

## Partitioning and eigenvalues

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EINDHOVEN UNIVERSITY OF TECHNOLOGY

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Partitioning and eigenvalues

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Let  $T$  be a matrix of size  $(n-m) \times n$ , whose rows form an orthonormal basis of the orthogonal complement of the row-space of  $S$ , then  $R := \begin{bmatrix} S \\ T \end{bmatrix}$  satisfies  $R^H = R^{-1}$ . Computing  $RAR^{-1}$  we obtain

$$RAR^{-1} = RAR^H = \begin{bmatrix} SAS^H & SAT^H \\ TAS^H & TAT^H \end{bmatrix}.$$

Now the theorem is proved, because the spectrum of any principal submatrix of a hermitian matrix interlaces the spectrum of that matrix, cf. [2], p. 119. Indeed,  $B$  is cospectral to  $SAS^H$ , which is a principal submatrix of the hermitian matrix  $RAR^{-1}$ , which is cospectral to  $A$ .  $\square$

Remark 1. If any block  $A_{ij}$  has a constant rowsum then  $AS^H D = S^H DB$ , as can easily be verified. If in addition  $B$  has eigenvalue  $\beta$ , whose eigenspace is spanned by the columns of  $X$ , say, then we have  $\lambda X = BX$ ,  $\lambda S^H DX = S^H DBX = AS^H DX$ . Hence the column-space of  $S^H DX$  is an eigenspace of  $A$  belonging to the eigenvalue  $\beta$ . So in this case the spectrum of  $B$  is a sub(multi)set of the spectrum of  $A$  (note that in this case we do not need to take  $A$  hermitian).

Remark 2. Let  $\bar{B}$ ,  $\bar{D}$  and  $\bar{S}$  be defined analogous to  $B$ ,  $D$  and  $S$ , but with respect to another partition of  $A$ , which is a refinement of the above partitioning. Then the spectrum of  $B$  interlaces the spectrum of  $\bar{B}$  (note that in an extremal case we have  $A = \bar{B}$ ). This can be proved in a similar way as above: first realize that  $DBD^{-1} = \bar{S}\bar{S}^H\bar{D}\bar{D}^{-1}\bar{S}\bar{S}^H$ , and  $\bar{S}\bar{S}^H\bar{S}\bar{S}^H = I$ , then let  $\bar{S}\bar{S}^H$  do the job.

Remark 3. Of course everything remains valid if "rowsum" is replaced by "columnsum".

### Literature

- [1] Hestenes, M.D. and D.G. Higman; Rank 3 groups and strongly regular graphs, Computers in Algebra and Number Theory, SIAM-AMS Proceedings, vol. IV, Amer. Math. Soc., (1971).
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