Voltage measurement in current zero investigations

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VOLTAGE MEASUREMENT IN CURRENT ZERO INVESTIGATIONS

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1. INTRODUCTION

Despite intensive theoretical and experimental work in the last decennia the process of current interruption in high voltage circuit breakers is least of all got to the bottom of understanding. The main reason is that the success or failure of interruption is mostly determined by phenomena occurring within a few or some tens of micro-seconds before and after current zero. These phenomena depend on the physical properties of the gas discharge as well as on the electrical characteristics of the circuit. Therefore the interruption is a complicated cooperation of a number of factors which assert their influence simultaneously or successively. Besides further extention of the available theories an amount of experimental work on circuit breakers, breaker models and gas discharges will have to got through for a more complete comprehension.
The most important expedient in these investigations are the oscillograms of current and voltage during the interruption period.

Modern cathode ray oscillography is developed adequately to registrate even the very rapid electrical transients with a sufficient rate of accuracy. However, the measuring circuits required to adapt the currents and voltages in the high voltage circuits to the ranges of the oscillograph ask for special attention.
For measuring the high transient voltages voltage dividers are used with the low voltage side connected to the oscillograph by means of a screened cable. These divider circuits must be suited to an extended frequency range, preferable from d.c. to $\geqslant$ 20 Mc/s, while higher frequencies may not give rise to high voltage peaks (e.g. owing to a resonance of the divider itself).
Another requirement is that the current through the voltage divider must be small in comparison with the current through the object under investigation. For this purpose a high impedance for low frequencies is necessary (e.g. $\geqq 10^5$ à 10^6 Ω/kV for 50 c/s) while the capacitance of the divider must be small with respect to the inherent capacitance of the interruption device or discharge chamber and the high voltage circuit in its direct vicinity. Usually a capacitance of 10 to 50 pF will be acceptable.
A peak break-down voltage of 50 to 100 KV will be sufficient for most investigations.

Voltage dividers as usually applied in surge voltage and short circuit testing are generally inadequate for fundamental research because of their low impedance and limited frequency range.

In this paper it will be studied which are the sources for inaccuracy in voltage measurement and by which constructive means the measuring errors can be limited.

2. REQUIREMENTS FOR THE DIVIDING CIRCUITS

2.1 The ratio between the applied voltage and the measured voltage

A voltage divider is composed of a high impedance Z_p ("primary impedance") in series with a small impedance Z_s ("secundary impedance"), fig. 2.1. In this chapter it is accepted that Z_s exclusively consists of passive elements. Then in principle Z_p and Z_s may be composed of one or more resistors and/or capacitors. In section 2.2 it will be demonstrated that an acceptable accuracy over the full frequency range can only be obtained with mixed (resistive-capacitive) dividers.

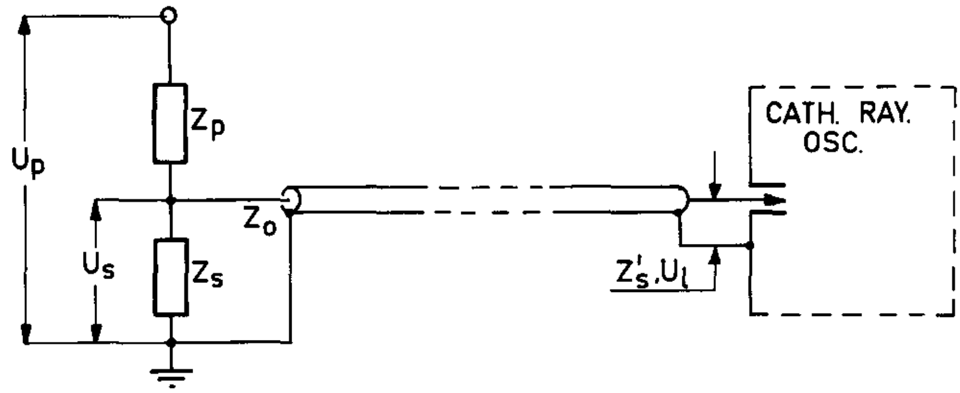

fig. 2.1.

When a "primary voltage" U_p is applied to the divider a "secundary voltage" U_s arises over Z_s, being a measure for the primary voltage. In the ideal case the ratio of the divider impedance and the secundary impedance will be a constant scalar for all frequencies and will be called furtheron the (ideal) divider-ratio A:

$$A \equiv \frac{Z_p(\text{ideal}) + Z_s(\text{ideal})}{Z_s(\text{ideal})} = \frac{U_p}{U_s} \qquad (2.1.)$$

Owing to their geometrical dimensions Z_p and Z_s are not ideal but have inherent inductances and parasitic capacitances. Also the impedance of the measuring cable and the input impedance of the oscillograph may interfere with the ideal circumstances so that the real secundary impedance measured at the oscillograph is not Z_s but a total secundary substitute impedance Z_s'. Therefore the ratio between the absolute values of the primary voltage and the voltage at the oscillograph (U_1) is a frequency dependent quantity which will be called the dividing-ratio $\bar{A}$:

$$\bar{A} \equiv \frac{|Z_p + Z_s'|}{|Z_s'|} = \frac{|U_p|}{|U_1|} \qquad (2.2.)$$

The relative dividing-ratio D, defined by:

$$D \equiv \frac{A}{\bar{A}} = \frac{|U_1|}{|U_p|} \cdot A \qquad (2.3.)$$

is a measure for the quality of the dividing circuit. $D = D(\omega)$ represents the frequency characteristic of the complete voltage dividing-circuit.
After treating the properties of the divider and the cable separately (sections 2.2 to 2.5 incl.) the frequency characteristic of the complete circuit will be considered in section 2.6.

2.2 The mixed divider

Each material point of a divider has a (small) stray capacitance with respect to the high voltage terminal and to ground. Therefore a pure resistive divider cannot be realized. A divider composed of large resistors will always obtain a mixed, resistive-capacitive character for high frequencies. For very high frequencies the capacitive character may dominate and the inherent inductances may also be of importance.

On the other hand a pure capacitive divider introduces large errors for low frequency signals owing to the parallel imput resistance of the oscillograph.
When a signal

$$U_p(t) = \sum_{i=1}^{k} U_i \sin(i\omega t + \psi_i) \qquad (2.4.)$$

is suddenly applied to a capacitive voltage divider connected to an oscillograph with parallel input resistance R_1, a secondary signal will be measured

$$U_s = \frac{1}{A} \sum_{i=1}^{k} U_i \left[F_i \sin(i\omega t + \psi_i + \varphi_i) + \varepsilon^{-t/\tau} \left\{ \sin \psi_i - F_i \sin(\psi_i + \varphi_i) \right] \qquad (2.5.)$$

with

$$A = \frac{C_p + C_s}{C_p} \qquad \varphi_i = tg^{-1} \frac{1}{i\omega\tau}$$

$$\tau = R_1(C_p + C_s) \qquad F_i = \frac{1}{\sqrt{\{1 + (1/i\omega\tau)^2\}}}$$

Apart from the transient behaviour there results a phase shift φ_i and a ratio error $1 - F_i$ of each harmonic component i.
Both errors may reach large values particularly for the basic harmonic component (industrial frequency) as may be seen from table 1, set up for $C_p + C_s = 10.000$ pF, $R_1 = 1$ MΩ., $\omega = 2\pi.50$ c/s.

component	ratio error	phase shift
i=1, (50 c/s)	$1 - F_1 = 4.7\%$	$\varphi_1 = 17°40'$
i=3, (150 c/s)	$1 - F_3 = 0.6\%$	$\varphi_3 = 6°3'$
i=5, (250 c/s)	$1 - F_5 \quad 0\%$	$\varphi_5 = 3°39'$

Table 1

Essentially a mixed divider may exist of a series or parallel RC-network. The series circuit meets the same objections as applicable to a resistive divider. For high frequency transients the total voltage arises across the resistors. So they must have fairly large dimensions which involves important stray capacitances.
From these brief considerations it can be concluded that only a mixed divider with a resistor branch and a capacitor branch in parallel will be able to meet the heavy duties when Z_p and Z_s are composed of passive elements only.

For exact measurements it should be required that the voltage across each divider element is proportional to the impedance of that element. Therefore no current may flow through the capacitive coupling between the resistor-branch and the capacitor-branch.
Thus at each fictive horizontal cut at a position i dividing the branches in parts R_{h_i}, R_{g_i} and C_{h_i}, C_{g_i} (fig. 2.2.) the general requirement

$$R_{h_i} C_{h_i} = R_{g_i} C_{g_i} = \text{constant} \qquad (2.6.)$$

must be fulfilled.

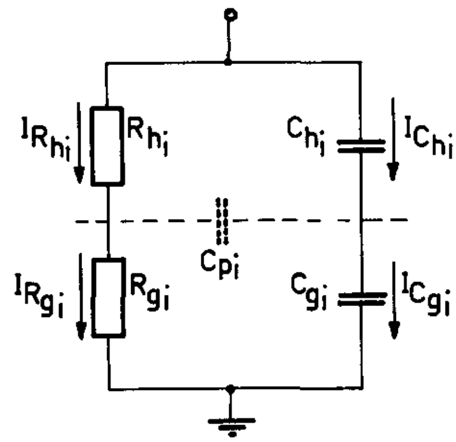

fig. 2.2.

To attain this a number of compensation methods are developed which are of great importance for dividers of large physical dimensions (extra high voltages), as applied in surge voltage testing. In voltage dividers for current zero investigations the influence of parasitic capacitive coupling can be held within acceptable limits by compact construction and by choice of a high value C_p in respect to the parallel capacitances.
Condition (2.6.) holds specially for the primary and secundary impedances:

$$R_pC_p = R_sC_s \tag{2.7.}$$

In the next section it will be showed that for certain measurements this condition (2.7.) must be fulfilled with an extremely high accuracy. Fluctuations in resistances and capacitances may occur owing to variations in temperature and humidity or to aging.
Therefore it may be preferable to fit a correction device for R_s and C_s at the secundary side. Furthermore space charges and partial discharges on primary elements should be prevent because they can cause a relatively large decrease of R_p.

2.3 Measuring errors due to inaccuracy of the RC-ratio

By closing Switch S in the circuit of fig. 2.3. a step voltage U_p is applied to a mixed divider. This leads to a secundary response

$$U_s = \frac{R_s}{R_p + R_s} U_p + \varepsilon^{-t/\tau}\left\{\frac{R_pC_p - R_sC_s}{(R_p+R_s)(C_p+C_s)} U_p - \frac{R_pC_p - R_sC_s}{R_s(C_p + C_s)} V_{c_s}\right\} \tag{2.8.}$$

with

$$\tau = \frac{R_pR_s}{R_p + R_s}(C_p + C_s) \tag{2.9.}$$

$$\frac{V_{c_p}}{V_{c_s}} = \frac{R_p}{R_s} \tag{2.10.}$$

V_{c_p}, V_{c_s} = voltage across C_p, C_s at $t \leqq 0$

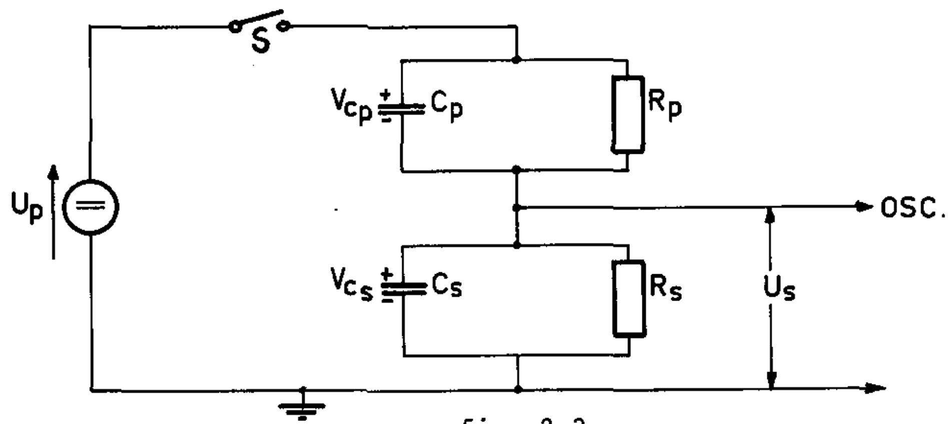

fig. 2.3.

Condition (2.10.) expresses that the voltage division before closing S was governed by the resistors.

If $V_{c_s} = V_{c_p} = 0$ equation (2.8.) indicates that the voltage division immediately after closing is controlled by the capacitors

$$U_s(0) = \frac{C_p}{C_p + C_s} U_p \qquad (2.11.)$$

Some time afterwards the stationary state is reached and the voltage ratio is determined by the resistors again

$$U_s(\infty) = \frac{R_s}{R_p + R_s} U_p \qquad (2.12.)$$

The transition occurs with a time constant $\tau \approx R_s C_s$. This time constant is generally fairly large, e.g. of the order of 10^{-2}s.
When the ideal divider-ratio A is defined by

$$A = \frac{R_p + R_s}{R_s} \qquad (2.13.)$$

the initial error shows to be

$$U_s(\infty) - U(0) = \frac{U_p}{A} \cdot \frac{R_s C_s - R_p C_p}{R_s (C_p + C_s)} = \frac{U_p}{A} \cdot \Delta \qquad (2.14.)$$

Due to $C_p \ll C_s$, Δ equals the relative error of the RC-ratio in good approximation

$$\Delta \approx 1 - \frac{R_p C_p}{R_s C_s} \qquad (2.15.)$$

Thus a small inaccuracy Δ in the RC-ratio will yield a similar small error in the measurement provided no initial charge on C_p and C_s exists (see fig. 2.4.).

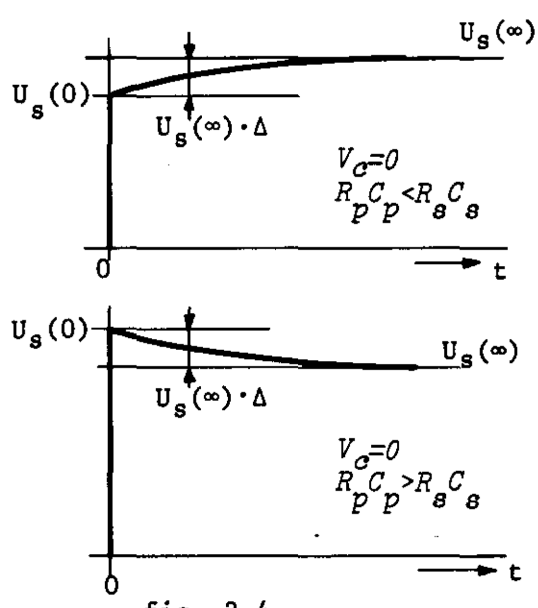

fig. 2.4.

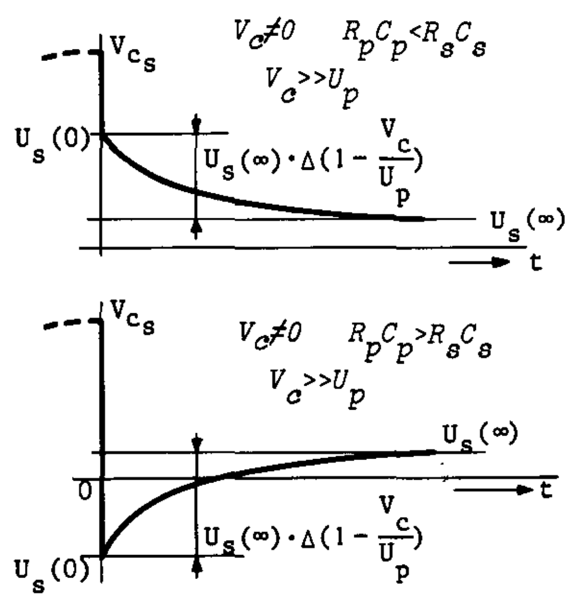

fig. 2.5.

However for $V_{c_s} \neq 0$ the initial secondary voltage equals

$$U_s(0) = U_p \left\{ \frac{C_p}{C_p + C_s} + \Delta \frac{V_{c_s}}{U_p} \right\} \qquad (2.16.)$$

For the stationary state eq. (2.12.) naturally remains valid so the initial error is now given by

$$U_s(\infty) - U(0) = \Delta \frac{U_p}{A} - \Delta V_{c_s} = \Delta \frac{U_p}{A} \left(1 - \frac{V_c}{U_p}\right) \qquad (2.17.)$$

with

$$V_c = V_{c_p} + V_{c_s}$$

From this appears that a small error Δ may give rise to an extensive error in the measured voltage in the case $V_c \gg U_p$ i.e. when $V_{c_s} \gg U_s(\infty)$, (see fig. 2.5.).

This condition may arise at a reignition in a circuitbreaker after a relatively slow increasing restriking voltage. (For phenomena occurring within a time $\ll \tau$ the divider keeps its capacitive character naturally.)

A mixed divider with $\Delta \neq 0$ gives rise to similar complications when used in investigations of transient phenomena in gas-discharges (e.g. in breaker models). Fig. 2.6. shows a possible circuit.

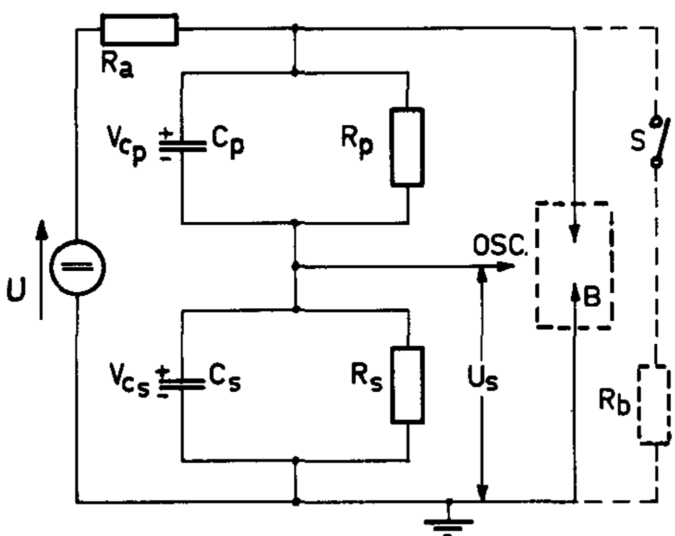

fig. 2.6.

Here B represents the chamber in which a discharge occurs due to an ignition, R_a is the current limiting resistor. For a first approximation the ignition may be represented by closing S in fig. 2.6., thus bringing a steady-state discharge resistance R_b into the circuit.

Now the solution for the secundary voltage U_s has a rather complicated form, which may be simplified by introducing

$$R_pC_p \approx R_sC_s$$
$$R_p \gg R_s \gg R_b$$
$$R_a \gg R_b$$

and the initial conditions $V_c = U$ and $V_{c_s} = \frac{R_s}{R_p + R_s} U = \frac{U}{A}$ for $t \leq 0$

Then

$$U_s = (K_1 + K_2 . \varepsilon^{-t/\tau_1} + K_3 . \varepsilon^{-t/\tau_2}) U \qquad (2.19.)$$

where

$$K_1 = \frac{1}{A} \cdot \frac{R_b(R_p + R_s)}{R_1(R_b + R_p + R_s) + R_b(R_p + R_s)} \approx \frac{1}{A} \cdot \frac{R_b}{R_a + R_b} \approx \frac{R_b}{AR_a}$$

$$K_2 \approx \left(1 - \frac{R_pC_p}{R_sC_s}\right) \cdot \frac{1}{A} = \frac{\Delta}{A}$$

$$K_3 \approx \frac{C_p}{C_p + C_s} \approx \frac{1}{A}$$

$$\tau_1 = R_sC_s \qquad \tau_2 = R_bC_p$$

With

$$U_s(\infty) \approx \frac{U}{A} \cdot \frac{R_b}{R_a}$$

equation (2.18.) yields

$$U_s \approx U_s(\infty) \left\{ 1 + \Delta \frac{R_a}{R_b} \cdot \varepsilon^{-t/\tau_1} + \frac{R_a}{R_b} \cdot \varepsilon^{-t/\tau_2} \right\} \qquad (2.19.)$$

So also for this situation a small inaccuracy Δ *may induce a large measuring error* $\Delta \cdot \frac{R_a}{R_b}$, *diminishing with the fairly large time constant* τ_1.

The third term of (2.19.) shows another effect which may lead to an incorrect interpretation of results in a certain type of measurements. Even for $\Delta = 0$ the stationary state will not immediately be reached because the initial charge of C_p and C_s transfers to a new state governed by the time constant τ_2. During this transition the voltage across the discharge chamber B decreases from the "open" voltage U to the discharge voltage after the reignition, being approximately $\frac{R_b}{R_a}$ U. When R_b and C_p are relatively large (e.g. $R_b \approx 5.10^3$ and $C_p \approx 200$ pF), τ_2 may be of the order of 1 μs. This time may be of the same order or even larger than the time required for the gas discharge under investigation to reach the (quasi-) steady state after reignition.

A large value R_b may be due to a large length, intensive cooling or a specific character of the gasdischarge. A large value C_p is not necessarily due to the divider itself. The total inherent ground capacitance of the discharge chamber and its direct vicinity, (C_B), participates in the adjustment because prior to the ignition also this parallel capacitor was charged to the initial voltage $V_c = U$. Taking this in account the time constant τ_2 changes to

$$\tau_2 = R_b(C_p + C_B) \qquad (2.20.)$$

In case the selfinductances of the capacitors and their connections to the discharge chamber are not negligible small, i.e. for

$$L_B > \frac{1}{4} R_b^2 C$$

it should be observed that the last term of (2.19.):

$$\frac{R_a}{R_b} \cdot \varepsilon^{-t/\tau_2} \qquad (2.21.)$$

has to be replaced by a damped oscillation

$$\frac{R_a}{R_b} \cdot \varepsilon^{-t/\tau_3} \cos \omega t \qquad (2.22.)$$

where $\tau_3 = \frac{L_B}{2 R_B}$, $\omega = (L_B C)^{-\frac{1}{2}}$ and $C = C_B + \frac{C_p C_s}{C_p + C_s} \approx C_B + C_p$

L_B is a substitution for the inductances of C_B, C_p, C_s , R_b and their mutual connections.

So the term (2.21.) or (2.22.) respectively indicates the way in which the behaviour of the gasdischarge under investigation is influenced by the electric circuit itself.

2.4 The inherent selfinductances

In the high frequency range the capacitive reactances of the primary and secundary divider side will usually be very much smaller than the resistances. Therefore the equivalent diagram for high frequencies takes the form of two LC-circuits in series, corresponding fig. 2.7.

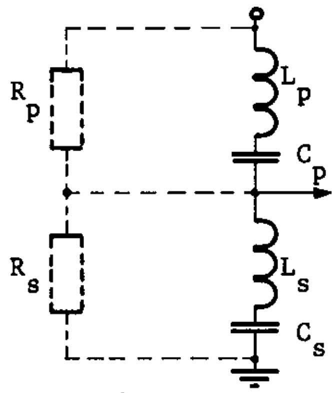

fig. 2.7.

Here L_p and L_s are the inherent selfinductances of the primary and secundary divider side. Because the resonant frequencies of the two series circuits determine the limitation of the frequency range (section 2.6) L_p and L_s are to be kept as small as possible.

Stretched conductors forming part of an electric circuit have a selfinductance per unit length which can be expressed by

$$L \approx 0{,}2 \,.\, \ln \alpha \cdot \frac{b}{d} \quad \mu H/m \qquad (2.23.)$$

Here b = mean width of the circuit (see fig. 2.8.).
d = diameter of the conductor
α = a constant depending on the shape of the circuit.

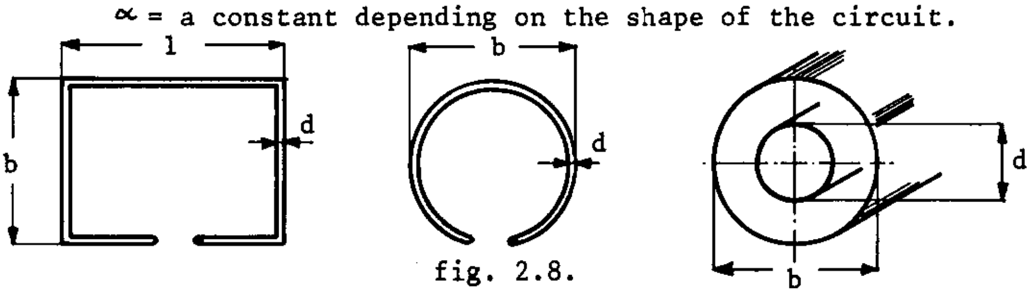

fig. 2.8.

For rectangular circuits α varies between 1,2 (for l=b, i.e. for a square circuit, see fig. 2.8.) and 2,6 (for $l \gg b$, i.e. for two long parallel conductors). For a circular circuit $\alpha \approx 1{,}1$ and for a coaxial system $\alpha \approx 1{,}3$. For high frequencies α may decrease to ≈ 30 % due to skin effect.

From these values it appears that the selfinductance of the measuring devices does not strongly depend on the shape of the circuits or the cross-section of the conductors because α is found under the log-sign.
Eq. (2.23.) provides values between 0,5 and 1,3 μH/m for the usual electrical circuits. (However coaxial cables have selfinductances which are considerably smaller, usually 0,25 to 0,6 μH per meter cable length.)

Because the inductances are proportional to the conductor length a compact construction is most important. Furthermore the connections to the high voltage circuit and to the central grounding point of the circuits must be kept as short as possible. A large diameter of the primary divider elements has only a relative advantage, especially as this give rise to enlargement of the parasitic capacitances. At the secundary side all possible means for small inductance must be employed because of the large capacitance C_s.
For a divider with C_p = 25 pF and L_p = 2 μH the primary resonant frequency is about 22 Mc/s. When the divider ratio A = 1000 an inductance L_s = 0.002 μH gives already the same secundary resonant frequency. Therefore a coaxial framing must be chosen. In that case the relatively small grounding capacitances are fixed and incorporated in the secundary capacitance C_s. Further reduction of L_s is obtained by dividing C_s in a number of parallel capacitances each having a small inherent selfinductance.

2.5 The measuring cable

For the frequency range under consideration the usual coaxial measuring cables may be considered as ideal (no-loss) transmission lines. The Laplace-transformed voltage and current equations at a certain position x of such a cable situated in a diagram according fig. 2.9. are *)

$$U(x,p) = U_s(p) \frac{Z}{Z_1 + Z} \cdot \frac{\varepsilon^{-px/v} + r_l \varepsilon^{-p(2l - x)/v}}{1 - r_1 r_l \varepsilon^{-2pl/v}} \qquad (2.24.)$$

$$I(x,p) = U_s(p) \frac{1}{Z_1 + Z} \cdot \frac{\varepsilon^{-px/v} - r_l \varepsilon^{-p(2l - x)/v}}{1 - r_1 r_l \varepsilon^{-2pl/v}} \qquad (2.25.)$$

with $Z = \sqrt{\frac{L_c}{C_c}}$ $v = \frac{1}{\sqrt{L_c C_c}}$

$r_1 = \frac{Z_1 - Z}{Z_1 + Z}$ $r_l = \frac{Z_l - Z}{Z_l + Z}$

p = Laplace operator
$U_s(p)$ = Transformed source voltage
Z_1 = Impedance operator of the source side
Z_l = Impedance operator of the load
Z = Characteristic impedance being a pure resistor for an ideal cable
r_1, r_l = Reflection coefficients
v = Velocity of propagation
L_c = Inductance per unit length of the cable
C_c = Capacitance per unit length of the cable
l = Cable length

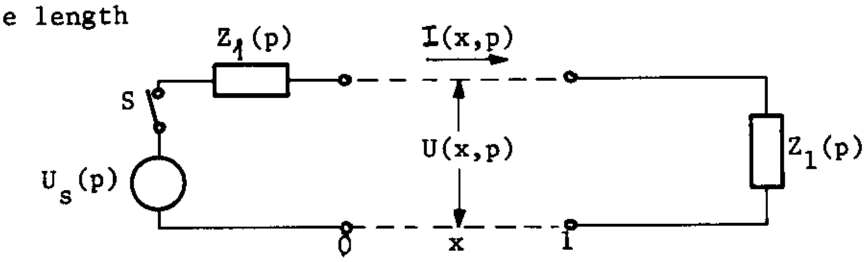

fig. 2.9.

*) See e.g. J.P. Schouten: "Operatorenrechnung", Springer Berlin (1961) (a book) p. 61 ff.

By putting x=l in eq. (2.24.) the voltage at the end of the cable is found

$$U_1(p) = U_s(p) \frac{Z}{Z_1 + Z} \frac{(1+r_1)\varepsilon^{-pl/v}}{1-r_1 r_1 \varepsilon^{-2pl/v}} \tag{2.26.}$$

Inverse transformation of this equation yields solutions which generally represent an infinity number of travelling waves.
These waves propagate in the cable with velocity v and are reflected at both terminals of the cable.
Two special cases are of interest for the voltage measuring techniques.

1. Characteristic load $Z_1 = Z$

Then $r_1 = 0$ (no reflection at the cable end) and eq. (2.26.) results in

$$U_1(p) = U_s(p) \frac{Z}{Z_1 + Z} \cdot \varepsilon^{-pl/v} \tag{2.27.}$$

Assuming Z_1 is a pure resistance R_1, inverse transformation of this formula yields:

$$U_1(t) = \frac{Z}{R_1 + Z} \cdot U_s(t-l/v) \tag{2.28.}$$

So in case of characteristic termination at the end the voltage shape is undisturbed and the amplitude does not depend on frequency. The applied signal arises at the end after a certain and constant time lag and is there completely absorbed.

2. Characteristic impedance at the source side $Z_1 = Z$

Now $r_1 = 0$ and equation (2.26.) yields

$$U_1(p) = U_s(p) \cdot \tfrac{1}{2}(1 + r_1)\varepsilon^{-pl/v} \tag{2.29.}$$

If $Z_1 = R_1$ is a pure resistance

$$U_1(t) = \tfrac{1}{2}\,(1 + r_1)\,U_s(t - l/v) \tag{2.30.}$$

and here again the voltage shape is undisturbed.
However, if $|Z_1| \gg Z$ then $r_1 \approx 1$ and the character of Z_1 has only little influence on the voltage at the end.

$$U_1(t) \approx U_s(t - l/v) \tag{2.31.}$$

This result is of great importance to the voltage dividers here considered. It shows that (in a certain limited frequency range) the requirement of true reproduction at the end of the cable can fairly be fulfilled by characteristic termination at the source side. In this situation the travelling waves are fully reflected at the end by the relatively high impedance Z_1 and consequently fully absorbed at the input by the characteristic impedance $Z_1 = Z$.

In practice the conditions $r_1 = 0$ or $r_1 = 0$ can never be perfectly fulfilled for all frequencies because of the resistance and leakage conductance of the cables presumed to be ideal and the inductance and capacitance of the characteristic resistor. The interference by those effects can be kept small for a relatively long frequency range by the use of short cable lengths. Maximum measuring accuracy can be gained by combining both cases 1 and 2, so for $Z_1 = Z_1 = Z$ as is well known from surge current measuring techniques.

2.6 The complete measuring circuit with a mixed divider

In the complete measuring circuit both the cable and the oscillograph input impedance are in parallel to the secundary divider side according fig. 2.10. Then condition (2.7.) $R_pC_p = R_sC_s$ must be extended to

$$R_pC_p = \frac{R_sR_1}{R_s + R_1}(C_s + lC_c + C_1) = R_s'C_s' \qquad (2.32.)$$

(Here is assumed $Z_1 \gg Z_1$)

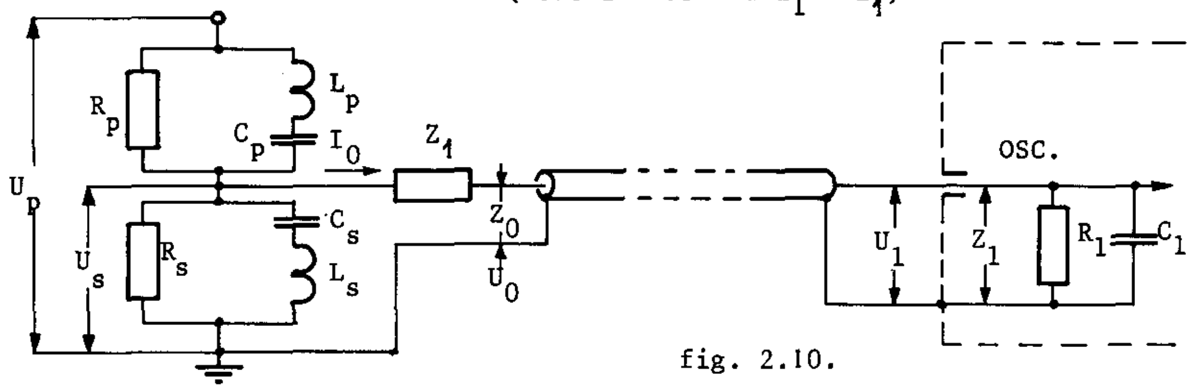

fig. 2.10.

With condition (2.32.) fulfilled the ideal divider ratio is represented by

$$A \equiv \frac{Z_p(\text{ideal}) + Z_s(\text{ideal})}{Z_s(\text{ideal})} = \frac{R_p + R_s'}{R_s'} = \frac{C_s' + C_p}{C_p} \qquad (2.33.)$$

At high frequencies the parasitic primary and secundary selfinductances L_p and L_s, the cable impedance and the input capacitance C_1 will disturbe the dividing ratio.

For the dividers here considered characteristic termination at the cable end is usually not possible for practical reasons. Since the current through the divider must be small with respect to the current through the test device under all transient circumstances to be recorded the primary resistance R_p must be very large. A divider current of 1 to 10 mA requires R_p to be of the order of 10^5 to $10^6\,\Omega$ per kV. C_p must be small in respect of the inherent parallel capacitance of the test device and its direct vicinity and therefore must be limited to some tens of pF's. On the other hand C_p may not be chosen too small due to the influence of the parasitic capacitances of C_p and R_p.

Due to the value of the order of $100\,\Omega$ of the characteristic impedance of coaxial cables it is for characteristic termination at the end required that R_s' should have at least the same value. This leads to $A \geqq 10^5$ and a secondary capacitance $C_p \geqq 10^6$ pF. This value is so high that in general the secondary resonance frequency does not occur beyond the required frequency range of the divider. Moreover it is doubtful whether calibration of a divider with such a high ratio over a wide frequency spectrum may be possible.

Characteristic termination at the cable input is less difficult. Where we apply an oscillograph with small input capacitance C_1 and a large input resistance R_1, the condition $|Z_1| \gg Z$ can be fulfilled over an extended frequency range. If C_s is sufficiently large moreover $|Z_s| \ll Z_1$ will be valid for high frequencies and $Z_1 = Z$ can be chosen. In this case equation (2.31.) shows in good approximation $|U_1| = |U_s|$.

The frequency characteristic, eq. (2.3.) is then given by

$$D = A \frac{|U_1|}{|U_p|} = A \frac{|U_s|}{|U_p|} = A \cdot \frac{|Z_s'|}{|Z_p + Z_s'|} \qquad (2.34.)$$

with

$$Z_s' = \frac{Z_s(Z + Z_o)}{Z_s + Z + Z_o} \qquad (2.35.)$$

$$Z_o = \frac{U_o}{I_o} = \text{input impedance of the cable} \qquad (2.36.)$$

The well known stationary solutions for the no-loss transmission line equations in terms of the voltages and currents at the input and the output side are

$$U_o = U_1 \sin\varphi + jI_1 \cos\varphi \qquad (2.37.)$$

$$I_o = I_1 \sin\varphi + j \frac{U_1}{Z} \cos\varphi \qquad (2.38.)$$

with $\varphi = \beta l$ and $\beta = \omega\sqrt{L_c C_c}$ = phase constant, (see e.g. lit. on page 10).

For low frequencies (2.35.) and (2.36.) can be simplified to $Z_o \approx R_1$ and $Z_s' \approx Z_s$, so (2.33.) with (2.34.) show D = 1.

For high frequencies $Z_1 = (j\omega C_1)^{-1}$ and from (2.37.) and 2.38.) can be deducted

$$Z_o = - jZ \text{ cotg } \{\varphi + \text{arctg } (\varphi \cdot \frac{C_1}{lC_c})\} \qquad (2.39.)$$

Generally $(C_1/lC_c) < 0.3$ so for the frequency range here considered (up to $\varphi = \pi/2$ approximately)

$$\bar{Z}_o \approx - jZ \text{ cotg } \varphi(1 + \frac{C_1}{lC_c}) = - jZ \text{ cotg } \psi \qquad (2.40.)$$

with $\psi = \varphi(1 + \frac{C_1}{lC_c})$

Comparison of this result with the impedance of the unloaded cable (I_1 = 0 in (2.37.) and (2.38.))

$$Z_o = - jZ \text{ cotg } \varphi \qquad (2.41.)$$

shows that the capacitive load of the oscillograph input in first approximation acts as an extension of the cable with C_1/C_c (meters).

For $\varphi(1 + \frac{C_1}{lC_c}) \rightarrow \pi/2$ we find $Z_o \rightarrow o$. Becuase $|Z_s| \ll Z$ equation (2.35.) still yields $Z_s' \approx Z_s$ and (2.34.) results in

$$D \approx A \frac{|Z_s|}{|Z_p + Z_s|} \qquad (2.42.)$$

However $Z_s = \frac{1}{j\omega C_s} + j\omega L_s$

and $Z_p = \frac{1}{j\omega C_p} + j\omega L_p$ for these high frequencies.

Therefore the frequency characteristic shows 2 extrema namely (see fig. 2.11.):

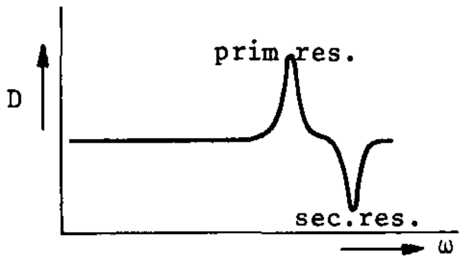

fig. 2.11.

1) a maximum for $\omega(L_p + L_s) = \frac{1}{\omega(C_p + C_s)}$ ("primary resonance")
2) a minimum for $\omega L_s = \frac{1}{\omega C_s}$ ("secondary resonance")

These extrema limit the useful frequency range in the upper range. A high primary resonance peak leads easily to an incorrect interpretation of results and even to zero-line drift on the records. Therefore it seems expedient to bring both extrema together as much as possible, so that they can balance each other. An ideal solution is usually not possible because the selfinductances and therefore the resonant frequencies depend somewhat on the place of the divider in the main circuit.

3. CONCLUSIONS IN RESPECT OF DIVIDER CONSTRUCTIONS

Below the most important requirements for the construction of mixed voltage dividers for voltages up to 100 kV and frequencies from d.c. to $\geqslant 20$ Mc/s are shortly summarized. Because some of these requirements are in contradiction to each other one will always have to seek for the most ideal compromise.

1. The RC-products of the primary and secondary divider side should be equal, so

$$R_pC_p = R_sC_s$$

In case a low voltage should be measured immediately following a much higher voltage of a realtively long duration (at least of the order of R_sC_s) this requirement must be fulfilled with an extremely high accuracy.

2. The influence of the parasitic capacitances must be reduced by a compact construction and a relatively high value for C_p.

3. To prevent interference of the transient phenomena to be measured by the divider itself R_p must be very large (10^5 to $10^6\,\Omega$ per kV) and C_p must be relatively small (10 to 50 pF).

4. Since characteristic termination at the end is not possible (because of conclusions 3 and 7), a characteristic series-resistor Z must be employed at the input side of the coaxial measuring cable.
To prevent disturbing reflections a high value C_s and a short cable length must be chosen.

5. To obtain an acceptable accuracy even for the highest frequencies $Z \ll (\omega C_1)^{-1}$ should be fulfilled always. Because C_1 is determined by the input capacitance of the oscillograph, a cable with a small characteristic impedance Z is preferable.

6. The high side of the frequency range is limited by the resonances of the primary and the secundary divider circuits.
   Therefore the inherent selfinductances of the divider elements must be kept as small as possible. C_p and C_s must be chosen as small as is acceptable with respect to conclusions 2 and 4.

7. The choice of a very large divider ratio A is not possible because a large C_s is not allowed (conclusion 6). Moreover an extremely high A would give rise to extensive difficulties in the calibration. Usually the order of magnitude of the ratio must be limited to 1000.