

# Measurements executed on a 108D pump with and without air chamber

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**MEASUREMENTS EXECUTED ON A 108D PUMP  
WITH AND WITHOUT AIR CHAMBER**

**J. DIEPENS,  
P. BEEKMAN,**

**February 1989**

**R 981 D**

**WIND ENERGY GROUP  
Technical University Eindhoven  
Faculty of Physics  
Laboratory of Fluid Dynamics and Heat Transfer  
P.O. Box 513  
5600 MB Eindhoven, the Netherlands**



Consultancy Services  
Wind Energy  
Developing Countries

p.o. box 85  
3800 ab amersfoort  
holland

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## 1. INTRODUCTION

In the past CWD pumps always have been equipped with air chambers. This was done because an air chamber smooths the pulsating flow of the pump and due to this the dynamic forces are reduced.

A problem with the actually air chambers is to keep the air in the air chamber and the supply of air to the air chamber.

To solve this problem much investigation is necessary. To show the importance of finding a way out to make air chambers available in the field, some tests have been done on a CWD 108 deepwell pump with and without air chamber.

The results of these tests can be compared with each other.

## 2. DESCRIPTION OF THE TEST STAND

The pump test stand used was the so-called CWD 5001 test rig, see fig. 2.1. For this test rig the CWD 5001 windmill was used, in which the rotor was replaced by a DC motor with speed control.

The stroke can be adjusted from 80 to 200 mm with steps of 40 mm. The speed range is from 0 upto 3.3 rps. Depending on the load the maximum rps can be lower than 3.3 rps.

The force in the pump rod is measured with a force transducer type E. Brosa + 50 kN. The flow is measured with an inductive flow meter: Disco mag. DMI 6531/H50 Endress + Hauser. The data acquisition was done with a IBM.XT personal computer equipped with a Metrabyte Dash 16 data acquisition board, an Intel 8087 Math co-processor and a Hercules graphics card.

To collect and process data use was made of the scientific software package ASYST.

An analysis of the accuracy of measurement is given in Annex IV.

The well under the test rig has a depth of 33 meter. The CWD 108D pump was installed at a depth of 27.6 meters ground level to foot valve. The static head was taken at 26.4 meters. For the rising main 2" GI pipe was used with a 3/4" CWD pump rod inside. From the delivery T to the de-aeration vessel 3 meter 2" plastic hose was used. The stroke was adjusted at 200 mm

CWD 5001 TEST RIG

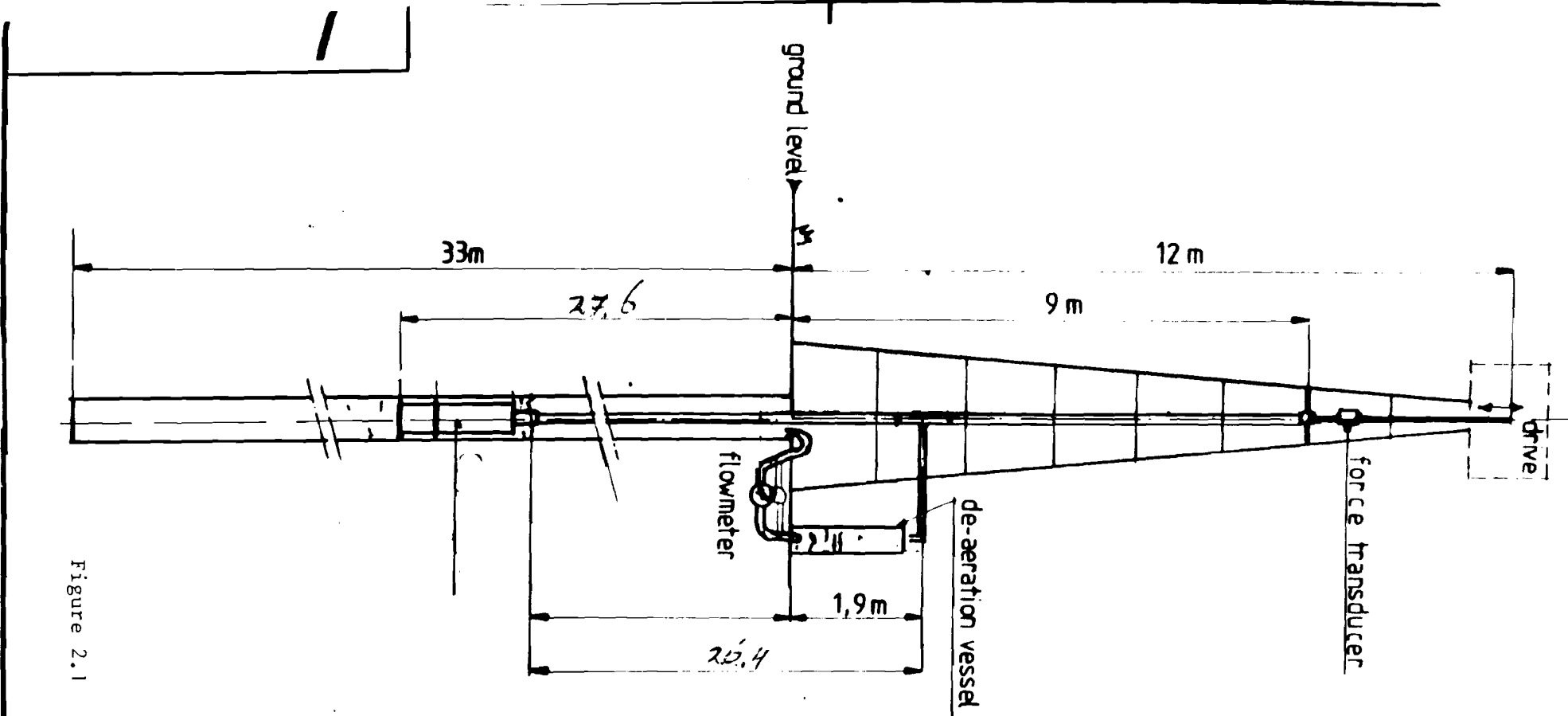




Figure 2.1

If not shown apply		Tolerance	Welds	Roughness	Modification description		Date	Initials		
 <p>CONSULTANCY SERVICES CWD ENERGY DEVELOPMENT SERVICES</p>	<p>P.O. BOX 55 2500 AB AMERSFOORT THE NETHERLANDS</p>	Material:		Subject:	Quantity per assembly	Drawn by				
		Treatment:				Date				
<p>This drawing is the property of CWD and can only be used, copied or issued to others, wholly or in parts, with our prior written permission. This drawing has been carefully reviewed and believed to be accurate, complete and reliable, however, no responsibility is assumed for the operability of any product made by means of this drawing.</p>						Approved by				
Format		Scale	Drawing Standards	 <p>American projection</p>			Date			
A4			ISO		All dimensions in mm		Mass:		kg	
						Drawing No				

### **3. DESCRIPTION OF THE MEASURED CONFIGURATIONS**

#### **3.1 Configuration 02 (fig. 2.1)**

Pump tested: CWD 108D pump

Installation depth: 27.6 m foot valve to ground level

Imersion depth: 3.1 m

Static head: 26.4 m

Static pump rod force: 2.3 KN

Internal diameter pump: 108 mm

Internal diameter rising main: 52 mm (2" GI pipe)

External diameter pump rod: 27 mm (3/4" GI pipe)

Stroke: 200 mm

Air chamber: yes 16 l

Valve lifting height:     piston valve 3.5 mm  
                                  foot valve  3.5 mm

**3.2 Configuration 07 (fig. 2.2)**

Pump testes: CWD 108D pump

Installation depth: 27.6 m foot valve to ground level

Imersion depth: 3.4 m

Static head: 26.1 m

Static pump rod force: 2.4 KN

Internal diameter pump: 108 mm

Internal diameter rising main: 52 mm (2" GI pipe)

External diameter pump rod: 27 mm (3/4" GI pipe)

Stroke: 200 mm

Air chamber: no

Valve lifting height: piston valve 3.5 mm

foot valve 3.5 mm



#### 4. ELUCIDATION OF THE MEASUREMENTS

Each measurement has an index number according the following format:

pxxx yy zz  
 A B C

where in A: p = for pump

xxx = diameter pump in mm

B: yy = configuration

C: zz = number of measurement

The results of the measurements are given in the annexes.

Configuration 02 Annex I p10802.....

Configuration 02 Annex II p10807.....

The results are represented as indicator diagrams for each rotation speed measured.

Also the total results,  $F_{\max}-F_{\min}$ ,  $\eta_{\text{vol}}$  and  $\eta_{\text{mech}}$  are given as function of the rotational speed. In the indicator diagrams the value of the pump rod weight ( $G_{\text{pr}}$ ) and pump rod weight + static force ( $F_{\text{st}} + G_{\text{pr}}$ ) are marked.

##### 4.1 Configuration 02

Force, flow and speed are measured for  $n=0.1$  to 1.47 rps (maximum attainable) in steps of 0.1 rps. The results of the measurements are given in indicator diagrams (annex I-1 to I-13) Out of these measured values the force step peak peak ( $\Delta F = F_{\max}-F_{\min}$ ) mechanical efficiency ( $\eta_{\text{mech}}$ ) and the volumetric efficiency ( $\eta_{\text{vol}}$ ) are calculated. Annexes I-14 to I-18 give the results of  $\Delta F$ ,  $\eta_{\text{mech}}$  and  $\eta_{\text{vol}}$  versus the rotational speed. The best fitting polynomial approximations have been calculated and are for  $0.5 \leq \omega \leq 9.5$ .

$$\Delta F = 0.01553 \omega^2 + 0.565 \omega + 2.818 \quad [\text{kN}]$$

$$\eta_{\text{mech}} = 0.00195 \omega^3 - 0.0358 \omega^2 + 0.196 \omega + 0.644 \quad [-]$$

$$\eta_{\text{vol}} = 0.0022 \omega^3 - 0.0437 \omega^2 + 0.236 \omega + 0.524 \quad [-]$$

In the force signal two vibrations can be seen with different frequency one in the upward stroke and the other in the downward stroke.

The natural frequency of pump rod and the accelerated water mass (assumed an ideal air chamber behaviour) is:

$$f_o = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$k = \frac{E_{\text{pr}} \cdot A_{\text{pr}}}{l_{\text{pr}}}$$

$$m = m_{\text{wc}} + m_p + \frac{1}{3} m_{\text{pr}}$$

$$f_o = \text{natural frequency} \quad [\text{Hz}]$$

$$E_{\text{pr}} = \text{elasticity modulus pump rod} \quad [\text{N/m}^2]$$

$$A_{\text{pr}} = \text{cross section pump rod} \quad [\text{m}^2]$$

$$l_{\text{pr}} = \text{length pump rod} \quad [\text{m}]$$

$$m_{\text{wc}} = \text{mass water in cilinder} \quad [\text{kg}]$$

$$m_p = \text{mass piston} \quad [\text{kg}]$$

$$m_{\text{pr}} = \text{mass pump rod} \quad [\text{kg}]$$

upward stroke

$$k = \frac{2.1 \times 10^{11} \cdot 2.02 \times 10^{-4}}{36} = 1.18 \times 10^6 \text{ N/m}^w$$

$$m = 6 + 3 + 1/3 \cdot 65 = 30.6 \text{ kg}$$

$$f_o = \frac{1}{2\pi} \sqrt{\frac{1.18 \times 10^6}{30.6}} = 31 \text{ Hz}$$

downward stroke

$$k = 1.18 \times 10^6$$

$$m = 0 + 3 + 1/3 \cdot 65 = 24.6 \text{ kg}$$

$$f_o = \frac{1}{2\pi} \sqrt{\frac{1.18 \times 10^6}{24.6}} = 34 \text{ Hz}$$

The frequencies found in the upward stroke range from 22 to 25.6 Hz and in the downward stroke from 22.4 to 36.7 Hz.

From this result it can be concluded that the vibrations in the force are caused by the give of the pump rod and accelerated water mass and that a model with lumped parameters can be used for pumps with air chambers.

A strange valve behaviour is observed at rotational speeds above 1.3 rps. A suddenly delay in valve closure and opening angle is found for  $n < 1.3$  rps.

Up to now no good explanation can be given for this strange valve behaviour.

## 4.2 Configuration 07

Force, flow and speed are measured for  $n=0.1$  to  $n=0.7$  rps. The results of the measurements are given in indicator diagrams (annex II-1 to II-5). Out of these measured values the force step peak-peak ( $\Delta F = F_{\max} - F_{\min}$ ), mechanical efficiency ( $\eta_{\text{mech}}$ ) and the volumetric efficiency are calculated. Annexes II-6 to II-10 give the results of  $\Delta F$ ,  $\eta_{\text{mech}}$  and  $\eta_{\text{vol}}$  versus the rotational speed.

The best fitting polynomial approximations have been calculated and are for  $0.5 \leq \omega \leq 5$  rad/s.

$$\Delta F = 0.168 \omega^2 + 0.550 \omega + 3.509 \quad [\text{kN}]$$

$$\eta_{\text{mech}} = 0.0013 \omega^3 - 0.141 \omega^2 + 0.506 \omega + 0.273 \quad [-]$$

$$\eta_{\text{vol}} = 0.010 \omega^3 - 0.104 \omega^2 + 0.381 \omega + 0.489 \quad [-]$$

In the force signal a vibration with a circle frequency of  $\approx 3.1$  Hz is found during upward stroke.

The natural frequency of pump rod and accelerated water mass is

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{k}{m^*}}$$

$$m^* = m_{\text{wc}} + m_{\text{p}} + 1/3 m_{\text{pr}} + \frac{A_{\text{p}}^2}{A_{\text{rm}}} \times l_{\text{rm}} \cdot \rho_{\text{w}}$$

$$l_{\text{rm}} = \text{length rising main} \quad [\text{m}]$$

$$\rho_{\text{w}} = \text{density water} \quad [\text{kg/m}^3]$$

$$k = \frac{2.1 \times 10^{11} \times 2.02 \times 10^{-4}}{36} = 1.18 \times 10^6 \text{ N/m}^2$$

$$m^* = 1200 \text{ kg}$$

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{1.18 \times 10^6}{1200}} = 5 \text{ Hz}$$

In this case the computed and measured frequencies are not equal and the lumped parameter model used is not correct. A more sophisticated model has to be used for pumps without air chambers.

## **5. CONCLUSIONS**

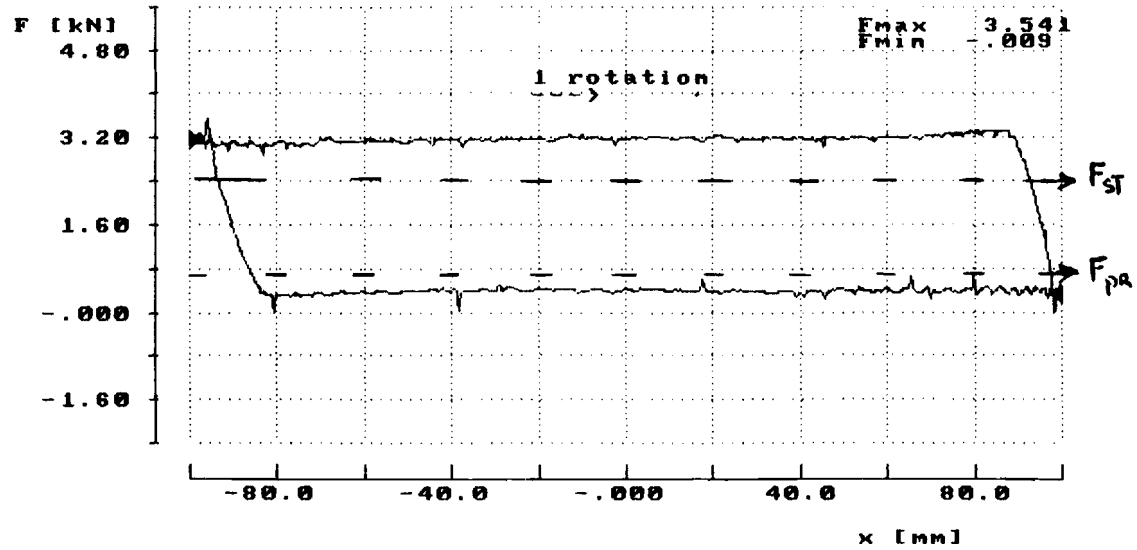
- Using a pump without an air chamber or with a not working air chamber leads to very high forces, see Annex III-3.
- Comparison of the measurements of configuration 02 and 07 shows no differences between the volumetrical and mechanical efficiency, see Annex III I-2.
- If air chambers are used a lumped parameter model can be used to compute pump rod forces.
- If no air chambers are used the lumped parameter model is not correct, a model including waves in pump rod and water colom will probably fit better to the measured results.

filename --> B:P1080227.\_

Pin [W]:  
154.470  
Pout[W]:  
129.753

Evol :  
.912  
Emech :  
.840

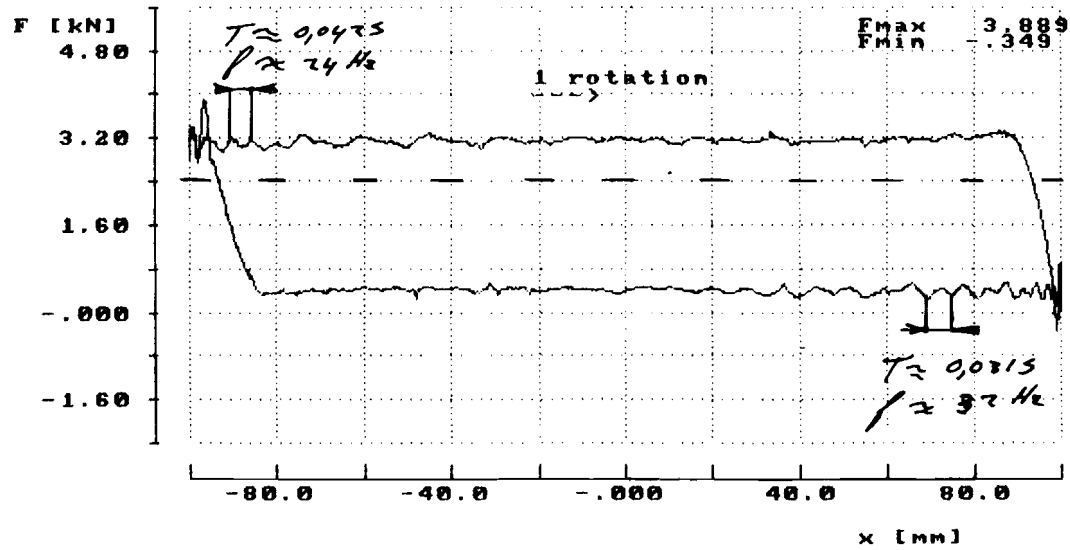
head [m] :  
26.4  
pump [mm] :  
108.0  
stroke [ml]:  
.2



rotationspeed (rps): .300  
meanflow (dm<sup>3</sup>/s) : .501 sample frequency (Hz): 613.961

filename --> B:P1080226.\_

Pin [W]:  
 203.971  
 Pout[W]:  
 178.175  
 Evol :  
 .941  
 Emech :  
 .874  
 head [m] :  
 26.4  
 pump [mm] :  
 108.0  
 stroke [m]:  
 .2



rotationspeed (rps): .399  
 meanflow (dm3/s) : .688 sample frequency (Hz): 817.310

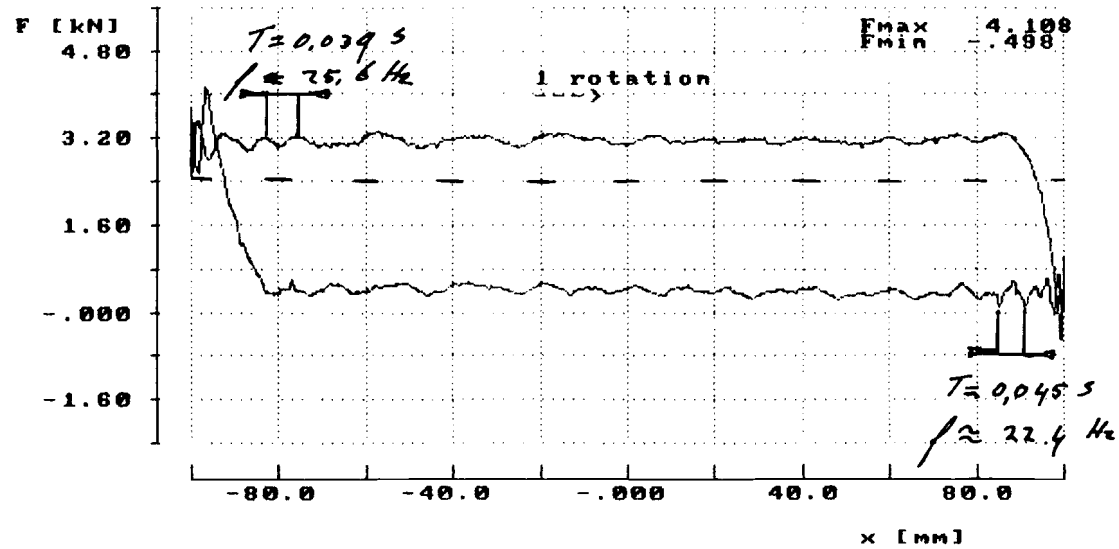


filename --> B:P1080225.\_

Pin [W]:  
252.771  
Pout[W]:  
231.159

Evol :  
.978  
Erech :  
.914

head [m] :  
26.4  
pump [mm] :  
100.0  
stroke [m]:  
.2



rotationspeed (rps): .498  
meanflow (dm<sup>3</sup>/s) : .893 sample frequency (Hz): 1019.754

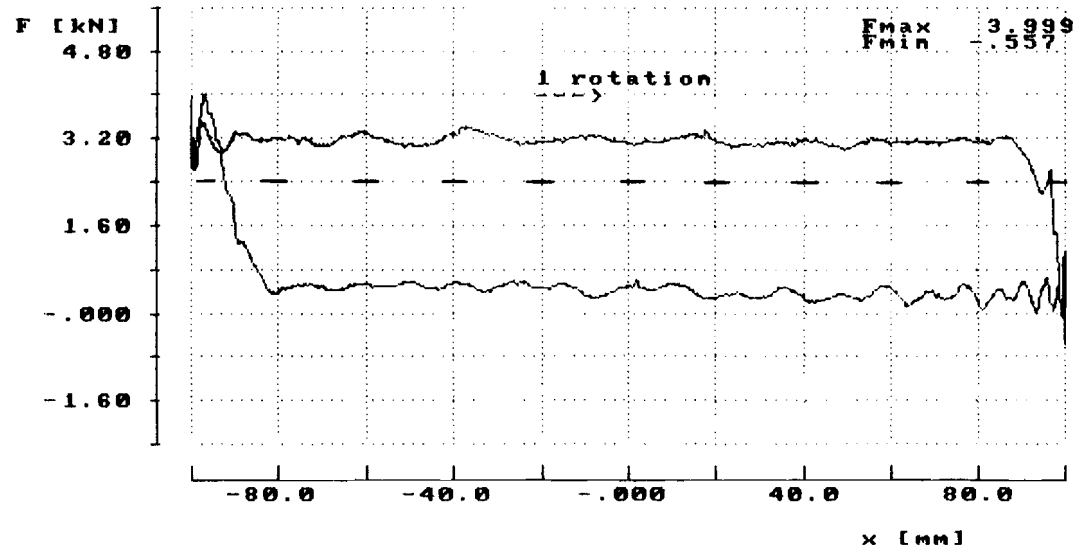
4

filename --> B:P1000224.\_

Pin [W]:  
305.789  
Pout[W]:  
276.686

Evol :  
.971  
Emech :  
.905

head [m] :  
26.4  
pump [mm] :  
100.0  
stroke [m]:  
.2



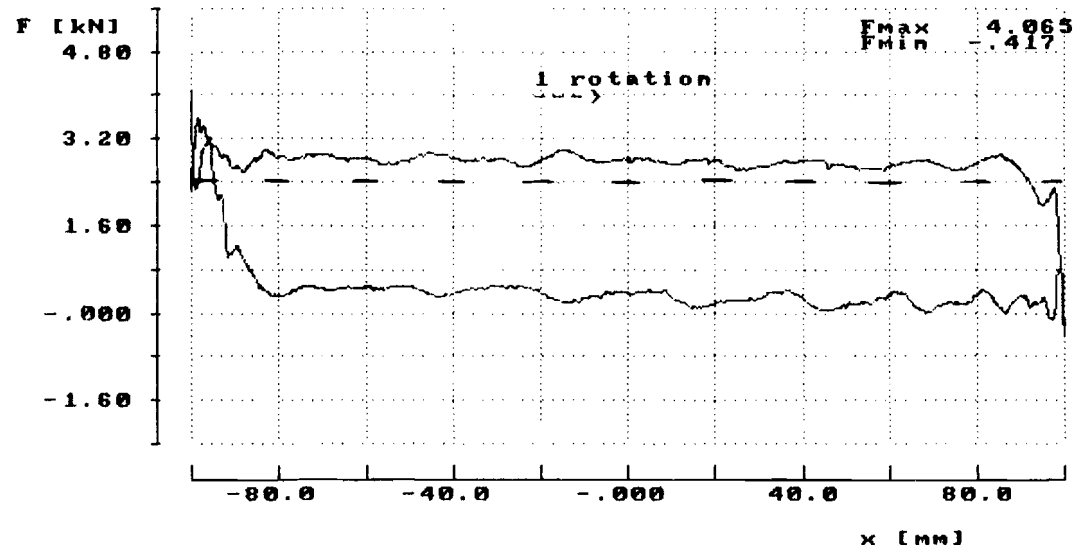
rotationspeed (rps): .601  
meanflow (dm3/s) : 1.068 sample frequency (Hz): 1230.031

filename --> B:P1080223.\_

Pin [W]:  
324.970  
Pout[W]:  
366.669

Evol :  
1.105  
Emech :  
1.128

head [m] :  
26.4  
pump [mm] :  
108.0  
stroke [m]:  
.2



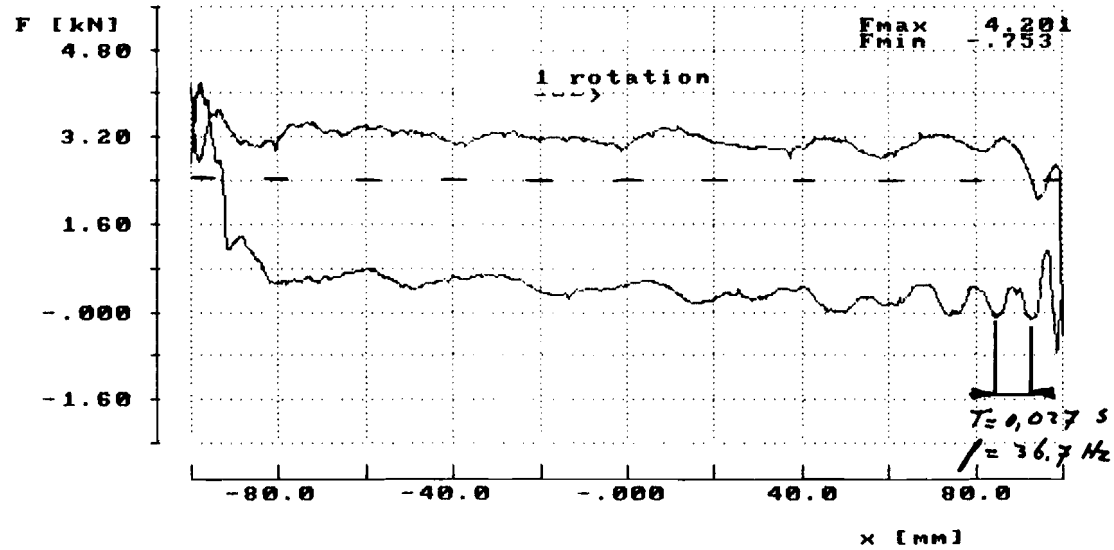
rotationspeed (rps): .700  
meanflow (dm3/s) : 1.416 sample frequency (Hz): 1432.777

filename --> B:P1000222.\_

Pin [W]:  
421.548  
Pout[W]:  
364.096

Evol :  
.941  
Emech :  
.864

head [m] :  
26.4  
pump [mm] :  
100.0  
stroke [m]:  
.2



rotationspeed (rps): .816  
meanflow (dm3/s) : 1.406 sample frequency (Hz): 1670.167

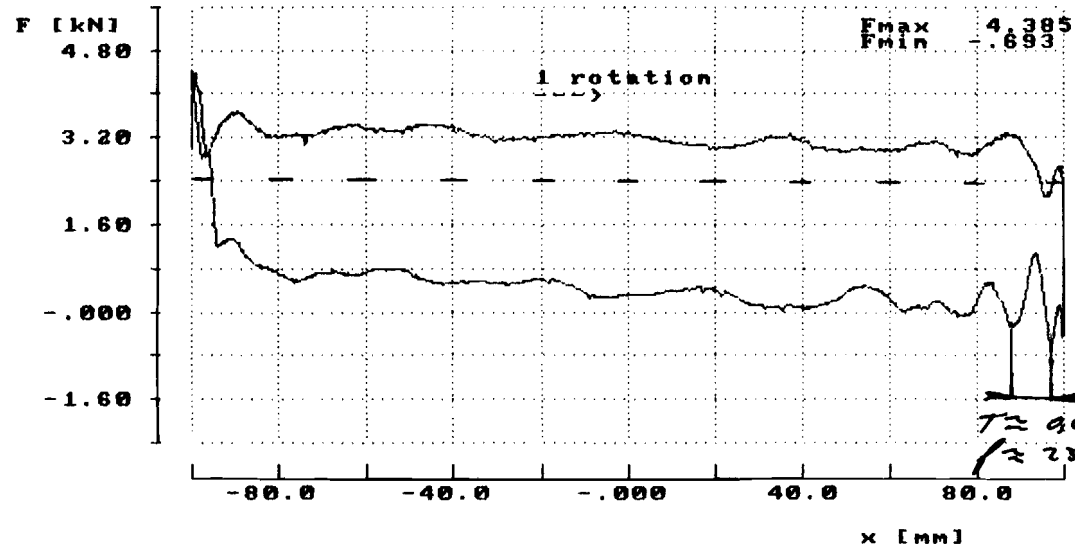
7

filename --> B:P1080221.\_

Pin [W]:  
478.351  
Pout[W]:  
400.931

Evol :  
.939  
Emech :  
.838

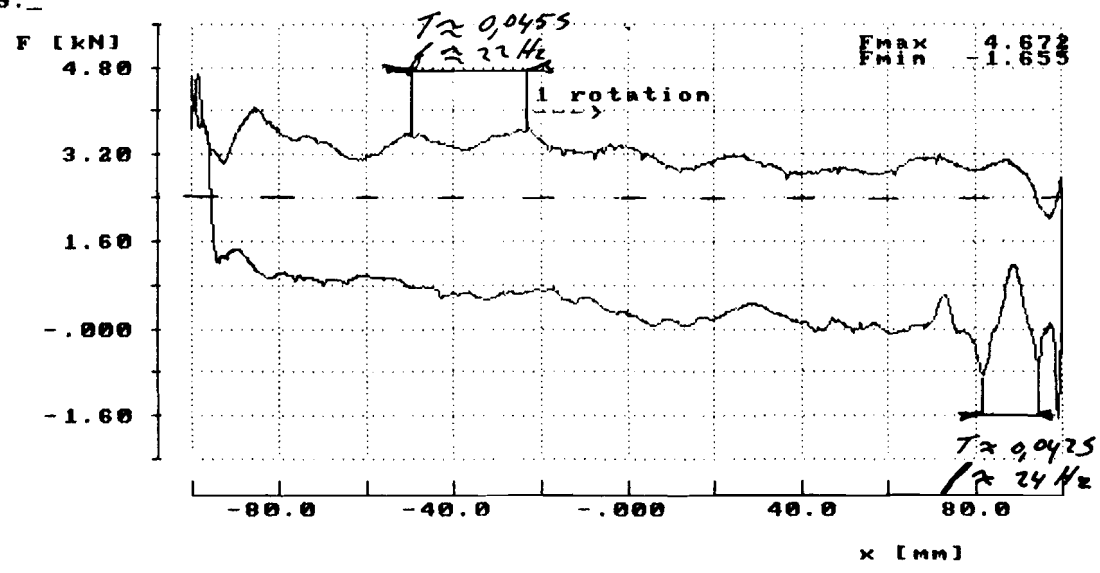
head [m] :  
26.4  
pump [mm] :  
108.0  
stroke [m]:  
.2



rotationspeed (rps): .900  
meanflow (dm3/s) : 1.548 sample frequency (Hz): 1843.390

filename --> B:P1000219.\_

Pin [W]:  
586.637  
Pout[W]:  
487.155  
Evol :  
1.029  
Emech :  
.830  
head [m] :  
26.4  
pump [mm] :  
100.0  
stroke [m]:  
.2



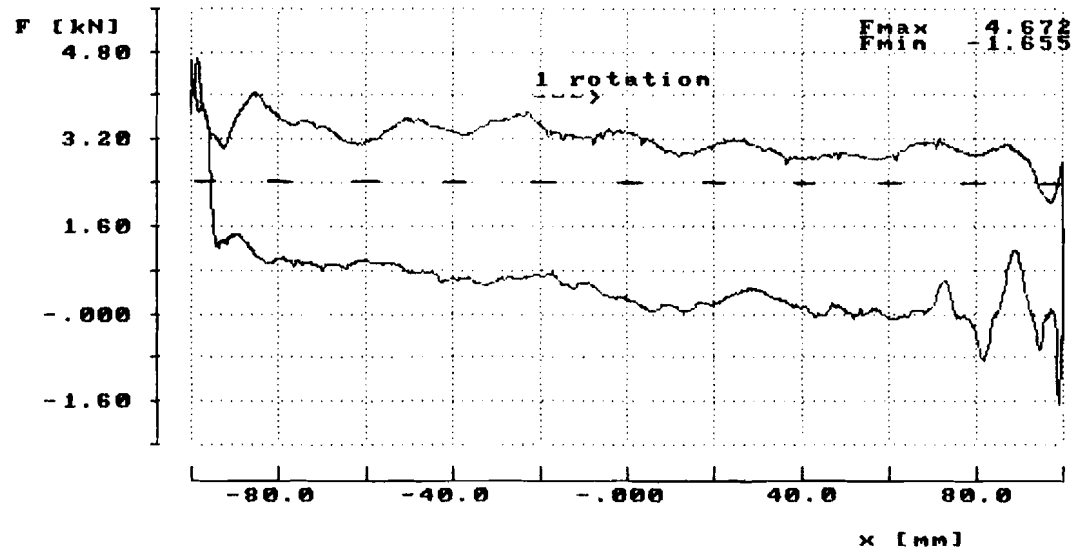
rotationspeed (rps): .998  
meanflow (dm3/s) : 1.881 sample frequency (Hz): 2043.123

filename --> B:P1000220.\_

Pin [W]:  
506.637  
Pout[W]:  
487.155

Evol :  
.934  
Emech :  
.830

head [m] :  
26.4  
pump [mm] :  
100.0  
stroke [m]:  
.2



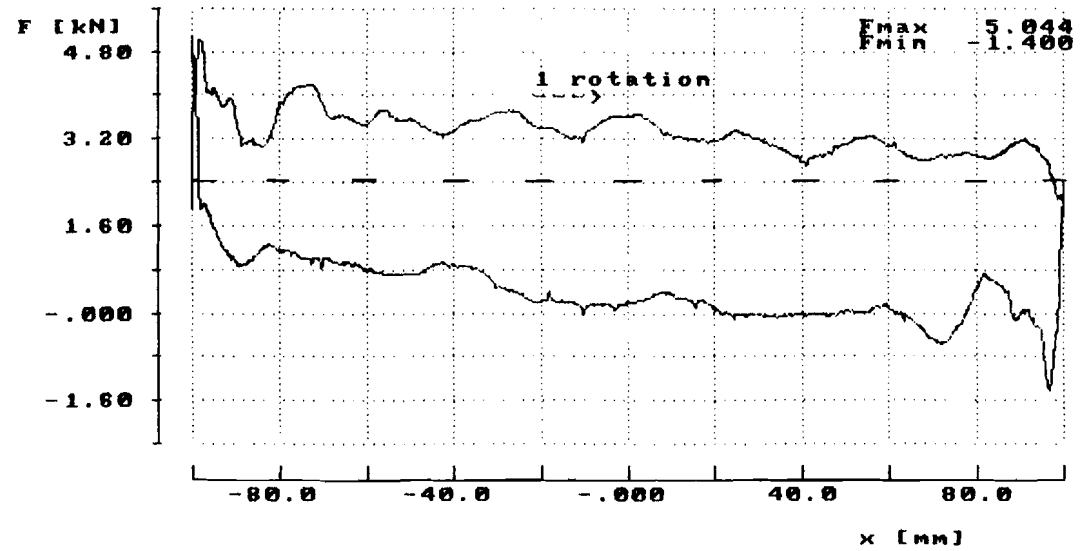
rotationspeed (rps): 1.099  
meanflow (dm3/s) : 1.881 sample frequency (Hz): 2251.593

filename --> B:P1080213.\_

Pin [W]:  
699.984  
Pout[W]:  
548.840

Evol :  
.963  
Erech :  
.784

head [m] :  
26.4  
pump [mm] :  
108.0  
stroke [m]:  
.2



rotationspeed (rps): 1.201  
meanflow (dm3/s) : 2.119 sample frequency (Hz): 2459.761

OK COMPLETE

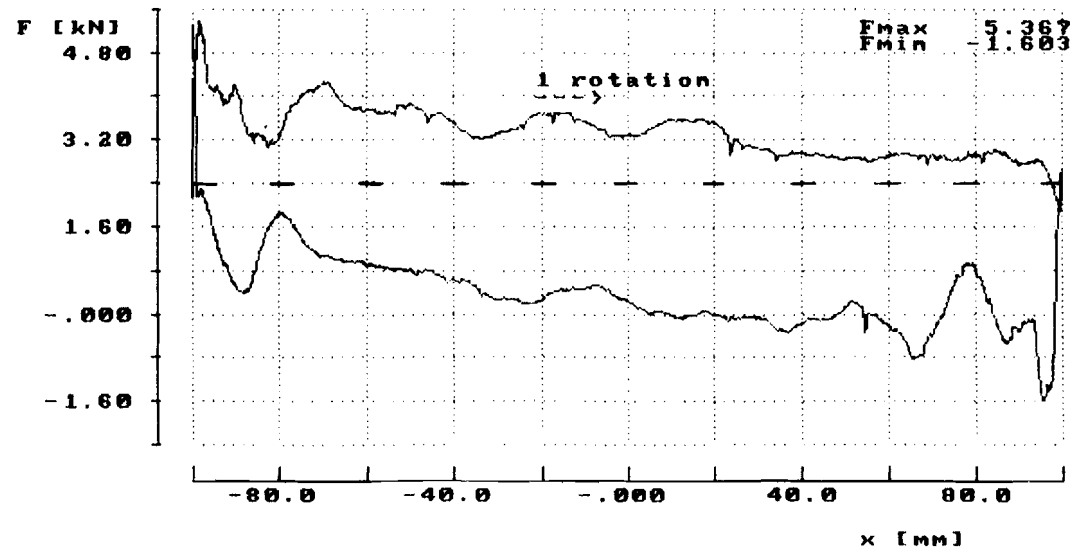


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Pin [W]:  
774.194  
Pout[W]:  
585.532

Evol :  
.944  
Emech :  
.756

head [m] :  
26.4  
pump [mm] :  
108.0  
stroke [m]:  
.2



rotationspeed (rps): 1.307  
meanflow (dm3/s) : 2.261 sample frequency (Hz): 2678.666

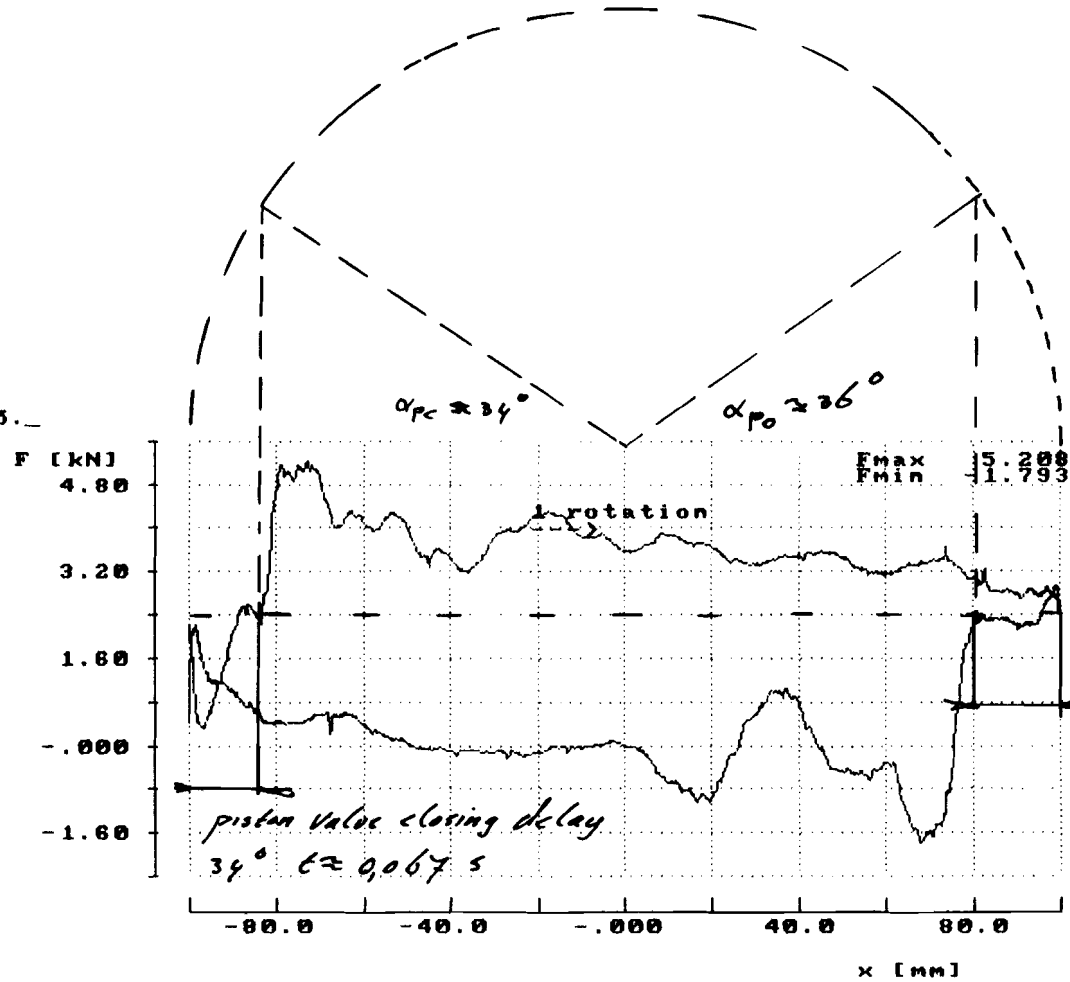
filename --> B:P1000215.\_

Pin [W]:  
894.716  
Pout[W]:  
625.543

Evol :  
.941  
Emech :  
.699

head [m] :  
26.4  
pump [mm] :  
108.0  
stroke [m]:  
.2

rotationspeed (rps): 1.400  
meanflow (dm3/s) : 2.415 sample frequency (Hz): 2867.964

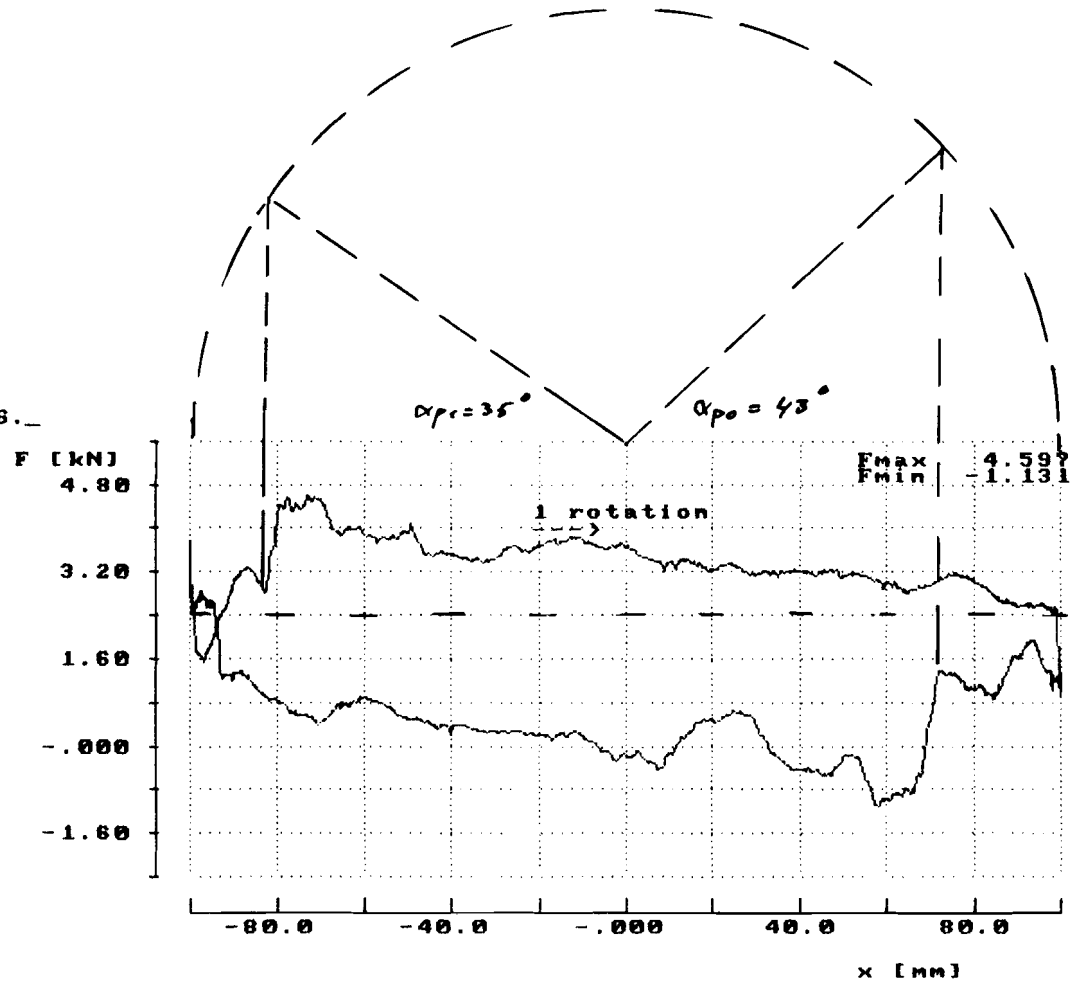


filename --> B:P1080216.\_

Pin [W]:  
846.209  
Pout[W]:  
631.023

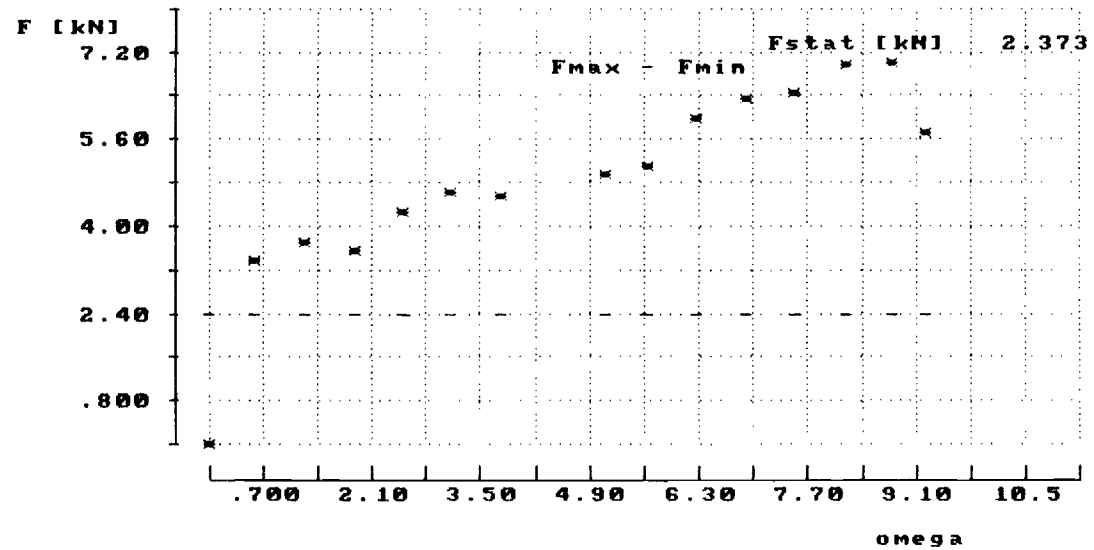
Evol :  
.904  
Emech :  
.746

head [m] :  
26.4  
pump [mm] :  
108.0  
stroke [m]:  
.2



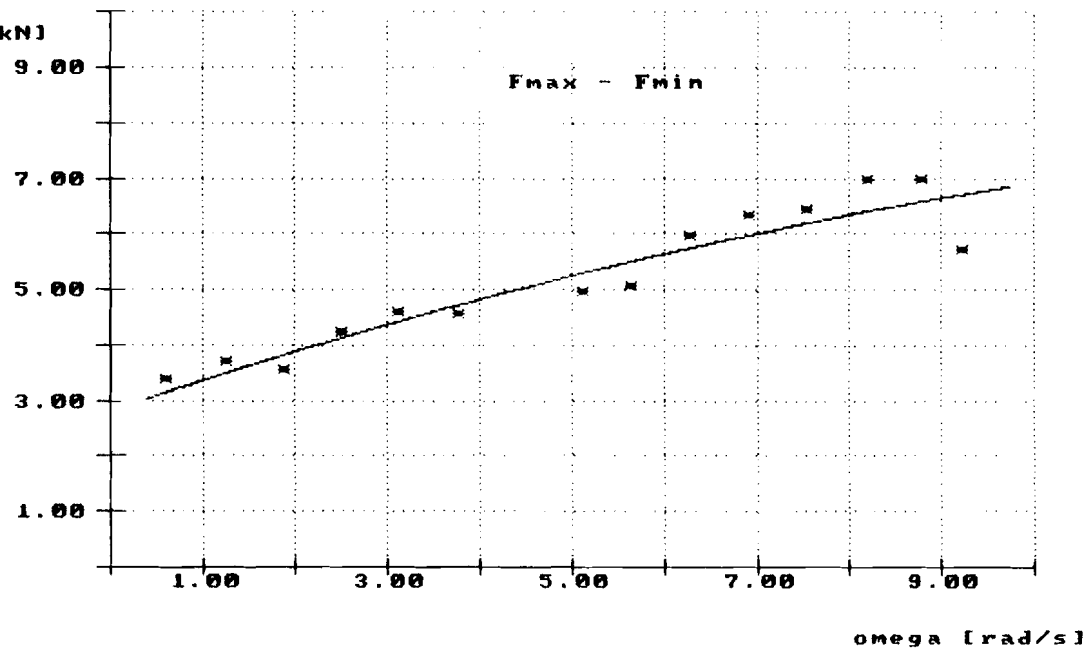
rotationspeed (rps): 1.471  
meanflow (dm<sup>3</sup>/s) : 2.437 sample frequency (Hz): 3012.567

filename --&gt; B:P10002T\_

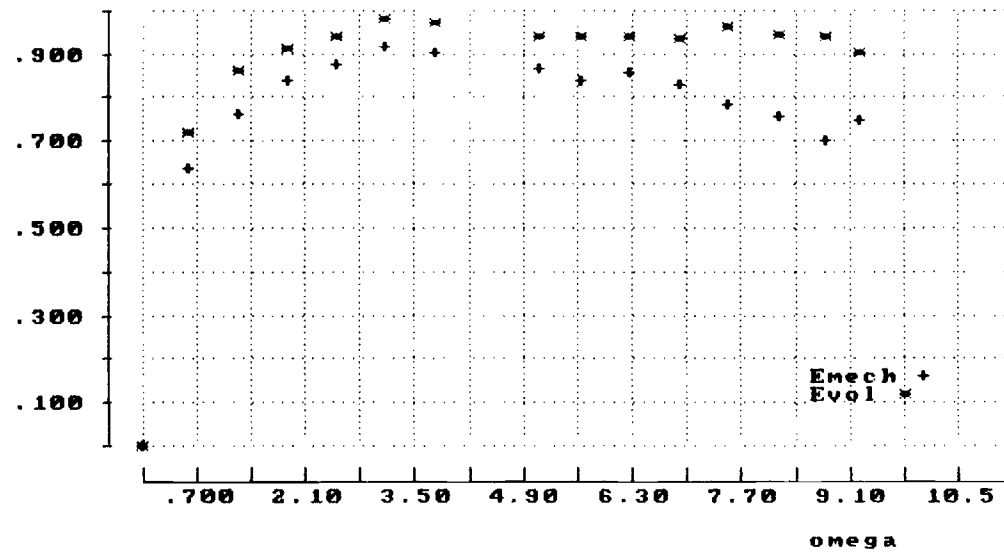


filename --> P1000903\_

differences : F [kN]  
n (rad/s) %  
9.2 17.2  
.6 6.9  
1.3 5.5  
6.3 3.7  
6.9 5.5  
5.7 8.6  
5.1 7.1  
1.9 7.8  
3.8 3.8  
3.1 3.8  
2.5 2.4  
1.9 7.8  
degree  
: 2  
coefficients  
:  
-.01553  
.56506  
2.81776



filename --&gt; B:P10802I\_

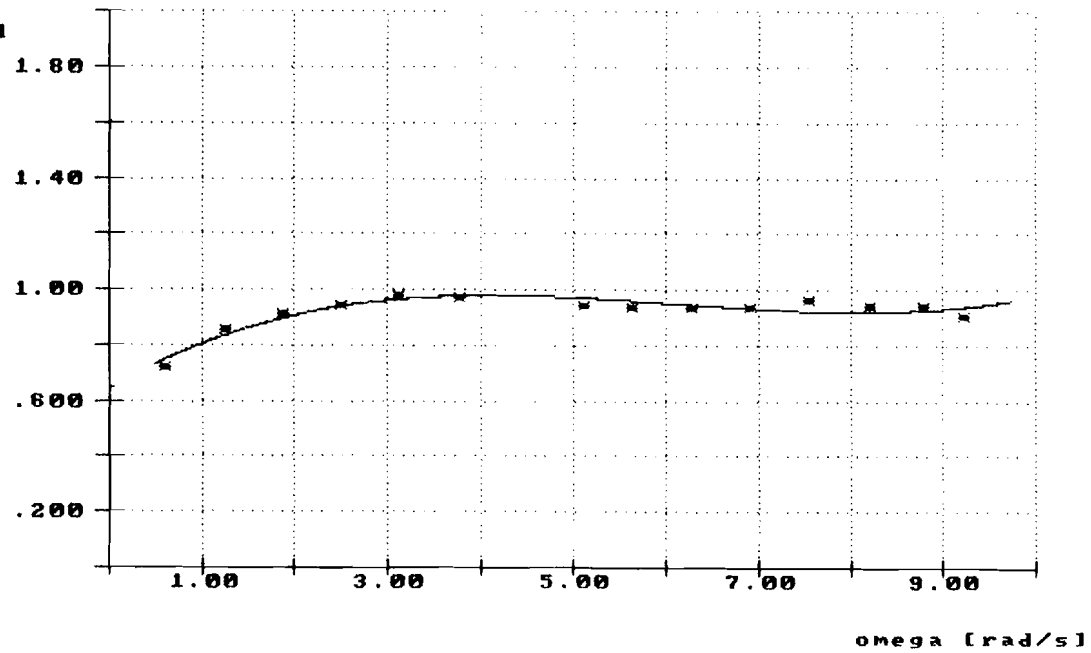


L

filename --> P1080903\_

differences :  
n (rad/s) %  
9.2 4.0  
.6 4.4  
1.3 2.7  
6.3 1.0  
6.9 .0  
5.7 2.3  
5.1 3.2  
1.9 1.4  
3.8 .8  
3.1 1.2  
2.5 .0  
1.9 1.4  
degree  
: 3  
coefficients  
:  
.00195  
-.03579  
.19590  
.64388

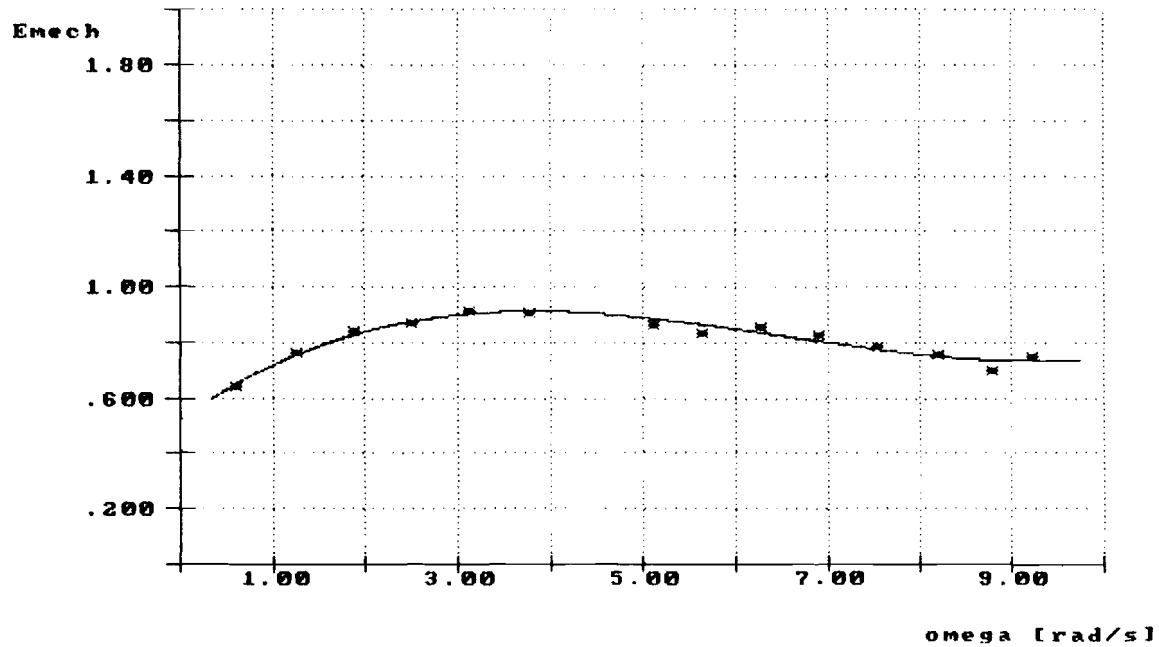
Evol



L

filename --> P1080903\_

differences :  
n (rad/s) %  
9.2 2.0  
.6 2.2  
1.3 .8  
6.3 2.4  
6.9 3.1  
5.7 3.3  
5.1 2.7  
1.9 1.3  
3.8 1.0  
3.1 1.1  
2.5 .4  
1.9 1.3  
degree  
: 3  
coefficients  
:  
.00222  
-.04368  
.23632  
.52445



L

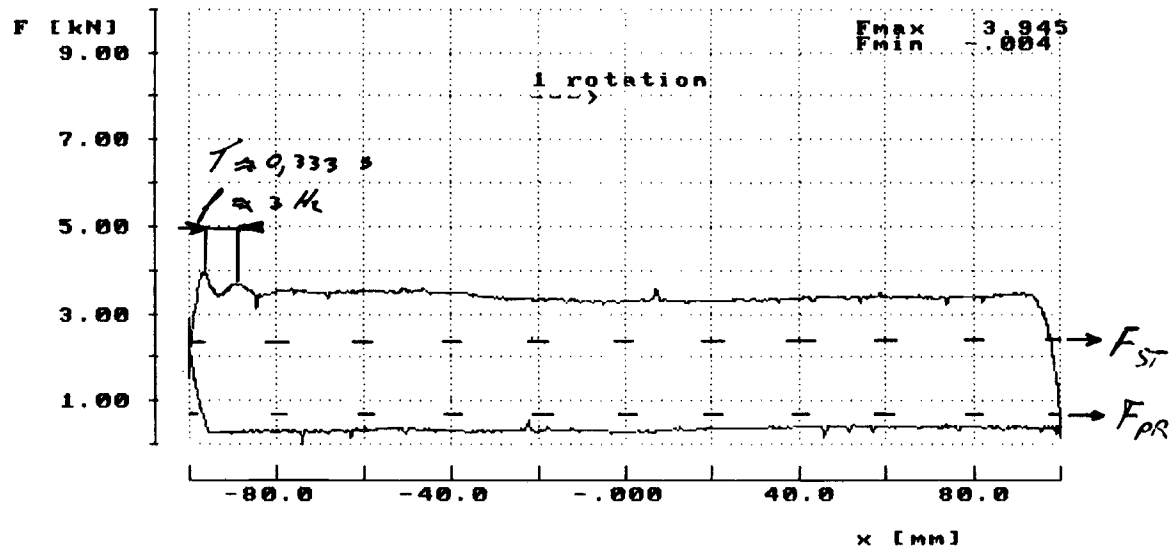


filename --> P1080706.\_

Pin [W]:  
60.201  
Pout[W]:  
32.353

Evol :  
.689  
Emech :  
.537

head [m] :  
26.1  
pump [mm] :  
108.0  
stroke [m]:  
.2



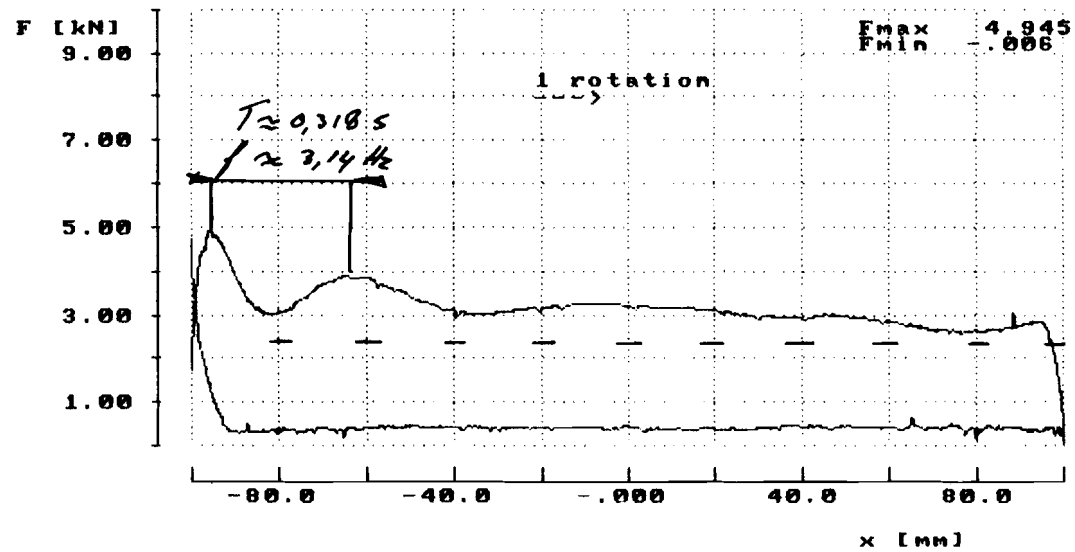
rotationspeed (rps): .100  
meanflow (dm3/s) : .126 sample frequency (Hz): 204.855

filename --> P1080703.\_

Pin [W]:  
160.030  
Pout[W]:  
141.598

Evol :  
1.016  
Emech :  
.885

head [m] :  
26.1  
pump [mm] :  
108.0  
stroke [m]:  
.2



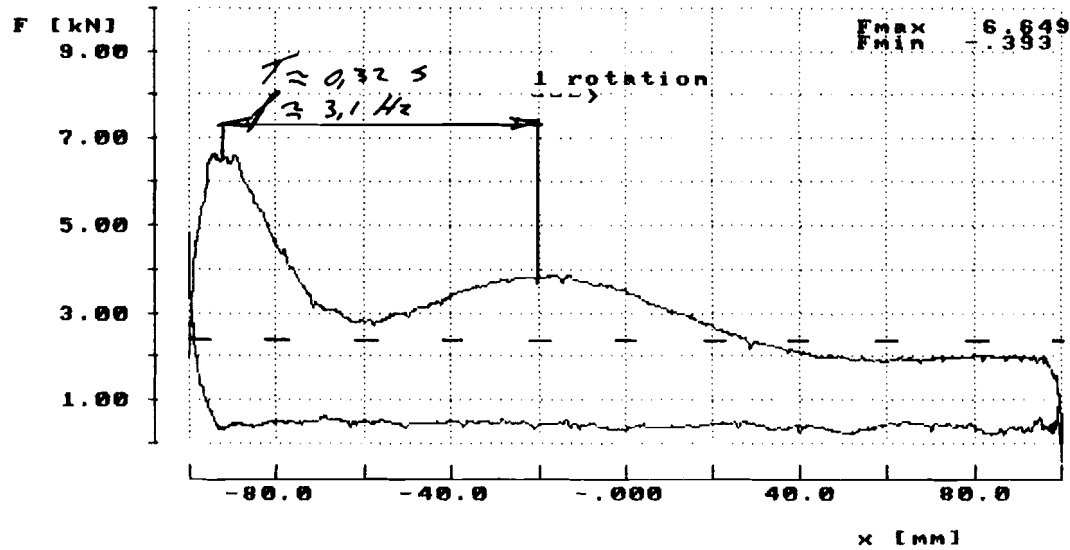
rotationspeed (rps): .297  
meanflow (dm<sup>3</sup>/s) : .553 sample frequency (Hz): 608.539

filename --> P1000701.\_

Pin [W]:  
261.048  
Pout[W]:  
236.791

Evol :  
1.009  
Enech :  
.907

head [m] :  
26.1  
pump [mm] :  
100.0  
stroke [m]:  
.2



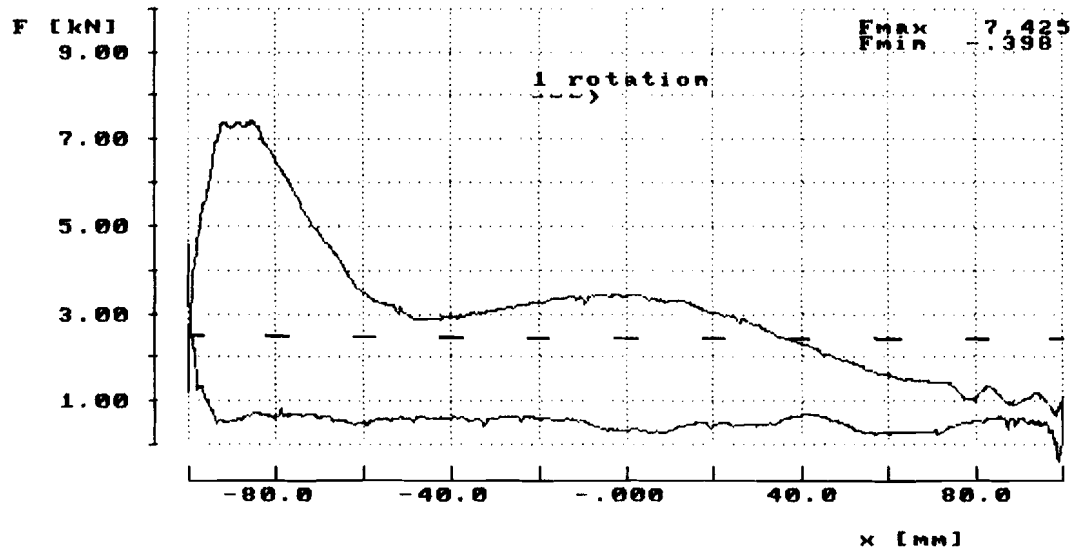
rotationspeed (rps): .500  
meanflow (dm3/s) : .925 sample frequency (Hz): 1024.273

filename --> P1000705.\_

Pin [W]:  
311.505  
Pout[W]:  
271.368

Evol :  
.984  
Emech :  
.871

head [m] :  
26.1  
pump [mm] :  
100.0  
stroke [m]:  
.2



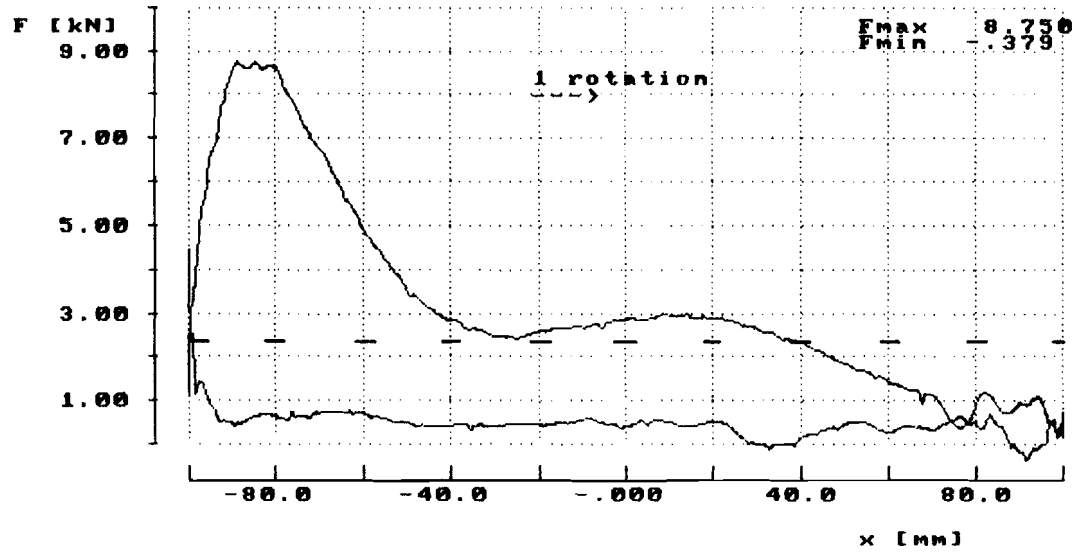
rotationspeed (rps): .600  
meanflow (dm3/s) : 1.060 sample frequency (Hz): 1229.127

filename --> P1000702.\_

Pin [W]:  
385.543  
Pout[W]:  
328.929

Evol :  
1.010  
Emech :  
.853

head [m] :  
26.1  
pump [mm] :  
100.0  
stroke [m]:  
.2



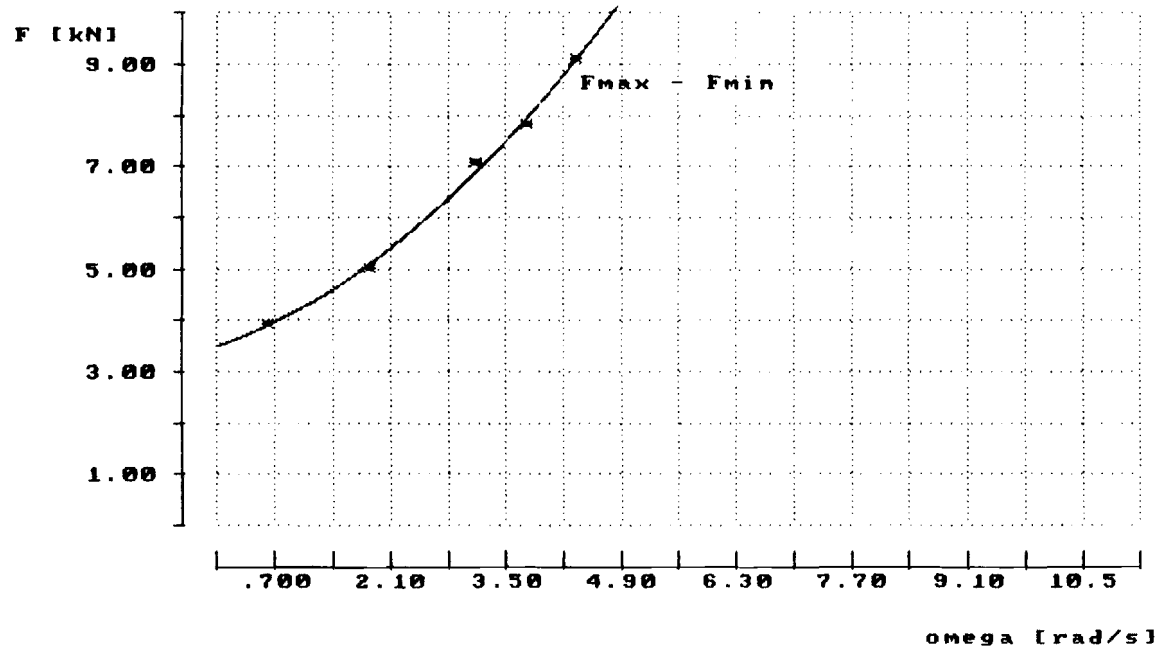
rotationspeed (rps): .694  
meanflow (dm3/s) : 1.285 sample frequency (Hz): 1421.932

filename --> B:PI0807I\_

differences :  
n (rad/s) %

.63	.7
1.87	2.0
3.14	2.7
3.77	1.9
4.36	.3

degree  
: 2  
coefficients  
:  
.168  
.550  
3.509



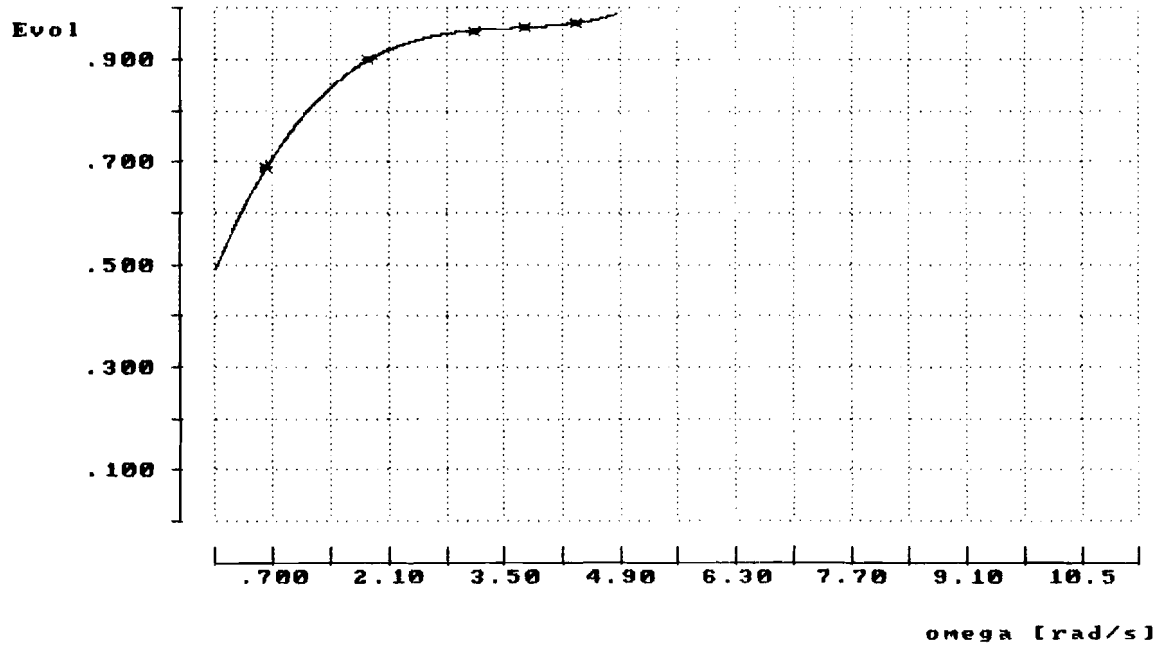
filename --> B:P10807I\_

differences :  
n (rad/s) %

.63	.0
1.87	.1
3.14	.2
3.77	.2
4.38	.1

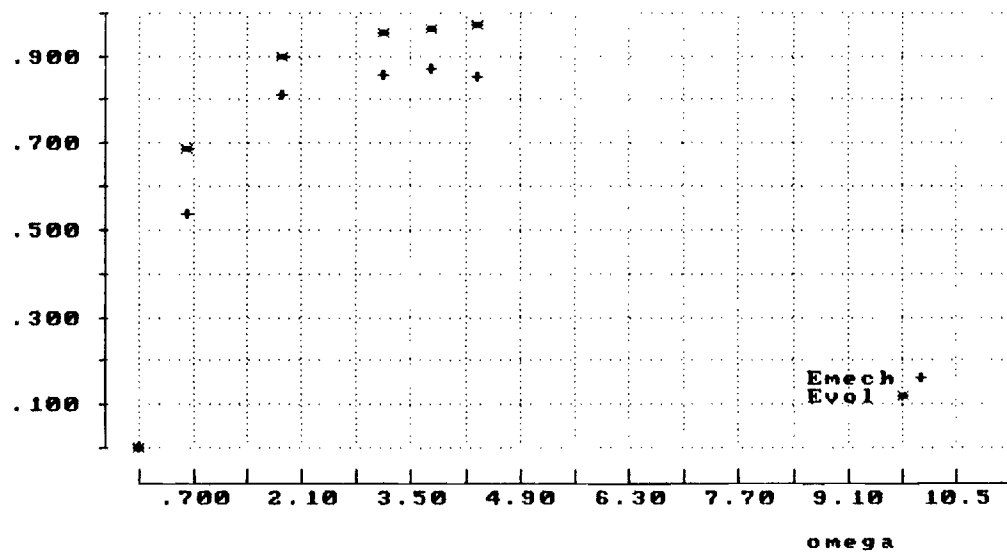
degree : 3  
coefficients :

.010
-.104
.381
.489



L

filename --> B:P10807I\_



L



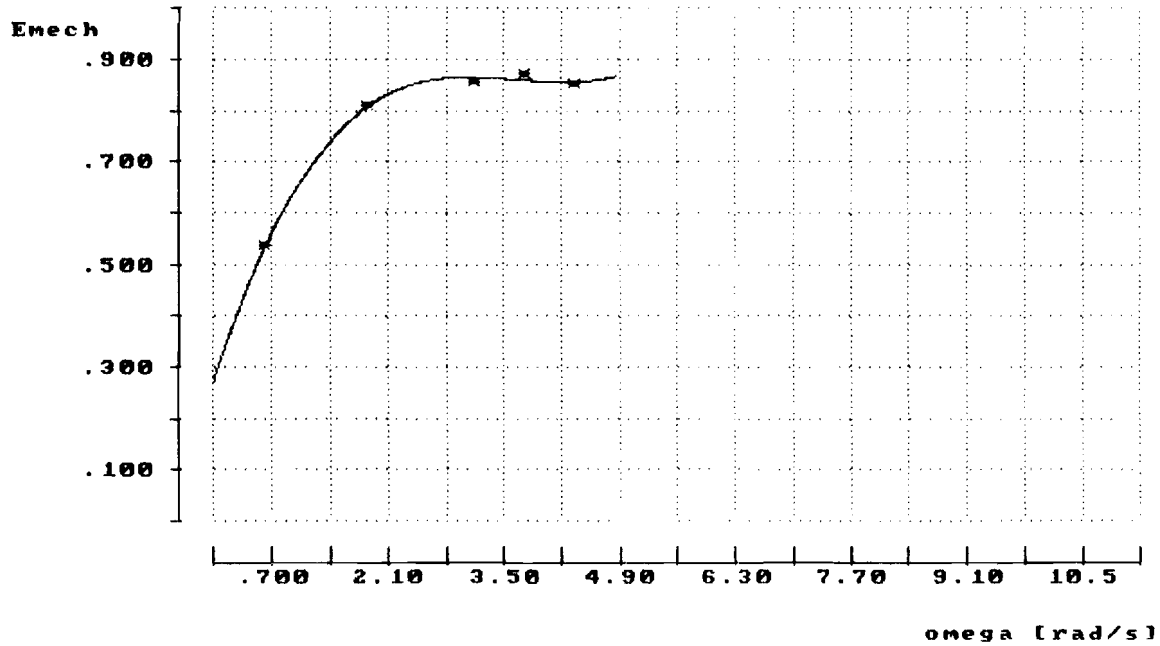
filename --> B:P10807T\_

differences :  
n (rad/s) %

.63	.1
1.87	.4
3.14	1.2
3.77	1.3
4.36	.4

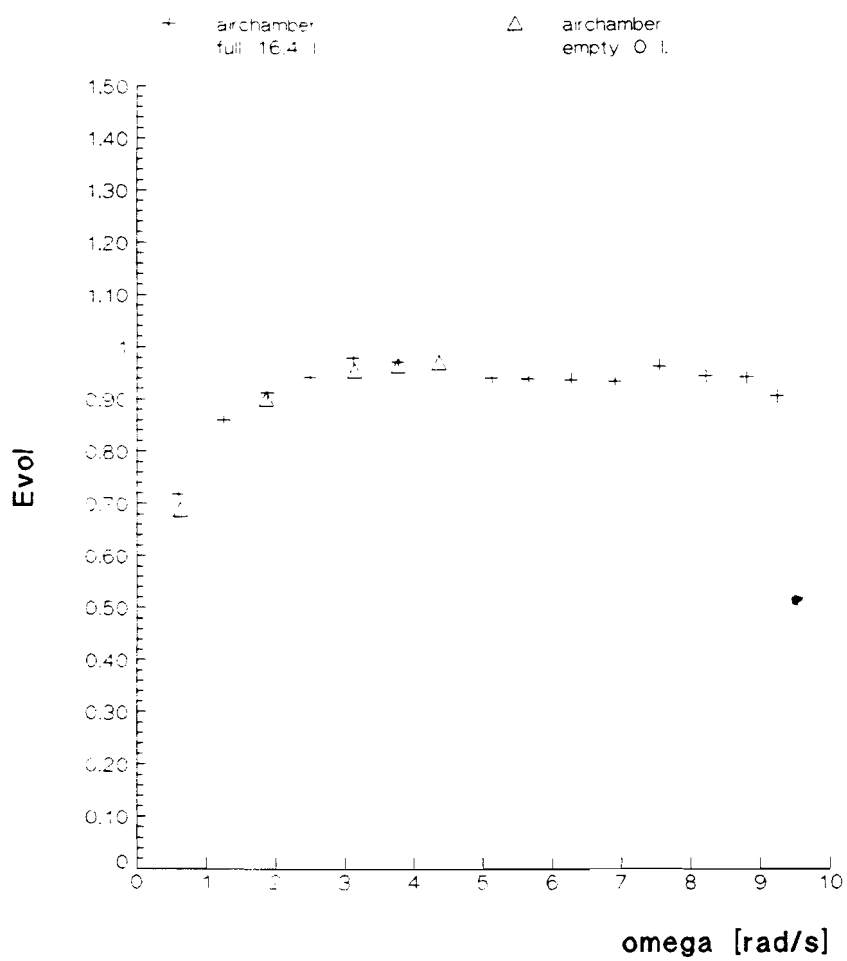
degree  
: 3  
coefficients  
:

.013
-.141
.506
.273

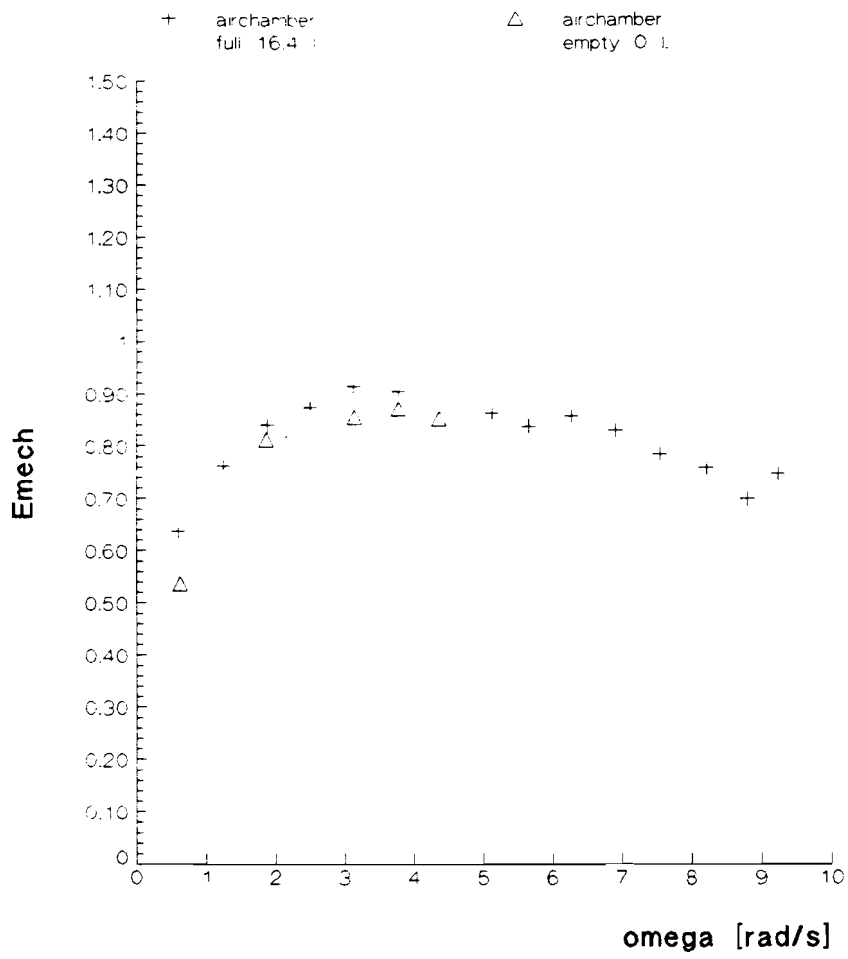


L

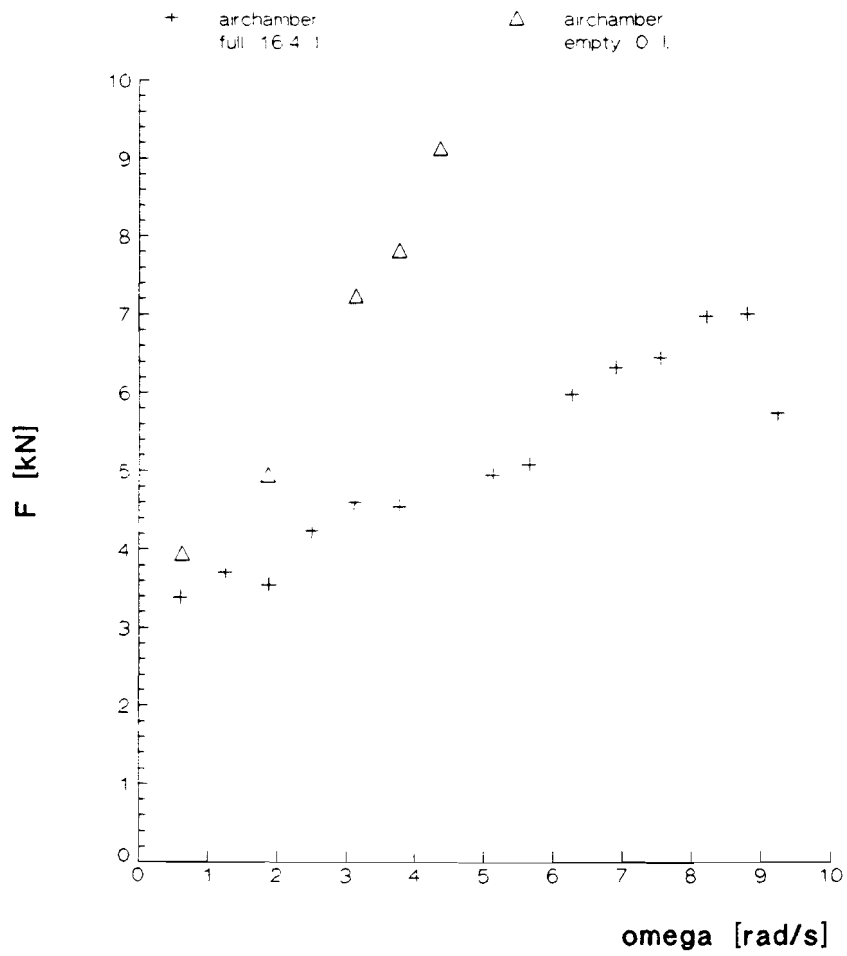
# Evol



### Emech



$$F = F_{\max} - F_{\min}$$



## ANNEX IV

### ANALYSIS OF ERRORS IN MEASURED SIGNALS

#### Random errors

##### 1. Measured quantities with absolute error

Head:  $H \pm 0.05$  [m]  
 Stroke:  $s \pm 0.2$  [mm]  
 Pump diameter:  $D \pm 1$  [mm]  
 Rotational speed:  $H \pm 0.05$  [m]

##### 2. Signal relative errors

Error in force (F) signal 1% of full-scale output  
 Error in flow (q) signal 2% of full-scale output  
 Error in speed (v) signal 1% of full-scale output

##### 3. Hardware relative error

DASH 16 interface 0.01%

The error in calculated quantities is a combination of errors from the measured quantities, signals and hardware.

Two calculated quantities are  $E_{\text{mech}}$  and  $E_{\text{vol}}$ . In this case only the relative error has been taken into account. The absolute error can be calculated from the relative error by multiplying the error with the actually calculated or measured quantity.

$$\text{Relative error: } \frac{\Delta E_{\text{mech}}}{|E_{\text{mech}}|} \quad \text{Absolute error: } \frac{\Delta E_{\text{mech}}}{|E_{\text{mech}}|} * |E_{\text{mech}}|$$

Error in  $E_{\text{mech}}$ :

$$E_{\text{mech}}: \quad = \frac{\overline{P_{\text{out}}}}{\overline{P_{\text{in}}}}$$

$$\overline{P_{\text{out}}} = \rho g H \cdot \bar{q}$$

$$\overline{P_{\text{in}}} = \overline{F \cdot v}$$

First the error in the mean-force, -speed and -flow must be know. After that it is possible to calculate the error in the input/output power.

The speed, flow and force have been sampled into the computer. Each sample has his own error. One measurement exist of 2048 samples. Data in a file on disk is a mean of  $m$  measurements. One sample from a measured signal,  $F$  (or  $v$ ,  $q$ ), is called  $F_{ij}$ .

The relative force error is:

$$\frac{\Delta F_{ij}}{|F_{ij}|} = 0.01$$

$i$  = number of measurements, 1 to  $m$

$j$  = sample number, 1 to 2048 per measurement

The means of  $m$  measurements reduce the absolute error per sample with a factor  $\sqrt{m}$ :

$$\overline{F}_j = \frac{1}{m} \sum_{i=1}^m F_{ij}$$

$$\Delta \overline{F}_j = \frac{\Delta F_{ij}}{\sqrt{m}}$$

$$\Delta \overline{v}_j = \frac{\Delta v_{ij}}{\sqrt{m}}$$

$$\Delta \overline{q}_j = \frac{\Delta q_{ij}}{\sqrt{m}}$$

The relative error:

$$\frac{\Delta \overline{F}_j}{|\overline{F}_j|} = \frac{\Delta F_{ij}}{\sqrt{m} |F_{ij}|}$$

$$\frac{\Delta \overline{v}_j}{|\overline{v}_j|} = \frac{\Delta v_{ij}}{\sqrt{m} |v_{ij}|}$$

$$\frac{\Delta \overline{q}_j}{|\overline{q}_j|} = \frac{\Delta q_{ij}}{\sqrt{m} |q_{ij}|}$$

The result of this calculation is an average sample  $\overline{F}_j$ ,  $\overline{v}_j$  or  $\overline{q}_j$  with a relative error which is lower as the relative error of the rough signal.

The mean input power per sample is:  $\overline{P}_{in j} = \overline{F_j^* v_j}$ .

The total average input power over 2048 samples is:

$$\overline{P}_{in} = \frac{1}{2048} \sum_{j=1}^{2048} \left[ \overline{F_j^* v_j} \right] = \overline{F^* v}.$$

The relative error:

$$\frac{\Delta P_{\text{in}}}{|P_{\text{in}}|} = \frac{\Delta F}{|F|} + \frac{\Delta v}{|v|}$$

$$\begin{aligned} \frac{\Delta E_{\text{mech}}}{|E_{\text{mech}}|} &= \frac{\Delta P_{\text{in}}}{|P_{\text{in}}|} + \frac{\Delta H}{|H|} + \frac{\Delta \bar{q}}{|\bar{q}|} = \frac{\Delta F}{|F|} + \frac{\Delta v}{|v|} + \frac{\Delta q}{|q|} + \frac{\Delta H}{|H|} \\ &= \frac{0.01}{\sqrt{m}} + \frac{0.01}{\sqrt{m}} + \frac{0.02}{\sqrt{m}} + \frac{0.05}{H} = 4 \left[ \frac{0.01}{\sqrt{m}} \right] + \frac{0.05}{H} \end{aligned}$$

Error in  $E_{\text{vol}}$ :

$$E_{\text{vol}}: \quad \frac{\bar{q}}{n \pi s} = \frac{\bar{q}}{[D/2]^2 s n \pi}$$

$$\begin{aligned} \frac{\Delta E_{\text{vol}}}{|E_{\text{vol}}|} &= \frac{2\Delta D}{|D|} + \frac{\Delta s}{|s|} + \frac{\Delta n}{|n|} + \frac{\Delta \bar{q}}{|\bar{q}|} \\ &= \frac{2}{|D|} + \frac{0.2}{|s|} + \frac{0.001}{n} + \frac{0.02}{\sqrt{m}} \end{aligned}$$

In the configurations as described in chapter [3] the relative errors are:

configuration 02/07

$$\frac{\Delta E_{\text{mech}}}{|E_{\text{mech}}|} = 0.01$$

$$\frac{\Delta E_{\text{vol}}}{|E_{\text{vol}}|} = 0.026$$