

Implementation of Pacejka's analytical motorcycle model

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IMPLEMENTATION OF PACEJKA's ANALYTICAL MOTORCYCLE MODEL

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Eindhoven , October 2006

CHAPTER 1

INTRODUCTION

The analysis of the single track vehicle model is more difficult than the double track automobile model. While only the lateral and yaw degrees of freedom are minimally needed for the steady-state cornering analysis for an automobile, a single track vehicle requires the roll degree of freedom for the steady-state cornering analysis and steer angle as a free motion variable for the stability analysis. Moreover in reality, the torsional behaviour of the front frame with respect to the main frame about a perpendicular axis to steering axis has a great effect on the stability.

The driver also influences the stability behaviour of an automobile and a motorcycle. So the theoretical model of the human driver should be introduced for the stability analysis for both models. Although the driver normally uses the steering wheel to control the automobile direction of motion, the rider of the motorcycle can apply steer angle or steer torque and lean angle or lean torque to steer and stabilize the motorcycle.

With the development of computers, simulating the dynamic behaviour of the vehicle is possible and becomes an important in research field. Simulation of the theoretically introduced model of the vehicle predicts its dynamic behaviour without building a prototype. This reduces the cost and the design times.

A number of researches have been made to establish the theoretical motorcycle model which simulates the motorcycle behaviour. One of them is TNO's research which has achieved to build a multi-body SimMechanics model of a motorcycle. While this model is very accurate, calculation time is too long for real-time application. There are several analytical models described in the literature. One of them is Pacejka's Tyre and Motorcycle Model described in Tyre and Vehicle Dynamics of Hans B. Pacejka. It may be suitable as the basis for a real-time simulation model for steering behaviour.

During this traineeship, Pacejka's analytical motorcycle model is established in Matlab and then compared to the SimMechanics model in order to assess the validity range and calculation efficiency. Depending on the results, Pacejka's model is simplified to achieve real-time performance. Here firstly the parameters used in the analytical model are derived and then accuracy is checked. The obtained graphs and results will be compared with the results from an already designed SimMechanics motorcycle simulation model. At the beginning, the steering behaviour of the motorcycle model is analyzed.

The objective of this traineeship is :

- To set up the analytical model
- To transfer parameters from the SimMechanics model to the analytical model
- To make the benchmarking for accuracy and calculation efficiency

This report is organized as follows. In chapter 2, SimMechanics and analytical motorcycle models are described. In chapter 3, generalized coordinates of the analytical motorcycle model are defined and then the derivation of the required equations in order

to calculate these variables in the time domain is explained. In chapter 4, benchmarking is made to control for the accuracy and calculation efficiency of the analytical model. So the parameters from the SimMechanics model are transferred to the analytical model. And then two different types of input are applied to both models separately. First of all, a step steer torque is applied to the handlebar in both models and then their responses are compared. Secondly, SimMechanics and analytical motorcycle models are modified in order to apply a steer angle and steer rate as an input and then their responses are compared. With these results conclusions are drawn and recommendations for future research are given in chapter 5.

CHAPTER 2

MODEL DESCRIPTION

In this chapter, SimMechanics and analytical motorcycle model are described separately. Motorcycle's geometry, bodies involved in the motorcycle model and tyre characteristic have a great effect on the responses of a motorcycle. So SimMechanics motorcycle model's geometry, bodies and tyre properties are defined in section 2.1. Herein, a stabilizing controller to stabilize the motor and a PD controller to keep the motorcycle's velocity constant are described. In section 2.2, the properties of the analytical motorcycle motor are explained. The forces and moments acting to the motorcycle are defined in order to calculate the motorcycle's response for an applied input.

2.1 SimMechanics Motorcycle Model

In 1983, Cornelis Koenen developed a model which represents the dynamic behaviour of a motorcycle. A Simmechanics model based on Koenen's work was developed in the multibody toolbox of Matlab/Simulink, SimMechanics by Willem Versteden in 2005.

The multibody model is built with respect to an orthogonal axis system (O,x,y,z). The origin O of this axis system lies in the contact point between the rear tyre and the ground plane. The gravity (g) is in the $-z$ direction. The multibody model consists of eight rigid bodies which are interconnected by kinematic constraints. Except from the front suspension which is a one degree of freedom translational joint all the joints in the model are one degree of freedom (dof) revolute joints. The model with bodies, steering axis, twist axis, body lean axis, pitch axis and sign convention is shown in figure 2.1.1

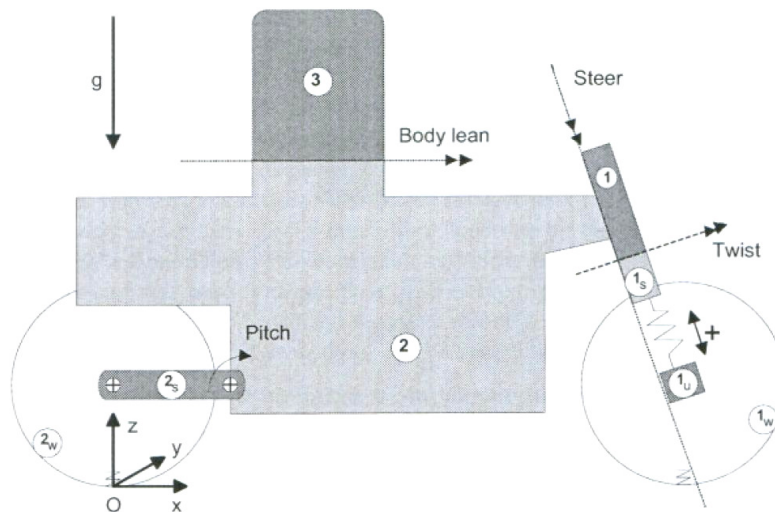


Figure 2.1.1 : SimMechanics's Motorcycle Model [2]

All parts are assumed to be infinitely stiff. The main frame (2) is the basis part of the model and connected to the ground by means of 6 dof joint. To be able simulate the real rider behaviour, the rider body is split up in two parts. The lower part of the rider body is assumed to be rigidly connected to main frame (2), the upper part (3) rotates about an axis (body lean axis) which is horizontal in the initial condition. The rear wheel (2w) is connected to the main frame with a sprung and damped swing arm. This swing arm enables the rear wheel to rotate around a point on the main body and in the plane of symmetry called as pitch movement. The rear wheel (2w) rotates its own axle. Steer axis is located at the front end of the main frame. The steer body (1), twist body (1s), front unsprung mass (1u) and front wheel (1w) together rotate relative to the main mass about a steering axis. The twist body (1s), front unsprung mass (1u) and front wheel (1w) rotate with respect to the twist axis which is perpendicular to the steering axis. Their rotation is in the out of the plane of symmetry of the motorcycle. If there is no twist angle the front suspension is modeled as a translatory movement of the front unsprung mass (1u) and front wheel (1w) perpendicular to the steering axis.

The geometry and rotation axis of the motorcycle should be defined. All of them are shown in the following figure.

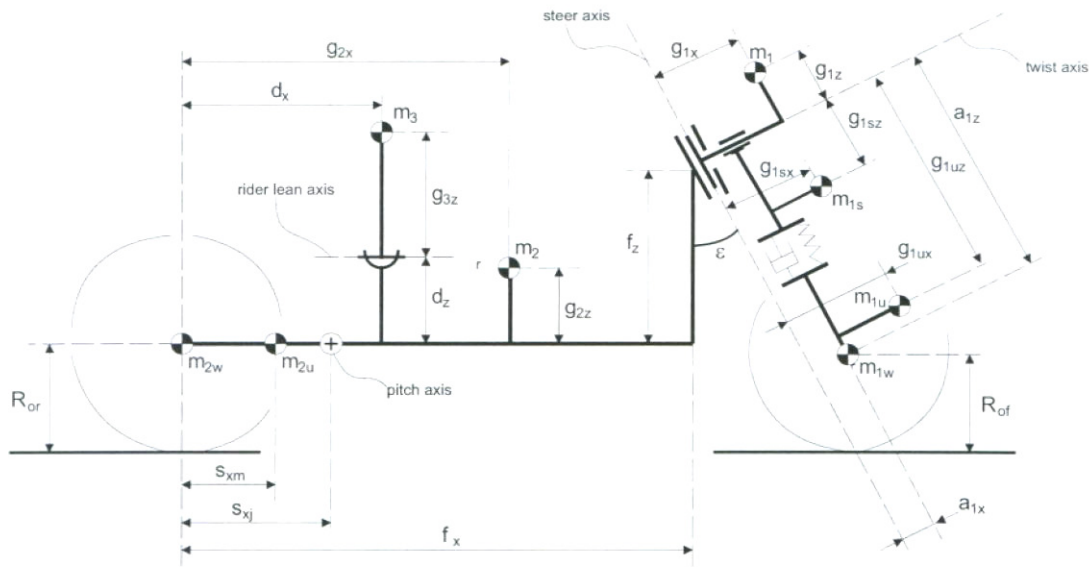
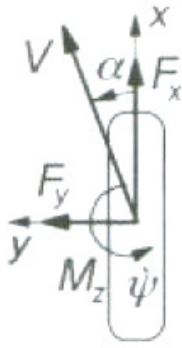
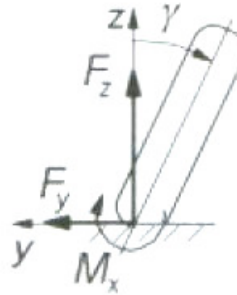


Figure 2.1.2 : SimMechanics's Motorcycle Geometry [2]

The MF Tyre model is used in the motorcycle model in order to describe the behaviour of the motorcycle tyres. It is assumed that slope on the road is zero that is only flat road surfaces are considered during the analysis. Moreover rolling resistance on the road is neglected by tuning of rolling resistance coefficients (QSY1, QSY2, QSY3 and QSY4). The ISO sign convention for force, moment and wheel slip of a tyre is used throughout SimMechanics analysis. This sign convention is depicted in figure 2.1.3.



**Side Angle
(Top View)**



**Inclination/Camber Angle
(Rear View)**

Figure 2.1.3 : The ISO Sign Convention for SimMechanics's Model [1]

When the motorcycle stays at the stationary position the air around it is assumed to be still relative to the ground plane. However speed up of the motorcycle will be increased by the stationary and non-stationary forces acting on it. Drag (F_d) and Lift (F_l) forces are taken into account among these forces and it is assumed that they are acting at a specified point of the main mass(2).

In motorcycle analysis, it is observed that there are three instability modes. The first one is the weave mode which is a relatively low frequency oscillatory motion where whole vehicle takes part. The capsize mode can become unstable beyond a relatively low critical velocity in certain cases. The third one which can become unstable in the range of 45 and 70 [kph] is the wobble mode which is a steering oscillation.

In real life, the rider of the motorcycle acts as a controller in order to stabilize the motorcycle by moving the center of gravity of upper body in the lateral direction and by applying a certain torque to the handle bar. In the SimMechanics model, camber angle, camber angle rate and steer angle rate are used to establish the stabilizing controller.

Rolling resistance force and aerodynamic drag force are applied to the motorcycle in the longitudinal direction. A PD controller is established by using the feedback of actual forward velocity and it applied a driving force to the rear wheel in order to keep the forward velocity of the motor constant.

2.2 Analytical Motorcycle Model

In figure 2.2.1, the motorcycle has been depicted while it moves at a roll angle ϕ of the mainframe and with a steer angle δ of the handlebar about a steering axis that shows a steering head (rake) angle ε with respect to a vertical line and a caster length t_c . The coordinate system is located at the reference point A which lies on the line of intersection of the plane of symmetry of the vehicle and the road plane and below the center of gravity of the mainframe. Point A moves in the longitudinal direction with a velocity of u and in the lateral direction with v . Moreover it rotates over the road surface

in the vertical direction with yaw angle (ψ) and yaw rate (r). ϕ which is called as mainframe roll angle is the angle between the plane of symmetry and the normal to the road surface. A part of the front frame rotates with twist angle β about twist axis. The upper part of the rider rotates with rider lean angle (ϕ_r) about the longitudinal axis. It is measured with respect to the vertical mainframe axis.

There are basically four bodies in the analytical model:

- Mainframe (M_m) including lower part of the rider and rear wheel
- Upper torso of the rider (M_r)
- Front upper frame (M_f)
- Front subframe including the front wheel (M_s)

Figure 2.2.1 dedicates important parameters used in the analytical analysis

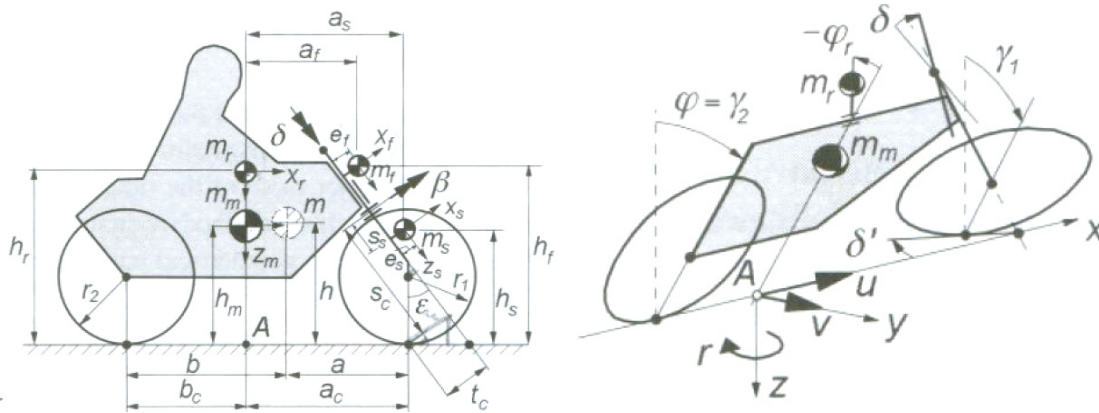


Figure 2.2.1 : Analytical Model Parameters [1]

To search force and moment response of the tyre, the analytical model is divided into two parts. In the first one which is a linear model, stiffness values depend on only vertical load and they remain constant with different slip and camber angle. On the other hand, the Magic Formula in a simplified version is used for the non-linear force and moment description in the non-linear model. In this report, the linear model is only used in the construction of the motorcycle analytical model.

For the transient responses, the relaxation length (σ) is used as a parameter in the first order differential equations which describe the responses of the transient slip or deflection angles α' and γ' . In steady-state condition, they are equal to the input slip and camber angles α and γ . The following equations are used to take the effect of relaxation length for the tyre i ($i = 1$ or 2).

$$\frac{1}{u} \sigma_{\alpha_i} \dot{\alpha}_i' + \alpha_i' = \alpha_i \quad (2.2.1)$$

$$\frac{1}{u} \sigma_{\gamma_i} \dot{\gamma}_i' + \gamma_i' = \gamma_i \quad (2.2.2)$$

The relaxation lengths depend on the normal load and are calculated by

$$\sigma_{\alpha i} = \sigma_{\gamma i} = f_{1i} F_{z_{io}} + f_{2i} (F_{z_i} - F_{z_{io}}) \quad (2.2.3)$$

Side force at small slip and camber angles:

$$F_{\gamma i} = C_{F\alpha i} \alpha_i' + C_{F\gamma i} \gamma_i' \quad (2.2.4)$$

Aligning torque changes with slip angle and camber angle. Moreover it is assumed that the lateral shift of the line of action of the longitudinal force (F_{x_i}) changes instantaneously with the camber angle and it has effect on the aligning torque (M_{z_i}).

$$M_{z_i} = -C_{M\alpha i} \alpha_i' + C'_{M\gamma i} \gamma_i' - r_{ci} F_{x_i} \gamma_i \quad (2.2.5)$$

The last important parameter is the overturning couple (M_{x_i}) and it is assumed that overturning couple depends only on the camber angle.

$$M_{x_i} = -C_{Mx\gamma i} \gamma_i \quad (2.2.6)$$

The stiffness coefficients are assumed to depend on the normal load as follows for the tyre i (i = 1 or 2).

$$C_{F\alpha i} = d_{1i} F_{z_{io}} + d_{2i} (F_{z_i} - F_{z_{io}}) \quad (2.2.7)$$

$$C_{F\gamma i} = d_{3i} F_{z_i} \quad (2.2.8)$$

$$C_{M\alpha i} = e_{1i} F_{z_i} \quad (2.2.9)$$

$$C'_{M\gamma i} = e_{2i} F_{z_i} \quad (2.2.10)$$

$$C_{Mx\gamma i} = e_{3i} F_{z_i} \quad (2.2.11)$$

It is assumed that aerodynamic drag force (F_d) acts to the motorcycle in the longitudinal backward direction at the pressure centre distance (h_d) above the road surface (in upright position) and aerodynamic lift force is neglected.

$$F_d = \frac{1}{2} \rho C_{da} u^2 \quad (2.2.12)$$

Where

ρ : air density

C_{da} : effective drag area

u : forward velocity

Because of the drag force (F_d) and the longitudinal tyre forces (F_{xi}), the load transfers from the front tyre to the rear tyre. The increase of the rear normal load is equal to the decrease of the front normal load.

$$F_{ax} = F_{x,tot} - F_d \quad (2.2.13)$$

Where

F_{ax} : The required force for the acceleration of the vehicle in the longitudinal direction

$F_{x,tot}$: The sum of the longitudinal tyre forces

$$F_{ax} = ma_x \quad (2.2.14)$$

Where

a_x : forward acceleration

For small roll angles, the vertical wheel loads :

$$F_{z1} = F_{z1o} - \Delta F_{z0} \quad (2.2.15)$$

$$F_{z2} = F_{z2o} - \Delta F_{z0} \quad (2.2.16)$$

With F_{z1o} and F_{z2o} are the initial wheel loads.

$$F_{z1o} = \frac{b}{l} mg \quad (2.2.17)$$

$$F_{z2o} = \frac{a}{l} mg \quad (2.2.18)$$

And ΔF_{z0} is the vertical load transfer from the front tyre to the rear tyre

$$\Delta F_{z0} = \frac{1}{l} (h_d F_d + h F_{ax}) \quad (2.2.19)$$

At the braking ($F_{x,tot} < 0$);

$$F_{x1} = \left(\frac{F_{z1}}{mg} \right) F_{x,tot}$$

$$F_{x2} = \left(\frac{F_{z2}}{mg} \right) F_{x,tot}$$

And at the driving ;

$$F_{x1} = 0$$

$$F_{x2} = F_{x,tot}$$

Different sign convention is used for the force, moment, slip and camber angles for the analytical model. The used sign convention is indicated in figure 2.2.2. When this sign convention is compared with the ISO sign convention, it is concluded that here lateral force (F_y) and self aligning torque (M_z) are defined in the opposite direction with respect to the lateral force and aligning torque in the ISO sign convention.

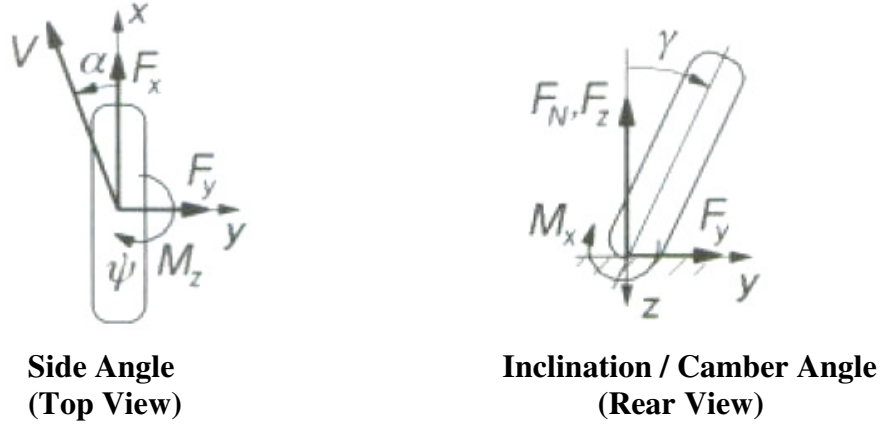


Figure 2.2.2 : Sign Convention For the Analytical Model [1]

The steer torque (M_δ) is considered as a stabilization controller. Moreover lateral velocity, yaw rate, roll angle, roll rate, steer angle and steer rate are also used to establish the stabilizing controller in the analytical equations. Their feedback control gains depend on the longitudinal velocity (u).

$$g_v = \frac{g_{vo}}{u} \quad g_r = \frac{g_{ro}}{u} \quad g_{d\phi} = g_{d\phi_0} \left[\frac{1 - \frac{u}{u_c}}{u} \right] \quad (2.2.20)$$

$$g_\phi = g_{\phi_0} \quad g_{d\delta} = \frac{g_{d\delta_0}}{u} \quad g_\delta = g_{\delta_0}$$

The parameters have been decided with trial and error by Pacejka. These parameters are used in the SimMechanics and analytical models when the steer torque is used as an input.

g_{vo}	-500
g_{ro}	100
$g_{d\phi_0}$	-900
g_{ϕ_0}	-10
$g_{d\delta_0}$	150
g_{δ_0}	50
u_c	15 m/s

Table 2.2.1 : Rider control gains g with cross-over velocity u_c [1]

It is concluded from chapter 2 that there are some differences between these models. The first difference is observed on the model geometry. The front side of the analytical motorcycle's geometry is slightly different from the SimMechanics model. So the parameters used to define the analytical model should be derived with respect to the parameters depicted in SimMechanics. The second difference is that while there are eight masses on SimMechanics, four masses are defined in the analytical model. A camber angle is used to turn the SimMechanics motorcycle and a stabilizing controller as function of camber angle, camber angle rate and steer angle rate is introduced to stabilize the motorcycle. However a steer torque is applied to the handlebar to turn the analytical motorcycle left or right and stabilizing controller is described as a function of lateral velocity, yaw rate, camber angle, camber rate, steer angle and steer rate. Another difference is observed in keeping the motorcycle's longitudinal velocity constant. A PD controller is developed for this function in SimMechanics. On the other hand, it is assumed that forward acceleration is equal to zero in the equations. It means that longitudinal velocity remains constant in the analytical motorcycle model.

CHAPTER 3

ANALYTICAL MODEL IMPLEMENTATION

For the dynamic analysis of the motorcycle, the modified equations of the Lagrange are derived to implement the analytical model. First of all, it is thought that a steer torque applied to the handlebar and a rider lean torque are the inputs for the system. So there are six generalized coordinates for this system. These are;

- Lateral velocity (v)
- Yaw velocity (r)
- Roll angle (φ)
- Rider roll angle (φ_r)
- Steer angle (δ)
- Twist angle (β)

The modified Lagrange equations are used to define the required equations in order to calculate the state variable. They are defined as follows;

$$\begin{aligned} \frac{d}{dt} \frac{\partial T}{\partial v} + r \frac{\partial T}{\partial u} &= Q_v \\ \frac{d}{dt} \frac{\partial T}{\partial r} - v \frac{\partial T}{\partial u} + u \frac{\partial T}{\partial v} &= Q_r \\ \frac{d}{dt} \frac{\partial T}{\partial \dot{q}_j} - \frac{\partial T}{\partial q_j} + \frac{\partial U}{\partial q_j} + \frac{\partial D}{\partial \dot{q}_j} &= Q_j \end{aligned} \quad (3.1)$$

In this expression, q_j defines the remaining generalized coordinates apart from v and

$r (= \dot{\psi})$ that is $q_j = \varphi, \varphi_r, \delta$ and β .

The kinetic energy (T), potential energy (U), Dissipation function (D) and Virtual work (ΔW) are derived with respect to these generalized coordinates to establish the equations.

The total kinetic energy (T) of six bodies including also front and rear tyre:

$$T = \frac{1}{2} \sum_{k=1}^6 m_k (u_k^2 + v_k^2 + w_k^2) + \frac{1}{2} \sum_{k=1}^6 (J_{xk} w_{xk}^2 + J_{yk} w_{yk}^2 + J_{zk} w_{zk}^2) - J_{mxz} w_{mz} w_{xm} \quad (3.2)$$

In (3.2), the products of inertia of bodies defined in the section 2.2 were neglected except from the inertia of mainframe (J_{mxz}).

The total potential energy (U) is written as follows;

$$U = m_m g z_m + m_r g z_r + m_f g z_f + m_s g z_s + \frac{1}{2} c_{\varphi_r} \varphi_r^2 + \frac{1}{2} c_{\beta} \beta^2 \quad (3.3)$$

Where z_m , z_r , z_f and z_s mean the height of center of gravity of the mainframe, rider, front upper frame and front sub-frame. In (3.3), c_{φ_r} and c_{β} define the stiffnesses of the rider lean angle and twist angle respectively.

The viscous damping around steering axes, twist axes and rider lean axes are defined by k_{δ} , k_{β} and k_{φ_r} respectively. They are used to define the dissipation function (D) as follows;

$$D = \frac{1}{2} (k_{\delta} \dot{\delta}^2 + k_{\beta} \dot{\beta}^2 + k_{\varphi_r} \dot{\varphi}_r^2) \quad (3.4)$$

The virtual work is derived with respect to the generalized forces (Q_j) as follows;

$$\Delta W = \sum_{j=1}^6 Q_j \Delta q_j \quad (3.5)$$

Where the generalized forces are written with respect to the generalized coordinates.

$$Q_v = F_{x1} \delta \cos \varepsilon - F_{x1} \beta \sin \varepsilon + F_{y1} + F_{y2}$$

$$Q_r = F_{x1} \delta (\cos \varepsilon) a_c - F_{x1} \beta (\sin \varepsilon) a_c + F_{x1} t_c \delta + F_{x1} s_c \beta + F_{y1} a_c + M_{z1} - F_{y2} b_c + M_{z2} + F_d h_d \varphi + a_x m_m h_m \varphi + a_x m_m y_m + a_x m_r h_r \varphi + a_x m_r s_r \varphi_r + a_x m_r y_r + a_x m_f h_f \varphi + a_x m_f e_f \delta + a_x m_s h_s \varphi + a_x m_s e_s \delta - a_x m_s s_s \beta$$

$$Q_{\varphi} = M_{x1} + M_{x2} + (F_d h_d + a_x m_m h_m + a_x m_r h_r + a_x m_f h_f + a_x m_s h_s) \left(-\frac{s_k}{l} \beta - \frac{h_k}{l} (\delta \sin \varepsilon + \beta \cos \varepsilon) \right)$$

$$Q_{\varphi_r} = M_{\varphi_r}$$

$$Q_{\delta} = F_{x1} h_{\beta} \beta + M_{z1} \cos \varepsilon + M_{x1} \sin \varepsilon + M_{\delta} + \{ F_d h_d + a_x (m_m h_m + m_r h_r + m_f h_f + m_s h_s) \} \left\{ -\frac{h_k}{l} (\varphi + \delta \sin \varepsilon + \beta \cos \varepsilon) \sin \varepsilon - \frac{s_k}{l} \beta \sin \varepsilon \right\}$$

$$Q_{\beta} = -M_{z1} \sin \varepsilon + M_{x1} \cos \varepsilon + \{ F_d h_d + a_x (m_m h_m + m_r h_r + m_f h_f + m_s h_s) \} \left\{ -\frac{h_k}{l} (\delta \sin \varepsilon + \varphi + \beta \cos \varepsilon) \cos \varepsilon - \frac{s_k}{l} (\varphi + \delta \sin \varepsilon + \beta \cos \varepsilon) \right\}$$

The modified Lagrange equations are carried out with these six generalized coordinates and the following expressions are obtained in order to solve the variables separately:

$$\begin{aligned}
& (m_m + m_f + m_s + m_r) \dot{v} + (m_m + m_f + m_s + m_r) ru + (m_f a_f + m_s a_s) \dot{r} + \\
& (m_m h_m + m_f h_f + m_s h_s + m_r h_r) \ddot{\varphi} + (m_r s_r) \ddot{\varphi}_r + (m_f e_f + m_s e_s) \ddot{\delta} - (m_s s_s) \ddot{\beta} - \\
& F_{x1} (\cos \varepsilon) \delta + F_{x1} (\sin \varepsilon) \beta - C_{F\alpha_1} \alpha_1' - C_{F\gamma_1} \gamma_1' - C_{F\alpha_2} \alpha_2' - C_{F\gamma_2} \gamma_2' = 0
\end{aligned} \tag{3.6}$$

$$\begin{aligned}
& (m_f a_f + m_s a_s) \dot{v} + (m_f a_f + m_s a_s) ur + (m_f a_f^2 + m_s a_s^2 + J_{mz} + \\
& (J_{fx} + J_{sx})(\sin \varepsilon)^2 + (J_{fz} + J_{sz})(\cos \varepsilon)^2) \dot{r} + (m_f h_f a_f + m_s h_s a_s - J_{mxz} + \\
& (J_{fz} + J_{sz} - J_{fx} - J_{sx})(\sin \varepsilon)(\cos \varepsilon)) \ddot{\varphi} - \left(\frac{J_{w_1 y}}{R_{w_1}} + \frac{J_{w_2 y}}{R_{w_2}} \right) u \dot{\varphi} + (m_f e_f a_f + \\
& m_s e_s a_s + (J_{fz} + J_{sz}) \cos \varepsilon) \ddot{\delta} - \frac{J_{w_1 y}}{R_{w_1}} u (\sin \varepsilon) \dot{\delta} - (m_s s_s a_s + J_{sx} \sin \varepsilon) \ddot{\beta} -
\end{aligned} \tag{3.7}$$

$$\begin{aligned}
& \frac{J_{w_1 y}}{R_{w_1}} u (\cos \varepsilon) \dot{\beta} - F_d h_d \varphi - F_{x1} (t_c + a_c \cos \varepsilon) \delta - F_{x1} (s_c - a_c \sin \varepsilon) \beta - \\
& a_x m h \varphi - a_x m_r s_r \varphi_r - a_x (m_f e_f + m_s e_s) \delta + a_x m_s s_s \beta - a_c C_{F\alpha_1} \alpha_1' - a_c C_{F\gamma_1} \gamma_1' + \\
& b_c C_{F\alpha_2} \alpha_2' + b_c C_{F\gamma_2} \gamma_2' + C_{M\alpha_1} \alpha_1' - C'_{M\gamma_1} \gamma_1' + r_{c1} F_{x1} (\varphi + \delta \sin \varepsilon + \beta \cos \varepsilon) + \\
& C_{M\alpha_2} \alpha_2' - C'_{M\gamma_2} \gamma_2' + r_{c2} F_{x2} \varphi = 0
\end{aligned}$$

$$\begin{aligned}
& (m_m h_m + m_f h_f + m_s h_s + m_r h_r) \dot{v} + (m_m h_m + m_f h_f + m_s h_s + m_r h_r + \frac{J_{w_1 y}}{R_{w_1}} + \\
& \frac{J_{w_2 y}}{R_{w_2}}) ur + (m_f h_f a_f + m_s h_s a_s - J_{mxz} + (J_{fz} + J_{sz} - J_{fx} - J_{sx})(\sin \varepsilon)(\cos \varepsilon)) \dot{r} \\
& + (m_f h_f^2 + m_s h_s^2 + m_m h_m^2 + m_r h_r^2 + J_{mx} + J_{rx} + (J_{fx} + J_{sx})(\cos \varepsilon)^2 + \\
& (J_{fz} + J_{sz})(\sin \varepsilon)^2) \ddot{\varphi} - (m_m h_m + m_f h_f + m_s h_s + m_r h_r) g \varphi + (J_{rx} + m_r s_r h_r) \ddot{\varphi}_r - \\
& m_r s_r g \varphi_r + (m_f e_f h_f + m_s e_s h_s + (J_{fz} + J_{sz}) \sin \varepsilon) \ddot{\delta} + \frac{J_{w_1 y}}{R_{w_1}} u (\cos \varepsilon) \dot{\delta} - (t_c F_{z1} + \\
& m_f e_f g + m_s e_s g) \delta - (m_s s_s h_s - J_{sx} \cos \varepsilon) \ddot{\beta} - \frac{J_{w_1 y}}{R_{w_1}} u (\sin \varepsilon) \dot{\beta} - (s_c F_{z1} - m_s s_s g) \beta + \\
& C_{Mx\gamma_1} (\varphi + \delta \sin \varepsilon + \beta \cos \varepsilon) + C_{Mx\gamma_2} \varphi = 0
\end{aligned} \tag{3.8}$$

$$\dot{\varphi} - \dot{\varphi} = 0 \tag{3.9}$$

$$\begin{aligned}
& (m_f e_f + m_s e_s) \dot{v} + (m_f e_f + m_s e_s + \frac{J_{w_1 y}}{R_{w_1}} \sin \varepsilon) u r + (m_f e_f a_f + m_s e_s a_s + \\
& (J_{fz} + J_{sz}) \cos \varepsilon) \dot{r} + (m_f e_f h_f + m_s e_s h_s + (J_{fz} + J_{sz}) \sin \varepsilon) \ddot{\varphi} - \frac{J_{w_1 y}}{R_{w_1}} u (\cos \varepsilon) \dot{\varphi} - \\
& (t_c F_{z1} + m_f e_f g + m_s e_s g) \varphi + (m_f e_f^2 + m_s e_s^2 + J_{fz} + J_{sz}) \ddot{\delta} + k_\delta \dot{\delta} - (t_c F_{z1} + m_f e_f g + \\
& m_s e_s g) (\sin \varepsilon) \delta - m_s e_s s_s \ddot{\beta} - \frac{J_{w_1 y}}{R_{w_1}} u \dot{\beta} - ((s_c F_{z1} - m_s s_s g) \sin \varepsilon + F_{x1} h_\beta) \beta + g_v v + g_r r + \\
& g_{d\varphi} \dot{\varphi} + g_\varphi \varphi + g_{d\delta} \dot{\delta} + g_\delta \delta + t_c C_{F\alpha_1} \alpha_1' + t_c C_{F\gamma_1} \gamma_1' + C_{M\alpha_1} (\cos \varepsilon) \alpha_1' - C'_{M\gamma_1} (\cos \varepsilon) \gamma_1' + \\
& r_{c1} F_{x1} (\cos \varepsilon) (\varphi + \delta \sin \varepsilon + \beta \cos \varepsilon) + C_{Mx\gamma_1} (\sin \varepsilon) (\varphi + \delta \sin \varepsilon + \beta \cos \varepsilon) - M_\delta = 0
\end{aligned} \tag{3.10}$$

$$\dot{\delta} - \dot{\delta} = 0 \tag{3.11}$$

$$\begin{aligned}
& -m_s s_s \dot{v} - (m_s s_s a_s + J_{sx} \sin \varepsilon) \dot{r} - (m_s s_s - \frac{J_{w_1 y}}{R_{w_1}} \cos \varepsilon) u r - (m_s s_s h_s - J_{sx} \cos \varepsilon) \ddot{\varphi} + \\
& \frac{J_{w_1 y}}{R_{w_1}} u (\sin \varepsilon) \dot{\varphi} - (s_c F_{z1} - m_s s_s g) \varphi - m_s e_s s_s \ddot{\delta} + \frac{J_{w_1 y}}{R_{w_1}} u \dot{\delta} - (s_c F_{z1} - m_s s_s g) (\sin \varepsilon) \delta + \\
& (m_s s_s^2 + J_{sx}) \ddot{\beta} + k_\beta \dot{\beta} + (c_\beta - (s_c F_{z1} - m_s s_s g) \cos \varepsilon) \beta + s_c C_{F\alpha_1} \alpha_1' + s_c C_{F\gamma_1} \gamma_1' - \\
& C_{M\alpha_1} (\sin \varepsilon) \alpha_1' + C'_{M\gamma_1} (\sin \varepsilon) \gamma_1' - r_{c1} F_{x1} (\sin \varepsilon) (\varphi + \delta \sin \varepsilon + \beta \cos \varepsilon) + \\
& C_{Mx\gamma_1} (\cos \varepsilon) (\varphi + \delta \sin \varepsilon + \beta \cos \varepsilon) = 0
\end{aligned} \tag{3.12}$$

$$\dot{\beta} - \dot{\beta} = 0 \tag{3.13}$$

$$\begin{aligned}
& m_r s_r \dot{v} + m_r s_r u r + (J_{rx} + m_r s_r h_r) \ddot{\varphi} - m_r s_r g \varphi + (J_{rx} + m_r s_r^2) \ddot{\varphi}_r + k_{\varphi_r} \dot{\varphi}_r + \\
& (c_{\varphi_r} - m_r s_r g) \varphi_r - M_r = 0
\end{aligned} \tag{3.14}$$

$$\dot{\varphi}_r - \dot{\varphi}_r = 0 \tag{3.15}$$

By considering the relaxation tyre effect, transient slip and camber angles are calculated from the following four equations:

$$\frac{\sigma_{\alpha_1}}{u} \dot{\alpha}_1' + \alpha_1' + \frac{v}{u} + \frac{a_c}{u} r - \frac{t_c}{u} \dot{\delta} - \frac{s_c}{u} \dot{\beta} - \delta \cos \varepsilon + \beta \sin \varepsilon = 0 \quad (3.16)$$

$$\frac{\sigma_{\gamma_1}}{u} \dot{\gamma}_1' + \gamma_1' - \varphi - \delta \sin \varepsilon - \beta \cos \varepsilon = 0 \quad (3.17)$$

$$\frac{\sigma_{\alpha_2}}{u} \dot{\alpha}_2' + \alpha_2' + \frac{v}{u} - \frac{b_c}{u} r = 0 \quad (3.18)$$

$$\frac{\sigma_{\gamma_2}}{u} \dot{\gamma}_2' + \gamma_2' - \varphi = 0 \quad (3.19)$$

To derive the above equations some assumptions are made;

- The lateral distance effect of combined main frame and rider body is neglected.
- The motorcycle goes with constant forward velocity of u in the longitudinal direction through the analysis. Longitudinal forward acceleration of a_x is zero.
- The products of inertia are neglected except J_{mxz} of the mainframe.

State vector and state equation are defined with respect to the generalized coordinates with the following expression;

$$\text{State vector : } x = [v; r; \dot{\varphi}; \varphi; \dot{\delta}; \delta; \dot{\beta}; \beta; \alpha_1'; \gamma_1'; \alpha_2'; \gamma_2'; \dot{\varphi}_r; \varphi_r]$$

$$\text{State equation : } A \dot{x} + Bx = 0$$

State equation is solved with 'lsim' function in the continuous time domain. And then the parameters defining the motorcycle state condition are calculated.

After defining the generalized coordinates, 14 equations are obtained by using the modified Lagrange equations and relaxation tyre equations. Moreover there are 14 variables in the state vector of x . So the state variables which are used to define the state of a motorcycle in time domain are easily calculated in Matlab.

CHAPTER 4

BENCHMARKING

To be able to compare the SimMechanics and analytical model results, first the parameters which affect the results should be the same. Moreover the same input should be applied to the models.

4.1 Preparing similar models : Simmechanics and analytical model have different parameters such as different number of body, coordinate system, moment of inertia, tyre properties and stabilization controller. To be able to compare both models they should be modified in order to get the same model types.

4.1.1 Motorcycle Parameters :

4.1.1.1 Mass Parameters :

While simmechanics model consists of 8 bodies, analytical model includes only four masses. The bodies on the analytical model are defined like:

- Mainframe include lower part of the rider and rear wheel

$$m_m = m_2 + m_{2u} + m_{2w}$$

- Front subframe includes front wheel

$$m_s = m_{1s} + m_{1u} + m_{1w}$$

- Upper torso of the rider

$$m_r = m_3$$

- Front upper frame

$$m_f = m_1$$

4.1.1.2 Geometrical Parameters :

For the analytical model, the origin O of the coordinate system is at the reference point A that lies on the line intersection of the plane of symmetry of the motorcycle and the road plane and is located in the upright position below the center of gravity of the mainframe. Positive z direction is in the downward direction. On the other hand, the origin O of the coordinate system on the simmechanics model is located at the contact point between the rear tyre and the ground plane. The gravity (g) is pointing in the $-z$ direction

The important geometrical difference between both models is observed on the front of motorcycle. The steer axis goes through at a distance of a_{1x} from the center of front tyre in the SimMechanics model. However the steer axis is located at a caster length

(t_c) from the contact point (C) for the analytical model. The caster length is derived as a function of a_{1x} and then calculated for given a_{1x} of motorcycle parameter. The geometrical difference for the front side of both model is shown in figure 4.1.1.

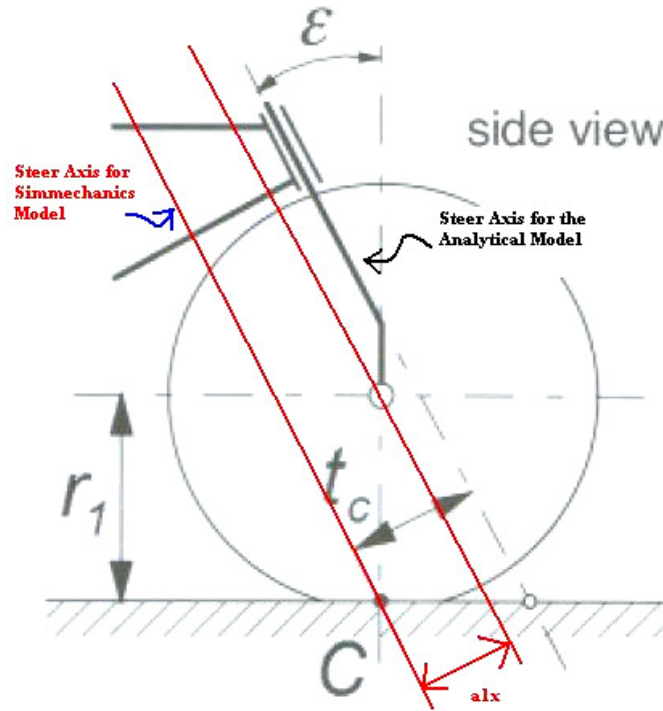


Figure 4.1.1. : Front side difference between two models [1],[2]

4.1.1.3 Inertial Parameters

In the dynamic behaviour of the motorcycle, the moments of inertia of the bodies have great effect. Inertias of all submasses are given with respect to the local axis system of each body in the Appendix A. All bodies are assumed to be symmetrical with respect to the vehicle center plane. The products of inertia will be neglected except J_{mxz} of the mainframe because y-axis is always perpendicular to the vehicle center plane. In analytical model, mainbody includes mainbody, swingarm and rear wheel. Therefore, their moments of inertia need to be translated to the center of mass of combined body in order make correct comparison. Here the Huygens-Steiner formula is used to calculate the moment of inertia.

$${}_o J = {}_{cm} J + m(\vec{r}_{cm} \cdot \vec{r}_{cm} I - \vec{r}_{cm} \vec{r}_{cm})$$

The Huygens-Steiner equation means that the inertia matrix with respect to an arbitrary reference point (${}_o J$) equals the inertia matrix with respect to the center of mass (${}_{cm} J$) plus the inertia matrix with respect to the arbitrary point of the mass (m) concentrated at the center of mass.

The same rule is also used for the calculation of the inertia of front subframe including twist body, unsprung mass and front wheel.

4.1.2 Motorcycle Tyre Parameters :

Tyre model has great influence on the dynamical behaviour of the motorcycle because tyres are seemed as force and moment producers. They linearly depend on side slip angle and camber angle for small slip and camber angle. Furthermore non-linear tyre forces and moments can also be described with the use of Magic Formula for large slip and camber angles. However this traineeship only focuses on the linear region and only linear analytical model is implemented.

While in the SimMechanics model the Magic Formula version 52 is used to calculate the stiffness coefficients depending on the vertical load, they are assumed linearly depend on the vertical load in the linear analytical model by neglecting the small effect of the lateral distortion due to the side force. Stiffness coefficients are defined by the (2.2.7), (2.2.8), (2.2.9), (2.2.10) and (2.2.11).

Pacejka has also introduced hypothetical front and rear tyre to use his analysis. So he calculated these coefficients to specify the tyre stiffnesses.

First of all, the magic formula parameters are changed in order to get a new tyre model where the stiffness coefficients linearly depend on the vertical tyre load. These files are called Dunlop_218FL_WV_last.tir for the front tyre and Dunlop_218L_WV_last.tir for the rear tyre. Stiffness coefficients distributions with respect to the vertical load in new tyre model are depicted in the following figures.

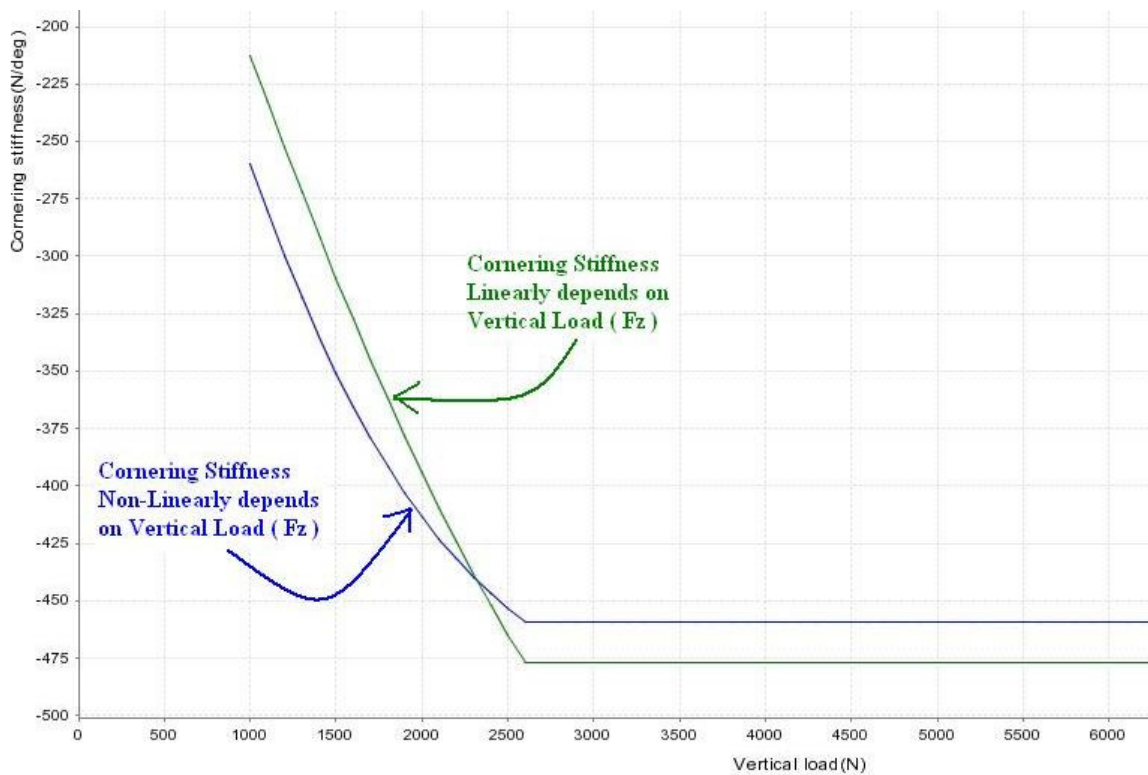


Figure 4.1.2 : $CF\alpha$ versus F_z for the front tyre

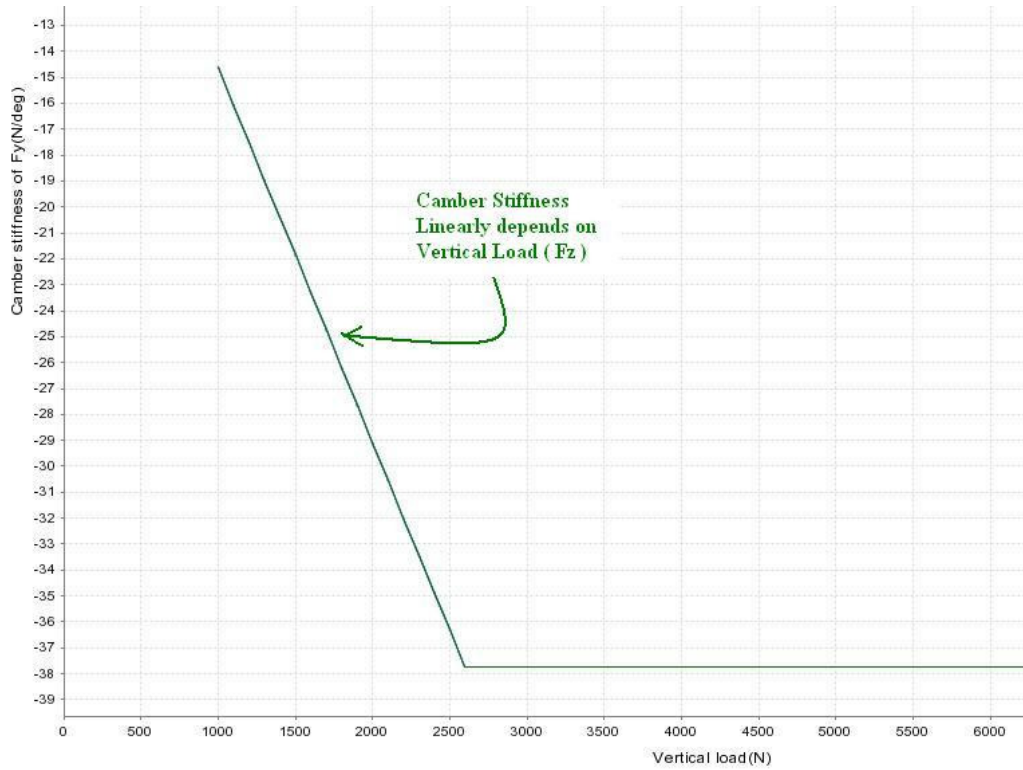


Figure 4.1.3 : CF_y versus F_z for the front tyre

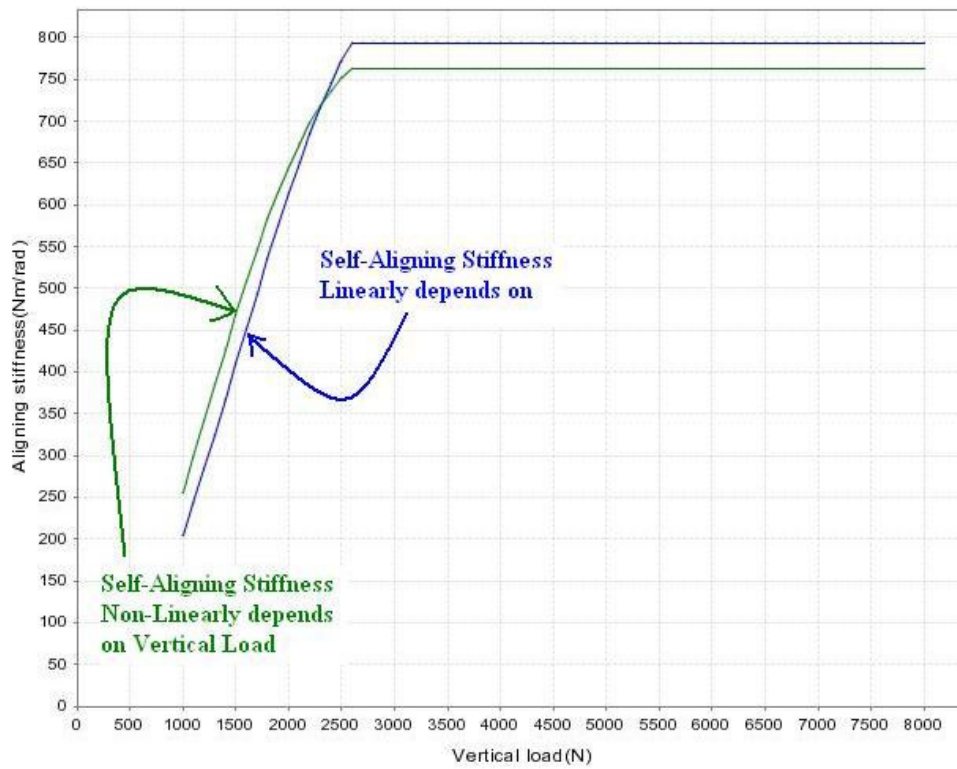


Figure 4.1.4 : CM_α versus F_z for the front tyre

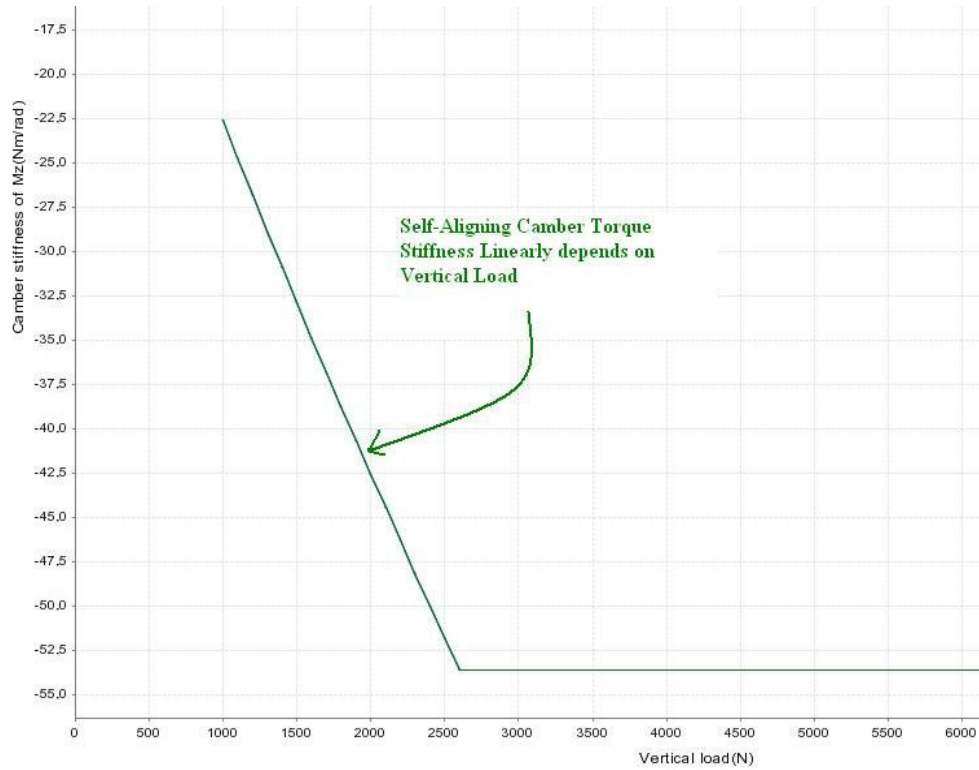


Figure 4.1.5 : CM_γ versus F_z for the front tyre

These two modified tyre models where cornering stiffness, camber stiffness, overturning couple stiffness, self-aligning cornering and camber stiffness linearly depend on vertical load are used on the simmechanics model for the front and rear tyre. For the analytical model, new coefficients which will be used to calculate tyre stiffnesses should be estimated by matching the force and moment distribution lines which are shown in the following figures for the front tyre by the solid and dashed lines. To get the same self-aligning cornering and camber stiffnesses, it is tried to make solid and dashed lines parallel to each other instead of matching them. Pacejka's and calculated parameters are also compared in table 4.1.1.

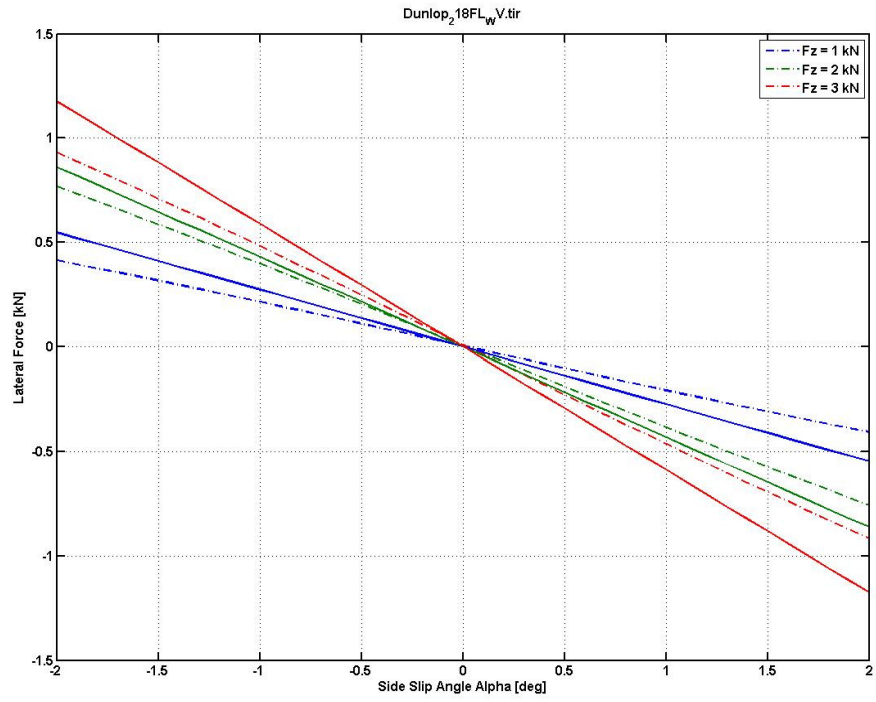


Figure 4.1.6 : Cornering Stiffness for the front tyre with Pacejka's parameters

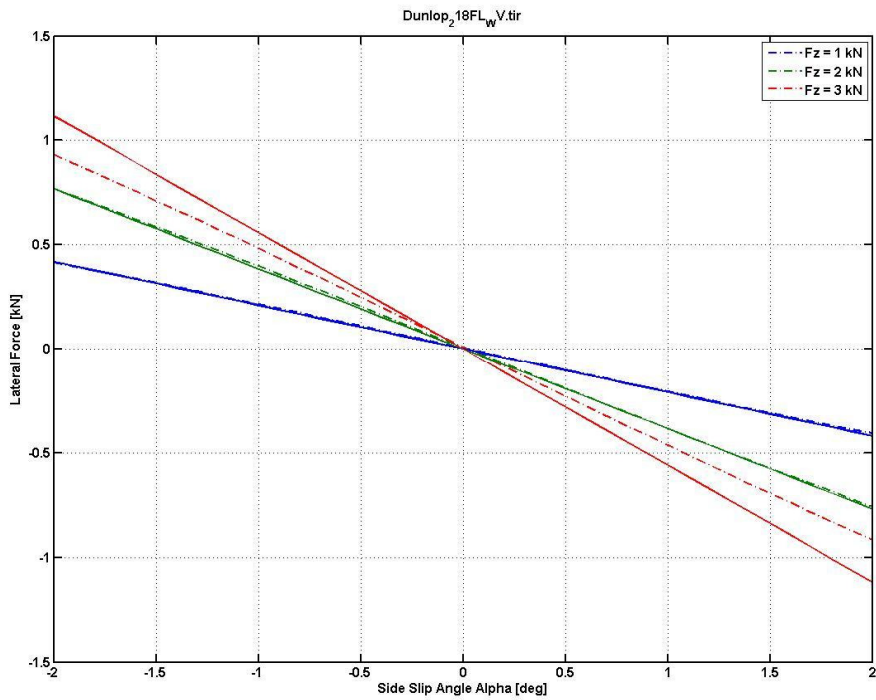


Figure 4.1.7 : Cornering Stiffness for the front tyre with new estimated parameters

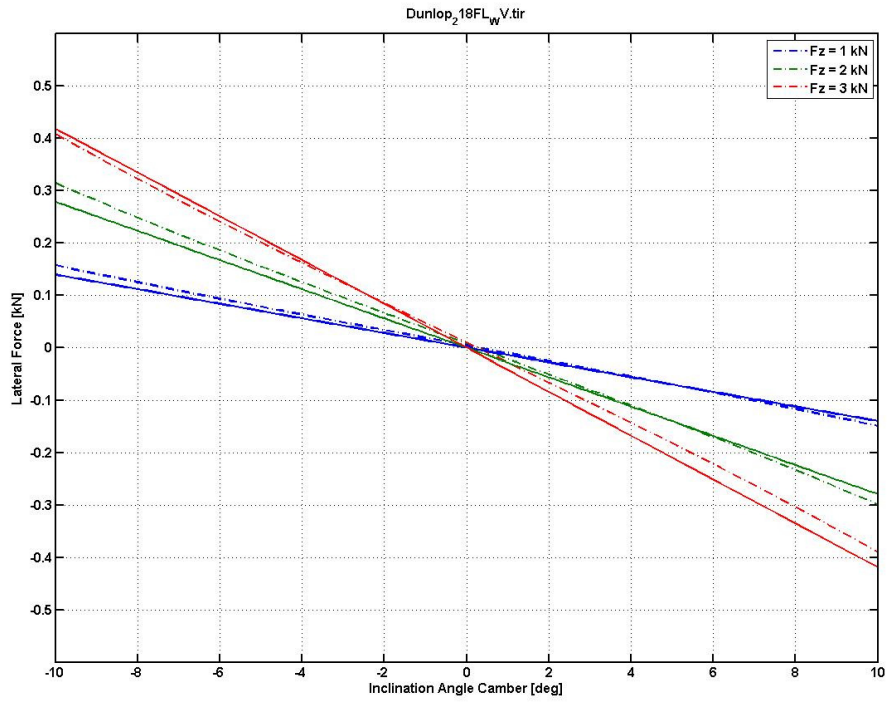


Figure 4.1.8 : Camber Stiffness for the front tyre with Pacejka's parameters

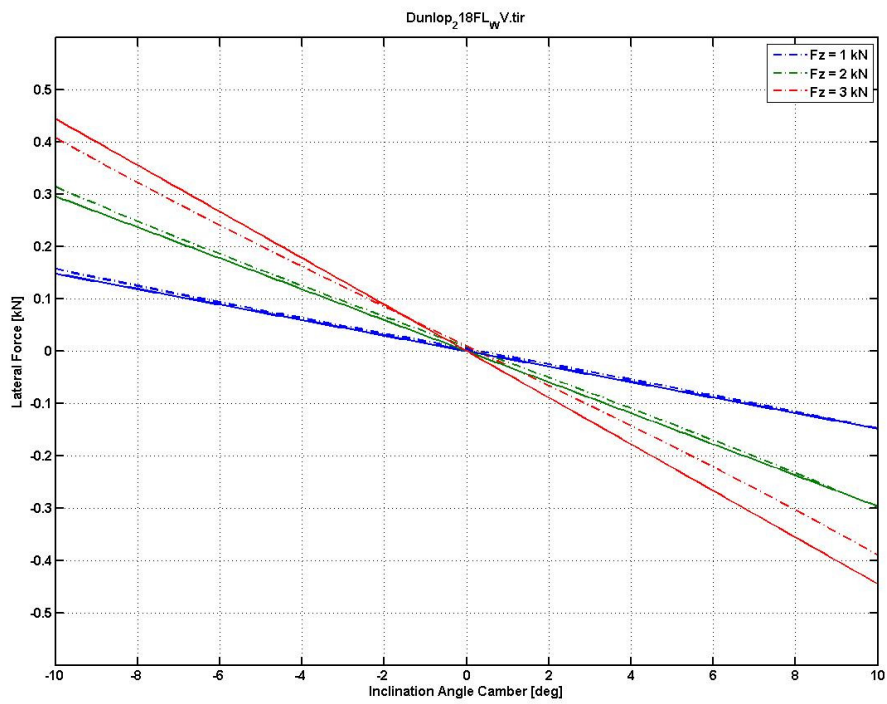


Figure 4.1.9 : Camber Stiffness for the front tyre with new estimated parameters

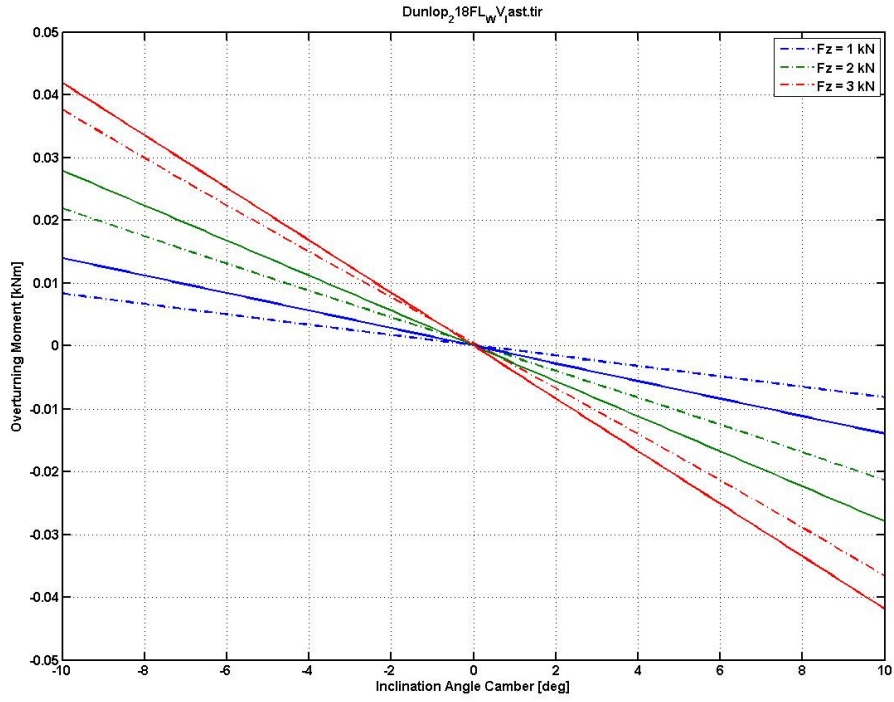


Figure 4.1.10 : Overturning Couple Stiffness for the front tyre with Pacejka's parameters

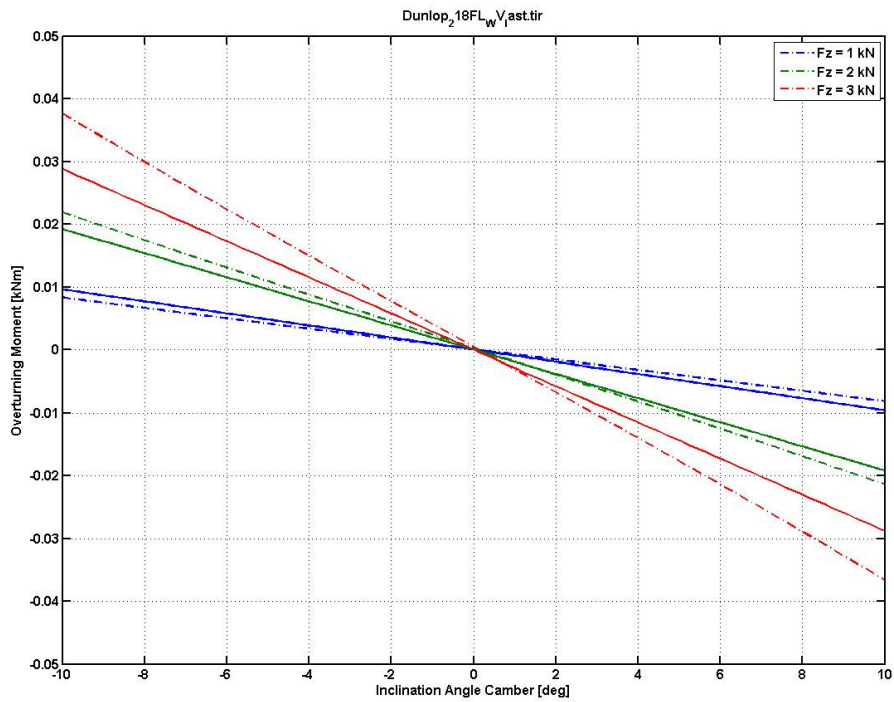


Figure 4.1.11 : Overturning Stiffness for the front tyre with new estimated parameters

Tyre Parameter Name	Pacejka's Value	Estimated Value
d_{11}	14	11.5
d_{12}	13	12
d_{21}	9	10
d_{22}	4	9.50
d_{31}	0.80	0.85
d_{32}	0.80	0.90
e_{11}	0.40	0.22
e_{12}	0.40	0.375
e_{21}	0.04	0.025
e_{22}	0.07	0.05
e_{31}	0.08	0.055
e_{32}	0.10	0.15

Table 4.1.1 : Parameters of front (,1) and rear (,2) tyre analytical motorcycle model

After getting the same tyre stiffnesses, other tyre parameters which influence the tyre behaviour are to be arranged in order to compare both models accurately. One of them is the tyre side characteristic which defines the orientation of the tyre. Symmetric tyre side is chosen in Willem's model since it assumes that there is no lateral force when the side slip is equal to zero. Moreover symmetric removes all asymmetric behaviours of the tyre. The second parameter is the slip force type. It is assumed that longitudinal forward velocity (u) is constant during the simulation so the longitudinal tyre force is required to keep the velocity constant. Combined type of slip forces is applied on Willem's model due to the interaction between longitudinal and lateral tyre forces. The other important parameter is dynamics which defines the type of contact dynamics. Pacejka takes the effect of tyre relaxation length in the equations and thinks that the relaxation lengths due to the side slip and camber angle depend on the normal load and are close to each other by disregarding the non-lagging part that exists in the response to camber changes. By choosing transient type of dynamic, tyre relaxation effects are also included in the Simmechanics analysis. Smooth and flat road surface are chosen in the simulation model.

4.1.3 Stabilizing Controller

As is explained in section 2.1, there are three instability modes in the motorcycle. They appear at different forward velocities. To be able to stabilize a motorcycle, steering torque should be applied to the handlebar. Willem uses reference camber which is applied

as an input, feedback camber angle, camber rate and steer rate to calculate the required steering torque. Steering torque is calculated with respect to the following equation.

$$T_{\delta} = K_1 \dot{\varphi} + K_2 (\varphi - \varphi_{ref}) + K_3 \dot{\delta}$$

Where $K_1 = 55$

$$K_2 = 250$$

$$K_3 = -10$$

However, Pacejka calculates the stabilizing steer torque as follows;

$$T_{\delta} = M_{\delta} - g_v v - g_r r - g_{d\varphi} \dot{\varphi} - g_{\varphi} \varphi - g_{d\delta} \dot{\delta} - g_{\delta} \delta$$

Where M_{δ} : Applied steer torque input to the handlebar

The coefficients used to calculate the feedback torque are calculated from the (2.2.20) and parameters in table 2.2.1.

Pacejka's controller with the parameters is applied to the simmechanics model.

4.2 Applied Test With Step Steer Torque

The responses of the motorcycle motion are investigated by applying a unit step steer torque (M_{δ}). After applying steer torque, first motorcycle is stabilized with the feedback control loop at the desired velocity. Here it is assumed that motorcycle is driven with constant longitudinal forward velocity (u) that is F_{ax} is equal to zero. In the analysis, a feedback controller which is defined in section 4.1.3 with the gains given in table 2.2.1 is used to stabilize the unwanted motorcycle motion.

To be able to compare the results accurately, SimMechanics model is to be modified such that the steer torque is applied as an input instead of the camber angle.

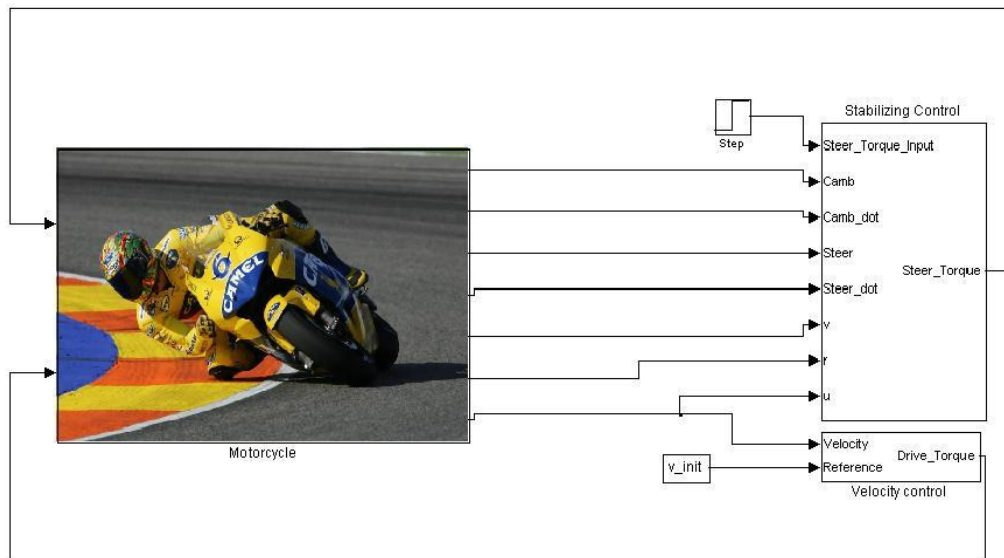


Figure 4.2.1 : Modified SimMechanics Model

60 [kph] of longitudinal motorcycle speed is considered and variations on the parameters are depicted and compared in the following figures.

The applied steer torque distribution is shown in figure 4.2.2. As can be seen from the following figure a unit step steer torque is applied to the motorcycle handlebar and kept constant during all simulation.

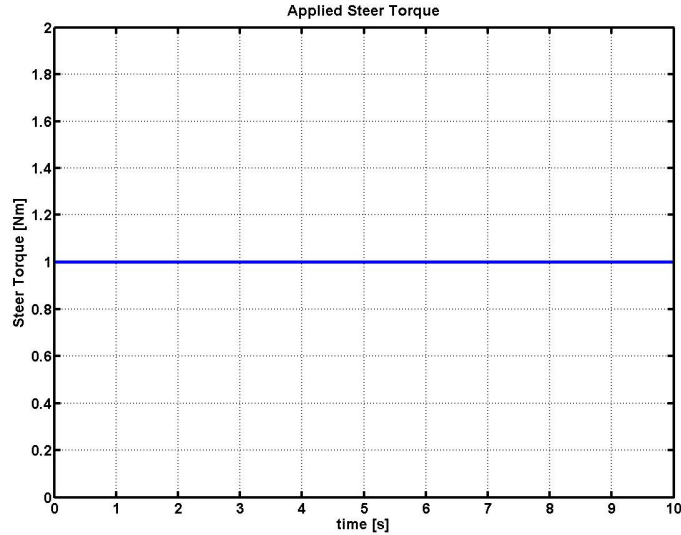


Figure 4.2.2 : Applied Steer Step Torque

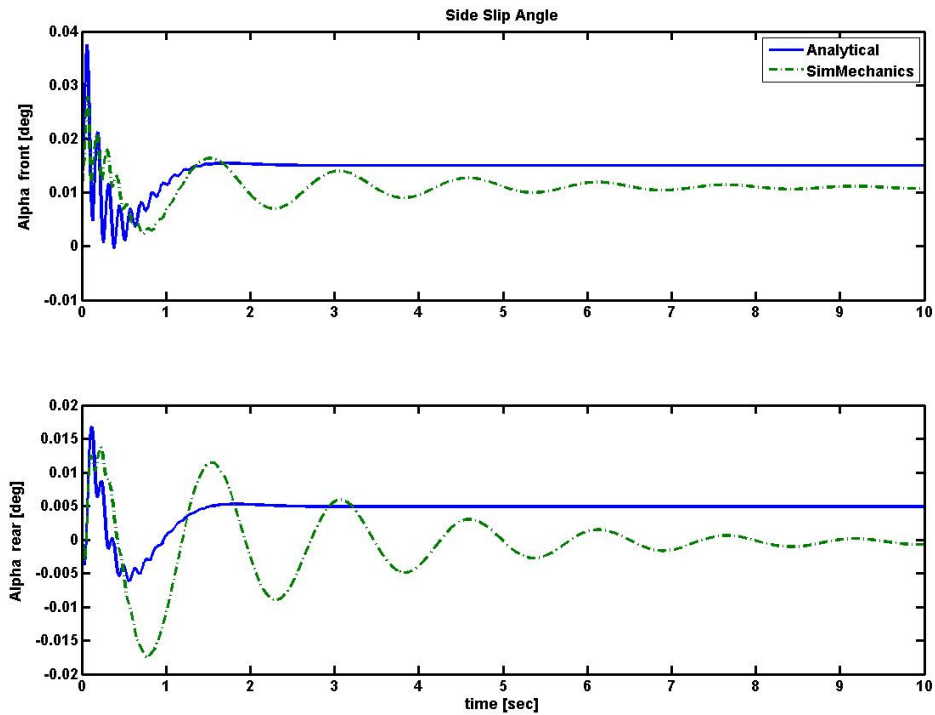


Figure 4.2.3 : Side slip angle distribution for the Analytical and SimMechanics model for 1 [Nm] Steer Torque

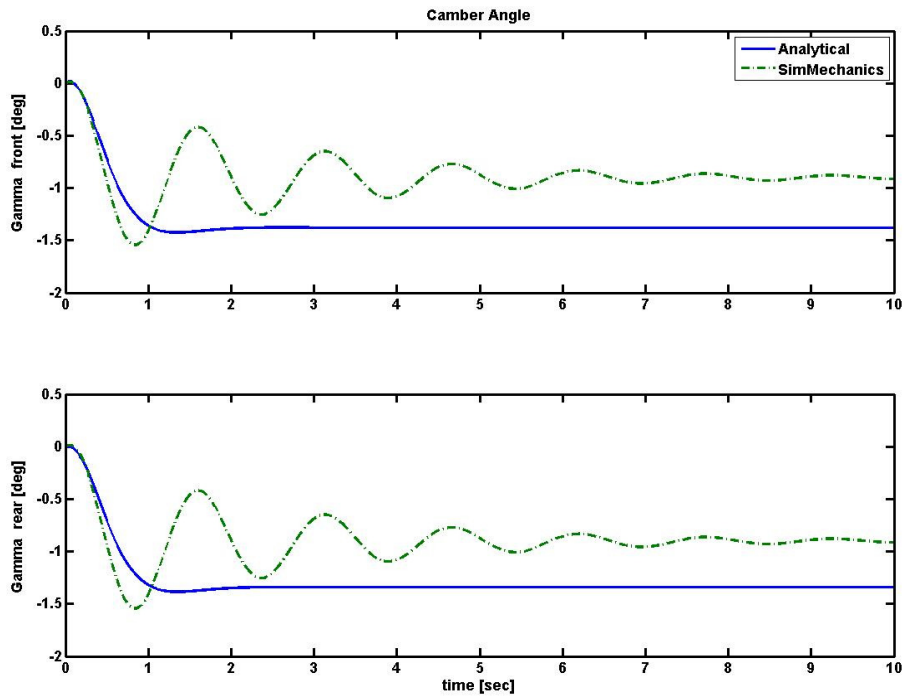


Figure 4.2.4 : Camber angle distribution for the Analytical and SimMechanics model for 1 [Nm] Steer Torque

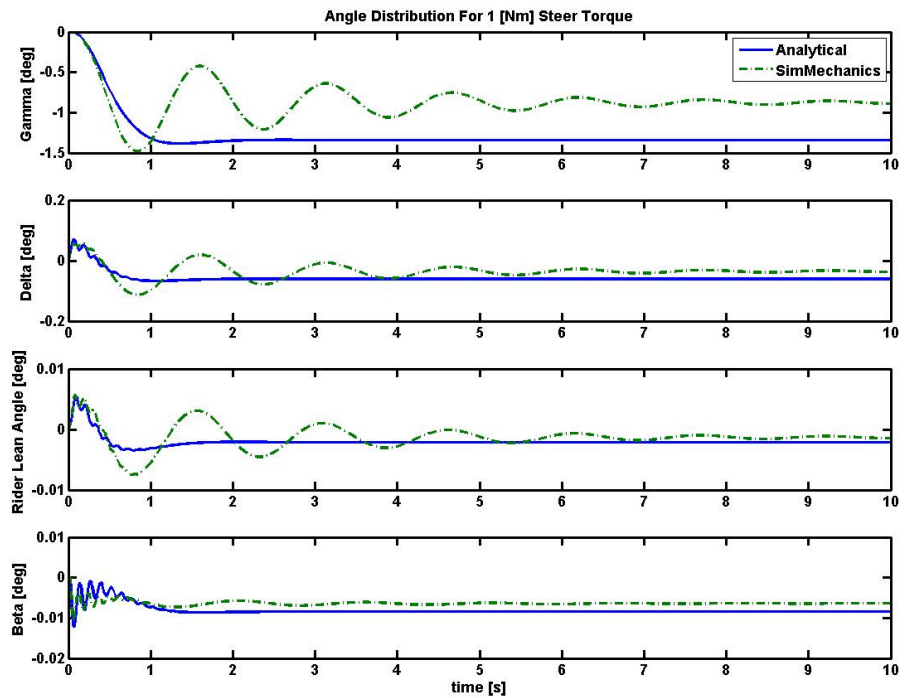


Figure 4.2.5 : Angle distribution for the Analytical and SimMechanics model for 1 [Nm] Steer Torque

It is observed from the figure 4.2.5 that when a positive steer torque (in the right direction) is applied to the handlebar, a positive steer angle (to the right) arises with a positive yaw rate (in the right direction). After a short time approximately 0.30 seconds sign changes are observed in the steer angle and yaw rate (r). And then negative camber angle (to the left direction) is build up and motorcycle will turn in the left direction. After 1-2 seconds motorcycle reaches to the steady-state condition.

It can be concluded from the above figures that the behaviour of both models is similar to each other. But their dynamic responses are not the same and errors are observed between two models. While the same tyre characteristic is tried to use in both models, its behaviour is not exactly the same. This is the first reason why the results are not the same. The second reason is that the gyroscopic effect is different when the steer torque is applied. The relaxation and stabilizing controller effect also influence the tyre behaviour.

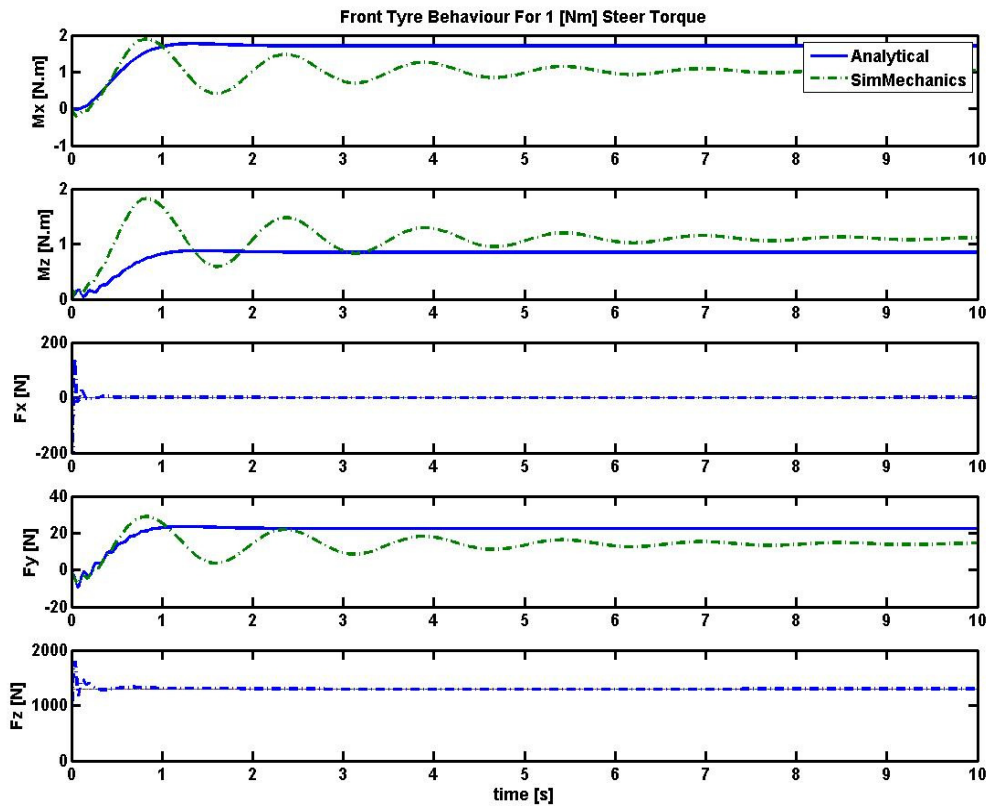


Figure 4.2.6 : Front Tyre Force and Moment distribution for the Analytical and SimMechanics model for 1 [Nm] Steer Torque

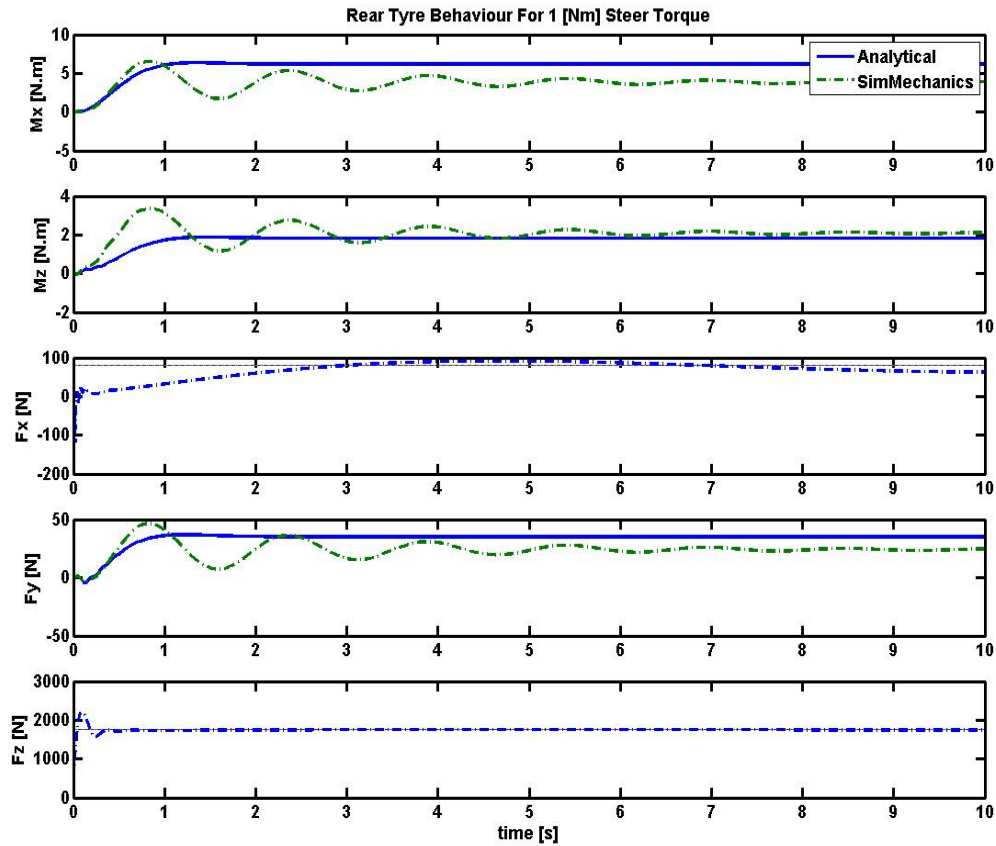


Figure 4.2.7 : Rear Tyre Force and Moment distribution for the Analytical and SimMechanics model for 1 [Nm] Steer Torque

To search the effect of the stabilizing controller and relaxation effect, they are disregarded on the analytical and simmechanics model. f_{1i} and f_{2i} coefficients are made very small value such as $0.15e-15$ and $0.1e-15$ respectively. By this way slip and camber relaxation lengths will be neglected in the analytical model. Moreover the dynamics property defining the type contact dynamics is chosen as steady-state for the front and rear tyre in simmechanics model. It means that no dynamic behaviour is included in the simmechanis analysis. The models are made open-loop by choosing the parameters given in table 2.2.1 as zero. Here, the open-loop means that there is no stabilizing controller in the motorcycle model.

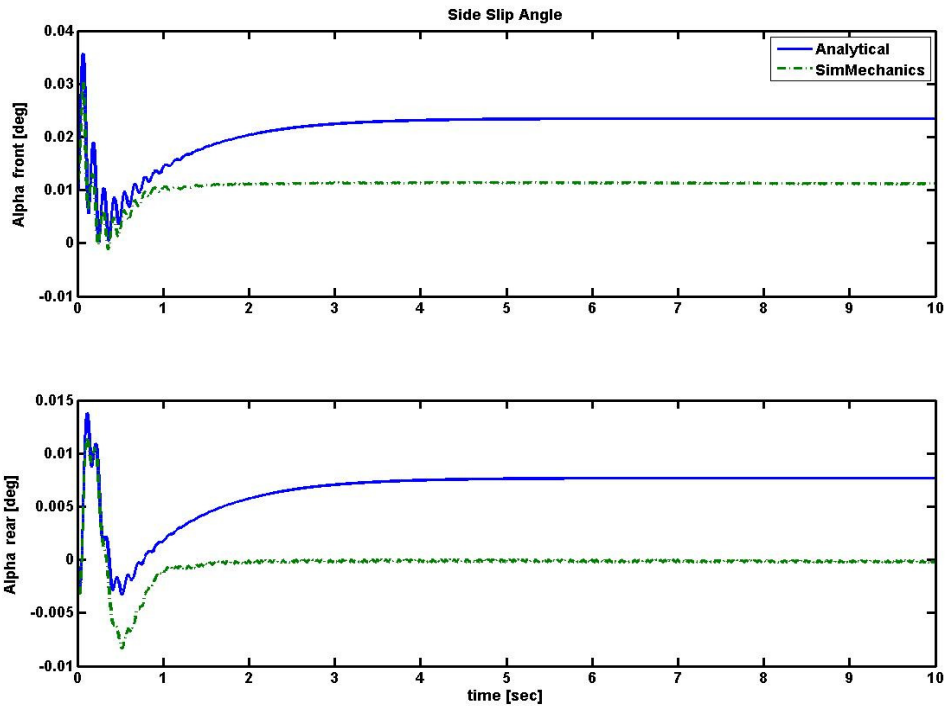


Figure 4.2.8 : Side slip angle distribution for the Open-Loop Analytical and SimMechanics model without relaxation for 1 [Nm] Steer Torque

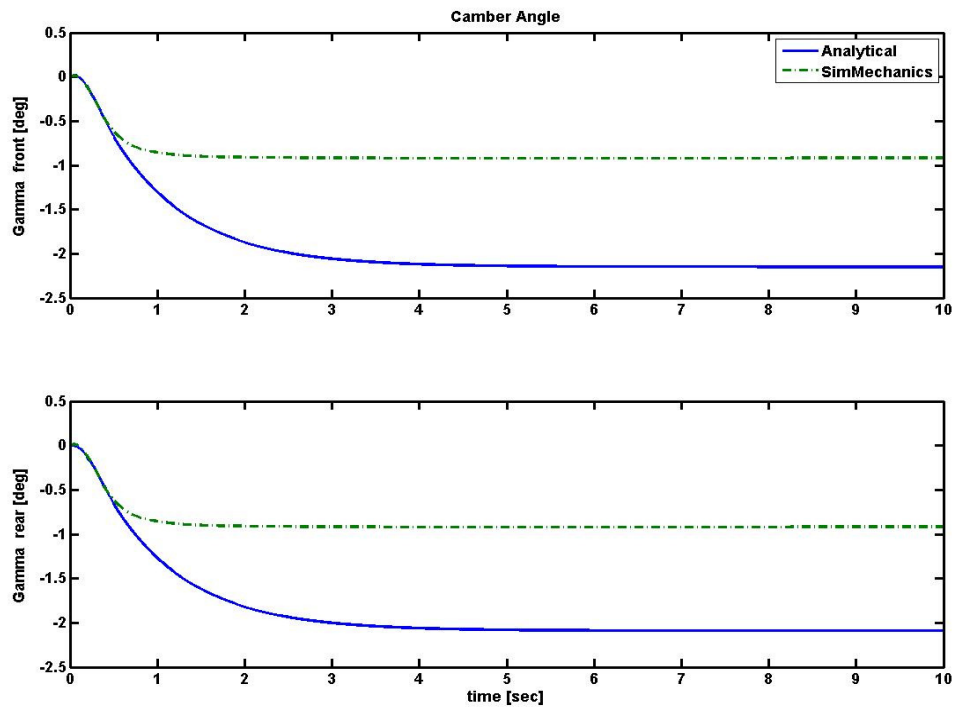


Figure 4.2.9 : Camber angle distribution for the Open-Loop Analytical and SimMechanics model without relaxation for 1 [Nm] Steer Torque

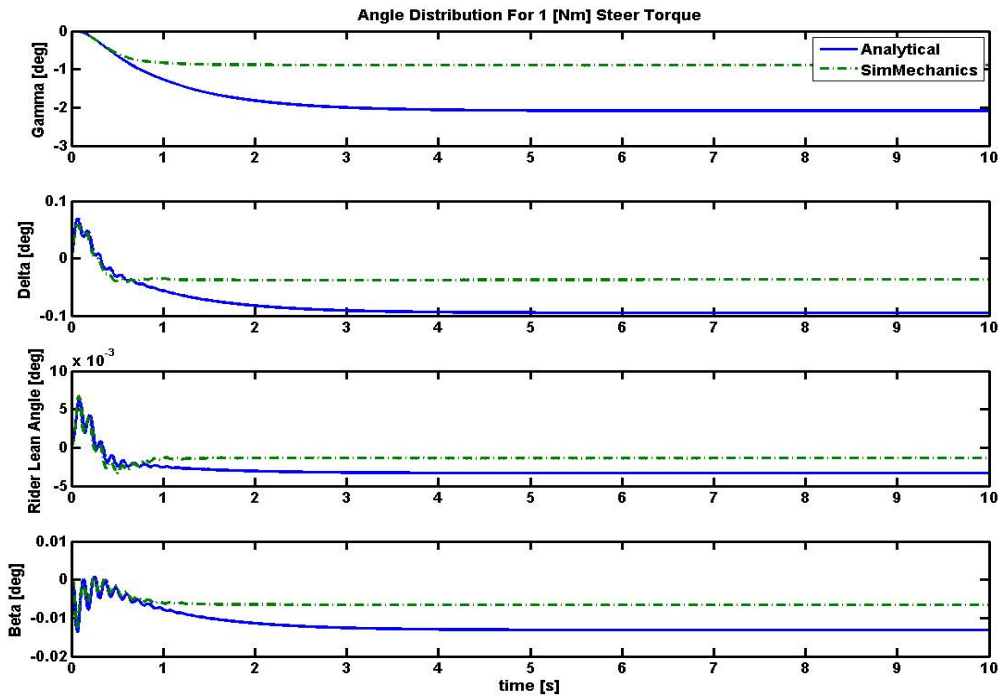


Figure 4.2.10 : Angle distribution for the Open-Loop Analytical and SimMechanics model without relaxation for 1 [Nm] Steer Torque

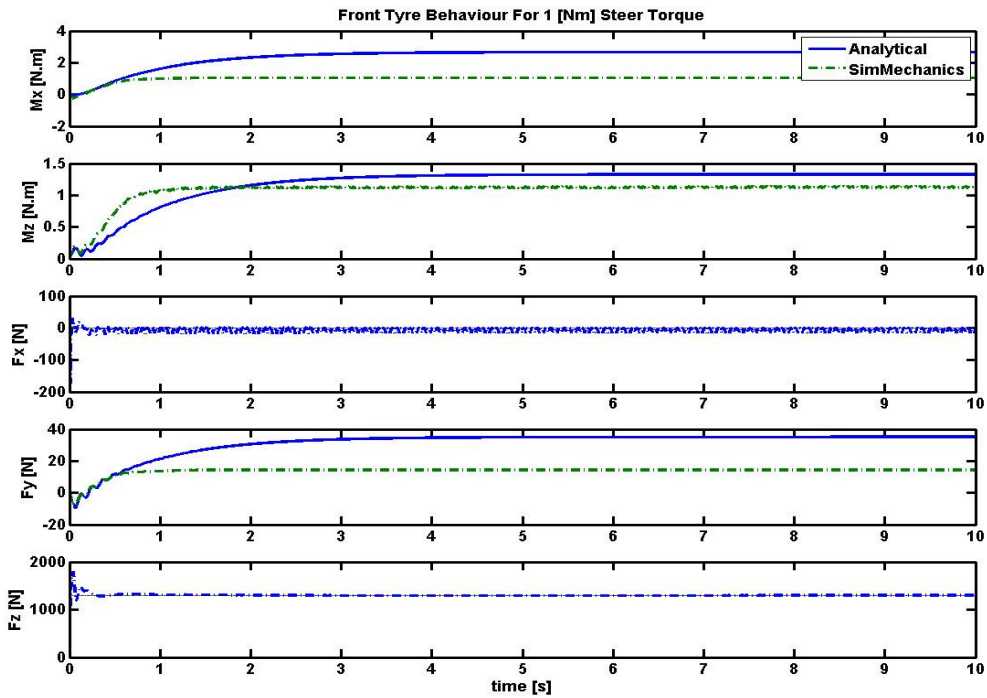


Figure 4.2.11 : Front Tyre Force and Moment distribution for the Open-Loop Analytical and SimMechanics model without relaxation for 1 [Nm] Steer Torque

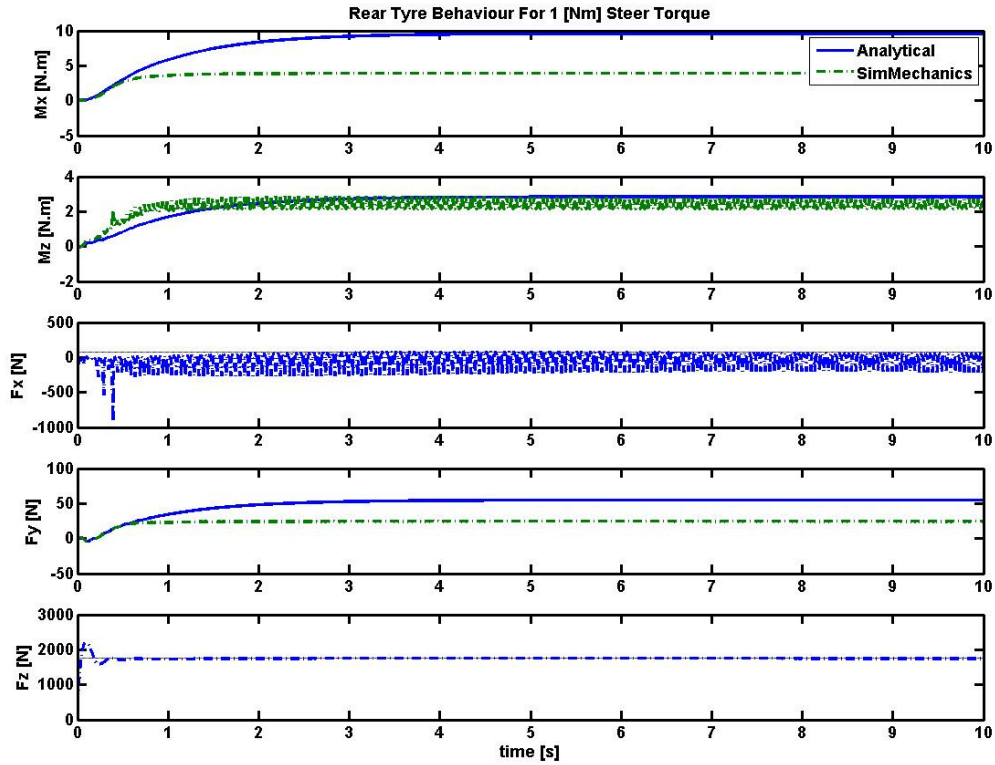


Figure 4.2.12 : Rear Tyre Force and Moment distribution for the Open-Loop Analytical and SimMechanics model without relaxation for 1 [Nm] Steer Torque

As can be seen from the figures of 4.2.8 , 4.2.9 and 4.2.10 slip angle, camber angle, rider lean angle and twist angle are approximately same until 0.5 [sec]. After that point, the differences are observed between two models. Here, the tyre properties used in the models are not exactly the same. So a test procedure is developed in the following section in order to be able to compare two models correctly.

4.3 Test Procedure :

The following modifications are applied to the SimMechanics and analytical model for the accurate comparison.

- The effect of the cornering, camber, overturning moment and self-aligning moment stiffness coefficients are neglected. By neglecting these parameters the tyre effect on the models will be also neglected. First of all, the lateral force (F_{yi}), the self-aligning moment (M_{zi}) and the overturning moment (M_{xi}) are made zero in the analytical model by choosing the parameters given in table 4.1.1 as zero. In the simmechanics model, none slip force option which means that there is no slip force on the tyre is chosen on the Delft-Tyre model.
- Controller also has effect on the motorcycle model. So the open loop motorcycle models are compared. Parameters given in table 2.2.1 are assumed to be zero in order to

change both models from the closed loop to the open loop system. However, the analytical motorcycle system goes to infinity too early without controller because of the relaxation effect. So the tyre relaxation effect is to be minimized in order to delay the unstable mode without the stability controller. The tyre relaxation effect is included in the analytical system by applying the equation 2.2.3. If the relaxation parameter f_{1i} and f_{2i} are entered as zero in order to make the relaxation lengths ($\sigma_{\alpha i}$ and $\sigma_{\gamma i}$) zero, the matrix of A used to define the state equation becomes singular.

$$Ax + Bx = Cu$$

Where x : state vector

u : input

So these parameters are assumed to be very small in order to solve this problem and neglect the relaxation effect.

- If a motorcycle goes with longitudinal velocity of (u) there is always an aerodynamic drag force (F_d) and lift force (F_l) acting on the motorcycle. They quadratically depend on the longitudinal velocity. Although the drag force is defined in both models, the lift force is only used in the simmechanics and its effect is neglected in the analytical model. So it is assumed that there is no lift force applied to the motor.
- The moment of inertia of the main body with respect to the x-axis is increased 1.000.000 times in order to prevent the early falling down of motorcycle.
- Different parameters can be applied as an input to turn the motorcycle. For example, a unit step steer torque was applied to the handlebar and then the motorcycle behaviour was analyzed with respect to this parameter. However it is difficult to measure the steer torque in the reality. Steer angle and steer angle rate can be easily measured simultaneously. So it is better to apply them as an input on the models. The steer angle is not used as a generalized coordinate anymore in order to derive the analytical equations since it is continuously measured and entered as an input. So the analytical equations are derived again with respect to the new generalized coordinates which are lateral velocity (v), yaw rate (r), roll angle(φ), rider lean angle (φ_r) and twist angle (β). Furthermore SimMechanis model is modified to make the steer angle and steer rate input as follows;

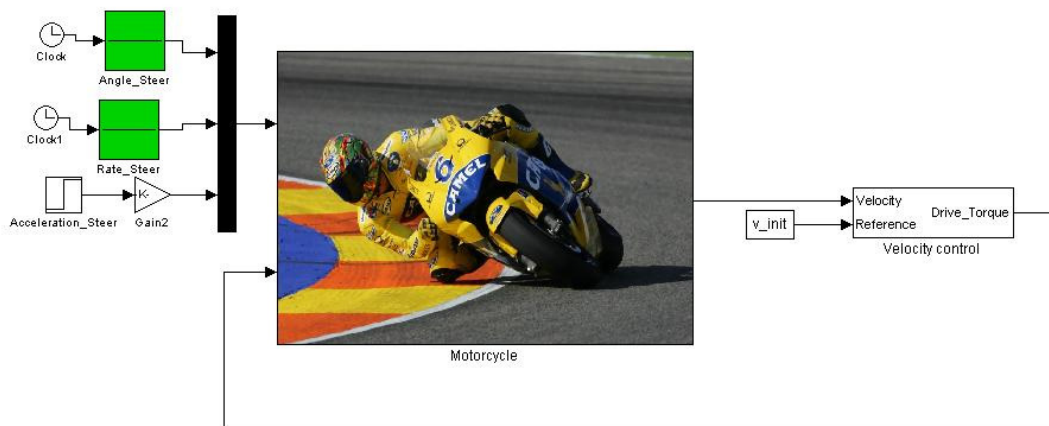


Figure 4.13 : Steer angle and Steer Rate Input on the SimMechanics Model

The following results are obtained by this test procedure without any applied tyre forces except from the longitudinal (F_{xi}) and vertical forces (F_{zi}) for the front and rear tyre.

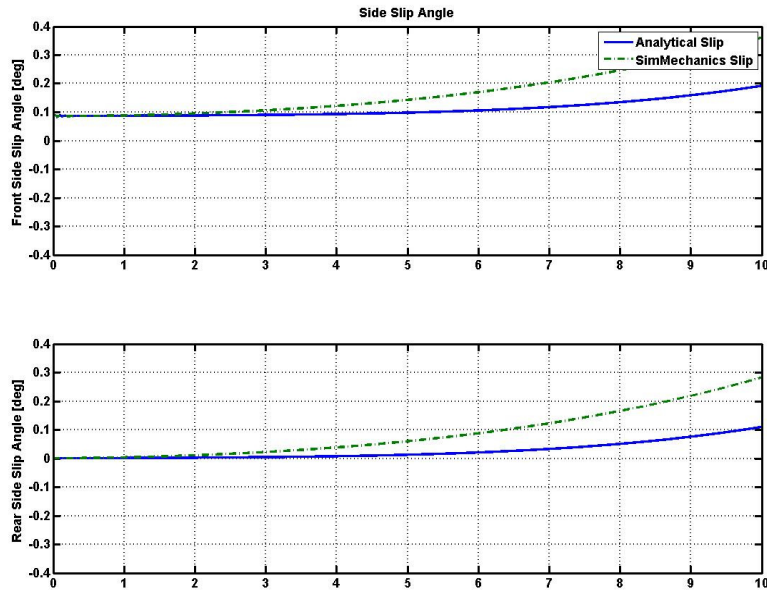


Figure 4.2.14 : Side slip angle distribution for the Open-Loop Analytical and SimMechanics model without tyre characteristics for 0.1 [deg] Steer Angle
 Although the side slip angle distribution for both models is approximately same until 2.5 [sec], after that point they become different with respect to each other.

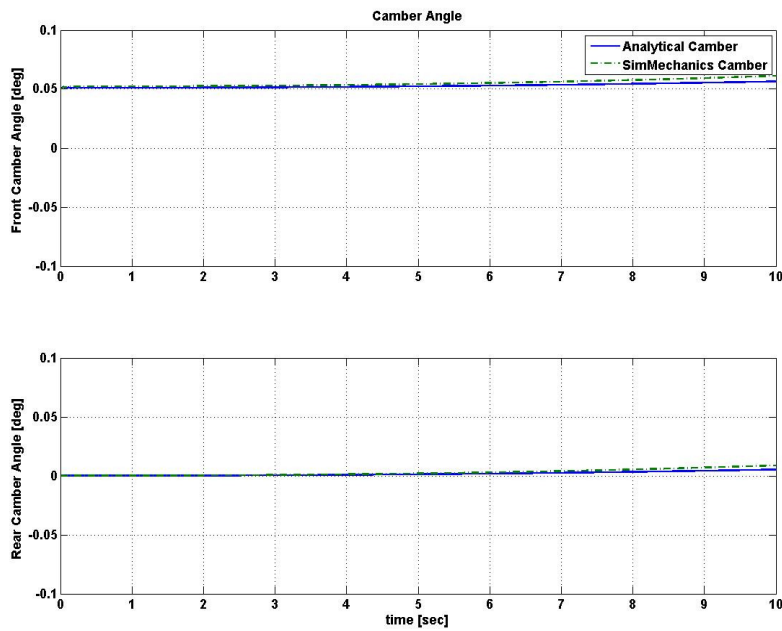


Figure 4.2.15 : Camber angle distribution for the Open-Loop Analytical and SimMechanics model without tyre characteristics for 0.1 [deg] Steer Angle

When the tyre characteristics are included in this analysis the following motor responses are obtained for both models.

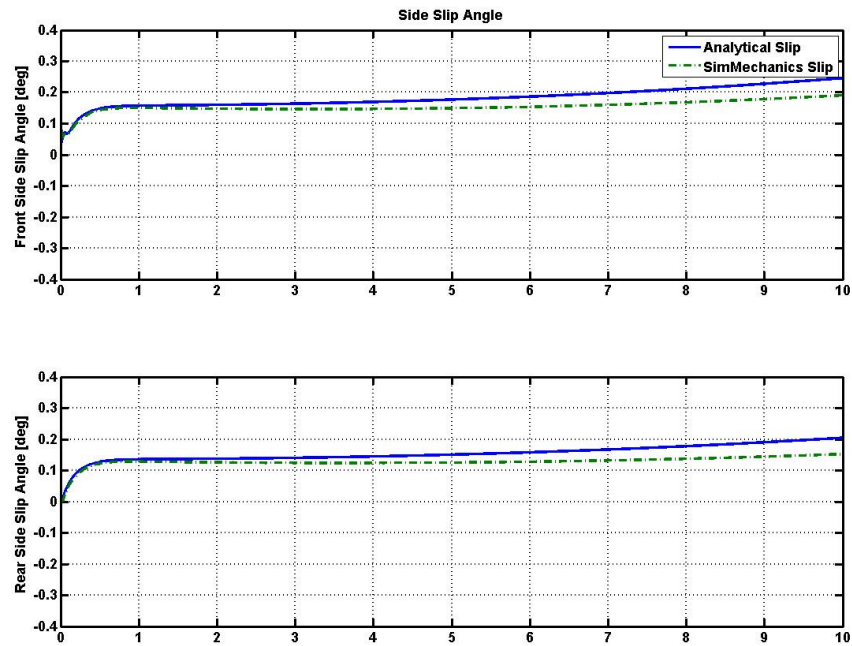


Figure 4.2.16 : Side slip angle distribution for the Open-Loop Analytical and SimMechanics model with tyre characteristics for 0.1 [deg] Steer Angle

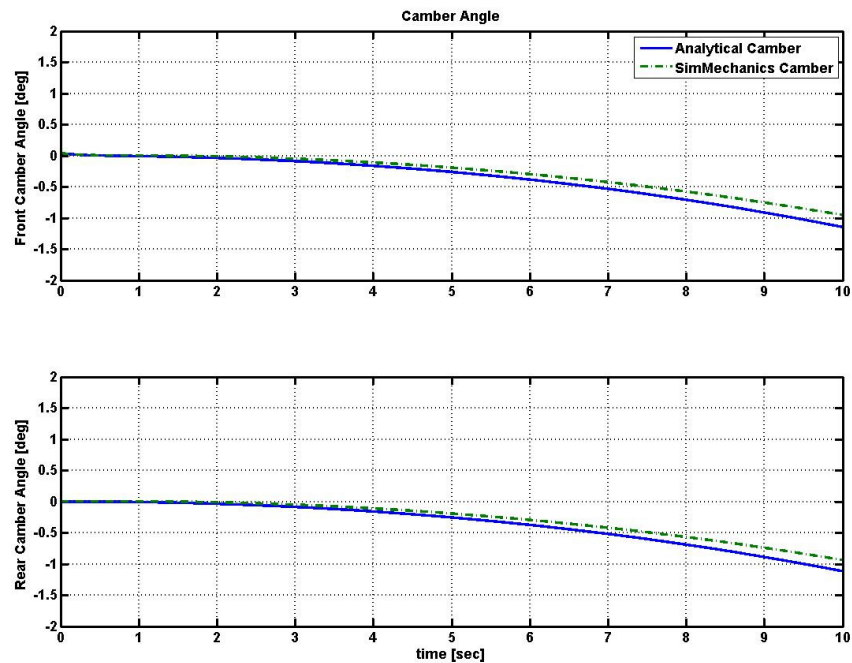


Figure 4.2.17 : Side slip angle distribution for the Open-Loop Analytical and SimMechanics model with tyre characteristics for 0.1 [deg] Steer Angle

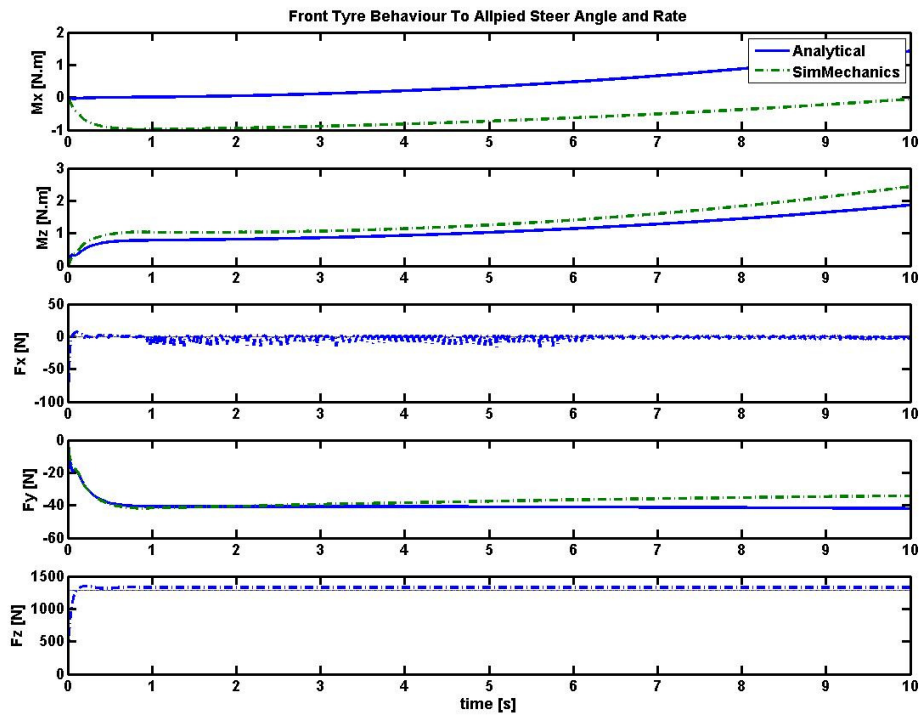


Figure 4.2.18 : Front tyre force and moment distribution for the Open-Loop Analytical and SimMechanics model with tyre characteristics for 0.1 [deg] Steer Angle

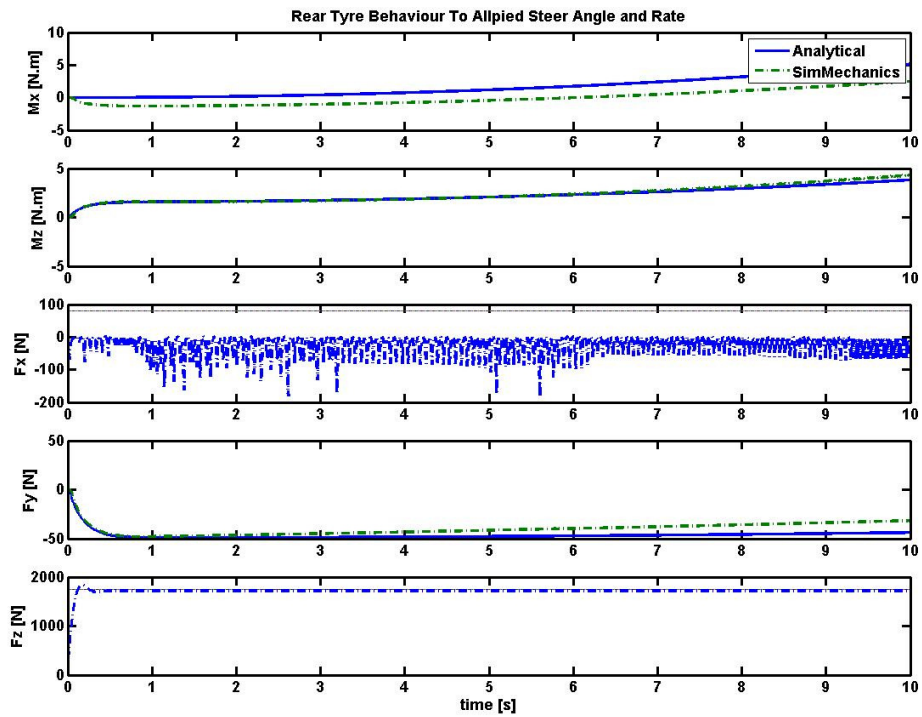


Figure 4.2.19 : Rear tyre force and moment distribution for the Open-Loop Analytical and SimMechanics model without tyre characteristics for 0.1 [deg] Steer Angle

CHAPTER 5

CONCLUSIONS and RECOMMENDATIONS

5.1 Conclusions

A lot of research has been done in order to develop a motorcycle model simulating of the real motor behaviour accurately. One of them was made by Willem who designed a simmechanics motorcycle model. While this model is very accurate, it requires too much calculation time. So it is difficult to apply this model in reality.

The main purpose of this traineeship is to implement Pacejka's analytical motorcycle model [1] and then check the calculation time and accuracy with respect to simmechanics [2].

First of all, it is assumed that step steer torque is applied to the handlebar as an input. And then state vector is defined with respect to the generalized coordinates which are lateral velocity (v), yaw rate (r), roll angle (φ), rider lean angle (φ_r), steer angle (δ) and twist angle (β). By using the modified Lagrange equations and relaxation length equations, there are enough equations to calculate the state variables in the time domain. This analysis is only made in the linear region where the stiffness coefficients are constant and they depend on the applied vertical forces. It is concluded from this analysis is that although their behaviour are similar to each other, their dynamics responses are not the same and errors are seen between two models. The relaxation, stabilizing controller, tyre characteristic and gyroscopic effect are included in this analysis. All of them affect the dynamic response of the motorcycle.

To be able correctly compare analytical and simmechanics model, a benchmarked is made and then a new test procedure is introduced for this purpose. It can be described as follows;

- A stabilizing controller should be applied to the motorcycle in order to prevent the weave, wobble and capsize unstable models. However it affects the dynamic behaviour of the motorcycle. So the models are modified to make them open-loop system. It means that there is no stabilizing controller on the model.
- When a motorcycle goes with a forward velocity (u), a drag and lift force proportional with the square of u are applied to the motorcycle. However the effect of the lift force is neglected in the analytical model. In comparison, it is assumed that only a drag force is applied to the motorcycle.
- The tyre behaviour is neglected also at the beginning and then it will be included in the comparison. By neglecting the tyre behaviour, the cornering, camber, overturning moment and self-aligning moment stiffnesses are not taken into account.
- The moment of inertia of mainframe with respect to x-axis is increase 1.000.000 times to prevent early falling down of motorcycle.

- At the beginning of the traineeship, step steer torque is applied as an input. But it is difficult to measure the applied steer torque continuously so it is decided to apply steer angle and steer rate to the handle bar. Steer angle is not generalized coordinate anymore. So modified Lagrange equations are derived with respect to new generalized coordinates.

When the kinematic behaviour of the models are compared with the side slip and camber angle, it is concluded that camber angle distribution of both model is very close to each other. On the other hand the difference between side slip angles becomes larger, since the motorcycle starts to lose its stability.

The biggest advantage of the analytical model is that its calculation time is smaller than the calculation time of simmechanics model.

5.2 Recommendations for future research

In future research, it should be developed a tyre model to use in the analytical model by using magic formula. The tyre character has a great effect on dynamic motorcycle response so exactly the same tyre behaviour should be used in both models. Furthermore, a new stabilizing controller should be introduced for the model where steer angle and steer rate are used as an input. After these modifications, analytical and simmechanics model can be compared more accurately.

LITERATURE

- [1] Hans B. Pacejka, Tyre and Vehicle Dynamics, p 511- 562 , Delft University of Technology, 2002.
- [2] W.D. Versteden, Improving a tyre model for motorcycle simulations, Mater's Thesis, Technical University of Eindhoven, June 2005.

APPENDICES

Appendix A : Motorcycle Model Parameters

All parameters of motorcycle are required in order to implement analytical motorcycle model in the matlab. They are specified in 'Improving a tyre model for motorcycle simulations' [2] in Appendix B. In figure A.1 a parameterized view of the motorcycle is shown and parameters are given below.

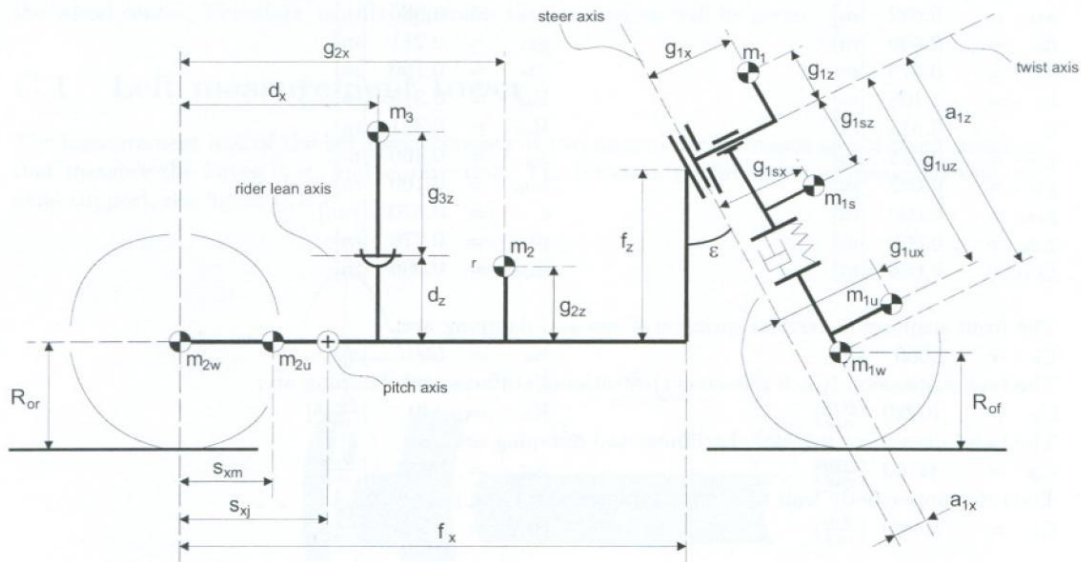


Figure A.1 : Motorcycle Model Geometry

Masses

m_1	=	13.1	kg	m_2	=	209.6	kg
m_{1s}	=	0	kg	m_{2u}	=	10.6	kg
m_{1u}	=	7.5	kg	m_{2w}	=	15	kg
m_{1w}	=	10	kg	m_3	=	44.5	kg

Moments of inertia

J_{x1}	=	0.46	kgm^2	J_{x1u}	=	0.29	kgm^2
J_{y1}	=	1.2	kgm^2	J_{y1u}	=	0.0	kgm^2
J_{z1}	=	0.21	kgm^2	J_{z1u}	=	0.29	kgm^2
J_{xz1}	=	0.0	kgm^2	J_{xz1u}	=	0.0	kgm^2
J_{x1s}	=	0.0	kgm^2	J_{x1w}	=	0.02	kgm^2
J_{y1s}	=	0.0	kgm^2	J_{y1w}	=	0.58	kgm^2
J_{z1s}	=	0.0	kgm^2	J_{z1w}	=	0.0	kgm^2
J_{xz1s}	=	0.0	kgm^2	J_{xz1w}	=	0.0	kgm^2
J_{x2}	=	15.23	kgm^2	J_{x2w}	=	0.02	kgm^2
J_{y2}	=	32.0	kgm^2	J_{y2w}	=	0.74	kgm^2
J_{z2}	=	19.33	kgm^2	J_{z2w}	=	0.0	kgm^2
J_{xz2}	=	-1.4	kgm^2	J_{xz2w}	=	0.0	kgm^2

Jx2u	=	0.37	kgm ²	Jx3	=	1.3	kgm ²
Jy2u	=	0.0	kgm ²	Jy3	=	2.1	kgm ²
Jz2u	=	0.37	kgm ²	Jz3	=	1.4	kgm ²
Jxz2u	=	0.0	kgm ²	Jxz3	=	-0.3	kgm ²

Geometry

e	=	0.520	rad	g1sz	=	0.000	m
fx	=	1.168	m	g1ux	=	0.066	m
fz	=	0.513	m	g1uz	=	0.632	m
g1x	=	0.015	m	a1x	=	0.066	m
g1z	=	0.032	m	a1z	=	0.632	m
g2x	=	0.680	m	pLx	=	0.770	m
g2z	=	0.211	m	pDz	=	0.900	m
g3z	=	0.190	m	sxm	=	0.100	m
dx	=	0.600	m	Rof	=	0.319	m
dz	=	0.679	m	Ror	=	0.321	m
g1sx	=	0.000	m	sjj	=	0.400	m

Twist stiffness and damping

C_{β}	=	34100	Nm/rad
K_{β}	=	99.7	Nm/srad

Rider upper body lean stiffness and damping

Cr	=	10000	Nm/rad
Kr	=	85.20	Nms/rad

Effective aerodynamic drag and lift areas

C_{DA}	=	0.488	m ²
C_{LA}	=	0.0	m ²