Finite wordlength effects in digital filters : a review

tu/e
Research Report
ISSN 0167-9708
Coden: TEUEDE

Eindhoven
University of Technology
Netherlands

Faculty of Electrical Engineering

Finite Wordlength in Digital Filters: A Review

by
H.J. Butterweck
J.H.F. Ritzerfeld
M.J. Werter

EUT Report 88-E-205
ISBN 90-6144-205-2
October 1988

Eindhoven University of Technology Research Reports

EINDHOVEN UNIVERSITY OF TECHNOLOGY

Faculty of Electrical Engineering
Eindhoven The Netherlands

ISSN 0167- 9708 Coden: TEUEDE

FINITE WORDLENGTH EFFECTS IN DIGITAL FILTERS:
A review

by

H.J. Butterweck
J.H.F. Ritzerfeld
M.J. Werter

EUT Report 88-E-205
ISBN 90-6144-205-2

Eindhoven
October 1988

CIP-GEGEVENS KONINKLIJKE BIBLIOTHEEK, DEN HAAG

Butterweck, H.J.

Finite wordlength effects in digital filters: a review /
by H.J. Butterweck, J.H.F. Ritzerfeld, M.J. Werter. -
Eindhoven: Eindhoven University of Technology, Faculty of
Electrical Engineering. - Fig. - (EUT report, ISSN 0167-9708;
88-E-205)
Met bibliografie, index, reg.
ISBN 90-6144-205-2
SISO 663.12 UDC 621.372.54.037.37.018.783(01) NUGI 832
Trefw.: digitale filters; bibliografieën.

ABSTRACT

A review is presented of recent work on quantization and overflow effects in digital filters. These unwanted non-linear phenomena include parasitic oscillations (limit cycles) and quantization noise. Modern stabilization methods and noise optimization strategies are discussed. A comprehensive bibliography contains the relevant original contributions dealing with the analysis of various finite wordlength effects and measures to reduce or avoid them.

Butterweck, H.J. and J.H.F. Ritzerfeld, M.J. Werter
FINITE WORDLENGTH EFFECTS IN DIGITAL FILTERS: A review.
Faculty of Electrical Engineering, Eindhoven University of Technology, The Netherlands, 1988.
EUT Report 88-E-205

The authors are with the

Group Electromagnetism and Circuit Theory,
Faculty of Electrical Engineering,
Eindhoven University of Technology,
P.O. Box 513,
5600 MB EINDHOVEN,
The Netherlands

CONTENTS

Finite wordlength effects in digital filters - a review

I. Introduction

In most applications signal processing in digital filters is intended to be performed in the form of *linear* operations, which for the important class of time-invariant systems are of the convolution type. The digital encoding of the various signals, however, implies that in general the required linearity can be achieved only to a certain degree. Fortunately, the deviation from the linear behaviour can be made arbitrarily small through choosing sufficiently long binary words. Yet there remain typical *finite-wordlength effects* that cause an actual digital filter to behave as a (weakly) *nonlinear* system.

Contrary to the finite wordlength of the signals to be processed the finite wordlength of the *filter coefficients* does not affect the linearity of the filter behaviour. This effect only amounts to restrictions on the linear filter characteristics, resulting in discrete grids of pole-zero patterns. Once a filter design with some combination of permitted coefficients[1] meets the required specifications (with regard to amplitude and phase characteristics) the actual filter performance differs from that predicted by linear theory only as to the previously mentioned nonlinear finite-wordlength effects. These effects, which divide into those due to "*signal quantization*" and those due to "*overflow*", form the subject matter of the present paper. Our interest in coefficient quantization is only indirect and stems from some relation between the sensitivity of the filter characteristics to parameter variations on the one hand and the generation of quantization noise due to signal quantization on the other. This relation states that in general low-sensitivity structures (allowing short coefficient words) are distinguished by low noise levels [11]-[17].

The majority of quantization and overflow phenomena can be derived from a simple model, in which appropriate nonlinear, memoryless components (NL) are inserted into an otherwise linear, idealized digital system.

[1] In the binary format a coefficient can only assume a value $p/2^n$ with $p \in Z$ and $n \in N$.

A typical NL characteristic is shown in Fig. 1.1; it is characterized by a fixed-point number representation (with 3 bits yielding 2^3 different signal levels), rounding R as quantization, and saturation as overflow correction.

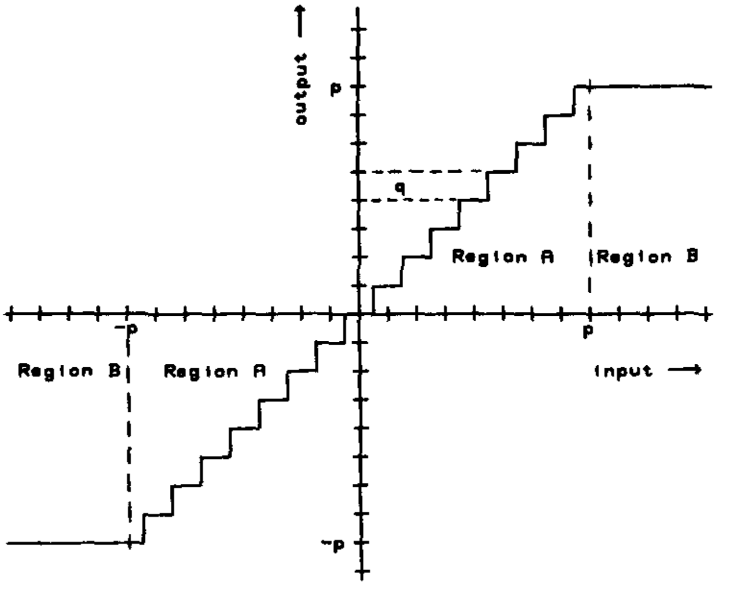

Fig. 1.1. Characteristic of a finite-wordlength nonlinearity.

Also other combinations can be conceived and will be studied in due course. Common to all these nonlinear characteristics are the following properties: a) For inputs whose magnitudes are smaller than p (Region A) the output is close to the input; the difference is that the former is machine-representable, while the latter is unrestricted; b) for all inputs whose magnitude is greater than p (Region B) the magnitude of the output cannot exceed p. Region A models quantization after multiplication by a constant factor, whereas Region B represents the correction required in connection with adder overflow.

The question where the various nonlinearities NL have to be inserted into the linear network, can be straightforwardly answered for any structure and its pertinent computation scheme. Care must be taken that every *feedback loop* must contain at least one NL element to avoid ever-increasing wordlengths. FIR filters without loops do not strictly require such elements; quantization and overflow correction is, however, often applied for intentional wordlength limitation. In any case, the AD-converter preceding a digital filter ultimately causes every digitally realized system with analogue input/output terminals to exhibit more or less nonlinear signal distortion.

In a common approximation, *quantization* and *overflow* are not only conceptually decoupled, but also analytically treated as *independent effects*. This implies that for large signals the fine quantization structure is neglected and the Region A-part of the nonlinear characteristic is replaced by a 45° straight line. Apparently this approximation can only be justified if the total number of quantization steps is large enough or, in other words, the binary words are sufficiently long. Even for this extreme case several authors have queried the validity of the decoupling approximation [18]-[22]. Indeed, there are overflow effects that can only be properly understood in connection with the quantization fine structure. As an example, consider a filter initially in the zero state and then excited by a short, strong pulse such that overflow occurs at some point inside the filter. Assume that the idealized (quantization-free) filter asymptotically returns to equilibrium (zero state), which implies "overflow stability" (cf. Section III).
Apparently, the filter has "forgotten" the overflow after a sufficiently long time. With quantization, the situation is not as simple: before excitation, the filter might (necessarily) oscillate in a limit cycle mode, while after overflow the filter does not recover to the zero state but again enters a limit cycle. The mode of oscillation can, however, be completely different from the former one. Because the filter never forgets the overflow, it has apparently to be considered as unstable.

Recently, chaotic overflow oscillations have been observed [23]. Also in that case the quantization has been neglected in the first instance. Taking the fine structure of the NL characteristic into account, the filters under consideration become finite-state machines with strictly periodic (non- chaotic) oscillations.

These examples belong to a small group of exceptional phenomena where the decoupling assumption fails even for a large dynamic range (long binary words). For most effects to be treated in this paper it is valid with sufficient accuracy.

The simple NL-model with a characteristic like that of Fig. 1.1 does not apply to all finite-wordlength mechanisms. This is particularly true for all types of "*controlled rounding*" (CR), in which the treatment of the least significant bit is not controlled by the signal to be quantized but by another signal. So it is often devised that an external, mostly stochastic signal controls the quantization or that an internal signal within the filter performs that task. More complicated schemes leave the decision about the rounding direction (upwards or downwards) to more than one control signal, one of which may be the signal to be quantized. All these methods are in current use to suppress quantization limit cycles and will be discussed in Section IV. We note that also a controlled overflow correction is conceivable, although attempts in this direction have not yet been reported.

II. Quantization and overflow characteristics

Returning to the NL model we have still to review other *characteristics for quantization and overflow* than that shown in Fig. 1.1. Although less frequently used than its fixed-point counterpart, a *floating-point realization* of a digital filter often deserves consideration. Also for this arithmetic finite-wordlength effects have to be reckoned with, including limit cycles [24] and quantization noise [25]. A completely different design approach of a more recent date makes use of *"residue arithmetic"*, (a number-theoretical tool). The associated finite-wordlength effects have not yet drawn too much attention [26]-[29].

For conventional fixed-point arithmetic we can mainly choose from three *quantization schemes* with specific individual merits: (a) rounding R, (b) magnitude truncation MT, (c) value truncation VT. Each method is characterized by a peculiar instruction rule concerning the direction of quantization (upwards or downwards): (a) for R towards the nearest machine-representable number (b) for MT towards zero (c) for VT always downwards. Let x and Q(x) denote the unquantized and quantized number, respectively, and let further $\varepsilon(x) = Q(x) - x$ denote the "quantization error", and q the quantization step size, then we have

$$\begin{aligned} |\varepsilon_R(x)| &\le q/2 \\ |\varepsilon_{MT}(x)| &< q \\ |\varepsilon_{VT}(x)| &< q \end{aligned} \qquad (2.1)$$

which admits the conclusion that rounding is the most attractive form of quantization with regard to the average error signal amplitude. The specific advantage of magnitude truncation lies in its inherent capability of limit cycle suppression (cf. Section IV), that follows from energy considerations in connection with the basic MT property $|Q_{MT}(x)| \le |x|$. Finally, value truncation is the natural quantization method for a two's complement arithmetic. Its formal treatment is similar to that of rounding due to the simple relation

$$Q_{VT}(x) = Q_R(x - q/2) \qquad (2.2)$$

stating that VT yields the same results as R after adding the constant signal q/2 to the unquantized signal.

Comparing the two main quantization schemes "*rounding*" and "*magnitude truncation*" we observe fundamental differences in their nonlinear signal processing behaviour, which follow from their error characteristics $\varepsilon(x)$, cf. Fig. 2.1.

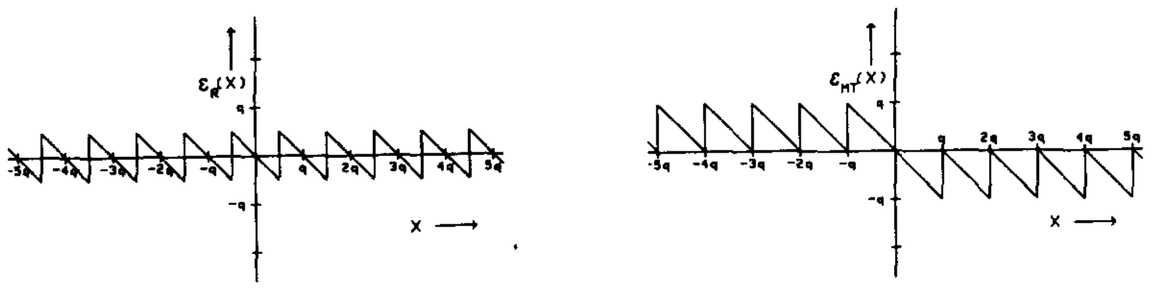

Fig. 2.1. Quantization error $\varepsilon(x)$ for rounding R and magnitude truncation MT.

It is true that both characteristics are strictly deterministic, i.e. with every x a unique error signal $\varepsilon(x)$ is associated. Nevertheless, we are inclined to attribute "quasi-random" features to the rounding characteristics in the following sense. If x(k) is assumed[2] to represent a stationary random process characterized by a probability density function P(x) and an autocorrelation function $s_{xx}(m) = E\{x(k)\, x(k-m)\}$ this process is transformed by the rounding error characteristic into another process $\varepsilon(k)$, which "almost always" has white-noise character with $s_{\varepsilon\varepsilon}(m) = q^2/12\ \delta(m)$ as well as a *uniform* probability distribution in the interval $-\frac{q}{2} \leq \varepsilon \leq \frac{q}{2}$. This property is the basis for the well-established *white-noise model* of the *rounding error* [30], which we also adopt in this paper. The reliability of this model improves with increasing level of the signal x(k) and with increasing spread of its power spectrum. It *fails* completely if x(k) varies *periodically*, associated with a line power spectrum. Then also $\varepsilon(k)$ is periodic and, hence, not noisy. Such a periodicity applies e.g. when a recursive filter oscillates in a *limit cycle* mode (cf. Section IV).

---

[2] The symbol k denotes the discrete time variable.

To analyze the corresponding error characteristic for *magnitude truncation* we first split it into two parts according to Fig. 2.2. The first part resembles the $\varepsilon_R(x)$ characteristic and will henceforth be referred to as the "quasi-rounding" component $\varepsilon_{QR}(x)$ of magnitude truncation. The white-noise model of the rounding error likewise applies to quasi-rounding, so that R and MT essentially differ in the second part of the MT error characteristic, the so-called "sign-part" $\varepsilon_{SGN}(x)$, (cf. Fig. 2.2).

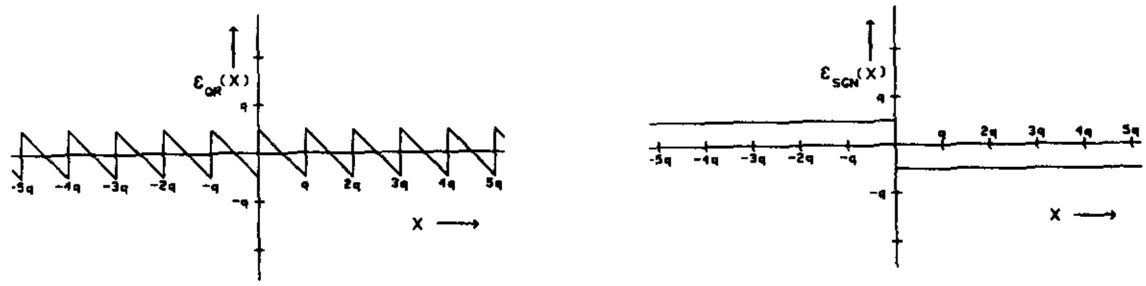

Fig. 2.2. Decomposition of the quantization error for magnitude truncation into a quasi-rounding and a sign part.

As to their signal-processing behaviour, the quasi-rounding and the sign part are basically dissimilar. While the former part lends itself to a modelling as an *additive* (white-)*noise source**, the latter remains an essentially nonlinear component whose output is strongly correlated with the input signal. In some applications a straight line through the origin with an appropriate negative slope can be advantageously split off from $\varepsilon_{SGN}(x)$, resulting in slight modifications of the filter coefficients and, ultimately, in effects of *detuning* (including Q-factor modifications). Apparently such detuning is level-dependent and decreases with increasing signal amplitude. What remains is a pure *nonlinear signal degradation*, that leads to a number of interference phenomena (including crosstalk [31]-[34]) and that has to be interpret ordinary distortion in the audio region.

*If the system contains more than one quantizer, the model is extended such that the various noise sources are uncorrelated.

While quantization has to be accepted as an unavoidable concomitant of any digital signal processing, the situation is less constraining with respect to *overflow*. Obviously, overflow can be completely avoided through using sufficiently small input signals: For a given impulse response (considered between the input terminal and a node of potential overflow) and for a prescribed overflow level an upper bound for the input signal can easily be derived [35].

Nevertheless, it is common practice to accept a small risk of overflow, occurring for very unfavourably chosen excitations. Thus the dynamic range of a filter is better exploited, ultimately resulting in a lower quantization noise level. This mild "scaling policy" consciously tolerates a small nonzero probability of overflow. So, infrequent overflows and accompanying interruptions of normal operation are accepted under the obvious tacit assumption that after each overflow the normal operation recovers, preferably with high speed.

The required recovery automatically leads to the paramount problem of *overflow stability*. To discuss this item we assume that the underlying idealized, linear system is stable and that quantization can be neglected (decoupling assumption). Then the stability problem is attacked in two steps, (a) under zero-input conditions, (b) under nonzero-input conditions. Stability according to (a) is defined as absence of spontaneous oscillations, particularly of periodic nature. A system stable in this sense is asymptotically (from a certain time instant k_0) overflow-free. Then it behaves linearly and (exponentially) approaches the equilibrium point in which all state variables become zero. Stability according to (b), the so-called "forced-response stability" is defined for a certain class U_0 of input signals $u(k)$. Such signals are defined with the aid of the idealized linear system and characterized by the property that for at least one initial condition the overflow threshold is never reached. The filter with overflow correction is then called "forced-response stable" if for any $u(k) \in U_0$ and any initial condition the response asymptotically $(k \to \infty)$ approaches the waveform of the linear counterpart.

So for the given class of input signals the actual filter eventually "forgets" former overflows and becomes overflow-free. Clearly, forced-response stability is a stronger condition than zero-input stability and includes the latter.

If the system is excited with a rather irregular waveform, zero-input stability will often suffice; only for periodic waveforms the stronger condition is strictly required.

Mainly three overflow characteristics have been proposed: (a) *saturation* (b) *zeroing* (c) *two's complement*, cf. Fig. 3.4 with $\nu = 0$. Saturation yields the smallest deviation from the normal operation, although during overflow the filter becomes more or less inoperative. It has also the best stability properties (cf. Section III). Zeroing means that the output is set to zero, if the input exceeds the overflow threshold; it can be easily generalized to reset *all* states, when *one* state exhibits overflow. Two's complement overflow amounts to a periodic continuation of the 45° straight line; its advantage lies in the automatic correction of intermediate overflows. With regard to stability it is the least favourable overflow correction so that the choice of the linear circuit is more restricted than for the other characteristics.

We conclude this section with a few remarks on the aim and organization of the paper. First of all, a comprehensive *bibliography* covers all nonlinear finite-wordlength effects in one-dimensional digital filters published in recognized journals and conference proceedings. Multidimensional filters and coefficient quantization have been left out of consideration. The text has been written in awareness of existing review articles [3]-[10] and should especially be viewed as an extension of Claasen's (et al) paper of 1976; in fact, it is a progress report covering the past twelve years. It should further be noted that not all aspects are treated with the same elaborateness. So, only a brief discussion is devoted to structure optimization with respect to quantization noise, mainly due to an exhaustive treatment of this subject in two recent textbooks [1],[2].
The references from [424] onward are recent contributions (published in the years 1987 and 1988) to non-linear effects in digital filters, which were added after the manuscript of this report was completed, and as such are not referenced in the text.
For ease of reference, a bibliography in alphabetical order of all authors is added.

III. Overflow oscillations

In *recursive* filters, quantization and overflow can lead to instabilities, even if the underlying linear filter is designed to behave stable. Instabilities due to quantization ("limit cycles") lead to relatively small deviations from the linear behaviour. While these effects will be treated in the next section, we now deal with those instabilities that are related to register *overflow*. The associated *oscillations* have large amplitudes; because of their disastrous effects on the filter behaviour they have to be absolutely avoided. One of the main factors determining their occurrence is the "overflow characteristic" (i.e. the way overflow is corrected), of which we treat the three commonly used types (a) saturation (b) zeroing (c) two's complement.

A. Zero-input oscillations

We begin with a study of *overflow oscillations* [38]-[69] in the original sense, i.e. for an otherwise unexcited digital system. In addition to this "zero-input" condition we assume that (a) overflow and quantization can be treated independently ("decoupling assumption") and (b) overflow correction is only required for signals entering a delay element. The latter assumption excludes all structures where intermediate overflows occur. For sake of conciseness, we restrict the following discussion to second-order sections with complex poles. Compared with real poles, complex conjugate pole-pairs generally favour all forms of parasitic oscillations (particularly for high Q-values) and thus deserve special consideration. In due course, we summarize more general results for higher-order sections and without reference to complex pole pairs.

The 2×1 *state vector* $\underline{x} = (x_1, x_2)^t$ in a second-order system satisfies the fundamental difference equation

$$\underline{x}(k+1) = F(A\,\underline{x}(k)) \tag{3.1}$$

where

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

denotes the *system matrix*, while $F(\bullet)$ stands for the overflow characteristic.

It is understood that $[F(A\,\underline{x})]_i = F(A\,\underline{x})_i$, i.e. the individual components of $A\,\underline{x}$ undergo the same memoryless and local (i.e. not controlled by other signals) overflow correction. The question to be analyzed is: Under which circumstances (choice of A, F and initial conditions) does (or does not) (3.1) admit periodic solutions?

Due to the overflow bound, which is henceforth normalized to unity, the state variables satisfy the condition $|x_i| \le 1$, resulting in a state vector confined to the interior of the *unit square* (cf. Fig. 3.1). Without overflow (i.e. as long as $|x_i| \le 1$) the solution of (3.1) is found as

$$\underline{x}(k) = \mathrm{Re}\{\tilde{X}(\underline{\xi}_r + j\underline{\xi}_i)e^{[(\Gamma+j\Omega)k+j\tilde{\varphi}]}\} = \tilde{X}e^{\Gamma k}[\underline{\xi}_r\cos(\Omega k+\tilde{\varphi})-\underline{\xi}_i\sin(\Omega k+\tilde{\varphi})], \qquad (3.2)$$

where $e^{\Gamma\pm j\Omega}$ denotes the complex eigenvalues of A and $\underline{\xi}_r \pm j\underline{\xi}_i$ denotes the pertinent eigenvectors. It is tacitly assumed that $\Gamma < 0$, expressing linear stability. Further, without loss of generality, the real and imaginary parts $\underline{\xi}_r$, $\underline{\xi}_i$ of the suitably normalized eigenvector are assumed to be orthogonal, i.e. $\underline{\xi}_r^t\underline{\xi}_i = 0$. (This freedom is provided by the indeterminacy of the complex magnitude of any eigenvector). Finally, the constants of integration $(\tilde{X},\tilde{\varphi})$ are determined by the initial conditions.

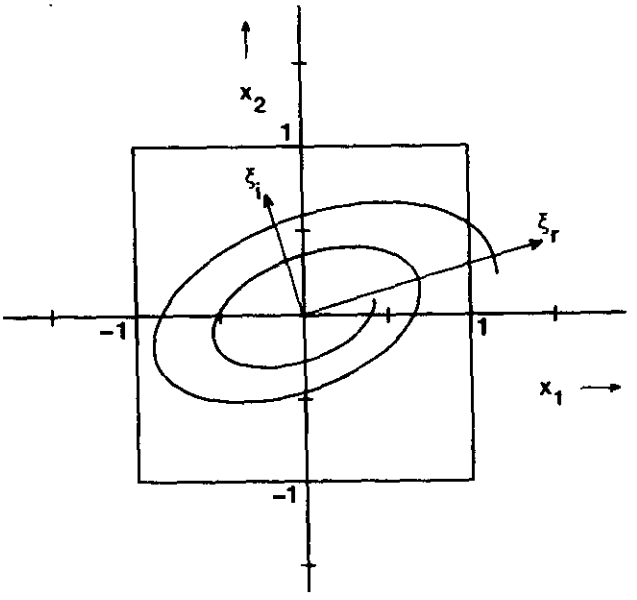

Fig. 3.1. Trajectory of the state vector $\underline{x}(k)$ in the phase plane and the overflow boundary.

If, for the time being, time k is viewed as a continuous variable, $\underline{x}(k)$ describes a trajectory in the phase plane. For the (unrealizable) case $r=0$ this would be an *ellipse* with main axes in the direction of $\underline{\xi}_r$ and $\underline{\xi}_i$. For $r<0$ (corresponding to poles *inside* the unit circle), we obtain a nonclosed, ellipse-like curve spiralling towards the origin, cf. Fig. 3.1.

Of course, these results only apply to the digital filter as long as overflow does not occur ($|x_i| \leq 1$). In general, this condition is not met for all initial conditions $\underline{x}(0)$ inside the unit square. Only the initial vectors $\underline{x}(0)$ of the region R of Fig. 3.2 lead to "allowed" $\underline{x}(k)$ for all (continuous) values[4] of k.

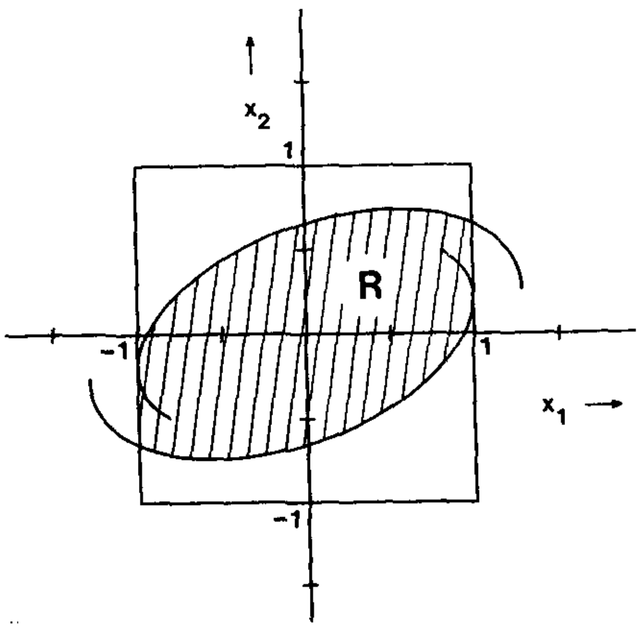

Fig. 3.2. Region R of initial states that never lead to overflow.

What occurs if $\underline{x}(0)$ is outside R? Then, at some time instant k, the linearly determined $\underline{x}(k)$ might leave the unit square, and *overflow correction* has to be applied. This correction introduces one of two basic state modifications:
(a) $\underline{x}$ is moved towards the origin (b) $\underline{x}$ is moved away from the origin.

[4]Note that the discrete character of k causes also some points outside R (but inside the square) to be allowed as initial conditions $\underline{x}(0)$, because parts of the continuous curves of Fig. 3.1. are not actually occupied.

Case (a) is wanted because it supports the natural linear motion; no oscillation occurs if all overflows are corrected this way. Case (b) is dangerous, because it compensates or even overcompensates the linear behaviour and, hence can (but need not) lead to oscillations. Of course, these statements ask for an unambiguous definition of "distance from the origin". Instead of the widely used euclidean norm our definition is guided by the linear state motion, according to (3.2).

Following

$$\underline{x} = X(\underline{\xi}_r \cos\varphi - \underline{\xi}_i \sin\varphi) \tag{3.3}$$

two variables X,φ can be associated with each state $\underline{x}$. Particularly, the variable X is determined from $\underline{x}$ as

$$X^2 = \left[\frac{\underline{\xi}_r^t\ \underline{x}}{\underline{\xi}_r^t\ \underline{\xi}_r}\right]^2 + \left[\frac{\underline{\xi}_i^t\ \underline{x}}{\underline{\xi}_i^t\ \underline{\xi}_i}\right]^2 . \tag{3.4}$$

Comparing (3.3) with the linear motion as described by (3.2) one recognizes $X = \tilde{X}e^{\Gamma k}$, i.e. a monotonically decreasing function $X(k)$. Combined with the fact that X^2 is a quadratic form in x_1, x_2 as formulated by (3.4), the parameter X^2 is a natural candidate for a Lyapounov function*. Observe that the curves $X = \text{const}$ constitute a family of "concentric" ellipses (with axes along $\underline{\xi}_r$ and $\underline{\xi}_i$) and that low-X ellipses are enclosed by high-X ellipses. Naturally, we choose X as the "distance from the origin".

Overflow correction is now visualized in Fig. 3.3. An uncorrected state point B is transformed into B', B'', B''' after applying saturation, zeroing, and two's complement, respectively. For this example all types lead to an increase of X and, hence, to a movement away from the origin. On the other hand, for point C this is only true for zeroing and two's complement.

*Other Lyapounov functions are discussed under the head "limit cycles", cf. Section IV.

For some ellipse geometries it is possible to use appropriate overflow characteristics such that the state *always* moves towards the origin and *oscillations are suppressed*. Obviously this is not the case for the arbitrarily oriented ellipse of Fig. 3.3. However, it is easily recognized that for an ellipse whose axes coincide with the $x_1 - x_2$-axes, each of the three overflow corrections satisfies the stability condition, while for an ellipse with a 45^o inclination stabilization can be obtained at least with a saturation characteristic.

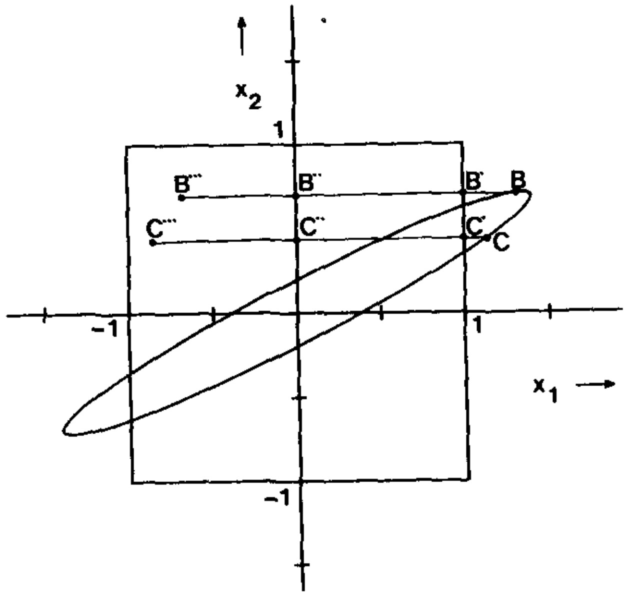

Fig. 3.3. Ellipse X = constant in the phase plane.

It should be noted that in this picture the potentiality for stabilizing overflow is determined by the eigenvectors of A and not by the eigenvalues. While the latter determine the speed with which the trajectories are traversed, the eigenvectors determine the appearance of the ellipse, i.e. the orientation of the axes and their length ratio. These parameters are essentially determined by the *filter structure*, examples of which are the (a) normal filter (b) wave digital filter (c) direct-form filter as depicted in Fig. 4.1. The corresponding system matrices are [42],[36],[37]

$$A = \frac{1}{2}\begin{bmatrix} a & \sqrt{-4b-a^2} \\ -\sqrt{-4b-a^2} & a \end{bmatrix}, \quad A = \frac{1}{2}\begin{bmatrix} a+b+1 & a+b-1 \\ a-b+1 & a-b-1 \end{bmatrix}, \quad A = \begin{bmatrix} a & b \\ 1 & 0 \end{bmatrix} \tag{3.5}$$

(a) (b) (c)

where $a = 2e^{\Gamma}\cos\Omega$, $b = -e^{2\Gamma}$ are the coefficients of the characteristic polynomial.

For (a) the ellipse degenerates into a circle, so that all overflow characteristics lead to stabilization. For (b) and high Q-values ($b \to -1$) the ellipse axes coincide with the $x_1 - x_2$-axes so that again all characteristics satisfy, while for (c) and high Q-values the ellipse axes have a 45° inclination so that stability is obtained with saturation. However, also for low Q-values and even for real poles stability can be guaranteed[6] [21],[22],[38].

Normal filters and wave-digital filters of orders higher than two can likewise be stabilized with all types of overflow characteristics [39]-[44]. However, higher-order direct-form filters are in general unstable with respect to over flow; high-period and chaotic oscillations have been observed in such structures [45]-[51], occasionally with oscillator applications in mind [52],[53]. Observe that every stability requirement yields sufficient conditions; often these conditions can be weakened with various analytic measures [54],[55] or with computer-generated Lyapounov functions [56],[57]. Attempts have also been reported with unconventional overflow characteristics [21],[58] and overflow signalling schemes [59]-[61]. Special investigations have been published on the stability properties of wave-digital filters [70]-[74], normal filters [75], lattice filters [76]-[79], block-state realizations [80]-[81], and multi-input-multi-output structures [82]-[83], while experimental results have been reported in [7]. Parasitic oscillations in more complicated systems, particularly those formed by single-input-single-output systems under looped conditions have been discussed in [47],[51],[84].

B. Forced-response stability

In the previous subsection we have discussed sufficient conditions guaranteeing that no zero-input overflow oscillations occur. The non-existence of such oscillations was viewed as an absolute design requirement that every usable filter has to meet.

---

[6] The pertinent proofs are constructed with other Lyapounov functions and other ellipse geometries, cf. eq. (4.8) of Section IV.

An ill-designed filter can exhibit autonomous oscillations under suitable initial conditions. Physically, these are e.g. determined through connecting the digital circuit to a power supply or as a residue of former (meanwhile terminated) input signals. Such an initial condition need not immediately cause overflow but can lead to it after a number of time steps. Thereafter overflow becomes periodic or asymptotically periodic or irregular (chaotic). All these instabilities are characterized by the non-existence of a time instant, after which overflow ceases to occur.

On the other hand, stability implies that such a time instant *does* exist. This requirement is also the starting point for the forced response stability to be discussed in this subsection [85]-[90].
Occasional overflows are allowed, but there has to exist a last overflow, after which the system behaves linearly and thus recovers from potential former overflows. Asympto-tically ($k \to \infty$) there remains the "forced response", which is independent of the initial conditions and, as such, not affected by all former overflows.

Stability in this sense depends upon the excitation. For each digital filter a (possibly empty) set of input signals exists for which stability holds. An apparent minimum requirement is that only such input signals $u(k)$ are admitted for which the associated linear filter (without overflow correction) does not exceed the overflow level after some time k_0. The ensemble of all such signals (with k_0 unspecified) is said to form the class U_0 (definition "A"). Besides this definition "A" an alternative definition "B" is in current use which examines $u(k)$ only for $k \geq k_0$. Following "B" we have $u(k) \in U_0$ iff there exists an initial condition at $k = k_0$ such that the linear filter does not exceed the overflow level for all $k \geq k_0$. Apparently, the past history of $u(k)$ in the "A" interpretation is condensed in the initial condition[7] according to "B" so that the "tails" of the "A" signals form the class U_0 in the "B" sense [86]-[87].

[7] In an uncontrollable system it can occur that not all initial conditions can be generated with the aid of suitable input signals. In such an exceptional case, the "B" definition is more general. This definition was already introduced in Section II.

A stable filter with overflow correction always exhibits a *finite* number of overflows (which may be zero or one in special cases) after $k = k_0$, which number depends upon the initial condition at k_0. Assuming that $u(k) \in U_0$, there is at least one initial condition (mostly a set of neighbouring initial conditions) with no overflow after $k = k_0$.

If stability in the above sense holds for all $u(k) \in U_0$, the filter is called "*forced-response stable*" with respect to U_0 [87]. Since excitations $u(k) \notin U_0$ are meaningless in the context of stability, the addition "with respect to U_0" is often omitted. Weaker forms of stability are found with respect to subsets of U_0 such as U_0^c with a scale factor c satisfying $0 \leq c < 1$. Compared with U_0 the signal amplitudes are reduced by a factor c such that $u(k)/c \in U_0$ [91].

In this notation $c = 0$ corresponds to "zero-input stability" being the weakest form of stability. It is somewhat surprising that systems whose stability is guaranteed only for zero input also behave stable for most excitations of practical importance. In fact, only periodic or almost-periodic* signals appear to be able to produce forced-response instabilities (with commensurate periods) in such systems.

Concerning the analytic investigations of forced-response stability it is a lucky circumstance that the nonzero-input problem can be transformed into a zero-input problem with time-varying nonlinearities [87]. Let $\underline{x}(k)$ and $\underline{\xi}(k)$ denote the state vectors of the actual and the idealized filter with excitation u(k) such that (cf. (3.1))

$$\begin{aligned} \underline{x}(k+1) &= F\{A\,\underline{x}(k) + \underline{b}\,u(k)\} \\ \underline{\xi}(k+1) &= A\,\underline{\xi}(k) + \underline{b}\,u(k) \end{aligned}$$

then the difference vector $\underline{d} = \underline{x} - \underline{\xi}$ satisfies the *homogeneous* difference equation

$$\underline{d}(k+1) = F\{\underline{\xi}(k+1) + A\underline{d}(k)\} - \underline{\xi}(k+1). \tag{3.6}$$

*As an example, we refer to [92]-[94], where an "irrationally" sampled continuous-time sinusoid has evoked instability.

Let us consider a certain component of $\underline{\xi}(k+1)$ and $A\underline{d}(k)$ and denote it provisionally by ν and δ, respectively. Then the same component of the right-hand term of (3.6) reads as $F\{\nu+\delta\}-\nu$, i.e. a time-varying (due to $\nu=\nu(k)$) nonlinear function of δ. With a linearly determined $\nu(k)$ the function $F\{\nu+\bullet\}-\nu$ is a shifted replica of $F\{\bullet\}$, with equal horizontal and vertical ν-shifts of the F-plot. Fig. 3.4 shows the result for the three basic overflow characteristics.

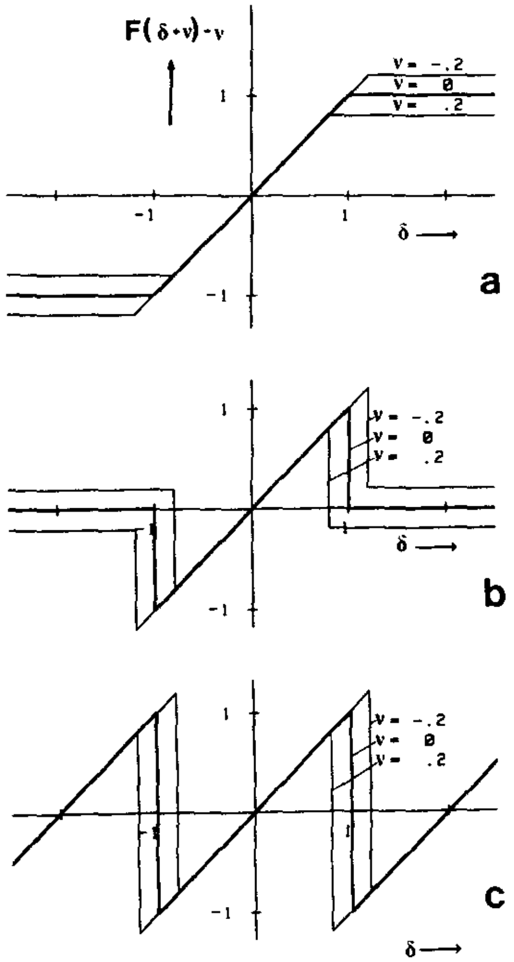

Fig. 3.4. Plots of $F\{\nu+\delta\}-\nu$ for (a) saturation, (b) zeroing (c) two's complement.

With the knowledge that for many structures (e.g. normal and wave-digital filters) the condition $|F(\delta)| \leq |\delta|$ ensures zero-input stability we can likewise conclude that (3.6) has a stable solution (with $\underline{d}(k) \to 0$ for $k \to \infty$) if $|F(\nu+\delta) - \nu| \leq |\delta|$. From Fig. 3.4 we conclude that this is true for saturation if $|\nu| < 1$, for zeroing if $|\nu| < 0.5$, and for two's complement if $\nu = 0$, i.e. for excitations that are elements of U_0, $U_0^{0.5}$, U_0^0, respectively, (in the sense of definition "A" as given above).

We conclude this section with some phenomena occurring in an *unstable* filter. For a given $u(k) \in U_0$ there exists a set of initial conditions, for which no overflow occurs. In general, there exists another set of initial conditions, which leads to a finite, nonzero number of overflows. Finally, due to the assumed instability, a third set of initial conditions gives rise to an infinite number of overflows. It is only in this situation that the instability becomes manifest. For a periodic excitation, the response, too, becomes asymptotically periodic, but the period need not be the same. Subharmonics can occur, but also completely different periods are observed [85]. In general, the asymptotic response is not unique, even if the periods of excitation and response are equal. Additional pulse excitations can lead to jump phenomena from one response to another [95],[96].

IV. Quantization limit cycles

Besides the large-amplitude overflow oscillations treated in the previous section, still other parasitic oscillations are observed in *recursive* digital structures, which have their origin in the quantization fine structure and, as a result, have relatively small amplitudes. These oscillations can occur under zero- or (nonzero)constant-input conditions and are generally called "*limit cycles*". Together with quantization noise (cf. Section V) they are considered as the most serious deviation from linear behaviour under normal operating conditions of a digital filter. In contrast with quantization noise, they can, however, be completely avoided. Unfortunately the involved techniques complicate the noise analysis such that a systematic noise minimization cannot be achieved with analytic tools. Thus in current literature we observe almost independent studies of limit cycle suppression and of noise optimization. The first problem mostly deals with quantization by magnitude truncation MT (or related methods) while the second is completely based on rounding R.

The main factor determining the occurrence of limit cycles is the quantization characteristic. In this section we mainly consider R and MT quantization; modified quantizations like controlled rounding CR and stochastic quantization require additional signals and, as such, more complicated descriptions than a simple characteristic.

A. Limit cycle suppression with Lyapounov and other deterministic methods

The analytical treatment of limit cycles resembles that of overflow oscillations. This implies an organization of the present section similar to that of Section III. Again we begin with second-order systems with complex poles under zero-input conditions, for which

$$\underline{x}(k+1) = f(A\underline{x}(k)) \tag{4.1}$$

likewise applies with the only modification that now $f(\bullet)$ is allowed to be a more general nonlinear vector function. The former strictly component-wise application of the scalar overflow characteristic $F(\bullet)$ is thus abandoned.

This generalization allows for the most general quantization scheme, in which not only state variables (= components of $\underline{x}$) are subject to quantization, but also intermediate products or sums.

Whereas in the overflow problem $\underline{x}$ denotes a continuous set of variables, quantization implies a discrete-amplitude character of $\underline{x}$ with all x_i integer multiples of the quantum q. Reckoning with the fact that all solutions of the homogeneous equation (4.1) are bounded for $k \to \infty$, any filter with quantized state variables can consequently be viewed as a *finite-state machine*.

While for any arbitrary initial condition $\underline{x}(0) \neq \underline{0}$ the state $\underline{x}(k)$ in a *linear filter* asymptotically approaches the origin ($\underline{x}(k) \to \underline{0}$ for $k \to \infty$), this is not the rule for the nonlinear filter described by (4.1). Instead, for some $k = k_0$ the state $\underline{x}(k)$ enters a limit cycle. This is a periodic motion characterized by N state points which are cyclically occupied by $\underline{x}(k)$. "*Accessible*" limit cycles can be entered from points outside the cycle which together with all their predecessors form a (mostly immense) set of state points to be assigned to such a cycle [97]-[99]. On the other hand, "*inaccessible*" cycles have to be started on the cycle itself. Limit cycles of period 1 consist of one point, which can be accessible or inaccessible. If and only if the origin $\underline{x} = \underline{0}$ is ultimately reached from *any* initial condition (implying accessibility of the origin) the filter is *limit-cycle free*.

Without any quantization (corresponding to the ideal, linear filter), the trajectory of the state vector $\underline{x}(k)$ would follow an *ellipse-like curve* spiralling towards the origin, as shown in Fig.3.1. In the actual filter, quantization introduces a slight modification of the state vector such that its quantized version becomes a point in the quantization grid, located in the close vicinity of the state before quantization. Like the overflow correction discussed in Section III, quantization can be associated with a state motion towards the origin or away from it. The first motion supports the linear motion and ensures *freedom of limit cycles* if quantization is always performed this way. Clearly, this rule provides a *sufficient* condition. Conversely, quantization correction away from the origin does not admit any conclusion: limit cycles can, but need not occur.

The above statements ask for a definition of "distance from the origin". In contrast with the straightforward definition of Section *III* we take a more general standpoint by identifying any "*Lyapounov energy function*" of the associated linear filter with the square of the distance from the origin. Such a function $P(\underline{x})$ is

(a) a quadratic form* $P = \underline{x}^t Q \underline{x}$, where

(b) Q is symmetrical and positive definite, corresponding to $P > 0$ for all $\underline{x} \neq \underline{0}$, and

(c) the system dynamics is such that P decreases with increasing time: $P(\underline{x}(k+1)) < P(\underline{x}(k))$.

For the linear system with $\underline{x}(k+1) = A\,\underline{x}(k)$ condition (c) reads as

(c') $Q - A^t Q A$ is positive definite.

All matrices Q satisfying conditions (b) and (c') are candidates for "*energy matrices*" defining an appropriate energy function. If, for one of such matrices, quantization lowers the energy P, freedom of limit cycles is guaranteed. In terms of (4.1) this condition reads

$$f^t(\underline{x}) Q\, f(\underline{x}) - \underline{x}^t Q \underline{x} < 0 \qquad \forall\, \underline{x} \tag{4.2}$$

If (4.2) holds, condition (c) for the energy function is satisfied in the linear *and* in the nonlinear filter, so that P is also a Lyapounov function of the nonlinear system. Care must be taken if $<$ is replaced by $=$ in (4.2) so that energy is not changed by quantization. If, moreover, "definite" in condition (c') is relaxed into "semi-definite", it can occur that energy remains constant, asscociated with the risk of a limit cycle. Such a situation occurs for a marginal choice of Q for which in the linear filter periods of low and high energy decrease alternate. The other extreme is a continuous decrease in exponential form, as found for the distance definition of Section III.

Historically, Lyapounov theory (with appropriate modifications) was first applied to *wave digital filters*, i.e. structures derived from classical LC-twoports. For the second-order section of Fig. 4.1b with A given in (3.5b), Q is advantageously chosen in the diagonal form

---

*Notice that P = const represents an ellipse in an x_1-x_2-plane.

$$Q = \frac{1}{2}\begin{bmatrix} 1+a-b & 0 \\ 0 & 1-a-b \end{bmatrix} . \tag{4.3}$$

With this choice the ellipses P = const have axes parallel to the coordinate axes. Further, the linear energy decrease per time step reads

$$P(\underline{x}(k+1)) - P(\underline{x}(k)) = -(1-a-b)(1+a-b)(1+b)[x_1(k)+x_2(k)]^2/4 \leq 0. \tag{4.4}$$

Observe that P is a marginal Lyapounov function, since for $x_1 + x_2 = 0$ the energy P remains constant. It appears, however, that one time step after this occurs, $x_1 + x_2 \neq 0$ so that energy again decreases. Applying MT quantization on the individual state variables reduces $|x_1|$ and $|x_2|$ and, consequently, also P, so that limit cycles are forbidden [70]-[73],[100],[101].

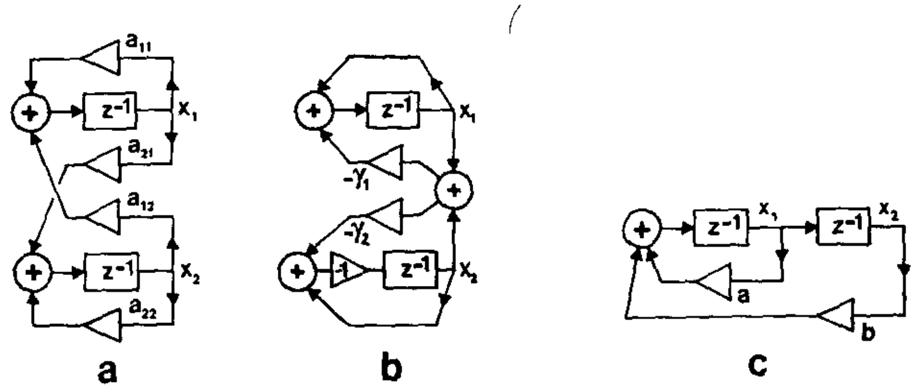

Fig 4.1. Recursive parts of second-order structures in normal form (a), wave digital form (b) and direct form (c).
The filter coefficients are interrelated by $a_{11} = a_{22} = a/2$;
$a_{12} = -a_{21} = \frac{1}{2}.\sqrt{-4b-a^2}$; $\gamma_1 = \frac{1}{2}(1-a-b)$, $\gamma_2 = \frac{1}{2}(1+a-b)$.

A widely used, straightforward design of a second-order section is in the *direct form* of Fig. 4.1c with A given by (3.5c). Here we choose advantageously

$$Q = \begin{bmatrix} 1-b & -a \\ -a & 1-b \end{bmatrix} \tag{4.5}$$

with ellipses $P = \text{const}$ under 45°. The linear energy decrease per time step is given by

$$P(\underline{x}(k+1)) - P(\underline{x}(k)) = -(1+b)[x_1(k+1) - x_1(k-1)]^2 \leq 0. \tag{4.6}$$

Although, due to $x_2(k+1) = x_1(k)$, only the state variable x_1 needs to be quantized, MT is unable to perform that task throughout without energy increase. This can be understood geometrically, since in parts of the ellipse MT causes a state motion away from the origin[10], cf. Fig. 4.2.

In an alternative approach, we combine the linear and nonlinear (quantization) operation, which at least yields a non-increasing energy function. First recall that $x_2(k+1) = x_1(k)$ so that some grid point M in the state space is always linearly transformed into a point on a straight line through the mirror point M^* with respect to the 45^0 line, cf. Fig. 4.2. Let M' denote the result of this linear transformation of M, and let M'' denote the result of the subsequent quantization, then M' lies on the line segment $x_2(k+1) = x_1(k)$ *inside* or on the Lyapounov ellipse (due to (4.6)) while M'' is desired to lie there, too.

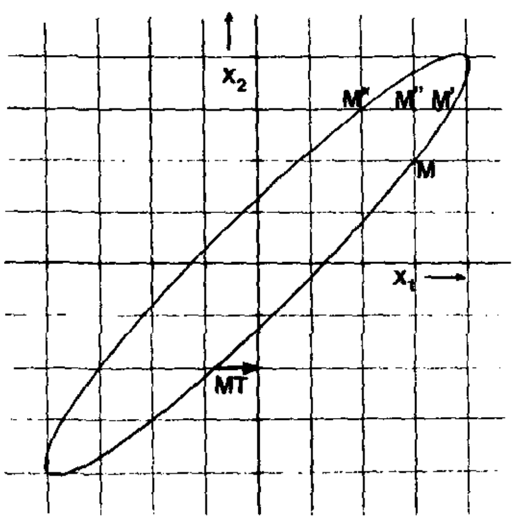

Fig. 4.2. State-space of a second-order direct form filter.

---

[10]Other (permitted) choices of Q-matrices might lead to such ellipses, that MT throughout reduces energy, cf. (4.8).

Realizing that, according to their definitions, M'' and M^* are grid points, we con-clude that M^* is a suitable candidate for M'' ,but also any (if existent) inter-mediate grid point between M' and M^*. To achieve minimum error, we choose for M'' the grid point nearest to M'. This construction yields the quantization rule that $x_1(k+1)$ has to be quantized "in the direction" of $x_2(k) = x_1(k-1)$.

Unfortunately, this "*controlled rounding*" CR does admit constant-energy limit cycles of periods 1 or 2, in which the state jumps between M and M^* or, for $M = M^*$, remains constant at M [102].

It is an advantage of CR that not only zero-input stability (with the exceptions mentioned) is achieved, but also stability under any *constant-input condition*, because the quantization rule only involves signal differences [102]. With some additional hardware, limit cycles of periods 1 and 2 can be suppressed, too, but at the expense of the general constant-input stability [103],[104]. This is not the case for CR applied to wave digital filters, in which stability holds for all constant inputs [105]-[108]. A number of other solutions are based upon the elementary insight that CR can be reduced to MT by first subtracting the control signal, then applying MT, and finally again adding the control signal. Structures thus derived from wave digital sections are often presented in multi-output form with lowpass, highpass, bandpass and allpass outputs [109]-[113]. Also second-order structures of the wave-digital type with only one MT quantizer have been devised, which are stable for zero-, constant-, and alternating input signals [114]-[116]. In these solutions, not the states are quantized, but some intermediate signal[11].

A third important class of second-order sections are the *normal* filters (also "couped sections") cf. Fig. 4.1a. In a certain sense, they exhibit the best overflow and limit cycle behaviour (and, moreover, excel with respect to quantization noise), a benefit that has to be paid for with twice the number of multiplications (4 instead of 2 for wave digital and direct-form sections).

The A matrix is given by (3.5a), the Q matrix is the unit matrix, and P is the square of the euclidean distance from the origin. The energy decrease per unit time reads

[11] This is an example of a more general vector function f(•) in (4.1).

$$P(\underline{x}(k+1)) - P(\underline{x}(k)) = -(1+b)(x_1^2(k) + x_2^2(k)) \leq 0 \qquad (4.7)$$

and vanishes only for $\underline{x}(k) = \underline{0}$. Thus, P is a regular Lyapounov function, whereas the corresponding functions for wave digital and direct-form filters only belong marginally to this class (in a strict sense, they are *not* Lyapounov functions). The curves P = constant now degenerate into concentric circles, and MT applied to the individual state variables ensures zero-input stability.

It is a common property of normal and wave digital filters that Q is a diagonal matrix and that P = constant are ellipses oriented parallel to the coordinate axes. Only *this* ellipse geometry allows for *MT quantizations* applied to the individual state variables, without risk of limit cycles [117]-[122]. The question arises: which A matrices admit a diagonal "energy matrix" Q? This problem has the solution [42],[43]

$$|a_{11}-a_{22}| + \det A < 1. \qquad (4.8)$$

All filters with matrices A satisfying (4.8) remain zero-input stable under MT quantization of the individual state variables and, for the same reason, under any overflow correction. Do the three basic section types satisfy (4.8)? The answer is "yes" for the normal form, "yes" iff $|a| - b < 1$ for the direct form, "almost yes" for the wave digital form with the inequality sign replaced by an equality sign (reflecting the marginal character of the Lyapounov function). Also "lattice filters" and "minimum-norm filters" [76],[44] satisfy (4.8).

Higher-order filters with orders $n > 2$ are often designed through appropriate generalizations of second-order sections. In the present state of the art, only the ladder filters, particularly the wave digital filters (WDF) appear to be sufficiently developed such that a direct n-th order approach is feasible. Due to the availability of a recent review article [36] on WDF design, including nonlinear parasitic effects, we can confine ourselves to the brief statement that, because of its inherent passivity properties, MT can often be (partly) replaced by R without risk of limit cycles, which results in lower quantization noise levels [123],[124].

In addition, investigations on limit cycles in floating-point arithmetic WDF design [125],[126] and in half-synchronic filters [74] as well as filters under looped conditions [84] deserve to be mentioned. Apart from the WDF approach, most higher-order filters are designed in parallel- or cascade form, in which cases knowledge about stability of second-order sections suffices.

Concerning stability with respect to constant (nonzero) inputs, which has so far been touched upon only in ad hoc situations, we mention a general principle to convert a stable *autonomous* system into a system with *input terminals* stable under any constant excitation. In more explicit terms, let an autonomous system satisfy (4.1) (where f has the original meaning of a scalar quantization characteristic) and let further the solution of (4.1) approach zero for $k \to \infty$ (expressing freedom of limit cycles), then through suitably supplying such a system with an input terminal, a constant-input stable system can be created as follows. At each quantization point i, some signal v_i is *added* after quantization, while the same signal is *subtracted* after the sum signal has passed the subsequent delay element. The pair of injected signals v_i is proportional to the input signal, $v_i(k) = b_i u(k)$, so that the state in the modified system satisfies

$$\underline{x}(k+1) = f\{A(\underline{x}(k) - \underline{b}\, u(k))\} + \underline{b}\, u(k). \tag{4.9}$$

For a *constant* excitation $u(k) = U$ the difference vector $\underline{x}(k) - \underline{b}U$ satisfies the original equation (4.1) so that the state vector asymptotically approaches the stationary solution $\underline{b}U$ without superimposed limit cycles [127]-[134].

So far, all limit-cycle suppressing mechanisms made use of MT (including the related CR) quantization, utilizing its energy reducing property. On the other hand, *rounding* R can amplify the signal magnitude by a factor $c \lesssim 2$, where the maximum factor $c = 2$ occurs for a signal magnitude equal to half a quantization step. In an attempt to achieve freedom of limit cycles also for R quantization, the nonlinear energy increase has to be compensated by an equal energy decrease associated with the linear filter operation. In concrete terms, the necessary damping finds expression in the condition $\|A\| < \frac{1}{2}$, where $\|A\|$ denotes the norm of the system matrix.

If this condition is not met by the design requirements, the matrix A can be transformed into some power A^L by means of a "block-state realization" or a "matrix-power feed-back" such that $\|A^L\| < \frac{1}{2}$ and R quantiza-tion can be applied without risk of limit cycles [80]-[83]. For high-Q filters this method, however, requires a great amount of hardware.

Besides the basic limit-cycle suppressing concepts discussed so far, a vast amount of ideas has been published dealing with special structures and more complicated (deterministic) stabilization methods. Due to space limitation, we can only present them in summarized form. Multirate filters [135], error feedback [136]-[138], digital incremental computers [139] and other special structures [140]-[143] belong to this category. Much attention has been paid to the stability of direct-form second-order sections in certain regions of the a-b-parameter plane and with respect to certain cycle periods (particularly periods 1 and 2). Most of the pertinent publications belong to the earlier period of research on nonlinear effects in recursive digital filters; as such, they have essentially contributed to our present understanding of these phenomena. With the advent of modern universal (i.e. for all coefficients and all periods) methods for limit cycle elimination the results of these investigations have to some extent lost their practical value [144]-[149]. In this context, frequency-domain criteria formed a powerful analytic tool to derive *sufficient* stability criteria [150]-[154]. Special investigations concern coupled-form filters [155], cascaded sections [156]-[158], sections with non-uniform internal wordlength [159],[160] and with small input signals [161].

The instabilities mentioned at the end of Section III have their counterparts also in the context of quantization effects. Under *periodic excitation* the solutions likewise need not be unique; jump phenomena and *subharmonics* (with relatively small amplitudes) result from such instabilities [162]-[167]. Measures for the suppression of such subharmonics have been proposed [168]-[170].

B. Limit cycle suppression with stochastic methods

Another way to eliminate the various types of limit cycles is to *control quantization* through an external *random signal*. This way potential conditions favourable to the occurrence of parasitic oscillations are irregularly disturbed which results in an asymptotic, albeit noisy approach of the zero state.

The disadvantages of such stochastic methods are evident: they require additional random sources (preferably independent sources for all quantizers) and, at a first glance, yield additional quantization noise. The latter point is, however, compensated by the flatness of the noise *spectrum* that contrasts with the (mostly) narrow bandwidth of the noise generated by MT quantization. Particularly in high-Q filters the ultimate noise contributions at the output terminals can thus be considerably smaller than those occurring with deterministic stabilization methods. Another advantage is the avoidance of cross-talk, as discussed in Section II.

The simplest method is *random rounding*, where the decision about the handling of the least significant bit is exclusively left to the exterior random signal [171],[172]. A variation on this strategy is found when the unquantized signal is supplemented with *random dither*, whose spectral distribution is flat and whose amplitude distribution is uniform in the interval [-q/2, q/2]. The complete signal is subsequently subject to R quantization [173],[174]. In contrast with this "uniform random dither", the former random rounding is occasionally referred to as "binary random dither". In rough terms, uniform r.d. has a better noise performance, whereas binary r.d. is superior with respect to limit cycle suppression. In a variant, uniform r.d. is subject to spectral shaping, particularly with a bandstop characteristic. The resulting "*bandstop dither*" has an improved noise performance, to be sure, but is costly to implement [175],[176].

Guided by the inherent properties of R and MT quantization, one can combine their respective merits into "*random quantization*", in which an external generator randomly switches between R and MT quantization, with comparatively short MT operating times. This way the excellent R noise properties are coupled with the stabilizing capability of MT quantization, which has to be paid for with a prolonged limit cycle expiration time [177]-[180].

C. Properties of limit cycles

So far attention was focused on the suppression of all forms of limit cycles. Occasionally, however, such parasitic oscillations are wanted (as in a digital oscillator) or at least tolerated (with the argument of saving additional hardware). In either case, it is worthwhile to get acquainted with *various properties* of limit cycles, if these can occur. This is particularly true for their ***maximum amplitude***, as its knowledge enables a user to apply a coarse requantization such that a potential limit cycle remains smaller than a quantization step. This "screening" of limit cycles is, in fact, the classical technique to cope with this phenomenon [181]. Usually, the requantization is only applied at the lowest signal levels (thus forming a threshold detector), although it then fails to work under constant-(nonzero) input conditions. In view of the modern suppression methods the "screening" method introduces a relatively high degree of signal distortion.

In second-order sections with complex poles at $z = e^{\Gamma \pm j\Omega}$ (with $\Gamma < 0$, $\Omega > 0$) the limit cycle amplitude is tightly bounded by the expression

$$q(\sin\Omega)^{-1} \sum_{k=0}^{\infty} e^{\Gamma k} |\sin(k+1)\Omega|$$

being the ℓ_1 norm $\|h\|_1$ of the filter's impulse response $h(k)$ (apart from the factor q). To the best of our knowledge, $\|h\|_1$ cannot, in general, be cast in closed form; this fact has motivated many authors to derive simpler upper bounds, more or less higher than $\|h\|_1$ [182]-[200].

Besides the amplitude, also the power of a limit cycle has an upper bound [201]-[203]; the same holds for certain norms of the output signal [204], [205]. Many papers deal with amplitude bounds in special structures, such as filters with error feed-back [206],[207], multirate filters [208], coupled-form filters [209]-[212], digital incremental computers [213]-[216], wave digital filters with internal oscillations [217], cascade sections [218]-[220]. Special attention has been devoted to rolling-pin limit cycles [221],[222] and to filters using floating-point arithmetic [223]-[228]. Amplitude bounds of limit cycles have also been derived with Lyapounov functions [229]-[232] and computer simulations [233]-[237].

Many investigators have studied special forms of limit cycles. Besides the aforementioned rolling-pin [221],[222] almost sinusoidal waveforms have been explored or simply presupposed [144]-[150],[189],[190],[209]-[216],[229]-[232]. In many structures periods 1 and 2 attract particular attention [146],[147],[211]-[220],[240]-[242] as well as other special periods [162]-[167],[201],[202],[229]-[239]. Symmetry considerations and associated pairing of limit cycles can be found in various papers [97],[229],[243]. Finally we refer to some experimental observations [157],[158],[218],[219].

V. Quantization noise

A. Error statistics

In contrast to the largely deterministic treatment of the non-linear effects of quantization and overflow in a zero-input or a constant-input situation, the error during normal operation of a digital filter, i.e. in the presence of a more or less arbitrary input signal, is generally treated on a stochastic basis. The keyword here is 'arbitrary': in strict mathematical terms only a random signal, when passed through a quantizer, can cause a random error. In case of a rounding quantizer, however, even a deterministic, non-periodic input may result in a random nature of the rounding error ϵ_R, which is then designated *quantization noise* or *roundoff noise*.
If the input to a rounder has a range of sufficiently many quantization steps and a sufficient spectral width, the white-noise model for the rounding error (cf. Section II) may be applied [30]. Specifically, the error is uniformly distributed in the range $[-q/2, q/2]$ if the error function $\epsilon_R(x)$ of Fig. 2.1 operates on a large number of different values of the argument x. This will be the case for any non-periodic signal, the wordlength of which is reduced by a sufficient number of bits. As a result, the error carries a power of $q^2/12$, where q is the quantization step size. In most practical cases successive errors are uncorrelated with each other and with the quantized signal [245]. In order to determine the total power of this quantization noise (or the noise spectrum) at the filter output, we simply add the contributions of the various quantizers each filtered linearly to the output.

The statistics of the truncation error ϵ_{MT} are not that readily evaluated. Of course the quasi-rounding part ϵ_{QR} of this error (cf. Fig. 2.2) is subject to the above observations concerning ϵ_R. The sign part ϵ_{SGN}, however, introduces an error that is strictly deterministic, leading to ordinary distortion and crosstalk as was already stated in Section II. Nevertheless, a statistical treatment of this error is worthwhile if the signal to be quantized is random. In a current analysis the filter input signal is assumed to be a gaussian random process [246], [248], [249]. Again, the sign part ϵ_{SGN} of the truncation error is decomposed into a linear part (with an appropriate negative slope) that is completely correlated with the quantized signal and an additive (uncorrelated) part that is interpreted as coloured noise. Figure 5.1 shows the quantization noise models for rounding and magnitude truncation, where s_{xx} denotes the autocorrelation function of any signal x.

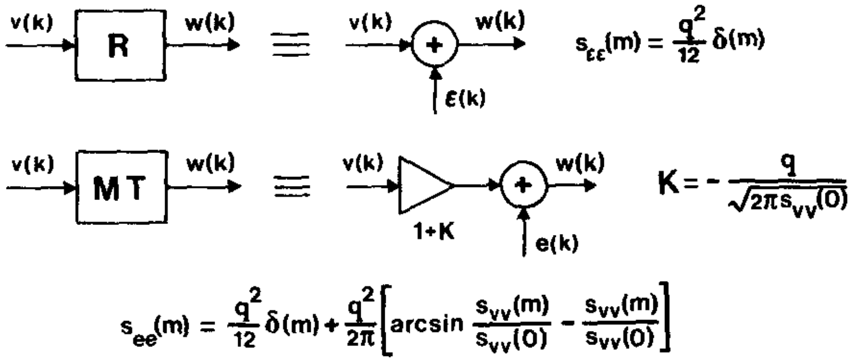

Fig. 5.1. Quantization noise models for R and MT. The model for the MT-error is valid for Gaussian excitation of the filter.

The multiplier 1+K causes a level-dependent (slight) modification of the filter coefficients and, ultimately, filter detuning, an effect which decreases with increasing signal amplitude. Moreover we observe that although the noise powers associated with the quasi-rounding part and the sign part of the truncation error are almost equal at their source, viz.

$$s_{ee}(0) = \frac{q^2}{12} + \frac{q^2}{2\pi}\cdot\left(\frac{1}{2}\pi - 1\right) \approx \frac{q^2}{12} + \frac{q^2}{11}, \tag{5.1}$$

the contribution of ε_{SGN} to the output noise power may substantially exceed that of ε_{QR}. This is the case e.g. for an MT-quantizer in the feedback loop of a high-Q filter: The frequency region in which the source noise spectrum is concentrated coincides with the passband of the linear system from quantizer to filter output. The source noise spectrum exhibits a similar resonance peak, as dictated by the filter poles [255].

The statistics of the quantization error are contained in a number of papers [245]-[270], [30], [25]. These include *random rounding* [247] (also ref. [171], [172]), which can be modelled similar to conventional rounding

(R), and an error analysis of the *distributed arithmetic* [256] as well as *floating point arithmetic* [265]-[270], [25]. *Controlled rounding* (CR) can be translated into MT [264] leading to comparable noise powers [250].

B. Optimal structures

Most papers on quantization noise adopt the white-noise model (implying R quantization) and deal with fixed-point arithmetic with a certain quantization step size q. The resulting noise power at the filter output depends on

- the filter parameters and consequently on the desired frequency response,
- the scaling criterion which is used,
- the filter structure.

Starting from a given filter structure (e.g. direct form, normal form, wave digital structure) we can significantly reduce the influence of the filter parameters on the output quantization noise power. This is done by means of *error-feedback*, which we will discuss in the next subsection. Conversely, given a certain (linear) system function, we may try to find the optimal structure, i.e. the structure achieving the lowest possible output noise power. This problem was first solved using a state-space description of a digital filter [289], [296], [305], and a great number of papers has been devoted to optimal and sub-optimal structures since that time [271]-[314]. Furthermore, error-feedback and structure optimization have been compared as to their noise reduction performance, or been considered in a combined strategy in order to achieve even lower noise levels [315]-[328].

As for scaling, clearly, the more conservative we choose our scaling rule, the less efficiently we exploit the machine-representable state-space, resulting in poor S/N-ratios. Conversely, if we scale too moderately, the probability of overflow will be intolerably high. Scaling is performed by normalizing the impulse responses $f_i(k)$ from filter input to the various internal nodes (indexed i) with respect to their l_p-*norm* $\|f_i\|_p$ (such that $\|f_i\|_p = 1$, where p can be any positive integer) or, similarly, by normalizing the corresponding frequency responses $F_i(e^{j\Omega})$ with respect to their L_q-*norm* $\|F_i\|_q$ (such that $\|F_i\|_q = 1$) [35].

The most conservative is l_1-scaling, by which absence of overflow is guaranteed for the total class of bounded input signals (i.e. $|u(k)| \leq 1$ or equivalently $\|u\|_\infty = 1$, where the overflow-level is set to unity). Next we have L_∞-scaling which prevents overflow for L_1-bounded input signals (which include unit-amplitude sinusoids). The most widely used is l_2-scaling (which is identical to L_2-scaling), primarily because it follows directly from the process of structure optimization. The following relations can be shown to hold [35].

$$\|f_i\|_2 = \left[\sum_{k=0}^{\infty} f_i^2(k)\right]^{1/2} \leq \|F_i\|_\infty = \max_{\Omega} |F_i(e^{j\Omega})| \leq \|f_i\|_1 = \sum_{k=0}^{\infty} |f_i(k)|. \quad (5.2)$$

For Gaussian random processes, we can interpret $\|f_i\|_2^{-1}$ as the scaling factor that ensures equal overflow probabilities for the i-th node register and the filter input register. Similarly, if we scale by setting $\delta \cdot \|f_i\|_2$ to unity, we can interpret δ as the number of standard deviations that can be represented in the i-th node register if the filter input is unit-variance white noise.

Before proceeding to a brief review on structure optimization, we point out that, historically, this was preceded by various strategies (heuristic as well as algorithmic using dynamic programming) to optimize a cascade connection of (in general) direct-form second-order sections. Higher-order filter design by combination of second-order sections is of interest in order to reduce the parameter sensitivity of a direct-form realization. Optimal structures, in the sense of low sensitivity and low noise, are achieved by a certain pole-zero pairing and ordering of the various sections, without changing their internal structure [350]-[366]. As a rule of thumb we may realize a complex conjugate pair of poles together with the nearest pair of zeros, working consecutively, starting with the highest-Q poles. A similar rule may be given for the correct section ordering [364].

Structure optimization (for a comprehensive treatment ref. [2]) is based on a state-variable description of a digital filter:

$$\underline{x}(k+1) = A\,\underline{x}(k) + \underline{b}\,u(k) \quad (5.3)$$

$$y(k) = \underline{c}^t\,\underline{x}(k) + d\,u(k) \quad (5.4)$$

A state-space filter is a direct realization of the state equations in which the components of the two (column) vectors $\underline{b}$ and $\underline{c}$ are the multipliers in the signal paths from the filter input ($u(k)$) to the inputs of the delay elements ($\underline{x}(k+1)$) and from the outputs of the delays ($\underline{x}(k)$) to the filter output ($y(k)$) respectively. The l_2-scaling constraint $\delta \cdot \|f_i\|_2 = 1$ can be met if we apply a diagonal coordinate transformation D^{-1} to the state $\underline{x}$ of a state-space filter to obtain a scaled state $\underline{x}'$:
$\underline{x}' = D^{-1} \cdot \underline{x}$, where $D = \mathrm{diag}\,\{\delta \cdot \|f_1\|_2, \delta \cdot \|f_2\|_2, \ldots, \delta \cdot \|f_n\|_2\}$, and n is the order of the filter. As a result the state transition matrix A undergoes a similarity transformation

$$A' = D^{-1} A D . \tag{5.5}$$

The trace and the determinant of a matrix (as well as its eigenvalues and hence the filter transfer function) are invariant under such a transformation. This is true for any non-singular transformation T^{-1} applied to the state $\underline{x}$. In fact, to find the optimal structure, we seek a coordinate transformation that minimizes the output noise power under the constraint that the filter remain properly scaled.

If we use one quantizer per state variable at the (double-precision) summation node preceding each delay element (cf. Fig. 4.1a for the second-order case), we can assume n rounding errors $\epsilon_1, \ldots, \epsilon_n$ to be injected at these nodes, each carrying a power of $q^2/12$. Specifically, the total noise variance at the output of the scaled filter is given by

$$\sigma^2_{total} = \frac{q^2}{12} \cdot \sum_{i=1}^{n} \|g_i'\|_2^2 = \frac{\delta^2 q^2}{12} \cdot \sum_{i=1}^{n} \|f_i'\|_2^2 \cdot \|g_i'\|_2^2 = \frac{\delta^2 q^2}{12} \cdot \sum_{i=1}^{n} \|f_i\|_2^2 \cdot \|g_i\|_2^2 , \tag{5.6}$$

where the impulse responses $g_i'(k)$ describe the paths from the various error sources to the output of the scaled filter. The first equality states that the (uncorrelated) contributions to the total output noise may be added. The second and third equality are valid, because for the scaled filter we have $\delta \cdot \|f_i'\|_2 = 1$ and $g_i'(k) = \delta \cdot \|f_i\|_2 \cdot g_i(k)$ since scaling affects f_i and g_i in a reciprocal manner, leaving their products unchanged. As a result, the order of the two operations 'scaling' and 'optimization' may be reversed;

instead of finding a non-singular transformation of the state $\underline{x}$ which minimizes $\Sigma \, \|g_i'\|_2^2$ under the constraint of proper scaling, we may optimize the unscaled filter, i.e. minimize $\Sigma \, \|f_i\|_2^2 \cdot \|g_i\|_2^2$ using an unconstrained transformation, and scale the resulting unscaled optimal filter in a final step, using the above diagonal transformation D^{-1}.

In order to facilitate the necessary calculations, two fundamental matrices K and W are introduced, which are positive definite and symmetric.

$$K = A K A^t + \underline{b}\, \underline{b}^t = \sum_{k=0}^{\infty} A^k \underline{b}\, \underline{b}^t (A^k)^t \qquad (5.7)$$

$$W = A^t W A + \underline{c}\, \underline{c}^t = \sum_{k=0}^{\infty} (A^k)^t \underline{c}\, \underline{c}^t A^k \qquad (5.8)$$

Note that the l_2-norms squared of the impulse responses f_i and g_i are on the main diagonal of K and W resp., so $K_{ii} = \|f_i\|_2^2$ and $W_{ii} = \|g_i\|_2^2$. The implicit definitions of K and W allow these norms to be computed by solving two sets of linear equations. This is an appealing property since we now can scale the filter and calculate the output roundoff noise using only linear algebra not involving computation of infinite summations.

As it turns out, a necessary and sufficient condition for optimality of an n-th order filter structure is that the matrices K and W are directly proportional after scaling, i.e. $W' = \rho^2 \delta^4 \cdot K'$. Since K' has equal elements on its main diagonal ($K'_{ii} = \delta^{-2}$), so must have W'. This means that the optimal filter is characterized by that the various noise sources at the quantizers contribute equally to the total output noise. The factor ρ^2 is called the noisegain of the optimal filter, a quantity which depends only upon the filter transfer function. The trace of W' ($tr(W') = n \cdot \rho^2 \delta^2$) is the sum of eigenvalues of W' and, in view of the above relation, $\rho\delta^2$ times the sum of square roots of eigenvalues of the product matrix $K'W'$. The eigenvalues of KW are invariant under a coordinate transformation ($K \rightarrow T^{-1} K\, T^{-t}$, $W \rightarrow T^t W\, T$ and $K\, W \rightarrow T^{-1}(K\, W)\, T$) and so we can calculate the noisegain and the output noise of the optimal filter without actually performing the optimization (which will be the case if we are only interested in what can be achieved):

$$\min \sigma^2_{total} = n \cdot \frac{\delta^2 q^2}{12} \cdot \left[\frac{1}{n} \cdot \sum_{i=1}^{n} \mu_i\right]^2, \tag{5.9}$$

where μ_i are the square roots of eigenvalues of $K\ W$, the so-called second-order modes of a filter. Hence, the noisegain of the optimal filter is equal to the arithmetic mean squared of the second-order modes of the filter.
In the case of second-order filters (n=2) the condition $W' = \rho^2 \delta^4 K'$ can also be written as $W' = \rho^2 \delta^4 M K' M$, where $M = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ i.e. a permutation matrix [296]. This condition is met if we choose $A'^t = M A' M$ and $\underline{c}' = \rho \delta^2 M \underline{b}'$ in (5.8). This means that a second-order optimal filter is characterized by[12] [296]

$$a_{11} = a_{22} \tag{5.10}$$

$$b_1 \cdot c_1 = b_2 \cdot c_2, \tag{5.11}$$

where a_{ij}, b_i, c_i are the elements of the state-transition matrix A, the input→state vector $\underline{b}$ and the state→output vector $\underline{c}$, respectively. In each second-order state-space filter we are free to choose the parameters a_{ij}, b_i and c_i to satisfy the above relations and will still be able to realize any second-order transfer function [282]. The optimal filter is obtained after scaling in a final step using $D = \delta \cdot \mathrm{diag}\ \{\sqrt{K_{11}}, \sqrt{K_{22}}\}$.

Again, higher-order filter design is based on combining second-order stages (in cascade or in parallel) but now this has a practical reason: The number of multiplications per output sample for an n-th order, minimum noise filter is $(n+1)^2$, whereas a cascade of second-order subfilters (of the state variable type) uses only $4n+1$ multipliers. The price we pay, of course, is a somewhat inferior performance of our design.[13] A connection of second-order sections optimized in isolation is called a sectional optimal structure. If we optimize a connection of second-order state-variable filters as a whole, the overall filter is called block optimal.

---

[12]The primes are omitted since both (5.10) and (5.11) are unaffected by scaling.

[13]Contrast this with direct-form filter design, where the cascade of second-order stages is highly superior to the n-th order design.

C. Error-feedback and related noise reduction strategies

Error or residue feedback [315]-[349] is a noise reduction strategy that makes use of the discarded bits of a quantization operation. The basic idea is to introduce zeros in the path from the error source to the filter output to compensate for the poles that tend to blow up the quantization noise, especially for narrow-band filters. Error-feedback leaves the filter response unchanged but alters the output noise spectrum, hence it is also designated *error spectrum shaping* (ESS). Most papers on ESS deal with the second-order direct form with first- or second-order error-feedback (the distinction depending on whether one or two past samples of the quantization error are retained and processed). The basic structure is shown in Fig. 5.2, where the quantization error $\varepsilon(k)$ is understood to be $y(k)-v(k)$.

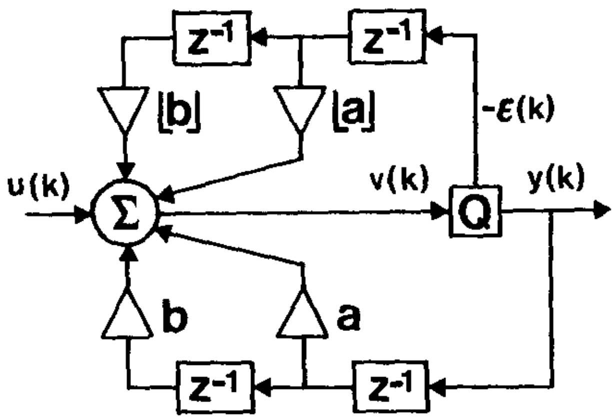

Fig. 5.2. Basic error-feedback structure.

The summation operates on double-precision numbers. The multipliers $\lfloor a \rfloor$ and $\lfloor b \rfloor$ have integer values nearest to a and b, resp., and as a result give rise to very simple additional hardware (shifters or inverters). Moreover, no extra quantization is required in the feedback loop of the error ε, which is a pseudo-double-precision number.
We can assume an error $\varepsilon(k)-\lfloor a \rfloor\varepsilon(k-1)-\lfloor b \rfloor\varepsilon(k-2)$ to be injected at the summing node of the linear (i.e. quantization-free) filter, yielding an output noise spectrum

$$\frac{q^2}{12}\cdot|H_\varepsilon(e^{j\Omega})|^2, \qquad \text{where} \qquad H_\varepsilon(z) = \frac{1-\lfloor a \rfloor\cdot z^{-1}-\lfloor b \rfloor\cdot z^{-2}}{1-a\cdot z^{-1}-b\cdot z^{-2}}. \qquad (5.12)$$

Error-feedback is most effective when a and b are close to integer values. This is the case e.g. for narrow-band low-pass filters (such as interpolation filters) which have poles clustered at $z=1$, so **a** and **b** will be close to 2 and −1, resp. The resulting error spectrum will be nearly white and its power highly insensitive to small changes in the parameters.[14]

For such filters two alternative structures have been proposed [369], which prove to be intimately related to error-feedback structures [318], [343]. The first of these splits up the parameters **a** and b into integer parts $\lfloor a \rfloor$, $\lfloor b \rfloor$ and decimal parts $a_1 := a - \lfloor a \rfloor$ and $b_1 := b - \lfloor b \rfloor$, introducing two quantizers in front of the multipliers a_1 and b_1 to allow for single-precision multipliers in combination with double-precision unit-delay elements. The resulting filter is basically a second-order error-feedback structure and as such performs equally well.

The second alternative [369] is based on the notion that a filter that has poles close to $z=1$ might profit from a change of origin in the complex z-plane, as much from the point of view of sensitivity as of roundoff noise. Introducing the complex variable $\hat{z} := z-1$ we can write

$$H(z) = \frac{1}{z^2 - a \cdot z - b} = \frac{1}{\hat{z}^2 + (2-a) \cdot \hat{z} + (1-a-b)} = \frac{1}{\hat{z}^2 - \hat{a} \cdot \hat{z} - \hat{b}} := \hat{H}(\hat{z}), \quad (5.13)$$

where $\qquad \hat{a} := a-2 \qquad$ and $\qquad \hat{b} := b+a-1.$

This $\hat{z}$ realization of the all-pole filter requires a realization of $\hat{z}^{-1}$, which is simply a delay with unity gain feedback. Such 'integrator' based structures (also known from digital incremental computers [367]) are studied in [367]-[378]. They can be interpreted as filter realizations using first-order error-feedback across each delay element [318]. Also, alternative arithmetics are considered (distributed, ROM/accumulator [338], [377]) in-

[14] Due to the *smoothing of the* resonance peak in the error spectrum, error-feedback will also significantly reduce the amplitude of possible limit cycles (though not the probability of their occurrence, since filters that are free of limit cycles may lose that property by applying error-feedback). Perfect error-feedback (i.e. without restricting the feedback coefficients to integers) would produce limit cycle-free filters but for the necessary double-precision quantizer(s) in the error loop. This is not surprising, since perfect error-feedback is just nothing more than double-precision arithmetic with a single-precision output [324].

cluding filter zeros (as well as poles) and even error-feedforward in order to optimally shape the error spectrum for such zeros [336].

To conclude this section, we mention some other papers on roundoff noise and its reduction. A special choice of structure, such as wave digital structures [379]-[389], may significantly reduce the quantization noise (as well as the parameter sensitivity). Various topics relating to roundoff noise are treated in [390]-[423], such as comparisons between different realizations, special arithmetics, sensitivity, scaling, simulation and measurement. This group contains contributions which do not necessarily fit into one of the main categories on quantization noise, i.e. statistics, optimization or reduction. Also some papers are included which study quantization and overflow stability as well as noise in special structures or even in a comparison between different solutions, but which cannot be designated as review articles such as [3]-[10].

BIBLIOGRAPHY

[1] L.B. Jackson, *Digital Filters and Signal Processing*, Dordrecht, The Netherlands: Kluwer Academic Publishers, 1986.

[2] R.A. Roberts and C.T. Mullis, *Digital Signal Processing*, Reading, Massachusetts, USA: Addison-Wesley Publishing Company, 1987.

Review papers on finite wordlength effects

[3] T.A.C.M. Claasen, W.F.G. Mecklenbräuker and J.B.H. Peek, "Effects of quantization and overflow in recursive digital filters," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-24, pp. 517-529, 1976.

[4] T.A.C.M. Claasen, W.F.G. Mecklenbräuker and J.B.H. Peek, "A survey of quantization and overflow effects in recursive digital filters," *Proc. IEEE Int. Symp. on Circuits and Systems*, Munich, Germany (New York, USA: IEEE 1976), pp. 620-624, 1976.

[5] J.F. Kaiser, "Quantization effects in digital filters," *Proc. IEEE Int. Symp. on Circuit Theory*, Toronto, Canada (New York, USA: IEEE 1973), pp. 415-417, 1973.

[6] J.F. Kaiser, "On the limit cycle problem," *Proc. IEEE Int. Symp. on Circuits and Systems*, Munich, Germany (New York, USA: IEEE 1976), pp. 642-645, 1976.

[7] A. Lacroix and N. Hoptner, "Simulation of digital filters with the aid of a universal program system," *Frequenz*, vol. 33, pp. 14-24, 1979.

[8] A.V. Oppenheim and C.J. Weinstein, "Effects of finite register length in digital filtering and the Fast Fourier Transform," *Proc. of the IEEE*, vol. 60, pp. 957-976, 1972.

[9] S.R. Parker, "Limit cycles and correlated noise in digital filters," *Digital Signal Processing*, ed. J.K. Aggarwal (North-Hollywood, California, USA: Western Periodicals 1979), pp. 117-179, 1979.

[10] G. Peceli, "Finite wordlength effects in digital filters," *Period. Polytech. Electr. Eng.*, vol. 28, pp. 191-200, 1984.

References pertaining to the first two introductory sections

[11] A. Fettweis, "On the connection between multiplier word length limitation and roundoff noise in digital filters," *IEEE Trans. Circuit Theory*, vol. CT-19, pp. 487-491, 1972.

[12] A. Fettweis, "Roundoff noise and attenuation sensitivity in digital filters with fixed-point arithmetic," *IEEE Trans. Circuit Theory*, vol. CT-20, pp. 174-175, 1973.

[13] J.D. Ledbetter and R. Yarlagadda, "Coefficient quantization effects on pole locations for state model digital filters," *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing*, Washington, D.C. (New York, USA: IEEE 1979), pp. 351-354, 1979.

[14] P.H. Lo and Y.C. Jenq, "Minimum sensitivity realization of second-order recursive digital filter," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-30, pp. 930-937, 1982.

[15] A. Mahanta, R.C. Agarwal and S.C. Dutta Roy, "FIR filter structures having low sensitivity and roundoff noise," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-30, pp. 913-920, 1982.

[16] A. Nishihara, "Low sensitivity second-order digital filters analysis and design in terms of frequency sensitivity," *Trans. Inst. Electron. & Commun. Eng. Jpn. Sect. E*, vol. E-67, pp. 433-439, 1984.

[17] N. Ohta and T. Higuchi, "Estimation of error variance due to coefficient quantization of digital filters excited by random signals,"

Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing, Hartford, Connecticut (New York, USA: IEEE 1977), pp. 615-618, 1977.

[18] I.W. Sandberg, "A theorem concerning limit cycles in digital filters," *Proc. 7th Annual Allerton Conf. on Circuit and System Theory*, Monticello, Illinois, ed. A.H. Hadad et al. (Urbana, Illinois: Dept. of Electr. Eng., Univ. of Illinois 1969), pp. 63-68, 1969.

[19] I.W. Sandberg, "The zero-input response of digital filters using saturation arithmetics," *IEEE Trans. Circuits & Syst.*, vol. CAS-26, pp. 911-915, 1979.

[20] P.K. Sim and K.K. Pang, "Design criterion for zero-input asymptotic overflow stability of recursive digital filters in the presence of quantization," *Proc. IEEE Int. Symp. on Circuits and Systems*, Kyoto, Japan (New York, USA: IEEE 1985), pp. 1607-1611, 1985.

[21] A.N. Willson, jr., "Limit cycles due to adder overflow in digital filters," *IEEE Trans. Circuit Theory*, vol. CT-19, pp. 342-346, 1972.

[22] A.N. Willson, jr., "Limit cycles due to adder overflow in digital filters," *Proc. IEEE Int. Symp. on Circuit Theory*, North-Hollywood, California (New York, USA: IEEE 1972), pp. 223-227, 1972.

[23] L.O. Chua and T. Lin, "Chaos in digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-35, pp. 648-659, 1988.

[24] D. Williamson, S. Sridharan and P.G. McCrea, "A new approach for block floating point arithmetic in recursive filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-32, pp. 719-722, 1985.

[25] A. Fettweis, "On properties of floating-point roundoff noise," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-22, pp. 149-151, 1974.

[26] A.Z. Baraniecki and G.A. Julien, "Quantization error and limit cycle analysis in residue number system coded recursive filters," *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing*, Paris, France (New York, USA: IEEE 1982), pp. 52-55, 1982.

[27] M.H. Etzel and W.K. Jenkins, "Error correction and suppression properties of RRNS digital filters," *Proc. IEEE Int. Symp. on Circuits and Systems*, Houston, Texas (New York, USA: IEEE 1980), pp. 1117-1120, 1980.

[28] M.H. Etzel and W.K. Jenkins, "The design of specialized classes for efficient recursive filter realizations," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-30, pp. 370-380, 1982.

[29] R. Ramnarayan and F. Taylor, "Limit cycles in large moduli residue number system digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-33, pp. 912-916, 1986.

[30] A. Sripad and D.L. Snyder, "A necessary and sufficient condition for quantization errors to be uniform and white," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-25, pp. 442-448, 1977.

[31] M.T. Abu-El-Ma'atti, "The intermodulation due to multicarrier quantization," *IEEE Trans. Commun.*, vol. COM-32, pp. 1211-1214, 1984.

[32] N.M. Blachman, "Third-order intermodulation due to quantization," *IEEE Trans. Commun.*, vol. COM-29, pp. 1386-1386, 1981.

[33] N.M. Blachman, "The intermodulation and distortion due to quantization of sinusoids," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-33, pp. 1417-1426, 1985.

[34] P.J.M. Simons and M.P.G. Otten, "Intermodulation due to magnitude truncation in digital filters," *Proc. 8th European Conf. on Circuit Theory and Design*, Paris, France, ed. R. Gerber (London, England: IEE 1987), pp. 723-728, 1987.

[35] L.B. Jackson, "On the interaction of roundoff noise and dynamic range in digital filters," *Bell Syst. Tech. J.*, vol. 49, pp. 159-184, 1970.

Papers on overflow oscillations and stability (referenced in section 3)

[36] A. Fettweis, "Wave digital filters: Theory and practice," *Proc. of the IEEE*, vol. 74, pp. 270-327, 1986.

[37] S.R. Parker and S.F. Hess, "Canonic realizations of second-order digital filters due to finite precision arithmetic," *IEEE Trans. Circuit Theory*, vol. CT-19, pp. 410-413, 1972.

[38] P.M. Ebert, J.E. Mazo and M.G. Taylor, "Overflow oscillations in recursive digital filters," *Bell Syst. Tech. J.*, vol. 48, pp. 2999-3020, 1969.

[39] C.W. Barnes, "Roundoff noise and overflow in normal digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-26, pp. 154-159, 1979.

[40] C.W. Barnes, "A parametric approach to the realization of second-order digital filter sections," *IEEE Trans. Circuits & Syst.*, vol. CAS-32, pp. 530-539, 1985.

[41] G. Meyer, "Limit cycles in digital filters with fixed-point arithmetic (in German)," *Nachrichtentech. Elektron.*, vol. 26, pp. 267-273, 1976.

[42] W.L. Mills, C.T. Mullis and R.A. Roberts, "Digital filter realizations without overflow oscillations," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-26, pp. 334-338, 1978.

[43] W.L. Mills, C.T. Mullis and R.A. Roberts, "Digital filter realizations without overflow oscillations," *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing*, Tulsa, Oklahoma (New York, USA: IEEE 1978), pp. 71-74, 1978.

[44] L.E. Turner and L.T. Bruton, "Elimination of granularity and overflow limit cycles in minimum norm recursive digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-27, pp. 50-53, 1980.

[45] J.E. Mazo, "On the stability of higher order digital filters which use saturation arithmetic," *Bell Syst. Tech. J.*, vol. 57, pp. 747-763, 1978.

[46] D. Mitra, "Criteria for determining if a high-order digital filter using saturation arithmetic is free of overflow oscillations," *Bell Syst. Tech. J.*, vol. 56, pp. 1679-1699, 1977.

[47] D. Mitra, "Large amplitude self-sustained oscillations in difference equations which describe digital filter sections using saturation arithmetic," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-25, pp. 134-143, 1977.

[48] D. Mitra, "Summary of some results on large amplitude, self-sustained oscillations in high order digital filter sections using saturation arithmetic," *Proc. IEEE Int. Symp. on Circuits and Systems*, Phoenix, Arizona (New York, USA: IEEE 1977), pp. 195-198, 1977.

[49] D. Mitra, "The absolute stability of high-order discrete-time systems utilizing the saturation nonlinearity," *IEEE Trans. Circuits & Syst.*, vol. CAS-25, pp. 365-371, 1978.

[50] D. Mitra, "Summary of results on the absolute stability of high-order, discrete-time systems utilizing the saturation nonlinearity," *Proc. IEEE Int. Symp. on Circuits and Systems*, New York, N.Y. (New York, USA: IEEE 1978), pp. 1029-1033, 1978.

[51] P.K. Sim and K.K. Pang, "On the asymptotic stability of an N-th order nonlinear recursive digital filter," *IREECON Int. Sydney '83: Digest of papers 19th Int. Electr. Convention and Exhibition*, Sydney, Australia (Sidney, Australia: Inst. of Radio and Electron. Eng. Australia 1983), pp. 108-110, 1983.

[52] H.D. Montgomery, "A non-linear digital oscillator," *Proc. IEEE Int. Conf. on Communications*, Philadelphia, Pennsylvania (New York, USA: IEEE 1972), pp. 33.3-33.8, 1972.

[53] A.N. Willson, jr., "A stability criterion for non-autonomous difference equations with applications to the design of a digital FSK oscillator," *IEEE Trans. Circuits & Syst.*, vol. CAS-21, pp. 124-130, 1974.
[54] T. Higuchi and H. Takeo, "A state-space approach for the elimination of limit cycles in digital filters with arbitrary structures," *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing*, Washington, D.C. (New York, USA: IEEE 1979), pp. 355-358, 1979.
[55] M. Kawamata and T. Higuchi, "A sufficient condition for the absence of overflow oscillations in arbitrary digital filters based on the element equations," *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing*, Denver, Colorado (New York, USA: IEEE 1980), pp. 85-88, 1980.
[56] K.T. Erickson and A.N. Michel, "Stability analysis of fixed-point digital filters using computer generated Lyapunov functions Part I: direct form and coupled form filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-32, pp. 113-132, 1985.
[57] K.T. Erickson and A.N. Michel, "Stability analysis of fixed-point digital filters using computer generated Lyapunov functions Part II: wave digital and lattice digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-32, pp. 132-139, 1985.
[58] B. Eckhardt and W. Winkelnkemper, "Implementation of a second-order digital filter section with stable overflow behaviour," *Nachrichtentech. Z.*, vol. NTZ-26, pp. 282-284, 1973.
[59] E. Auer, "New methods to guarantee overflow-stability for the canonic second-order digital filter structure," *Proc. 7th European Conf. on Circuit Theory and Design*, Prague, Czechoslovakia, ed. V. Zina and J. Krasil (Amsterdam, The Netherlands: North-Holland), pp. 477-480, 1985.
[60] A.G. Bolton, "A two's complement overflow limit cycle free digital filter structure," *IEEE Trans. Circuits & Syst.*, vol. CAS-31, pp. 1045-1046, 1984.
[61] V. Singh, "Realization of two's complement overflow limit cycle free state-space digital filters: a frequency-domain viewpoint," *IEEE Trans. Circuits & Syst.*, vol. CAS-33, pp. 1042-1044, 1986.
[62] C.W. Barnes and A.T. Fam, "Minimum norm recursive digital filters that are free of overflow limit cycles," *IEEE Trans. Circuits & Syst.*, vol. CAS-24, pp. 569-574, 1977.
[63] U. Bernhardt, H. Lubenow and H. Unger, "On avoiding limit cycles in digital filters (in German)," *Nachrichtentech. Elektron.*, vol. 27, pp. 433-435, 1977.
[64] L.M. Gol'denberg and B.D. Matyushkin, "Conditions for the absolute stability of processes in recursive digital filters," *Autom. & Remote Control*, vol. 40, pp. 1161-1169, 1979.
[65] K. Kurosawa, "Limit cycle and overflow free digital filters (in Japanese; English abstr.)," *Trans. Inst. Electron. & Commun. Eng. Jpn. Part A*, vol. A-65, pp 263-264. English abstract: *ibid. Sect. E*, vol. E-65, p. 180, 1982.
[66] P.H. Lo and Y.C. Jenq, "On the overflow problem in a second-order digital filter," *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing*, Atlanta, Georgia (New York, USA: IEEE 1981), pp. 1221-1226, 1981.
[67] V. Singh, "On the realization of two's complement overflow limit cycle free state-space digital filters," *Proc. of the IEEE*, vol. 74, pp. 1287-1288, 1986.
[68] Y.Z. Tsypkin, "Frequency criteria for the absolute stability of nonlinear sampled-data systems," *Autom. & Remote Control*, vol. 25, pp. 261-267, 1964.

[69] Y.Z. Tsypkin, "A criterion for absolute stability of automatic pulse systems with monotonic characteristics of the nonlinear element," *Sov. Phys.-Dokl.*, vol. 9, pp. 263-266, 1964.

[70] A. Fettweis, "Pseudopassivity, sensitivity and stability of wave digital filters," *IEEE Trans. Circuit Theory*, vol. CT-19, pp. 668-673, 1972.

[71] A. Fettweis and K. Meerkötter, "Suppression of parasitic oscillations in wave digital filters," *Proc. IEEE Int. Symp. on Circuits and Systems*, San Fransisco, California (New York, USA: IEEE 1974), pp. 682-686, 1974.

[72] A. Fettweis and K. Meerkötter, "Suppression of parasitic oscillations in wave digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-22, pp. 239-246, 1975.

[73] A. Fettweis and K. Meerkötter, "Correction to 'Suppression of parasitic oscillations in wave digital filters'," *IEEE Trans. Circuits & Syst.*, vol. CAS-22, p. 575, 1975.

[74] A. Fettweis and K. Meerkötter, "Suppression of parasitic oscillations in half-synchronic wave digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-23, pp. 125-126, 1976.

[75] J. Payan and J.I. Acha, "Parasitic oscillations in normalized digital structures," *Int. J. Electron.*, vol. 60, pp. 591-595, 1986.

[76] A.H. Gray, jr., "Passive cascaded lattice digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-27, pp. 337-344, 1980.

[77] D.T. Nguyen, "Overflow oscillations in digital lattice filters," *IEE Proc. G*, vol. 128, pp. 269-272, 1981.

[78] J.O. Smith, "Elimination of limit cycles in time-varying lattice filters," *Proc. IEEE Int. Symp. on Circuits and Systems*, San Jose, Califor nia (New York, USA: IEEE 1986), pp. 197-200, 1986.

[79] M. Takizawa, H. Kishi and N. Hamada, "Synthesis of lattice digital filters by the state-variable method," *Electron. & Commun. Jpn.*, vol. 65-A, no.4, pp. 27-36, 1982.

[80] C.W. Barnes and S. Shinnaka, "Finite word effects in block-state realizations of fixed-point digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-27, pp. 345-349, 1980.

[81] C.W. Barnes and S. Shinnaka, "Stability domains for second-order recursive digital filters in normal form with 'matrix power' feedback, *IEEE Trans. Circuits & Syst.*, vol. CAS-27, pp. 841-843, 1980.

[82] A.T. Fam, "Multiplexing preserving filters," *Proc. 12th Asilomar Conf. on Circuits, Systems and Computers*, Pacific Grove, California (North-Hollywood, California, USA: Western Periodicals), pp. 257-259, 1978.

[83] A.T. Fam and C.W. Barnes, "Nonminimal realizations of fixed-point digital filters that are free of all finite word-length limit cycles," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-27, pp. 149-153, 1979.

[84] A. Fettweis and K. Meerkötter, "On parasitic oscillations in digital filters under looped conditions," *IEEE Trans. Circuits & Syst.*, vol. CAS-24, pp. 475-481, 1977.

[85] T.A.C.M. Claasen and L.O.G. Kristiansson, "Improvement of overflow behaviour of second-order digital filters by means of error feedback," *Electron. Lett.*, vol. 10, pp. 240-241, 1974.

[86] T.A.C.M. Claasen, W.F.G. Mecklenbräuker and J.B.H. Peek, "Necessary and sufficient conditions for the absence of overflow phenomena in a second-order recursive digital filter," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-23, pp. 509-515, 1975.

[87] T.A.C.M. Claasen, W.F.G. Mecklenbräuker and J.B.H. Peek, "On the stability of the forced response of digital filters with overflow non-

linearities," *IEEE Trans. Circuits & Syst.*, vol. CAS-22, pp. 692-696, 1975.

[88] A.N. Willson, jr., "Some effects of quantization and adder overflow on the forced response of digital filters," *Bell Syst. Tech. J.*, vol. 51, pp. 863-887, 1972.

[89] A.N. Willson, jr., "Error-feedback circuits for digital filters," *Electron. Lett.*, vol. 12, pp. 450-452, 1976.

[90] A.N. Willson, jr., "Computation of the periods of forced overflow oscillations in digital filters," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-24, pp. 89-97, 1976.

[91] P.K. Sim and K.K. Pang, "Effects of input-scaling on the asymptotic overflow stability properties of second-order recursive digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-32, pp. 1008-1015, 1985.

[92] H. Samueli and A.N. Willson, jr., "Almost periodic forced overflow oscillations in digital filters," *Proc. IEEE Int. Symp. on Circuits and Systems*, Houston, Texas (New York, USA: IEEE), pp. 1108-1112, 1980.

[93] H. Samueli and A.N. Willson, jr., "Almost period P sequences and the analysis of forced overflow oscillations in digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-29, pp. 510-515 , 1982.

[94] H. Samueli and A.N. Willson, jr., "Nonperiodic forced overflow oscillations in digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-30, pp. 709-722 , 1983.

[95] A. Krishnan and R. Subramanian, "Jump phenomena in digital filters," *J. Inst. Electron. & Telecommun. Eng. (India)*, vol. 25, pp. 373-376, 1979.

[96] L.O.G. Kristiansson, "Jump phenomena in digital filters," *Electron. Lett.*, vol. 10, pp. 14-15, 1974.

Papers on quantization stability and limit cycles (referenced in section 4)

[97] T.A.C.M. Claasen, W.F.G. Mecklenbräuker and J.B.H. Peek, "Some remarks on the classification of limit cycles in digital filters," *Philips Res. Rep.*, vol. 28, pp. 297-305, 1973.

[98] D.C. Munson, jr., "Accessibility of zero-input limit cycles," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-29, pp. 1027-1032, 1981.

[99] D.C. Munson, jr., "Accessibility of zero-input limit cycles," *Proc. IEEE Int. Symp. on Circuits and Systems*, Chicago, Illinois (New York, USA: IEEE 1981), pp. 821-824, 1981.

[100] K. Meerkötter and W. Wegener, "A new second-order digital filter without parasitic oscillations," *Arch. Elektron. & Übertragungstech.*, vol. AEU-29, pp. 312-314, 1975.

[101] K. Meerkötter, "Realization of limit cycle-free second-order digital filters," *Proc. IEEE Int. Symp. on Circuits and Systems*, Munich, Germany (New York, USA: IEEE 1976), pp. 295-298, 1976.

[102] H.J. Butterweck, "Suppression of parasitic oscillations in second-order digital filters by means of a controlled-rounding arithmetic," *Arch. Elektron. & Übertragungstech.*, vol. AEU-29, pp. 371-374, 1975.

[103] D. Mitra and V.B. Lawrence, "Summary of results on controlled rounding arithmetics, for direct-form digital filters, that eliminate all self-sustained oscillations," *Proc. IEEE Int. Symp. on Circuits and Systems*, New York, N.Y. (New York, USA: IEEE 1978), pp. 1023-1028, 1978.

[104] D. Mitra and V.B. Lawrence, "Controlled rounding arithmetics for second-order direct-form digital filters, that eliminate all self-sustained oscillations," *IEEE Trans. Circuits & Syst.*, vol. CAS-28, pp. 894-905, 1981.

[105] G. Verkroost and H.J. Butterweck, "Suppression of parasitic oscillations in wave digital filters and related structures by means of controlled rounding," *Arch. Elektron. & Ubertragungstech.*, vol. AEU-30, pp. 181-186, 1976.
[106] G. Verkroost and H.J. Butterweck, "Suppression of parasitic oscillations in wave digital filters and related structures by means of controlled rounding," *Proc. IEEE Int. Symp. on Circuits and Systems*, Munich, Germany (New York, USA: IEEE 1976), pp. 628-629, 1976.
[107] G. Verkroost, "A general second-order digital filter with controlled rounding to exclude limit cycles for constant input signals," *IEEE Trans. Circuits & Syst.*, vol. CAS-24, pp. 428-431, 1977.
[108] G. Verkroost, "Een tweede-orde digitaal filter waarin door middel van gestuurde kwantisering 'limit cycles' voorkomen worden," *Nederl. Elektron. en Radio Genootschap NERG*, vol. 42, pp. 111-114, 1977.
[109] C. Eswaran and A. Antoniou, "Wave digital biquads that are free of limit cycles under zero- and constant-input conditions," *Proc. IEEE Int. Symp. on Circuits and Systems*, Montreal, Canada (New York, USA: IEEE 1984), pp. 723-726, 1984.
[110] H.K. Kwan, "A multi-output second-order digital filter structure for VLSI implementation," *IEEE Trans. Circuits & Syst.*, vol. CAS-32, pp. 108-109, 1985.
[111] H.K. Kwan, "A multi-output second-order digital filter without limit cycle oscillations," *IEEE Trans. Circuits & Syst.*, vol. CAS-32, pp. 974-975, 1985.
[112] H.K. Kwan, "A multi-output wave digital biquad using magnitude truncation instead of controlled rounding," *IEEE Trans. Circuits & Syst.*, vol. CAS-32, pp. 1185-1187, 1985.
[113] L.E. Turner, "Second-order recursive digital filter that is free from all constant-input limit cycles," *Electron. Lett.*, vol. 18, pp. 743-745, 1982.
[114] H.J. Butterweck, A.C.P. Van Meer and G. Verkroost, "New second-order digital filter sections without limit cycles," *Proc. EUSIPCO-83, 2nd European Signal Processing Conf.*, Erlangen, Germany, ed. H.W. Schüßler (Amsterdam, The Netherlands: North-Holland 1983), pp. 97-98, 1983.
[115] H.J. Butterweck, A.C.P. Van Meer and G. Verkroost, "New second-order digital filter sections without limit cycles," *IEEE Trans. Circuits & Syst.*, vol. CAS-31, pp. 141-146, 1984.
[116] E.S.K. Liu and L.E. Turner, "Quantisation effects in second-order wave digital filters," *Electron. Lett.*, vol. 19, pp. 487-488, 1983.
[117] M. Miyata, "Roundoff noise control in time domain for digital filters and oscillators," *Electron. & Commun. Jpn.*, vol. 63-A, no. 10, pp. 1-8, 1980.
[118] P.R. Moon, "Limit cycle suppression by diagonally dominant Lyapunov functions in state-space digital filters," *Proc. IEEE Int. Symp. on Circuits and Systems*, Montreal, Canada (New York, USA: IEEE 1984), pp. 1082-1085, 1984.
[119] V. Singh, "Formulation of a criterion for the absence of limit cycles in digital filters designed with one quantizer," *IEEE Trans. Circuits & Syst.*, vol. CAS-32, pp. 1062-1064, 1985.
[120] V. Singh, "A new realizability condition for limit cycle free state-space digital filters employing saturation arithmetic," *IEEE Trans. Circuits & Syst.*, vol. CAS-32, pp. 1070-1071, 1985.
[121] P.P. Vaidyanathan, "The discrete-time bounded-real lemma in digital filtering," *IEEE Trans. Circuits & Syst.*, vol. CAS-32, pp. 918-924, 1985.
[122] P.P. Vaidyanathan and V.C. Liu, "An improved sufficient condition for

absence of limit cycles in digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-34, pp. 319-322, 1987.

[123] M.J.J.C. Annegarn, "Chopping operations in wave digital filters," *Electron. Lett.*, vol. 11, pp. 378-380, 1975.

[124] G. Lucioni, "Alternative method to magnitude truncation in wave digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-34, pp. 106-107, 1987.

[125] C.D.R. De Vaal and R. Nouta, "Suppression of parasitic oscillations in floating point wave digital filters," *Proc. IEEE Int. Symp. on Circuits and Systems*, New York, N.Y. (New York, USA: IEEE), pp. 1018-1022, 1978.

[126] C.D.R. De Vaal and R. Nouta, "On the suppression of zero-input parasitic oscillations in floating point wave digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-27, pp. 144-145, 1980.

[127] P.S.R. Diniz and A. Antoniou, "On the elimination of constant-input limit cycles in digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-31, pp. 670-671, 1984.

[128] M. Kawamata, "Synthesis of limit cycle free digital filters based on the state equations (in Japanese; English abstract)," *Trans. Inst. Electron. & Commun. Eng. Jpn. Part A*, vol. A-63, pp. 870-877. English abstract: *ibid. Sect. E*, vol. E-63, p. 870, 1980.

[129] M. Kawamata and T. Higuchi, "A systematic approach to synthesis of limit cycle free digital filters," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-31, pp. 212-214, 1983.

[130] M. Kawamata and T. Higuchi, "Synthesis of limit cycle free state-space digital filters with minimum coefficient quantization error," *Proc. IEEE Int. Symp. on Circuits and Systems*, Newport Beach, California (New York, USA: IEEE 1983), pp. 827-830, 1983.

[131] M. Kawamata and T. Higuchi, "On the absence of limit cycles in a class of state-space digital filters which contains minimum noise realizations," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-32, pp. 928-930, 1984.

[132] A. Nishihara, "Design of limit cycle-free digital biquad filters," *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing*, Tokyo, Japan (New York, USA: IEEE 1986), pp. 517-520, 1986.

[133] L.E. Turner and L.T. Bruton, "Elimination of limit cycles recursive digital filters using a generalized minimum norm," *Proc. IEEE Int. Symp. on Circuits and Systems*, Chicago, Illinois (New York, USA: IEEE 1981), pp. 817-820, 1981.

[134] L.E. Turner, "Elimination of constant-input limit cycles in recursive digital filters using a generalised minimum norm," *IEE Proc. G*, vol. 130, pp. 69-77, 1983.

[135] K.M. Wong and R.A. King, "Method to suppress limit cycle oscillations in digital filter," *Electron. Lett.*, vol. 10, pp. 55-57, 1974.

[136] T.L. Chang, "Suppression of limit cycles in digital filters designed with one magnitude-truncation quantizer," *IEEE Trans. Circuits & Syst.*, vol. CAS-28, pp. 107-111, 1981.

[137] M. Renfors, B. Sikström and L. Wanhammer, "LSI implementation of limit cycle free digital filters using error-feedback techniques," *Proc. EUSIPCO-83, 2nd European Signal Processing Conf.*, Erlangen, Germany, ed. H.W. Schüßler (Amsterdam, The Netherlands: North-Holland 1983), pp. 107-110, 1983.

[138] S. Sridharan and D. Williamson, "Comments on 'Suppression of limit cycles in digital filters designed with one magnitude-truncation quantizer'," *IEEE Trans. Circuits & Syst.*, vol. CAS-31, pp. 235-236, 1984.

[139] I.H. Witten and P.G. McCrea, "Suppressing limit cycles in digital incremental computers," *IEEE Trans. Circuits & Syst.*, vol. CAS-28, pp. 723-730, 1981.

[140] V.B. Lawrence and D. Mitra, "Digital filters with control of limit cycles," *U.S. Patent* no. 4213187, 1980.
[141] V.B. Lawrence and E.A. Lee, "Quantization schemes for recursive digital filters," *Proc. IEEE Int. Symp. on Circuits and Systems*, Rome, Italy (New York, USA: IEEE 1982), pp. 690-694, 1982.
[142] L.E. Turner and L.T. Bruton, "Elimination of zero-input limit cycles by bounding the state transition matrix," *Proc. IEEE Int. Symp. on Circuits and Systems*, Phoenix, Arizona (New York, USA: IEEE 1977), pp. 199-202, 1977.
[143] L.E. Turner and L.T. Bruton, "Elimination of zero-input limit cycles by bounding the state transition matrix," *Int. J. Circuit Theory & Appl.*, vol. 7, pp. 97-111, 1979.
[144] T.A.C.M. Claasen, W.F.G. Mecklenbräuker and J.B.H. Peek, "Second-order digital filters with only one magnitude-truncation quantizer and having practically no limit cycles," *Electron. Lett.*, vol. 9, pp. 531-532, 1973.
[145] T.A.C.M. Claasen, W.F.G. Mecklenbräuker and J.B.H. Peek, "Remarks on the zero-input behaviour of second-order digital filters designed with one magnitude-truncation quantizer," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-23, pp. 240-242, 1975.
[146] R. Ishii, Y. Mitome and S. Kawasaki, "Limit cycles in a second-order all-pole digital system (in Japanese; English abstract)," *Trans. Inst. Electron. & Commun. Eng. Jpn. Part A*, vol. A-63, pp. 51-58. English abstract: *ibid. Sect. E*, vol. E-63, p. 120, 1980.
[147] C.Y. Kao, "An analysis of limit cycles due to sign-magnitude truncation in multiplication in recursive digital filters," *Proc. 5th Asilomar Conf. on Circuits, Systems and Computers*, Pacific Grove, California (North-Hollywood, California, USA: Western Periodicals 1971), pp. 349-353, 1971.
[148] A. Lepschy, G.A. Mian and U. Viaro, "Parameter space quantization in fixed-point digital filters," *Electron. Lett.*, vol. 22, pp. 384-386, 1986.
[149] A. Lepschy, G.A. Mian and U. Viaro, "Stability analysis of second-order direct form digital filters with two roundoff quantizers," *IEEE Trans. Circuits & Syst.*, vol. CAS-33, pp. 824-826, 1986.
[150] T.A.C.M. Claasen, W.F.G. Mecklenbräuker and J.B.H. Peek, "A comparison between the stability of second-order filters with various arithmetics," *Proc. 1st European Conf. on Circuit Theory and Design*, London, England (London, England: IEE 1974), pp. 354-358, 1974.
[151] T.A.C.M. Claasen, W.F.G. Mecklenbräuker and J.B.H. Peek, "Frequency domain criteria for the absence of zero-input limit cycles in nonlinear discrete-time systems, with applications to digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-22, pp. 232-239, 1975.
[152] E.D. De Luca and G.O. Martens, "A new coefficient quantization scheme to suppress limit cycles in state-variable digital filters," *Proc. 6th European Conf. on Circuit Theory and Design*, Stuttgart, Germany, ed. E. Lueder (Berlin, Germany: VDE-Verlag 1983), pp. 157-159, 1983.
[153] L.M. Gol'denberg, "Stability of recursive digital filters," *Avtom. & Telemekh.*, vol. 7, pp. 22-27, 1977.
[154] A.M.B. Pavani and J. Szczupak, "A mathematical model for digital filters under limit condition," *Proc. IEEE Int. Symp. on Circuits and Systems*, Kyoto, Japan (New York, USA: IEEE 1985), pp. 1615-1616, 1985.
[155] B.H. Nam and N.H. Kim, "Stability analysis of modified coupled-form digital filter using a constructive algorithm," *Trans. Korean Inst. Electr. Eng.*, vol. 34, pp. 430-435, 1985.
[156] E.P.F. Kan and J.K. Aggarwal, "Minimum-deadband design of digital fil-

ters," *IEEE Trans. Audio & Electroacoust.*, vol. AU-19, pp. 292-296, 1971.

[157] R.B. Kieburtz, "An experimental study of roundoff effects in a tenth-order recursive digital filter," *IEEE Trans. Commun.*, vol. COM-21, pp. 757-763, 1973.

[158] R.B. Kieburtz, "Rounding and truncation limit cycles in a recursive digital filter," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-22, p. 73, 1974.

[159] L.M. Gol'denberg and B.D. Matyushkin, "Stability of recursive second-order digital filters," *Avtom. & Telemekh.*, vol. 12, p. 5458, 1978.

[160] O. Monkewich and W. Steenaart, "Deadband effects and limit cycles in stored-product digital filters," *Proc. IEEE Int. Symp. on Circuits and Systems*, Chicago, Illinois (New York, USA: IEEE), pp. 813-816, 1981.

[161] Y.S. Itskovich, "Roundoff noise in digital filters with small input signal (transl. of: Radiotekhnika, vol. 37, no. 5, p. 39, May 1982)," *Telecommun. & Radio Eng. Part 2*, no.5, vol. 36/37, pp. 74-77, 1982.

[162] H.J. Butterweck, F.H.R. Lucassen and G. Verkroost, "Subharmonics and related quantization effects in periodically excited recursive digital filters," *Proc. EUSIPCO-83, 2nd European Signal Processing Conf.*, Erlangen, Germany, ed. H.W. Schüßler (Amsterdam, The Netherlands: North-Holland 1983), pp. 57-59, 1983.

[163] H.J. Butterweck, F.H.R. Lucassen and G. Verkroost, "Subharmonics and other quantization effects in periodically excited recursive digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-33, pp. 958-964, 1986.

[164] H.J. Butterweck, "Subharmonics and other quantization effects in periodically excited recursive digital filters," *Proc. IEEE Int. Symp. on Circuits and Systems*, San Jose, California (New York, USA: IEEE 1986), pp. 861-862, 1986.

[165] R. Ishii and M. Kato, "An effect of a limit cycle on an output signal," *Proc. IEEE Int. Symp. on Circuits and Systems*, Rome, Italy (New York, USA: IEEE 1982), pp. 1041-1044, 1982.

[166] D.R. Morgan and A. Aridgides, "Discrete-time distortion analysis of quantized sinusoids," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-33, pp. 323-326, 1985.

[167] W.H. Storzbach, "Forced oscillations in recursive digital filters," *Proc. IEEE Int. Symp. on Circuit Theory*, North-Hollywood, California (New York, USA: IEEE 1972), pp. 233-236, 1972.

[168] M.J. Werter, "Suppression of subharmonics in digital filters for discrete-time periodic input signals with period P or a divisor of P," *Proc. EUSIPCO-86, 3rd European Signal Processing Conf.*, The Hague, The Netherlands, ed. I.T. Young et al. (Amsterdam, The Netherlands: North-Holland 1986), pp. 179-182, 1986.

[169] M.J. Werter, "Digital filter sections which suppress subharmonics for discrete-time periodic input signals with period P or a divisor of P," *Proc. IEEE Int. Symp. on Circuits and Systems*, San Jose, California (New York, USA: IEEE 1986), pp. 863-866, 1986.

[170] M.J. Werter, "A new decomposition of discrete-time periodic signals," *Proc. 8th European Conf. on Circuit Theory and Design*, Paris, France, ed. R. Gerber (London, England: IEE 1987), pp. 139-144, 1987.

[171] M. Büttner, "A novel approach to eliminate limit cycles in digital filters with a minimum increase in the quantization noise," *Proc. IEEE Int. Symp. on Circuits and Systems*, Munich, Germany (New York, USA: IEEE 1976), pp. 291-294, 1976.

[172] M. Büttner, "Elimination of limit cycles in digital filters with very low increase in the quantization noise," *IEEE Trans. Circuits & Syst.*, vol. CAS-24, pp. 300-304, 1977.

[173] P. Rashidi and R.E. Bogner, "Suppression of limit cycles oscillations in second-order recursive digital filters," *Aust. Telecommun. Res.*, vol. 12, pp. 8-16, 1978.
[174] G. Spahlinger, "Suppression of limit cycles in digital filters by statistical rounding," *Proc. IEEE Int. Symp. on Circuits and Systems*, Kyoto, Japan (New York, USA: IEEE 1985), pp. 1603-1606, 1985.
[175] M.H. Ackroyd and H.M. Liu, "Limit cycle suppression in digital filters," *Saraga Memorial Colloquim on Electronic Filters*, London, England (London, England: IEE 1982), pp. 5/1-6, 1982.
[176] H.M. Liu and M.H. Ackroyd, "Suppression of limit cycles in digital filters by random dither," *Radio & Electron. Eng.*, vol. 53, pp. 235-240, 1983.
[177] R.B. Kieburtz and K.V. Mina, "Digital filter circuit," *U.S. Patent* no. 3906199, 1975.
[178] R.B. Kieburtz, K.V. Mina and V.B. Lawrence, "Control of limit cycles in recursive digital filters by randomized quantization," *Proc. IEEE Int. Symp. on Circuits and Systems*, Munich, Germany (New York, USA: IEEE 1976), pp. 624-627, 1976.
[179] R.B. Kieburtz, K.V. Mina and V.B. Lawrence, "Control of limit cycles in recursive digital filters by randomized quantization," *IEEE Trans. Circuits & Syst.*, vol. CAS-24, pp. 291-299, 1977.
[180] V.B. Lawrence and K.V. Mina, "Control of limit cycles oscillations in second-order recursive digital filters using constrained random quantization," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-26, pp. 127-134, 1978.
[181] C.Y. Kao, "Apparatus for suppressing limit cycles due to quantization in digital filters," *U.S. Patent* no. 3749895, 1973.
[182] A.I. Abu-El-Haija, "A tight bound on $\Sigma|h(n)|$ for general second-order $H(z)$," *IEEE Trans. Circuits & Syst.*, vol. CAS-29, pp. 492-497, 1982.
[183] U. Appel, "Bounds on second-order digital filter limit cycles," *IEEE Trans. Circuits & Syst.*, vol. CAS-22, pp. 630-632, 1975.
[184] A.A. Belal, "On the quantization error bounds in second-order digital filters with complex conjugate poles," *IEEE Trans. Circuits & Syst.*, vol. CAS-24, p. 45, 1977.
[185] T.A. Brubaker and J.N. Gowdy, "Limit cycles in digital filters," *IEEE Trans. Autom. Control*, vol. AC-17, pp. 675-677, 1972.
[186] T.L. Chang, "A note on upper bounds on limit cycles in digital filters," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-24, pp. 99-100, 1976.
[187] A.M. Fink, "A bound on quantization errors in second-order digital filters with complex poles that is tight for small theta," *IEEE Trans. Circuits & Syst.*, vol. CAS-23, pp. 325-326, 1976.
[188] D.G. He and S.K. Han, "Novel approximations to $\Sigma|h(n)|$ for second-order digital filters," *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing*, Tampa, Florida (New York, USA: IEEE 1985), pp. 1731-1734, 1985.
[189] L.B. Jackson, "An analysis of limit cycles due to multiplication rounding in recursive digital (sub-) filters," *Proc. 7th Annual Allerton Conf. on Circuit and System Theory*, Monticello, Illinois, ed. A.H. Hadad et al. (Urbana, Illinois: Dept. of Electr. Eng., Univ. of Illinois 1969), pp. 69-78, 1969.
[190] L.B. Jackson, "Comments on 'Quantizer-induced digital controller limit cycles'," *IEEE Trans. Autom. Control*, vol. AC-15, pp. 614-615, 1970.
[191] H. Kubota and S. Tsuji, "An upper bound on the RMS value of limit cycles in digital filters and a reduction method (in Japanese; English abstr.)," *Trans. Inst. Electron. & Commun. Eng. Jpn. Part A*, vol. A-64,

pp 63-70. English abstract: *ibid. Sect. E*, vol. E-64, p. 38, 1981.
[192] H. Kubota, T. Yoshida and S. Tsuji, "A consideration on limit cycles due to value-truncation errors in digital filters (in Japanese; English abstr.)," *Trans. Inst. Electron. & Commun. Eng. Jpn. Part A*, vol. A-64, pp 371-377. English abstract: *ibid. Sect. E*, vol. E-64, p. 365, 1981.
[193] J.L. Long and T.N. Trick, "An absolute bound on limit cycles due to roundoff errors in digital filters," *IEEE Trans. Audio & Electroacoust.*, vol. AU-21, pp. 27-30, 1973.
[194] J.L. Long and T.N. Trick, "A note on absolute bounds on quantization errors in fixed-point implementations of digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-22, pp. 567-570, 1975.
[195] S.R. Parker, "The phenomena of quantization error and limit cycles in fixed point digital filters," *Proc. 28th National Electronics Conf.*, Chicago, Illinois, ed. R.E. Horton (Oak Brook, Illinois, USA: National Electronics Conf. 1972), pp. 38-42, 1972.
[196] S.R. Parker and S. Yakowitz, "A general method for calculating quantization error bounds in fixed-point multivariable digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-22, pp. 570-572, 1975.
[197] B. Steinle and P. Gruber, "Comments on upper bounds on limit cycles in digital filters of second-order," *Signal Process.*, vol. 8, pp. 415-422, 1985.
[198] Z. Unver and K. Abdullah, "A tighter practical bound on quantization errors in second-order digital filters with complex conjugate poles," *IEEE Trans. Circuits & Syst.*, vol. CAS-22, pp. 632-633, 1975.
[199] J. Wisliceny, "Estimation of the maximum amplitude of quantisation-conditioned limit cycles in linear recursive digital filters (in German)," *Z. Elektr. Inf.- & Energietech.*, vol. 10, pp. 330-338, 1980.
[200] S. Yakowitz and S.R. Parker, "Computation of bounds for digital filter quantization errors," *IEEE Trans. Circuit Theory*, vol. CT-20, pp. 391-396, 1973.
[201] D. Mitra, "A bound on limit cycles in digital filters which exploits a particular structural property of the quantization," *IEEE Trans. Circuits & Syst.*, vol. CAS-24, pp. 581-589, 1977.
[202] D. Mitra, "A bound on limit cycles in digital filters which exploits a particular structural property of the quantization," *Proc. IEEE Int. Symp. on Circuits and Systems*, Phoenix, Arizona (New York, USA: IEEE 1977), pp. 605-610, 1977.
[203] I.W. Sandberg and J.F. Kaiser, "A bound on limit cycles in fixed-point implementations of digital filters," *IEEE Trans. Audio & Electroacoust.*, vol. AU-20, pp. 110-112, 1972.
[204] J.A. Heinen, "A bound on the norm of the individual state variables of a digital filter," *IEEE Trans. Circuits & Syst.*, vol. CAS-32, pp. 1073-1074, 1985.
[205] P.H. Lo and Y.C. Jenq, "An ℓm-norm bound for state variables in second-order recursive digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-28, pp. 1170-1171, 1981.
[206] A.I. Abu-El-Haija and A.M. Peterson, "On limit cycle amplitudes in error-feedback digital filters," *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing*, Atlanta, Georgia (New York, USA: IEEE 1981), pp. 1227-1230, 1981.
[207] B. Liu and M.R. Bateman, "Limit cycle bounds for digital filters with error spectrum shaping," *Proc. 14th Asilomar Conf. on Circuits, Systems and Computers*, Pacific Grove, California, ed. D.E. Kirk (North-Hollywood, California, USA: Western Periodicals 1981), pp. 215-218, 1980.
[208] L. Claesen, J. Vandewalle and H. De Man, "General bounds on parasitic oscillations in arbitrary digital filters and their application in

CAD," *Proc. IEEE Int. Symp. on Circuits and Systems*, Montreal, Canada (New York, USA: IEEE 1984), pp. 747-750, 1984.
[209] L.B. Jackson, "Limit cycles in state-space structures for digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-26, pp. 67-68, 1979.
[210] L.B. Jackson and N.H.K. Judell, "Addendum to 'Limit cycles in state-space structures for digital filters'," *IEEE Trans. Circuits & Syst.*, vol. CAS-27, p. 320, 1980.
[211] G.U. Jatnieks and B.A. Shenoi, "Zero-input limit cycles in coupled digital filters," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-22, pp. 146-149, 1974.
[212] P.K. Sim and K.K. Pang, "Quantization phenomena in a class of complex biquad recursive digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-33, pp. 892-899, 1986.
[213] A.I. Abu-El-Haija and A.M. Peterson, "Limit cycle oscillations in digital incremental computers," *IEEE Trans. Circuits & Syst.*, vol. CAS-25, pp. 902-908, 1978.
[214] A.I. Abu-El-Haija, "Correction to 'Limit cycle oscillations in digital incremental computers'," *IEEE Trans. Circuits & Syst.*, vol. CAS-26, p. 898, 1979.
[215] T. Thong and B. Liu, "Limit cycles in combinatorial filters using two's complement truncation arithmetic," *Proc. 8th Asilomar Conf. on Circuits, Systems and Computers*, Pacific Grove, California (North-Hollywood, California, USA: Western Periodicals 1975), pp. 51-55, 1974.
[216] T. Thong and B. Liu, "Limit cycles in combinatorial implementation of digital filters," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-24, pp. 248-256, 1976.
[217] S.R. Parker and F.A. Perry, "Hidden limit cycles and error bounds in wave digital filters," *Proc. IEEE Int. Symp. on Circuits and Systems*, Tokyo, Japan (New York, USA: IEEE 1979), pp. 372-373, 1979.
[218] M. Büttner and J. Schloss, "On the experimental investigations of bounds of the amplitude of limit cycles," *Proc. 2nd European Conf. on Circuit Theory and Design*, Genova, Italy (London, England: IEE 1976), pp. 709-716, 1976.
[219] M. Büttner, "Some experimental results concerning random-like noise and limit cycles in recursive digital filters," *Nachrichtentech. Z.*, vol. NTZ-28, pp. 402-406, 1975.
[220] G.A. Maria and M.M. Fahmy, "Limit cycle oscillations in a cascade of first- and second-order digital filter sections," *IEEE Trans. Circuits & Syst.*, vol. CAS-22, pp. 131-134, 1975.
[221] V.B. Lawrence and K.V. Mina, "A new and interesting class of limit cycles," *Proc. IEEE Int. Symp. on Circuits and Systems*, Phoenix, Arizona (New York, USA: IEEE 1977), pp. 191-194, 1977.
[222] V.B. Lawrence and K.V. Mina, "A new and interesting class of limit cycles in recursive digital filters," *Bell Syst. Tech. J.*, vol. 58, pp. 379-408, 1979.
[223] T. Kaneko, "Limit cycle oscillations in floating-point digital filters," *IEEE Trans. Audio & Electroacoust.*, vol. AU-12, pp. 100-106, 1973.
[224] A. Lacroix, "Limit cycles in floating point digital filters," *18th Midwest Symp. on Circuits and Systems*, Montreal, Canada, ed. M.N.S. Swamy (North-Hollywood, California, USA: Western Periodicals, 1975), pp. 475-479, 1975.
[225] A. Lacroix, "Underflow limit cycles in floating point digital filters," *Proc. Florence Conf. on Digital Signal Processing*, Florence, Italy, pp. 75-84, 1975.
[226] A. Lacroix, "Limit cycles in floating point digital filters," *Arch.*

Elektron. & Übertragungstech., vol. AEU-30, pp. 277-284, 1976.

[227] D. Mitra and J.R. Boddie, "Limit cycles in floating point digital filters," *Proc. IEEE Int. Symp. on Circuits and Systems*, Tokyo, Japan (New York, USA: IEEE 1979), pp. 374-377, 1979.

[228] I.W. Sandberg, "Floating-point roundoff accumulation in digital filter realizations," *Bell Syst. Tech. J.*, vol. 46, pp. 1775-1791, 1967.

[229] S.R. Parker and S.F. Hess, "Limit cycle oscillations in digital filters," *IEEE Trans. Circuit Theory*, vol. CT-18, pp. 687-697, 1971.

[230] S.R. Parker and S.F. Hess, "Heuristic bands for the frequency of digital oscillators due to quantization noise," *Electron. Lett.*, vol. 8, pp. 86-87, 1972.

[231] M.H. Rahman, G.A. Maria and M.M. Fahmy, "Bounds on zero-input limit cycles in all-pole digital filters," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-24, pp. 189-192, 1976.

[232] S.A. White, "Quantizer-induced digital controller limit cycles," *IEEE Trans. Autom. Control*, vol. AC-14, pp. 430-432, 1969.

[233] E. Auer, "A method for the determination of all limit cycles," *Proc. 6th European Conf. on Circuit Theory and Design*, Stuttgart, Germany, ed. E. Lueder (Berlin, Germany: VDE-Verlag 1983), pp. 154-156, 1983.

[234] P. Gruber, "The determination of limit cycles in nonlinear sampled data systems," *Proc. 1976 Joint Automatic Control Conf.*, W-Lafayette, Indiana (New York, USA: ASME 1976), pp. 424-433, 1976.

[235] D.C. Munson, jr., "Determining exact maximum amplitude limit cycles in digital filters," *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing*, Atlanta, Georgia (New York, USA: IEEE 1981), pp. 1231-1234, 1981.

[236] D.C. Munson, jr., J.H. Strickland and T.P. Walker, "Maximum amplitude zero-input limit cycles in digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-31, pp. 266-275, 1984.

[237] K.P. Prasad and P.S. Reddy, "Limit cycles in second-order digital filters," *J. Inst. Electron. & Telecommun. Eng. (India)*, vol. 26, pp. 85-86, 1980.

[238] H.A. Ojongbede, "Limit cycle constraints for recursive digital filter design," *Electron. Lett.*, vol. 6, pp. 698-700, 1970.

[239] E.C. Tan, "Limit cycles and similarity transformation of simple digital circuits," *Int. J. Electron.*, vol. 55, pp. 767-773, 1983.

[240] K.M. Adams, "Elementary limit cycles with applications to the testing of digital circuits," *Proc. IEEE Int. Symp. on Circuits and Systems*, Rome, Italy (New York, USA: IEEE 1982), pp. 1237-1240, 1982.

[241] K.M. Adams, "Elementary limit cycles with applications to the testing of digital circuits," *IEEE Trans. Circuits & Syst.*, vol. CAS-30, pp. 809-814, 1983.

[242] T.L. Chang and C.S. Burrus, "Oscillations caused by quantization in digital filters," *Proc. IEEE Int. Symp. on Circuit Theory*, North-Hollywood, California (New York, USA: IEEE 1972), pp. 228-232, 1972.

[243] D.C. McLernon and R.A. King, "Additional properties of one-dimensional limit cycles," *IEE Proc. G*, vol. 133, pp. 140-144, 1986.

[244] H.J. Butterweck, "Quantization effects in various second-order digital filters: a comparitive study," *Proc. 5th European Conf. on Circuit Theory and Design*, The Hague, The Netherlands, ed. R. Boite, P. Dewilde (Delft, The Netherlands: Delft Univ. Press 1981), pp. 863-864, 1981.

Papers on quantization noise (pertaining to section 5)

[245] C.W. Barnes, B.N. Tran and S.H. Leung, "On the statistics of fixed-point roundoff error," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-33, pp. 595-606, 1985.

[246] H.J. Butterweck, "On the quantization noise contributions in digital filters which are uncorrelated with the output signal," *IEEE Trans. Circuits & Syst.*, vol. CAS-26, pp. 901-910, 1979.

[247] A.C. Callahan, "Random rounding: some principles and applications," *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing*, Philadelphia, Pennsylvania (New York, USA: IEEE 1976), pp. 501-504, 1976.

[248] T.A.C.M. Claasen, W.F.G. Mecklenbräuker and J.B.H. Peek, "Quantization noise analysis for fixed point digital filters using magnitude truncation for quantization," *IEEE Trans. Circuits & Syst.*, vol. CAS-22, pp. 887-895, 1975.

[249] T.A.C.M. Claasen, W.F.G. Mecklenbräuker and J.B.H. Peek, "Quantization errors in digital filters using magnitude truncation quantization," *Proc. IEEE Int. Conf. on Communications*, San Francisco, California (New York, USA: IEEE 1975), pp. 31/ 17-21, 1975.

[250] T.A.C.M. Claasen, "Quantization noise analysis of digital filters with controlled quantization," *Electron. Lett.*, vol. 12, pp. 46-48, 1976.

[251] T.A.C.M. Claasen and A. Jongepier, "Model for the power spectral density of quantization noise," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-29, pp. 914-917, 1981.

[252] G. Dehner, "On the noise behaviour of digital filter blocks of second order," *Arch. Elektron. & Übertragungstech.*, vol. AEÜ-30, pp. 394-398, 1976.

[253] B. Eckhardt, "On the roundoff error of a multiplier," *Arch. Elektron. & Übertragungstech.*, vol. AEÜ-29, pp. 162-164, 1975.

[254] B. Eckhardt and H.W. Schüßler, "On the quantization error of a multiplier," *Proc. IEEE Int. Symp. on Circuits and Systems*, Munich, Germany (New York, USA: IEEE 1976), pp. 634-637, 1976.

[255] R.W.C. Groen, A.W.M. Van den Enden and J.H.F. Ritzerfeld, "On the calculation of quantization noise in digital filters caused by magnitude truncation," *Proc. 8th European Conf. on Circuit Theory and Design*, Paris, France, ed. R. Gerber (London, England: IEE), pp. 717-722, 1987.

[256] K.M. Kammeyer, "Quantization error analysis of the distributed arithmetic," *IEEE Trans. Circuits & Syst.*, vol. CAS-24, pp. 681-689, 1977.

[257] B. Liu and M.E. Van Valkenburg, "On roundoff error of fixed-point digital filters using sign-magnitude truncation," *Proc. IEEE Int. Symp. on Electrical Network Theory*, London, England (New York, USA: IEEE 1971), pp. 68-69, 1971.

[258] B. Liu and M.E. Van Valkenburg, "On roundoff error of fixed-point digital filters using sign-magnitude truncation," *IEEE Trans. Circuit Theory*, vol. CT-19, pp. 536-537, 1972.

[259] A.J. McWilliam and B.J. Stanier, "Roundoff noise prediction in short-wordlength fixed-point digital filters," *IEE J. Electron. Circuits & Syst.*, vol. 2, pp. 9-15, 1978.

[260] D.R. Morgan and W.J. Cassarly, "Effect of wordlength truncation on quantized gaussian random variables," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-34, pp. 1004-1006, 1986.

[261] S.R. Parker, "Correlated noise in digital filters," *Proc. IEEE Int. Symp. on Decision and Control, including 13th Symp. on Adaptive Processes*, Phoenix, Arizona (New York, USA: IEEE 1974), pp. 114-115, 1974.

[262] S.R. Parker and P.E. Girard, "Correlated noise due to roundoff in

fixed-point digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-23, pp. 204-211, 1976.

[263] I. Tokaji and C.W. Barnes, "Roundoff error statistics for a continuous range of multiplier coefficients," *IEEE Trans. Circuits & Syst.*, vol. CAS-34, pp. 52-59, 1987.

[264] G. Verkroost and G.J. Bosscha, "On the measurement of quantization noise in digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-31, pp. 222-223, 1984.

[265] H. Haznedar, "Errors in pseudo fixed-point recursive digital filters with quantization after final accumulation," *Proc. 14th Annual Pittsburgh Conf. on Modeling and Simulation*, vol. 14, pp. 1429-1434, 1983.

[266] E.P.F. Kan and J.K. Aggarwal, "Error analysis of digital filter employing floating-point arithmetic," *IEEE Trans. Circuit Theory*, vol. CT-18, pp. 678-686, 1971.

[267] B. Liu and T. Kaneko, "Error analysis of digital filters realized with floating-point arithmetic," *Proc. of the IEEE*, vol. 57, pp. 1735-1747, 1969.

[268] A.V. Oppenheim, "Realization of digital filters using block-floating-point arithmetic," *IEEE Trans. Audio & Electroacoust.*, vol. AU-18, pp. 130-136, 1970.

[269] E.I. Verriest, "Error analysis of linear recursions in floating point," *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing*, Tampa, Florida (New York, USA: IEEE 1985), pp. 1711-1714, 1985.

[270] C.J. Weinstein and A.V. Oppenheim, "A comparison of roundoff noise in floating point and fixed-point digital filter realizations," *Proc. of the IEEE*, vol. 57, pp. 1181-1183, 1969.

[271] M. Akamine and T. Higuchi, "Synthesis of minimum quantization error digital filters using floating point arithmetic," *Electron. & Commun. Jpn.*, vol. 66-A, no. 10, pp. 29-38, 1983.

[272] M. Akamine and T. Higuchi, "State-space approach to synthesis of minimum quantization error digital filter using floating point arithmetic," *Proc. IEEE Int. Symp. on Circuits and Systems*, Montreal, Canada (New York, USA: IEEE 1984), pp. 1002-1005, 1984.

[273] M. Arjmand and R.A. Roberts, "Reduced multiplier, low roundoff noise digital filters," *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing*, Washington, D.C. (New York, USA: IEEE 1979), pp. 344-346, 1979.

[274] C.W. Barnes and S.H. Leung, "Use of transversal recursive structures for efficient realization of low-noise digital filters with decimated output," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-28, pp. 645-651, 1980.

[275] C.W. Barnes and T. Miyawaki, "Roundoff noise invariants in normal digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-29, pp. 251-256, 1982.

[276] C.W. Barnes, "On the design of optimal state-space realizations of second-order digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-31, pp. 602-608, 1984.

[277] C.W. Barnes, "Computationally efficient second-order digital filter sections with low roundoff noise gain," *IEEE Trans. Circuits & Syst.*, vol. CAS-31, pp. 841-847, 1984.

[278] D.V. Bhaskar-Rao, "A study of coefficient quantization errors in state-space digital filters," *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing*, Tampa, Florida (New York, USA: IEEE 1985), pp. 1715-1718, 1985.

[279] D.V. Bhaskar-Rao, "Analysis of coefficient quantization errors in state-space digital filters," *IEEE Trans. Acoust., Speech & Signal*

Process., vol. ASSP-34, pp. 131-139, 1986.

[280] B.W. Bomar, "Minimum roundoff noise digital filters with some power-of-two coefficients," *IEEE Trans. Circuits & Syst.*, vol. CAS-31, pp. 833-840, 1984.

[281] B.W. Bomar, "Minimum roundoff noise digital filters with power-of-two coefficients," *Proc. IEEE Int. Symp. on Circuits and Systems*, Montreal, Canada (New York, USA: IEEE 1984), pp. 998-1001, 1984.

[282] B.W. Bomar, "New second-order state-space structures for realizing low roundoff noise digital filters," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-33, pp. 106-110, 1985.

[283] B.W. Bomar, "Computationally efficient low roundoff noise second-order state-space structures," *IEEE Trans. Circuits & Syst.*, vol. CAS-33, pp. 35-41, 1986.

[284] D.S.K. Chan, "Constrained minimization of roundoff noise in fixed-point digital filters," *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing*, Washington, D.C. (New York, USA: IEEE 1979), pp. 335-339, 1979.

[285] P.S.R. Diniz and A. Antoniou, "New improved state-space digital filter structures," *Proc. IEEE Int. Symp. on Circuits and Systems*, Kyoto, Japan (New York, USA: IEEE 1985), pp. 1599-1602, 1985.

[286] P.S.R. Diniz and A. Antoniou, "More economical state-space digital filter structures which are free of constant-input limit cycles," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-34, pp. 807-815, 1986.

[287] A.P. Gerheim, "Numerical solution of the Lyapunov equation for narrow-band digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-31, pp. 991-992, 1984.

[288] A.P. Gerheim, "Calculation of quantization noise at non-storage nodes," *IEEE Trans. Circuits & Syst.*, vol. CAS-31, pp. 1054-1055, 1984.

[289] S.Y. Hwang, "Roundoff noise in state-space digital filtering: A general analysis," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-24, pp. 256-262, 1976.

[290] S.Y. Hwang, "Roundoff noise minimization in state-space digital filtering," *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing*, Philadelphia, Pennsylvania (new York, USA: IEEE 1976), pp. 498-500, 1976.

[291] S.Y. Hwang, "Minimum uncorrelated unit noise in state-space digital filtering," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-25, pp. 273-281, 1977.

[292] M. Iwatsuki, M. Kawamata and T. Higuchi, "Synthesis of minimum sensitivity structures in linear systems using controllability and observability measures," *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing*, Tokyo, Japan (New York, USA: IEEE 1986), pp. 501-504, 1986.

[293] L.B. Jackson, "Roundoff noise bounds derived from coefficient sensitivities for digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-23, pp. 481-485, 1976.

[294] L.B. Jackson, "Lower bounds on the roundoff noise from digital filters in cascade or parallel form," *Proc. IEEE Int. Symp. on Circuits and Systems*, Munich, Germany (New York, USA: IEEE 1976), pp. 638-641, 1976.

[295] L.B. Jackson, A.G. Lindgren and Y. Kim, "Synthesis of state-space digital filters with low roundoff noise and coefficient sensitivity," *Proc. IEEE Int. Symp. on Circuits and Systems*, Phoenix, Arizona (New York, USA: IEEE 1977), pp. 41-44, 1977.

[296] L.B. Jackson, A.G. Lindgren and Y. Kim, "Optimal synthesis of second-order state-space structures for digital filters," *IEEE Trans. Circuits*

& *Syst.*, vol. CAS-26, pp. 149-153, 1979.

[297] M. Kawamata and T. Higuchi, "A unified approach to the optimal synthesis of fixed-point state-space digital filters," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-33, pp. 911-920, 1985.

[298] W.H. Ku and S.M. Ng, "Floating-point coefficient sensitivity and roundoff noise of recursive digital filters realized in ladder structures," *IEEE Trans. Circuits & Syst.*, vol. CAS-22, pp. 927-936, 1975.

[299] E.A. Lee and D.G. Messerschmitt, "On quantization effects in state-variable filter implementations," *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing*, Tampa, Florida (New York, USA: IEEE 1985), pp. 1719-1722, 1985.

[300] S.H. Leung, "State-space realization of digital filters with stationary inputs," *Proc. IEEE Int. Symp. on Circuits and Systems*, Newport Beach, California (New York, USA: IEEE 1983), pp. 811-814, 1983.

[301] W.L. Mills, C.T. Mullis and R.A. Roberts, "Normal realizations of IIR digital filters," *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing*, Washington, D.C. (New York, USA: IEEE 1979), pp. 340-343, 1979.

[302] W.L. Mills, C.T. Mullis and R.A. Roberts, "Low roundoff noise and normal realizations of fixed-point IIR digital filters," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-29, pp. 893-903, 1981.

[303] S.K. Mitra and J. Fadavi-Ardekani, "A new approach to the design of cost-optimal low-noise digital filters," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-29, pp. 1172-1176, 1981.

[304] C.T. Mullis and R.A. Roberts, "Roundoff noise in digital filters: Frequency transformations and invariants," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-24, pp. 539-550, 1976.

[305] C.T. Mullis and R.A. Roberts, "Synthesis of minimum roundoff noise fixed-point digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-23, pp. 551-561, 1976.

[306] C.T. Mullis and R.A. Roberts, "Filter structures which minimize roundoff noise in fixed-point digital filters," *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing*, Philadelphia, Pennsylvania (New York, USA: IEEE 1976), pp. 505-508, 1976.

[307] A.A. Sakla and E. El-Masry, "Optimal roundoff noise selection in class of minimal canonical nonproduct of multiplier digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-32, pp. 409-412, 1985.

[308] V. Tavsanoglu and L. Thiele, "Simultaneous minimization of roundoff noise and sensitivity in state-space digital filters," *Proc. IEEE Int. Symp. on Circuits and Systems*, Newport Beach, California (New York, USA: IEEE 1983), pp. 815-818, 1983.

[309] V. Tavsanoglu and L. Thiele, "Optimal design of state-space digital filters by simultaneous minimization of sensitivity and roundoff noise," *IEEE Trans. Circuits & Syst.*, vol. CAS-31, pp. 884-888, 1984.

[310] V. Tavsanoglu, "Explicit evaluation of K and W matrices for second-order digital filters," *Proc. 7th European Conf. on Circuit Theory and Design*, Prague, Czechoslovakia, ed. V. Zina and J. Krasil (Amsterdam, The Netherlands: North-Holland 1985), pp. 488-491, 1985.

[311] L. Thiele, "On the realization of minimum sensitivity and minimum round-off noise state-space discrete systems," *Proc. 6th European Conf. on Circuit Theory and Design*, Stuttgart, Germany, ed. E. Lueder (Berlin, Germany: VDE-Verlag 1983), pp. 151-153, 1983.

[312] L. Thiele, "Design of sensitivity and roundoff noise optimal state-space discrete systems," *Int. J. Circuit Theory & Appl.*, vol. 12, pp. 39-46, 1984.

[313] J. Zeman and A.G. Lindgren, "Fast digital filters with low roundoff noise," *IEEE Trans. Circuits & Syst.*, vol. CAS-28, pp. 716-723, 1981.

[314] J. Zeman and A.G. Lindgren, "Fast state-space decimator with very low roundoff noise," *Signal Process.*, vol. 3, pp. 377-388, 1981.

[315] C.W. Barnes, "Error-feedback in normal realization of recursive digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-28, pp. 72-75, 1981.

[316] T.L. Chang, "Comparison of roundoff noise variances in several low roundoff noise digital filter structures," *Proc. of the IEEE*, vol. 68, pp. 173-174, 1980.

[317] T.L. Chang, "Correction to 'Comparison of roundoff noise variances in several low roundoff noise digital filter structures'," *Proc. of the IEEE*, vol. 68, p. 1167, 1980.

[318] T.L. Chang, "On low roundoff noise and low sensitivity digital filter structures," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-29, pp. 1077-1080, 1981.

[319] T.L. Chang, "A unified analysis of roundoff noise reduction in digital filters," *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Process.*, Atlanta, Georgia (New York, USA: IEEE), pp. 1209-1212, 1981.

[320] W.E. Higgins and D.C. Munson, jr., "Noise reduction strategies for digital filters: Error spectrum shaping versus the optimal linear state-space formulation," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-30, pp. 963-973, 1982.

[321] W.E. Higgins and D.C. Munson, jr., "Optimal error spectrum shaping for cascade-form digital filters," *Proc. IEEE Int. Symp. on Circuits and Systems*, Rome, Italy (New York, USA: IEEE 1982), pp. 1029-1032, 1982.

[322] W.E. Higgins and D.C. Munson, jr., "A section ordering strategy for cascade-form digital filters using error spectrum shaping," *Proc. IEEE Int. Symp. on Circuits and Systems*, Newport Beach, California (New York, USA: IEEE 1983), pp. 835-838, 1983.

[323] W.E. Higgins and D.C. Munson, jr., "Optimal and suboptimal error spectrum shaping for cascade-form digital fiters," *IEEE Trans. Circuits & Syst.*, vol. CAS-31, pp. 429-437, 1984.

[324] C.T. Mullis and R.A. Roberts, "An interpretation of error spectrum shaping in digital filters," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-30, pp. 1013-1015, 1982.

[325] M. Renfors, "Roundoff noise in error-feedback state-space filters," *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing*, Boston, Massachussets (New York, USA: IEEE 1983), pp. 619-622, 1983.

[326] P.P. Vaidyanathan, "On error spectrum shaping in state-space digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-32, pp. 88-92, 1985.

[327] D. Williamson, "Roundoff noise minimization and pole-zero sensitivity in fixed-point digital filters using residue feedback," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-34, pp. 1210-1220, 1986.

[328] D. Williamson, "Minimum roundoff noise fixed-point digital filters using integer residue feedback," *Proc. IEEE Int. Symp. on Circuits and Systems*, San Jose, California (New York, USA: IEEE), pp. 867-870, 1986.

[329] A.I. Abu-El-Haija and A.M. Peterson, "An approach to eliminate roundoff errors in digital filters," *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing*, Tulsa, Oklahoma (New York, USA: IEEE 1978), pp. 75-78, 1978.

[330] A.I. Abu-El-Haija and A.M. Peterson, "An approach to eliminate roundoff errors in digital filters," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-27, pp. 195-198, 1979.

[331] A.I. Abu-El-Haija, "On implementing error-feedback digital filters," *25th Midwest Symp. on Circuits and Systems*, Houghton, Michigan, ed. R.E. Stuffle, P.H. Lewis (North-Hollywood, California, USA: Western

Periodicals, 1982), pp. 140-147, 1982.
[332] A.I. Abu-El-Haija, "Error-feedback digital filters with minimum limit cycle oscillations," *Proc. EUSIPCO-83, 2nd European Signal Processing Conf.*, Erlangen, Germany, ed. H.W. Schüßler (Amsterdam, The Netherlands: North-Holland 1983), pp. 111-114, 1983.
[333] A.I. Abu-El-Haija, "Determining coefficients of error-feedback digital filters to obtain minimum roundoff errors with minimum complexity," *Proc. IEEE Int. Symp. on Circuits and Systems*, Newport Beach, California (New York, USA: IEEE 1983), pp. 819-822, 193.
[334] T.L. Chang, "A low roundoff noise digital filter structure," *Proc. IEEE Int. Symp. on Circuits and Systems*, New York, N.Y. (New York, USA: IEEE 1978), pp. 1004-1008, 1978.
[335] T.L. Chang, "Error-feedback digital filters," *Electron. Lett.*, vol. 15, pp. 348-349, 1979.
[336] T.L. Chang, "Error spectrum shaping structures for digital filters," *Proc. 13th Asilomar Conf. on Circuits, Systems and Computers*, Pacific Grove, California, ed. S.P. Chan (North-Hollywood, California, USA: Western Periodicals 1979), pp. 279-283, 1979.
[337] T.L. Chang, "Comments on 'An approach to eliminate roundoff errors in digital filters' (Author's reply: A.I. Abu-El-Haija)," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-28, p. 244, 1980.
[338] T.L. Chang and S.A. White, "An error cancellation digital filter structure and its distributed arithmetic implementation," *IEEE Trans. Circuits & Syst.*, vol. CAS-28, pp. 339-342, 1981.
[339] P.S.R. Diniz and A. Antoniou, "Low-sensitivity digital filter structures which are amenable to error-spectrum shaping," *IEEE Trans. Circuits & Syst.*, vol. CAS-32, pp. 1000-1007, 1985.
[340] P.G. McCrea and I.H. Witten, "Reducing noise in recursive digital filters by residue retention," *Electron. Lett.*, vol. 14, pp. 686-688, 1978.
[341] L. Mintzer and J.P. Strauss, "A practical digital filter with near optimal rounding noise cancellation," *Proc. 13th Asilomar Conf. on Circuits, Systems and Computers*, Pacific Grove, California, ed. S.P. Chan (North-Hollywood, California, USA: Western Periodicals 1979), pp. 263-266, 1979.
[342] D.C. Munson, jr. and B. Liu, "Narrow-band recursive filters with error spectrum shaping," *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing*, Washington, D.C. (New York, USA: IEEE 1979), pp. 367-370, 1979.
[343] D.C. Munson, jr. and B. Liu, "Narrow-band recursive filters with error spectrum shaping," *IEEE Trans. Circuits & Syst.*, vol. CAS-28, pp. 160-163, 1981.
[344] T. Thong and B. Liu, "Error spectrum shaping in narrow-band recursive filters," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-25, pp. 200-203, 1977.
[345] D. Williamson, P.G. McCrea and S. Sridharan, "Residue feedback in digital filters using fractional coefficients and block floating point arithmetic," *Proc. IEEE Int. Symp. on Circuits and Systems*, Newport Beach, California (New York, USA: IEEE 1983), pp. 831-834, 1983.
[346] D. Williamson and S. Sridharan, "Residue feedback in digital filters using fractional feedback coefficients," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-33, pp. 477-483, 1985.
[347] D. Williamson and S. Sridharan, "An approach to coefficient wordlength reduction in digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-32, pp. 893-903, 1985.
[348] D. Williamson and S. Sridharan, "Residue feedback in ladder and lattice

filter structures," *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing*, Tampa, Florida (New York, USA: IEEE 1985), pp. 53-56, 1985.

[349] D. Williamson and S. Sridharan, "Error-feedback in a class of orthogonal polynomial digital filter structures," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-34, pp. 1013-1016, 1986.

[350] D.S.K. Chan and L.R. Rabiner, "Theory of roundoff noise in cascade realizations of finite impulse response digital filters," *Bell Syst. Tech. J.*, vol. 52, pp. 329-345, 1973.

[351] D.S.K. Chan and L.R. Rabiner, "An algorithm for minimizing roundoff noise in cascade realizations of finite impulse response digital filters," *Bell Syst. Tech. J.*, vol. 52, pp. 347-385, 1973.

[352] G. Dehner, "A contribution to the optimization of roundoff noise in recursive digital filters," *Arch. Elektron. & Ubertragungstech.*, vol. AEU-29, pp. 505-510, 1975.

[353] K. Hirano, H. Sakaguchi and B. Liu, "Optimization of recursive cascade filters," *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing*, Philadelphia, Pennsylvania (New York, USA: IEEE 1976), pp. 513-516, 1976.

[354] S.Y. Hwang, "On optimization of cascade fixed-point digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-21, pp. 163-166, 1974.

[355] L.B. Jackson, "Roundoff noise analysis for fixed-point digital filters realized in cascade or parallel form," *IEEE Trans. Audio & Electroacoust.*, vol. AU-18, pp. 107-122, 1970.

[356] W. Jaschinski and K.A. Owenier, "Some results in optimization of roundoff noise for fixed-point digital filters realized in cascade form," *Arch. Elektron. & Ubertragungstech.*, vol. AEU-31, pp. 111-115, 1977.

[357] S. Kawarai, "Advantageous estimation formula of the least roundoff noise for the cascade fixed-point digital filters," *Proc. IEEE Int. Symp. on Circuits and Systems*, Kyoto, Japan (New York, USA: IEEE 1985), pp. 511-512, 1985.

[358] H. Lanfer and E. Lueder, "Minimizing the roundoff noise in digital filters by the branch and bound method," *Int. J. Circuit Theory & Appl.*, vol. 7, pp. 179-186, 1979.

[359] T.R. Lapp and R.A. Gabel, "An algorithm for optimally ordering the sections of a cascade digital filter," *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing*, Philadelphia, Pennsylvania (New York, USA: IEEE 1976), pp. 517-520, 1976.

[360] W.S. Lee, "Optimization of digital filters for low roundoff noise," *Proc. IEEE Int. Symp. on Circuit Theory*, Toronto, Canada (New York, USA: IEEE 1973), pp. 381-383, 1973.

[361] W.S. Lee, "Optimization of digital filters for low roundoff noise," *IEEE Trans. Circuits & Syst.*, vol. CAS-21, pp. 424-431, 1974.

[362] B. Liu and A. Peled, "Heuristic optimization of the cascade realization of fixed-point digital filters," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-23, pp. 464-473, 1975.

[363] E. Lueder, H. Hug and W. Wolf, "Minimizing the roundoff noise in digital filters by dynamic programming," *Frequenz*, vol. 29, pp. 211-214, 1975.

[364] E. Lueder, "Digital filter processing with improved accuracy," *Proc. 5th European Conf. on Circuit Theory and Design*, The Hague, The Netherlands, ed. R. Boite, P. Dewilde (Delft, The Netherlands: Delft Univ. Press 1981), pp. 25-33, 1981.

[365] M.H. Rahman and M.M. Fahmy, "A roundoff noise minimization technique for cascade realization of digital filters," *Proc. IEEE Int. Symp. on Circuits and Systems*, Phoenix, Arizona (New York, USA: IEEE 1977), pp.

45-48, 1977.
[366] K. Steiglitz and B. Liu, "An improved algorithm for ordering poles and zeros of fixed-point recursive digital filters," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-24, pp. 341-343, 1976.
[367] A.I. Abu-El-Haija, K. Shenoi and A.M. Peterson, "Digital filter structures having low errors and simple hardware implementation," *IEEE Trans. Circuits & Syst.*, vol. CAS-25, pp. 593-599, 1978.
[368] J.I. Acha and J. Payan, "Low-noise structures for narrow-band recursive digital filters without overflow oscillations," *IEEE Trans. Circuits & Syst.*, vol. CAS-34, pp. 96-99, 1987.
[369] R.C. Agarwal and C.S. Burrus, "New recursive digital filter structures having very low sensitivity and roundoff noise," *IEEE Trans. Circuits & Syst.*, vol. CAS-22, pp. 921-927, 1975.
[370] M. Bhattacharya, R.C. Agarwal and S.C. Dutta Roy, "Alternative low-noise realizations for narrow-band recursive digital filters," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-32, pp. 659-661, 1984.
[371] M. Bhattacharya and R.C. Agarwal, "Cascade realization of linear phase FIR filters with low sensitivity," *Signal Process.*, vol. 9, pp. 245-251, 1985.
[372] M. Bhattacharya, R.C. Agarwal and S.C. Dutta Roy, "On realization of low-pass and high-pass recursive filters with low sensitivity and low roundoff noise," *IEEE Trans. Circuits & Syst.*, vol. CAS-33, pp. 425-428, 1986.
[373] L. Bhuyan and B.N. Chatterji, "Very low sensitivity recursive digital filter structures," *Electron. Lett.*, vol. 17, pp. 348-350, 1981.
[374] L.T. Bruton, "Low sensitivity digital ladder filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-22, pp. 168-176, 1975.
[375] D.C. Munson, jr. and B. Liu, "Low-noise realizations for digital filters with poles near the unit circle," *Proc. 16th Ann. Allerton Conf. on Communication, Control and Computing*, Monticello, Illinois, USA, ed. M.B. Parsley, J.B. Cruz, jr., pp. 372-381, 1978.
[376] D.C. Munson, jr. and B. Liu, "Low-noise realizations for narrow-band recursive digital filters," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-28, pp. 41-54, 1980.
[377] D.C. Munson, jr. and B. Liu, "ROM/ACC realization of digital filters for poles near the unit circle," *IEEE Trans. Circuits & Syst.*, vol. CAS-27, pp. 147-151, 1980.
[378] N. Nishimura, K. Hirano and R.N. Pal, "A new class of very low sensitivity and low roundoff noise recursive digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-28, pp. 1152-1157, 1981.
[379] A. Antoniou and M.G. Rezk, "A comparison of cascade and wave fixed-point digital filter structures," *IEEE Trans. Circuits & Syst.*, vol. CAS-27, pp. 1184-1194, 1980.
[380] A. Fettweis, "On sensitivity and roundoff noise in wave digital filters," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-22, pp. 383-384, 1974.
[381] D. van Haften and P.M. Chirlian, "An analysis of errors in wave digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-28, pp. 154-160, 1981.
[382] J.L. Long and T.N. Trick, "Sensitivity and noise comparison of some fixed-point recursive digital filter structures," *Proc. IEEE Int. Symp. on Circuits and Systems*, Newton, Massachussets (New York, USA: IEEE 1975), pp. 56-59, 1975.
[383] K.K. Pang and B. Treloar, "On the dynamic range problem of wave digital filters," *IREECON Int. Sydney '83: Digest of papers 19th Int. Electr. Convention and Exhibition*, Sydney, Australia (Sydney, Australia: Inst.

of Radio and Electron. Eng. Australia 1983), pp. 95-97, 1983.
[384] K.K. Pang and B. Treloar, "An investigation of the dynamic range problem in wave digital filters," *IREECON Int. Sydney '83: Digest of papers 19th Int. Electr. Convention and Exhibition*, Sydney, Australia (Sydney, Australia: Inst. of Radio and Electron. Eng. Australia 1983), pp. 538-541, 1983.
[385] K. Renner and S.C. Gupta, "Reduction of roundoff noise in wave digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-21, pp. 305-310, 1974.
[386] H. Tatangsurja and W. Steenaart, "A comparison of stored-product and wave digital filters: coefficient accuracy and roundoff noise," *27th Midwest Symp. on Circuits and Systems*, Morgantown, West Virginia, ed. R.E. Swartwout (North-Hollywood, California, USA: Western Periodicals, 1984), pp. 612-614, 1984.
[387] U. Ullrich, "Calculation of roundoff noise in wave digital filters with fixed-point arithmetic," *Proc. 2nd European Conf. on Circuit Theory and Design*, Genova, Italy (London, England: IEE 1976), pp. 444-449, 1976.
[388] U. Ullrich, "Roundoff noise and dynamic range of wave digital filters," *Signal Process.*, vol. 1, pp. 45-64, 1979.
[389] W. Wegener, "On the design of wave digital lattice filters with short coefficient wordlengths and optimal dynamic range," *IEEE Trans. Circuits & Syst.*, vol. CAS-25, pp. 1091-1098, 1978.
[390] A. Antoniou, C. Charalambous and Z. Motamedi, "Two methods for the reduction of quantization effects in recursive digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-30, pp. 160-167, 1983.
[391] R. Ansari and B. Liu, "A class of low-noise computationally efficient recursive digital filters with applications to sampling rate alterations," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-33, pp. 90-97, 1985.
[392] A. Anuff and C.Y. Kao, "Comments on 'On the evaluation of roundoff noise in digital filters'," *IEEE Trans. Circuits & Syst.*, vol. CAS-23, p. 573, 1976.
[393] J. Beller, "Digital filter design with low roundoff noise," *Proc. 6th European Conf. on Circuit Theory and Design*, Stuttgart, Germany, ed. E. Lueder (Berlin, Germany: VDE-Verlag 1983), pp. 148-150, 1983.
[394] F. Bonzanigo, "Comment on 'Roundoff noise and attenuation sensitivity in digital filters with fixed-point arithmetic' (Author's reply: A. Fettweis)," *IEEE Trans. Circuits & Syst.*, vol. CAS-21, pp. 809-811, 1974.
[395] G. Buchholz, "Simulation of digital filters (in German)," *Nachrichtentech. Elektron.*, vol. 32, pp. 60-62, 1982.
[396] P. Calcagno, E. Garetti and A.R. Meo, "Optimization and comparison of fixed-point implementations for first- and second-order digital blocks," *IEEE Trans. Audio & Electroacoust.*, vol. AU-19, pp. 314-322, 1971.
[397] C. Charalambous, A. Antoniou and Z. Motamedi, "Two methods for the reduction of quantization effects in recursive digital filters," *Proc. IEEE Int. Symp. on Circuits and Systems*, Rome, Italy (New York, USA: IEEE 1982), pp. 1049-1052, 1982.
[398] S. Chu and C.S. Burrus, "Roundoff noise in multirate digital filters," *Circuits Syst. & Signal Process.*, vol. 3, pp. 419-434, 1984.
[399] A. Fettweis, "On the evaluation of roundoff noise in digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-22, p. 896, 1975.
[400] S.Y. Hwang, "Dynamic range constraint in state-space digital filtering," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-23, pp. 591-593, 1975.
[401] S.Y. Hwang, "Comments on 'Reduction of roundoff noise in wave digital

filters'," *IEEE Trans. Circuits & Syst.*, vol. CAS-22, p. 764, 1975.

[402] W.K. Jenkins and B.J. Leon, "An analysis of quantization error in digital filters based on interval algebras," *IEEE Trans. Circuits & Syst.*, vol. CAS-22, pp. 223-232, 1975.

[403] W.K. Jenkins, "Recent advances in residue number techniques for recursive digital filtering," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-27, pp. 19-30 , 1979.

[404] N.G. Kingsbury, "Digital filter second-order element with low quantising noise for poles and zeros at low frequencies," *Electron. Lett.*, vol. 9, pp. 271-273, 1973.

[405] H.K. Kwan, "Amplitude scaling of arbitrary linear digital networks," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-32, pp. 1240-1242, 1984.

[406] J.W.K. Lam, V. Ramachandran and M.N.S. Swamy, "Comparison of the effects of quantization on digital filters," *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing*, Atlanta, Georgia (New York, USA: IEEE 1981), pp. 1213-1216, 1981.

[407] J.W.K. Lam, V. Ramachandran and M.N.S. Swamy, "Comparison of the effects of quantization on digital filters," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-30, pp. 1010-1013, 1982.

[408] B. Liu, "Effects of finite wordlength on the accuracy of digital filters - A review," *IEEE Trans. Circuit Theory*, vol. CT-18, pp. 670-677, 1971.

[409] B. Liu and R. Ansari, "Quantization effects in computationally efficient realizations of recursive digital filters," *Proc. IEEE Int Symp. on Circuits and Systems*, Rome, Italy (New York, USA: IEEE 1982), pp. 716-720, 1982.

[410] E.S.K. Liu and L.E. Turner, "Stability, dynamic range and roundoff noise in a new second-order recursive digital filter," *Proc. IEEE Int. Symp. on Circuits and Systems*, Rome, Italy (New York, USA: IEEE 1982), pp. 1045-1048, 1982.

[411] E.S.K. Liu and L.E. Turner, "Stability, dynamic range and roundoff noise in a new second-order recursive digital filter," *IEEE Trans. Circuits & Syst.*, vol. CAS-30, pp. 815-821, 1983.

[412] J.D. Markel and A.H. Gray, jr., "Roundoff noise characteristic of a class of orthogonal polynomial structures," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-23, pp. 473-486, 1975.

[413] S.K. Mitra, K. Hirano and H. Sakaguchi, "A simple method of computing the input quantization and multiplication roundoff errors in a digital filter," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-22, pp. 326-329, 1974.

[414] S.K. Mitra and K. Mondal, "A novel approach to recursive digital filter realization with low roundoff noise," *Proc. IEEE Int. Symp. on Circuits and Systems*, Munich, Germany (New York, USA: IEEE), pp. 299-302, 1976.

[415] O. Monkewich and W. Steenaart, "Stored product digital filtering with non-linear quantization," *Proc. IEEE Int. Symp. on Circuits and Systems*, Munich, Germany (New York, USA: IEEE 1976), pp. 157-160, 1976.

[416] R.K. Patney, "A different look at roundoff noise in digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-27, pp. 59-62, 1980.

[417] A. Peled and B. Liu, "A new hardware realization of digital filters," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-22, pp. 456-462, 1974.

[418] V.M. Solev'ev, "Method for improving the accuracy of digital filters," *Autom. & Remote Control*, vol. 45, pp. 855-858, 1984.

[419] J. Szczupak and S.K. Mitra, "Recursive digital filters with low roundoff noise," *Int. J. Circuit Theory & Appl.*, vol. 5, pp. 275-286, 1977.

[420] F. Taylor and J.W. Marshall, "Computer-aided design and analysis of standard IIR architectures: part I," *IEEE Circuits & Syst. Mag.*, vol. 3, no.4, pp. 2-6, 1981.

[421] T. Thong, "Finite wordlength effects in the ROM digital filter," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-24, pp. 436-437, 1976.

[422] T.N. Trick, J. Yau and E. Sanchez-Sinencio, "Simulation of fixed-point digital filter structures (computer aided design)," *Proc. IEEE Int. Symp. on Circuits and Systems*, Tokyo, Japan (New York, USA: IEEE 1979), pp. 370-371, 1979.

[423] P.P. Vaidyanathan, S.K. Mitra and Y. Neuvo, "A new approach to the realization of low-sensitivity IRR digital filters," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-34, pp. 350-361, 1986.

Recent papers (1987/88) on finite wordlength effects

[424] J.I. Acha, "Overflow and roundoff analysis of a very-low sensitivity digital filter structure," *Int. J. Electron.*, vol. 62, pp. 115-119, 1987.

[425] G. Amit and U. Shaked, "Small roundoff noise realization of fixed-point digital filters and controllers," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-36, pp. 880-891, 1988.

[426] E. Auer, "Digital filter structures free of limit cycles," *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing*, Dallas, Texas (New York, USA: IEEE 1987), pp. 904-907, 1987.

[427] J. Beller, "Roundoff noise reduction in digital filters by matrix transformation," *Proc. 8th European Conf. on Circuit Theory and Design*, Paris, France, ed. R. Gerber (London, England: IEE), pp. 369-374, 1987.

[428] R. Boite, X.L. He and J.P. Renard, "On the floating-point realizations of digital filters," *Proc. EUSIPCO-88, 4th European Signal Processing Conf.*, Grenoble, France, ed. J.L. Laccrime et.al. (Amsterdam, The Netherlands: North-Holland 1988), pp. 1501-1504, 1988.

[429] B.W. Bomar and R.D. Joseph, "Calculation of L_∞ norms for scaling second-order state-space digital filter sections," *IEEE Trans. Circuits & Syst.*, vol. CAS-34, pp. 983-984, 1987.

[430] P. Burrascano, G. Martinelli and G. Orlandi, "Low-sensitivity digital filters based on zero extraction," *IEEE Trans. Circuits & Syst.*, vol. CAS-34, pp. 1581-1586, 1987.

[431] C. Eswaran, A. Antoniou and K. Manivannan, "Universal digital biquads which are free of limit cycles," *IEEE Trans. Circuits & Syst.*, vol. CAS-34, pp. 1243-1248, 1987.

[432] A. Fettweis, "Passivity and losslessness in digital filtering," *Arch. Elektron. & Ubertragungstech.*, vol. AEU-42, pp. 1-8, 1988.

[433] B.D. Green and L.E. Turner, "New limit cycle bounds for digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-35, pp. 365-374, 1988.

[434] U. Kleine and T.G. Noll, "On the forced-response stability of wave digital filters using carry-save arithmetic," *Arch. Elektron. & Ubertragungstech.*, vol. AEU-41, pp. 321-324, 1987.

[435] H.K. Kwan and W. Lee, "Low sensitivities and low roundoff noise realizations of sharp cut-off second-order digital filters," *Proc. IEEE Pacific Rim Conf. on Comm., Computers and Signal Processing*, Victoria, Canada (New York, USA: IEEE 1987), pp. 231-234, 1987.

[436] A. Lepschy, G.A. Mian and U. Viaro, "Stability of coupled-form digital filters with roundoff quantization," *Alta Freq.*, vol. 56, pp. 357-360, 1987.

[437] A. Lepschy, G.A. Mian and U. Viaro, "A contribution to the stability

analysis of second-order direct-form digital filters with magnitude truncation," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-35, pp. 1207-1210, 1987.

[438] A. Lepschy, G.A. Mian and U. Viaro, "Effects of quantization in second-order fixed-point digital filters with two's complement truncation quantizers," *IEEE Trans. Circuits & Syst.*, vol. CAS-35, pp. 461-466, 1988.

[439] A. Lepschy, G.A. Mian and U. Viaro, "Parameter plane quantisation induced by the signal quantisation in second-order fixed-point digital filter with one quantiser," *Signal Process.*, vol. 14, pp. 103-106, 1988.

[440] V.C. Liu and P.P. Vaidyanathan, "Circulant and skew-circulant matrices as new normal-form realization of IIR digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-35, pp. 625-635, 1988.

[441] A.A. Petrovskiy and Y.A. Ganushkin, "The signal-quantizing noise variance in second-order digital recursive filters," *Telecommun. & Radio Eng. Part 2*, vol. 40, pp. 86-89, 1987.

[442] T. Saramaki, T. Yu and S.K. Mitra, "Very low sensitivity realization if IIR digital filters using a cascade of complex all-pass structures," *IEEE Trans. Circuits & Syst.*, vol. CAS-34, pp. 876-886, 1987.

[443] P.K. Sim and K.K. Pang, "Conditions for overflow stability of a class of complex biquad digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-34, pp. 471-479, 1987.

[444] P.K. Sim, "Relationship between input-scaling and stability of the forced response of recursive digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-35, pp. 506-511, 1988.

[445] V. Singh, "A new proof of the discrete-time bounded-real lemma and lossless bounded-real lemma," *IEEE Trans. Circuits & Syst.*, vol. CAS-34, pp. 960-962, 1987.

[446] V. Tavsanoglu, "The necessary and sufficient conditions for minimum roundoff noise in second-order state-space digital filters and their optimal synthesis," *IEEE Trans. Circuits & Syst.*, vol. CAS-34, pp. 669-677, 1987.

[447] I. Tokaji and C.W. Barnes, "Minimum unit noise gain in non-minimal state-space realizations of digital filters," *IEEE Trans. Circuits & Syst.*, vol. CAS-35, pp. 455-457, 1988.

[448] P.P. Vaidyanathan, "Low-noise and low sensitivity digital filters," *Handbook of Digital Signal Processing: Engineering Applications*, ed. D.F. Elliot, San diego, California, USA: Academic Press, 1987.

[449] D. Williamson, "Delay replacement in direct form structures," *IEEE Trans. Acoust., Speech & Signal Process.*, vol. ASSP-36, pp. 453-460, 1988.

AUTHOR INDEX

recursive digital filters with applications to sampling rate alterations," 1985.
[409] Ansari, R., see *B. Liu.*
[379] Antoniou, A. and M.G. Rezk, "A comparison of cascade and wave fixed-point digital filter structures," 1980.
[390] Antoniou, A., C. Charalambous and Z. Motamedi, "Two methods for the reduction of quantization effects in recursive digital filters," 1983.
[397] Antoniou, A., see *C. Charalambous.*
[127] Antoniou, A., see *P.S.R. Diniz.*
[285] Antoniou, A., see *P.S.R. Diniz.*
[286] Antoniou, A., see *P.S.R. Diniz.*
[339] Antoniou, A., see *P.S.R. Diniz.*
[109] Antoniou, A., see *C. Eswaran.*
[431] Antoniou, A., see *C. Eswaran.*
[392] Anuff, A. and C.Y. Kao, "Comments on 'On the evaluation of roundoff noise in digital filters'," 1976.
[183] Appel, U., "Bounds on second-order digital filter limit cycles," 1975.
[166] Aridgides, A., see *D.R. Morgan.*
[273] Arjmand, M. and R.A. Roberts, "Reduced multiplier, low roundoff noise digital filters," 1979.
[233] Auer, E., "A method for the determination of all limit cycles," 1983.
[59] Auer, E., "New methods to guarantee overflow-stability for the canonic second-order digital filter structure," 1985.
[426] Auer, E., "Digital filter structures free of limit cycles," 1987.
[26] Baraniecki, A.Z. and G.A. Julien, "Quantization error and limit cycle analysis in residue number system coded recursive filters," 1982.
[62] Barnes, C.W. and A.T. Fam, "Minimum norm recursive digital filters that are free of overflow limit cycles," 1977.
[39] Barnes, C.W., "Roundoff noise and overflow in normal digital filters," 1979.
[274] Barnes, C.W. and S.H. Leung, "Use of transversal recursive structures for efficient realization of low-noise digital filters with decimated output," 1980.
[80] Barnes, C.W. and S. Shinnaka, "Finite word effects in block-state realizations of fixed-point digital filters," 1980.
[81] Barnes, C.W. and S. Shinnaka, "Stability domains for second-order recursive digital filters in normal form with 'matrix power' feedback," 1980.
[315] Barnes, C.W., "Error-feedback in normal realization of recursive digital filters," 1981.
[275] Barnes, C.W. and T. Miyawaki, "Roundoff noise invariants in normal digital filters," 1982.
[276] Barnes, C.W., "On the design of optimal state-space realizations of second-order digital filters," 1984.
[277] Barnes, C.W., "Computationally efficient second-order digital filter sections with low roundoff noise gain," 1984.
[40] Barnes, C.W., "A parametric approach to the realization of second-order digital filter sections," 1985.
[245] Barnes, C.W., B.N. Tran and S.H. Leung, "On the statistics of fixed-point roundoff error," 1985.
[83] Barnes, C.W., see *A.T. Fam.*
[263] Barnes, C.W., see *I. Tokaji.*
[447] Barnes, C.W., see *I. Tokaji.*
[207] Bateman, M.R., see *B. Liu.*
[184] Belal, A.A., "On the quantization error bounds in second-order digital filters with complex conjugate poles," 1977.

[393] Beller, J., "Digital filter design with low roundoff noise," 1983.
[427] Beller, J., "Roundoff noise reduction in digital filters by matrix transformation," 1987.
[63] Bernhardt, U., H. Lubenow and H. Unger, "On avoiding limit cycles in digital filters (in German)," 1977.
[278] Bhaskar-Rao, D.V., "A study of coefficient quantization errors in state-space digital filters," 1985.
[279] Bhaskar-Rao, D.V., "Analysis of coefficient quantization errors in state-space digital filters," 1986.
[370] Bhattacharya, M., R.C. Agarwal and S.C. Dutta Roy, "Alternative low-noise realizations for narrow-band recursive digital filters," 1984.
[371] Bhattacharya, M. and R.C. Agarwal, "Cascade realization of linear phase FIR filters with low sensitivity," 1985.
[372] Bhattacharya, M., R.C. Agarwal and S.C. Dutta Roy, "On realization of low-pass and high-pass recursive filters with low sensitivity and low roundoff noise," 1986.
[373] Bhuyan, L. and B.N. Chatterji, "Very low sensitivity recursive digital filter structures," 1981.
[32] Blachman, N.M., "Third-order intermodulation due to quantization," 1981.
[33] Blachman, N.M., "The intermodulation and distortion due to quantization of sinusoids," 1985.
[227] Boddie, J.R., *see D. Mitra.*
[173] Bogner, R.E., *see P. Rashidi.*
[428] Boite, R., X.L. He and J.P. Renard, "On the floating-point realizations of digital filters," 1988.
[60] Bolton, A.G., "A two's complement overflow limit cycle free digital filter structure," 1984.
[280] Bomar, B.W., "Minimum roundoff noise digital filters with some power-of two coefficients," 1984.
[281] Bomar, B.W., "Minimum roundoff noise digital filters with power-of-two coefficients," 1984.
[282] Bomar, B.W., "New second-order state-space structures for realizing low roundoff noise digital filters," 1985.
[283] Bomar, B.W., "Computationally efficient low roundoff noise second-order state-space structures," 1986.
[429] Bomar, B.W. and R.D. Joseph, "Calculation of L_∞ norms for scaling second-order state-space digital filter sections," 1987.
[394] Bonzanigo, F., "Comment on 'Roundoff noise and attenuation sensitivity in digital filters with fixed-point arithmetic' (Author's reply: A. Fettweis)," 1974.
[264] Bosscha, G.J., *see G. Verkroost.*
[185] Brubaker, T.A. and J.N. Gowdy, "Limit cycles in digital filters," 1972.
[374] Bruton, L.T., "Low sensitivity digital ladder filters," 1975.
[44] Bruton, L.T., *see L.E. Turner.*
[133] Bruton, L.T., *see L.E. Turner.*
[142] Bruton, L.T., *see L.E. Turner.*
[143] Bruton, L.T., *see L.E. Turner.*
[395] Buchholz, G., "Simulation of digital filters (in German)," 1982.
[430] Burrascano, P., G. Martinelli and G. Orlandi, "Low-sensitivity digital filters based on zero extraction," 1987.
[369] Burrus, C.S., *see R.C. Agarwal.*
[242] Burrus, C.S., *see T.L. Chang.*
[398] Burrus, C.S., *see S. Chu.*
[102] Butterweck, H.J., "Suppression of parasitic oscillations in second-

order digital filters by means of a controlled-rounding arithmetic," 1975.
[246] Butterweck, H.J., "On the quantization noise contributions in digital filters which are uncorrelated with the output signal," 1979.
[244] Butterweck, H.J., "Quantization effects in various second-order digital filters: a comparitive study," 1981.
[162] Butterweck, H.J., F.H.R. Lucassen and G. Verkroost, "Subharmonics and related quantization effects in periodically excited recursive digital filters," 1983.
[114] Butterweck, H.J., A.C.P. Van Meer and G. Verkroost, "New second-order digital filter sections without limit cycles," 1983.
[115] Butterweck, H.J., A.C.P. Van Meer and G. Verkroost, "New second-order digital filter sections without limit cycles," 1984.
[164] Butterweck, H.J., "Subharmonics and other quantization effects in periodically excited recursive digital filters," 1986.
[163] Butterweck, H.J., F.H.R. Lucassen and G. Verkroost, "Subharmonics and other quantization effects in periodically excited recursive digital filters," 1986.
[105] Butterweck, H.J., *see G. Verkroost.*
[106] Butterweck, H.J., *see G. Verkroost.*
[219] Büttner, M., "Some experimental results concerning random-like noise and limit cycles in recursive digital filters," 1975.
[171] Büttner, M., "A novel approach to eliminate limit cycles in digital filters with a minimum increase in the quantization noise," 1976.
[218] Büttner, M. and J. Schloss, "On the experimental investigations of bounds of the amplitude of limit cycles," 1976.
[172] Büttner, M., "Elimination of limit cycles in digital filters with very low increase in the quantization noise," 1977.
[396] Calcagno, P., E. Garetti and A.R. Meo, "Optimization and comparison of fixed-point implementations for first- and second-order digital blocks," 1971.
[247] Callahan, A.C., "Random rounding: some principles and applications," 1976.
[260] Cassarly, W.J., *see D.R. Morgan.*
[350] Chan, D.S.K. and L.R. Rabiner, "Theory of roundoff noise in cascade realizations of finite impulse response digital filters," 1973.
[351] Chan, D.S.K. and L.R. Rabiner, "An algorithm for minimizing roundoff noise in cascade realizations of finite impulse response digital filters," 1973.
[284] Chan, D.S.K., "Constrained minimization of roundoff noise in fixed-point digital filters," 1979.
[242] Chang, T.L. and C.S. Burrus, "Oscillations caused by quantization in digital filters," 1972.
[186] Chang, T.L., "A note on upper bounds on limit cycles in digital filters," 1976.
[334] Chang, T.L., "A low roundoff noise digital filter structure," 1978.
[335] Chang, T.L., "Error-feedback digital filters," 1979.
[336] Chang, T.L., "Error spectrum shaping structures for digital filters," 1979.
[316] Chang, T.L., "Comparison of roundoff noise variances in several low roundoff noise digital filter structures," 1980.
[317] Chang, T.L., "Correction to 'Comparison of roundoff noise variances in several low roundoff noise digital filter structures'," 1980.
[337] Chang, T.L., "Comments on 'An approach to eliminate roundoff errors in digital filters' (Author's reply: A.I. Abu-El-Haija)," 1980.
[136] Chang, T.L., "Suppression of limit cycles in digital filters designed

with one magnitude-truncation quantizer," 1981.
[318] Chang, T.L., "On low roundoff noise and low sensitivity digital filter structures," 1981.
[319] Chang, T.L., "A unified analysis of roundoff noise reduction in digital filters," 1981.
[338] Chang, T.L. and S.A. White, "An error cancellation digital filter structure and its distributed arithmetic implementation," 1981.
[397] Charalambous, C., A. Antoniou and Z. Motamedi, "Two methods for the reduction of quantization effects in recursive digital filters," 1982.
[390] Charalambous, C., *see A. Antoniou.*
[373] Chatterji, B.N., *see L. Bhuyan.*
[381] Chirlian, P.M., *see D. van Haften.*
[398] Chu, S. and C.S. Burrus, "Roundoff noise in multirate digital filters," 1984.
[23] Chua, L.O. and T. Lin, "Chaos in digital filters," 1988.
[97] Claasen, T.A.C.M., W.F.G. Mecklenbräuker and J.B.H. Peek, "Some remarks on the classification of limit cycles in digital filters," 1973.
[144] Claasen, T.A.C.M., W.F.G. Mecklenbräuker and J.B.H. Peek, "Second-order digital filters with only one magnitude-truncation quantizer and having practically no limit cycles," 1973.
[85] Claasen, T.A.C.M. and L.O.G. Kristiansson, "Improvement of overflow behaviour of second-order digital filters by means of error feedback," 1974.
[150] Claasen, T.A.C.M., W.F.G. Mecklenbräuker and J.B.H. Peek, "A comparison between the stability of second-order filters with various arithmetics," 1974.
[86] Claasen, T.A.C.M., W.F.G. Mecklenbräuker and J.B.H. Peek, "Necessary and sufficient conditions for the absence of overflow phenomena in a second-order recursive digital filter," 1975.
[87] Claasen, T.A.C.M., W.F.G. Mecklenbräuker and J.B.H. Peek, "On the stability of the forced response of digital filters with overflow nonlinearities," 1975.
[145] Claasen, T.A.C.M., W.F.G. Mecklenbräuker and J.B.H. Peek, "Remarks on the zero-input behaviour of second-order digital filters designed with one magnitude-truncation quantizer," 1975.
[151] Claasen, T.A.C.M., W.F.G. Mecklenbräuker and J.B.H. Peek, "Frequency domain criteria for the absence of zero-input limit cycles in nonlinear discrete-time systems, with applications to digital filters," 1975.
[248] Claasen, T.A.C.M., W.F.G. Mecklenbräuker and J.B.H. Peek, "Quantization noise analysis for fixed point digital filters using magnitude truncation for quantization," 1975.
[249] Claasen, T.A.C.M., W.F.G. Mecklenbräuker and J.B.H. Peek, "Quantization errors in digital filters using magnitude truncation quantization," 1975.
[250] Claasen, T.A.C.M., "Quantization noise analysis of digital filters with controlled quantization," 1976.
[3] Claasen, T.A.C.M., W.F.G. Mecklenbräuker and J.B.H. Peek, "Effects of quantization and overflow in recursive digital filters," 1976.
[4] Claasen, T.A.C.M., W.F.G. Mecklenbräuker and J.B.H. Peek, "A survey of quantization and overflow effects in recursive digital filters," 1976.
[251] Claasen, T.A.C.M. and A. Jongepier, "Model for the power spectral density of quantization noise," 1981.
[208] Claesen, L., J. Vandewalle and H. De Man, "General bounds on parasitic oscillations in arbitrary digital filters and their application in CAD," 1984.
[152] De Luca, E.D. and G.O. Martens, "A new coefficient quantization scheme

to suppress limit cycles in state-variable digital filters," 1983.
[208] De Man, H., *see L. Claesen.*
[125] De Vaal, C.D.R. and R. Nouta, "Suppression of parasitic oscillations in floating point wave digital filters," 1978.
[126] De Vaal, C.D.R. and R. Nouta, "On the suppression of zero-input parasitic oscillations in floating point wave digital filters," 1980.
[352] Dehner, G., "A contribution to the optimization of roundoff noise in recursive digital filters," 1975.
[252] Dehner, G., "On the noise behaviour of digital filter blocks of second order," 1976.
[127] Diniz, P.S.R. and A. Antoniou, "On the elimination of constant-input limit cycles in digital filters," 1984.
[285] Diniz, P.S.R. and A. Antoniou, "New improved state-space digital filter structures," 1985.
[339] Diniz, P.S.R. and A. Antoniou, "Low-sensitivity digital filter structures which are amenable to error-spectrum shaping," 1985.
[286] Diniz, P.S.R. and A. Antoniou, "More economical state-space digital filter structures which are free of constant-input limit cycles," 1986.
[370] Dutta Roy, S.C., *see M. Bhattacharya.*
[372] Dutta Roy, S.C., *see M. Bhattacharya.*
[15] Dutta Roy, S.C., *see A. Mahanta.*
[38] Ebert, P.M., J.E. Mazo and M.G. Taylor, "Overflow oscillations in recursive digital filters," 1969.
[58] Eckhardt, B. and W. Winkelnkemper, "Implementation of a second-order digital filter section with stable overflow behaviour," 1973.
[253] Eckhardt, B., "On the roundoff error of a multiplier," 1975.
[254] Eckhardt, B. and H.W. Schüßler, "On the quantization error of a multiplier," 1976.
[307] El-Masry, E., *see A.A. Sakla.*
[56] Erickson, K.T. and A.N. Michel, "Stability analysis of fixed-point digital filters using computer generated Lyapunov functions Part I: direct form and coupled form filters," 1985.
[57] Erickson, K.T. and A.N. Michel, "Stability analysis of fixed-point digital filters using computer generated Lyapunov functions Part II: wave digital and lattice digital filters," 1985.
[109] Eswaran, C. and A. Antoniou, "Wave digital biquads that are free of limit cycles under zero- and constant-input conditions," 1984.
[431] Eswaran, C., A. Antoniou and K. Manivannan, "Universal digital biquads which are free of limit cycles," 1987.
[27] Etzel, M.H. and W.K. Jenkins, "Error correction and suppression properties of RRNS dgital filters," 1980.
[28] Etzel, M.H. and W.K. Jenkins, "The design of specialized classes for efficient recursive filter realizations," 1982.
[303] Fadavi-Ardekani, J., *see S.K. Mitra.*
[220] Fahmy, M.M., *see G.A. Maria.*
[231] Fahmy, M.M., *see M.H. Rahman.*
[365] Fahmy, M.M., *see M.H. Rahman.*
[82] Fam, A.T., "Multiplexing preserving filters," 1978.
[83] Fam, A.T. and C.W. Barnes, "Nonminimal realizations of fixed-point digital filters that are free of all finite word-length limit cycles," 1979.
[62] Fam, A.T., *see C.W. Barnes.*
[11] Fettweis, A., "On the connection between multiplier word length limitation and roundoff noise in digital filters," 1972.
[70] Fettweis, A., "Pseudopassivity, sensitivity and stability of wave digital filters," 1972.

[12] Fettweis, A., "Roundoff noise and attenuation sensitivity in digital filters with fixed-point arithmetic," 1973.
[25] Fettweis, A., "On properties of floating-point roundoff noise," 1974.
[380] Fettweis, A., "On sensitivity and roundoff noise in wave digital filters," 1974.
[71] Fettweis, A. and K. Meerkötter, "Suppression of parasitic oscillations in wave digital filters," 1974.
[399] Fettweis, A., "On the evaluation of roundoff noise in digital filters," 1975.
[72] Fettweis, A. and K. Meerkötter, "Suppression of parasitic oscillations in wave digital filters," 1975.
[73] Fettweis, A. and K. Meerkötter, "Correction to 'Suppression of parasitic oscillaions in wave digital filters'," 1975.
[74] Fettweis, A. and K. Meerkötter, "Suppression of parasitic oscillations in half-synchronic wave digital filters," 1976.
[84] Fettweis, A. and K. Meerkötter, "On parasitic oscillations in digital filters under looped conditions," 1977.
[36] Fettweis, A., "Wave digital filters: Theory and practice," 1986.
[432] Fettweis, A., "Passivity and losslessness in digital filtering," 1988.
[187] Fink, A.M., "A bound on quantization errors in second-order digital filters with complex poles that is tight for small theta," 1976.
[359] Gabel, R.A., *see T.R. Lapp.*
[441] Ganushkin, Y.A., *see A.A. Petrovskiy.*
[396] Garetti, E., *see P. Calcagno.*
[287] Gerheim, A.P., "Numerical solution of the Lyapunov equation for narrow-band digital filters," 1984.
[288] Gerheim, A.P., "Calculation of quantization noise at non-storage nodes," 1984.
[262] Girard, P.E., *see S.R. Parker.*
[153] Gol'denberg, L.M., "Stability of recursive digital filters," 1977.
[159] Gol'denberg, L.M. and B.D. Matyushkin, "Stability of recursive second-order digital filters," 1978.
[64] Gol'denberg, L.M. and B.D. Matyushkin, "Conditions for the absolute stability of processes in recursive digital filters," 1979.
[185] Gowdy, J.N., *see T.A. Brubaker.*
[76] Gray, jr., A.H., "Passive cascaded lattice digital filters," 1980.
[412] Gray, jr., A.H., *see J.D. Markel.*
[433] Green, B.D. and L.E. Turner, "New limit cycle bounds for digital filters," 1988.
[255] Groen, R.W.C., A.W.M. Van den Enden and J.H.F. Ritzerfeld, "On the calculation of quantization noise in digital filters caused by magnitude truncation," 1987.
[234] Gruber, P., "The determination of limit cycles in nonlinear sampled data systems," 1976.
[197] Gruber, P., *see B. Steinle.*
[385] Gupta, S.C., *see K. Renner.*
[381] Haften, D. van and P.M. Chirlian, "An analysis of errors in wave digital filters," 1981.
[79] Hamada, N., *see M. Takizawa.*
[188] Han, S.K., *see D.G. He.*
[265] Haznedar, H., "Errors in pseudo fixed-point recursive digital filters with quantization after final accumulation," 1983.
[188] He, D.G. and S.K. Han, "Novel approximations to $\Sigma|h(n)|$ for second-order digital filters," 1985.
[428] He, X.L., *see R. Boite.*
[204] Heinen, J.A., "A bound on the norm of the individual state variables of

a digital filter," 1985.
[37] Hess, S.F., *see S.R. Parker.*
[229] Hess, S.F., *see S.R. Parker.*
[230] Hess, S.F., *see S.R. Parker.*
[320] Higgins, W.E. and D.C. Munson, jr., "Noise reduction strategies for digital filters: Error spectrum shaping versus the optimal linear state-space formulation," 1982.
[321] Higgins, W.E. and D.C. Munson, jr., "Optimal error spectrum shaping for cascade-form digital filters," 1982.
[322] Higgins, W.E. and D.C. Munson, jr., "A section ordering strategy for cascade-form digital filters using error spectrum shaping," 1983.
[323] Higgins, W.E. and D.C. Munson, jr., "Optimal and suboptimal error spectrum shaping for cascade-form digital filters," 1984.
[54] Higuchi, T. and H. Takeo, "A state-space approach for the elimination of limit cycles in digital filters with arbitrary structures," 1979.
[271] Higuchi, T., *see M. Akamine.*
[272] Higuchi, T., *see M. Akamine.*
[292] Higuchi, T., *see M. Iwatsuki.*
[55] Higuchi, T., *see M. Kawamata.*
[129] Higuchi, T., *see M. Kawamata.*
[130] Higuchi, T., *see M. Kawamata.*
[131] Higuchi, T., *see M. Kawamata.*
[297] Higuchi, T., *see M. Kawamata.*
[17] Higuchi, T., *see N. Ohta.*
[353] Hirano, K., H. Sakaguchi and B. Liu, "Optimization of recursive cascade filters," 1976.
[413] Hirano, K., *see S.K. Mitra.*
[378] Hirano, K., *see N. Nishimura.*
[7] Hoptner, N., *see A. Lacroix.*
[363] Hug, H., *see E. Lueder.*
[354] Hwang, S.Y., "On optimization of cascade fixed-point digital filters," 1974.
[400] Hwang, S.Y., "Dynamic range constraint in state-space digital filtering," 1975.
[401] Hwang, S.Y., "Comments on 'Reduction of roundoff noise in wave digital filters'," 1975.
[289] Hwang, S.Y., "Roundoff noise in state-space digital filtering: A general analysis," 1976.
[290] Hwang, S.Y., "Roundoff noise minimization in state-space digital filtering," 1976.
[291] Hwang, S.Y., "Minimum uncorrelated unit noise in state-space digital filtering," 1977.
[146] Ishii, R., Y. Mitome and S. Kawasaki, "Limit cycles in a second-order all-pole digital system (in Japanese; English abstract)," 1980.
[165] Ishii, R. and M. Kato, "An effect of a limit cycle on an output signal," 1982.
[161] Itskovich, Y.S., "Roundoff noise in digital filters with small input signal (transl. of: Radiotekhnika, vol. 37, no. 5, p. 39, May 1982)," 1982.
[292] Iwatsuki, M., M. Kawamata and T. Higuchi, "Synthesis of minimum sensitivity structures in linear systems using controllability and observability measures," 1986.
[189] Jackson, L.B., "An analysis of limit cycles due to multiplication rounding in recursive digital (sub-) filters," 1969.
[35] Jackson, L.B., "On the interaction of roundoff noise and dynamic range in digital filters," 1970.

[190] Jackson, L.B., "Comments on 'Quantizer-induced digital controller limit cycles'," 1970.
[355] Jackson, L.B., "Roundoff noise analysis for fixed-point digital filters realized in cascade or parallel form," 1970.
[293] Jackson, L.B., "Roundoff noise bounds derived from coefficient sensitivities for digital filters," 1976.
[294] Jackson, L.B., "Lower bounds on the roundoff noise from digital filters in cascade or parallel form," 1976.
[295] Jackson, L.B., A.G. Lindgren and Y. Kim, "Synthesis of state-space digital filters with low roundoff noise and coefficient sensitivity," 1977.
[209] Jackson, L.B., "Limit cycles in state-space structures for digital filters," 1979.
[296] Jackson, L.B., A.G. Lindgren and Y. Kim, "Optimal synthesis of second-order state-space structures for digital filters," 1979.
[210] Jackson, L.B. and N.H.K. Judell, "Addendum to 'Limit cycles in state-space structures for digital filters'," 1980.
[1] Jackson, L.B., *Digital Filters and Signal Processing,* 1986.
[356] Jaschinski, W. and K.A. Owenier, "Some results in optimization of roundoff noise for fixed-point digital filters realized in cascade form," 1977.
[211] Jatnieks, G.U. and B.A. Shenoi, "Zero-input limit cycles in coupled digital filters," 1974.
[402] Jenkins, W.K. and B.J. Leon, "An analysis of quantization error in digital filters based on interval algebras," 1975.
[403] Jenkins, W.K., "Recent advances in residue number techniques for recursive digital filtering," 1979.
[27] Jenkins, W.K., *see M.H. Etzel.*
[28] Jenkins, W.K., *see M.H. Etzel.*
[14] Jenq, Y.C., *see P.H. Lo.*
[66] Jenq, Y.C., *see P.H. Lo.*
[205] Jenq, Y.C., *see P.H. Lo.*
[251] Jongepier, A., *see T.A.C.M. Claasen.*
[429] Joseph, R.D., *see B.W. Bomar.*
[210] Judell, N.H.K., *see L.B. Jackson.*
[26] Julien, G.A., *see A.Z. Baraniecki.*
[5] Kaiser, J.F., "Quantization effects in digital filters," 1973.
[6] Kaiser, J.F., "On the limit cycle problem," 1976.
[203] Kaiser, J.F., *see I.W. Sandberg.*
[256] Kammeyer, K.M., "Quantization error analysis of the distributed arithmetic," 1977.
[156] Kan, E.P.F. and J.K. Aggarwal, "Minimum-deadband design of digital filters," 1971.
[266] Kan, E.P.F. and J.K. Aggarwal, "Error analysis of digital filter employing floating-point arithmetic," 1971.
[223] Kaneko, T., "Limit cycle oscillations in floating-point digital filters," 1973.
[267] Kaneko, T., *see B. Liu.*
[147] Kao, C.Y., "An analysis of limit cycles due to sign-magnitude truncation in multiplication in recursive digital filters," 1971.
[181] Kao, C.Y., "Apparatus for suppressing limit cycles due to quantization in digital filters," 1973.
[392] Kao, C.Y., *see A. Anuff.*
[165] Kato, M., *see R. Ishii.*
[128] Kawamata, M., "Synthesis of limit cycle free digital filters based on the state equations (in Japanese; English abstract)," 1980.
[55] Kawamata, M. and T. Higuchi, "A sufficient condition for the absence of

overflow oscillations in arbitrary digital filters based on the element equations," 1980.
[129] Kawamata, M. and T. Higuchi, "A systematic approach to synthesis of limit cycle free digital filters," 1983.
[130] Kawamata, M. and T. Higuchi, "Synthesis of limit cycle free state-space digital filters with minimum coefficient quantization error," 1983.
[131] Kawamata, M. and T. Higuchi, "On the absence of limit cycles in a class of state-space digital filters which contains minimum noise realizations," 1984.
[297] Kawamata, M. and T. Higuchi, "A unified approach to the optimal synthesis of fixed-point state-space digital filters," 1985.
[292] Kawamata, M., *see M. Iwatsuki.*
[357] Kawarai, S., "Advantageous estimation formula of the least roundoff noise for the cascade fixed-point digital filters," 1985.
[146] Kawasaki, S., *see R. Ishii.*
[157] Kieburtz, R.B., "An experimental study of roundoff effects in a tenth-order recursive digital filter," 1973.
[158] Kieburtz, R.B., "Rounding and truncation limit cycles in a recursive digital filter," 1974.
[177] Kieburtz, R.B. and K.V. Mina, "Digital filter circuit," 1975.
[178] Kieburtz, R.B., K.V. Mina and V.B. Lawrence, "Control of limit cycles in recursive digital filters by randomized quantization," 1976.
[179] Kieburtz, R.B., K.V. Mina and V.B. Lawrence, "Control of limit cycles in recursive digital filters by randomized quantization," 1977.
[155] Kim, N.H., *see B.H. Nam.*
[295] Kim, Y., *see L.B. Jackson.*
[296] Kim, Y., *see L.B. Jackson.*
[243] King, R.A., *see D.C. McLernon.*
[135] King, R.A., *see K.M. Wong.*
[404] Kingsbury, N.G., "Digital filter second-order element with low quantising noise for poles and zeros at low frequencies," 1973.
[79] Kishi, H., *see M. Takizawa.*
[434] Kleine, U. and T.G. Noll, "On the forced-response stability of wave digital filters using carry-save arithmetic," 1987.
[95] Krishnan, A. and R. Subramanian, "Jump phenomena in digital filters," 1979.
[96] Kristiansson, L.O.G., "Jump phenomena in digital filters," 1974.
[85] Kristiansson, L.O.G., *see T.A.C.M. Claasen.*
[298] Ku, W.H. and S.M. Ng, "Floating-point coefficient sensitivity and roundoff noise of recursive digital filters realized in ladder structures," 1975.
[191] Kubota, H. and S. Tsuji, "An upper bound on the RMS value of limit cycles in digital filters and a reduction method (in Japanese; English abstract)," 1981.
[192] Kubota, H., T. Yoshida and S. Tsuji, "A consideration on limit cycles due to value-truncation errors in digital filters (in Japanese; English abstract)," 1981.
[65] Kurosawa, K., "Limit cycle and overflow free digital filters (in Japanese; English abstract)," 1982.
[405] Kwan, H.K., "Amplitude scaling of arbitrary linear digital networks," 1984.
[110] Kwan, H.K., "A multi-output second-order digital filter structure for VLSI implementation," 1985.
[111] Kwan, H.K., "A multi-output second-order digital filter without limit cycle oscillations," 1985.
[112] Kwan, H.K., "A multi-output wave digital biquad using magnitude trunca-

tion instead of controlled rouning," 1985.
[435] Kwan, H.K. and W. Lee, "Low sensitivities and low roundoff noise realizations of sharp cut-off second-order digital filters," 1987.
[224] Lacroix, A., "Limit cycles in floating point digital filters," 1975.
[225] Lacroix, A., "Underflow limit cycles in floating point digital filters," 1975.
[226] Lacroix, A., "Limit cycles in floating point digital filters," 1976.
[7] Lacroix, A. and N. Hoptner, "Simulation of digital filters with the aid of a universal program system," 1979.
[406] Lam, J.W.K., V. Ramachandran and M.N.S. Swamy, "Comparison of the effects of quantization on digital filters," 1981.
[407] Lam, J.W.K., V. Ramachandran and M.N.S. Swamy, "Comparison of the effects of quantization on digital filters," 1982.
[358] Lanfer, H. and E. Lueder, "Minimizing the roundoff noise in digital filters by the branch and bound method," 1979.
[359] Lapp, T.R. and R.A. Gabel, "An algorithm for optimally ordering the sections of a cascade digital filter," 1976.
[221] Lawrence, V.B. and K.V. Mina, "A new and interesting class of limit cycles," 1977.
[180] Lawrence, V.B. and K.V. Mina, "Control of limit cycles oscillations in second-order recursive digital filters using constrained random quantization," 1978.
[222] Lawrence, V.B. and K.V. Mina, "A new and interesting class of limit cycles in recursive digital filters," 1979.
[140] Lawrence, V.B. and D. Mitra, "Digital filters with control of limit cycles," 1980.
[141] Lawrence, V.B. and E.A. Lee, "Quantization schemes for recursive digital filters," 1982.
[178] Lawrence, V.B., *see R.B. Kieburtz.*
[179] Lawrence, V.B., *see R.B. Kieburtz.*
[103] Lawrence, V.B., *see D. Mitra.*
[104] Lawrence, V.B., *see D. Mitra.*
[13] Ledbetter, J.D. and R. Yarlagadda, "Coefficient quantization effects on pole locations for state model digital filters," 1979.
[299] Lee, E.A. and D.G. Messerschmitt, "On quantization effects in state-variable filter implementations," 1985.
[141] Lee, E.A., *see V.B. Lawrence.*
[435] Lee, W., *see H.K. Kwan.*
[360] Lee, W.S., "Optimization of digital filters for low roundoff noise," 1973.
[361] Lee, W.S., "Optimization of digital filters for low roundoff noise," 1974.
[402] Leon, B.J., *see W.K. Jenkins.*
[148] Lepschy, A., G.A. Mian and U. Viaro, "Parameter space quantization in fixed-point digital filters," 1986.
[149] Lepschy, A., G.A. Mian and U. Viaro, "Stability analysis of second-order direct form digital filters with to roundoff quantizers," 1986.
[436] Lepschy, A., G.A. Mian and U. Viaro, "Stability of coupled-form digital filters with roundoff quantization," 1987.
[437] Lepschy, A., G.A. Mian and U. Viaro, "A contribution to the stability analysis of second-order direct-form digital filters with magnitude truncation," 1987.
[438] Lepschy, A., G.A. Mian and U. Viaro, "Effects of quantization in second-order fixed-point digital filters with two's complement truncation quantizers," 1988.
[439] Lepschy, A., G.A. Mian and U. Viaro, "Parameter plane quantisation in-

duced by the signal quantisation in second-order fixed-point digital filter with one quantiser," 1988.
[300] Leung, S.H., "State-space realization of digital filters with stationary inputs," 1983.
[245] Leung, S.H., *see C.W. Barnes.*
[274] Leung, S.H., *see C.W. Barnes.*
[23] Lin, T., *see L.O. Chua.*
[295] Lindgren, A.G., see *L.B. Jackson.*
[296] Lindgren, A.G., *see L.B. Jackson.*
[313] Lindgren, A.G., *see J. Zeman.*
[314] Lindgren, A.G., *see J. Zeman.*
[267] Liu, B. and T. Kaneko, "Error analysis of digital filters realized with floating-point arithmetic," 1969.
[408] Liu, B., "Effects of finite wordlength on the accuracy of digital filters - A review," 1971.
[257] Liu, B. and M.E. Van Valkenburg, "On roundoff error of fixed-point digital filters using sign-magnitude truncation," 1971.
[258] Liu, B. and M.E. Van Valkenburg, "On roundoff error of fixed-point digital filters using sign-magnitude truncation," 1972.
[362] Liu, B. and A. Peled, "Heuristic optimization of the cascade realization of fixed-point digital filters," 1975.
[207] Liu, B. and M.R. Bateman, "Limit cycle bounds for digital filters with error spectrum shaping," 1980.
[409] Liu, B. and R. Ansari, "Quantization effects in computationally efficient realizations of recursive digital filters," 1982.
[391] Liu, B., see *R. Ansari.*
[353] Liu, B., see *K. Hirano.*
[342] Liu, B., see *D.C. Munson, jr..*
[343] Liu, B., see *D.C. Munson, jr..*
[375] Liu, B., see *D.C. Munson, jr..*
[376] Liu, B., see *D.C. Munson, jr..*
[377] Liu, B., see *D.C. Munson, jr..*
[417] Liu, B., see *A. Peled.*
[366] Liu, B., see *K. Steiglitz.*
[215] Liu, B., see *T. Thong.*
[216] Liu, B., see *T. Thong.*
[344] Liu, B., see *T. Thong.*
[410] Liu, E.S.K. and L.E. Turner, "Stability, dynamic range and roundoff noise in a new second-order recursive digital filter," 1982.
[116] Liu, E.S.K. and L.E. Turner, "Quantisation effects in second-order wave digital filters," 1983.
[411] Liu, E.S.K. and L.E. Turner, "Stability, dynamic range and roundoff noise in a new second-order recursive digital filter," 1983.
[176] Liu, H.M. and M.H. Ackroyd, "Suppression of limit cycles in digital filters by random dither," 1983.
[175] Liu, H.M., *see M.H. Ackroyd.*
[440] Liu, V.C. and P.P. Vaidyanathan, "Circulant and skew-circulant matrices as new normal-form realization of IIR digital filters," 1988.
[122] Liu, V.C., *see P.P. Vaidyanathan.*
[66] Lo, P.H. and Y.C. Jenq, "On the overflow problem in a second-order digital filter," 1981.
[205] Lo, P.H. and Y.C. Jenq, "An ℓm-norm bound for state variables in second-order recursive digital filters," 1981.
[14] Lo, P.H. and Y.C. Jenq, "Minimum sensitivity realization of second-order recursive digital filter," 1982.
[193] Long, J.L. and T.N. Trick, "An absolute bound on limit cycles due to

roundoff errors in digital filters," 1973.
[194] Long, J.L. and T.N. Trick, "A note on absolute bounds on quantization errors in fixed-point implementations of digital filters," 1975.
[382] Long, J.L. and T.N. Trick, "Sensitivity and noise comparison of some fixed-point recursive digital filter structures," 1975.
[63] Lubenow, H., *see U. Bernhardt.*
[162] Lucassen, F.H.R., *see H.J. Butterweck.*
[163] Lucassen, F.H.R., *see H.J. Butterweck.*
[124] Lucioni, G., "Alternative method to magnitude truncation in wave digital filters," 1987.
[363] Lueder, E., H. Hug and W. Wolf, "Minimizing the roundoff noise in digital filters by dynamic programming," 1975.
[364] Lueder, E., "Digital filter processing with improved accuracy," 1981.
[358] Lueder, E., *see H. Lanfer.*
[15] Mahanta, A., R.C. Agarwal and S.C. Dutta Roy, "FIR filter structures having low sensitivity and roundoff noise," 1982.
[431] Manivannan, K., *see C. Eswaran.*
[220] Maria, G.A. and M.M. Fahmy, "Limit cycle oscillations in a cascade of first- and second-order digital filter sections," 1975.
[231] Maria, G.A., *see M.H. Rahman.*
[412] Markel, J.D. and A.H. Gray, jr., "Roundoff noise characteristic of a class of orthogonal polynomial structures," 1975.
[420] Marshall, J.W., *see F. Taylor.*
[152] Martens, G.O., *see E.D. De Luca.*
[430] Martinelli, G., *see P. Burrascano.*
[64] Matyushkin, B.D., *see L.M. Gol'denberg.*
[159] Matyushkin, B.D., *see L.M. Gol'denberg.*
[45] Mazo, J.E., "On the stability of higher order digital filters which use saturation arithmetic," 1978.
[38] Mazo, J.E., *see P.M. Ebert.*
[340] McCrea, P.G. and I.H. Witten, "Reducing noise in recursive digital filters by residue retention," 1978.
[24] McCrea, P.G., *see D. Williamson.*
[345] McCrea, P.G., *see D. Williamson.*
[139] McCrea, P.G., *see I.H. Witten.*
[243] McLernon, D.C. and R.A. King, "Additional properties of one-dimensional limit cycles," 1986.
[259] McWilliam, A.J. and B.J. Stanier, "Roundoff noise prediction in short-wordlength fixed-point digital filters," 1978.
[3] Mecklenbräuker, W.F.G., *see T.A.C.M. Claasen.*
[4] Mecklenbräuker, W.F.G., *see T.A.C.M. Claasen.*
[86] Mecklenbräuker, W.F.G., *see T.A.C.M. Claasen.*
[87] Mecklenbräuker, W.F.G., *see T.A.C.M. Claasen.*
[97] Mecklenbräuker, W.F.G., *see T.A.C.M. Claasen.*
[144] Mecklenbräuker, W.F.G., *see T.A.C.M. Claasen.*
[145] Mecklenbräuker, W.F.G., *see T.A.C.M. Claasen.*
[150] Mecklenbräuker, W.F.G., *see T.A.C.M. Claasen.*
[151] Mecklenbräuker, W.F.G., *see T.A.C.M. Claasen.*
[248] Mecklenbräuker, W.F.G., *see T.A.C.M. Claasen.*
[249] Mecklenbräuker, W.F.G., *see T.A.C.M. Claasen.*
[100] Meerkötter, K. and W. Wegener, "A new second-order digital filter without parasitic oscillations," 1975.
[101] Meerkötter, K., "Realization of limit cycle-free second-order digital filters," 1976.
[71] Meerkötter, K., *see A. Fettweis.*
[72] Meerkötter, K., *see A. Fettweis.*

[73] Meerkötter, K., *see A. Fettweis.*
[74] Meerkötter, K., *see A. Fettweis.*
[84] Meerkötter, K., *see A. Fettweis.*
[396] Meo, A.R., *see P. Calcagno.*
[299] Messerschmitt, D.G., *see E.A. Lee.*
[41] Meyer, G., "Limit cycles in digital filters with fixed-point arithmetic (in German)," 1976.
[148] Mian, G.A., *see A. Lepschy.*
[149] Mian, G.A., *see A. Lepschy.*
[436] Mian, G.A., *see A. Lepschy.*
[437] Mian, G.A., *see A. Lepschy.*
[438] Mian, G.A., *see A. Lepschy.*
[439] Mian, G.A., *see A. Lepschy.*
[56] Michel, A.N., *see K.T. Erickson.*
[57] Michel, A.N., *see K.T. Erickson.*
[42] Mills, W.L., C.T. Mullis and R.A. Roberts, "Digital filter realizations without overflow oscillations," 1978.
[43] Mills, W.L., C.T. Mullis and R.A. Roberts, "Digital filter realizations without overflow oscillations," 1978.
[301] Mills, W.L., C.T. Mullis and R.A. Roberts, "Normal realizations of IIR digital filters," 1979.
[302] Mills, W.L., C.T. Mullis and R.A. Roberts, "Low roundoff noise and normal realizations of fixed-point IIR digital filters," 1981.
[177] Mina, K.V., *see R.B. Kieburtz.*
[178] Mina, K.V., *see R.B. Kieburtz.*
[179] Mina, K.V., *see R.B. Kieburtz.*
[180] Mina, K.V., *see V.B. Lawrence.*
[221] Mina, K.V., *see V.B. Lawrence.*
[222] Mina, K.V., *see V.B. Lawrence.*
[341] Mintzer, L. and J.P. Strauss, "A practical digital filter with near optimal rounding noise cancellation," 1979.
[146] Mitome, Y., *see R. Ishii.*
[46] Mitra, D., "Criteria for determining if a high-order digital filter using saturation arithmetic is free of overflow oscillations," 1977.
[47] Mitra, D., "Large amplitude self-sustained oscillations in difference equations which describe digital filter sections using saturation arithmetic," 1977.
[48] Mitra, D., "Summary of some results on large amplitude, self-sustained oscillations in high order digital filter sections using saturation arithmetic," 1977.
[201] Mitra, D., "A bound on limit cycles in digital filters which exploits a particular structural property of the quantization," 1977.
[202] Mitra, D., "A bound on limit cycles in digital filters which exploits a particular structural property of the quantization," 1977.
[49] Mitra, D., "The absolute stability of high-order discrete-time systems utilizing the saturation nonlinearity," 1978.
[50] Mitra, D., "Summary of results on the absolute stability of high-order, discrete-time systems utilizing the saturation nonlinearity," 1978.
[103] Mitra, D. and V.B. Lawrence, "Summary of results on controlled rounding arithmetics, for direct-form digital filters, that eliminate all self-sustained oscillations," 1978.
[227] Mitra, D. and J.R. Boddie, "Limit cycles in floating point digital filters," 1979.
[104] Mitra, D. and V.B. Lawrence, "Controlled rounding arithmetics for second-order direct-form digital filters, that eliminate all self-sustained oscillations," 1981.

[140] Mitra, D., *see V.B. Lawrence.*
[413] Mitra, S.K., K. Hirano and H. Sakaguchi, "A simple method of computing the input quantization and multiplication roundoff errors in a digital filter," 1974.
[414] Mitra, S.K. and K. Mondal, "A novel approach to recursive digital filter realization with low roundoff noise," 1976.
[303] Mitra, S.K. and J. Fadavi-Ardekani, "A new approach to the design of cost-optimal low-noise digital filters," 1981.
[442] Mitra, S.K., *see T. Saramaki.*
[419] Mitra, S.K., *see J. Szczupak.*
[423] Mitra, S.K., *see P.P. Vaidyanathan.*
[117] Miyata, M., "Roundoff noise control in time domain for digital filters and oscillators," 1980.
[275] Miyawaki, T., *see C.W. Barnes.*
[414] Mondal, K., *see S.K. Mitra.*
[415] Monkewich, O. and W. Steenaart, "Stored product digital filtering with non-linear quantization," 1976.
[160] Monkewich, O. and W. Steenaart, "Deadband effects and limit cycles in stored-product digital filters," 1981.
[52] Montgomery, H.D., "A non-linear digital oscillator," 1972.
[118] Moon, P.R., "Limit cycle suppression by diagonally dominant Lyapunov functions in state-space digital filters," 1984.
[166] Morgan, D.R. and A. Aridgides, "Discrete-time distortion analysis of quantized sinusoids," 1985.
[260] Morgan, D.R. and W.J. Cassarly, "Effect of wordlength truncation on quantized gaussian random variables," 1986.
[390] Motamedi, Z., *see A. Antoniou.*
[397] Motamedi, Z., *see C. Charalambous.*
[304] Mullis, C.T. and R.A. Roberts, "Roundoff noise in digital filters: Frequency transformations and invariants," 1976.
[305] Mullis, C.T. and R.A. Roberts, "Synthesis of minimum roundoff noise fixed-point digital filters," 1976.
[306] Mullis, C.T. and R.A. Roberts, "Filter structures which minimize roundoff noise in fixed-point digital filters," 1976.
[324] Mullis, C.T. and R.A. Roberts, "An interpretation of error spectrum shaping in digital filters," 1982.
[42] Mullis, C.T., *see W.L. Mills.*
[43] Mullis, C.T., *see W.L. Mills.*
[301] Mullis, C.T., *see W.L. Mills.*
[302] Mullis, C.T., *see W.L. Mills.*
[2] Mullis, C.T., *see R.A. Roberts.*
[375] Munson, jr., D.C. and B. Liu, "Low-noise realizations for digital filters with poles near the unit circle," 1978.
[342] Munson, jr., D.C. and B. Liu, "Narrow-band recursive filters with error spectrum shaping," 1979.
[376] Munson, jr., D.C. and B. Liu, "Low-noise realizations for narrow-band recursive digital filters," 1980.
[377] Munson, jr., D.C. and B. Liu, "ROM/ACC realization of digital filters for poles near the unit circle," 1980.
[98] Munson, jr., D.C., "Accessibility of zero-input limit cycles," 1981.
[99] Munson, jr., D.C., "Accessibility of zero-input limit cycles," 1981.
[235] Munson, jr., D.C., "Determining exact maximum amplitude limit cycles in digital filters," 1981.
[343] Munson, jr., D.C. and B. Liu, "Narrow-band recursive filters with error spectrum shaping," 1981.
[236] Munson, jr., D.C., J.H. Strickland and T.P. Walker, "Maximum amplitude

zero-input limit cycles in digital filters," 1984.
[320] Munson, jr., D.C., *see W.E. Higgins.*
[321] Munson, jr., D.C., *see W.E. Higgins.*
[322] Munson, jr., D.C., *see W.E. Higgins.*
[323] Munson, jr., D.C., *see W.E. Higgins.*
[155] Nam, B.H. and N.H. Kim, "Stability analysis of modified coupled-form digital filter using a constructive algorithm," 1985.
[423] Neuvo, Y., *see P.P. Vaidyanathan.*
[298] Ng, S.M., *see W.H. Ku.*
[77] Nguyen, D.T., "Overflow oscillations in digital lattice filters," 1981.
[16] Nishihara, A., "Low sensitivity second-order digital filters analysis and design in terms of frequency sensitivity," 1984.
[132] Nishihara, A., "Design of limit cycle-free digital biquad filters," 1986.
[378] Nishimura, N., K. Hirano and R.N. Pal, "A new class of very low sensitivity and low roundoff noise recursive digital filters," 1981.
[434] Noll, T.G., *see U. Kleine.*
[125] Nouta, R., *see C.D.R. De Vaal.*
[126] Nouta, R., *see C.D.R. De Vaal.*
[17] Ohta, N. and T. Higuchi, "Estimation of error variance due to coefficient quantization of digital filters excited by random signals, 1977.
[238] Ojongbede, H.A., "Limit cycle constraints for recursive digital filter design," 1970.
[268] Oppenheim, A.V., "Realization of digital filters using block-floating-point arithmetic," 1970.
[8] Oppenheim, A.V. and C.J. Weinstein, "Effects of finite register length in digital filtering and the Fast Fourier Transform," 1972.
[270] Oppenheim, A.V., *see C.J. Weinstein.*
[430] Orlandi, G., *see P. Burrascano.*
[34] Otten, M.P.G., *see P.J.M. Simons.*
[356] Owenier, K.A., *see W. Jaschinski.*
[378] Pal, R.N., *see N. Nishimura.*
[383] Pang, K.K. and B. Treloar, "On the dynamic range problem of wave digital filters," 1983.
[384] Pang, K.K. and B. Treloar, "An investigation of the dynamic range problem in wave digital filters," 1983.
[20] Pang, K.K., *see P.K. Sim.*
[51] Pang, K.K., *see P.K. Sim.*
[91] Pang, K.K., *see P.K. Sim.*
[212] Pang, K.K., *see P.K. Sim.*
[443] Pang, K.K., *see P.K. Sim.*
[229] Parker, S.R. and S.F. Hess, "Limit cycle oscillations in digital filters," 1971.
[195] Parker, S.R., "The phenomena of quantization error and limit cycles in fixed point digital filters," 1972.
[37] Parker, S.R. and S.F. Hess, "Canonic realizations of second-order digital filters due to finite precision arithmetic," 1972.
[230] Parker, S.R. and S.F. Hess, "Heuristic bands for the frequency of digital oscillators due to quantization noise," 1972.
[261] Parker, S.R., "Correlated noise in digital filters," 1974.
[196] Parker, S.R. and S. Yakowitz, "A general method for calculating quantization error bounds in fixed-point multivariable digital filters," 1975.
[262] Parker, S.R. and P.E. Girard, "Correlated noise due to roundoff in fixed-point digital filters," 1976.
[9] Parker, S.R., "Limit cycles and correlated noise in digital filters,"

1979.
[217] Parker, S.R. and F.A. Perry, "Hidden limit cycles and error bounds in wave digital filters," 1979.
[200] Parker, S.R., *see S. Yakowitz.*
[416] Patney, R.K., "A different look at roundoff noise in digital filters," 1980.
[154] Pavani, A.M.B. and J. Szczupak, "A mathematical model for digital filters under limit condition," 1985.
[75] Payan, J. and J.I. Acha, "Parasitic oscillations in normalized digital structures," 1986.
[368] Payan, J., *see J.I. Acha.*
[10] Peceli, G., "Finite wordlength effects in digital filters," 1984.
[3] Peek, J.B.H., *see T.A.C.M. Claasen.*
[4] Peek, J.B.H., *see T.A.C.M. Claasen.*
[86] Peek, J.B.H., *see T.A.C.M. Claasen.*
[87] Peek, J.B.H., *see T.A.C.M. Claasen.*
[97] Peek, J.B.H., *see T.A.C.M. Claasen.*
[144] Peek, J.B.H., *see T.A.C.M. Claasen.*
[145] Peek, J.B.H., *see T.A.C.M. Claasen.*
[150] Peek, J.B.H., *see T.A.C.M. Claasen.*
[151] Peek, J.B.H., *see T.A.C.M. Claasen.*
[248] Peek, J.B.H., *see T.A.C.M. Claasen.*
[249] Peek, J.B.H., *see T.A.C.M. Claasen.*
[417] Peled, A. and B. Liu, "A new hardware realization of digital filters," 1974.
[362] Peled, A., *see B. Liu.*
[217] Perry, F.A., *see S.R. Parker.*
[206] Peterson, A.M., *see A.I. Abu-El-Haija.*
[213] Peterson, A.M., *see A.I. Abu-El-Haija.*
[329] Peterson, A.M., *see A.I. Abu-El-Haija.*
[330] Peterson, A.M., *see A.I. Abu-El-Haija.*
[367] Peterson, A.M., *see A.I. Abu-El-Haija.*
[441] Petrovskiy, A.A. and Y.A. Ganushkin, "The signal-quantizing noise variance in second-order digital recursive filters," 1987.
[237] Prasad, K.P. and P.S. Reddy, "Limit cycles in second-order digital filters," 1980.
[350] Rabiner, L.R., *see D.S.K. Chan.*
[351] Rabiner, L.R., *see D.S.K. Chan.*
[231] Rahman, M.H., G.A. Maria and M.M. Fahmy, "Bounds on zero-input limit cycles in all-pole digital filters," 1976.
[365] Rahman, M.H. and M.M. Fahmy, "A roundoff noise minimization technique for cascade realization of digital filters," 1977.
[406] Ramachandran, V., *see J.W.K. Lam.*
[407] Ramachandran, V., *see J.W.K. Lam.*
[29] Ramnarayan, R. and F. Taylor, "Limit cycles in large moduli residue number system digital filters," 1986.
[173] Rashidi, P. and R.E. Bogner, "Suppression of limit cycles oscillations in second-order recursive digital filters," 1978.
[237] Reddy, P.S., *see K.P. Prasad.*
[428] Renard, J.P., *see R. Boite.*
[325] Renfors, M., "Roundoff noise in error-feedback state-space filters," 1983.
[137] Renfors, M., B. Sikström and L. Wanhammer, "LSI implementation of limit cycle free digital filters using error-feedback techniques," 1983.
[385] Renner, K. and S.C. Gupta, "Reduction of roundoff noise in wave digital filters," 1974.

[379] Rezk, M.G., *see A. Antoniou.*
[255] Ritzerfeld, J.H.F., *see R.W.C. Groen.*
[2] Roberts, R.A. and C.T. Mullis, *Digital Signal Processing*, 1987.
[273] Roberts, R.A., *see M. Arjmand.*
[42] Roberts, R.A., *see W.L. Mills.*
[43] Roberts, R.A., *see W.L. Mills.*
[301] Roberts, R.A., *see W.L. Mills.*
[302] Roberts, R.A., *see W.L. Mills.*
[304] Roberts, R.A., *see C.T. Mullis.*
[305] Roberts, R.A., *see C.T. Mullis.*
[306] Roberts, R.A., *see C.T. Mullis.*
[324] Roberts, R.A., *see C.T. Mullis.*
[353] Sakaguchi, H., *see K. Hirano.*
[413] Sakaguchi, H., *see S.K. Mitra.*
[307] Sakla, A.A. and E. El-Masry, "Optimal roundoff noise selection in class of minimal canonical nonproduct of multiplier digital filters," 1985.
[92] Samueli, H. and A.N. Willson, jr., "Almost periodic forced overflow oscillations in digital filters," 1980.
[93] Samueli, H. and A.N. Willson, jr., "Almost period P sequences and the analysis of forced overflow oscillations in digital filters," 1982.
[94] Samueli, H. and A.N. Willson, jr., "Nonperiodic forced overflow oscillations in digital filters," 1983.
[422] Sanchez-Sinencio, E., *see T.N. Trick.*
[228] Sandberg, I.W., "Floating-point roundoff accumulation in digital filter realizations," 1967.
[18] Sandberg, I.W., "A theorem concerning limit cycles in digital filters," 1969.
[203] Sandberg, I.W. and J.F. Kaiser, "A bound on limit cycles in fixed-point implementations of digital filters," 1972.
[19] Sandberg, I.W., "The zero-input response of digital filters using saturation arithmetics," 1979.
[442] Saramaki, T., T. Yu and S.K. Mitra, "Very low sensitivity realization if IIR digital filters using a cascade of complex all-pass structures," 1987.
[218] Schloss, J., *see M. Büttner.*
[254] Schüßler, H.W., *see B. Eckhardt.*
[425] Shaked, U., *see G. Amit.*
[211] Shenoi, B.A., *see G.U. Jatnieks.*
[367] Shenoi, K., *see A.I. Abu-El-Haija.*
[80] Shinnaka, S., *see C.W. Barnes.*
[81] Shinnaka, S., *see C.W. Barnes.*
[137] Sikström, B., *see M. Renfors.*
[51] Sim, P.K. and K.K. Pang, "On the asymptotic stability of an N-th order nonlinear recursive digital filter," 1983.
[20] Sim, P.K. and K.K. Pang, "Design criterion for zero-input asymptotic overflow stability of recursive digital filters in the presence of quantization," 1985.
[91] Sim, P.K. and K.K. Pang, "Effects of input-scaling on the asymptotic overflow stability properties of second-order recursive digital filters," 1985.
[212] Sim, P.K. and K.K. Pang, "Quantization phenomena in a class of complex biquad recursive digital filters," 1986.
[443] Sim, P.K. and K.K. Pang, "Conditions for overflow stability of a class of complex biquad digital filters," 1987.
[444] Sim, P.K., "Relationship between input-scaling and stability of the forced response of recursive digital filters," 1988.

[34] Simons, P.J.M. and M.P.G. Otten, "Intermodulation due to magnitude truncation in digital filters," 1987.
[119] Singh, V., "Formulation of a criterion for the absence of limit cycles in digital filters designed with one quantizer," 1985.
[120] Singh, V., "A new realizability condition for limit cycle free state-space digital filters employing saturation arithmetic," 1985.
[61] Singh, V., "Realization of two's complement overflow limit cycle free state-space digital filters: a frequency-domain viewpoint," 1986.
[67] Singh, V., "On the realization of two's complement overflow limit cycle free state-space digital filters," 1986.
[445] Singh, V., "A new proof of the discrete-time bounded-real lemma and lossless bounded-real lemma," 1987.
[78] Smith, J.O., "Elimination of limit cycles in time-varying lattice filters," 1986.
[30] Snyder, D.L., *see A. Sripad.*
[418] Solev'ev, V.M., "Method for improving the accuracy of digital filters," 1984.
[174] Spahlinger, G., "Suppression of limit cycles in digital filters by statistical rounding," 1985.
[138] Sridharan, S. and D. Williamson, "Comments on 'Suppression of limit cycles in digital filters designed with one magnitude-truncation quantizer'," 1984
[24] Sridharan, S., *see D. Williamson.*
[345] Sridharan, S., *see D. Williamson.*
[346] Sridharan, S., *see D. Williamson.*
[347] Sridharan, S., *see D. Williamson.*
[348] Sridharan, S., *see D. Williamson.*
[349] Sridharan, S., *see D. Williamson.*
[30] Sripad, A. and D.L. Snyder, "A necessary and sufficient condition for quantization errors to be uniform and white," 1977.
[259] Stanier, B.J., *see A.J. McWilliam.*
[160] Steenaart, W., *see O. Monkewich.*
[415] Steenaart, W., *see O. Monkewich.*
[386] Steenaart, W., *see H. Tatangsurja.*
[366] Steiglitz, K. and B. Liu, "An improved algorithm for ordering poles and zeros of fixed-point recursive digital filters," 1976.
[197] Steinle, B. and P. Gruber, "Comments on upper bounds on limit cycles in digital filters of second-order," 1985.
[167] Storzbach, W.H., "Forced oscillations in recursive digital filters," 1972.
[341] Strauss, J.P., *see L. Mintzer.*
[236] Strickland, J.H., *see D.C. Munson, jr..*
[95] Subramanian, R., *see A. Krishnan.*
[406] Swamy, M.N.S., *see J.W.K. Lam.*
[407] Swamy, M.N.S., *see J.W.K. Lam.*
[419] Szczupak, J. and S.K. Mitra, "Recursive digital filters with low round-off noise," 177.
[154] Szczupak, J., *see A.M.B. Pavani.*
[54] Takeo, H., *see T. Higuchi.*
[79] Takizawa, M., H. Kishi and N. Hamada, "Synthesis of lattice digital filters by the state-variable method," 1982.
[239] Tan, E.C., "Limit cycles and similarity transformation of simple digital circuits," 1983.
[386] Tatangsurja, H. and W. Steenaart, "A comparison of stored-product and wave digital filters: coefficient accuracy and roundoff noise," 1984.
[308] Tavsanoglu, V. and L. Thiele, "Simultaneous minimization of roundoff

noise and sensitivity in state-space digital filters," 1983.
[309] Tavsanoglu, V. and L. Thiele, "Optimal design of state-space digital filters by simultaneous minimization of sensitivity and roundoff noise," 1984.
[310] Tavsanoglu, V., "Explicit evaluation of K and W matrices for second-order digital filters," 1985.
[446] Tavsanoglu, V., "The necessary and sufficient conditions for minimum roundoff noise in second-order state-space digital filters and their optimal synthesis," 1987.
[420] Taylor, F. and J.W. Marshall, "Computer-aided design and analysis of standard IIR architectures: part I," 1981.
[29] Taylor, F., *see R. Ramnarayan.*
[38] Taylor, M.G., *see P.M. Ebert.*
[311] Thiele, L., "On the realization of minimum sensitivity and minimum round-off noise state-space discrete systems," 1983.
[312] Thiele, L., "Design of sensitivity and roundoff noise optimal state-space discrete systems," 1984.
[308] Thiele, L., *see V. Tavsanoglu.*
[309] Thiele, L., *see V. Tavsanoglu.*
[215] Thong, T. and B. Liu, "Limit cycles in combinatorial filters using two's complement truncation arithmetic," 1974.
[421] Thong, T., "Finite wordlength effects in the ROM digital filter," 1976.
[216] Thong, T. and B. Liu, "Limit cycles in combinatorial implementation of digital filters," 1976.
[344] Thong, T. and B. Liu, "Error spectrum shaping in narrow-band recursive filters," 1977.
[263] Tokaji, I. and C.W. Barnes, "Roundoff error statistics for a continuous range of multiplier coefficients," 1987.
[447] Tokaji, I. and C.W. Barnes, "Minimum unit noise gain in non-minimal state-space realizations of digital filters," 1988.
[245] Tran, B.N., *see C.W. Barnes.*
[383] Treloar, B., *see K.K. Pang.*
[384] Treloar, B., *see K.K. Pang.*
[422] Trick, T.N., J. Yau and E. Sanchez-Sinencio, "Simulation of fixed-point digital filter structures (computer aided design)," 1979.
[193] Trick, T.N., *see J.L. Long.*
[194] Trick, T.N., *see J.L. Long.*
[382] Trick, T.N., *see J.L. Long.*
[191] Tsuji, S., *see H. Kubota.*
[192] Tsuji, S., *see H. Kubota.*
[68] Tsypkin, Y.Z., "Frequency criteria for the absolute stability of non-linear sampled-data systems," 1964.
[69] Tsypkin, Y.Z., "A criterion for absolute stability of automatic pulse systems with monotonic characteristics of the nonlinear element," 1964.
[142] Turner, L.E. and L.T. Bruton, "Elimination of zero-input limit cycles by bounding the state transition matrix," 1977.
[143] Turner, L.E. and L.T. Bruton, "Elimination of zero-input limit cycles by bounding the state transition matrix," 1979.
[44] Turner, L.E. and L.T. Bruton, "Elimination of granularity and overflow limit cycles in minimum norm recursive digital filters," 1980.
[133] Turner, L.E. and L.T. Bruton, "Elimination of limit cycles recursive digital filters using a generalized minimum norm," 1981.
[113] Turner, L.E., "Second-order recursive digital filter that is free from all constant-input limit cycles," 1982.
[134] Turner, L.E., "Elimination of constant-input limit cycles in recursive digital filters using a generalised minimum norm," 1983.

[433] Turner, L.E., *see B.D. Green.*
[116] Turner, L.E., *see E.S.K. Liu.*
[410] Turner, L.E., *see E.S.K. Liu.*
[411] Turner, L.E., *see E.S.K. Liu.*
[387] Ullrich, U., "Calculation of roundoff noise in wave digital filters with fixed-point arithmetic," 1976.
[388] Ullrich, U., "Roundoff noise and dynamic range of wave digital filters," 1979.
[63] Unger, H., *see U. Bernhardt.*
[198] Unver, Z. and K. Abdullah, "A tighter practical bound on quantization errors in second-order digital filters with complex conjugate poles," 1975.
[121] Vaidyanathan, P.P., "The discrete-time bounded-real lemma in digital filtering," 1985.
[326] Vaidyanathan, P.P., "On error spectrum shaping in state-space digital filters," 1985.
[423] Vaidyanathan, P.P., S.K. Mitra and Y. Neuvo, "A new approach to the realization of low-sensitivity IRR digital filters," 1986.
[448] Vaidyanathan, P.P., "Low-noise and low sensitivity digital filters," 1987.
[122] Vaidyanathan, P.P. and V.C. Liu, "An improved sufficient condition for absence of limit cycles in digital filters," 1987.
[440] Vaidyanathan, P.P., *see V.C. Liu.*
[114] Van Meer, A.C.P., *see H.J. Butterweck.*
[115] Van Meer, A.C.P., *see H.J. Butterweck.*
[257] Van Valkenburg, M.E., *see B. Liu.*
[258] Van Valkenburg, M.E., *see B. Liu.*
[255] Van den Enden, A.W.M., *see R.W.C. Groen.*
[208] Vandewalle, J., *see L. Claesen.*
[105] Verkroost, G. and H.J. Butterweck, "Suppression of parasitic oscillations in wave digital filters and related structures by means of controlled rounding," 1976.
[106] Verkroost, G. and H.J. Butterweck, "Suppression of parasitic oscillations in wave digital filters and related structures by means of controlled rounding," 1976.
[107] Verkroost, G., "A general second-order digital filter with controlled rounding to exclude limit cycles for constant input signals," 1977.
[108] Verkroost, G., "Een tweede-orde digitaal filter waarin door middel van gestuurde kwantisering 'limit cycles' voorkomen worden," 1977.
[264] Verkroost, G. and G.J. Bosscha, "On the measurement of quantization noise in digital filters," 1984.
[114] Verkroost, G., *see H.J. Butterweck.*
[115] Verkroost, G., *see H.J. Butterweck.*
[162] Verkroost, G., *see H.J. Butterweck.*
[163] Verkroost, G., *see H.J. Butterweck.*
[269] Verriest, E.I., "Error analysis of linear recursions in floating point," 1985.
[148] Viaro, U., *see A. Lepschy.*
[149] Viaro, U., *see A. Lepschy.*
[436] Viaro, U., *see A. Lepschy.*
[437] Viaro, U., *see A. Lepschy.*
[438] Viaro, U., *see A. Lepschy.*
[439] Viaro, U., *see A. Lepschy.*
[236] Walker, T.P., *see D.C. Munson, jr..*
[137] Wanhammer, L., *see M. Renfors.*
[389] Wegener, W., "On the design of wave digital lattice filters with short

coefficient wordlengths and optimal dynamic range," 1978.

[100] Wegener, W., *see K. Meerkötter.*

[270] Weinstein, C.J. and A.V. Oppenheim, "A comparison of roundoff noise in floating point and fixed-point digital filter realizations," 1969.

[8] Weinstein, C.J., *see A.V. Oppenheim.*

[168] Werter, M.J., "Suppression of subharmonics in digital filters for discrete-time periodic input signals with period P or a divisor of P," 1986.

[169] Werter, M.J., "Digital filter sections which suppress subharmonics for discrete-time periodic input signals with period P or a divisor of P," 1986.

[170] Werter, M.J., "A new decomposition of discrete-time periodic signals," 1987.

[232] White, S.A., "Quantizer-induced digital controller limit cycles," 1969.

[338] White, S.A., *see T.L. Chang.*

[345] Williamson, D., P.G. McCrea and S. Sridharan, "Residue feedback in digital filters using fractional coefficients and block floating point arithmetic," 1983.

[24] Williamson, D., S. Sridharan and P.G. McCrea, "A new approach for block floating point arithmetic in recursive filters," 1985.

[346] Williamson, D. and S. Sridharan, "Residue feedback in digital filters using fractional feedback coefficients," 1985.

[347] Williamson, D. and S. Sridharan, "An approach to coefficient wordlength reduction in digital filters," 1985.

[348] Williamson, D. and S. Sridharan, "Residue feedback in ladder and lattice filter structures," 1985.

[327] Williamson, D., "Roundoff noise minimization and pole-zero sensitivity in fixed-point digital filters using residue feedback," 1986.

[328] Williamson, D., "Minimum roundoff noise fixed-point digital filters using integer residue feedback," 1986.

[349] Williamson, D. and S. Sridharan, "Error-feedback in a class of orthogonal polynomial digital filter structures," 1986.

[449] Williamson, D., "Delay replacement in direct form structures," 1988.

[138] Williamson, D., *see S. Sridharan.*

[21] Willson, jr., A.N., "Limit cycles due to adder overflow in digital filters," 1972.

[22] Willson, jr., A.N., "Limit cycles due to adder overflow in digital filters," 1972.

[88] Willson, jr., A.N., "Some effects of quantization and adder overflow on the forced response of digital filters," 1972.

[53] Willson, jr., A.N., "A stability criterion for non-autonomous difference equations with applications to the design of a digital FSK oscillator," 1974.

[89] Willson, jr., A.N., "Error-feedback circuits for digital filters," 1976.

[90] Willson, jr., A.N., "Computation of the periods of forced overflow oscillations in digital filters," 1976.

[92] Willson, jr., A.N., *see H. Samueli.*

[93] Willson, jr., A.N., *see H. Samueli.*

[94] Willson, jr., A.N., *see H. Samueli.*

[58] Winkelnkemper, W., *see B. Eckhardt.*

[199] Wisliceny, J., "Estimation of the maximum amplitude of quantisation-conditioned limit cycles in linear recursive digital filters (in German)," 1980.

[139] Witten, I.H. and P.G. McCrea, "Suppressing limit cycles in digital incremental computers," 1981.

[340] Witten, I.H., *see P.G. McCrea.*
[363] Wolf, W., *see E. Lueder.*
[135] Wong, K.M. and R.A. King, "Method to suppress limit cycle oscillations in digital filter," 1974.
[200] Yakowitz, S. and S.R. Parker, "Computation of bounds for digital filter quantization errors," 1973.
[196] Yakowitz, S., *see S.R. Parker.*
[13] Yarlagadda, R., *see J.D. Ledbetter.*
[422] Yau, J., *see T.N. Trick.*
[192] Yoshida, T., *see H. Kubota.*
[442] Yu, T., *see T. Saramaki.*
[313] Zeman, J. and A.G. Lindgren, "Fast digital filters with low roundoff noise," 1981.
[314] Zeman, J. and A.G. Lindgren, "Fast state-space decimator with very low roundoff noise," 1981.

ISSN 0167-9708
Coden: TEUEDE

(188) Jóźwiak, J.
THE FULL DECOMPOSITION OF SEQUENTIAL MACHINES WITH THE STATE AND OUTPUT BEHAVIOUR REALIZATION.
EUT Report 88-E-188. 1988. ISBN 90-6144-188-9

(189) Pineda de Gyvez, J.
ALWAYS: A system for wafer yield analysis.
EUT Report 88-E-189. 1988. ISBN 90-6144-189-7

(190) Siuzdak, J.
OPTICAL COUPLERS FOR COHERENT OPTICAL PHASE DIVERSITY SYSTEMS.
EUT Report 88-E-190. 1988. ISBN 90-6144-190-0

(191) Bastiaans, M.J.
LOCAL-FREQUENCY DESCRIPTION OF OPTICAL SIGNALS AND SYSTEMS.
EUT Report 88-E-191. 1988. ISBN 90-6144-191-9

(192) Worm, S.C.J.
A MULTI-FREQUENCY ANTENNA SYSTEM FOR PROPAGATION EXPERIMENTS WITH THE OLYMPUS SATELLITE.
EUT Report 88-E-192. 1988. ISBN 90-6144-192-7

(193) Kersten, W.F.J. and G.A.P. Jacobs
ANALOG AND DIGITAL SIMULATION OF LINE-ENERGIZING OVERVOLTAGES AND COMPARISON WITH MEASUREMENTS IN A 400 kV NETWORK.
EUT Report 88-E-193. 1988. ISBN 90-6144-193-5

(194) Hosselet, L.M.L.F.
MARTINUS VAN MARUM: A Dutch scientist in a revolutionary time.
EUT Report 88-E-194. 1988. ISBN 90-6144-194-3

(195) Bondarev, V.N.
ON SYSTEM IDENTIFICATION USING PULSE-FREQUENCY MODULATED SIGNALS.
EUT Report 88-E-195. 1988. ISBN 90-6144-195-1

(196) Liu Wen-Jiang, Zhu Yu-Cai and Cai Da-Wei
MODEL BUILDING FOR AN INGOT HEATING PROCESS: Physical modelling approach and identification approach.
EUT Report 88-E-196. 1988. ISBN 90-6144-196-X

(197) Liu Wen-Jiang and Ye Dau-Hua
A NEW METHOD FOR DYNAMIC HUNTING EXTREMUM CONTROL, BASED ON COMPARISON OF MEASURED AND ESTIMATED VALUE.
EUT Report 88-E-197. 1988. ISBN 90-6144-197-8

(198) Liu Wen-Jiang
AN EXTREMUM HUNTING METHOD USING PSEUDO RANDOM BINARY SIGNAL.
EUT Report 88-E-198. 1988. ISBN 90-6144-198-6

(199) Jóźwiak, L.
THE FULL DECOMPOSITION OF SEQUENTIAL MACHINES WITH THE OUTPUT BEHAVIOUR REALIZATION.
EUT Report 88-E-199. 1988. ISBN 90-6144-199-4

(200) Huis in 't Veld, R.J.
A FORMALISM TO DESCRIBE CONCURRENT NON-DETERMINISTIC SYSTEMS AND AN APPLICATION OF IT BY ANALYSING SYSTEMS FOR DANGER OF DEADLOCK.
EUT Report 88-E-200. 1988. ISBN 90-6144-200-1

(201) Woudenberg, H. van and R. van den Born
HARDWARE SYNTHESIS WITH THE AID OF DYNAMIC PROGRAMMING.
EUT Report 88-E-201. 1988. ISBN 90-6144-201-X

(202) Engelshoven, R.J. van and R. van den Born
COST CALCULATION FOR INCREMENTAL HARDWARE SYNTHESIS.
EUT Report 88-E-202. 1988. ISBN 90-6144-202-8

(203) Delissen, J.G.M.
THE LINEAR REGRESSION MODEL: Model structure selection and biased estimators.
EUT Report 88-E-203. 1988. ISBN 90-6144-203-6

(204) Kalasek, V.K.I.
COMPARISON OF AN ANALYTICAL STUDY AND EMTP IMPLEMENTATION OF COMPLICATED THREE-PHASE SCHEMES FOR REACTOR INTERRUPTION.
EUT Report 88-E-204. 1988. ISBN 90-6144-204-4

ISSN 0167-9708
Coden: TEUEDE

(205) Butterweck, H.J. and J.H.F. Ritzerfeld, M.J. Werter
FINITE WORDLENGTH EFFECTS IN DIGITAL FILTERS: A review.
EUT Report 88-E-205. 1988. ISBN 90-6144-205-2

(206) Bollen, M.H.J. and G.A.P. Jacobs
EXTENSIVE TESTING OF AN ALGORITHM FOR TRAVELLING-WAVE-BASED DIRECTIONAL DETECTION AND PHASE-SELECTION BY USING TWONFIL AND EMTP.
EUT Report 88-E-206. 1988. ISBN 90-6144-206-0

(207) Schuurman, W. and M.P.H. Weenink
STABILITY OF A TAYLOR-RELAXED CYLINDRICAL PLASMA SEPARATED FROM THE WALL BY A VACUUM LAYER.
EUT Report 88-E-207. 1988. ISBN 90-6144-207-9

(208) Lucassen, F.H.R. and H.H. van de Ven
A NOTATION CONVENTION IN RIGID ROBOT MODELLING.
EUT Report 88-E-208. 1988. ISBN 90-6144-208-7

(209) Jóźwiak, L.
MINIMAL REALIZATION OF SEQUENTIAL MACHINES: The method of maximal adjacencies.
EUT Report 88-E-209. 1988. ISBN 90-6144-209-5

(210) Lucassen, F.H.R. and H.H. van de Ven
OPTIMAL BODY FIXED COORDINATE SYSTEMS IN NEWTON/EULER MODELLING.
EUT Report 88-E-210. 1988. ISBN 90-6144-210-9

(211) Boom, A.J.J. van den
H_∞-CONTROL: An exploratory study.
EUT Report 88-E-211. 1988. ISBN 90-6144-211-7