The relation between effective deformation and micro vickers hardness in a state of large plastic deformation

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THE RELATION BETWEEN EFFECTIVE DEFORMATION AND MICRO VICKERS HARDNESS IN A STATE OF LARGE PLASTIC DEFORMATION.

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SUMMARY

It is shown that both Voce and Palm's and Ludwik's equations describe the relation between hardness and effective deformation in a defective way in the case of large deformation as prevails in punching.
Generalisation of Nadai's equation however renders a reliable non-linear function between hardness and effective stress.
The result is applied to the punching process giving an expression for the punching force and an indication towards the physical background of the shearing factor in punching.

RÉSUMÉ

C'est montré que les deux équations de Voce et Palm et de Ludwik descrivent la rélation entre la dureté et la déformation d'une façon défective dans le cas de déformation large comme se trouve au perçage au poinçoin.
D'autre part, généralisation de l'équation de Nadai fournit une sûre fonction non-linéaire entre la dureté et la charge effective. Le résultat est appliqué au procès de perçage, donnant une expression pour la force de perçage et une indication vers le fond physique de l'élément de cisaillement au perçage.

ZUSAMMENFASSUNG

Es wird gezeigt dass die Gleichungen nach Voce und Palm, und Ludwik den zusammenhang zwischen Härte und logarithmischer Formänderung, für grosse Verformungsgrade wie z.B. beim Stanzen, nicht richtig zu beschreiben vermögen.

Eine Verallgemeinerung der Nadai'schen Fliesskurve liefert jedoch eine zuverlässige nicht-lineäre Beziehung zwischen Härte und Vergleichsspannung. Wird das Ergebnis beim Stanzprozess angewendet so ergibt sich ein Ausdruck für die erforderliche Stanzkraft, und ist eine beschränkte physikalische Deutung des Scherfaktors möglich.

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1. *Introduction*

It is a well known fact that due to work hardening the hardness of metals is increased by deformation.
Therefore the measurement of micro hardness is applied to determine the distribution of deformation in forming processes. {1, 2, 3}.
As however the analytical relations available to describe the dependence of hardness on deformation {4.5} are only valid in a limited range of deformation investigations have been carried out aiming at obtaining more general validity.
By performing tensile tests combined with hardness measurements on the tensile specimen the strain (effective deformation) can be related to hardness. The validity of the expressions obtained has been checked in a case of large plastic deformation as prevails in the punching process.

2. *Voce's equation*

Voce {5} formulates

$$\bar{\sigma} = \bar{\sigma}_\infty - (\sigma_\infty - \sigma_o)\, e^{-\bar{\varepsilon}/\varepsilon_c} \qquad (1)$$

Palm {4} gives a complementary function in terms of hardness

$$HV = HV_\infty - (HV_\infty - HV_o)^{-\bar{\varepsilon}/\varepsilon_c'} \qquad (2)$$

and shows that holds

$$\varepsilon_c = \varepsilon_c'$$

Hence it follows

$$HV = C_1 + C_2 \bar{\sigma} \qquad (3)$$

By means of tensile tests of which fig. 1 shows the results and next subjecting the experimental data to non-linear regression analysis the constants in eq. 1 have been determined as listed in table 1.
Regression analysis has also been applied to experimental data obtained by hardness measurements on tensile test specimen according to eq. 3 from which fig. 2 results. The values of the constants have also been listed in table 1.

Table 1.

material	σ_∞ [N/mm²]	σ_0 [N/mm²]	ε_c [-]	load [gf]	HV_∞ [N/mm²]	H_0 [N/mm²]	C_1 [N/mm²]	C_2 [-]
low carbon steel	610	210	0.200	100	2400	1200	610	2.93
KMS 63 H	890	250	0.751	50	2590	1140	590	2.24

However when applying eq.3 to a state of large plastic deformation as prevails in the region of shearing in the punching process as indicated in fig. 5 it is found for the carbon steel investigated

$$HV_{max} > 3000 \text{ N/mm}^2 > H_\infty$$

from which it follows

$$\bar{\sigma}_{max} > 800 \text{ N/mm}^2 > \sigma_\infty$$

This is not compatible with eq. 1 and table 1 from which is taken the condition

$$\bar{\sigma}_{max} < \sigma_\infty = 610 \text{ N/mm}^2$$

For this reason eqs. 1 and 2 are considered not being applicable

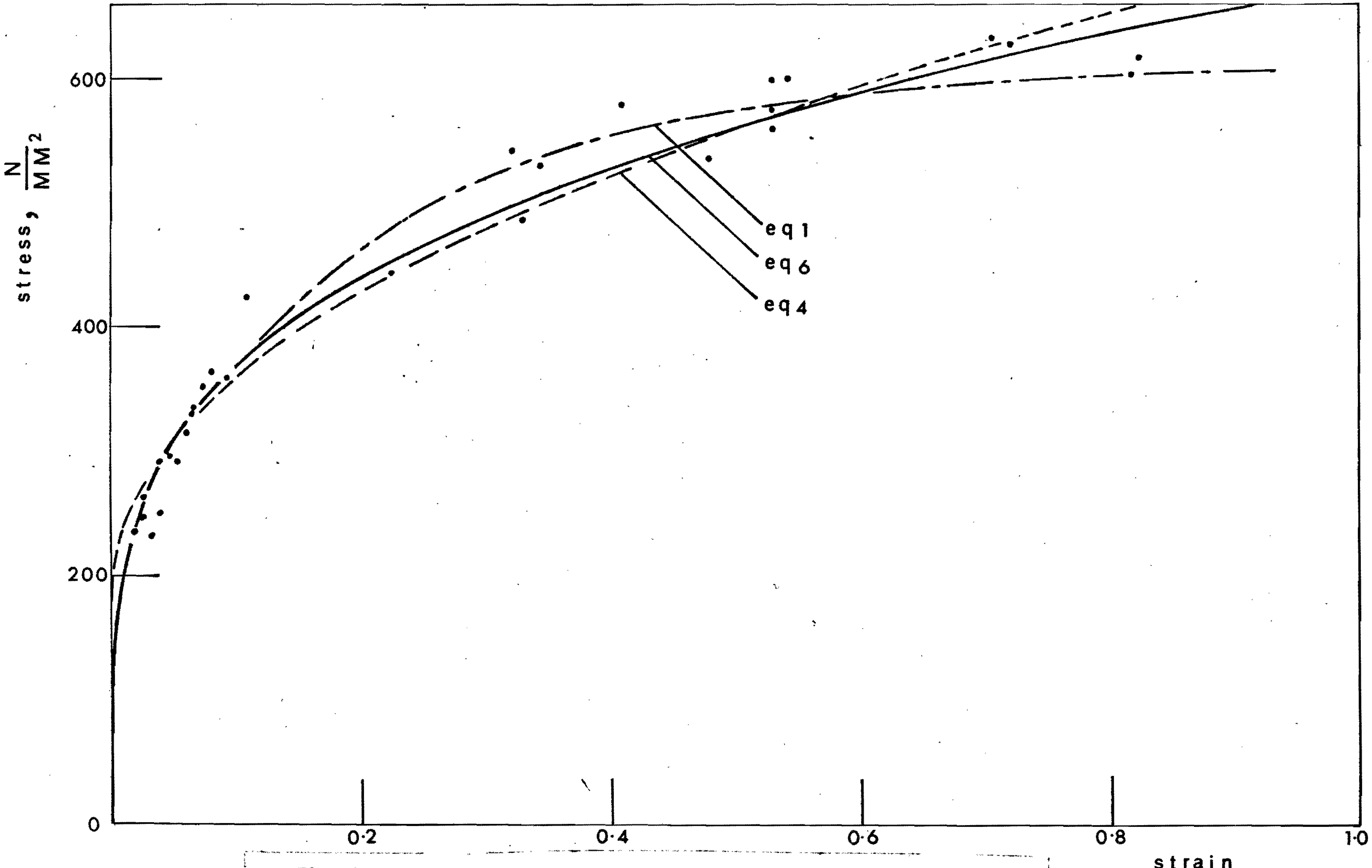

Fig. 1. Comparison between the deformation equations (work-hardening functions) according to Voce (eq.1), Ludwik (eq. 4) and Nadai (eq. 6), resp. for low carbon steel.

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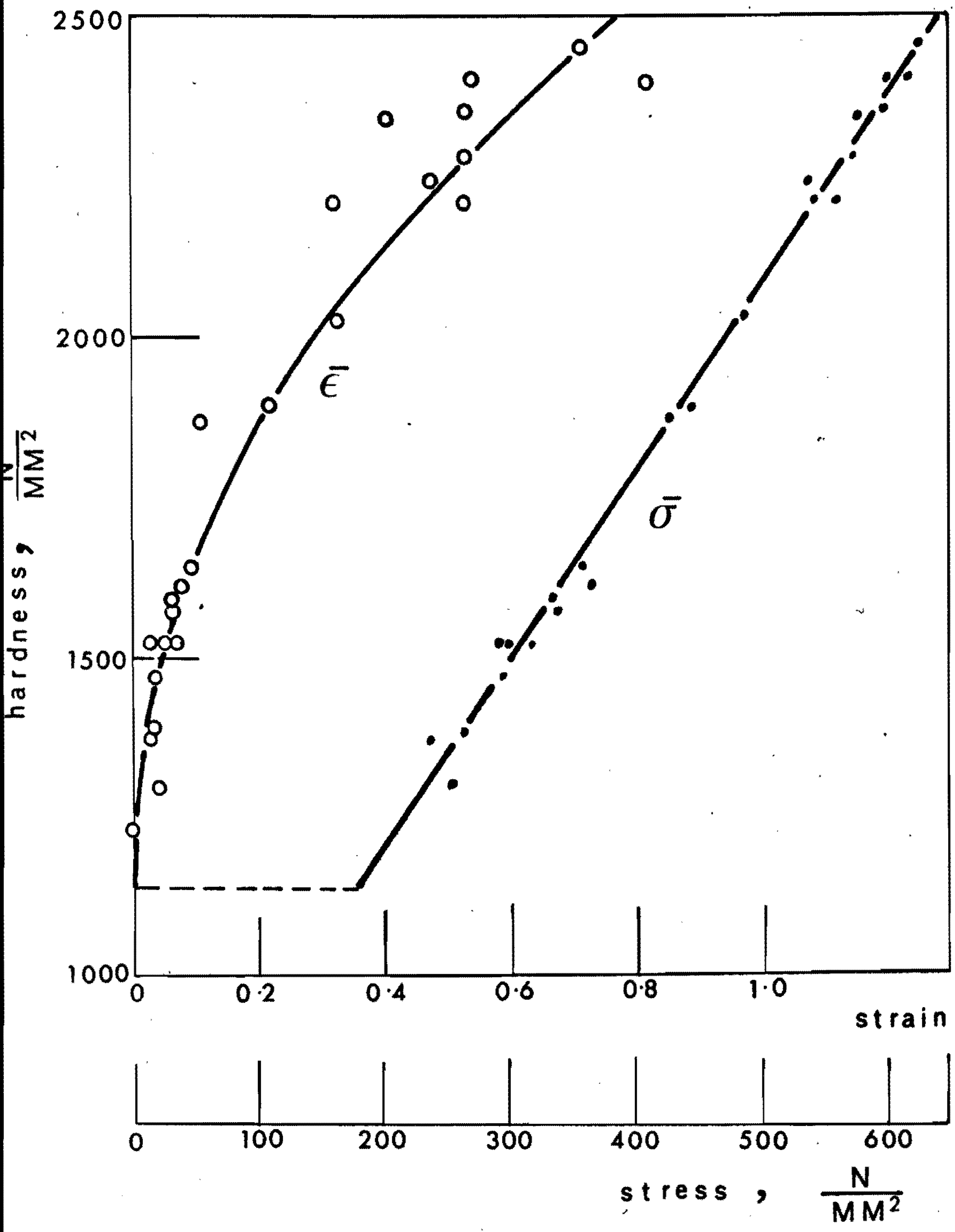

Fig. 2. Micro Vickers hardness as a function both of effective deformation (eq. 5) and effective stress (eq.3) (table 2).

to a state of large deformation {6}.

3. *Ludwik's equation*

According to literature Ludwik proposed {6}

$$\bar{\sigma} = \sigma_0 + K\{\bar{\varepsilon}\}^{1} \qquad (4)$$

Combining eqs. 3 and 4 as shown in fig. 2 it follows

$$HV = H_0 + H_1\{\bar{\varepsilon}\}^{1} \qquad (5)$$

Application of regression analysis to the experimentel data of tensile tests renders the constants of eq. 4 as listed in table 2.
The fact that the yield stress σ_0 proves to be negative creates a doubt of physical correctness.
Combining however the results of tensile tests on the one hand with hardness measurements on the other, as shown in fig. 2 it is concluded that holds

$$\sigma_0 = 178\ N/mm^2$$

Introducing the latter value in the regression analysis the constants of eqs. 4 and 5 have been re-determined as shown in table 2.

Table 2.

material	eq.	σ_0 [N/mm²]	K [N/mm²]	1 [-]	load [gf]	H_0 [N/mm²]	H_1 [N/mm²]	C_1 [N/mm²]	C_2 [-]
low	4	-820	1480	0.09	100	-	-	-	-
carbon steel	3,4,5	178	530	0.465	100	1135	1550	610	2.93

Fig. 6 however shows that hardness measurements in punching nevertheless indicate that Ludwik's equation does not hold in a state of large deformation.

4. *Nadai's equation*

Starting from Nadai's equation

$$\bar{\sigma} = C\,\{\bar{\varepsilon}\}^{n} \qquad (6)$$

it is assumed that analogous to Palm's equation the dependence of hardness on effective deformation is governed by

$$HV = H\,(\bar{\varepsilon} + \varepsilon_H)^{n} \qquad (7)$$

Here $\bar{\varepsilon}$ stands for the effective deformation generated by the forming process while ε_H represents the average effective deformation locally added by the indentation connected with the hardness measurement.

It is remarked that the latter value depends on the width of the indentation and on the local distribution of deformation as controlled by the workhardening properties of the material.

From fig. 3 it follows

$$\Delta\sigma_H = \bar{\sigma}_1 - \bar{\sigma} = C\,(\bar{\varepsilon} + \varepsilon_H)^{n} - C\,\{\bar{\varepsilon}\}^{n}$$

or

$$\frac{\Delta\sigma_H + \bar{\sigma}}{C} = (\bar{\varepsilon} + \varepsilon_H)^{n} \qquad (8)$$

Substitution in eq. 7 yields

$$HV = \frac{H}{C}\,(\bar{\sigma} + \Delta\sigma_H) \qquad (9)$$

From the experimental data as shown in fig. 4 it is derived

$$\Delta\sigma_H = \frac{a^2}{\bar{\sigma} + a}$$

It follows the quantity (a) being a constant.

$$HV = \frac{H}{C}\left(\bar{\sigma} + \frac{a^2}{\bar{\sigma} + a}\right) \qquad (10)$$

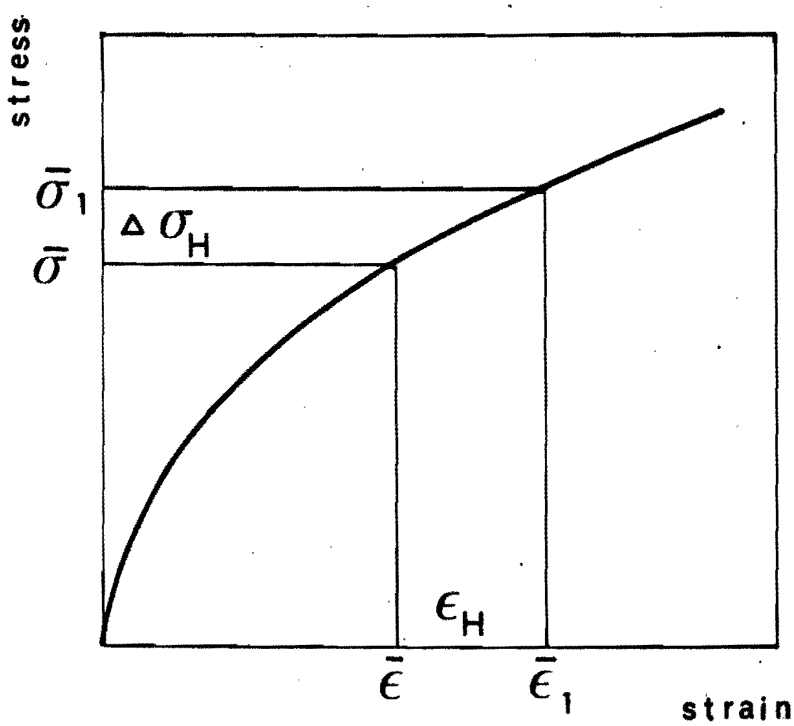

Fig. 3. Definition of the dependence of $\Delta\bar{\sigma}_H$ on ε_H (eq.8).

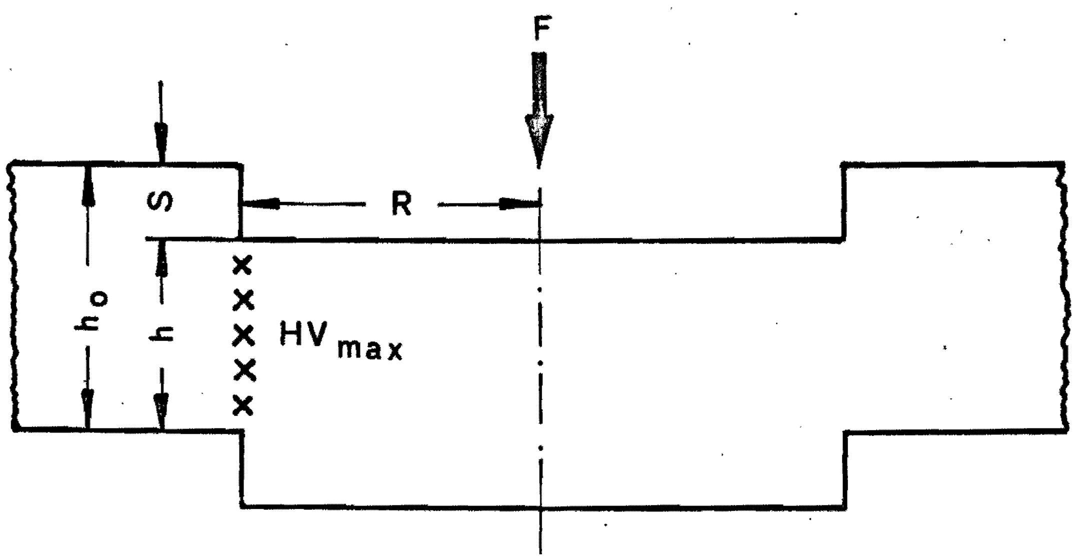

Fig. 5. Principle of hardness measurement in punching.
Definition of the geometrical quantity h_o/h.

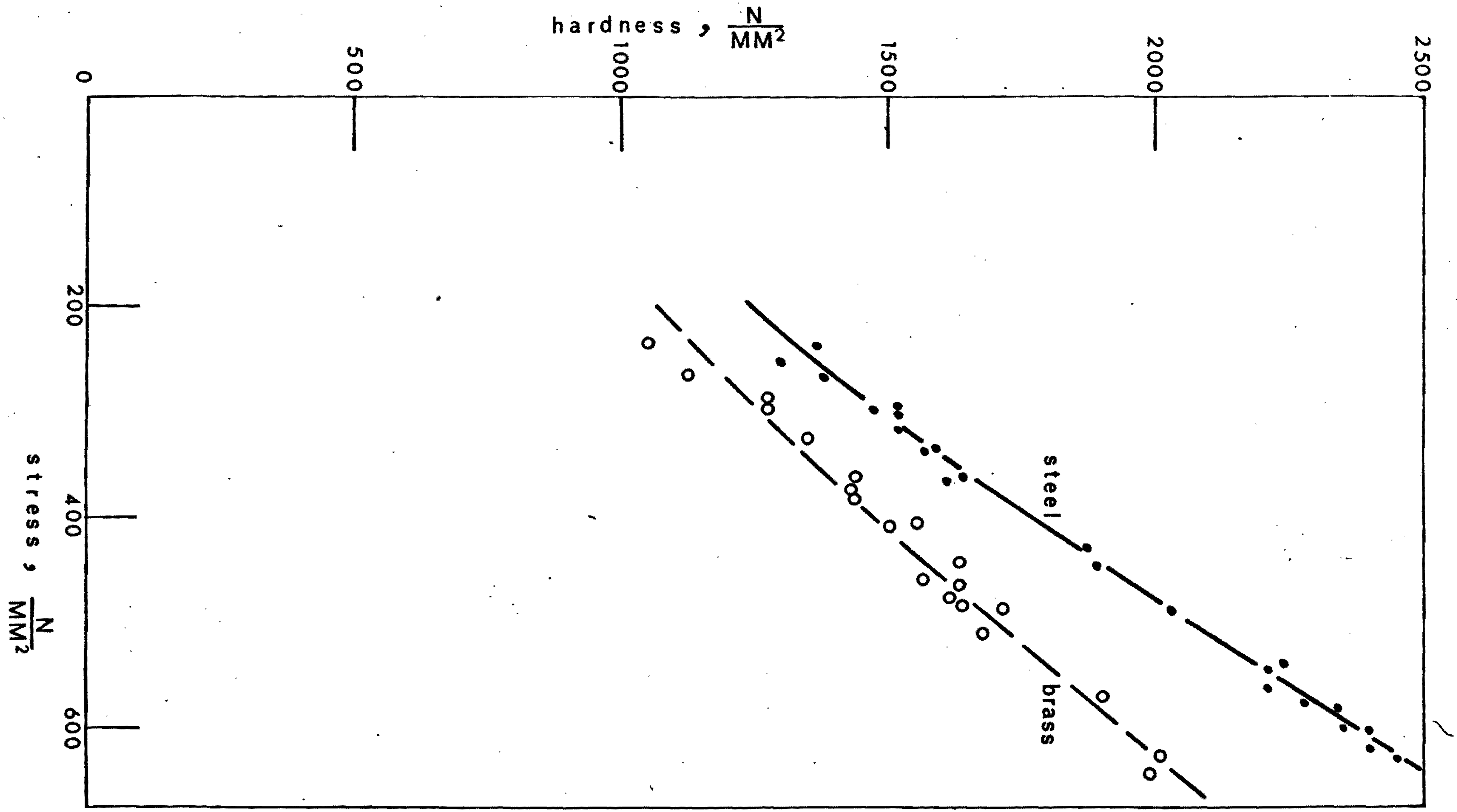

Fig. 4. Hardness as a function of effective stress for low carbon steel and brass-KMS 63 H. (eq. 10, table 3).

It is observed that the presence of initial deformation $\bar{\varepsilon}_o$ of the material changes eq. 6 into

$$\bar{\sigma} = C\,(\bar{\varepsilon} + \bar{\varepsilon}_o)^n \qquad (11)$$

not affecting the result formulated in eq. 10.
Table 3 shows the results of regression analysis applied to the eqs. 6, 10 and 11.

Table 3.

material	C [N/mm^2]	n [-]	$\bar{\varepsilon}_o$ [-]	load [gf]	$H/_C$ [-]	a [N/mm^2]
low carbon steel	680	0.264	0	100	3.47	270
KMS 63 H	710	0.495	0.11	50	2.75	310

It is remarked that the quantity $\frac{H}{C}$ in eq. 10 has the same meaning as $\frac{\bar{HV}_\infty}{\sigma_\infty}$ in eqs. 1 and 2 {4}. The result obtained proving that the relation between hardness and effective stress is non-linear as expressed by eq. 10 is supported by experimental data shown in literature {7}.

5. *Verification in punching*

Bij executing hardness measurements in specimen obtained by punching according to the principle shown in fig. 5 it can be checked wether eqs. 3,4,5 and eqs. 6,10 will hold in the case of large plastic flow in a state of combined stress.
The maximum value of hardness in the shearing zone has been measured in a process of incremental punching.
Making use of eqs. 3 and 10 the effective stress $\bar{\sigma}_{max}$ is calculated and is plotted against the geometrical quantity h_o/h as a measure of deformation. The results are shown in fig. 6 using Ludwik's

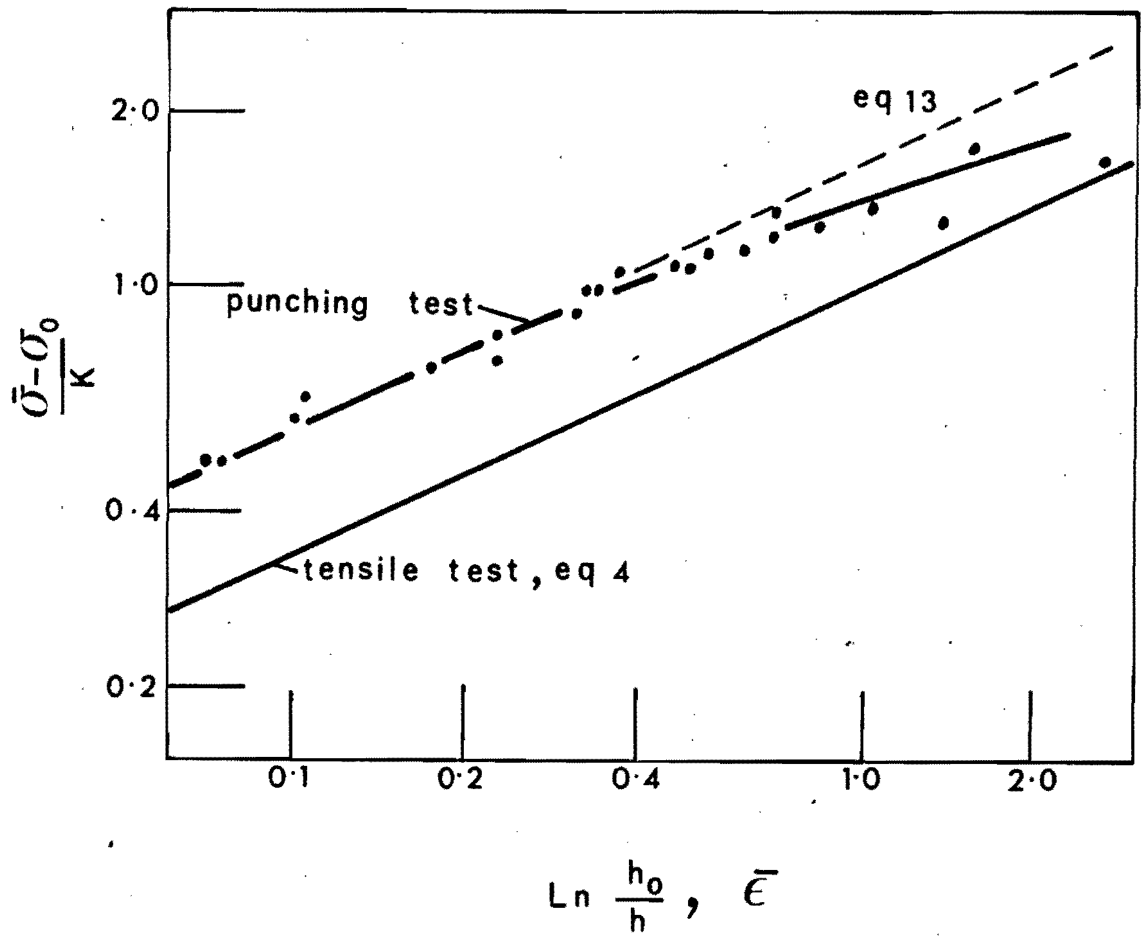

Fig. 6. Result of applying Ludwik's equation (eq. 13) to punching.

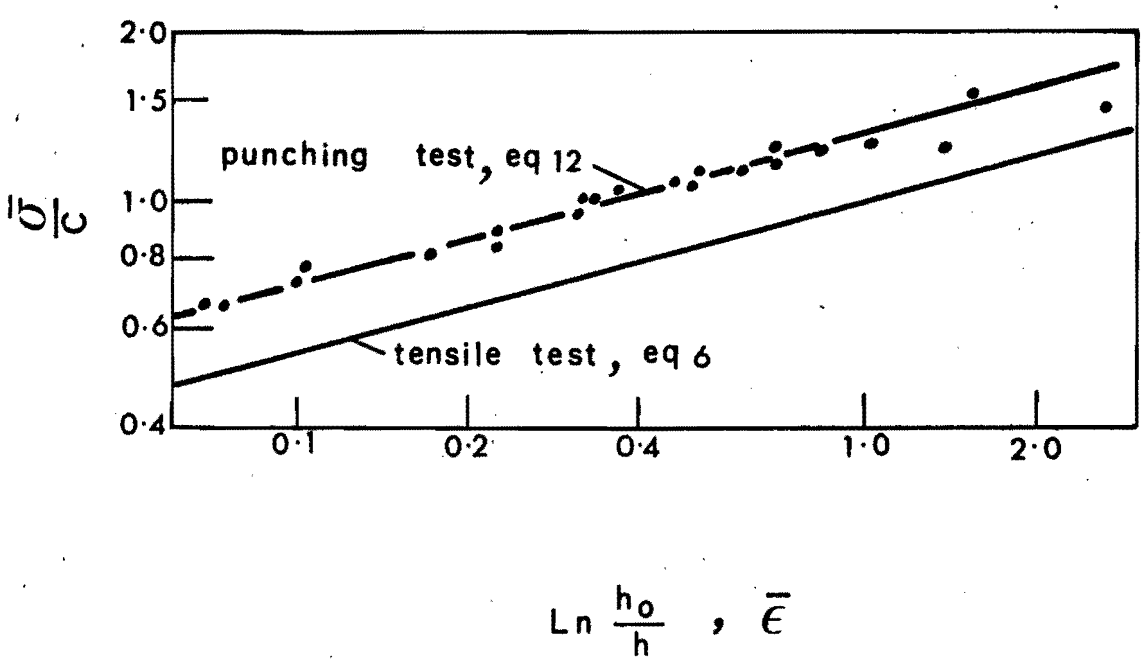

Fig. 7. Result of applying Nadai's equation (eq. 12) to punching.

equation and in fig. 7 when applying Nadai's equation.
The latter curve can analytically be represented by

$$\frac{\bar{\sigma}_{max}}{C} = \{k \ln \frac{h_o}{h}\}^{n} \qquad (12)$$

covering the entire range of deformation investigated while the former

$$\frac{\bar{\sigma} - \sigma_o}{K} = \{ k' \ln \frac{h_o}{h} \}^{1} \qquad (13)$$

deviates considerably from the measuring points in the region of large strains.

When assuming that the mechanics of punching is based on pure shearing in a state of complete plasticity the instantaneous punching force is given by

$$F = \frac{2\pi}{\sqrt{3}} Rh\, C \{ k \ln \frac{h_o}{h} \}^{n} \qquad (14)$$

and hence the maximum force by

$$F_{max} = \frac{2\pi}{\sqrt{3}} R\, h_o\, C \{ \frac{n}{e} \}^{n} k^{n} \qquad (15)$$

when neglecting the influence of friction.
In table 4 comparitive experimental data {8} have been listed.

Table 4.

material	C [N/mm²]	n [-]	k [-]	$F_{eq.15}$ [N.10⁴]	$F_{measured}$ [N.10⁴]	F [%]
Al 99.3	148	0.264	2.92	3.85	4.06	5.4
C -10	695	0.106	2.92	20.0	21.6	7.4
Ma -8	683	0.218	2.92	18.1	18.8	3.7
Low carbon steel	680	0.264	2.92	3.54	3.94	10.0

From the theory of plastic instability it is easily shown that the tensile strength of a material can be expressed in terms of

$$\sigma_{B_o} = C\left\{\frac{n}{e}\right\}^{n} \tag{16}$$

or

$$\sigma_B = C\, e^{\bar{\varepsilon}_o} \left\{\frac{n}{e}\right\}^{n} \tag{17}$$

in the case that initial deformation $\bar{\varepsilon}_o$ is present.
Substitution in eq. 15 yields

$$F_{max} = 2\pi\, R\, h_o\, \sigma_{B_o} \frac{k^n}{\sqrt{3}} \tag{18}$$

or

$$F_{max} = 2\pi\, R\, h_o\, S_f\, \sigma_{B_o} \tag{19}$$

Where

$$S_f = \frac{k^n}{\sqrt{3}} \tag{20}$$

represents the shearing factor in punching.
Obviously k is approximately a constant characteristic for the process describing the relation between the geometrical measure of deformation h_o/h and some average of effective deformation physically being present in the entire shearing zone.

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