.
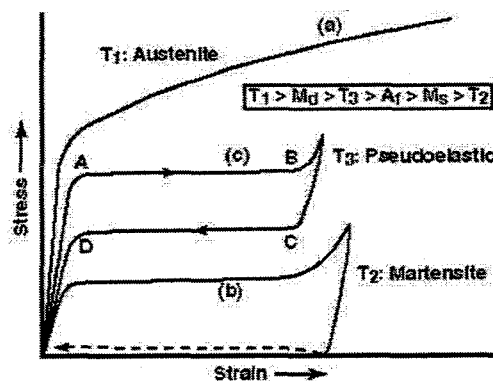
Figure 1.2: Super-elastic behavior of the shape memory alloys

Cooling the SMA wire with a constant applied stress from Austenitic to Martensite state, there is a phase transformation characterized by a martensite start temperature M_{ss} and a martensite finish temperature M_{sf}, which are functions of the applied stress. The transformation strain is greater than the thermal strain corresponding to the same temperature difference required by the phase transformation.

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1.2 Problem definition

Shape Memory Alloys (SMA) are widely used now days, especially in a passive way in Medical Applications. The improvement of control for active use of SMA actuators implies the use of well define models. These models are going to be utilized in the design of the controllers. So far, different types of models describing the behavior of the SMA wire has been used, some of the parameters used in these models are not well defined yet, it is what brings us to the purpose of acquiring greater knowledge of the parameters involved in the models. In this research, attention will be paid to chose a model that describes the SMA wire behavior and try to defined the most needed features on it.

As described in the previous section, the working principle of SMA is based on the shape memory effect, where the SMA wire is heated to make it going back to its initial shape. A common way of raising the temperature of the SMA wire is applying current through it. Thus the model to be used has to be able to depict the relation between the current and temperature.

There exist several methods to estimate model parameters based on measurements. In this research special attention will be the Least Square Estimators (LSE). It is possible to conclude that knowing the parameters of the SMA wire and their relations we have defined a model which would allow to go further in research of controlling SMA wires actively.

1.3 Project survey

Now we have defined the problem, the selection of the model describing the SMA wire behavior is the first issue. After looking for available models in the literature, the model selected is the one used by Franken[1] in his research. The features to define in the model are the parameters (c and τ) which are temperature dependent. Arriving the need to perform temperature measurements.

The measurement of the temperature directly in the wire is the main initiative, it will be done in two different ways. First of all under controlled conditions, and later on in normal operation of the SMA wire. The first experiment (under controlled conditions) is aimed to find the temperature resistance relation. Knowing the relationship between temperature and resistance, in a further stage we think it is possible to use it to control actively the SMA wire, because of the direct relation between current - resistance and temperature. The second experiment (normal operation) is used to find the parameters c and τ. Measuring the applied current and the resultant temperature increments and using LSE it is possible to determine the best fitted values for the unknown parameters.

Chapter 2

Model of the system

As a result of seeking for possible model representations of the SMA wire behavior, two main proposals are raised. The first model uses ideas from statistical thermodynamics and describes the evolution of two martensite fractions. The second model describes the thermo-mechanical behavior of a SMA wire based in the general engineering equations for stress and stress.

2.1 Model of Achenbach and Muller

The basic idea depicted in this model[2] is to assume that the phase transition is a thermally activated process, which is visualized as changes in the energy stored in the SMA structure.
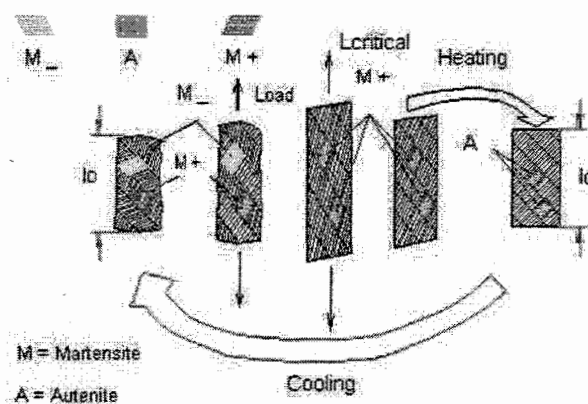

Figure 2.1: Layer structure of a SMA and the Shape Memory Effect

These changes in the energy see Figure 2.1 are calculated like the variation in the shear length Δ, which is obtained from the elongation (D). The sum of the length changes of the individual layers is:

$$D = N\{X_A(\Delta_A) + X_+(\Delta_+ + X_-(\Delta_-))\} \qquad (2.1)$$

N: is the number of lattice layers X_A, X_+, X_- denote the volume fractions of the corresponding phases, and the bracketed quantities are the expectation values of the length changes in the phases. They are calculated from statistical thermodynamics, e.g.:

$$(\Delta_A) = \frac{\int_{-\Delta_s}^{+\Delta_s} \Delta exp\left[-\frac{\phi(\Delta,P)d\Delta}{k\theta}\right]}{\int_{-\Delta_s}^{+\Delta_s} exp\left[-\frac{\phi(\Delta,P)d\Delta}{k\theta}\right]} \qquad (2.2)$$

θ Is the wire temperature, k is Boltzmann's constant and $\phi(\Delta, P)$ is the potential energy seen by a layer. It depends on the layer shear length Δ and is given by a triple-well function with each well corresponding to one of the three phases, see Figure 2.2. In the presence of an external load P, this function has to be superposed by the work done by the load, which in the one-dimensional case is simply -PΔ. For the determination
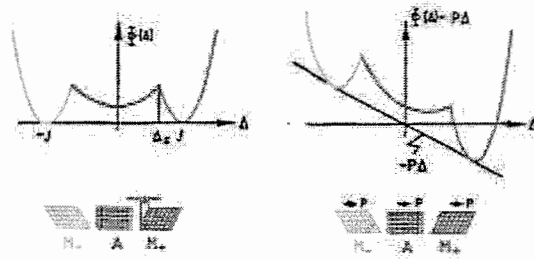
Figure 2.2: Three phases of a lattice particle and corresponding potential energy

of the phase fractions the model assumes the shear lengths of the layers to fluctuate about their equilibrium values in the potential wells. Occasionally, the layers are able to overcome the barriers between the wells, and this gives rise to time rates of change for the phase fractions X_+ and X_- according to:

$$\begin{aligned}
\dot{X}_+ &= -X_+ p^{+A} + X_A p^{+A} \\
\dot{X}_- &= -X_- p^{-A} + X_A p^{-A}
\end{aligned} \qquad (2.3)$$

The quantities $p^{\alpha\beta}$ are the transition probabilities from phase α to phase β, which also can be calculated from statistical thermodynamics, e.g:

$$p^{+A} = \sqrt{\frac{k\theta}{2m\Pi}} \frac{exp\,[-\frac{\phi(\Delta_s,P)}{k\theta}]}{\int_{+\Delta_s}^{\infty} exp\,[-\frac{\phi(\Delta,P)}{k\theta}]\,d\theta}\,exp - \frac{A(1-2X_A)}{k\Theta} \qquad (2.4)$$

A is the interfacial energy coefficient responsible for the alloy's hysteretic behavior and m is the mass of the layer.

This model includes also the possibility of electric heating equation(2.5), which is crucial for the SMA wire application as actuator. This extension makes it necessary to consider the balance of energy, which reads:

$$mc\theta = \alpha(\theta - T_{env}(t)) + j(t) - \dot{X}_+ H_+(P) - \dot{X}_- H_-(P) - A(1-2\dot{X}_A) \qquad (2.5)$$

It is readily interpreted as follows, the temperature in the wire changes due to:

- Heat exchange with the environment at temperature $T_{env}(t)$
- The Joule heating j(t) produced by the electric current and the latent heats due to the transitions
- The third and fourth term H_{+-} represent the leading parts of the latent heat, and the last terms the part due to creation and annihilation of interfaces between austenitic and martensite layers
- c is the specific heat and a the thermal conductivity coefficient.

The above equations together with appropriate initial conditions and prescribed function j(t), it can be solved for the resulting change D(t) equation(2.1).

Concluding this model allows to specify at each instant the location of the transformation phase of the SMA wire while is heated by a current. Nevertheless, a disadvantage is the little knowledge about the statistical relations needed to define the probabilities used in the model. Due to this fact a simpler approach is chosen, in the next section.

2.2 Model of van der Wijst

The model of Wijst[3] describes the thermo-mechanical behavior of a SMA wire, using the Stress-Strain engineering and the Heat Balance equations.

The stress and strain are related to the wire length (l) and the tensional force (f), both quantities can be measured to calculate stress and strain with the following equations:

$$e = \frac{1}{2}[[\frac{l}{l_{ref}}]^2 - 1]$$

$$p = [\frac{fl_{ref}}{A_{ref}l}] \qquad (2.6)$$

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Where l_{ref} and A_{ref} are the initial length the cross-sectional area of the wire, respectively.

Knowing that the easiest way to heat up the SMA wire is applying a current and using the heat balance, it is possible to arrive to the model describing the behavior of the SMA wire while working.

The heat balance is a result of the stored, lost, conducted and generated power. The power generated is:

$$Pgenerated = I^2 * R$$
$$R = \frac{r_{ref}}{A_{ref} * l} \tag{2.7}$$

Where, I is the applied electric current, R the total electric resistance of the wire, r_{rel} the specific electric resistance and ℓ and A the length and cross sectional of the wire respectively.

The power lost due to convection is:

$$Plost = h(\theta - T_{env})d_s * l \tag{2.8}$$

Where, h is the heat transfer coefficient depending on the environmental circumstances, θ the surface temperature of the wire and T_{env} the temperature of the environment, d_s the circumference and ℓ the length of the wire.

The stored power is:

$$Pstored = \rho c A l \dot{\theta} \tag{2.9}$$

Where ρ is the density and c is the specific heat capacity of the wire, the conducted power and $\dot{\theta}$.

Finally, the heat balance results in:

$$\tau \dot{\theta} + (\theta - T_{env}) = cI^2 \; with \; \tau = \frac{\rho c A}{h d_s} c = \frac{r_{rel}}{A h d_s} \tag{2.10}$$

In previous works the thermal parameters c and τ are treated as constant, but actually they are dependent of temperature and the crystal structure of the wire. Therefore, the aim is to try to find a way to estimate these parameters when the SMA wire is acting under diverse conditions.

This is the model selected to work with due to its reliability and it means the possibility to study the behavior of the SMA wire while is excited with a current, by simply measuring the temperature.

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Chapter 3

Design of Experiments

Departing from the knowledge of the model, the course is to design an experimental set up and a procedure that allows to determine the most important features used in the model describing the SMA wire. To explore the response of the SMA wire two options are shown in the table below.
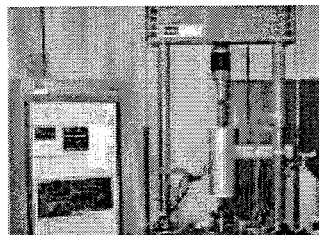
Figure 3.1: Controlled chamber

From these options, the controlled chamber Figure 3.1 is chosen to perform the temperature - resistance measurements. The controlled chamber will be used under some changes, they are going to be clarified later on.

In order to determine the temperature of the SMA caused by the applied current. The definition of a relation among them is needed. Reminding the Ohm's Law, it is known that the current flow in a conductor depends on the resistance of it. That is why, it is aim to measure the resistance of the wire and see how it is changing.

Controlled chamber	Strain-Stress-Temperature measurement (S.S.T)
The SMA wire is heated with a controlled applied current. Cooling with liquid CO_2. Constant load is placed.	The SMA wire is heated with a controlled applied current. Constant load is placed.
Temperature measurement: Using a Thermocouple touching the SMA wire	Temperature measurement: Thermocouple principium, using the wire as conductor, and reading the drop voltage directly on it.
Strain measurement: Length relation.	Strain measurement: Length relation.
Stress measurement: Length relation and applied force.	Stress measurement: Length relation and applied force.
Advantages: Controlled environmental conditions.	Advantages: The temperature measurement is direct in the SMA wire
Disadvantages: The temperature measurement is not direct in the SMA wire.	Disadvantages: Uncontrolled environmental conditions.
See Figure 3.1.	See Fig.3.2

Table 3.1: Alternative Set Up

3.1 Measuring internal Resistance of the SMA wire

The resistance of the SMA wire will mainly depend on the phase fractions of martensite and austenitic present in the SMA wire. Thus, every phase has a different resistance. Therefore, knowing that the transition between phases is a thermally activated process, the purpose is to find the temperature and resistance relation.

In order to find the relation between the temperature and the resistance in a SMA wire, the SMA wire is heated uniformly at a pre-defined temperature in a range of 100°- 30°. This is performed in an oven which allows the control of temperature. At the same time another device measures the resistance of the SMA wire.

The resistance has been measured using a four-wire configuration; This configuration comprises Figure 3.3 : two of those wires are connected to the SMA, one is conducting a small amount of current (1mA) and the other one carries the resultant voltage difference generated in the SMA wire from the heating at a constant temperature.
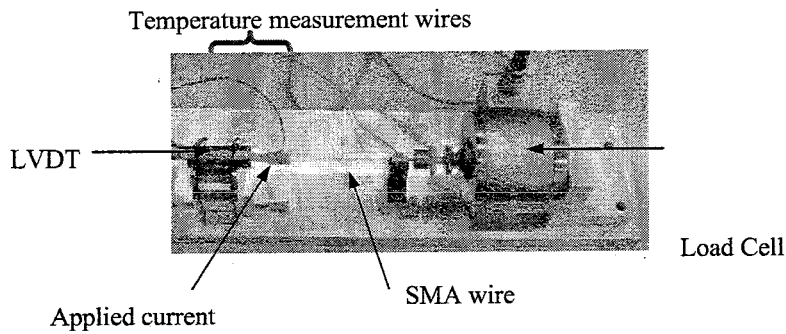
Temperature measurement wires
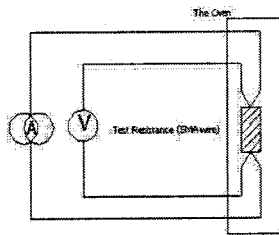

Figure 3.2: Strain-Stress-Temperature

Figure 3.3: Four wire configuration

With the measured values of current and voltage, the resistance is calculated using the Ohm's law for steady state conditions:

$$R = V * I \qquad (3.1)$$

V: Voltage (mV)   I: Current (mA)

3.1.1   Description of the test

The test consists of the application of a current through the SMA wire while the voltage drop across it is measured. In order to get the resistance value, a DMM 196 sensor is used connecting its terminals as shown in Figure 3.3.

Previously, the DMM 196 must be calibrated. To that end, two known resistors of 67 ohm and 2.4 ohm are used. After that, the DMM 196 terminals are connected to the SMA wire in the oven.

The SMA wire lies on a polymer base frame in order to avoid contact with other metals present in the oven.

The temperature control inside the oven has an accuracy of the $\pm 2°$ Celsius. To get a constant temperature of $100°$ in the SMA wire it was necessary to wait around 10 minutes. After that the setpoint temperature is decreased in steps of $10°$.

The applied current through the SMA wire is ImA and the expected voltage drop is around 500mV.

Two different ranges of temperature measurement are used: Between $100°$ and $50°$ and between $50°$ and $30°$. When the temperature reading inside the oven becomes stable, the resistance value is read from the DMM 196 sensor. Figure 3.4 shows some data values acquired in this way.
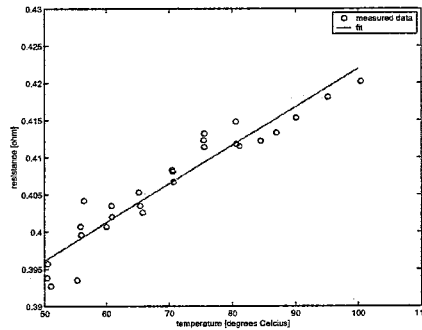
Figure 3.4: Resistance Measurements under controlled temperature

Equation (3.2) gives a relation between temperature and resistance by using Least Squares:

$$R = 0,005 * \Theta + 0.3703 \qquad (3.2)$$

This equation describes a liner relationship between the temperature and resistance. This can be noticed under controlled conditions. The response under uncontrolled conditions will be analyzed later.

3.2 Temperature measurements on the SMA wire

The most common method of measuring temperature uses a thermocouple [4]. It consist of a junction of two dissimilar metals, which returns a voltage output proportional
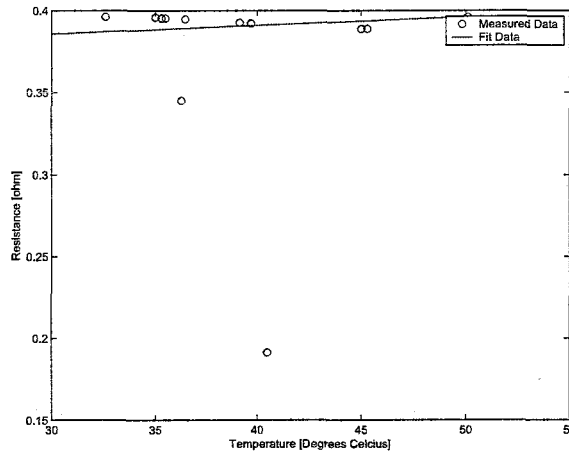
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Figure 3.5: Resistance Measurements under controlled temperature

to the difference in temperature between the hot junction and the lead wires (cold junction). The selection of the appropriate thermocouple depends on the temperature range expected in the measurements, the necessary insulation and the size of the SMA wire (or object to be measured). It is important to ensure a good contact between the hot junction and the SMA wire, for temperature measurement.

3.2.1 First experiment

The thermocouple K[1] is placed in contact with the SMA wire to measure the temperature of the SMA wire while the current is increased. Due to the low precision of the thermocouple, every time the temperature changes the reading goes to zero and start rising again (See Figure 3.6). It supposes a delay in the measurement and inaccuracy. It could be caused by the excess of mass in the hot junction which causes reduction of heat transfer through the SMA wire and the thermocouple.

The measurements are performed under different conditions in order to appreciate the alteration of the resistance, when applying different trajectories for the applied current and/or when the current magnitude changes, see Table 3.2.

Regarding what it was studied before, measuring the resistance under controlled conditions, the resistance rises while the temperature increases. Whereas, under uncontrolled conditions the resistance keeps constant no matter the changes in the temperature.
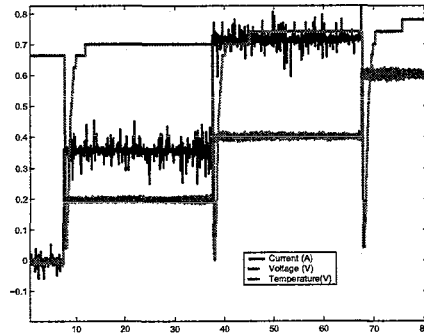
---

[1]Straight thermocouple

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Figure 3.6: Temperature measurements on the SMA wire

It could be due to the fact that the SMA wire is reacting to the environment changes or probably to the thermocouple inaccuracy. That is why it is intended to perform new measurements to get a better relation between temperature and resistance.

Input	Resistance (Ω)	Input	Resistance (Ω)
Step	1.8	Ramp	1.8

Table 3.2: Resistance measurements

3.2.2 Second experiment

To obtain a better contact between the thermocouple and the SMA wire a smaller size of thermocouple K is chosen. To ensure good contact, the thermocouple is welded in the SMA wire by means of a laser shot avoiding additions of mass in the hot junction. It is important to notice that the circuit supplying the current to the SMA wire has to be floating, to guarantee a current flow just through the SMA wire. This is accomplished by using a high resistance circuit for the thermocouple sensor (some $k\Omega$).

The measurements and simulations are going to be shown in the next chapter.

Chapter 4

Simulations and Measurements

The heat balance equation is used to simulate the output response of the the SMA wire. The model is implemented in Matlab Simulink as depicted here.
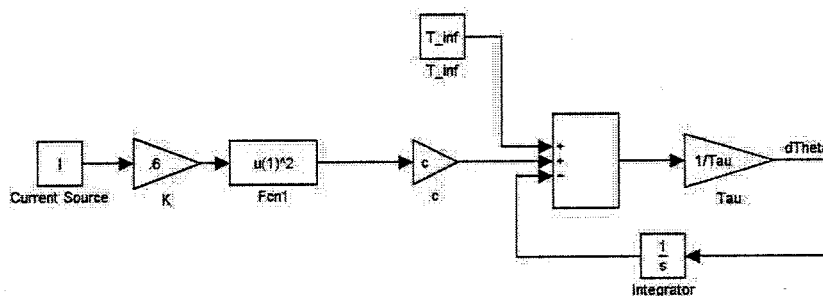

Figure 4.1: SMA model

Current source: Input current, step trajectorie

K: Constant to adjust the model and measurement data, because the measurements where performed with the current not with the current squared

τ and c: Parameters to determine

Using the data obtained from the model, the parameters c and τ are calculated applying the *least square* estimation method. Later on these parameters would be obtained from the measurements of temperature and current.

As it was mentioned before, trying to find the relation between temperature and resistance is the goal. The heat balance model formally establishes such relation. However, some parameters are not well defined in the model (c and τ). Thus, in order to define these variables Parameter Estimation[7] techniques are used.

4.1 Least square estimator

The least square estimator consist of finding the best-fit value of an unknown. The best-fit value is the value that has the minimal sum of the deviations squared (least square error) from a given set of data.

The implementation of the least square estimator is as follows. The model equation is:

$$\tau\dot{\theta} + (\theta - T_{env}) = cI^2 \Rightarrow \theta - T_{env} = -\tau\dot{\theta} + cI^2$$
$$\theta - T_{env} = (-\dot{\theta}\ I^2)\begin{pmatrix} \tau \\ c \end{pmatrix} \tag{4.1}$$

Now denote the model output $\theta - T_{env}$ as $\hat{z} = (\theta - T_{env})_{model} = H\hat{x}$ where $H = (-\dot{\theta}\ I^2)$ is the vector of measurement input data and $\hat{x}$ the parameter to estimate. Denote $z = (\theta - T_{env})_{measured}$ as the measured output, then the estimation error is[7]:

$$e = z - \hat{z}$$
$$e = z - H\ \hat{x}$$
$$J = e^T\ e = (z - H\ \hat{x})^T\ (z - H\ \hat{x})$$
$$= z^T\ z - z^T\ H\ \hat{x} - \hat{x}^T\ H^T\ z + \hat{x}^T\ H^T\ H\ \hat{x}$$

To minimize the error :
$$\frac{\delta J}{\delta x} = -z^T\ H - (H^T z)^T + (H^T\ H\ \hat{x})^T + \hat{x}^T\ H^T\ H \tag{4.2}$$
$$= -z^T\ H - z^T\ H + \hat{x}^T\ H^T\ H + \hat{x}^T\ H^T\ H = 0$$

The resulting least squares estimate is :
$$\hat{x} = (H^T\ H)^{-1}\ H^T\ z^T$$

Subsequently, the measurement values for temperature and current are filled in the matrices H and z; it is possible to find the estimated values for the parameters c and τ, they are shown in the next section. The implementation in Matlab is shown in the appendix (Least-Square Matlab implementation).

4.2 Measurement results

With the obtained measurements of Current and Temperature and using the least square estimator, the parameters c and τ were estimated. The obtained values are compiled in the Table. 4.1.
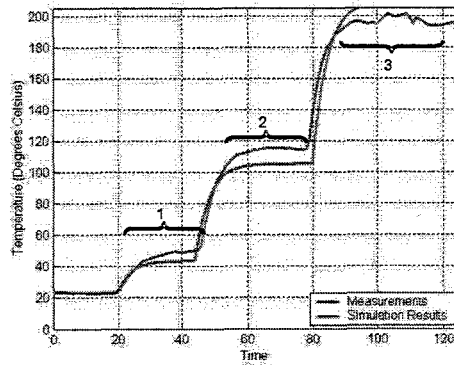

Figure 4.2: Temperature with a step input

	τ [s]	c $[^\circ A^{-2}]$
Step input 1	6.1040	50.7216
Step input 2	2.6700	81.9804
Step input 3	3.9479	109.3072
Step input 1	6.1981	50.5173
Step input 2	4.6338	80.6604
Step input 3	4.0265	107.3465
Average value	4.5967	80.0889

Table 4.1: Estimated parameters c and τ

The estimation of the parameters is done in three intervals (see Figure 4.2), in order to see the variation of the parameters in different range of temperature and with different measurement data.

Thus comparing the result values for c and τ at interval 1 of both measurement sets, the difference is not relevant. However, comparing the parameters for different range of temperature (interval 1 vs. interval 2), the difference is more considerable. It could be of interest for future work to find out the reason of this difference.

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Looking at the Figure 4.2, the response of the temperature in reality is slower than in the simulation. It shows that the model could have a lack in the description of the phase transformation acting in the SMA wire. Nevertheless, the tracking of the model is well obtained.

Finally the values for c and τ that are used in the model, are the result of averaging the estimated parameters, for τ is 4.5967 $^{\circ}A^{-2}$ and for c is 80.0889 s.

Chapter 5

Conclusions and Recommendations

In order to enlarge the knowledge of Shape Memory Alloys (SMA) this research found: well defined models, describing the behavior of SMA and the knowledge acquisition of the parameters present in the models.

The models available in the literature showed different chances to describe the behavior of the SMA. Two models were mainly studied. The first one, which uses statistical thermodynamics and the second one which uses energy balance. Coming to the conclusion that the second one called The Heat Balance equation, describes well the SMA behavior.

It is known that SMA are thermally actively alloys and learning that current is the easiest way to increase the temperature in a controlled way. Thus, the relation between current and temperature becomes significant. That is why a first approach used is to find this relation. Coming to the conclusion that there is a substantial difference in the behavior of the SMA while acting under controlled and uncontrolled conditions. Due to the mentioned differences, it is decided to use the Heat balance equation to describe the behavior of the SMA.

Once the model is available, the estimation of the unknown parameters takes place. Thus, performing temperature and current measurements, and using the Least Square Estimators (LSE) structure, the parameters c and τ were determined.

An important achievement through this research is: the possibility to perform temperature measurements, directly in the SMA. These temperature measurements showed, the complexity and fast response of the alloy to simple changes in the applied current.

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Appendices

```
clear all;
close all;
% First load the measurement data of Temperature and
% Current in the workspace:
load Volt_Temp.m;
% Initial conditions:
T_inf = 25;              % Temperature of the environment
                         % (Degrees Celsius)
R=.27;                   % Resistor used in parallel with the SMA wie to be able
                         % to measure the voltage using the NI-DAQ
T=ch2;                   % T =  Temperature measurements
V=ch4;                   % V =  Voltage measurements
I=V/R;                   % Calculation of the current

% LSQ

a=size(T);
T_r=(T(1:a-1));          % Adjusting matrix dimensions, for coupling with the
I_r=(I(1:a-1));          % derivative of the temperature calculation (dT)
dT=diff(T);              % Calculating the temperature derivative
y=[T_r-T_inf*ones(a-1,1)]'; % Measurement model
u=[-dT I_r];
d=u'*y';
p=inv(u'*u);
theta_est=p*u'*y'       % Estimated parameters c and Tau
```

Transpose of a Matrix:

```
(AB)^{T}  = B^{T} A^{T}
(ABC)^{T} = C^{T} B^{T} A^{T}
(A + B)^{T} = A^{T} + B^{T}
```