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On the Calculation of Stability Charts on the Basis of the Damping and the Stiffness of the Cutting Process

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SUMMARY. This report deals with a new method for calculating stability charts. Simple experiments, measuring only frequencies, yield the values necessary to establish the threshold of stability. Now, the dynamic cutting coefficient can be determined. A very good resemblance between the calculated values and the experimental ones is shown for cutting speeds higher than 60 m/min.

RESUME. Ce rapport traite d'une nouvelle méthode de calculer des graphiques de stabilité. Des expériences simples, seulement le mesurage des fréquences, donnent les valeurs nécessaires pour fixer la limite de stabilité. Ensuite le coefficient dynamique de cisaillement peut être déterminé. Une bonne concordance entre les valeurs calculées et les valeurs obtenues par les expériences est arrivée en cas que les vitesses de cisaillement seront plus hautes que 60 m/min.

ZUSAMMENFASSUNG. Dieser Bericht beschäftigt sich mit einer neu entwickelten Methode zur rechnerischen Ermittlung von Stabilitätsdiagrammen. Die zur Bestimmung der Stabilitätsgrenze erforderlichen Werte gehen aus einfachen Frequenzmessungen hervor. Daraus lassen sich die dynamischen Schnittkraftkoeffizienten bestimmen. An einem Beispiel wird für Schnittgeschwindigkeiten über 60 m/min gezeigt, daß eine gute Übereinstimmung zwischen rechnerisch und experimentell ermittelten Werten besteht.

1. INTRODUCTION

THERE are two main directions in the field of dynamic stability-tests of machine tools[1].

First of all there is a method characterised by measuring the transfer function of the tool.

Then, the critical depth of cut is determined with the equation

$$b_{cr} = \frac{1}{2(-R_n)k_d}$$

The quantity k_d seems to depend upon the geometry of the cutting process as well as upon the work material. R_n is the maximum negative real part of the polar curve.

The second method simply consists of carrying out experiments in order to establish the critical depth of cut for standardized conditions. The progress of the investigations concerning cutting stability is mainly obstructed by an insufficient knowledge of the k_d value. The latter value depends on many quantities such as feed, cutting speed, tool wear, geometry of the tool, and workpiece material. In particular, the influence of the work material on cutting stability makes it difficult to compare results from cutting tests according to both methods.

2. THE INCREMENTAL CUTTING STIFFNESS

Peters and Vanherck[2] assume that it is permissible to take the incremental cutting stiffness k_i for the already mentioned k_d value. Thus, they calculate the critical depth of cut with the relation

$$b_{cr} = \frac{1}{2(-R_n)k_i}$$

The numerical values of k_i are obtainable by static cutting tests. Figure 1 shows, for orthogonal cutting,

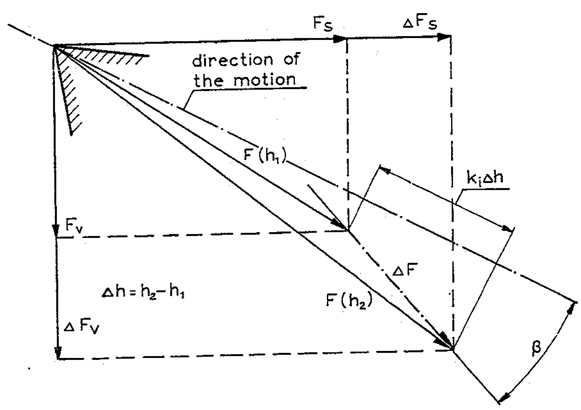

Fig. 1. *Determination of the incremental cutting stiffness according to the method of Peters and Vanherck.*

the change ΔF of the resultant cutting force due to an increase Δh of the feed. The incremental cutting stiffness may be defined as

$$k_i = k_{st} \cos \beta$$

where

$$k_{st} = \frac{\Delta F}{\Delta h}$$

β represents the angle between the vector ΔF and the direction of motion of the tool.

Peters and Vanherck evaluated the calculated b_{cr} values with experimental data from a special tool holder. A fairly good resemblance between both results was found. However, experiments carried out in the Laboratory of Production Engineering at the Eindhoven University of Technology, using the same tool holder, did not confirm the usefulness of the method to the same extent.

The cutting forces were measured with a three-component dynamometer which has its first natural frequency at approximately 1·5 kHz. Our results are shown in Fig. 2. In general, the calculated b_{cr} values, are considerably smaller than the experimental ones.

Among other things, Fig. 3 shows our curves for k_i according to the method of Peters and Vanherck, belonging to the calculated stability chart of Fig. 2.

It should be noticed that all our tests are carried out for the following conditions:

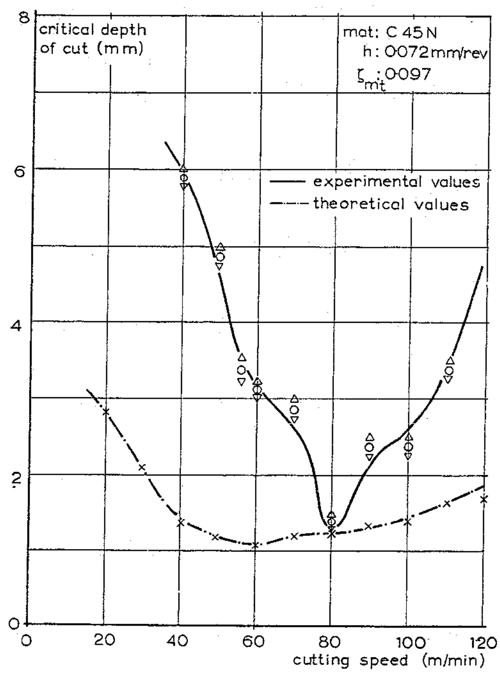

Fig. 2. The experimental and calculated stability chart (Peters method). The quantity ζ_{mt} represents the average value of the damping ratio when the carriage is moving.

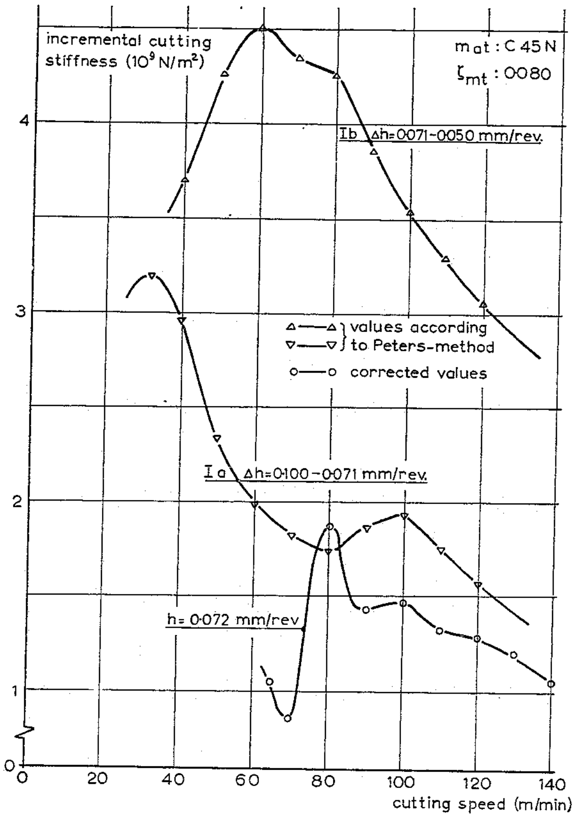

Fig. 3. The incremental cutting stiffness vs. cutting speed according to the method from Peters and Vanherck (I) and according to the new method (II).

orthogonal cutting

workpiece material: C45 N

tool: standard throw-away type carbide tool tip P30

geometry: $\alpha = 5°$, $\gamma = 6°$, $\kappa = 90°$, $\kappa' = 30°$, $r_\varepsilon = 0·4$ mm, $\lambda = 0°$

nominal feed: 0·072 mm/rev.

3. THE DAMPING OF THE CUTTING PROCESS

3.1 *General*

Many investigators in this field have already perceived the existence of damping in the cutting process. In this context the most well-known relation $\Delta F = k_1 \Delta h + k_2 \Delta \dot{x} + k_3 \Delta \Omega$ is given by Tobias[3].

However, experiments in order to obtain numerical values for this damping phenomenon are found to be difficult. Consequently, reliable values for the process damping caused by the workpiece material are not available at the present time.

The test rig which is used in cooperative work for the CIRP Ma Group for investigations of susceptibility to chatter of materials allowed the possibility of carrying out experiments in order to obtain numerical values of the damping ratio during turning operations.

3.2 *Process damping and incremental stiffness as basic quantities for stability charts*

Tlusty *et al.*[4] derived

$$T_c = \frac{1}{2(-R)}$$

where T_c represents the transfer function of the cutting process while R is the real part of the transfer function T_m of the machine tool.

When k_i is supposed to be independent of the depth of cut b the dynamic cutting force can be written as

$$T_c \Delta h = b k_i \Delta h$$

Hence, it follows on the threshold of stability

$$b_{cr} k_i = \frac{1}{2(-R_n)}$$

For a single-degree-of-freedom system we can derive

$$R_n = -\frac{1}{k}\frac{1}{4\zeta(1+\zeta)} \simeq -\frac{1}{4\zeta k} = -\frac{1}{2\rho_t \omega_0}$$

where k is the equivalent stiffness of the machine tool, and ρ_t the equivalent damping constant in the direction of the feed, ζ the damping ratio, and ω_0 the angular velocity at natural frequency. Now, we can write

$$b_{cr} k_i = \rho_t \omega_0$$

If, during turning operations, damping is added to the system (ρ_c) it will be necessary to increase b in order to reach instability, or

$$b_{cr} k_i = (\rho_t + \rho_c)\omega_0 = \rho_s \omega_0$$

If we excite the tool by a pulse during cutting, it is possible to measure the displacement response before regeneration occurs. Now, we can calculate the total damping ratio of the system with the aid of the consecutive amplitude ratio

$$\frac{A_n}{A_0} = \exp\left(-\frac{\pi \rho_s n}{\omega_d m}\right)$$

where n is the number of periods, and m is the equivalent mass. The value ω_d is characterised by

$$\omega_d = \omega_0 \sqrt{(1-\zeta_s^2)}$$

Thus, the amplitude ratio yields

$$\frac{A_n}{A_0} = \exp\left(-\frac{\pi n \rho_s}{\omega_0 m}\frac{1}{\sqrt{(1-\zeta_s^2)}}\right) \simeq \exp(-2\pi n \zeta_s)$$

or

$$\zeta_s = \frac{\ln (A_0/A_n)}{2\pi n}$$

It has been noticed that the overall damping ratio of the system ζ_s can be written as

$$\zeta_s = \frac{\rho_s}{2\sqrt{[m(k+bk_i)]}}$$

Consequently

$$\rho_s = 2\zeta_s \sqrt{[m(k+bk_i)]}$$

Now, on the threshold of stability the following relations will exist

$$b_{cr} k_i = \rho_s \omega_0 = 2\zeta_s \sqrt{(k^2 + b_{cr} k k_i)}$$

$$b_{cr} k_i = 2\zeta_s^2 k \left\{1 + \sqrt{\left(1+\frac{1}{\zeta_s^2}\right)}\right\}$$

$$b_{cr} k_i = 2k\{\zeta_s + \zeta_s^2 + \ldots\}$$

Finally, with $k = m\omega_0^2$ we find

$$b_{cr} k_i \simeq 2m\omega_0^2 \zeta_s$$

Once more, we will assume that k_i is not influenced by b. If the stability chart under certain conditions is known, it is possible to calculate the values of k_i with the aid of the ζ_s-values obtained by means of the logarithmic decrement. This k_i value will be unique and not be influenced by the dynamic behaviour of the tool. The k_i and ζ_s values obtained can be used for predicting stability charts for every machine tool of which the transfer function is known. It should be noticed that where the direction of ΔF is unknown, it is difficult to extrapolate k_i to any other direction than the main one of the rig.

The assumption that the cutting process will add damping to the vibratory system is confirmed by the results shown in Fig. 4 and Fig. 5. These results are obtained from experiments measuring the pulse response of the test rig during cutting. Figure 6 shows a typical example of such a response.

3.3 *Experimental approach of the problem*

Considering the system on the threshold of stability, it follows

$$b_{cr} k_i = \rho_s \omega_0$$

In general, the following relation exists

$$\Delta k = m(\omega_n^2 - \omega_0^2)$$

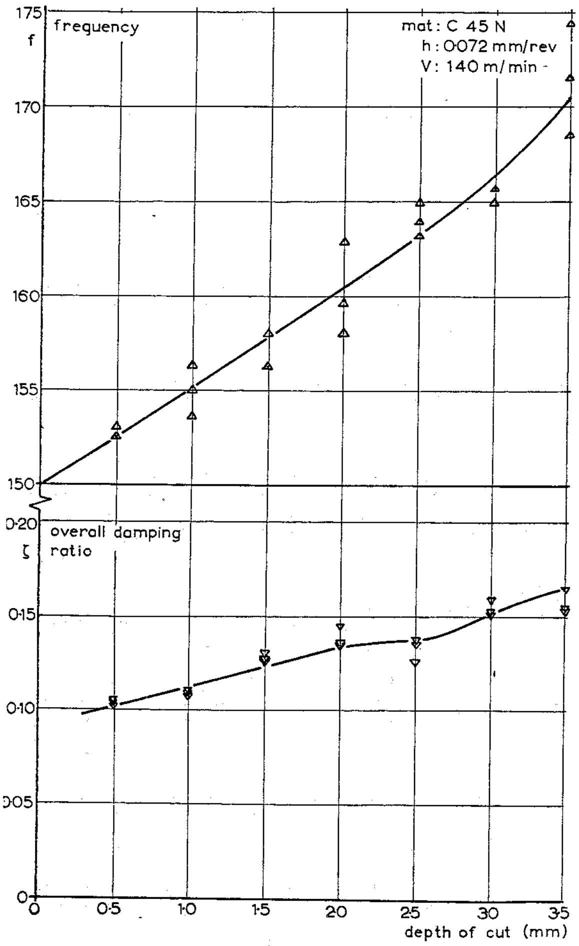

Fig. 4. The overall damping ratio and the frequency of the pulse response vs. depth of cut.

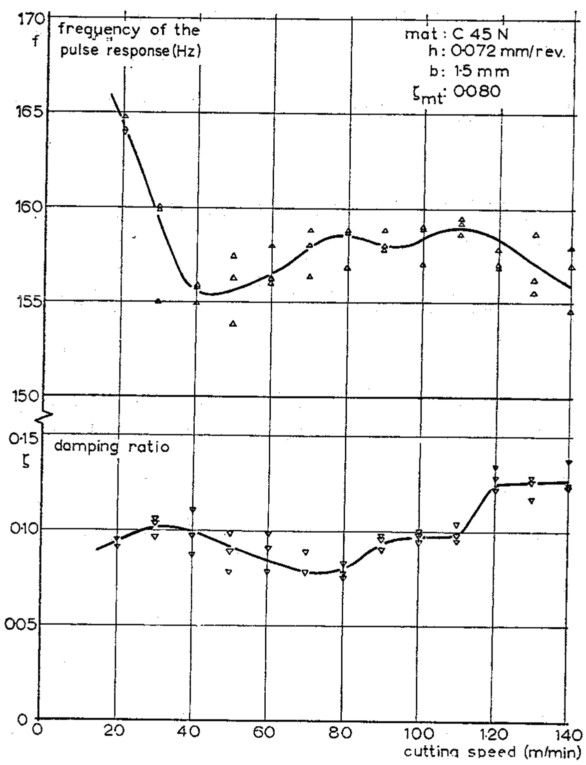

Fig. 5. The damping ratio and the frequency of the overall system vs. cutting speed.

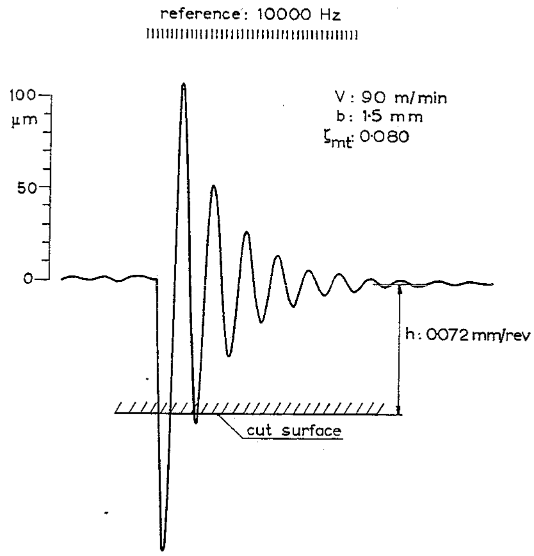

Fig. 6. Example of pulse response. The mass of the tool holder = 14·35 kg.

where Δk is the amount of stiffness to be added in order to obtain the natural frequency of the system at ω_n.

After pulse-excitation during cutting, the angular velocity of the motion will be

$$\omega_c = \omega_n \sqrt{(1-\zeta_s^2)}$$

If $b = 0$ and the carriage of the lathe moves, the next relation will also be valid

$$\omega_{mt} = \omega_{0m} \sqrt{(1-\zeta_{mt}^2)}$$

It has to be noticed that in this case ω_{0m} represents the angular velocity at natural frequency of the tool while the carriage moves. The corresponding damping ratio is ζ_{mt}. Consequently

$$k_m = m\omega_{0m}^2$$

It is established that the transfer function of the machine tool depends upon the velocity of the carriage. The magnitude of the change in compliance will be influenced by ζ_t and to some extent also by the carriage speed[5].

Assuming $\zeta_s^2 \ll 1$ and $\zeta_{mt}^2 \ll 1$, it is possible to obtain approximately the process stiffness with the equation

$$bk_i = m(\omega_c^2 - \omega_{mt}^2)$$

Thus, writing for the overall damping in practice

$$\rho_s = \rho_{mt} + \rho_c$$

on the threshold of stability the next relation will be valid

$$m\{\omega_c^2(b,V) - \omega_{mt}(b,V)\} = \rho_s \omega_{0m}$$

We consider the feed as a parameter for this relation. Consequently

$$m(\omega_c^2-\omega_{mt}^2) = 2\zeta_s\sqrt{[m(k_m+b_{cr}k_i)]}\,\omega_{0m}$$

$$\omega_c^2-\omega_{mt}^2 = 2\zeta_s\frac{\omega_c}{\sqrt{(1-\zeta_s^2)}}\frac{\omega_{mt}}{\sqrt{(1-\zeta_{mt}^2)}}$$

$$= 2A\omega_c\omega_{mt}$$

This yields

$$\frac{\omega_c}{\omega_{mt}} = A+\sqrt{(1+A^2)}$$

Assuming $A^2 \ll 1$, it follows

$$\frac{\omega_c}{\omega_{mt}} = A+1+\tfrac{1}{2}A^2+\ldots \simeq 1+A$$

$$\frac{\omega_c}{\omega_{mt}} \simeq 1+\frac{\zeta_s}{\sqrt{(1-\zeta_s^2)}\sqrt{(1-\zeta_{mt}^2)}}$$

or in practice

$$\zeta_s \simeq \frac{\omega_c}{\omega_{mt}}-1$$

If the experimental results satisfy this equation at the threshold of stability, the validity of the theory in the preceding pages has been demonstrated.

Figure 7 shows the stability chart for $\zeta_{mt} = 0{\cdot}080$. The curve for ζ_s, made with the aid of the logarithmic decrement, for the corresponding cutting-data of the stability chart of Fig. 7, is shown in Fig. 8. A second curve in the diagram of Fig. 8 shows the calculated values. For practical reasons, during experiments, the values for the parameter b_{cr} are replaced by values which are a shade smaller. The third curve shows that ζ_{mt} slightly depends upon the cutting speed. Figure 9 gives the curve showing the frequency vs. cutting speed at b_{cr}, while another curve shows the frequency for $b = 0$. A very good resemblance between the calculated values and the experimental ones can be established for cutting speeds higher than 60 m/min.

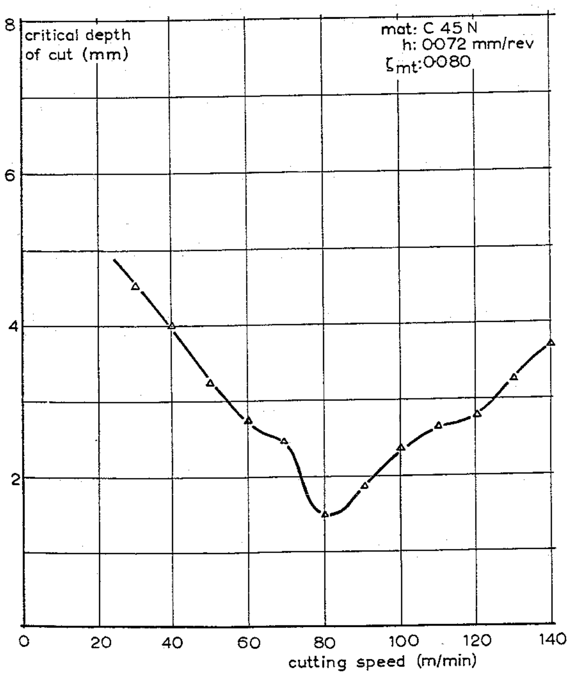

Fig. 7. The experimental stability chart for ζ_{mt} = 0.080.

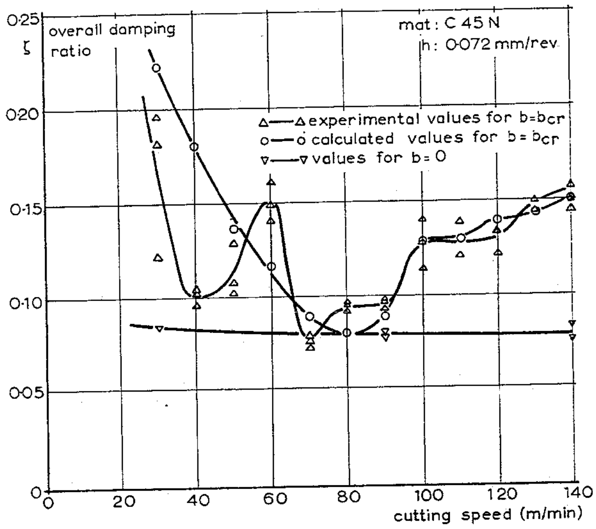

Fig. 8. The overall damping ratio at the threshold of stability and the damping ratio of the moving tool (b=0) vs. cutting speed.

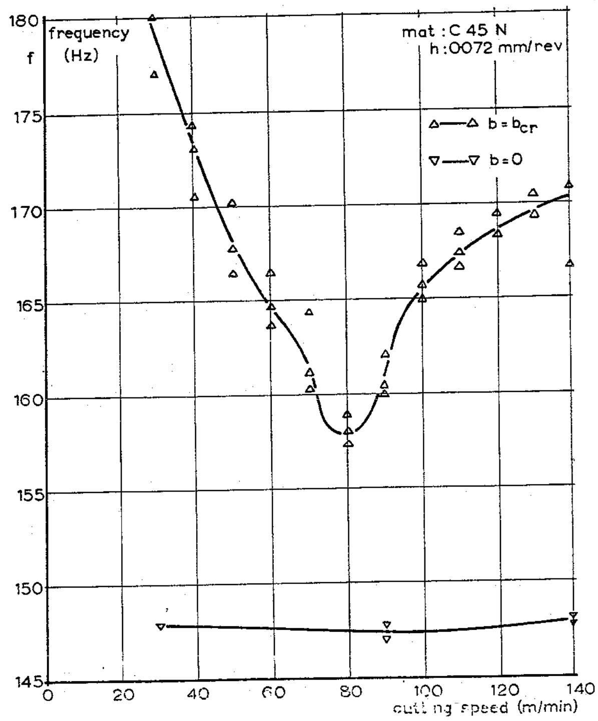

Fig. 9. The frequency at the threshold of stability and the frequency for b = 0 vs. cutting speed.

So, it appears that the right slope of the stability chart is not only defined by the compliance of the tool and an incremental cutting stiffness k_i, but also by a process damping.

Although, at low cutting speeds, the calculated values of ζ_s do not agree well with the experimental ones, it is doubtless that a process damping will also have great influence in this range of cutting speeds. Up to now, a further explanation cannot be given.

3.4 *The calculation of k_i when process damping is taken into account*

From the foregoing, it is clear that for cutting speeds larger than 60 m/min, the new method gives a good resemblance with the experimental results according to

$$\zeta_s = \frac{\omega_c}{\omega_{mt}} - 1$$

Consequently, the new method will give the best k_i values for cutting speeds higher than the mentioned ones. The preceding theory shows

$$b_{cr}k_i = \rho_s\omega_{0m} = \frac{\rho_s}{\sqrt{(mk_m)}} m\omega_{0m}^2$$

or

$$b_{cr}k_i = 2\zeta_s \frac{\sqrt{[m(k_m+b_{cr}k_i)]}}{\sqrt{(mk_m)}} m\omega_{0m}^2$$

$$b_{cr}k_i = 2\zeta_s m\omega_c\omega_{0m} \simeq 2\left(\frac{\omega_c}{\omega_{mt}} - 1\right) m\omega_c\omega_{0m}$$

with the approximation $\omega_{mt} \simeq \omega_{0m}$, the incremental cutting stiffness will be

$$k_i = \frac{2m\omega_c(\omega_c - \omega_{mt})}{b_{cr}}$$

Figure 3 shows the calculated k_i values.

A considerable difference is to be seen between the latter values and those calculated according to Peters-method.

3.5 *The influence of the wear of the tool on the process damping and on the incremental cutting stiffness*

Figure 10 shows the curves of the overall damping ratio and of the frequency taken from the pulse-response versus tool wear when the cutting speed V = 76 m/min and b = 1·25 mm. It appears from the results that the damping will not increase up to 0·2 mm flank wear (VB) of the tool. However, for values higher than 0·2 mm, the damping will suddenly increase up to two or three times its initial value. This influence is taken into account during all experiments mentioned in this paper by keeping the wear of the tool within the range of 0·1 to 0·2 mm.

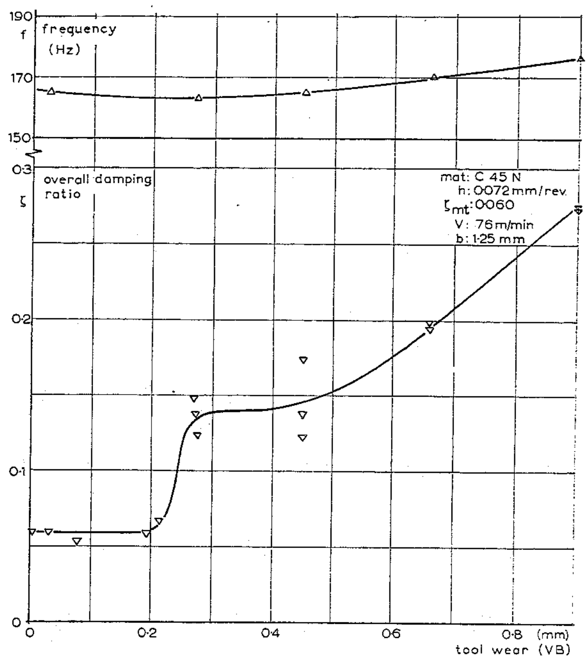

Fig. 10. The overall damping ratio and the frequency of the system vs. tool wear.

Where the change in frequency is only small, and this change can be explained to a large extent by the increasing damping, we can conclude that the incremental cutting stiffness calculated according to the new method will not be influenced by the wear of the tool.

4. CONCLUSIONS

For cutting speeds higher than 60 m/min the method proposed in this paper gives, up to now, the best analytical approximation for the experimental results. Only frequencies have to be measured in order to get all the information necessary for calculation of the incremental cutting stiffness and the process damping. A further advantage of this method is, that these experiments, i.e. measuring frequencies, can be done easily in each laboratory.

Furthermore, this method gives the possibility of attaining real values of the damping and the stiffness of materials. These values will be unique and they will not be influenced by the dynamic data of the tool. Now, we can derive the dynamic cutting-coefficient, as we can compare materials on susceptibility to chatter. It is possible to use the results for predicting the dynamic behaviour of machine tools during cutting if the directions of motion of the tool and of the test rig are the same, or if the direction of the dynamic force is known.

At low cutting speeds, we cannot calculate properly the stability chart only with the values for the process damping and those for the cutting stiffness.

The k_i values will not be influenced by the wear of the tool, while the damping may increase to high values.

In the beginning, the experiments mentioned in this paper did not yield good reproducible values. This was on account of the temperature of the workpiece, which increases during cutting.

The influence has been eliminated by standardizing the experimental conditions.

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