

Formation and trajectory control of multiple dynamical systems

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Formation and Trajectory Control of multiple dynamical systems

F. Kamalizadeh DCT 2005.152

Traineeship report

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Abstract

In recent years a lot of effort is put into making the transition from individually operating dynamical systems to cooperation of multiple dynamical systems within a group. Besides providing straightforward advantages as increased effectiveness and performance, cooperation between systems can also lead to performing complex tasks that cannot be realised with a single system. In this report possibilities of multi-system control are examined with particular focus on movement of individual systems in group formations. The main objective is to present a control strategy that is able to force a group of dynamical systems to move according to a desired trajectory in a desired formation layout and ensure stability of the overall dynamics.

The control strategy presented in this report observes the group of dynamical systems as one large single system and moreover separates the overall movement and the formation of the group with each other. This is realised with a coordinate transformation, which transforms the dynamics of the n m-degree-of-freedom (m-DOF) systems into a m-DOF average system and a (n-1)m formation system. Here the average system represents the movement of the group and logically the formation system coincides with the dynamics of the group formation. Advantage of this division is that it decouples the two main objectives, group maneuvering and formation control, making control independent from each other and therefore more convenient. Using Lyapunov theory supported by Lasalle's Invariance Principle asymptotic stability is proven for the average and formation systems and thus the overall group dynamics.

This approach assumes that the dynamics of the individual systems are exactly known and perfectly modelled, but in practice this is generally not the case. When modeling imprecisions are present, stable group dynamics cannot be promised with this control approach and implementation possibly leads to undesired behavior or even instability. This problem is solved by implementing a new control approach based on sliding mode theory, which can reckon with these imprecisions. This nonlinear control method makes it possible to maintain stability and consistent performance in the face of modeling imprecisions.

To provide quantitative results and support the analytical findings in this report, also simulation are performed with both control approaches in a Matlab/Simulink environment and their results are presented and discussed. The simulations performed with the first control approach as with the sliding mode approach provide good results and show convergence of the group to the desired group and formation trajectories.

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Chapter 1

Introduction

Recently interest in control of multiple cooperative robotic systems has increased considerably, the vision being that multiple robots can perform tasks faster and more efficiently than a single robot. Moreover cooperation of multiple robotic systems expands the overall workspace and therefore can perform tasks that cannot be dealt with utilizing a single robot. Also technology developments such as faster computation and less expensive and improved communication possibilities have boosted this interest; reduced cost of robotic hardware enables the use of teams of robots and as a result multi-robot control can be applied effectively for large populations of robots. Application of such systems can be done in a large array of different work areas. Nowa-days predominantly research focus is on defense purposes, such as search-and-rescue and attack missions for Unmanned Aerial Vehicles (UAVs), but multi-robot applications arose interest in other areas like production processes, geological research and household use and are becoming more and more common.

The research conducted in this report is realized by a cooperation between Eindhoven University of Technology (Department of Mechanical Engineering) and the RMIT University, Melbourne Australia (Sir Lawrence Wackett Centre for Aerospace Design Technology). These institutes have given the opportunity to carry out a traineeship at the Sir Lawrence Wackett Centre for a period of three months. The RMIT University was specifically interested in looking at the possibilities of cooperation between multiple UAVs, since this can be very useful in various scenarios, such as surveillance of a certain area and attacking enemy targets. As a large variety of UAVs exist, from a control point of view it is more interesting to look at a more general system structure, instead of the dynamic model of a specific UAV. Therefore the choice is made for this report to aim at dynamical systems with structures as

$$M_i(\vec{q}_i)\vec{q}_i + C_i(\vec{q}_i, \vec{q}_i)\vec{q}_i = T_i + F_i \qquad i = 1, ..., n$$
(1.1)

where $\vec{q}_i, \dot{\vec{q}_i}$ and $\ddot{\vec{q}_i}$ are the configuration, velocity and acceleration vectors of the i^{th} system, $M_i(\vec{q}_i)$ and $C_i(\vec{q}_i, \dot{\vec{q}_i})$ are the inertia and Coriolis matrices and T_i and F_i are the control actions and the environmental disturbances on the i^{th} system, respectively. Since there exist a large variety of possible systems and it is nearly impossible to examine all of them, this distinction is made to still be able to study the possibilities of cooperative control for these specific systems. However it should be noted that the control schemes provided in this report can be extended to other dynamical systems, e.g. UAVs and mobile robots.

The main goal in cooperative control in general is to design the available controllers in such a way that would lead to that two or more dynamical systems work together in performing a desired task. These cooperative control schemes are not just a straightforward extension of common

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controllers for a single system. Unlike these controllers, the controllers of cooperative systems also have to take into account influences form the other systems in the group (e.g. their positions and velocities) besides their own system behavior, since these are exactly the parts that are essential for cooperation. This asks for a different control approach. The problem of cooperation of dynamical systems in this report is, as mentioned before, restricted to systems with structures as in (I.I). Note that the model structures must be equal, but that the system parameters may vary. Moreover the systems are assumed to be fully actuated and also hold the same number of degrees of freedoms (dofs). In addition the accelerations, velocities and positions of all the systems are assumed to be available.

For the cooperative scheme the control scheme proposed by Lee et al. in [8] is used as a foundation. They provide a control design that enables a group of dynamical systems to maneuver according to a desired trajectory while the formation is ensured to converge to a prescribed formation structure. This is made possible by decomposing the dynamics of the individual systems into an average system for trajectory control and a formation system for formation control. Then by designing suitable controllers for the two new independent systems, one can proof convergence to the desired trajectory and formation.

The control scheme mentioned above is based on the assumption that the exact dynamic models for the systems is known. In general this is not the case and models contain inaccuracies with respect to their practical counterpart. This report will propose a control approach which is able to prove asymptotic stability of the systems in the face of model imprecisions. This alteration of the control approach is based on sliding mode theory.

The main goal of this report is to design a control approach, which is able to control a group of dynamical systems, with structures as in (I.I), to follow a desired trajectory while moving in a prescribed formation layout and analytically prove overall stability of the group of systems. Moreover the objective is to analytically prove the above mentioned in the presence of model inaccuracies. A contribution of this report is the addition of a control scheme, which can cope with possible modeling imprecisions. Another contribution is that the simulations performed in Matlab/Simulink show the performance and outcome of the presented cooperative control schemes and provide insight for practical implementation of the control schemes.

This report is organized as follows. In chapter 2 a literature study will be presented to gather insight about the subject and reason the chosen control approach. This approach will be further analyzed and ,together with a Lyapunov-based stability analysis, is introduced in chapter 3. The control approach presented in chapter 3 shares several resemblances with research done at TU/e [16]. In chapter 4 this is analyzed in detail to further investigate the relations between the two control methods. Furthermore the control strategy from chapter 3 assumes perfect knowledge about the system dynamics is known and in chapter 5 the study is extended by presenting a cooperative control approach, which can induce stability in the presence of system uncertainties. Simulations of the various control approaches are presented in chapter 6. And finally recommendations for future work and concluding remarks are provided.

Chapter 2

Literature Study

2.1 Introduction

Coordinating the motion of multiple robots is one of the fundamental problems in robotics. In the past some research has been done about the subject, but research in this area has really taken a flight just recently. Strikingly there has been interest in the subject from various different fields of science, with quite a lot of different views on the matter. The strategies of cooperation cover diverse disciplines as artificial intelligence, animal biology, control and artificial life. In the literature study performed for this report I have come across vastly different approaches. In the following chapter several approaches will be independently discussed to get insight into the procedures and the ideas are more specifically clarified by treating several examples. Finally a control strategy, which is most suitable for dealing with the formulated objectives, will be chosen from the presented options and substantiated. This strategy will be used in the remainder of this report.

2.2 Other Research

Although our main interest lays at treating the cooperation problem in a control manner, this section mentions some alternative ways for directing a group of systems. This is to emphasize the fact that interest in controlling group behavior is not only restricted to control researchers, but is found interesting in many different scientific areas.

A way of looking at the problem is by investigating nature's way of group performance. The strategies presented in [21] and [5] are based on behavior observed in the animal kingdom. Nature has some amazing phenomena of cooperative behavior among animals. The synchronous motion of a flock of birds, cooperation in ant colonies and behavior of shoals of fish are just several examples of this. Van Dyke Parunak et al. [21] have developed a technology for coordinating the movements of multiple unmanned aerial vehicles based on ant-behavior. Ants perform impressive ways of coordination without direct inter-agent coordination by sensing and on itself depositing so-called pheromones (sent markers) in the environment. The pheromones can specify a location as being interesting, attractive or non-interesting and so a network of paths is constructed that connect their nests with their goals. Converting this natural phenomenon to a mathematical point of view, equations are presented which describe the evolution and propagation of these pheromones and behavior of ant colonies can be mimicked by robotic systems. Another way of controlling groups of systems is presented in [24],[13] and [25]. Here they see the problem as an independent case and try to solve it by making use of self defined cost functions and multi-criteria algorithms. A way of analyzing stability is to investigate the convergence of a multi-criteria algorithm. In [24] for example, Vincent and Rubin present a framework for multiple systems searching for a target in a specific area. An algorithm is defined which consists of parameters as draw distance, velocity and turning radius of an UAV. This algorithm is then analyzed for the tradeoff between search time and number of UAVs deployed and optimized for the desired objectives.

2.3 Control Research

Besides the approaches described in the previous paragraph, there also is another movement, which is more appealing for control engineers. In this section several researches are discussed, which have attempted to deal with the cooperation problem by means of using control theories. These can vary from fairly simple linear control to complex nonlinear approaches based on for example Lyapunov theory [2] or Model Predictive Control [22]. Most of the work in this area is built around two themes; centralized knowledge and distributed knowledge. In the first theme the assumption is made that there exists a unit that has global information of the world and decides the behavior of each of the robots. In distributed knowledge the robot is only directly aware of its neighbors and the formation therefore consists of a more local control. Each robot observes its direct environment and, dependent on the control strategy, can receive other environmental information from other robots, but the motion of the robot is decided only by its local neighborhood. On a lower control level, inside formations motion can be determined in two ways. The first procedure consists of a so-called master-slave configuration, where each robot determines its position with respect to a leader robot. The leader describes the desired motion, which is to be followed by the other robots and is not responsible for keeping formation. The cooperative or mutual configuration is the other method. Here there are interactions between all the robots. The collective sharing of information among each other results into equality of all the robots in the group, i.e. no ranking. All the robots can influence the group behavior and therefore all the robots determine together the overall dynamics of the group.

In [20] Sketjne et al. propose to solve the formation control problem of a fleet of vessels by defining two main control schemes. The formation is seen as a single unit, where all the vessels have the same priority. They define a formation reference point (FRP), which describes the trajectory of the formation. Each individual ship will have a relative position to this reference point and the first control scheme ensures that the ships converge to their positions in the formation and stay at their respective position relative to the reference point. The second control scheme is present to ensure that the FRP will move along a desired path with the specified velocity. In these control schemes information about one's position is not shared among the other vessels. Also the control (with FRP as a leader) can be seen as a variation on the master-slave control configuration. A backstepping design together with Lyapunov theory are proposed to solve the formation maneuver problem and to proof stability of the overall system. All this leads to a nonlinear control design method and simulations demonstrate good performance of the controller.

Lawton et al. present in [7] a decentralized control law for mobile robots to stay in formation and converge to a desired location. This control law is based on feedback on the relative motion (formation keeping) and global motion (convergence to location) of each robot. Hereto each robot determines the distances between its two nearest neighbors. In the paper the control laws are

developed for robots with the following dynamics

$$\begin{aligned} \dot{x} &= v \cos(\theta) \\ \dot{y} &= v \sin(\theta) \\ \dot{\theta} &= \omega \\ \dot{v} &= \frac{F}{m} \\ \dot{\omega} &= \frac{\tau}{J} \end{aligned}$$
(2.1)

where (x, y) are the position of the center of the robot, θ is the orientation, v and ω are the tangential and the angular velocity of the robot respectively, τ is the torque input, F the force input, m the mass and J is the moment of inertia. For formation keeping Lawton et al. do not look at the center of the robots, but a specific position on the robots (point on the wheel-axis) to avoid to have to use nonholonomic control laws. This point is defined as

$$x_h = x + L\cos(\theta)$$

$$y_h = y + L\sin(\theta)$$
(2.2)

Differentiating (2.2) twice gives the following equations of motion

$$\begin{bmatrix} \ddot{x}_h \\ \ddot{y}_h \end{bmatrix} = R(\theta) \begin{bmatrix} \frac{F}{m} - L\omega^2 \\ \frac{\tau L}{J} + v\omega \end{bmatrix}$$
(2.3)

with

$$R(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

It is clear that the terms $\frac{F}{m}$ and $\frac{\tau L}{J}$ in (2.3) can be freely chosen due to the inputs F and τ . Input-output linearization of the equations of motion is possible by carefully using these terms. Choosing

$$\begin{bmatrix} \frac{F}{m} \\ \frac{\tau L}{J} \end{bmatrix} = \begin{bmatrix} L\omega^2 \\ -v\omega \end{bmatrix} + R(-\theta) \begin{bmatrix} u_x \\ u_y \end{bmatrix}$$
(2.4)

and substituting (2.4) in (2.3) finally leads to the input-output linearized system

$$\begin{bmatrix} \ddot{x}_h \\ \ddot{y}_h \end{bmatrix} = \begin{bmatrix} u_x \\ u_y \end{bmatrix}$$
(2.5)

As stated before there are two control objectives, namely converging to a desired location and keeping formation. In the paper they create two different error functions for both these objectives. In (2.6) these error functions are shown.

$$E_{G} = \sum_{i=1}^{N} \tilde{r}_{i}^{T} \tilde{r}_{i}$$

$$E_{F} = \sum_{i=1}^{N} (\tilde{r}_{i} - \tilde{r}_{i+1})^{T} (\tilde{r}_{i} - \tilde{r}_{i+1})$$
(2.6)

where $r_i = (x_{hi}, y_{hi})^T$, $r_{id} = (x_{hid}, y_{hid})^T$ represents the desired location and $\tilde{r}_i = r_i - r_{id}$. The first equation, E_G , expresses the distance between the robot and the desired location. E_F shows

in what extent the robots stay in formation. E_F is zero if and only if $\tilde{r}_i = \tilde{r}_{i+1}$ and that means that $E_F = 0$ when the robots are at the same relative formation as the desired formation at the end $(r_i - r_{i+1} = r_{id} - r_{i+1,d})$. Combining these error functions in a total error results in

$$E = k_F E_F + k_G E_G \tag{2.7}$$

where $k_F \ge 0$ and $k_G \ge 0$ can be used to put more emphasis on one of the objectives. The control objective is to select $u = (u_x, u_y)^T$ to cause (2.7) to go zero asymptotically. They propose for the control input

$$u_{i} = -k_{F}(\tilde{r}_{i} - \tilde{r}_{i+1}) - k_{F}(\tilde{r}_{i} - \tilde{r}_{i-1}) - k_{G}\tilde{r}_{i} - d_{F}\dot{r}_{i}$$
(2.8)

The terms $-k_G \tilde{r}_i$ and $-d_F \dot{r}_i$ drive the i^{th} robot to the desired location. The robots are forced to stay in formation by the two remaining terms $-k_F(\tilde{r}_i - \tilde{r}_{i+1})$ and $-k_F(\tilde{r}_i - \tilde{r}_{i-1})$, which equalize $\tilde{r}_i, \tilde{r}_{i-1}$ and \tilde{r}_{i+1} . By demanding that $k_F \ge 0$ and $k_G > 0$, Lawton et al. prove that $E \to 0$ asymptotically using Lyapunov theory and Lasalle's Invariance Principle.

Control of multiple robotic systems has also been treated by Rodriquez-Angeles and Nijmeijer. They have addressed the case of master-slave configurations [17] as well as cooperative behavior of robots [16,14]. In [16] they present a controller to achieve cooperative behavior among multiple robots. These robots share a common desired trajectory q_d . Ultimately the objective of the controller is to synchronize the position and velocity q_i , \dot{q}_i of the i^{th} robot with the desired trajectory and with the position and velocity q_j , \dot{q}_j of the other robots, i.e. $q_i \rightarrow q_d$, $q_i \rightarrow q_j$. In this approach all position information of all the robots has to be communicated to each robot. The dynamic model of the p robots is considered to be

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i \qquad i = 1, ..., p$$
(2.9)

where $M_i(q_i)$ is the inertia matrix, $C_i(q_i, \dot{q}_i)$ represents the Coriolis matrix and $g_i(q_i)$ denotes the gravity forces. The system input provided by the controller τ_i is given by

$$\tau_i = M_i(q_i)\ddot{q}_{ri} + C_i(q_i, \dot{q}_i)\dot{q}_{ri} + g_i(q_i) - K_{d,i}\dot{s}_i - K_{p,i}s_i \qquad i = 1, ..., p$$
(2.10)

where $K_{p,i}$ and $K_{d,i}$ are gain matrices, s_i and \dot{s}_i are synchronization errors for position and velocity respectively and are defined as

$$s_i = q_i - q_{ri}, \qquad \dot{s}_i = \dot{q}_i - \dot{q}_{ri}$$
 (2.11)

Here q_{ri} and \dot{q}_{ri} are reference signals for the systems. These reference signals are defined as the desired trajectory q_d minus an interaction term between the robots

$$q_{ri} = q_d - \sum_{j=1, j \neq i}^p K_{i,j}(q_i - q_j)$$
(2.12)

where $K_{ij} \ge 0$ are diagonal matrices with which the amount of interactions between robots can be determined. The same structure goes for \dot{q}_{ri} and \ddot{q}_{ri} , where respectively derivatives and second derivatives are taken for q_d , q_i and q_j . (2.12) captures an interesting topic; on the one hand q_{ri} is forced to follow q_d and on the other hand the robots are supposed to synchronize to each other's state ($q_i \rightarrow q_j$). Substituting (2.10) and (2.12) into (2.9) and rearranging leads to

$$M_i(q_i)\ddot{s}_i = -C_i(q_i, \dot{q}_i)\dot{s}_i - K_{d,i}\dot{s}_i - K_{p,i}s_i \qquad i = 1, ..., p$$
(2.13)

Stability for (2.13) is proven using Lyapunov theory. As a Lyapunov function is taken

$$V(s,\dot{s}) = \sum_{i=1}^{p} \left(\frac{1}{2}\dot{s}_{i}^{T}M_{i}(q_{i})\dot{s}_{i} + \frac{1}{2}s_{i}^{T}K_{p,i}s_{i}\right)$$
(2.14)

which is positive definite if all $K_{p,i} > 0, s_i, \dot{s}_i$ and $V(s_i, \dot{s}_i) = 0$ only if $s_i, \dot{s}_i = 0$. Determining the time derivative of (2.14) gives

$$\dot{V}(s,\dot{s}) = -\sum_{i=1}^{p} \dot{s}_{i}^{T} K_{d,i} \dot{s}_{i}$$
(2.15)

 $\dot{V}(s, \dot{s})$ is negative semi-definite for all $K_{d,i} > 0$ and hence s_i, \dot{s}_i are stable. With the help of Barbalat's lemma asymptotic stability is proven for s, \dot{s} . For asymptotic stable synchronization of the robots another mathematical condition must hold (see [14] for more detail). This eventually leads to that $q_i^{(r)} \rightarrow q_d^{(r)}$ and $q_i^{(r)} \rightarrow q_j^{(r)}$ for all i, j = 1, ..., p, r = 0, 1. To analyze the results, this mutual control scheme has been simulated in Matlab Simulink. The

To analyze the results, this mutual control scheme has been simulated in Matlab Simulink. The strategy, for this specific case, is designed for two mechanical systems (see (2.9)) to converge to the desired trajectory q_d and to each other's trajectory $(q_1 \rightarrow q_2)$. The systems have I degree of freedom and their respective system matrices are set as $M_1 = M_2 = 5$ [kg], $C_1 = C_2 = 3$ [kg] and $g_1 = g_2 = 0$ [m/s^2]. The initial conditions are chosen as $q_1(0) = -3$ [m], $q_2(0) = 3$ [m] and $\dot{q}_1(0) = \dot{q}_2(0) = 0$ [m/s] respectively and the control matrices as $K_{p,1} = K_{p,2} = 10$, $K_{d,1} = K_{d,2} = 5$ and $K_{1,2} = K_{2,1} = 1$. In figure 2.1 one can see the trajectories of q_d , q_1



Figure 2.1: Mutual synchronization; time-position plot for q_d, q_1 and q_2

and q_2 using values for the control variables ensuring asymptotic stability. The desired trajectory trajectory q_d is chosen as $sin(t) + 1/2sin(5t - 5/2\pi)$ and from the figure it can be seen that q_1 and q_2 simultaneously converge to each other and to q_d . The synchronization of the systems' velocities (\dot{q}_1, \dot{q}_2) is also shown in figure 2.2 and furthermore the behavior of the synchronization errors is plotted at the right-hand side of the figure. These synchronization errors are defined as $e_{12} = q_1 - q_2$, $e_{11} = q_1 - q_d$ and $e_{22} = q_2 - q_d$. Performance of the synchronization can be optimized by tuning the control variables $K_{p,i}, K_{d,i}$ and $K_{i,j}$.

The paper of Lee et al. [8] presents a control scheme that enables a group of systems to maneuver



Figure 2.2: Mutual synchronization. Left: time-velocity plot, right: time-synchronization error plot

according to a desired trajectory while the individual systems converge to a prescribed formation structure. The dynamics of the group is decomposed into two parts: the average system that represents overall motion of the group and the shape system that represents the group formation. The major objective of Lee et al. is to design a decomposition that separates the dynamics of a group of n general m-DOF mechanical systems into a m-DOF average system, representing the overall motion of the group, and a m(n-1)-DOF shape system, which governs the group formation structure. This partition makes it possible to autonomously control the formation and trajectory tracking of the systems. They consider the following shape of the m-DOF systems

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i = T_i + F_i \qquad i = 1, ..., n$$
(2.16)

where q_i and \dot{q}_i are the configuration and velocity of the i^{th} spacecraft, $M_i(q_i)$ and $C_i(q_i, \dot{q}_i)$ are the inertia and Coriolis matrices and T_i and F_i are the control actions and the environmental disturbances on the i^{th} agent, respectively. As so-called shape variables, for formation control,

$$q_E^T = [q_1^T - q_2^T, q_2^T - q_3^T, ..., q_{n-1}^T - q_n^T]$$
(2.17)

is chosen. Hence, q_E represents relative displacements between successive agents. Besides the formation variables q_E another coordinate q_L , which represents the average system variables, is defined. This q_L represents the motion of the center of mass of the group of systems. With a decomposition matrix S(q) the state of the differential equations can be transformed from $\vec{q}^T = [\vec{q}_1^T, ..., \vec{q}_n^T]$ to q_L and q_E . Using the decomposition matrix S(q) the dynamics of the group can be decoupled into a m-DOF average system and the m(n-1)-DOF shape system

$$M_L(q)\ddot{q}_L + C_L(q,\dot{q})\dot{q}_L + C_{LE}(q,\dot{q})\dot{q}_E = T_L + F_L$$

$$M_E(q)\ddot{q}_E + C_E(q,\dot{q})\dot{q}_E + C_{EL}(q,\dot{q})\dot{q}_L = T_E + F_E$$
(2.18)

These new systems possess equivalent model structures as usual mechanical systems. Thus, it is possible to utilize a variety of well known control schemes, such as for example PD-control and feedback linearization, to achieve the desired objectives.

A subclass of control for multiple robots is the potential field approach. Here the general idea is that in a certain environment every obstacle has a negative potential field that exerts a repulsive

force and every goal has a positive potential field that exerts an attractive force. That way it is possible to navigate a system through the environment. In [10] Nguyen et al. make use of this idea to control a mobile robot through an environment with obstacles. A decentralized control method for nonlinear second order systems is presented, which consists of virtual potentials that are a function of distance;

$$\frac{d\vec{z}_i}{dt} = \vec{w}_i$$

$$m\frac{d\vec{w}_i}{dt} = \alpha \vec{z}_i - \nabla_{\vec{z}_i} \sum_{\substack{j(j \neq i) \\ j(j \neq i)}} \left[-V_a(||\Delta \vec{z}_{ij}||) + V_r(||\Delta \vec{z}_{ij}||) \right]$$
(2.19)

where $\triangle \vec{z}_{ij}$ is the difference between the positions of the i^{th} and j^{th} robot, \vec{w}_i represent the position and velocity vector of the i^{th} robot respectively, and V_a, V_r are the attractive and repulsive potential functions. The virtual attractive and repulsive potentials are used to direct robots towards goals (attractive potentials) while avoiding obstacles (repulsive potentials). Although several experiments show the desired results, stability is not proven in [Io]. The decentralized control method of Nguyen et al. is similar to Leonard and Fiorelli [9] in that both of these methods use artificial potentials. But unlike Nguyen, Leonard and Fiorelli use artificial potentials to direct the group of vehicles in formation. A potential, V_I , is present to achieve a desired formation by creating inter-vehicle forces. This potential is constructed in such a way, that its gradient represents an attracting force when the relative distance between two UAVs is between user-specified distances d_0 and d_1 , a repelling force when the relative distance is smaller than d_0 and no force when the distance is longer than d_1 (see figure 2.3). This potential structure leads to that the potential function is minimal when two UAVs are positioned a distance d_0 apart from each other. This quality is essential and is exploited in the system input signals, which is explained in the next paragraph. The same construction holds for another potential V_h , with the only difference that the design



Figure 2.3: schematic representation of artificial potential V_I . The black dots stand for UAVs. When the relative distance between UAV₁ and another UAV is smaller than d_0 , a repelling force is applied to both UAVs, when the relative distance is between d_0 and d_1 an attractive force is felt and when the relative distance is larger than d_1 no forces are applied.

parameters used here are h_0 , h_1 instead of d_0 , d_1 . This potential causes forces between robots and so-called virtual leaders. These virtual leaders are moving reference points which define the overall trajectory of the formation and therefore herd the group of robots. Result of these potentials is that a minimum of the sum of all potentials will occur at inter-vehicle distances d_0 and distances to virtual leaders h_0 . Defining the robots as point masses, the differential equations become

$$\ddot{q}_i = -\sum_{j \neq i}^N \nabla_{q_i} V_I(|q_{ij}|) - \sum_{k=1}^M \nabla_{q_i} V_h(|h_{ik}|) - f_{v_i}$$
(2.20)

where N and M are the number of robots and virtual leaders, q_i the position of robot i and $|q_{ij}|$ and $|h_{ik}|$ the distance between robots i and j and the distance between robot i and virtual leader k, respectively. This construction of the system input forces the robot to go the position where the sum of the potentials is minimal, which occurs when $|q_{ij}| = d_0$ and $|h_{ik}| = h_0$. The additional input f_{v_i} is added for stability purposes. Using Control Lyapunov Functions, Leonard et al. prove stability of these non-autonomous nonlinear dynamical systems.

2.4 Conclusion

From this chapter it can be concluded that there are numerous ways to look at the cooperation problem and every approach has its advantages and drawbacks. A comparison of the methods has eventually led to making use of the decomposition strategy from [8] for trying to achieve the objective of this report. The main reason for this choice is the fact that the proposed theory is highly suitable for formation control of dynamical systems and the fact that it decouples formation control from trajectory control. With this approach it is possible to force a group of systems to move into a desired formation and follow a desired trajectory. In the following chapter the control approach will be presented in more detail and its stability will be analyzed.

Chapter 3

Theory And Stability Analysis

3.1 Introduction

In this chapter the chosen control strategy from chapter 2 will be presented and analyzed in more detail. This study will be followed by a stability analysis of the control strategy based on Lyapunov's stability theory. Finally conclusions are made about the presented control strategy and the stability of the group of dynamical systems.

3.2 Coordinate Transformation

The control scheme laid out in this chapter has as objectives to enable a group of individual systems to maneuver according to a desired trajectory and while tracking a desired trajectory converge to a prescribed formation structure. Intuitively it would be pleasant if these objectives could be controlled separately, since then a control effort for one objective cannot influence the course of the other objective and ease of overall control improves greatly. The first step is the decoupling of the group of individual systems into two parts: an average system that represents overall motion of the group and a shape system that represents the group formation. This is achieved by introducing a coordinate transformation, which will be dealt with now. Consider the following general shape of the m-degree-of-freedom (m-DOF) systems

$$M_i(\vec{q}_i)\ddot{\vec{q}}_i + C_i(\vec{q}_i, \dot{\vec{q}}_i)\dot{\vec{q}}_i = T_i + F_i \qquad i = 1, ..., n$$
(3.1)

where $\vec{q}_i, \dot{\vec{q}_i}$ and $\ddot{\vec{q}_i}$ are the configuration, velocity and acceleration vectors of the i^{th} system, $M_i(\vec{q}_i)$ and $C_i(\vec{q}_i, \dot{\vec{q}_i})$ (mxm-matrices) are the inertia and Coriolis matrices and T_i and F_i (mx1-matrices) are the control actions and the environmental disturbances on the i^{th} system, respectively. The coordinate transform enables to change the perspective from $\vec{q}^T = [\vec{q}_1^T, ..., \vec{q}_n^T]$ (mnx1) to the coordinate \vec{q}_E , which is the formation system state, and \vec{q}_L representing the motion of the average system. \vec{q}_E is defined as follows

$$\vec{q}_{E}^{T} = [\vec{q}_{E_{1}}^{T}, \vec{q}_{E_{2}}^{T}, ..., \vec{q}_{E_{n-1}}^{T}] = [\vec{q}_{1}^{T} - \vec{q}_{2}^{T}, \vec{q}_{2}^{T} - \vec{q}_{3}^{T}, ..., \vec{q}_{n-1}^{T} - \vec{q}_{n}^{T}]$$
(3.2)

Hence, \vec{q}_E (m(n-1)x1) represents the relative displacements between successive systems. This leads to the the fact that the individual systems determine their positions relative to their direct neighbor. This can be seen as a matter of local cooperative behavior. It can easily be seen that the center of mass (CoM) of the group of systems is $[\sum_{i=1}^n M_i(\vec{q}_i)]^{-1}[\sum_{i=1}^n M_i\vec{q}_i]$. For example, the

CoM of two individual I-DOF systems is $\frac{M_1q_1+M_2q_2}{M_1+M_2}$. Clearly the average motion of the group of systems in formation can be described accordingly and thus $\vec{q_L}$ (*m*x1) is defined as

$$\vec{q}_L = \left[\sum_{i=1}^n M_i(\vec{q}_i)\right]^{-1} \left[\sum_{i=1}^n M_i \vec{q}_i\right]$$
(3.3)

As will become clear later on, it is more convenient to describe the new coordinates in their velocity form and with this define a transition matrix $S(\vec{q})$ as

$$\begin{bmatrix} \dot{\vec{q}}_{L} \\ \dot{\vec{q}}_{E_{1}}^{T} \\ \dot{\vec{q}}_{E_{2}}^{T} \\ \vdots \\ \vdots \\ \dot{\vec{q}}_{E_{n-1}}^{T} \end{bmatrix} = \underbrace{\begin{bmatrix} \phi_{1}(\vec{q}) & \phi_{2}(\vec{q}) & \dots & \phi_{n-1}(\vec{q}) & \phi_{n}(\vec{q}) \\ I & -I & \dots & 0 & 0 \\ 0 & I & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & -I & 0 \\ 0 & 0 & \dots & I & -I \end{bmatrix}}_{S(\vec{q})} \begin{bmatrix} \dot{\vec{q}}_{1} \\ \dot{\vec{q}}_{2} \\ \dot{\vec{q}}_{3} \\ \vdots \\ \vdots \\ \dot{\vec{q}}_{n} \end{bmatrix}$$
(3.4)

where $\phi_i(\vec{q}) = [\sum_{j=1}^n M_j(\vec{q}_j)]^{-1} M_i(\vec{q}_i)$. In (3.4) the velocity space of (3.1) is decomposed into the velocity of the average system $\dot{\vec{q}}_L$ and the formation system $\dot{\vec{q}}_E$. Here one can also see that $\dot{\vec{q}}_L$ represents the weighted average of the velocity of each system;

$$\dot{\vec{q}}_L = \left[\sum_{j=1}^n M_j(\vec{q}_j)\right]^{-1} \left[M_1(\vec{q}_1)\dot{\vec{q}}_1 + \dots M_n(\vec{q}_n)\dot{\vec{q}}_n\right],\tag{3.5}$$

thus representing the overall motion of the group. Using $S(\vec{q})$ (*mnxmn*) from (3.4) the transformed system inputs and environmental disturbances can be found accordingly

$$\begin{bmatrix} T_L \\ \uparrow \\ T_E \\ \downarrow \end{bmatrix} = S^{-T}(\vec{q}) \begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_n \end{bmatrix}, \begin{bmatrix} F_L \\ \uparrow \\ F_E \\ \downarrow \end{bmatrix} = S^{-T}(\vec{q}) \begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_n \end{bmatrix}$$
(3.6)

Note that the inverse of $S(\vec{q})$ must exist for (3.6), which is the case since the matrix $S(\vec{q})$ is regular.

The result of the coordinate transform can now be shown by making use of (3.4) and (3.6); substitution into the individual systems of (3.1) leads to

$$S^{-T}(\underline{M}(S^{-1}\ddot{\vec{q}_a} + \dot{S}^{-1}\dot{\vec{q}_a}) + \underline{C}S^{-1}\dot{\vec{q}_a}) = S^{-T}\underline{T} + S^{-T}\underline{F}$$
(3.7)

where $\vec{q}_a^T = [\vec{q}_L^T \quad \vec{q}_E^T]$ and \underline{M} (mnxmn), \underline{C} (mnxmn), \underline{T} (mnx1) and \underline{F} (mnx1) are the specific matrices of the combined individual systems of (3.1) with state \vec{q} . For clarification, $\underline{M} = \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}$,

 $\underline{C} = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix}, \underline{T} = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} \text{ and } \underline{F} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \text{ for the case of two 1-DOF individual systems.}$ Rewriting (3.7) gives

$$S^{-T}\underline{M}S^{-1}\ddot{\vec{q}_a} + (S^{-T}\underline{M}\dot{S}^{-1} + S^{-T}\underline{C}S^{-1})\dot{\vec{q}_a} = \underline{T}_a + \underline{F}_a$$
(3.8)

where $\underline{T}_a = \begin{bmatrix} T_L & T_E \end{bmatrix}^T$ and $\underline{F}_a = \begin{bmatrix} F_L & F_E \end{bmatrix}^T$.

The outcome of the coordinate transformation is shown schematically in figure 3.1 to illustrate the whole transformation. Here one can see that the dynamics of the group has been partially

$\begin{bmatrix} \mathbf{M}_{\mathbf{L}} & 0 \\ \mathbf{m} \mathbf{x} \mathbf{m} \\ \mathbf{m} \end{bmatrix} = \begin{bmatrix} \mathbf{m} \mathbf{x} (\mathbf{n} \cdot \mathbf{l}) \mathbf{m} \\ \mathbf{m} \mathbf{x} \mathbf{m} \end{bmatrix}$	$\vec{\vec{q}}_{L}_{{}_{\mathfrak{m}\mathfrak{x}1}}$			C _{LE} m x (n-1)m	$\vec{q}_{L}_{m\mathrm{x}1}$		T _L mx1		F _L mx1
0 M _E ↓ ↓ (n-1)m x (n-1)m	$\ddot{ec{q}_E}_{ ext{(n-1)m x l}}$	+	C _{EL} (n-1)m x m	C _E (n-1)m x (n-1)m	$\dot{ec{q}_E}_{ ext{(n-1)m x 1}}$	=	T _E (n-1)m x 1	+	F _E (n-1)m x 1

Figure 3.1: Overview of coordinate transform

decoupled into a m-DOF average system (3.9) and a m(n-1)-DOF formation system (3.10)

$$M_L(\vec{q})\ddot{\vec{q}}_L + C_L(\vec{q}, \dot{\vec{q}})\dot{\vec{q}}_L + C_{LE}(\vec{q}, \dot{\vec{q}})\dot{\vec{q}}_E = T_L + F_L$$
(3.9)

$$M_E(\vec{q})\ddot{\vec{q}}_E + C_E(\vec{q},\vec{q})\dot{\vec{q}}_E + C_{EL}(\vec{q},\vec{q})\dot{\vec{q}}_L = T_E + F_E \tag{3.10}$$

Complete decoupling can be achieved by feedforwarding the terms $C_{LE}(\vec{q}, \vec{q})\dot{\vec{q}}_E$ and $C_{EL}(\vec{q}, \dot{\vec{q}})\dot{\vec{q}}_L$ as part of the system inputs T_L and T_E , respectively. Observation of these new systems leads to the notion that they mimic the dynamics of generally well-known mechanical systems. This is very useful, since now it is possible to utilize a variety of well known control schemes, such as for example PD-control and feedback linearization, to achieve the desired objectives.

3.3 Control Scheme and Stability Analysis

In this section the control approach of the average and formation system (3.9) and (3.10) will be discussed and analysed. The objective is to prove asymptotic stability for both these systems with the help of Lyapunov's Stability Theory.

Since the systems (3.9) and (3.10) resemble general mechanical systems a wide variety of control options are available. All having its advantages and disadvantages the choice is made to perform so-called feedback linearization on the systems, because it is a relatively straightforward method for this class of nonlinear systems in combination with its promising results. The general idea of this approach is to transform nonlinear system dynamics into linear system dynamics, so that linear control techniques can be applied. This is done by using state feedback.

Defining $\underline{T}_a = \begin{bmatrix} T_L & T_E \end{bmatrix}^T$, The nonlinear terms in (3.9) and (3.10) can be canceled by choosing \underline{T}_a as

$$\underline{T}_{a} = \begin{bmatrix} C_{L} & C_{LE} \\ C_{EL} & C_{E} \end{bmatrix} \dot{\vec{q}}_{a} - \underline{F}_{a} + \begin{bmatrix} M_{L} & 0 \\ 0 & M_{E} \end{bmatrix} \vec{v}$$
(3.11)

where $\vec{v} = \begin{bmatrix} \vec{v}_L & \vec{v}_E \end{bmatrix}^T$ (*nm*x1) is the new vector input. Substitution of (3.11), which is also known as the computed torque, into (3.9) and (3.10) yields

$$\begin{aligned} \ddot{\vec{q}}_L &= \vec{v}_L \\ \ddot{\vec{q}}_E &= \vec{v}_E \end{aligned} \tag{3.12}$$

Hence choosing \underline{T}_a as in (3.11) results into the transition from nonlinear to linear system dynamics and the possibility to utilize linear control techniques now. Introducing

$$\vec{e} = \begin{bmatrix} \vec{e}_L \\ \vec{e}_E \end{bmatrix} = \begin{bmatrix} \vec{q}_L - \vec{q}_L^d \\ \vec{q}_E - \vec{q}_E^d \end{bmatrix} \qquad (nmx1)$$
(3.13)

as being the error vector, the new control input \vec{v} is defined as

$$\vec{v} = \begin{bmatrix} \vec{q}_L^d\\ \vec{q}_E^d \end{bmatrix} - \begin{bmatrix} K_v^L & 0\\ 0 & K_v^E \end{bmatrix} \dot{\vec{e}} - \begin{bmatrix} K_p^L & 0\\ 0 & K_p^E \end{bmatrix} \vec{e}$$
(3.14)

where \ddot{q}_L and \ddot{q}_E represent the desired accelerations of the average system and the desired accelerations between successive systems, respectively. Also the two matrices at the right side of the equation represent the new constant controller matrices, which in the remainder of this report are respectively referred to as K_v^a and K_p^a . Substitution of (3.14) into (3.12) and rearranging of the equations eventually leads to the error dynamics

$$\ddot{\vec{e}} + K_v^a \dot{\vec{e}} + K_p^a \vec{e} = \vec{0}$$
(3.15)

To prove asymptotic stability for (3.15) Lyapunov's Direct Method (LDM) is used. As a Lyapunov function the 'energy' function of (3.15) is selected:

$$V(\vec{e}, \dot{\vec{e}}) = \frac{1}{2} \dot{\vec{e}} \cdot \dot{\vec{e}} + \frac{1}{2} \vec{e}^T K_p^a \vec{e}$$
(3.16)

,which is a often used structure when dealing with stability analyses based on Lyapunov theory. According to LDM it is suffice to proof asymptotic stability if the Lyapunov function (3.16) and its time derivative satisfy the following conditions

•
$$V(\vec{0},\vec{0}) = \vec{0}, \quad V(\vec{e},\dot{\vec{e}}) > \vec{0} \quad outside \quad (\vec{0},\vec{0})$$

• $\dot{V}(\vec{0},\vec{0}) = \vec{0}, \quad \dot{V} < \vec{0} \quad outside \quad (\vec{0},\vec{0})$
(3.17)

The point $(\vec{0}, \vec{0})$ corresponds with the equilibrium point of (3.15); $\vec{e}_{eq} = (\vec{e}, \vec{e}) = (\vec{0}, \vec{0})$. It is clear that (3.16) is zero in its equilibrium point and positive definite outside this point. Furthermore the derivative of (3.16) is

$$\dot{V} = \dot{\vec{e}} \cdot \ddot{\vec{e}} + \dot{\vec{e}}^T K_p^a \dot{\vec{e}}$$
(3.18)

Substitution of (3.15) into \dot{V} results into

$$\dot{V} = -\dot{\vec{e}}^T K_v^a \dot{\vec{e}} \tag{3.19}$$

Here equation (3.19) is not strictly negative definite, because there are no negative terms in \vec{e} present; \dot{V} can equal zero for values outside the equilibrium point and therefore asymptotic stability can not be proven using LDM only. Fortunately, there are several theorems that make it possible to conclude asymptotic stability for these cases, e.g. Lasalle's Invariance Principle for autonomous systems and Barbalat's Lemma for non-autonomous cases. Since we are dealing with autonomous dynamics, the error dynamics is time invariant, Lasalle's Invariance Principle (LIP) may be used to still be able to prove asymptotic stability for (3.15). This principle says to observe the set of all points, Q, for which $\dot{V} = 0$. Then defining M as the largest invariant set in

Q, LIP says that all solutions will asymptotically converge to M as $t \to \infty$. $\dot{V} = 0$ only if $\dot{\vec{e}} = \vec{0}$. Translating this to the error dynamics for this case gives

$$\ddot{\vec{e}} + K_n^a \vec{e} = \vec{0} \tag{3.20}$$

The left side of equation (3.20) is nonzero as long as $\vec{e} \neq 0$. Thus the only and therefore automatically the largest invariant set M contains only one position, being $(\vec{e} = 0, \dot{\vec{e}} = 0)$, which is equal to the equilibrium point \vec{e}_{eq} . Therefore it can be concluded that all solutions will asymptotically converge to the equilibrium point \vec{e}_{eq} and therefore (3.15) is asymptotically stable. This means that $\vec{q}_L \rightarrow \vec{q}_L^d$ and $\vec{q}_E \rightarrow \vec{q}_E^d$ asymptotically as time goes to infinity.

In the above analysis the assumption is made that the dynamic models of the systems are exactly known. In practice, generally this is not the case. A drawback of feedback linearization is that no robustness is guaranteed if the model is not exact, due to for example parameter uncertainties or unmodeled dynamics. Therefore applying this control strategy in this form will possibly lead to undesired behavior or even instability in practice. In chapter 5 this problem wil be examined and a new control strategy will be presented which will be able to stabilize dynamic models even in the presence of model uncertainties.

3.4 Conclusion

The presented coordinate transformation has allowed to decompose the dynamics of a group of n general m-DOF mechanical systems into a m-DOF average system, representing the overall motion of the group, and a m(n-I)-DOF formation system, which governs the group formation structure. This partition makes it possible to autonomously control the formation and trajectory tracking of the systems. Since the two new systems mimic the structure of common mechanical systems a range of well known control schemes can be used to achieve the desired control aims. A feedback linearization approach is used and with the help of Lyapunov's Stability Theory and Lasalle's Invariance Principle asymptotic stability is proven for the average and formation systems.

Chapter 4

Comparison TU/e research

4.1 Introduction

The control scheme presented in chapter 3 shares several similarities with the research done at the Technische Universiteit of Eindhoven by Rodriguez-Angeles and Nijmeijer, described among others in [16]. To acquire insight to what extent these control approaches resemble each other, the two control approaches are analyzed in detail and compared with each other in this chapter. After this conclusions will be given as to which control approach is more suitable for the problem presented in this report.

4.2 Control approaches

Although discussed in the previous chapters, the design of the two control schemes will be recapitulated here to present a clear overview. With this discussion a foundation is laid for the comparison of the approaches in the next section.

4.2.1 Rodriquez-Angeles strategy

The work done by Rodriguez-Angeles and Nijmeijer in [16] deals with a much more complex problem than that of the research done by Lee in [8]. The main goal of Rodriguez-Angeles et al. is to solve the problem of position synchronization of two or more cooperative robot systems, in the case when only position measurements are available. In other words, to design the controllers for all the systems in the group, such that position q_i and velocity \dot{q}_i of the i^{th} robot converge to the commonly desired trajectory q_d, \dot{q}_d and q_j, \dot{q}_j of the j^{th} robot, with $j \neq i, j = 1, ..., p$. The imposed constraint that only position measurements are available complicates the synchronization problem considerably. Since the other state variables (velocities and accelerations) are needed for synchronization of the systems, nonlinear model-based observers are implemented to estimate these unavailable variables. But because in this report we assume that all state variables are available, these nonlinear observers are not needed and a simplified version of the control scheme can be considered.

Rodriguez-Angeles et al. consider systems with dynamic models

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i \qquad i = 1, ..., p$$
(4.1)

where $M_i(q_i)$ is the inertia matrix, $C_i(q_i, \dot{q}_i)$ represents the Coriolis matrix and $g_i(q_i)$ denotes the gravity forces. The system input τ_i for the individual systems is proposed as

$$\tau_i = M_i(q_i)\ddot{q}_{ri} + C_i(q_i, \dot{q}_i)\dot{q}_{ri} + g_i(q_i) - K_{d,i}\dot{s}_i - K_{p,i}s_i \qquad i = 1, ..., p$$

$$s_i = q_i - q_{ri} \qquad (4.2)$$

where $K_{p,i}$ and $K_{d,i}$ are positive definite gain matrices and s_i , the synchronization errors, stand for the difference between the position of the i^{th} system and the so-called reference signal q_{ri} , which is defined as

$$q_{ri} = q_d - \sum_{j=1, j \neq i}^{p} K_{i,j}(q_i - q_j)$$
(4.3)

This reference signal q_{ri} makes it possible to generate interactions between the individual systems, where with $K_{i,j}$ the amount of desired interactions can be determined. This definition is essential for guaranteeing the synchronous behavior of the systems. Note that with the error definitions $e_{ii} = q_i - q_d$ and $e_{ij} = q_i - q_j$, $s_i = q_i - q_{ri}$ can also be written as

$$s_i = e_{ii} + \sum_{j=1, j \neq i}^p K_{i,j} e_{ij}$$
(4.4)

This notation of s_i will be convenient for the comparison of the two control schemes in the next section. Substitution of (4.2) and (4.3) into the differential equations of (4.1) leads to the synchronization error dynamics

$$M_i(q_i)\ddot{s}_i + C_i(q_i, \dot{q}_i)\dot{s}_i + K_{d,i}\dot{s}_i + K_{p,i}s_i = 0 \qquad i = 1, ..., p$$
(4.5)

Since the couplings between the systems in the group are solely modeled by the synchronization error \vec{s} and its derivatives, the synchronization error dynamics for all the systems are decoupled. With the help of Lyapunov theory and Barbalat's lemma asymptotic stability is proven for s, \dot{s} . For asymptotic stable synchronization of the systems another mathematical condition must hold (see [16] for more detail). This eventually leads to that $q_i^{(r)} \rightarrow q_d^{(r)}$ and $q_i^{(r)} \rightarrow q_j^{(r)}$ for all i, j = 1, ..., p, r = 0, 1.

4.2.2 Lee strategy

This approach is discussed thoroughly in chapter 3 but will be repeated here shortly to provide a clear view. The approach from chapter 3 is applicable to systems with dynamical models as

$$M_i(\vec{q}_i)\vec{q}_i + C_i(\vec{q}_i, \vec{q}_i)\vec{q}_i = T_i + F_i \qquad i = 1, ..., n$$
(4.6)

The main objective of this control scheme is to control a group of dynamical systems with structures as in (4.6) to follow a desired trajectory while moving in a prescribed formation layout. With a special decomposition the dynamics of the group of systems is decoupled into a group motion-part and a formation-part. This transformation makes it possible to independently control the formation and the trajectory of the group and is realized by transforming the available coordinates $\vec{q} = \begin{bmatrix} \vec{q_1} & \vec{q_2} & \dots & \vec{q_n} \end{bmatrix}^T$ to a set of new group coordinates:

$$\vec{q}_E^T = [\vec{q}_1^T - \vec{q}_2^T, \vec{q}_2^T - \vec{q}_3^T, ..., \vec{q}_{n-1}^T - \vec{q}_n^T]$$

$$\vec{q}_L = [\sum_{i=1}^n M_i(\vec{q}_i)]^{-1} [\sum_{i=1}^n M_i \vec{q}_i]$$
(4.7)

Here q_E , used in the formation-dynamics, stands for the relative displacements between successive systems and q_L , utilized in the group motion-dynamics, represents the motion of the center of mass of the group. This coordinate transformation is realized by introducing the decomposition matrix $S(\vec{q})$, which describes the correlation between the old and new coordinates:

$$\dot{\vec{q}}_a = S(\vec{q})\dot{\vec{q}} \tag{4.8}$$

 $S(\vec{q}) = \begin{bmatrix} \phi_1(\vec{q}) & \phi_2(\vec{q}) & \dots & \phi_{n-1}(\vec{q}) & \phi_n(\vec{q}) \\ I & -I & \dots & 0 & 0 \\ 0 & I & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & -I & 0 \\ 0 & 0 & \dots & I & -I \end{bmatrix}$

where $\dot{\vec{q}}_a = \begin{bmatrix} \dot{\vec{q}}_L & \dot{\vec{q}}_E \end{bmatrix}^T$ and $\phi_i(\vec{q}) = [\sum_{j=1}^n M_j(\vec{q}_j)]^{-1} M_i(\vec{q}_i)$. Utilizing the decomposition from (4.8) in (4.6), eventually leads to the new overall dynamics

$$\underline{M}_{a}\ddot{\vec{q}_{a}} + \underline{C}_{a}\dot{\vec{q}_{a}} = \underline{T}_{a} + \underline{F}_{a} \tag{4.9}$$

with \underline{M}_a and \underline{C}_a being the decomposed system matrices of the group and \underline{T}_a and \underline{F}_a the decomposed input and environmental disturbances matrices. Choosing the new input \underline{T}_a as

$$\underline{T}_a = \underline{C}_a \dot{\vec{q}}_a - \underline{F}_a + \underline{M}_a (\ddot{\vec{q}}_a^d - K_v^a \dot{\vec{e}} - K_p^a \vec{e})$$
(4.10)

and substituting into and rearranging (4.9) yields

$$\ddot{\vec{e}} + K_v^a \dot{\vec{e}} + K_p^a \vec{e} = \vec{0}$$
(4.11)

Here $\vec{e} = \vec{q_a} - \vec{q_a^d} = \begin{bmatrix} \vec{q_L} - \vec{q_L^d} \\ \vec{q_E} - \vec{q_E^d} \end{bmatrix}$ and $\vec{q_L^d}, \vec{q_E^d}$ represent the desired motion of the center of mass of the group and the desired distances between successive systems, respectively.

With Lyapunov's Direct Method and Lasalle's Invariance Principle asymptotic stability is proven for (4.11) and that means that $\vec{q}_L \rightarrow \vec{q}_L^d$ and $\vec{q}_E \rightarrow \vec{q}_E^d$ asymptotically as time goes to infinity.

4.3 Comparison of approaches

When one puts the two control schemes side by side the first striking resemblance is that both schemes assume the same dynamic models for the systems, being

$$M_{i}(\vec{q}_{i})\vec{q}_{i} + C_{i}(\vec{q}_{i},\vec{q}_{i})\vec{q}_{i} = T_{i} + F_{i}$$

$$M_{i}(q_{i})\vec{q}_{i} + C_{i}(q_{i},\dot{q}_{i})\dot{q}_{i} + g_{i}(q_{i}) = \tau_{i}$$
(4.12)

Note that although formulated differently, $g_i(q_i)$ and F_i represent the same class of forces. The works of Rodriguez-Angeles et al. and Lee et al. are both focussed on robotic systems, particularly robot manipulators and therefore both have based their research on the universally known dynamical structures of these systems. Also both approaches are based on cooperation between the individual systems. However the approaches use this collaboration for different purposes and

here is where the largest distinction between the control schemes can be made. The approach of Rodriquez-Angeles et al. looks at the systems individually and the interactions with each other. Besides independent convergence of all the systems to a common desired trajectory $(q_i \rightarrow q_d)$, most importantly the systems also observe the motions of the other systems and synchronize their trajectories with each other $(q_i \rightarrow q_j)$. Because of this mutual synchronization the approach in this configuration is not capable to control the systems to move in (non-autonomous) formations. The approach from section 4.2.2 on the other hand focuses on the movement of the systems are seen together as one large system. Thereto no individual forces are defined, but general group movement controls and formation controls. Mutual synchronization is not sought after in this approach and even undesired in some cases, e.g. non-autonomous formations. Despite this difference the controllers from the approaches are structurally very similar;

$$T_{i} = M_{i}(\vec{q}_{i})\ddot{\vec{q}}_{i}^{d} + C_{i}(\vec{q}_{i}, \dot{\vec{q}}_{i})\ddot{\vec{q}}_{i} - F_{i} - M_{i}k_{v}I(\dot{\vec{q}}_{i} - \dot{\vec{q}}_{i}^{d}) - M_{i}k_{p}I(\vec{q}_{i} - \vec{q}_{i}^{d})$$

$$\tau_{i} = M_{i}(q_{i})\ddot{q}_{ri} + C_{i}(q_{i}, \dot{q}_{i})\dot{q}_{ri} + g_{i}(q_{i}) - K_{d,i}(\dot{q}_{i} - \dot{q}_{ri}) - K_{p,i}(q_{i} - q_{ri})$$
(4.13)

For comparison, the group control input \underline{T}_a of the Lee-approach is transformed back to the level of the individual systems.¹ If it is assumed that q_{ri} from the second equation is equal to \bar{q}_i^d from the first equation in (4.13) the two control inputs are in fact exactly equal, except for the difference between $C_i(\vec{q_i}, \vec{q_i})\vec{q_i}$ and $C_i(q_i, \dot{q_i})\dot{q_{ri}}$, which is structurally unimportant. Rodriguez-Angeles et al. have decided to use the Coriolis matrices of the systems in their synchronization error dynamics (see (4.5)), where Lee et al. choose to fully compensate these Coriolis matrices resulting into that no system matrices are present in the error dynamics (see (4.11)). The Coriolis matrices are able to assist the control matrices $K_{d,i}$, as a result of what less control effort is needed with Rodriquez-Angeles compared to the full compensation of Lee et al.. The choice of Lee et al. on the other hand is most likely done to make the error dynamics autonomous. The only significant difference between the two control approaches is the difference between the definitions q_{ri} and \bar{q}_i^d . The reference signal of Rodriquez-Angeles defined as $q_{ri} = q_d - \sum_{j=1, j \neq i}^p K_{i,j}(q_i - q_j)$ and the desired trajectories \vec{q}_i^d of the Lee-approach once again show the different objectives of the control approaches as mentioned earlier. With Rodriquez-Angeles the systems individually converge to a common desired trajectory q^d and also synchronize their movements with each other, where with Lee no mutual synchronization is present and the systems converge to their individually desired trajectory \bar{q}_i^d . It must be noted that these desired trajectories \bar{q}_i^d are linked, due to the definitions of \vec{q}_E .

4.4 Mathematical conversion

Since the two control approaches share several important similarities, in this section an attempt is made to adapt the Lee-approach to mimic the actions of the Rodriquez-approach.

Although the approach of Lee does not encompass mutual synchronization of the systems, it is able to approximate this quality of the Rodriquez-Angeles approach. Hereto some assumptions must be made. The first one is to assume that $K_v^a = k_v I$ and $K_p^a = k_p I$ with k_v, k_p being constant

¹For further details about the inverse transformation of \underline{T}_a see Appendix A

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scalars. This can be utilized in (4.11) and writing down this equation in detail yields

$$\frac{\sum_{i=1}^{n} M_{i}(\vec{q_{i}})\vec{q_{i}}}{\sum_{i=1}^{n} M_{i}(\vec{q_{i}})} - \ddot{\vec{q}}_{L}^{d} + k_{v}\left(\frac{\sum_{i=1}^{n} M_{i}(\vec{q_{i}})\vec{q_{i}}}{\sum_{i=1}^{n} M_{i}(\vec{q_{i}})} - \dot{\vec{q}}_{L}^{d}\right) + k_{p}\left(\frac{\sum_{i=1}^{n} M_{i}(\vec{q_{i}})\vec{q_{i}}}{\sum_{i=1}^{n} M_{i}(\vec{q_{i}})} - \vec{q}_{L}^{d}\right) = 0$$

$$\frac{\ddot{\vec{q}}_{i} - \ddot{\vec{q}}_{i+1} - \ddot{\vec{q}}_{E_{i}}^{d} + k_{v}\left(\dot{\vec{q}}_{i} - \dot{\vec{q}}_{i+1} - \dot{\vec{q}}_{E_{i}}^{d}\right) + k_{p}\left(\vec{q}_{i} - \vec{q}_{i+1} - \vec{q}_{E_{i}}^{d}\right) = 0$$

$$\frac{\ddot{\vec{q}}_{i-1} - \ddot{\vec{q}}_{n} - \ddot{\vec{q}}_{E_{n-1}}^{d} + k_{v}\left(\dot{\vec{q}}_{n-1} - \dot{\vec{q}}_{n} - \dot{\vec{q}}_{E_{n-1}}^{d}\right) + k_{p}\left(\vec{q}_{n-1} - \vec{q}_{n} - \vec{q}_{E_{n-1}}^{d}\right) = 0$$

$$(4.14)$$

To make sure that all individual systems follow the same common desired trajectory as in the control approach of Rodriquez-Angeles et al., the desired formation trajectories must be set to zero, i.e. $q_{E_i}^{(r)} = 0$, i = 1, ..., n - 1, r = 0, 1, 2. Substituting this choice into (4.15) results into

$$\ddot{\vec{q}}_{i} - \ddot{\vec{q}}_{i+1} + k_{v}(\dot{\vec{q}}_{i} - \dot{\vec{q}}_{i+1}) + k_{p}(\vec{q}_{i} - \vec{q}_{i+1}) = 0$$

$$\vdots$$

$$\ddot{\vec{q}}_{n-1} - \ddot{\vec{q}}_{n} + k_{v}(\dot{\vec{q}}_{n-1} - \dot{\vec{q}}_{n}) + k_{p}(\vec{q}_{n-1} - \vec{q}_{n}) = 0$$
(4.16)

From the equations of (4.16) can be easily concluded that when the control gains $k_p > 0$ and $k_v > 0$ that all the differential equations are exponentially stable. This then implies that for $t \to \infty$, $\vec{q}_i^{(r)} \to \vec{q}_{i+1}^{(r)}$, i = 1, ..., n - 1, r = 0, 1 and this on itself implies that the coordinates of all systems are connected to each other. Although it is not the same synchronization as in Rodriquez-Angeles' work $(q_i^{(r)} \to q_j^{(r)})$, this can be seen as a means of mutual synchronization. Since the coordinates of all the systems are connected to each other, suppose that this common coordinate is denoted as \vec{q}_g , i.e. $\vec{q}_i \to \vec{q}_{i+1} \to \vec{q}_{i+2} ... \to \vec{q}_n \to \vec{q}_g$. Applying this to the differential equation in (4.14), results in

$$\ddot{\vec{q}}_g - \ddot{\vec{q}}_L^d + k_v (\dot{\vec{q}}_g - \dot{\vec{q}}_L^d) + k_p (\vec{q}_g - \vec{q}_L^d) = 0$$
(4.17)

This differential equation is, analogous to (4.16), also asymptotically stable if $k_v > 0, k_p > 0$, which means that $\bar{q}_g^{(r)} \to \bar{q}_L^{(r)d} r = 0, 1$ as $t \to \infty$.

The analysis performed in this section show that the two control approaches can provide nearly similar controls despite their differences. However complete adaptation to eachothers control schemes cannot be managed. Therefore the control schemes can be categorized as consisting of the same basis structure, but each diverge from there on out.

4.5 Conclusion

The control approaches of Lee et al. and Rodriquez-Angeles et al. are discussed in section 4.2.2 and 4.2.1 respectively. In section 4.3 the two control approaches are compared and several differences as well as comparisons are highlighted and discussed. The key difference between the work of Rodriquez and Lee lies in the aim of the approaches; Rodriquez-Angeles is mainly interested in mutual synchronization, where Lee is more interested in the group dynamics of the individual systems. Despite these differences, the control schemes are able to provide nearly similar controls due to significant resemblances, which is demonstrated in section 4.4. When considering the main objective of this report, trajectory and formation control of a group of systems, the approach of Lee is more suitable to utilize and is therefore used in the remainder of this report.

Chapter 5

Sliding Mode Control

5.1 Introduction

In chapter 3 a control strategy was presented, which was based on the assumption that the exact dynamic models were known for the real systems. This chapter will elaborate on this by introducing a control approach which will be able to stabilize the systems in the presence of model imprecisions. This approach is known as Sliding Mode Control (SMC). First a general introduction to SMC will be given, after which the control scheme for the specific systems will be derived. This section will be followed by a stability analysis of the control approach and finally conclusions are made about the results of the implementation of the robust controller.

5.2 SMC Theory

So far the assumption was that the dynamic models presented in (3.1) were exact representations of the real systems. In general this is not the case and models contain uncertainties with respect to their practical counterpart. These model uncertainties come in two types; parametric uncertainties and unmodeled dynamics. Neglecting these uncertainties can lead to undesired behavior or even instability of systems. Therefore these inaccuracies must be taken into account. Sliding mode control, a robust control method, makes this possible. The structure of a SMC controller generally consists of a input-output linearization part together with additional terms which account for model uncertainties. This approach introduces a term, the sliding surface, which makes it possible to transfer a n^{th} order control problem into a 1^{st} order control problem and hence simplifies the control process. Ultimately SMC makes it possible to prove stability in the presence of parametric uncertainties. However this control method is not generally applicable. To be able to use this control method the specific system must be of minimum phase, i.e. the zero dynamics of the system must be stable.

The above mentioned control method can be utilized in this case and so sliding mode control will be applied on the systems of (3.9) and (3.10). The sliding mode design presented in this chapter is based on the theory provided in [19]. For convenience the differential equations are noted here again;

$$\underline{M}_a \vec{q}_a + \underline{C}_a \vec{q}_a = \underline{T}_a + \underline{F}_a \tag{5.1}$$

For SMC defining the so-called sliding surface is an essential part of the control approach. The sliding surface S(t) for this case is defined as

$$\vec{s} = \dot{\vec{e}} + \lambda I \vec{e} = 0 \tag{5.2}$$

where λ is a positive constant scalar and \vec{e} is the error vector as defined earlier in (3.13). Equation (5.2) can be seen as a linear differential equation whose unique equilibrium solution is $\vec{e} = \vec{0}$, with the assumption that the initial conditions $\vec{q}_a(0) = \vec{q}_a^d(0)$. Therefore remaining on the sliding surface S(t) is the same as the tracking problem of (\vec{q}_a, \vec{q}_a) to $(\vec{q}_a^d, \dot{q}_a^d)$, since the solution of (5.2) means that $\vec{q}_a \rightarrow \vec{q}_a^d$ and $\dot{\vec{q}}_a \rightarrow \dot{\vec{q}}_a^d$ asymptotically. Therefore the control problem can be reduced to controlling towards $\vec{s} = 0$. The controller should force all solutions to converge to and stay on the sliding surface S(t), i.e. converge to the origin of the (\vec{e}, \vec{e}) -plane.

5.3 Control Scheme and Stability Analysis

In this section the sliding mode control scheme for the specific systems will be presented and the stability of the control approach will be examined. The stability analysis will be based on Lyapunov stability theory.

As in chapter 3 Lyapunov's Direct Method is used for the stability study. The selected Lyapunov function for the *s*-dynamics is

$$V(\vec{s}) = \frac{1}{2}\vec{s}^T \underline{M}_a \vec{s} \tag{5.3}$$

This function is zero in the equilibrium point $\vec{s} = 0$ and positive definite in \vec{s} . Note that \underline{M}_a is a symmetric, positive definite matrix. Therefore (5.3) satisfies the stability conditions and qualifies for a Lyapunov function for the \vec{s} -dynamics. Furthermore differentiating (5.3) yields

$$\dot{V}(\vec{s}, \vec{s}) = \vec{s}^T \underline{M}_a \dot{\vec{s}} + \frac{1}{2} \vec{s}^T \underline{\dot{M}}_a \vec{s}$$
(5.4)

The derivative of \vec{s} can be obtained using (5.2) and substituting this in (5.4) gives

$$\dot{V} = \vec{s}^T \underline{M}_a (\ddot{\vec{q}}_a - \ddot{\vec{q}}_a^d + \lambda I \dot{\vec{e}}) + \frac{1}{2} \vec{s}^T \underline{\dot{M}}_a \vec{s}$$
(5.5)

From the differential equations of (5.1), $\ddot{\vec{q}}_a$ can be determined and applying this and rearranging \dot{V} results in

$$\dot{V} = \vec{s}^T \left(-\underline{C}_a \dot{\vec{q}}_a + \underline{T}_a + \underline{F}_a - \underline{M}_a (\ddot{\vec{q}}_a^d - \lambda I \dot{\vec{e}}) \right) + \frac{1}{2} \vec{s}^T \underline{\dot{M}}_a \vec{s}$$
(5.6)

To proof asymptotic stability of the sliding surface S(t) with LDM, \dot{V} must be zero in the equilibrium point $\vec{s} = 0$ and negative definite in \vec{s} . In the following paragraph a construction for the controller \underline{T}_a is presented such that these demands are met.

We now propose the control input \underline{T}_a to be of the form

$$\underline{T}_{a} = \underline{\hat{T}}_{a} - \underline{k} \operatorname{sign}(\vec{s}) = \underline{\hat{C}}_{a} \dot{\vec{q}}_{a} - \underline{\hat{F}}_{a} + \underline{\hat{M}}_{a} (\ddot{\vec{q}}_{a}^{d} - \lambda I \dot{\vec{e}}) - \frac{1}{2} \underline{\dot{\hat{M}}}_{a} \vec{s} - \underline{k} \operatorname{sign}(\vec{s})$$
(5.7)

where $\underline{\hat{M}}_a, \underline{\hat{C}}_a$ and $\underline{\hat{F}}_a$ are estimates of the system matrices and \underline{k} (*nm*x1) is an arbitrary control vector. Also the introduced sign-function is defined as

$$sign(\vec{s}) = +1 \quad \text{if } \vec{s} > 0$$

$$sign(\vec{s}) = -1 \quad \text{if } \vec{s} < 0$$

$$sign(\vec{s}) = 0 \quad \text{if } \vec{s} = 0$$
(5.8)

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Note that the system parameters are assumed to be estimated by an available system identification method. \underline{k} sign(\vec{s}) from (5.7), a characteristic slide mode term, is used to make the sliding surface invariant and $\underline{\hat{T}}_a$ would be the controller which would make \dot{V} equal to zero if the system dynamics were known exactly. By using the definition of \underline{T}_a from (5.7), the derivative of the Lyapunov function becomes

$$\dot{V} = \vec{s}^{T} \underbrace{\left[-(\underline{C}_{a} - \underline{\hat{C}}_{a})\dot{\vec{q}}_{a} + (\underline{F}_{a} - \underline{\hat{F}}_{a}) + (\underline{M}_{a} - \underline{\hat{M}}_{a})(\ddot{\vec{q}}_{a}^{d} - \lambda I\dot{\vec{e}}) \right]}_{\alpha} + \frac{1}{2}\vec{s}^{T} \underbrace{(\underline{\dot{M}}_{a} - \underline{\dot{M}}_{a})}_{\beta} \vec{s} - \underline{k} |\vec{s}|$$

$$(5.9)$$

Although the system dynamics are not known exactly, it is assumed that information is available about the extent of the inaccuracies; the estimation errors are bounded and known. These assumptions are essential for proving stability of the sliding surface \vec{s} . By stating that

$$\begin{aligned} |\alpha| &\le m_1 \\ \beta &\le m_2 \end{aligned} \tag{5.10}$$

where m_1, m_2 are known functions and defining the control vector <u>k</u> from (5.7) as

$$\underline{k} = m_1 + \frac{1}{2}m_2|\vec{s}| + \gamma \tag{5.11}$$

the derivative of the Lyapunov function $V(\vec{s})$ becomes

$$\dot{V} = \underbrace{\vec{s}^{T} \alpha + \frac{1}{2} \vec{s}^{T} \beta \vec{s} - m_{1} |\vec{s}| - \frac{1}{2} m_{2} |\vec{s}|^{T} |\vec{s}|}_{\varphi} - \gamma |\vec{s}|$$
(5.12)

With help of (5.10) one can conclude that $\varphi \leq 0$ and consequently \dot{V} ultimately becomes

$$\dot{V} \le -\gamma |\vec{s}| \tag{5.13}$$

The choice of the controller \underline{T}_a has resulted in making \dot{V} zero in the equilibrium point $\vec{s} = 0$ and negative definite in \vec{s} . Hereto LDM's stability conditions are met and consequently the sliding surface S(t) is proved to be asymptotically stable for all \vec{s} , i.e. $\vec{q}_a \to \vec{q}_a^d$ and $\dot{\vec{q}}_a \to \dot{\vec{q}}_a^d$ asymptotically as $t \to \infty$.

5.4 Adaptation of SMC controller

As can be seen from \underline{T}_a the control law has to be discontinuous across the surface S(t) to account for model inaccuracies. Practical implementation of such a control law generally leads to undesired behavior, because of the difficulty of applying a practical discontinuous function due to finite precision. This can lead to chattering, where the control input will switch continuously across the sliding surface. This high control activity strains systems heavily and causes fast wear of instruments and therefore is undesirable. By replacing the sign-function by a saturation-function (see figure 5.1) the chattering problem can be eliminated. This leads to a boundary layer with thickness 2Φ around the sliding surface S(t) resulting in that outside the boundary layer the control input \underline{T}_a remains the same, guaranteeing convergence of all solutions to the boundary layer, but



Figure 5.1: Left; Saturation function $f(s) = -sat(\Phi^{-1}s)$ Right; Boundary layer with thickness 2Φ , formed due the implementation of a saturation function, reference [19] chapter 7

that inside the boundary layer convergence to the sliding surface cannot be guaranteed. As a consequence the switch from a sign function $sign(\vec{s})$ to a saturation function $sat(\Phi^{-1}\vec{s})$ results in going from theoretically perfect convergence to a bounded convergence, i.e. the sliding surface, and conversely the error \vec{e} , does not become zero, but stays within a known area around zero. Whereas with the sign-function the sliding surface S(t) was an invariant set, the application of a saturation-function makes the boundary layer the invariant set.

5.5 Conclusion

The control approach of chapter 3 was unable to guarantee stability of the overall dynamics in the presence of modeling uncertainties. Since in practice these uncertainties can be found frequently, this chapter has presented a control approach, the sliding mode control, which is able to cope with system uncertainties. This approach introduces a term, the sliding surface, which transfers the n^{th} order control problem into a 1^{st} order control problem. Since this surface is defined as in (5.2), choosing a control input that forces all trajectories to go to the sliding surface makes the systems stable. Analytically, asymptotic stability is proven for the systems in equation (3.1) using Lyapunov's Direct Method. Ultimately utilizing sliding mode control leads to asymptotic stability and consistent performance in the face of modeling imprecisions. In the next chapter simulations will be performed of the presented control approaches to support the analytical findings.

Chapter 6

Simulation Results

6.1 Introduction

In the previous chapters is demonstrated that the presented control approaches are able to stabilize the systems of (3.9) and (3.10), even in case of model inaccuracies. In this chapter the closed loop systems are simulated in Matlab/Simulink to present quantitative results of the control approaches and confirm the analytical findings. First of all the general structure of the Matlab/Simulink model will be presented, after which results of several different control approaches will be given and discussed. Finally theory and simulations are compared and conclusions will be made about the outcome of the simulations.

6.2 Simulink build

To be able to perform simulations of the control strategy, the average and formation systems have been implemented in a Matlab/Simulink environment (see figure 6.1). The model basically



Figure 6.1: Simulation model for performing simulations (Matlab/Simulink)

consists of three main parts; the desired trajectory, the control input \underline{T}_a and the average and formation systems. The trajectory-subsystem is a straightforward part containing $\bar{q}_a^{d}, \dot{\bar{q}}_a^{d}$ and \ddot{q}_a^{d} . Depending on the used control approach, the control input-part is either the modeling of the approach of chapter 3 (3.11) or the sliding mode controller with sign-function (??) or saturation-function. Since this part contains basic modeling blocks without any elements that are worth mentioning it is not illustrated here. The systems-part, on the other hand, is presented in figure

6.2. It has as inputs \underline{T}_a and \underline{F}_a and provides as outputs the vectors \vec{q}_a and $\dot{\vec{q}}_a$. Note that the systems are built using integrator-blocks. This leads to more accurate results than the use of differentiators. Also the initial conditions for \vec{q}_a and $\dot{\vec{q}}_a$ are added to the second and first integrator-block, respectively. The control loop of the simulink-model then works as follows; the loop starts



Figure 6.2: The differential equations of the average and formation systems built in Matlab/Simulink

with the integrator blocks of the systems-part, which provide the initial \vec{q}_a and \vec{q}_a . Together with the desired trajectory $(\vec{q}_a^d, \dot{\vec{q}}_a^d, \ddot{\vec{q}}_a^d)$, they are sent to the control input-part and used as inputs. This part then computes the required control input \underline{T}_a to stabilize the systems. Simultaneously with the environmental disturbances term \underline{F}_a , \underline{T}_a is on itself fed to the systems-part. This part is then able to compute the new \vec{q}_a and $\dot{\vec{q}}_a$, which is the last step of the control loop. From here on the loop starts all over again.

Note that the constant terms for the simulations, such as initial conditions and controller gains, are provided in a Matlab m-file, which has to be adopted before simulations are started.

6.3 Simulations

In this section the different control approaches will be simulated, starting with the controller from chapter 3. Hereafter the sliding mode controller with sign-function and with saturation-function will be discussed. For all these control schemes the same dynamical systems and the same desired formations and trajectories will be used in the simulations, so as to make comparisons with more ease.

The group of systems is chosen to exist of four members with respectively two degrees of freedom each, i.e. n = 4 and m = 2. The dynamic models used in the simulations are considered as simple frictionless point masses. These assumptions lead to the following system dynamics

$$\underline{M}_{i}\vec{q}_{i} = T_{i} \quad i = 1, ..., 4 \tag{6.1}$$

where \underline{M}_i (2x2), the inertia matrix, is a positive definite diagonal matrix and $\ddot{\vec{q}}_i$ (2x1) is the acceleration vector of the i^{th} system. For the four members of the group the inertia matrices are chosen arbitrarily as

$$\underline{M}_i = m_i I \quad i = 1, \dots, 4 \tag{6.2}$$

with I (2x2) being the unity matrix and $m_1 = 42 \, [kg], m_2 = 20 \, [kg], m_3 = 32 \, [kg]$ and $m_4 = 12 \, [kg]$.

The desired formation of the group is chosen to be non-autonomous, i.e. the formation is time dependent and changes its shape during simulations. The desired formation starts with a line

formation, gradually changes into a square formation and after a while finally transforms into a rectangular structure (see figure 6.3). A smooth corresponding formation trajectory is generated



Figure 6.3: Schematic presentation of desired formation structure

using the ref3-design tool¹. Moreover, the desired average trajectory of the group of systems is defined as a linear increasing velocity trajectory (\dot{q}_L^d), which is to be followed in formation;

$$\vec{q}_L^d = \begin{bmatrix} 0,05t^2\\0,05t^2 \end{bmatrix}, \dot{\vec{q}}_L^d = \begin{bmatrix} 0,1t\\0,1t \end{bmatrix}, \ddot{\vec{q}}_L^d = \begin{bmatrix} 0,1\\0,1 \end{bmatrix}$$
(6.3)

These desired formation and average trajectories are shown in more detail in appendix B. Furthermore the initial positions of the four systems are considered to be

$$\vec{q}_{1_{init}} = \begin{bmatrix} 0 & 0 \end{bmatrix}^{T}$$

$$\vec{q}_{2_{init}} = \begin{bmatrix} -2 & -1 \end{bmatrix}^{T}$$

$$\vec{q}_{3_{init}} = \begin{bmatrix} -3 & 2 \end{bmatrix}^{T}$$

$$\vec{q}_{4_{init}} = \begin{bmatrix} -1 & 3 \end{bmatrix}^{T}$$
(6.4)

Also all initial velocities \dot{q}_i and accelerations \ddot{q}_i are set equal to zero. The simulations in Matlab/Simulink are performed using the Runga-Kutta ODE solver method(ode4) with a stepsize of 0.01 seconds and a simulation time of 50 seconds.

6.3.1 Computed Torque Control

In this section simulations, with the desired trajectory and formation described above, are carried out using the control scheme from chapter 3. Hence it is considered that the systems are modelled perfectly and all system parameters are known. The controller gains of the control matrices, K_p^a and K_v^a respectively, are set equal for the average system and formation system, leading to that

$$\begin{aligned} K_p^a &= k_p I \\ K_v^a &= k_v I, \end{aligned} \tag{6.5}$$

where I (8x8) is the unity matrix and the controller gains are set as $k_p = 5$ and $k_v = 10$. In figure 6.4 the individual trajectories are shown in which the controlled systems travel. The x and y axis represent the two dofs, whereas the z-axis displays the time development. As can

 $^{^{1}}$ Ref-3 is a third degree reference trajectory design tool for Matlab/Simulink, which is developed at the Technical University of Eindhoven. For further details see http://www.dct.tue.nl/home of ref3.htm



Figure 6.4: x-y-time trajectory plot of group obtained by utilizing the control scheme from chapter 3

be seen the systems converge quickly from their initial positions to the desired line formation. After staying in this formation for the first ten seconds, a change in formation is clearly apparent; from t = 10 seconds a change in formation is started, which results in a square formation with inter-agent distances of 10 m at t = 21 seconds. This formation is then again held for ten seconds, where at t = 31 seconds another formation change is initiated. This eventually leads to a rectangular formation from t = 45 seconds.

With the error plots of figure 6.5 one can analyse the quantitative behavior of the specific control scheme. Here the right figure provides a closer look at the first eight seconds of the total error plot displayed at the left side. From the initial errors the errors converge to zero quickly; from t = 13



Figure 6.5: left: error \vec{e} versus time plot of simulations performed with chapter 3 control scheme, right: zoomed in view of left figure

seconds the errors are $O(10^{-13} \text{m})$ and stay at this level in the remainder of the simulations. It must be noted that at the time instances when the formation changes (t = 10s, t = 21s and t = 31s), the errors are temporarily increased to a maximum of $2.10^{-9}m$. These errors can be

ascribed to the chosen desired trajectories (see Appendix B). The systems are unable to perfectly follow the change of the desired formation trajectories, resulting into an increase of the error levels. By proper controller tuning or by choosing less demanding desired trajectories (e.g. lower acceleration levels) these effects can be decreased or even diminished.

In figure 6.6 also the control input \underline{T}_a is displayed. For the systems to keep up with the linear



Figure 6.6: control input \underline{T}_a versus time plot of simulations with chapter 3 control scheme

increasing desired group velocity \dot{q}_L^d it is necessary to provide a constant control input to the systems; as can be seen, the control input \vec{T}_L for the average system converges to a constant nonzero value accordingly. As for the control input \vec{T}_E for the formation system, this understandably converges to zero when the formation is desired to be constant. For the transitions of formations, for example from line to square formation, a change in \vec{T}_E is noticeable and it ultimately converges back to zero when the new formation is achieved. In figure 6.5 as in figure 6.6 overshoot is present. Since the controller gains K_p and K_v are chosen randomly, it is most likely that the systems are not controlled optimally and proper tuning of these gains will lead to less or no overshoot (tuning of damping term K_v) and an overall reduction of the present error levels.

The simulation results presented above support the analytical findings from chapter 3. Choosing the control gains in accordance with the demands form chapter 3 the systems converge from their initial positions to the desired average and formation trajectories. Hence simulations show that the control scheme is able to stably control a group of dynamical systems to follow a desired trajectory while moving in a desired formation.

6.3.2 Sliding mode control

Analogously to the previous section, this paragraph presents simulation results for the case when utilizing the robust controllers from chapter 5. Here the assumption is made that the dynamic models of the system are not known precisely and some uncertainty is present. With an estimation algorithm the unknown model parameters can be approximated. The sliding mode controller makes use of these approximations to ultimately stabilize the dynamical systems in the presence

of modeling uncertainties. The estimated system matrices have the same structure as in (6.2);

$$\underline{\hat{M}}_{i} = \hat{m}_{i}I \quad i = 1, ..., 4 \tag{6.6}$$

and its estimated parameters are set as $\hat{m}_1 = 37 [kg], \hat{m}_2 = 24 [kg], \hat{m}_3 = 35 [kg]$ and $\hat{m}_4 = 9 [kg]$. Furthermore the several sliding mode parameters are chosen as $\lambda = 2, \gamma = 200, m_1 = 120$ and $m_2 = 0$. Performing simulations with the presented data yields the results shown in figure 6.7. Here the left plot displays the error development over time, whereas the plot at the right of figure 6.7 shows the control input \underline{T}_a during the simulations. The error plot shows that all error



Figure 6.7: Simulations performed with sliding mode controller with sign-function, left: error \vec{e} versus time plot right: control input T_a versus time plot

terms from \vec{e} eventually tend to zero. After reaching zero the error terms stay within a region of 0.06 m around zero. From this observation can be concluded that the sliding mode controller is able to stabilize the individual systems, with which the analytical findings from chapter 5 are confirmed. However the theory from chapter 5 showed perfect converge, i.e $\vec{e} \rightarrow 0$ and stays zero, while in the simulations the group of systems maintain a nonzero error. This difference between theory and simulations can be attributed to the simulation conditions; The high frequent control input, discussed hereafter, experiences problems due to the fact that its frequency is higher than the sample frequency. Consequently the input is not calculated correctly, which results into less performance. Tests have shown that the error \vec{e} becomes less when the sample time of the simulations is lowered. Note also that total convergence of \vec{e} is achieved later compared to the simulations performed with the computed torque controller from the previous section.

The input plot at the right side of figure 6.7 clearly shows the undesired phenomena of chattering earlier mentioned in chapter 5. This high controller activity can be eliminated by replacing the sign function $sign(\vec{s})$ in \underline{T}_a with the saturation function $sat(\phi^{-1}\vec{s})$ as suggested in section 5.2. The results of this adaptation are shown in figure 6.8. Here the error and control input plots are shown using a sliding mode controller with a saturation function. The error plots of the sliding mode controller with saturation function show great resemblance with each other. Only significant difference is that the region of convergence is smaller with the sliding mode controller with saturation function, that is 0.004 m compared to 0.06 m with the original sign function. This observation contradicts with the theory from chapter 5, which showed that implementation of the saturation function lead to convergence to within a guaranteed region rather than the perfect tracking with the sign function. But this contradiction can again be explained



Figure 6.8: Simulations performed with sliding mode controller with saturation-function ($\Phi^{-1} = 1$), left: error \vec{e} versus time plot right: control input T_a versus time plot

by looking at the simulation conditions. The control scheme with the sign-function experiences problems due to the sample time of the simulations as is mentioned earlier. On the other hand the control scheme with the saturation-function is not influenced by the simulation conditions; the smooth control input from figure 6.8 can be correctly calculated leading to a proper representation of the control scheme. This discrepancy causes the divergent results between theory and simulations.

Whereas the difference between the error plots of figure 6.7 and figure 6.8 is relatively small, the differences between the inputs are evidently visible. The chattering phenomena from figure 6.7 has been completely eliminated by the substitution of the sign function with the saturation function as can be seen in figure 6.8. The implementation of a saturation function creates a boundary layer around the sliding surface, as a result of which switching of the control input does not occur anymore. The use of a saturation function instead of a sign-function results into a more desirable, smooth control input while guaranteeing nearly equivalent performance and thus satisfies its purpose.

Furthermore it is interesting to analyze the course of the sliding mode \vec{s} over time. From figure



Figure 6.9: plots of the sliding surface \vec{s} with sliding mode controller with sign-function (left) and with saturation-function (right)

6.9 one can see that the sliding mode of the controller with sign function and saturation function show great resemblance. Overall convergence of \vec{s} is performed nearly identical. Analogous to the error plots, there is a difference between the regions of convergence. The sliding mode \vec{s} of the controller with sign function stays within a region of 0.2 around zero, where as the convergence region with a saturation function is 0.008. A slight difference that is also present is that the development of the sliding mode \vec{s} with saturation function has a smoother course compared with the sliding mode with sign function. The sign function in \underline{T}_a leads to the chattering shown in figure 6.7. This effect makes itself felt in the trajectories of the systems and so also in the error vector $\vec{e} = \vec{q}_a - \vec{q}_a^d$. And since \vec{s} is defined as in (5.2), a linear combination of \vec{e} and $\dot{\vec{e}}$, it eventually leads to a less smooth course of \vec{s} than with the use of the saturation function.

6.4 Conclusion

In chapter 3 and chapter 5 several control schemes were presented for control of multiple dynamical systems and together with this stability analyses were shown proving asymptotic stability. To provide quantitative results and support the analytical findings simulations were performed in this chapter. Simulations with the computed torque control approach from chapter 3 show convergence of the individual systems to the desired trajectories with a maximum error of 1.10^{-13} m. This is in accordance with the stability analysis from chapter 3. The simulations with the sliding mode controller also show stable overall dynamics. With the estimated system parameters the sliding mode controller is able to control a group of dynamical systems in following desired group and formation trajectories in the presence of modeling inaccuracies. Besides this, the adaptation of the robust controller to solve the problem of chattering also show good results in the simulations; the high frequency controller activity is substituted by a less demanding smooth control input. Besides the similarities between theory and simulations, also some differences are present. The stability analysis of chapter 5 for the robust controller with sign function proved that the sliding mode \vec{s} (and thus the error \vec{e}) converges to zero and stays zero, i.e. $\vec{s} = 0$ was a invariant set. However in the simulations the sliding mode does converge to zero, but instead of staying zero varies around zero with a margin of 0.06. Also where with the robust controller with sign function perfect convergence is proven, utilizing the saturation function zero error can not be proven. Instead the theory stated that the error stays within a region of 2Φ around zero. This is another discrepancy between theory and simulations, since during the simulations the error of the control approach with saturation function is smaller than the error of the control approach with sign function. These contradictions between theory and simulations are caused by the relatively low sample time in the simulations, which lead to an incorrect representation of the sliding mode controller with sign function.

Chapter 7

Conclusions and recommendations

7.1 Conclusions

In this section conclusions are provided regarding the presented work. The main objective of this report was to present a control scheme, which is able to stably control a group of dynamical systems into following a desired trajectory while moving in a prescribed formation layout. Moreover the objective was to analytically proof the above mentioned in the presence of model inaccuracies. From the analysis from chapter 3 can be concluded that the control scheme results into asymptotically stable group dynamics and thus is able to answer the aforementioned objective. The same result holds for the analysis in chapter 5, where a control scheme is presented which accounts for model inaccuracies. These analytical findings are supported by the various simulations in chapter 6.

After a literature study regarding cooperative control of multiple dynamical systems, an approach was chosen which appeared most suitable for the main objective of this report. The major attraction of this approach is that it separates the two goals (group trajectory control and formation control) from each other, which makes overall control easier and more effective. This decomposition is realized by introducing a coordinate transformation, which transforms the individual system coordinates into newly defined group coordinates. Feedback linearization of the new group dynamics results into a computed torque controller and together with Lyapunov's Direct Method and Lasalle's Invariance Principle asymptotic stability for the group of dynamical systems is proven. Simulations with this control approach ,discussed in chapter 6, show good results and confirm the analytical findings.

In addition to this control approach, which assumed perfect knowledge of the system dynamics, a more extensive approach is introduced in chapter 5. In this chapter the control approach from chapter 3 is extended to be able to prove asymptotic stability in the face of modeling imprecisions. For this sliding mode theory is used. The structure of a SMC controller generally consists of an input-output linearization part together with additional terms which account for model uncertainties. Using Lyapunov's Direct Method sliding mode control is able to prove asymptotic stability for the group of systems in the presence of modeling imprecisions. The simulations performed in section 6.3.2 show convergence to the desired trajectories, but do not confirm the perfect converge promised by the theory from chapter 5. This difference between theory and simulations can be attributed to the simulation conditions; The sliding mode control approach with sign function results into a high frequent control input. This input experiences problems due to the fact that its frequency is higher than the sample frequency used in the simulations. Consequently the control

input is not calculated correctly, which results in less performance.

The high control activity, denoted as chattering, is also clearly present in the simulation results. The suggested solution to substitute the sign function from the control input into a saturation function eliminates this chattering phenomena as predicted. The high frequency control input is altered into a smooth control input, but goes at the cost of a minor increase of the errors. Ultimately utilizing sliding mode control leads to asymptotic stability of the group of dynamical systems and consistent performance in the face of modeling imprecisions.

7.2 Recommendations

In this report several aspects of cooperative control of multiple dynamical systems have been discussed. But still many aspects are left untouched. Here several suggestions will be presented to extend the presented work in this interesting field.

The control schemes presented and suggested in this report are all based on the fact that the full state of all the systems are known. Although not unthinkable the direct availability of this information is uncommon. Velocity measuring equipment are not used widely nowadays and are often excluded to save costs. The requirement of this can be avoided, either by differentiating and filtering the position measurements or by using system-observers to estimate the missing state variables. The first method is quite straightforward and gives good results in practice. Drawbacks of this method are that if the position measurements are contaminated with measurement noise it can lead to noisy results. Furthermore this technique makes it difficult to guarantee stability analytically. Although quite more complicated, a observer-based estimator is able to provide good estimates along with providing a systematic way for proving stability. Therefore it is suggested to utilize this last method to extend the research and be able to stably control multiple systems even when the full states of the systems are not known. A good starting point is to study the work of Rodriquez-Angeles and Nijmeijer (among others in [15]), since they have successfully used observer-based estimators and also have many parallels with the dynamical framework presented in this report.

Another way of extending this work is by tuning the controller to optimally put into use the possibilities of this control approach, minimizing the errors. The focus of this report was to analyze controlling a group of multiple dynamical systems. Hereto the control parameters are arbitrarily chosen with only criteria guaranteeing stability of the systems. Proper tuning of the control parameters lies beyond the scope of this report and therefore a study, which primarily focusses at optimal controller tuning is suggested.

The dynamical systems suggested in this report are fairly simple dynamical systems. In practice however, systems are always subjected to restrictions, e.g. saturated control inputs and position limitations. Such constraints can complicate the presented control problem considerably. But to be able to provide a more practicable control approach these practical implications must be accounted for. And therefore it would be interesting to examine the influences of these constraints on the overall dynamics and presented control schemes.

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Appendix A

In this appendix the control input \underline{T}_a of the group of systems is converted to the equivalent control inputs T_i for the individual systems. This is done to be able to compare the control approaches from Rodriquez-Angeles et al. and Lee with each other in chapterrodleecon.

The differential equations for the group of dynamical systems are defined in chapter 3 as

$$\underline{M}_a \vec{q}_a + \underline{C}_a \vec{q}_a = \underline{T}_a + \underline{F}_a \tag{7.1}$$

and the control input is set as

$$\underline{T}_a = \underline{C}_a \dot{\vec{q}}_a - \underline{F}_a + \underline{M}_a (\ddot{\vec{q}}_a^d - K_v^a \dot{\vec{e}} - K_p^a \vec{e})$$
(7.2)

With the knowledge that $\vec{q}_a = S\vec{q}$ and $\underline{T} = S^T \underline{T}_a$ the control inputs for the individual systems can be extracted from (7.2), resulting in

$$\underline{T} = S^T T_a = \underline{C}\dot{\vec{q}} - \underline{F} + \underline{M}\ddot{\vec{q}}^d - \underline{M}S^{-1}K_v^a S(\dot{\vec{q}} - \dot{\vec{q}}^d) - \underline{M}S^{-1}K_p^a S(\vec{q} - \vec{q}^d)$$
(7.3)

Intermezzo

With S being a matrix and S^{-1} the inverse matrix of S, linear algebra states that if and only if the matrix V = vI, I being the unity matrix and v is a constant scalar, that $S^{-1}VS = V$

Since $K_p^a = k_p I$ and $K_v^a = k_v I$ the intermezzo can be used and applying this on (7.3) leads to

$$\underline{\Gamma} = \underline{C}\dot{\vec{q}} - \underline{F} + \underline{M}\ddot{\vec{q}}^d - \underline{M}K_v^a(\dot{\vec{q}} - \dot{\vec{q}}^d) - \underline{M}K_p^a(\vec{q} - \vec{q}^d)$$
(7.4)

Furthermore transforming back (7.1) to differential equations for the individual systems leads to

$$\underline{M}\ddot{\vec{q}} + \underline{C}\dot{\vec{q}} = \underline{T} + \underline{F}$$
(7.5)

And eventually substitution of (7.4) into these differential equations and rearranging eventuates in

$$\ddot{\vec{q}} - \ddot{\vec{q}}^d + K_v^a (\dot{\vec{q}} - \dot{\vec{q}}^d) + K_p^a (\vec{q} - \vec{q}^d) = 0$$
(7.6)

Appendix B

The desired group trajectory \vec{q}_L^d utilized in the simulations of chapter 6 is shown here in figure 7.2. Note that this desired trajectory for the average system is equal for both degrees of freedom.



Figure 7.1: plots of the desired position, velocity and acceleration for the average system, which are used during simulations

In the simulations also a non-autonomous formation is utilized. This desired formation starts off with a line formation, gradually changes into a square formation and after a while finally transforms into a rectangular structure. The desire formation trajectory consists of three terms, namely $\vec{q}_E^d = \begin{bmatrix} \vec{q}_{E_1}^d & \vec{q}_{E_2}^d & \vec{q}_{E_3}^d \end{bmatrix}^T$. In figure 7.2 and 7.3 the two degrees of freedom of $\vec{q}_{E_2}^d$ are shown. Moreover $\vec{q}_{E_1}^d$ and $\vec{q}_{E_3}^d$ are identical and constant during the simulations, $\vec{q}_{E_1}^d = \vec{q}_{E_3}^d = \begin{bmatrix} 0 \\ 10 \end{bmatrix}$.

APPENDIX B



Figure 7.2: plots of the desired position, velocity and acceleration of $q_{E_{2x}}$ for the formation system, which are used during simulations



Figure 7.3: plots of the desired position, velocity and acceleration of $q_{E_{2y}}$ for the formation system, which are used during simulations