UBET

Research on friction influenced elements

Appendix B : Analysis

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July 90

VF code D1/D2/D3 WPA nr 865

Contents

Enclosure 1 **Literature study of UBET**

In this enclosure, the author wants to give an overview of the development of UBET, with the aid of the papers published about UBET.
A number between brackets refers to the references.

In 1974 McDermott and Bramley [1], [2] from the University of Leeds presented for the first time their updated version of the concept of Kudo [32]. In the new approach eight elemental rings, each capable of being linked to all others, were considered and included not only rectangular but also triangular and circular cross–sectioned elements, which therefore enabled forging with significant draft and radii to be analysed.
Two modes of deformation were considered, inward flow and outward flow.
For each of these eight elements a general admissible velocity field was obtained. (kinematically compatible both within itself and with the external applied forces)
For calculating the rate of internal energy dissipation, Hill [44] was used :

$$P = \sigma \cdot \int_V \cdot \dot{\bar{\epsilon}} \cdot dV + m \cdot \sigma \cdot \int_S |v| dS, \; m = 1 \text{ in case of an interregion}$$

boundary, else m is the friction coefficient.

This method considered the forging only at the end of the process when all the die cavity was filled, i.e. maximum load.
Furthermore, the importance of optimum flash design was stressed, and a start was made for optimum flash design.

In the years between 1974 and 1976 McDermot, Cramphorn and Bramley concluded that there were still some limitations, such as the technique only being applicable to axisymmetric forgings, only being usefull to analyse press formed parts, the program being

written in ALGOL, elements being too much simplified, limited industrial verification of the technique, subdivision into elements of a part being difficult and arduous.
In 1976 they present a paper [3], which stressed the above and included the solutions for the limitations. By introducing general rectangular and triangular elements, which allowed flow over all boundaries, problems, arisen by too simplified velocity fields, were overcome. Furthermore, velocity optimization was introduced, as well as computerized elemental subdivision.
An analysis for a generalized circular cross–sectioned element had not been developed, but in stead of this an arc now was approximated by straight lines, and the authors stressed that the error introduced by doing so would be outweighted by far by the accuracy of the optimizational facility.
Like the former two papers, this paper also included a comparison of experimental and theoretical data. A reasonable agreement was shown. A large amount of the experimental data was aquired from industry, without being sure about the correctness of the data. Furthermore a UBET program for plane strain was developed, and the idea of linking both programs is mentioned.

At the MTDR conference in 1977 Cramphorn and Bramley presented a paper [4] which was similar to [3], but it became possible to optimize power consumption towards more unknown velocities. The number of unknown velocities was given by the number of internal boundaries minus the number of elements.
In addition to this, for hot working processes, in which the flow stress is mainly dependent on the strain rate, a stress strain–rate relation was included in the program. For other processes it remained possible to enter a constant flow stress.

At the MTDR conference in 1979 Bramley and Osman presented a paper [5] in which the incremental solution was shown. The principle of the incremental solution is to keep the optimum velocity field constant during a unit of time increment. After this unit of time increment, the new component geometry is known, and this forms the start for a new increment. In this way it's possible to predict metal flow during a forming proces. Furthermore the advantage in analysis of the double acting concept was shown, which replaced the concept of single action.

In 1980, Bramley and Thornton presented two papers [6], [7] concerning the implementation of strain–hardening, and the effect on metal flow. Strain history was determined by using an average total strain in a region.

During the year 1982, Bramley and Osman presented two papers [8], [9] to conferences, considering the development of flowline prediction and grid distortion in UBET, both derived from the optimum velocity field.
A reasonable agreement between experimental and predicted values was shown.

In the years 1984 and 1985, Bramley, Osman and Ghobrial [11],[12],[13], and in 1987, Bramley and Osman [16], presented four papers, concerning preform design, for which a new part of UBET was developed namely the reverse method. In the reverse method, the simulation process starts from the final forging shape, with the velocities reversed, in such a way that during decrementation the dies move outward while the material inside the dies moves inward. With this method it was possible to design optimum preforms for a forging process, and by using this method material wastage could be reduced, because flash dimension could be optimised.

At the NAMRC conference Christensen, Bay, Osman and Bramley [14] presented their recent work on calculation of local surface stresses. In order to calculate the local surface stress, the velocity change approach was used. This technique consists of introducing a velocity change on an imaginary elelement. By taking the average of a positive and a negative velocity change, extra shear losses drop out of the equations. Local pressure can now be detected by analysis of the difference in the power consumption.

In 1987, Lugora, Bramley and Osman [15] presented a paper to a conference, in which 3 dimensional block–type elements were introduced in order to describe asymmetric forging processes. The 3 dimensional analysis requires, unlike the axisymmetric UBET program, manual subdivision of the component. Furthermore it was noted that elements to describe the link between axisymetric and block–type elements had to be developed. Research of asymmetric components was carried out by separating axisymmetric and block–type elements, and analysing them apart from eachother.

Enclosure 2 Description of the optimizers

E2.1. The simplex optimizer

A regular simplex in n dimensions is n+1 mutually equidistant points so that for n = 2 it forms an equilateral triangle, for n = 3 it forms a regular tetrahedron and so on. A very useful property of the simplex method is that a new simplex can be formed out of the previous one by the addition of only a single new point.

The method begins by setting up a regular simplex in the space of the independent variables, and evaluation of the function in each vertex. In the following iteration steps, each time the vertex, which results in the worst function evalution, is replaced by it's image in the centroid of the remaining vertices. See figure 1 for n = 2.

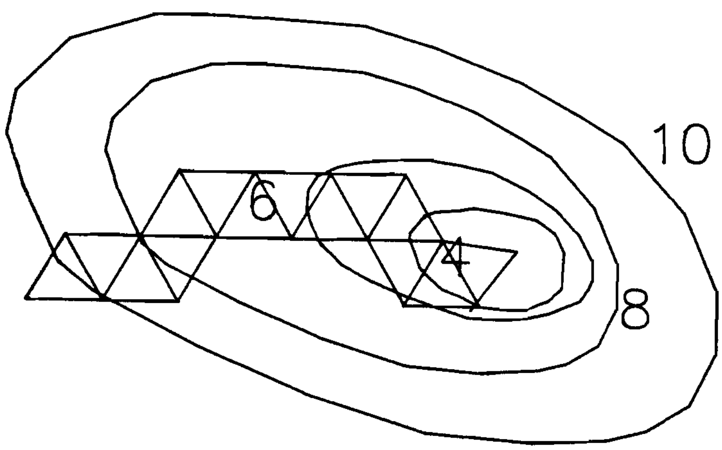


figure 1: regular simplex for 2 dimensions

So for each iteration step it only takes one function evaluation.

Note : If in iteration step k the worst vertex is denoted by x_1, then in iteration step k+1 x_1 will be replaced by x_2. If x_2 is the worst vertex of iteration step k+1, then the search for the optimum will start to oscilate between x_1 and x_2, if no prevention for such a case is taken. If one of the vertices is situated in the neighbourhood of the minimum, it will remain a vertex of the simplex and the search will rotate around it.

Spendley deduced that the maximum expected age of any vertex could be approximately represented by :

$$\alpha = 1.65 \cdot n + 0.05 \cdot n^2$$

so if a vertex exceeds this age it is reasonable to conclude that the vertex is in the neighbourhood of the minimum. At that moment the distance between the vertices is reduced and the search goes on with a smaller design.

Furthermore, it is possible to improve the effectivity of the Simplex method by using the version proposed by Nelder and Mead (1965). In this proposal the regularity of the simplex is abandonned and the simplex rescales itself according to the local geometry of the function. Supposing that for iteration i the vertices of the simplex are $v_0, v_1, \dots, v_n$ and the corresponding function values $F_0, F_1, \dots, F_n$ are ordered in such a way that :

$$F_n > F_{n-1} > \dots\dots > F_1 > F_0.$$

then v_0 is the best vertex and v_n the worst. Let m be the centroid of the rest of the vertices :

$$m_k = \frac{1}{n} \cdot \sum_{j=0}^{n-1} v_{jk} \qquad k = 1, \dots n.$$

Then, as in the original simplex method, the worst vertex v_n is to be replaced and a simple reflection is tried first :

$$v_{new} = m + \alpha \cdot (m - v_n)$$

α reflection coefficient.

Now there are three possible cases to be considered :

v_{new} is such a point that $F_0 < F_{new} < F_{n-1}$; $F_{new} < F_0$; $F_{new} > F_{n-1}$.

In the case of $F_0 < F_{new} < F_{n-1}$, v_{new} replaces v_n and the iteration is complete.

See figure 2 for details in the case of n = 2.

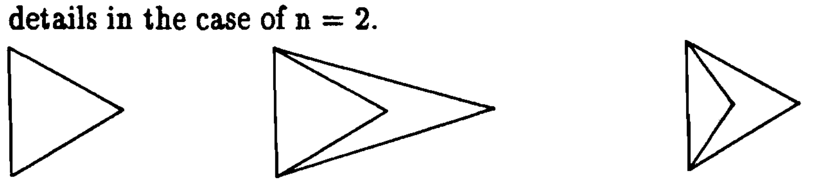


figure 2: scaling of simplex

In case $F_{new} < F_0$, then the iteration has produced a new best point and it may be useful to expand the design by defining a new point :

$$v_{exp} = m + \beta\cdot(v_{new} - m) \ , \beta > 1 \text{ expansion coefficient.}$$

Then, if $F_{exp} < F_0$, the expansion is considered to be succesfull and v_{exp} replaces v_n. Otherwise the expansion has failed and v_n is replaced by v_{new}.

In case $F_{new} > F_{n-1}$ it is probable that the size of the design is too large to make any progress, and it is tried to make further progress by contraction.

A new point is defined by :

$$v_{con} = m + \gamma\cdot(v_n - m) \text{ if } F_n < F_{new}, \text{ and}$$

$$v_{con} = m + \gamma\cdot(v_{new} - m) \text{ if } F_n > F_{new}.$$

$0 < \gamma < 1$ is the contraction coefficient.

If $F_{con} < \min (F_n, F_{new})$ then the contraction has succeeded and v_{con} replaces v_n.

The convergence criterion is based on the variation in the function values over the simplex and the search is ended when the standard deviation falls below a preassigned limit.

$$\sigma^2 = \sum_{i=0}^{n} \frac{(F_i - \overline{F})^2}{n} \quad , \ \overline{F} \text{ mean of function values.}$$

E2.2. The Davidon–Fletcher–Powell optimizer

The Davidon–Fletcher–Powell optimizer (DFP for short) is a quasi Newton method for optimizing, and in particular a member of the Broyden family of quasi Newton optimizers. The Newton method model is obtained from a truncated Taylor series expansion of F(v) about v^k in the k–th iteration, which can be written as :

$$F(x^k+\delta) \approx q^k(\delta) = F^k + g^k\cdot\delta + 0.5\cdot\delta^T\cdot G^k\cdot\delta.$$

where : $\delta = x - x^k$, and $q^k(\delta)$ is the resulting quadratic approximation for iteration k. The next iterate x^{k+1} is taken to be $x^k+\delta^k$, where the correction δ^k minimizes $q^k(\delta)$.

Besides the function this method also requires the first and second derivative of F to be available at any point, in order to calculate $q^k(\delta)$, which is defined by the coefficients F^k, g^k and G^k. The method is only well defined in the cases of G^k being positive definite, because only then $q^k(\delta)$ has a unique minimizer. In this case δ^k is defined by the condition that $\nabla q^k(\delta^k)=0$.

An iteration step of Newton's method looks like :

solve $G^k \cdot \delta = -g^k$,

set $x^{k+1} = x^k + \delta^k$.

A disadvantage of Newton's method is the fact that G^k may not be positive definite when x^k is far from the solution. Even if G^k is positive definite convergence may not occur, in fact F^k may not even decrease.

This latter possibility can be avoided by applying Newton's method with line search, in which the Newton correction is used to generate a search direction :

$s^k = -(G^k)^{-1} \cdot g^k$.

This is then used to find α^k by minimizing $f(x^k + \alpha \cdot s^k)$ with respect to α. Even when G^k is not positive definite, it is problably still possible to calculate s^k, and search in $\pm\, s^k$, but according to literature the relevance of searching in such a direction is questionable.

A major disadvantage of Newton's method is the fact that the method requires first and second derivatives. (for instance, in UBET these are not available).

This disadvantage is avoided in the quasi Newton methods. This type of method is like Newton's method with line search except that $(G^k)^{-1}$ is approximated by a symmetric positive definite matrix H^k, which is corrected or updated in each iteration step.

An iteration step of a quasi Newton method looks like :

set $s^k = -\, H^k \cdot g^k$,

line search along s^k giving $x^{k+1} = x^k + \alpha^k \cdot s^k$,

update H^k giving H^{k+1}.

The most important operation in this is the updating of H. The DFP method uses the next updating formula :

$$H^{k+1} = H + \frac{\delta \cdot \delta^T}{\delta^T \cdot \gamma} - \frac{H \cdot \gamma \cdot \gamma^T \cdot H}{\gamma^T \cdot H \cdot \gamma}$$

in which $\delta^k = \alpha^k \cdot s^k = x^{k+1} - x^k,$

$\gamma^k = g^{k+1} - g^k.$

Note : In the updating formula the superscript k is dropped, at the right hand side of the equation.

E2.3. Determination of the first derivative

The method is still using the first derivative. In the implementation of Song Bin the first derivative is calculated numerically. Because the implementation was originaly meant for a computer with a Distributed Array Processor (DAP for short), the formula :

$$F'(x) = \frac{F(x-h) - F(x)}{h},$$

was used with a very small interval h, which had the advantage of using a little amount of CPU time, but the disadvantage of demanding a very long datalenght, namely 48 bit at the least. By transferring to a computer with a smaller datalenght, and using a very small increment for h, the calculation of the value of the first derivative was no longer reliable. In order to improve the calculation of the value of the first derivative, the formula :

$$F'(x) = \frac{F(x+h) - F(x-h)}{2 \cdot h}, \text{ (central differentation)}$$

was used and increased the value of h. Unfortunately, by doing so, it is not possible to achieve more than 3 accurate figures for the first derivative, and the number of function calculations increases with a factor 2.

E2.4. Extrapolation according to Richardson

The next step is applying Richardson's rule to the calculation of the derivatives:

suppose $F'(x) = \beta$,

Numerical calculation will perform :

$$F'(x) = \frac{F(x+h) - F(x-h)}{2 \cdot h} = \beta + \epsilon(h),$$

suppose

$$h_n = (\tfrac{1}{2})^n, \text{ so :}$$

$$\beta_0 = \beta + \epsilon_0, \quad \beta_n = \beta + \epsilon_n.$$

By using the central differentiation it is possible to obtain a better estimation for the first derivative, because, by decreasing h with a factor two, the error in the computed result will decrease with a factor 4, so :

$$\beta_n - \beta_{n-1} = \beta + \epsilon_n - (\beta + \epsilon_{n-1}),$$

$$\epsilon_n - \epsilon_{n-1} = \beta_n - \beta_{n-1},$$

$$\epsilon_n - 4 \cdot \epsilon = \beta_n - \beta_{n-1},$$

$$\epsilon_n = \tfrac{1}{3} \cdot (\beta_{n-1} - \beta_n).$$

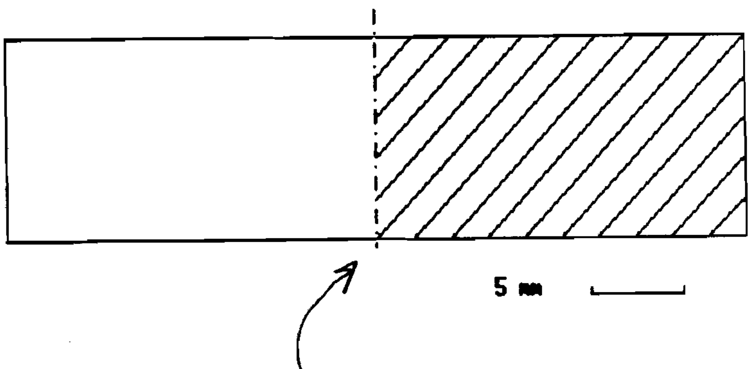
So a better estimation for the first derivative is obtained by :

$$\beta_n^1 = \beta_n - \epsilon_n,$$

$$\beta_n^1 = \beta_n + \tfrac{1}{3} \cdot (\beta_n - \beta_{n-1}).$$

This is h^2 extrapolation. Furthermore, it is possible to build a Romberg schedule for h^4, h^6, h^8 extrapolation and so on, in order to obtain even better results for the value of the first derivative. This is based on calculating a better estimation for β by extrapolating the results of previous calculations.

E2.5. Plots of products



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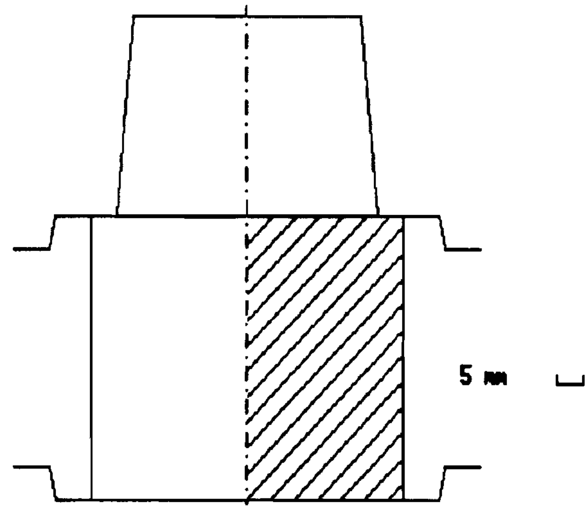
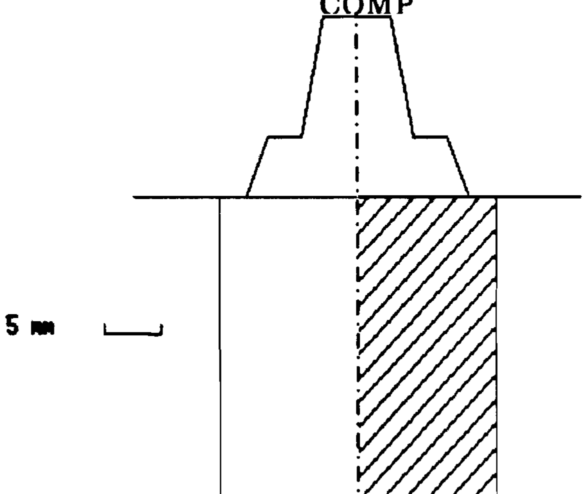


FCAP



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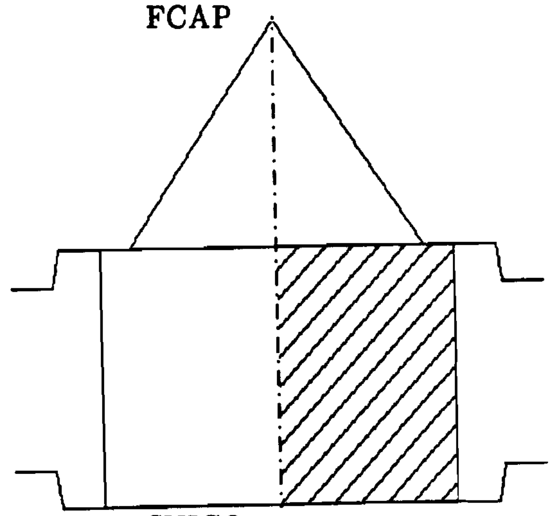
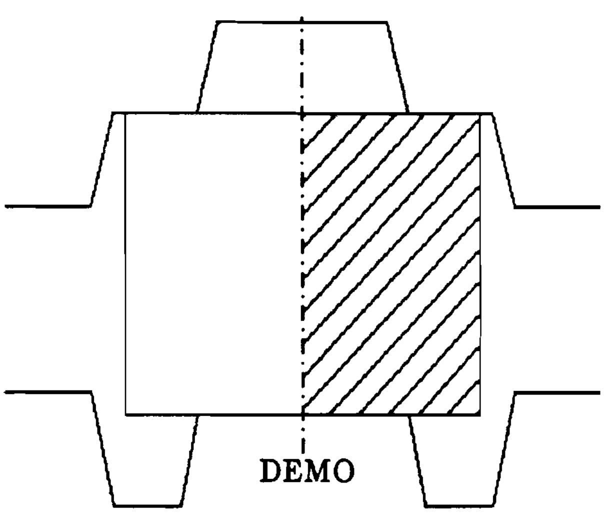


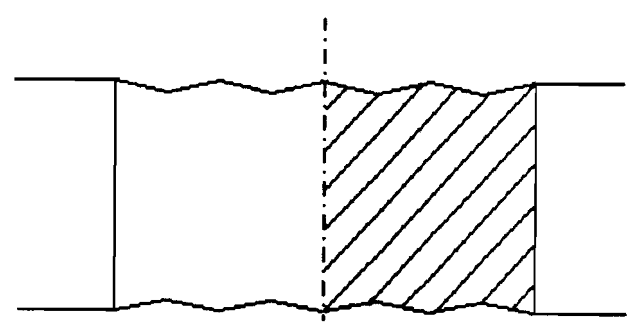
SNICO1

5 mm



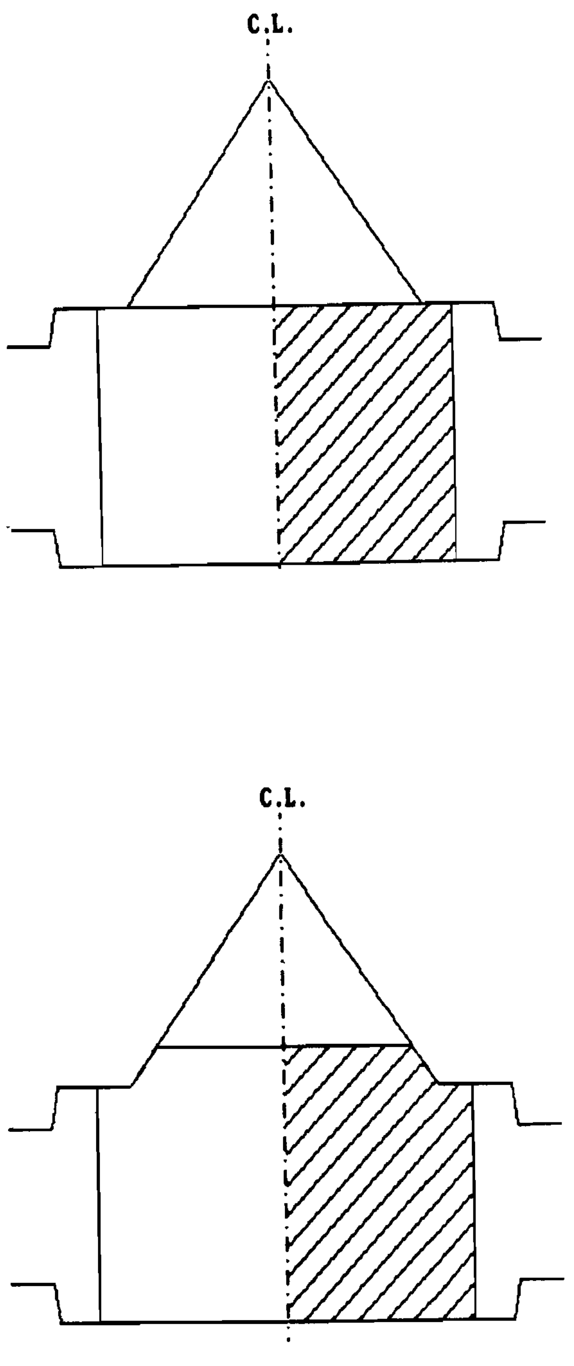
DEMO

10 mm



FGRID 5 mm

E2.6. Differences in metal flow



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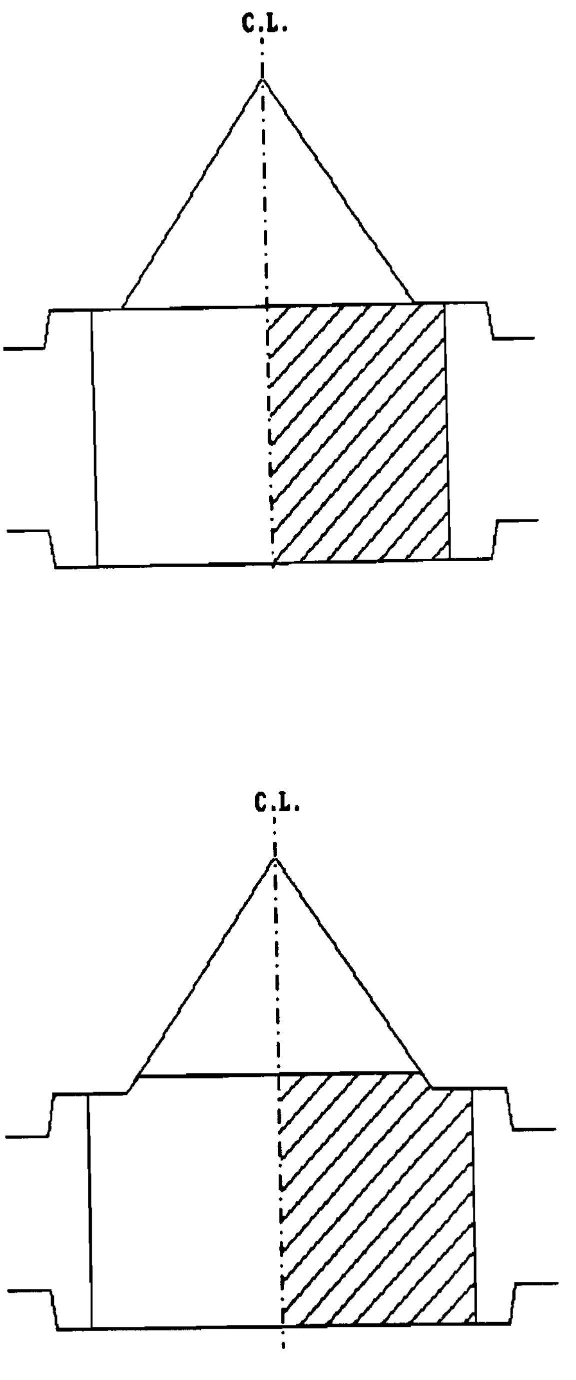

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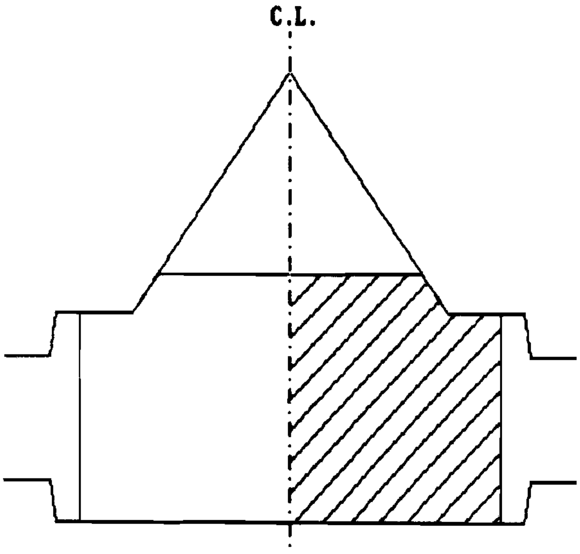

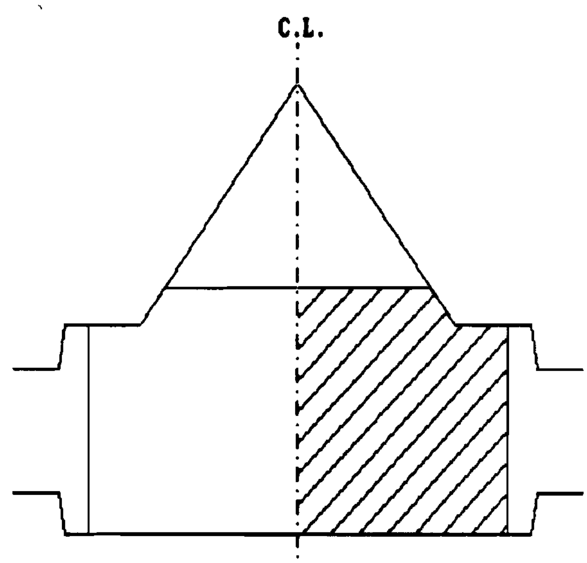

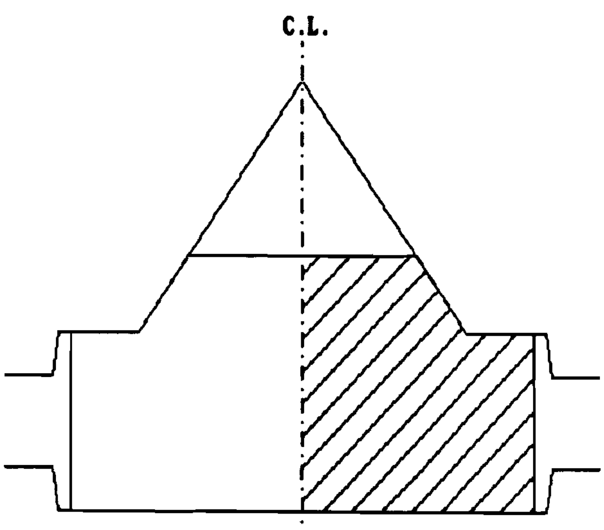

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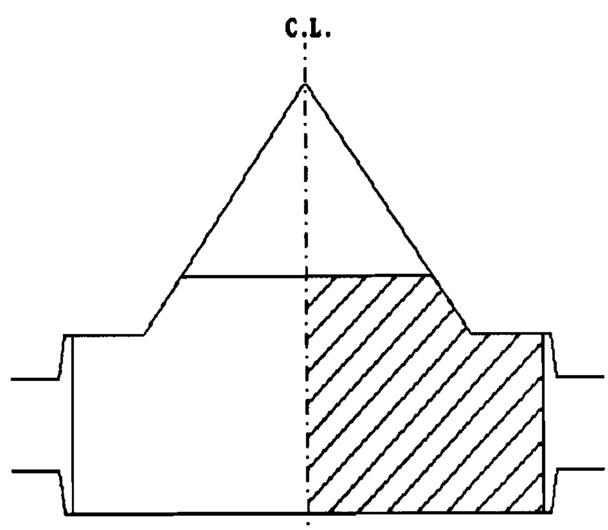

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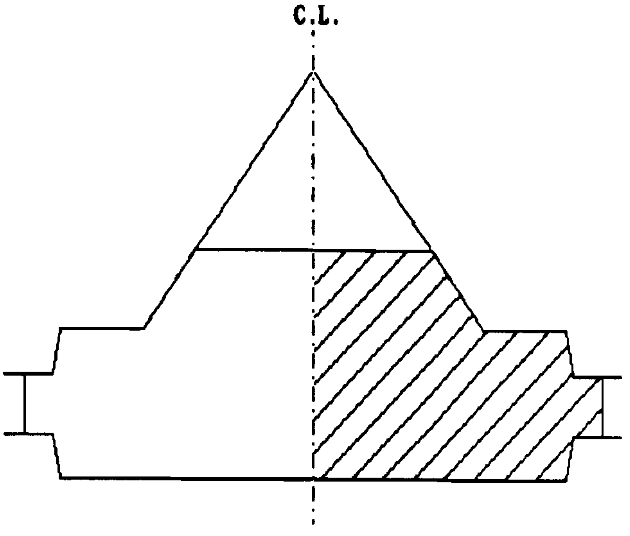

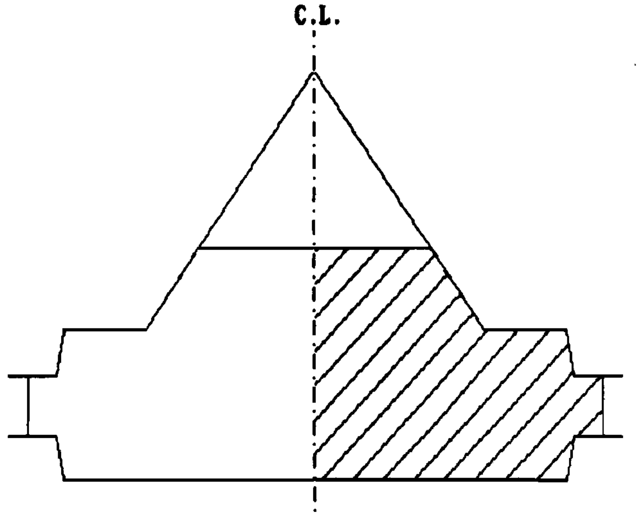

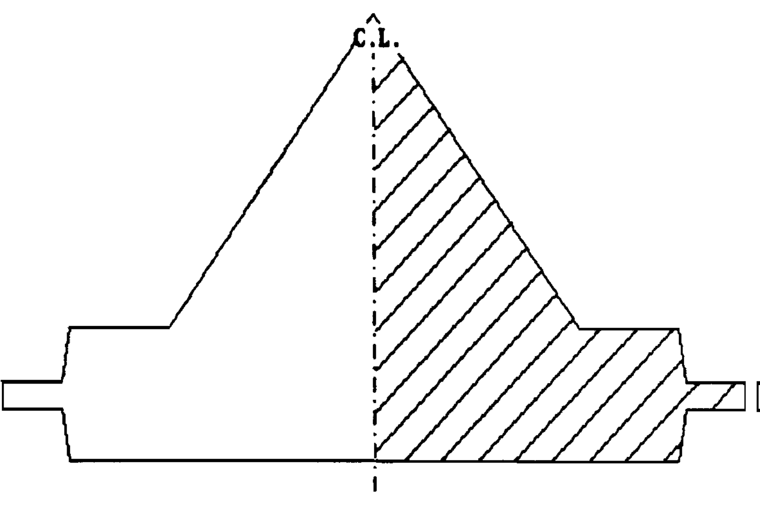

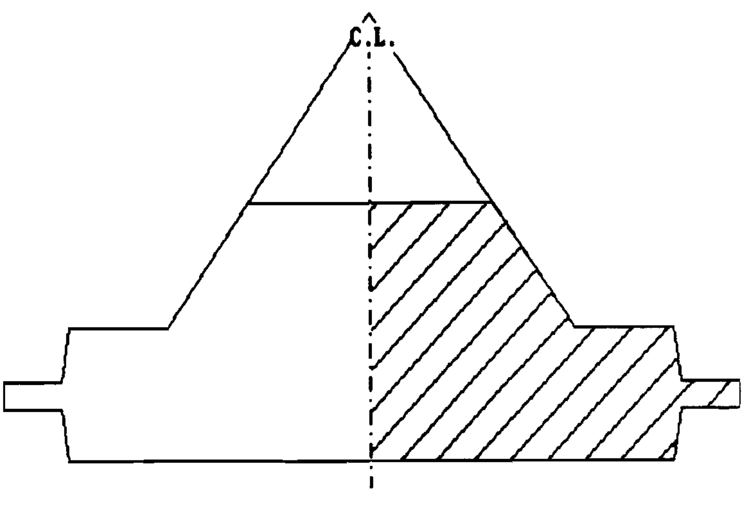

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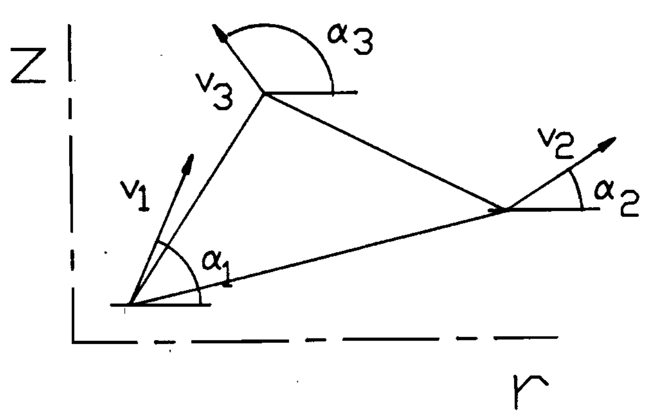
Enclosure 3 Nodal point approach

E3.1. Introduction

As outlined in chapter 3, one of the ways of modelling a forging, with the achievement of bulging, is the use of triangular elements. Because of the problems appearing during the conventional analysis of triangular elements, now an analysis is introduced which makes use of linear interpolation of both magnitude and direction of the velocities between the nodal points.



$$v_{1r} = v_1 \cdot \cos(\alpha_1)$$
$$v_{1z} = v_1 \cdot \sin(\alpha_1)$$
$$v_{2r} = v_2 \cdot \cos(\alpha_2)$$
$$v_{2z} = v_2 \cdot \sin(\alpha_2)$$
$$v_{3r} = v_3 \cdot \cos(\alpha_3)$$
$$v_{3z} = v_3 \cdot \sin(\alpha_3)$$

figure 1: definition of variables

With the use of interpolation between the nodal points, it is possible to determine the velocity inside the element at any position, while also the boundary conditions, at the nodal points and the inter–elemental boundaries, fully agree.

Because of the use of linear interpolation, triangular elements are used, due to the fact that a straight plane is described when 3 points are known.

E3.2. Theoretical analysis

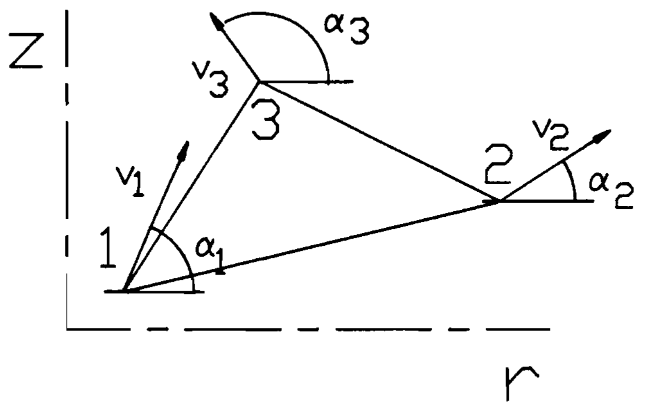


figure 2: triangular element, nodal point approach

E3.2.1. Derivation of the equation for the velocity plane

side $\underline{12} = (r_1 - r_2)\cdot i + (z_1 - z_2)\cdot j + (v_1 - v_2)\cdot k.$

side $\underline{23} = (r_3 - r_2)\cdot i + (z_3 - z_2)\cdot j + (v_3 - v_2)\cdot k.$

The next step is the calculation of the normal to the plane.

$$\vec{n} = \underline{23} * \underline{12}$$

$$\vec{n} = \det \begin{vmatrix} i & j & k \\ r_3 - r_2 & z_3 - z_2 & v_3 - v_2 \\ r_1 - r_2 & z_1 - z_2 & v_1 - v_2 \end{vmatrix}$$

$$\begin{aligned} \vec{n} = & \left[(z_3 - z_2)\cdot(v_1 - v_2) - (z_1 - z_2)\cdot(v_3 - v_2) \right]\cdot(r - r_2) \\ - & \left[(r_3 - r_2)\cdot(v_1 - v_2) - (r_1 - r_2)\cdot(v_3 - v_2) \right]\cdot(z - z_2) \\ + & \left[(r_3 - r_2)\cdot(z_1 - z_2) - (r_1 - r_2)\cdot(z_3 - z_2) \right]\cdot(v - v_2) \end{aligned}$$

Suitably rearranged in the form :

$$v = c_1 \cdot r + c_2 \cdot z + c_3$$

$$c_1 = -\frac{(z_3-z_2)\cdot(v_1-v_2) - (z_1-z_2)\cdot(v_3-v_2)}{(r_3-r_2)\cdot(z_1-z_2) - (r_1-r_2)\cdot(z_3-z_2)}$$

$$c_2 = \frac{(r_3-r_2)\cdot(v_1-v_2) - (r_1-r_2)\cdot(v_3-v_2)}{(r_3-r_2)\cdot(z_1-z_2) - (r_1-r_2)\cdot(z_3-z_2)}$$

$$c_3 = v_2 - c_2 \cdot z_2 - c_1 \cdot r_2$$

E3.2.2. <u>Derivation of the equation for the angle plane</u>

side $\underline{12} = (r_1-r_2)\cdot i + (z_1-z_2)\cdot j + (\alpha_1-\alpha_2)\cdot k.$

side $\underline{23} = (r_3-r_2)\cdot i + (z_3-z_2)\cdot j + (\alpha_3-\alpha_2)\cdot k.$

The next step is the calculation of the normal to the plane.

$$\vec{n} = \underline{23} * \underline{12}$$

$$\vec{n} = \det \begin{vmatrix} i & j & k \\ r_3-r_2 & z_3-z_2 & \alpha_3-\alpha_2 \\ r_1-r_2 & z_1-z_2 & \alpha_1-\alpha_2 \end{vmatrix}$$

$$\begin{aligned} \vec{n} = & \left[(z_3-z_2)\cdot(\alpha_1-\alpha_2) - (z_1-z_2)\cdot(\alpha_3-\alpha_2)\right]\cdot(r-r_2) \\ - & \left[(r_3-r_2)\cdot(\alpha_1-\alpha_2) - (r_1-r_2)\cdot(\alpha_3-\alpha_2)\right]\cdot(z-z_2) \\ + & \left[(r_3-r_2)\cdot(z_1-z_2) - (r_1-r_2)\cdot(z_3-z_2)\right]\cdot(\alpha-\alpha_2) \end{aligned}$$

Suitably rearranged in the form :

$$\alpha = c_4 \cdot r + c_5 \cdot z + c_6$$

$$c_4 = -\frac{(z_3 - z_2)\cdot(\alpha_1 - \alpha_2) - (z_1 - z_2)\cdot(\alpha_3 - \alpha_2)}{(r_3 - r_2)\cdot(z_1 - z_2) - (r_1 - r_2)\cdot(z_3 - z_2)}$$

$$c_5 = \frac{(r_3 - r_2)\cdot(\alpha_1 - \alpha_2) - (r_1 - r_2)\cdot(\alpha_3 - \alpha_2)}{(r_3 - r_2)\cdot(z_1 - z_2) - (r_1 - r_2)\cdot(z_3 - z_2)}$$

$$c_6 = \alpha_2 - c_5 \cdot z_2 - c_4 \cdot r_2$$

E3.2.3. Velocity field

The velocity field is now fully determined by :

$$v_r = v \cdot \cos(\alpha)$$

$$v_z = v \cdot \sin(\alpha)$$

$$v = c_1 \cdot r + c_2 \cdot z + c_3$$

$$\alpha = c_4 \cdot r + c_5 \cdot z + c_6$$

E3.2.4. Strain rates

When the velocity field is known, the strain rates can be obtained.

$$\dot{\epsilon}_r = \frac{\partial v_r}{\partial r} = \frac{\partial}{\partial r}\left[v\cdot\cos(\alpha) \right] = \frac{\partial v}{\partial r}\cdot\cos(\alpha) - \frac{\partial \alpha}{\partial r}\cdot v\cdot\sin(\alpha)$$

$$\dot{\epsilon}_r = c_1\cdot\cos(\alpha) - v\cdot c_4\cdot\sin(\alpha)$$

$$\dot{\epsilon}_z = \frac{\partial v_z}{\partial z} = \frac{\partial}{\partial z}\left[v\cdot\sin(\alpha) \right] = \frac{\partial v}{\partial z}\cdot\sin(\alpha) + \frac{\partial \alpha}{\partial z}\cdot v\cdot\cos(\alpha)$$

$$\dot{\epsilon}_z = c_2\cdot\sin(\alpha) + v\cdot c_5\cdot\cos(\alpha)$$

$$\dot{\epsilon}_\varphi = \frac{v_r}{r} = \frac{v}{r}\cdot\cos(\alpha)\;)$$

$$\dot{\gamma}_{rz} = \frac{1}{2}\cdot\left[\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z} \right]$$

$$\dot{\gamma}_{rz} = \frac{1}{2}\cdot\left[(c_2+v\cdot c_4)\cdot\cos(\alpha) + (c_1-v\cdot c_5)\cdot\sin(\alpha) \right]$$

E3.2.5. Effective strain

Furthermore, with the aid of the strain rates, the effective strain rate can be calculated according to the formula :

$$\dot{\bar{\epsilon}} = \sqrt{\frac{2}{3}\cdot\left[\dot{\epsilon}_r^2 + \dot{\epsilon}_z^2 + \dot{\epsilon}_\varphi^2\right] + \frac{4}{3}\left[\dot{\gamma}_{rz}^2\right]}$$

E3.2.6. Local volume invariance

$$\dot{\epsilon}_r + \dot{\epsilon}_z + \dot{\epsilon}_\varphi = 0$$

According to the presented analysis :

$$\dot{\epsilon}_r + \dot{\epsilon}_z + \dot{\epsilon}_\varphi = \left[c_1 + v\cdot c_5 + \frac{v}{r}\right]\cdot\cos(\alpha) + \left[c_1 + v\cdot c_4\right]\cdot\sin(\alpha)$$

As can be seen easily, when r equals 0, the right hand side of this equation, goes to infinity, and this implies that local volume invariance does not stick, and that the velocity field is not kinematically admissible.

E3.2.7. Global volume invariance

The volume of an axisymmetric closed ring with a triangular cross–section is described by :

$$V = \frac{\pi}{3}\cdot \sum^{i}\left[r_{i+1}\cdot z_i - r_i\cdot z_{i+1} \right]\cdot\left[r_i + r_{i+1} \right]$$

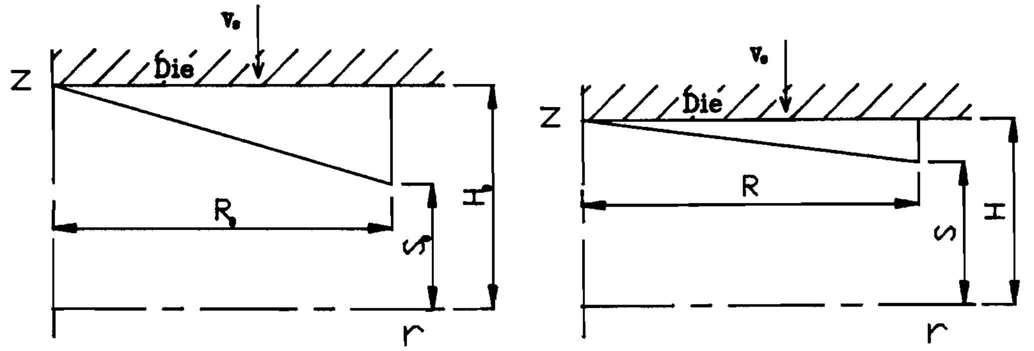
i cyclic, if i = 1,2,3 then i+1 = 2,3,1

If we model a forging with the use of triangular elements, we will always know the initial positions of the nodal points, and from those we can calculate the volume of the ring before deformation. After deformation the volume has to remain constant. By doing so, we assume that the boundaries of the element remain straight. Even if the boundaries of the element obtain a certain curvature during deformation, we can still keep the error small, by using small increments.

E3.3 Single elements

In order to gain more understanding about the validity of this analysis, the deformation of a single element is studied. In this, three different cases can be separated, namely : frictionless, sticking friction and intermediate friction conditions.

E3.3.1. Frictionless



undeformed state — deformed state

figure 3: description of geometry

In case of a frictionless forging like shown in figure 3, it is well known that the deformation pattern is uniform. This means for the analysis that $r_2 = r_3 = R$.

E3.3.1.1. Global volume invariance

$$V = \frac{\pi}{3}\cdot\Sigma^{i}\left[r_{i+1}\cdot z_i - r_i\cdot z_{i+1} \right]\cdot\left[r_i + r_{i+1} \right]$$

So in the undeformed state :

$$V_o = \frac{2}{3}\cdot\pi\cdot R_o^2\cdot(h_o - s_o)$$

And in the deformed state :

$$V_{def} = \frac{2}{3}\cdot\pi\cdot R^2\cdot(h-s)$$

When the above set of equations is rearranged, the result is :

$$R = R_o\cdot\sqrt{\frac{h_o-s_o}{h-s}}$$

And because only the distance h – s is important, we can use $h = h_o$, without changing the validity of this approach.

$$R = R_o\cdot\sqrt{\frac{h_o-s_o}{h_o-s}}$$

E3.3.1.2. <u>Velocity and angle planes</u>

$$v_1 = 0$$

$$v_2 = \frac{\partial R}{\partial t} = \frac{\partial R}{\partial s}\cdot\frac{\partial s}{\partial t}$$

$$v_{3r} = \frac{\partial R}{\partial t} = \frac{\partial R}{\partial s}\cdot\frac{\partial s}{\partial t} \qquad v_{3z} = \frac{\partial s}{\partial t}$$

$$v_3 = \sqrt{v_{3r}^2 + v_{3z}^2} = \frac{\partial s}{\partial t}\cdot\sqrt{1 + \left[\frac{\partial R}{\partial s}\right]^2}$$

$$\frac{\partial R}{\partial s} = \frac{R_o}{2\cdot(h_o-s)}\cdot\sqrt{\frac{h_o-s_o}{h_o-s}}$$

$$\frac{\partial s}{\partial t} = v_s \qquad s = v_s\cdot t + s_o$$

$$\alpha_1 = 0$$

$$\alpha_2 = 0$$

$$\alpha_3 = \operatorname{arccot}\left[\frac{\partial R}{\partial s}\right]$$

$$v = c_1 \cdot r + c_2 \cdot z + c_3$$

$$c_1 = \frac{v_2}{R}$$

$$c_2 = \frac{v_3 - v_2}{s - h_o}$$

$$c_3 = v_2 - c_2 \cdot z_2 - c_1 \cdot r_2$$

$$\alpha = c_4 \cdot r + c_5 \cdot z + c_6$$

$$c_4 = 0$$

$$c_5 = \frac{\alpha_3}{s - h_o}$$

$$c_6 = - c_5 \cdot z_2$$

E3.3.1.3. Strain rates

$$\dot{\epsilon}_r = c_1 \cdot \cos(\alpha)$$
$$\dot{\epsilon}_z = c_2 \cdot \sin(\alpha) + v \cdot c_5 \cdot \cos(\alpha)$$
$$\dot{\epsilon}_\varphi = \frac{v}{r} \cdot \cos(\alpha)$$
$$\dot{\gamma}_{rz} = \frac{1}{2} \cdot \left[c_2 \cdot \cos(\alpha) + (c_1 - v \cdot c_5) \cdot \sin(\alpha) \right]$$

E.3.3.1.4. Effective strain rate

$$\dot{\bar{\epsilon}} = \sqrt{\frac{2}{3} \cdot \left[\dot{\epsilon}_r^2 + \dot{\epsilon}_z^2 + \dot{\epsilon}_\varphi^2\right] + \frac{2}{3}\left[(\dot{\gamma}_{rz}^2\right]}$$

E3.3.1.5. Deformation power

$$P_{def} = \int \sigma \cdot \dot{\bar{\epsilon}} \cdot dV$$

For ideal plastic material behaviour and by using the axisymmetry :

$$P_{def} = 2 \cdot \pi \cdot \sigma_0 \cdot \int\int \cdot \dot{\bar{\epsilon}} \cdot r \cdot dr \cdot dz$$

Integration boundaries :

$0 \leq r \leq R$ $\quad h_0 - \frac{r}{R} \cdot (h_0 - s) \leq z \leq h_0$

E3.3.1.6. Friction power

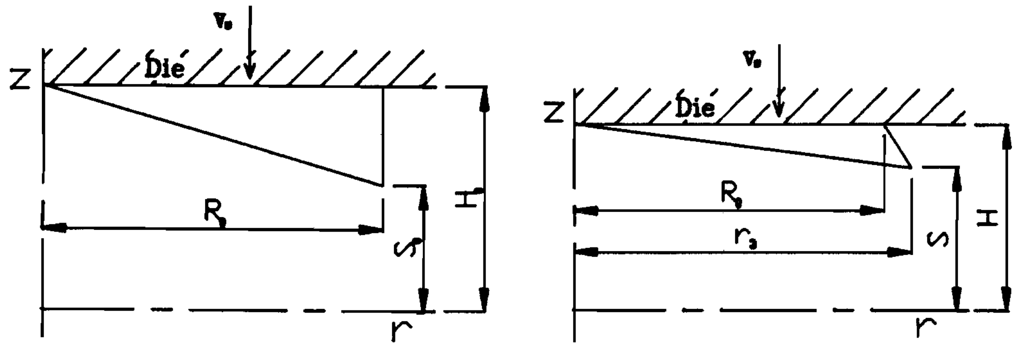
$$P_{fri} = m \cdot \frac{\sigma_0}{\sqrt{3}} \int |\Delta v_t| \cdot dA$$

Because $m = 0$:

$$P_{fri} = 0$$

E3.3.2. Sticking friction

The second case to be considered is sticking friction. In this case it is known that particles of the forging product which are in contact with the die, remain at the same place.



undeformed state deformed state

figure 4: description of geometry

E3.3.2.1. Global volume invariance

Volume in the undeformed state :

$$V_0 = \frac{2}{3}\cdot\pi\cdot R_0^2\cdot(h_0-s_0)$$

Volume in the deformed state, using the fact that $r_2 = R_0$:

$$V_{def} = \frac{1}{3}\cdot\pi\cdot R_0^2\cdot\left[(h-s) + \frac{r_3}{R_0}\cdot(h-s)\right]$$

Global volume invariance gives, rearranged :

$$r_3 = R_0\cdot\frac{2\cdot(h_0-s_0) - (h-s)}{h_0-s}$$

Again we can assume that there is a fixed die, so $h = h_0$:

$$r_3 = R_0\cdot\frac{h_0-2\cdot s_0-s}{h_0-s}$$

E3.3.2.2. Velocity and angle planes

$$v_1 = 0$$

$$v_2 = 0$$

$$v_{3r} = \frac{\partial r_3}{\partial t} = \frac{\partial r_3}{\partial s} \cdot \frac{\partial s}{\partial t}$$

$$v_{3z} = \frac{\partial s}{\partial t}$$

$$v_3 = \sqrt{v_{3r}^2 + v_{3z}^2} = \frac{\partial s}{\partial t} \cdot \sqrt{1 + \left[\frac{\partial r_3}{\partial s}\right]^2}$$

$$\frac{\partial r_3}{\partial s} = \frac{2 \cdot R_o \cdot (h_o - s_o)}{(h_o - s)^2}$$

$$\frac{\partial s}{\partial t} = v_s \qquad s = v_s \cdot t + s_o$$

$$\alpha_1 = 0$$

$$\alpha_2 = 0$$

$$\alpha_3 = \operatorname{arccot}\left[\frac{\partial r_3}{\partial s}\right]$$

$$v = c_1 \cdot r + c_2 \cdot z + c_3$$

$$c_1 = 0$$

$$c_2 = \frac{v_3}{s-h_o}$$

$$c_3 = -c_2 \cdot z_2$$

$$\alpha = c_4 \cdot r + c_5 \cdot z + c_6$$

$$c_4 = 0$$

$$c_5 = \frac{\alpha_3}{s-h_o}$$

$$c_6 = -c_5 \cdot z_2$$

E3.3.2.3. Strain rates

$$\dot{\epsilon}_r = 0$$
$$\dot{\epsilon}_z = c_2 \cdot \sin(\alpha) + v \cdot c_5 \cdot \cos(\alpha)$$
$$\dot{\epsilon}_\varphi = \frac{v}{r} \cdot \cos(\alpha)$$
$$\dot{\gamma}_{rz} = \frac{1}{2} \cdot \left[c_2 \cdot \cos(\alpha) - v \cdot c_5 \cdot \sin(\alpha) \right]$$

E3.3.2.4. Effective strain rate

$$\dot{\bar{\epsilon}} = \sqrt{\frac{2}{3} \cdot \left[\dot{\epsilon}_r^2 + \dot{\epsilon}_z^2 + \dot{\epsilon}_\varphi^2 \right] + \frac{4}{3} \left[(\dot{\gamma}_{rz}^2 \right]}$$

E3.3.2.5. Deformation power

$$P_{def} = \int \sigma \cdot \dot{\bar{\epsilon}} \cdot dV$$

For ideal plastic material behaviour and by using the axisymmetry :

$$P_{def} = 2 \cdot \pi \cdot \sigma_0 \cdot \int\int \cdot \dot{\bar{\epsilon}} \cdot r \cdot dr \cdot dz$$

Integration boundaries :

$$0 \leq r \leq r_3 \qquad h_0 - \frac{r}{r_3} \cdot (h_0 - s) \leq z \leq h_0$$

E3.3.2.6. Friction power

$$P_{fri} = m \cdot \frac{\sigma_0}{\sqrt{3}} \int |\Delta v_t| \cdot dA$$

Because the velocity of the deforming material at the die–material interface equals zero (sticking friction) :

$$P_{fri} = 0$$

E3.3.3. Intermediate friction

The third case to be considered is intermediate friction conditions. In this case there is no additional information about the position of the particles at the die–material interface.

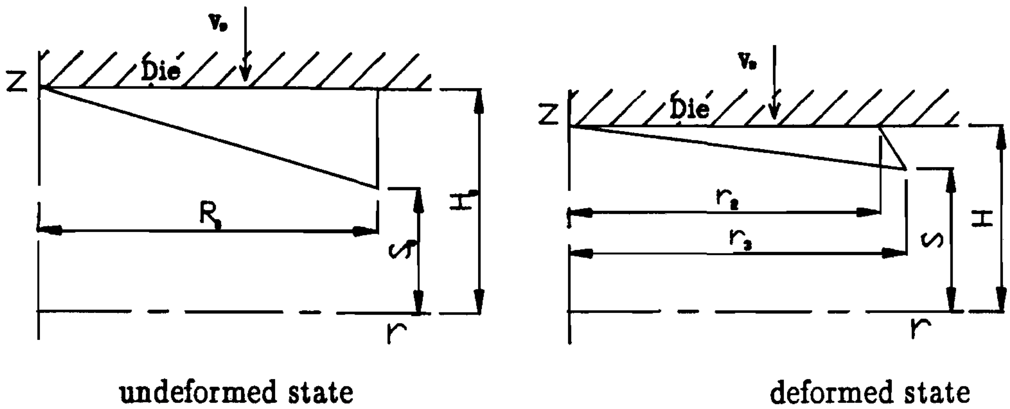


figure 5: description of geometry

E3.3.3.1. Global volume invariance

Volume in the undeformed state :

$$V_0 = \frac{2}{3}\cdot\pi\cdot R_0^2\cdot(h_0-s_0)$$

Volume in the deformed state :

$$V_{def} = \frac{1}{3}\cdot\pi\cdot\left[\, r_2\cdot r_3\cdot(h-s) + r_2^2\cdot(h-s)\,\right]$$

Global volume invariance gives, rearranged :

$$r_3 = \frac{2\cdot R_0^2\cdot(h_0-s_0) - r_2^2\cdot(h-s)}{r_2\cdot(h-s)}$$

Again we can assume that there is a fixed die, so $h = h_0$:

$$r_3 = \frac{2\cdot R_0^2\cdot(h_0-s_0) - r_2^2\cdot(h_0-s)}{r_2\cdot(h_0-s)}$$

E3.3.3.2. Calculation of r_2

In order to calculate the unknown coordinates of the nodal points after deformation, there are 2 possibilities. The first one is the usual upper bound method of calculating power consumption, depending on the unknown coordinates, and optimising the power consumption with respect to the unknowns. The second possibility is the appliance of a friction rule.

A friction rule describes the relation between friction factor and displacement of material at the die–material interface. I.e. when the friction factor equals zero, the deformation pattern will be homogeneous, but in case of sticking friction, material at the die–material interface will not move. A possible friction rule could be the linear interpolation between the two extreme cases outlined above.

Making use of linear interpolation between zero friction and sticking friction, the equation for r_2 , depending on friction condition, becomes :

$$r_2 = R_0 + (1-m)\cdot R_0 \cdot \left[\sqrt{\frac{h_0 - s_0}{h_0 - s}} - 1 \right]$$

In this formula, the first R_0 denotes the original position of the r coordinate of nodal point 2, while the following part denotes the displacement depending on the friction factor m.

E3.3.3.3. Velocity and angle planes

$$v_1 = 0$$

$$v_2 = \frac{\partial r_2}{\partial t} = \frac{\partial r_2}{\partial s} \cdot \frac{\partial s}{\partial t}$$

$$v_{3r} = \frac{\partial r_3}{\partial t} = \frac{\partial r_3}{\partial s} \cdot \frac{\partial s}{\partial t}$$

$$v_{3z} = \frac{\partial s}{\partial t}$$

$$v_3 = \sqrt{v_{3r}^2 + v_{3z}^2} = \frac{\partial s}{\partial t} \cdot \sqrt{1 + \left[\frac{\partial r_3}{\partial s}\right]^2}$$

$$\frac{\partial r_2}{\partial s} = \frac{(1-m) \cdot R_o}{2 \cdot (h_o - s)} \cdot \sqrt{\frac{h_o - s_o}{h_o - s}}$$

$$\frac{\partial r_3}{\partial s} = \frac{2 \cdot R_o^2 \cdot (h_o - s_o)}{r_2 \cdot (h_o - s)^2} - \frac{2 \cdot R_o^2 \cdot (h_o - s_o)}{r_2^2 \cdot (h_o - s)} \cdot \frac{\partial r_2}{\partial s} - \frac{\partial r_2}{\partial s}$$

$$\frac{\partial s}{\partial t} = v_s \qquad s = v_s \cdot t + s_o$$

$$\alpha_1 = 0$$

$$\alpha_2 = 0$$

$$\alpha_3 = \operatorname{arccot}\left[\frac{\partial r_3}{\partial s}\right]$$

$$v = c_1 \cdot r + c_2 \cdot z + c_3$$

$$c_1 = \frac{v_2}{r_2}$$

$$c_2 = \frac{r_2 \cdot v_3 - r_3 \cdot v_2}{r_2 \cdot (s - h_o)}$$

$$c_3 = v_2 - c_2 \cdot z_2 - c_1 \cdot r_2$$

$$\alpha = c_4 \cdot r + c_5 \cdot z + c_6$$

$$c_4 = 0$$

$$c_5 = \frac{\alpha_3}{s - h_o}$$

$$c_6 = -c_5 \cdot z_2$$

E3.3.3.4. Strain rates

$$\dot{\epsilon}_r = c_1 \cdot \cos(\alpha)$$
$$\dot{\epsilon}_z = c_2 \cdot \sin(\alpha) + v \cdot c_5 \cdot \cos(\alpha)$$
$$\dot{\epsilon}_\varphi = \frac{v}{r} \cdot \cos(\alpha)$$
$$\dot{\gamma}_{rz} = \frac{1}{2} \cdot \left[c_2 \cdot \cos(\alpha) + (c_1 - v \cdot c_5) \cdot \sin(\alpha) \right]$$

E3.3.3.5. Effective strain rate

$$\dot{\bar{\epsilon}} = \sqrt{\frac{4}{3}\cdot\left[\dot{\epsilon}_r^2 + \dot{\epsilon}_z^2 + \dot{\epsilon}_\varphi^2\right] + \frac{4}{3}\left[\dot{\gamma}_{rz}^2\right]}$$

E3.3.3.6. Deformation power

$$P_{def} = \int \sigma\cdot\dot{\bar{\epsilon}}\cdot dV$$

For ideal plastic material behaviour, and using axisymmetry :

$$P_{def} = 2\cdot\pi\cdot\sigma_0\cdot\int\int\cdot\dot{\bar{\epsilon}}\cdot r\cdot dr\cdot dz$$

Integration boundaries :

$$0 \leq r \leq r_2 \qquad h_0 - \frac{r}{r_3}\cdot(h_0 - s) \leq z \leq h_0$$

$$r_2 \leq r \leq r_3 \qquad h_0 - \frac{r}{r_3}\cdot(h_0 - s) \leq z \leq h_0 - \frac{r - r_2}{r_3 - r_2}\cdot(h_0 - s)$$

E3.3.3.7. Friction power

$$P_{fri} = m\cdot\frac{\sigma_0}{\sqrt{3}}\int|\Delta v_t|\cdot dA$$

At the die–material interface, $z = h_0$, so the slip velocity is ;

$$|\Delta v_t| = v\cdot\cos(\alpha) \qquad (z = h_0)$$

$$|\Delta v_t| = \left[c_1\cdot r + c_2\cdot h_0 + c_3\right]\cdot\cos\left[c_5\cdot h_0 + c_6\right]$$

Using this and axisymmetry, P_{fri} becomes :

$$P_{fri} = 2\cdot\pi\cdot m\cdot\frac{\sigma_0}{\sqrt{3}}\int\left[c_1\cdot r + c_2\cdot h_0 + c_3\right]\cdot\cos\left[c_5\cdot h_0 + c_6\right]\cdot r\cdot dr$$

Integration boundary :

$$0 \leq r \leq r_2$$

Enclosure 4: Elements with internal degrees of freedom

E4.1. Introduction

Another way of modelling forging processes with the achievement of bulging is the use of elements which have internal bulging parameters included in their velocity field.
With the aid of these elements a process can be analysed, and the power consumption can be minimised with respect to these internal degrees of freedom.

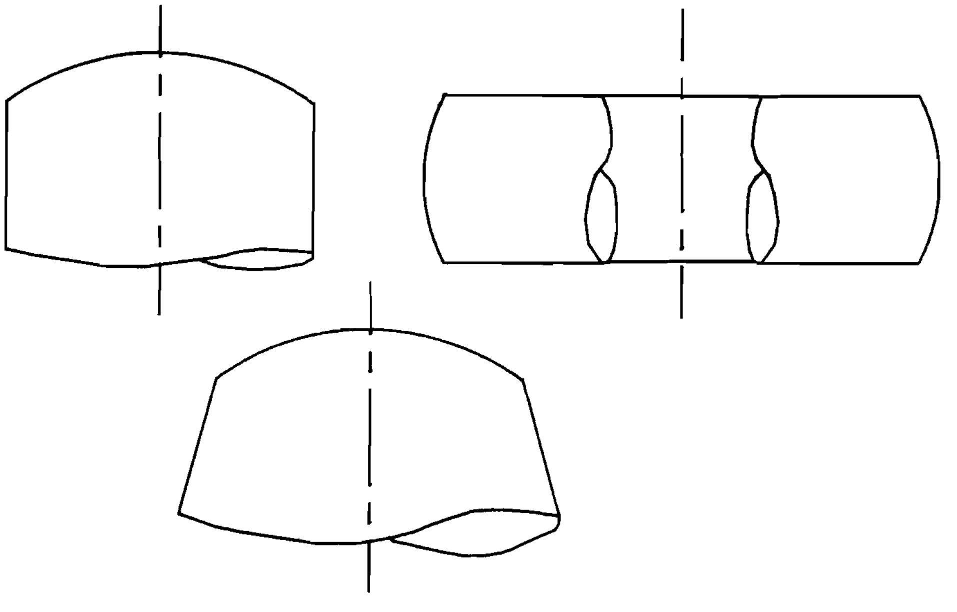


figure 1 examples of bulging elements

In figure 1 above, some examples of bulging elements are plotted.
Some of those elements are analysed on the following pages. The author is aware of the fact that his selection is not complete, but only a start.

E4.2. Kobayashi's method

When elements are developed which have the possibility of bulging, it becomes necessary for analysing purposes to connect them together correctly. This means that at the connecting surfaces (discontinuity surface) the perpendicular velocity component is continuous over the connecting surface.

In normal analysis, this property is commonly used as a method to determine the velocity components in neighbouring elements.

Kobayashi [30],[31] first introduced a method of working the other way around. His method consists of assuming the velocity fields in two neighbouring regions, followed by determination of the position of the discontinuity surface by demanding continuity of the perpendicular velocity component over the discontinuity surface. In his analysis this method is used in order to be able to use more complicated velocity fields.

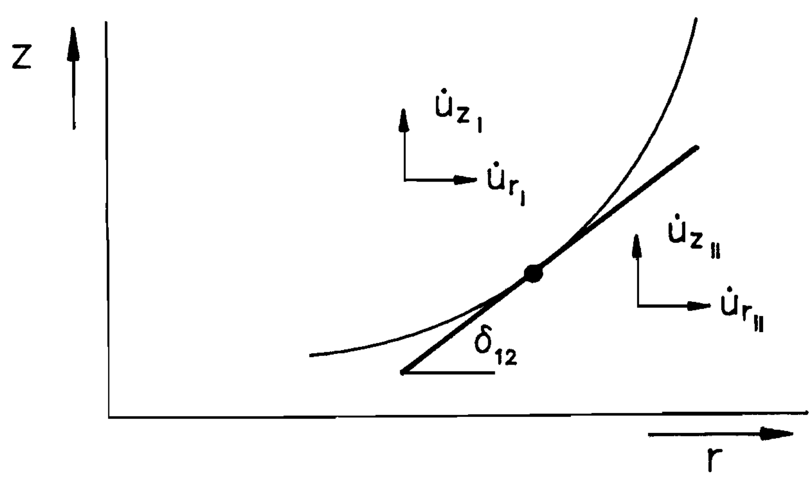


figure 2 discontinuity surface

Continuity over a discontinuity surface (see figure 2) :

$$\dot{u}_{zI}\cdot\cos\delta_{12}-\dot{u}_{rI}\cdot\sin\delta_{12}=\dot{u}_{zII}\cdot\cos\delta_{12}-\dot{u}_{rII}\cdot\sin\delta_{12}$$

When both sides of the equation are divided by $\cos\delta_{12}$:

$$\dot{u}_{zI}-\dot{u}_{rI}\cdot\tan\delta_{12}=\dot{u}_{zII}-\dot{u}_{rII}\cdot\tan\delta_{12}$$

Suitably rearranged :

$$(\dot{u}_{rII}-\dot{u}_{rI})\cdot\tan\delta_{12}=\dot{u}_{zII}-\dot{u}_{zI}$$

$$\tan\delta_{12}=\frac{\dot{u}_{zII}-\dot{u}_{zI}}{\dot{u}_{rII}-\dot{u}_{rI}}$$

Due to the fact that the first derivative in a point equals the tangent of the line at that point :

$$\tan\delta_{12}=\frac{\partial z_{12}}{\partial r}$$

$$\frac{\partial z_{12}}{\partial r}=\frac{\dot{u}_{zII}-\dot{u}_{zI}}{\dot{u}_{rII}-\dot{u}_{rI}}$$

At this stage a differential equation is obtained, which gives us, when combined with a boundary condition, the position of the discontinuity surface. Although the derived differential equations are solved analytically in Kobayashi's analysis, this is not necessary.

E4.3. Strategy

Firstly the velocity fields for the elements have to be derived. A start is made by assuming the axial or radial velocity. After this, with the aid of local volume invariance it is possible to obtain the velocity component in the other direction. Finally, it has to checked if the velocity field fits in with the boundary conditions.

Local volume invariance :

$$\mathrm{div}(\underline{\dot{U}}) = 0.$$

Axisymmetric processes :

$$\frac{\partial}{\partial\theta} = 0$$

Local volume invariance for axisymmetric processes :

$$\frac{\partial \dot{u}_r}{\partial r} + \frac{\partial \dot{u}_z}{\partial z} + \frac{\dot{u}_r}{r} = 0$$

Secondly, the strain rates have to be obtained from the velocity field. From these strainrates it then is possible to calculate the effective strain rate.

$$\dot{\epsilon}_r = \frac{\partial \dot{u}_r}{\partial r} \qquad \dot{\epsilon}_z = \frac{\partial \dot{u}_z}{\partial z} \qquad \dot{\epsilon}_\theta = \frac{\dot{u}_r}{r} \qquad \gamma_{rz} = 0.5 \cdot \left[\frac{\partial \dot{u}_r}{\partial z} + \frac{\partial \dot{u}_z}{\partial r} \right]$$

Effective strain rate :

$$\dot{\bar{\epsilon}} = \sqrt{ \frac{2}{3} \cdot \left[\dot{\epsilon}_r^2 + \dot{\epsilon}_z^2 + \dot{\epsilon}_\varphi^2 \right] + \frac{4}{3} \left[\dot{\gamma}_{rz}^2 \right] }$$

Final stage in the analysis of a single element is the calculation of the power consumption, i.e. deformation power, shear power and friction power.

Deformation power :

$$P_D = \int_V \sigma \cdot \dot{\bar{\epsilon}} \cdot dV$$

Friction power :

$$P_F = \frac{m}{\sqrt{3}} \cdot \int_A \sigma \cdot |\Delta \underline{v}_s| \cdot dA$$

Shear power :

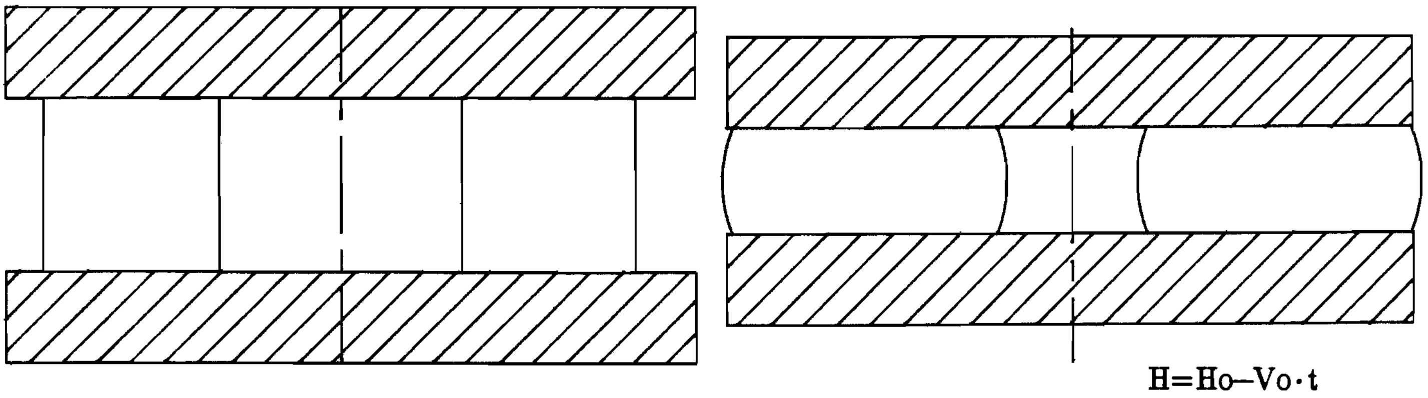
$$P_F = \frac{1}{\sqrt{3}} \cdot \int_S \sigma \cdot |\Delta \underline{v}_t| \cdot dS$$

All power calculations are performed numerically. Equations in this enclosure are merely presented for the determination of the integration intervals.

In most analyses, some boundaries can not be derived analytically, but have to be solved numerically by integrating the velocity field with respect to time, in case of an outer surface.

In case of an internal boundary (discontinuity surface) the integration boundary has to be determined according to Kobayashi's method.

E4.4. Element1 : Ring (upset element)



a : original state

b : deformed state

figure 3: description of geometry

E4.4.1. Velocity field

From literature [23] it is known that, apart from a parameter describing the amount of bulging, there also exists a neutral radius.

Definition : $\dot{u}_r\,(r = Rn) = 0$

Assumption for radial velocity :

$$\dot{u}_r = A\cdot r\cdot(1-3\cdot\beta\cdot z^2)\cdot(r - \frac{R_n^2}{r})$$

In the formula above, β is a parameter describing the severity of the bulging, while R_n describes the position of the neutral surface.

Local volume invariance for axisymmetric processes gives :

$$\dot{u}_z = -2\cdot A\cdot z\cdot(1-\beta\cdot z^2)$$

The parameter A can be determined with the aid of the boundary conditions for $z = 0.5\cdot H$ and $z = -0.5\cdot H$.

$$\dot{u}_z(z = 0.5\cdot H) = 0.5\cdot Vo$$

$$A = \frac{Vo}{2\cdot H\cdot(1-0.25\cdot\beta\cdot H^2)}$$

E4.4.2. Strainrates and effective strainrate

Radial strainrate

$$\dot{\epsilon}_r = \frac{\delta \dot{u}_r}{\delta r} = A \cdot \left[1-3 \cdot \beta \cdot z^2 \right] \cdot \left[1 + \frac{R_n^2}{r^2} \right]$$

Axial strainrate

$$\dot{\epsilon}_z = \frac{\delta \dot{u}_z}{\delta z} = -2 \cdot A \cdot \left[1-3 \cdot \beta \cdot z^2 \right]$$

Angle strainrate

$$\dot{\epsilon}_\theta = \frac{\dot{u}_r}{r} = A \cdot \left[1-3 \cdot \beta \cdot z^2 \right] \cdot \left[1 - \frac{R_n^2}{r^2} \right]$$

Shear strainrate

$$\gamma_{rz} = 0.5 \cdot \left[\frac{\delta \dot{u}_r}{\delta z} + \frac{\delta \dot{u}_z}{\delta r} \right] = -3 \cdot A \cdot \beta \cdot z \cdot \left[1 - \frac{R_n^2}{r} \right]$$

Effective strainrate

$$\dot{\bar{\epsilon}} = \sqrt{ \frac{2}{3} \cdot \left[\dot{\epsilon}_r^2 + \dot{\epsilon}_z^2 + \dot{\epsilon}_\varphi^2 \right] + \frac{4}{3} \left[\dot{\gamma}_{rz}^2 \right] }$$

E4.4.3. Power consumption

Deformation power :

$$P_D = \int_V \sigma \cdot \dot{\bar{\epsilon}} \cdot dV \qquad dV = 2 \cdot \Pi \cdot r \cdot dr \cdot dz$$

$$P_D = 4 \cdot \Pi \cdot \int_0^{H/2} \int_{Ri}^{Ru} \bar{\sigma} \cdot \dot{\bar{\epsilon}} \cdot r \cdot dr \cdot dz \qquad Ru = Ru(z),\ Ri = Ri(z)$$

For ideal plastic material behaviour this becomes :

$$P_D = 4 \cdot \Pi \cdot \sigma_0 \cdot \int_0^{H/2} \int_{Ri}^{Ru} \dot{\bar{\epsilon}} \cdot r \cdot dr \cdot dz \qquad Ru = Ru(z),\ Ri = Ri(z)$$

Friction power :

$$P_F = \frac{m}{\sqrt{3}} \cdot \int_S \sigma \cdot |\Delta \underline{v}| \cdot dS \qquad dS = 2 \cdot \Pi \cdot r \cdot dr$$

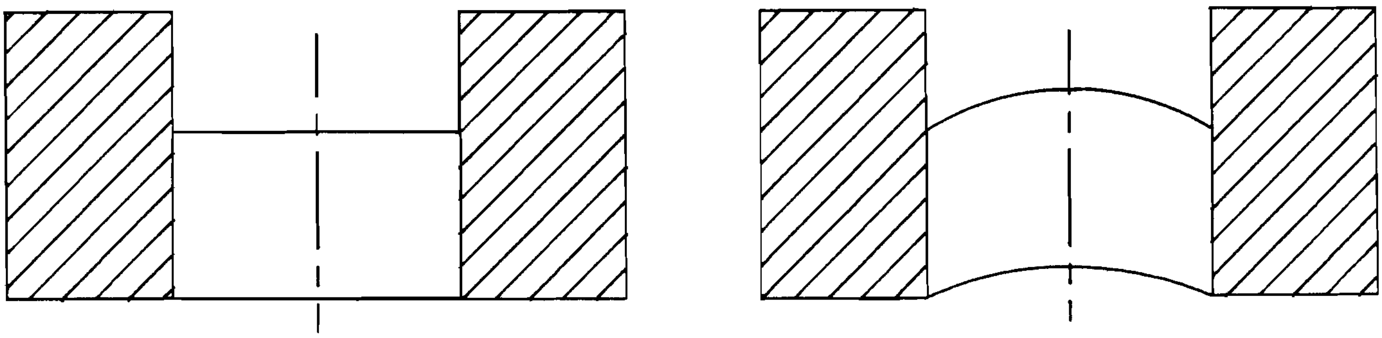
For ideal plastic material behaviour this becomes :

$$P_F = 4 \cdot \Pi \cdot \frac{m}{\sqrt{3}} \cdot \int_{Rs}^{Rm} \sigma \cdot A \cdot r \cdot \left[1 - 3 \cdot \beta \cdot z^2 \right] \cdot \left[r - \frac{R_n^2}{r} \right] \cdot r \cdot dr$$

$$Rm = Ru(0.5 \cdot H)$$

$$Rs = Ri(0.5 \cdot H)$$

E4.5. Element 2 : Billet through tube



a : original state   b : deformed state

figure 4: description of geometry

E4.5.1. Velocity field

In this case, the only deformation will be some shearing as a result of friction at at the die material interface.

Assumption for axial velocity :

$$\dot{u}_z = C \cdot \left[1 - \gamma \cdot r^2 \right]$$

In the formula above, γ is a parameter describing the severity of the bulging.

Local volume invariance for axisymmetric processes gives :

$$\dot{u}_r = 0$$

The parameter C can be determined in analysis with the aid of the global volume invariance.

E4.5.2. Strainrates and effective strainrate

Radial strainrate

$$\dot{\epsilon}_r = 0$$

Axial strainrate

$$\dot{\epsilon}_z = 0$$

Angle strainrate

$$\dot{\epsilon}_\theta = 0$$

Shear strainrate

$$\gamma_{rz} = 0.5 \cdot \left[\frac{\delta \dot{u}_r}{\delta z} + \frac{\delta \dot{u}_z}{\delta r} \right] = -C \cdot \gamma \cdot r$$

Effective strainrate

$$\dot{\bar{\epsilon}} = \sqrt{ \frac{2}{3} \cdot \left[\dot{\epsilon}_r^2 + \dot{\epsilon}_z^2 + \dot{\epsilon}_\varphi^2 \right] + \frac{4}{3} \left[\dot{\gamma}_{rz}^2 \right] }$$

$$\dot{\bar{\epsilon}} = \frac{2}{\sqrt{3}} \cdot C \cdot \gamma \cdot r$$

E4.5.3. Power consumption

Deformation power :

$$P_D = \int_V \bar{\sigma} \cdot \dot{\bar{\epsilon}} \cdot dV \qquad dV = 2 \cdot \Pi \cdot r \cdot dr \cdot dz$$

$$P_D = 4 \cdot \Pi \cdot \int_{Zd}^{Zu} \int_0^{Ri} \bar{\sigma} \cdot \dot{\bar{\epsilon}} \cdot r \cdot dr \cdot dz \qquad Zu = Zu(r),\ Zd = Zd(r)$$

For ideal plastic material behaviour this becomes :

$$P_D = 4 \cdot \Pi \cdot \sigma_0 \cdot \int_0^{H/2} \int_{Ri}^{Ru} \dot{\bar{\epsilon}} \cdot r \cdot dr \cdot dz \qquad Ru = Ru(z),\ Ri = Ri(z)$$

Friction power :

$$P_F = \frac{m}{\sqrt{3}} \cdot \int_S \sigma \cdot |\Delta \underline{v}| \cdot dS \qquad dS = 2 \cdot \Pi \cdot r \cdot dz$$

For ideal plastic material behaviour this becomes :

$$P_F = 4 \cdot \Pi \cdot \frac{m}{\sqrt{3}} \cdot \sigma_0 \cdot \int_{H/2}^{Zs} C \cdot (1 - \gamma \cdot r^2) \cdot r \cdot dz \qquad Rs = Zu(Ri)$$

E4.6. Element 3 : Connecting billet

a : original state b : deformed state

figure 5: description of geometry

E4.6.1. Velocity field

At this moment two different elements have been defined. Now an element is introduced which is capable of connecting radial flow (element 1) and axial flow (element 2). The new element also has to contain a parameter which can be influenced by friction.

Assumption for radial velocity :

$$\dot{u}_r = -B \cdot r \cdot z \cdot (1-\beta \cdot r^2)$$

In the formula above, β is a parameter describing the severity of the bulging.

Local volume invariance for axisymmetric processes gives :

$$\dot{u}_z = B \cdot z^2 \cdot (1-\beta \cdot r^2)$$

The parameter B can be determined with the aid of global volume invariance.

E4.6.2. Strainrates and effective strainrate

Radial strainrate

$$\dot{\epsilon}_r = \frac{\delta \dot{u}_r}{\delta r} = -B \cdot z \cdot \left[1-3 \cdot \beta \cdot r^2\right]$$

Axial strainrate

$$\dot{\epsilon}_z = \frac{\delta \dot{u}_z}{\delta z} = 2 \cdot B \cdot z \cdot \left[1-2 \cdot \beta \cdot r^2\right]$$

Angle strainrate

$$\dot{\epsilon}_\theta = \frac{\dot{u}_r}{r} = -B \cdot z \cdot \left[1-\beta \cdot r^2\right]$$

Shear strainrate

$$\gamma_{rz} = 0.5 \cdot \left[\frac{\delta \dot{u}_r}{\delta z} + \frac{\delta \dot{u}_z}{\delta r}\right] = -\frac{1}{2} \cdot B \cdot r \cdot \left[1 - \beta \cdot r^2\right] - 2 \cdot \beta \cdot B \cdot r \cdot z^2$$

Effective strainrate

$$\dot{\bar{\epsilon}} = \sqrt{\frac{2}{3} \cdot \left[\dot{\epsilon}_r^2 + \dot{\epsilon}_z^2 + \dot{\epsilon}_\varphi^2\right] + \frac{4}{3}\left[\dot{\gamma}_{rz}^2\right]}$$

E4.6.3. Power consumption

Deformation power :

$$P_D = \int_V \sigma \cdot \dot{\bar{\epsilon}} \cdot dV \qquad dV = 2 \cdot \Pi \cdot r \cdot dr \cdot dz$$

$$P_D = 4 \cdot \Pi \cdot \int_0^{Zu} \int_0^{Ru} \bar{\sigma} \cdot \dot{\bar{\epsilon}} \cdot r \cdot dr \cdot dz \qquad Ru = Ru(z),\ Zu = Zu(r)$$

For ideal plastic material behaviour this becomes :

$$P_D = 4 \cdot \Pi \cdot \sigma_0 \cdot \int_0^{Zu} \int_0^{Ru} \dot{\overline{\epsilon}} \cdot r \cdot dr \cdot dz \qquad Ru = Ru(z),\ Zu = Zu(r)$$

Friction power :

$$P_F = 0$$

E4.7. Additional elements

Following upon the 3 elements described on the previous pages, the analysis for axial axial compression of a disc and as an extension to this axial compression of a ring is under process. Furthermore, a trapezoid disk element for axial flow is under study.

Enclosure 5: Modelling of an upsetting

E5.1. Introduction

In order to be able to compare the results of modelling as described in the enclosures 3 and 4, simple upsetting is modelled with the nodal point approach as well as the method with internal degrees of freedom in the velocity field. Furthermore, Avitzur's model for simple upsetting was used for comparison.

E5.2. Avitzur's model

This analysis has been included in this work in order to give a complete view.
Source : Avitzur [23].

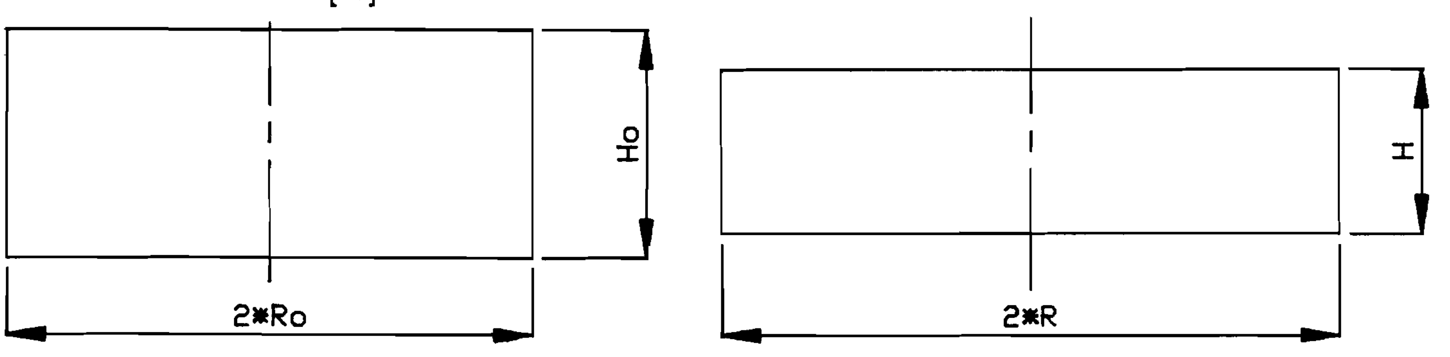


figure 1: description of geometry

E5.2.1. Velocity field

$$\dot{u}_z = -A \cdot z$$
$$\dot{u}_r = \frac{1}{2} \cdot A \cdot r$$

Use of boundary condition $\dot{u}_z = -\frac{V_o}{2}$ gives :

$$A = \frac{V_o}{h}$$

E5.2.2 Global volume invariance

$$V_o = 2 \cdot \pi \cdot R_o^2 \cdot h_o$$
$$V = 2 \cdot \pi \cdot R^2 \cdot h$$
$$R = \frac{R_o \cdot \sqrt{h_o}}{\sqrt{h}}$$

E5.2.3. Strain rates

$$\dot{\epsilon}_r = \frac{1}{2} \cdot A$$
$$\dot{\epsilon}_z = -A$$
$$\dot{\epsilon}_\varphi = \frac{1}{2} \cdot A$$
$$\dot{\gamma}_{rz} = 0$$

E5.2.4. Effective strain rate

$$\dot{\bar{\epsilon}} = \sqrt{\frac{2}{3} \cdot \left[\dot{\epsilon}_r^2 + \dot{\epsilon}_z^2 + \dot{\epsilon}_\varphi^2 \right] + \frac{4}{3} \left[\dot{\gamma}_{rz}^2 \right]}$$

$$\dot{\bar{\epsilon}} = A$$

E5.2.5. Deformation power

$$P_{def} = \int \sigma \cdot \dot{\bar{\epsilon}} \cdot dV \qquad dV = 2 \cdot \pi \cdot r \cdot dr \cdot dz$$

$$P_{def} = 2 \cdot \pi \cdot \sigma_o \cdot \int\int \cdot \dot{\bar{\epsilon}} \cdot r \cdot dr \cdot dz \qquad 0 \leq r \leq R \qquad 0 \leq z \leq h$$

$$P_{def} = 2 \cdot \pi \cdot \sigma_o \cdot \int\int \cdot A \cdot r \cdot dr \cdot dz$$

$$P_{def} = \frac{1}{2} \cdot \pi \cdot \sigma_o \cdot V_o \cdot R^2$$

E5.2.6. Friction power

$$P_{fri} = m \cdot \frac{\sigma_o}{\sqrt{3}} \int |\Delta v_t| \cdot dA \qquad dA = 2 \cdot \pi \cdot r \cdot dr$$

$$|\Delta v_t| = \frac{1}{2} \cdot A \cdot r$$

$$P_{fri} = 2 \cdot \pi \cdot m \cdot \frac{\sigma_o}{\sqrt{3}} \int \frac{1}{2} \cdot A \cdot r^2 \cdot dr \qquad 0 \leq r \leq R$$

$$P_{fri} = \pi \cdot m \cdot \frac{\sigma_o}{3\sqrt{3}} \cdot \frac{V_o}{h} \cdot R^3$$

E5.3. Nodal point approach

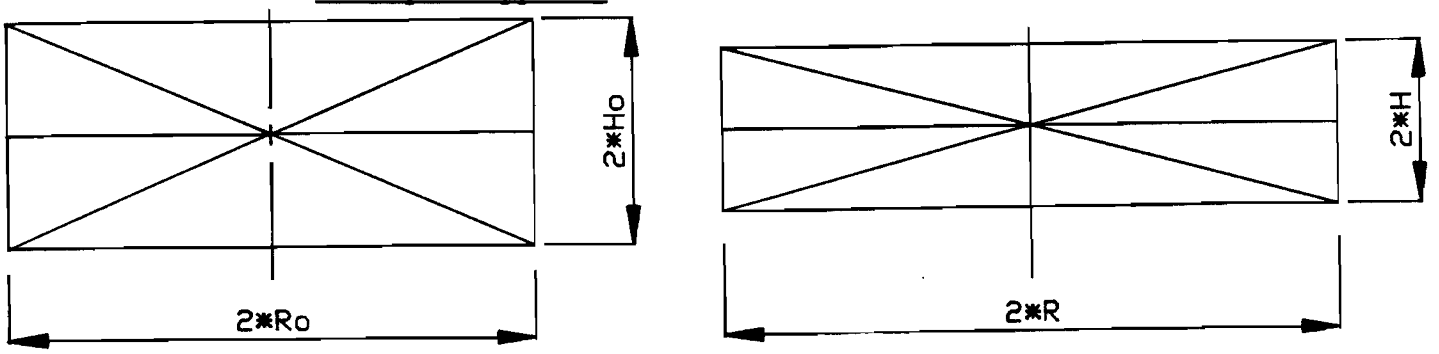


figure 2: description of geometry

nodal point	: 1 $v_r = 0$	$v_z = -v_0/2$
nodal point	: 2 $v_r = ?$	$v_z = -v_0/2$
nodal point	: 3 $v_r = 0$	$v_z = 0$
nodal point	: 4 $v_r = ?$	$v_z = 0$

E5.3.1. Coordinates of the nodal points after deformation

Global volume invariance element 1

undeformed volume:

$$V_0 = \frac{\pi}{6} \cdot h_0 \cdot R_0^2$$

deformed volume:

$$V = \frac{\pi}{6} \cdot h \cdot r_2^2$$

Combination of those two equations gives:

$$r_2 = \frac{R_0 \cdot \sqrt{h_0}}{\sqrt{h}}$$

Global volume invariance element 2

undeformed volume:

$$V_o = \frac{\pi}{3} \cdot h_o \cdot R_o^2$$

deformed volume:

$$V = \frac{\pi}{6} \cdot h \cdot \left[r_4^2 + r_2 \cdot r_4 \right]$$

Combination of those two equations gives, together with the formula for the radius r_2:

$$r_4 = \frac{R_o \cdot \sqrt{h_o}}{\sqrt{h}}$$

E5.3.2. Velocities

When the unknown coordinates of the nodal points are known as a function of time, then the unknown velocities can be calculated by differentiating them with respect to time.

$$v_{2r} = \frac{\partial r_2}{\partial t} = \frac{\partial r_2}{\partial h} \cdot \frac{\partial h}{\partial t}$$

$$v_{2z} = \frac{\partial h}{\partial t}$$

$$v_2 = \sqrt{v_{2r}^2 + v_{2z}^2} = \frac{\partial h}{\partial t} \cdot \sqrt{\frac{1}{2} + \left[\frac{\partial r_2}{\partial h}\right]^2}$$

$$\frac{\partial r_2}{\partial h} = \frac{-R_o \cdot \sqrt{h_o}}{2 \cdot h \cdot \sqrt{h}}$$

$$\frac{\partial h}{\partial t} = -v_o \qquad h = h_o - v_o \cdot t$$

$$\alpha_2 = -\operatorname{arccot}\left[\frac{h\cdot\sqrt{h}}{R_o\cdot\sqrt{h_o}}\right]$$

$$v_{4r} = \frac{\partial r_4}{\partial t} = \frac{\partial r_4}{\partial h}\cdot\frac{\partial h}{\partial t}$$

$$v_{4z} = 0$$

$$v_2 = v_{2r}$$

$$\frac{\partial r_4}{\partial h} = \frac{-R_o\cdot\sqrt{h_o}}{2\cdot h\cdot\sqrt{h}}$$

E5.3.3. Element I

E5.3.3.1. Velocity and angle planes

$$v = c_1\cdot r + c_2\cdot z + c_3$$

$$c_1 = \frac{2\cdot v_2 - v_o}{2\cdot r_2}$$

$$c_2 = \frac{v_o}{h}$$

$$c_3 = v_2 - c_2\cdot z_2 - c_1\cdot r_2$$

$$\alpha = c_4 \cdot r + c_5 \cdot z + c_6$$

$$c_4 = \frac{\pi + 2 \cdot \alpha_2}{2 \cdot r_2}$$

$$c_5 = \frac{-\pi}{h}$$

$$c_6 = \alpha_2 - c_5 \cdot z_2 - c_4 \cdot r_2$$

E5.3.3.2. Strain rates

$$\dot{\epsilon}_r = c_1 \cdot \cos(\alpha) - v \cdot c_4 \cdot \sin(\alpha)$$
$$\dot{\epsilon}_z = c_2 \cdot \sin(\alpha) + v \cdot c_5 \cdot \cos(\alpha)$$
$$\dot{\epsilon}_\varphi = \frac{v}{r} \cdot \cos(\alpha))$$
$$\dot{\gamma}_{rz} = \frac{1}{2} \cdot \left[(c_2 + v \cdot c_4) \cdot \cos(\alpha) + (c_1 - v \cdot c_5) \cdot \sin(\alpha) \right]$$

E5.3.3.3. Effective strain rate

$$\dot{\bar{\epsilon}} = \sqrt{\frac{2}{3} \cdot \left[\dot{\epsilon}_r^2 + \dot{\epsilon}_z^2 + \dot{\epsilon}_\varphi^2 \right] + \frac{4}{3} \left[\dot{\gamma}_{rz}^2 \right]}$$

E5.3.3.4. Deformation power

$$P_{def} = \int \sigma \cdot \dot{\bar{\epsilon}} \cdot dV$$

$$P_{def} = 2 \cdot \pi \cdot \sigma_o \cdot \int\int \cdot \dot{\bar{\epsilon}} \cdot r \cdot dr \cdot dz \qquad 0 \leq r \leq r_2 \qquad \frac{r}{r_2} \cdot h \leq z \leq h$$

E5.3.3.5. Friction power

$$P_{fri} = m \cdot \frac{\sigma_o}{\sqrt{3}} \int |\Delta v_t| \cdot dA$$

$$|\Delta v_t| = v \cdot \cos(\alpha) \qquad (z = h_o) \qquad 0 \leq r \leq r_2$$

$$P_{fri} = 2 \cdot \pi \cdot m \cdot \frac{\sigma_o}{\sqrt{3}} \int \left[c_1 \cdot r + c_2 \cdot h_o + c_3\right] \cdot \cos\left[c_5 \cdot h_o + c_6\right] \cdot r \cdot dr$$

E5.3.4. Element II

E5.3.4.1. Velocity and angle planes

$$v = d_1 \cdot r + d_2 \cdot z + d_3$$

$$d_1 = \frac{v_4}{r_4}$$

$$d_2 = \frac{r_4 \cdot v_2 - r_2 \cdot v_4}{r_4 \cdot h/2}$$

$$d_3 = 0$$

$$\alpha = d_4 \cdot r + d_5 \cdot z + d_6$$

$$d_4 = 0$$

$$d_5 = \frac{2 \cdot \alpha_2}{h}$$

$$d_6 = 0$$

E5.3.4.2. Strain rates

$$\dot{\epsilon}_r = d_1 \cdot \cos(\alpha)$$

$$\dot{\epsilon}_z = d_2 \cdot \sin(\alpha) + v \cdot d_5 \cdot \cos(\alpha)$$

$$\dot{\epsilon}_\varphi = \frac{v}{r} \cdot \cos(\alpha)$$

$$\dot{\gamma}_{rz} = \frac{1}{2} \cdot \left[d_2 \cdot \cos(\alpha) + (d_1 - v \cdot d_5) \cdot \sin(\alpha) \right]$$

E5.3.4.3. Effective strain rate

$$\dot{\bar{\epsilon}} = \sqrt{\frac{2}{3} \cdot \left[\dot{\epsilon}_r^2 + \dot{\epsilon}_z^2 + \dot{\epsilon}_\varphi^2 \right] + \frac{4}{3} \left[\dot{\gamma}_{rz}^2 \right]}$$

E5.3.4.4. Deformation power

$$P_{def} = \int \sigma \cdot \dot{\bar{\epsilon}} \cdot dV$$

$$P_{def} = 2 \cdot \pi \cdot \sigma_0 \cdot \int\int \cdot \dot{\bar{\epsilon}} \cdot r \cdot dr \cdot dz \qquad 0 \le r \le r_4 \qquad \frac{r}{r_4} \cdot h \ge z \ge 0$$

E5.4. Internal degrees of freedom

When element 1 as described in enclosure 5 is used and Rn, Ri(z) and Rm are put to zero, the resulting element describes simple upsetting.

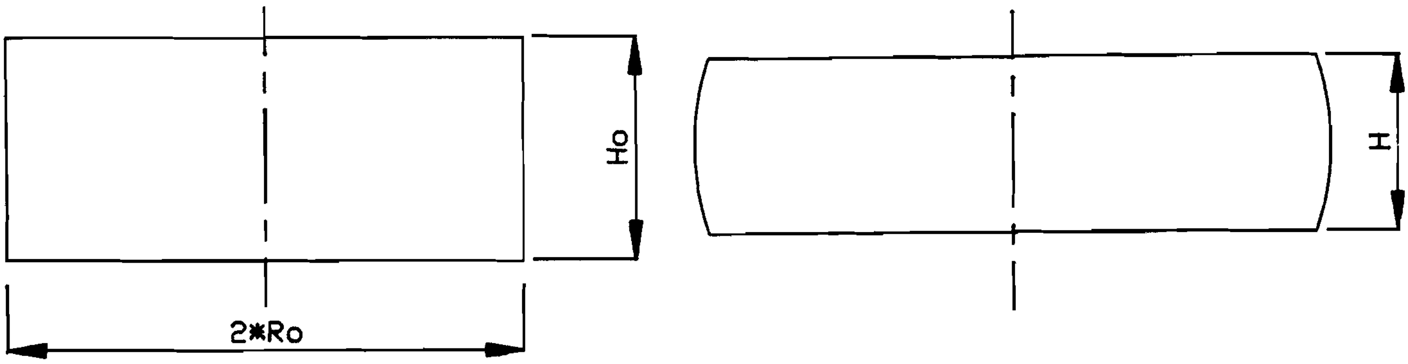


figure 3: description of geometry

E5.4.1. Velocity field

$$\dot{u}_r = A \cdot r \cdot (1-3 \cdot \beta \cdot z^2)$$

In the formula above, β is a parameter describing the severity of the bulging.

Local volume invariance for axisymmetric processes gives :

$$\dot{u}_z = -2 \cdot A \cdot z \cdot (1-\beta \cdot z^2)$$

The parameter A can be determined with the aid of the boundary conditions for $z = 0.5 \cdot H$ and $z = -0.5 \cdot H$.

$$\dot{u}_z(z = 0.5 \cdot H) = -0.5 \cdot Vo$$

$$A = \frac{Vo}{2 \cdot H \cdot (1-0.25 \cdot \beta \cdot H^2)}$$

E5.4.2. Strainrates

$$\dot{\epsilon}_r = \frac{\delta \dot{u}_r}{\delta r} = A \cdot \left[1 - 3 \cdot \beta \cdot z^2 \right]$$

$$\dot{\epsilon}_z = \frac{\delta \dot{u}_z}{\delta z} = -2 \cdot A \cdot \left[1 - 3 \cdot \beta \cdot z^2 \right]$$

$$\dot{\epsilon}_\varphi = \frac{\dot{u}_r}{r} = A \cdot \left[1 - 3 \cdot \beta \cdot z^2 \right]$$

$$\gamma_{rz} = 0.5 \cdot \left[\frac{\delta \dot{u}_r}{\delta z} + \frac{\delta \dot{u}_z}{\delta r} \right] = -3 \cdot A \cdot \beta \cdot z$$

E5.4.3. Effective strainrate

$$\dot{\bar{\epsilon}} = \sqrt{\frac{2}{3} \cdot \left[\dot{\epsilon}_r^2 + \dot{\epsilon}_z^2 + \dot{\epsilon}_\varphi^2 \right] + \frac{4}{3} \left[\dot{\gamma}_{rz}^2 \right]}$$

E5.4.4. Deformation power

$$P_D = \int_V \sigma \cdot \dot{\bar{\epsilon}} \cdot dV \qquad dV = 2 \cdot \Pi \cdot r \cdot dr \cdot dz$$

$$P_D = 2 \cdot \Pi \cdot \int_0^{H/2} \int_0^{Ru} \bar{\sigma} \cdot \dot{\bar{\epsilon}} \cdot r \cdot dr \cdot dz \qquad Ru = Ru(z)$$

For ideal plastic material behaviour this becomes :

$$P_D = 2 \cdot \Pi \cdot \sigma_0 \cdot \int_0^{H/2} \int_0^{Ru} \dot{\bar{\epsilon}} \cdot r \cdot dr \cdot dz \qquad Ru = Ru(z)$$

E5.4.5. Friction power

$$P_F = \frac{m}{\sqrt{3}} \cdot \int_S \sigma \cdot |\Delta \underline{v}| \cdot dS \qquad dS = 2 \cdot \Pi \cdot r \cdot dr$$

For ideal plastic material behaviour this becomes :

$$P_F = 2 \cdot \Pi \cdot \frac{m}{\sqrt{3}} \cdot \sigma \cdot A \cdot \int_0^{Rm} r \cdot \left[1-0.75 \cdot \beta \cdot h^2 \right] \cdot r \cdot dr$$

$$Rm = Ru(0.5 \cdot H)$$

E5.5. Nodal point approach, 4 elements

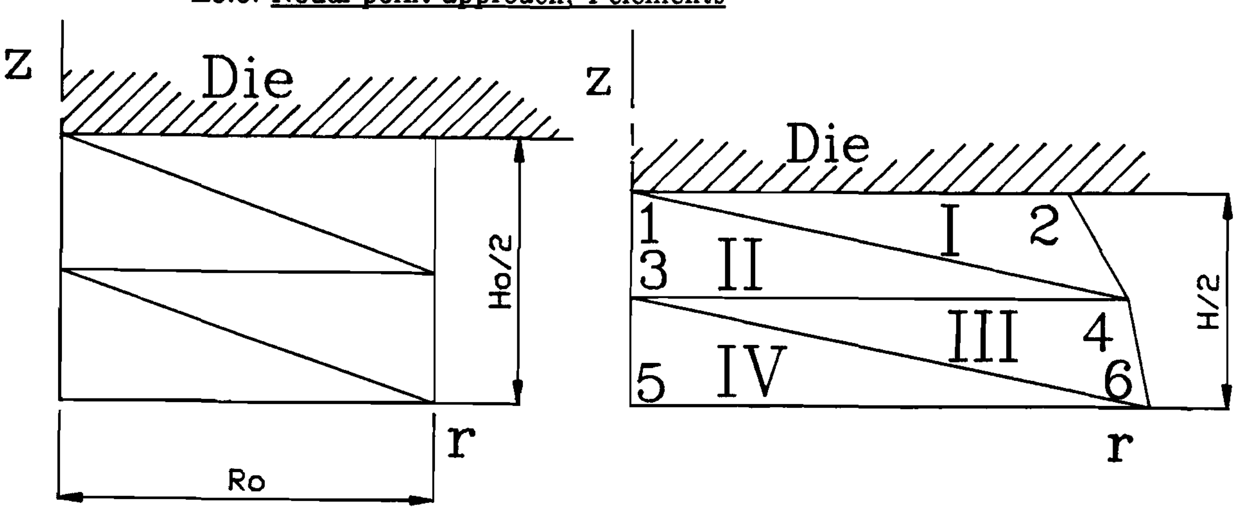


figure 4: description of geometry

Element I:

$$V_{def} = \frac{\pi}{3} \cdot \left[r_2^2 \cdot \frac{h}{2} + r_2 \cdot r_4 \cdot \frac{h}{2} - r_2^2 \cdot z_4 - r_2 \cdot r_4 \cdot z_4 \right]$$
$$V_o = \frac{\pi}{3} \cdot \left[\frac{1}{2} \cdot h_o \cdot r_o^2 \right]$$
$$\frac{1}{2} \cdot h_o \cdot r_o^2 = r_2^2 \cdot \frac{h}{2} + r_2 \cdot r_4 \cdot \frac{h}{2} - r_2^2 \cdot z_4 - r_2 \cdot r_4 \cdot z_4$$

Element II:

$$V_{def} = \frac{\pi}{3}\cdot\left[r_4^2\cdot\frac{h}{2} - r_4^2\cdot z_3 \right]$$
$$V_o = \frac{\pi}{3}\cdot\left[\frac{1}{4}\cdot h_o\cdot r_o^2 \right]$$
$$\frac{1}{4}\cdot h_o\cdot r_o^2 = r_4^2\cdot\left[\frac{h}{2} - z_3 \right]$$

Element III:

$$V_{def} = \frac{\pi}{3}\cdot\left[r_4^2\cdot z_3 + r_4\cdot r_6\cdot z_4 + r_6^2\cdot z_4 - r_6^2\cdot z_3 \right]$$
$$V_o = \frac{\pi}{3}\cdot\left[\frac{1}{2}\cdot h_o\cdot r_o^2 \right]$$
$$\frac{1}{2}\cdot h_o\cdot r_o^2 = \left[r_4^2\cdot z_3 + r_4\cdot r_6\cdot z_4 + r_6^2\cdot z_4 - r_6^2\cdot z_3 \right]$$

Element IV:

$$V_{def} = \frac{\pi}{3}\cdot\left[r_6^2\cdot z_3 \right]$$
$$V_o = \frac{\pi}{3}\cdot\left[\frac{1}{4}\cdot h_o\cdot r_o^2 \right]$$
$$\frac{1}{4}\cdot h_o\cdot r_o^2 = r_6^2\cdot z_3$$

Choose r_6 as parameter:

$$z_3 = \frac{h_o\cdot r_o^2}{4\cdot r_6^2}$$

$$r_4 = r_o\cdot\sqrt{\frac{h_o}{2\cdot h - 4\cdot z_4}}$$

$$z_4 = \frac{\frac{1}{2}\cdot h_o\cdot r_o^2 - r_4^2\cdot z_3 + r_6^2\cdot z_3}{r_4\cdot r_6 + r_6^2}$$

In order to find r_2, the following quadratic equation has to be solved:

$$0 = r_2^2\cdot\left[\frac{1}{2}\cdot h - z_4 \right] + r_2\cdot\left[\frac{1}{2}\cdot h\cdot r_4 - z_4\cdot r_4 \right] + \frac{1}{2}\cdot h_o\cdot r_o^2$$

Enclosure 6: Combined pen extrusion

E6.1. Velocity fields

The velocity field for element I is equal to the velocity field as described in E4.3., the ring upset element.

$$\dot{u}_r = A \cdot r \cdot (1-3 \cdot \beta \cdot z^2) \cdot (r - \frac{R_n^2}{r})$$

In the formula above, β is a parameter describing the severity of the bulging, while R_n describes the position of the neutral surface.

Local volume invariance for axisymmetric processes gives :

$$\dot{u}_z = -2 \cdot A \cdot z \cdot (1-\beta \cdot z^2)$$

The parameter A can be determined with the aid of the boundary conditions for $z = 0.5 \cdot H$ and $z = -0.5 \cdot H$.

$$\dot{u}_z(z = 0.5 \cdot H) = 0.5 \cdot Vo$$

$$A = \frac{Vo}{2 \cdot H \cdot (1-0.25 \cdot \beta \cdot H^2)}$$

The velocity field for element II is equal to the velocity field as described in E4.5., the connecting billet.

$$\dot{u}_r = -B \cdot r \cdot z \cdot (1-\beta \cdot r^2)$$

In the formula above, β is a parameter describing the severity of the bulging.

Local volume invariance for axisymmetric processes gives :

$$\dot{u}_z = B \cdot z^2 \cdot (1-\beta \cdot r^2)$$

As outlined in E4.5.,the constant B has to be determined with the aid of global volume invariance.

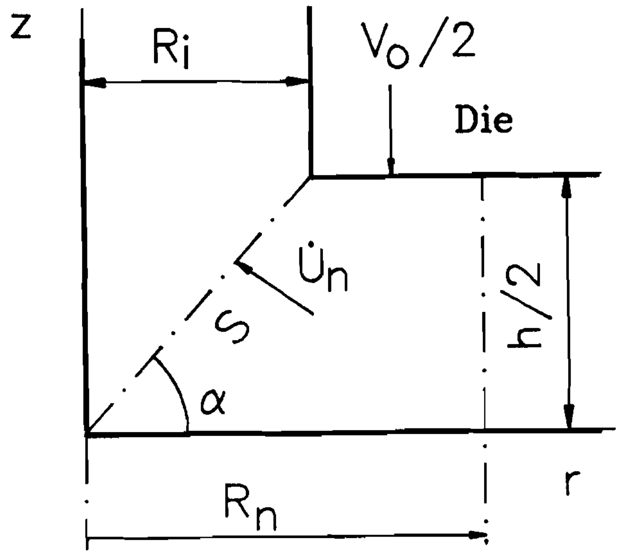


figure 1: description of geometry

The amount of material flowing into the forging at z = h/2, between r = Ri and r = Rn, equals the amount of flow normal to the line $z = \frac{r \cdot h}{2 \cdot Ri}$.

$$V_{in} = \Pi \cdot \frac{Vo}{2} \cdot \left[R_n^2 - R_i^2 \right]$$

$$V_{out} = \int \dot{u}_n \cdot dA \qquad dA = 2 \cdot \Pi \cdot r \cdot ds$$

$$\dot{u}_n = \dot{u}_z \cdot \cos\alpha - \dot{u}_r \cdot \sin\alpha \qquad ds = \frac{dr}{\cos\alpha}$$

$$\tan\alpha = \frac{h}{2 \cdot R_i}$$

$$V_{out} = 2 \cdot \Pi \cdot \int \left[B \cdot z^2 \cdot \left[1 - 2 \cdot \beta \cdot r^2\right] \cdot \cos\alpha + B \cdot r \cdot z \cdot \left[1 - \beta \cdot r^2\right] \cdot \sin\alpha \right] \cdot r \cdot ds$$

$$ds = \frac{dr}{\cos\alpha}$$

$$V_{out} = 2 \cdot \Pi \cdot \int \left[B \cdot z^2 \cdot \left[1 - 2 \cdot \beta \cdot r^2\right] \cdot \cos\alpha + B \cdot r \cdot z \cdot \left[1 - \beta \cdot r^2\right] \cdot \sin\alpha \right] \cdot r \cdot \frac{dr}{\cos\alpha}$$

Suitably rearranged :

$$V_{out} = 2 \cdot \Pi \cdot \int \left[B \cdot z^2 \cdot \left[1 - 2 \cdot \beta \cdot r^2\right] + B \cdot r \cdot z \cdot \left[1 - \beta \cdot r^2\right] \cdot \tan\alpha \right] \cdot r \cdot dr$$

$$z = \frac{r \cdot h}{2 \cdot Ri}$$

$$V_{out} = 2\cdot\Pi\cdot\int\left[B\cdot\left[\frac{h\cdot r}{2\cdot R_i}\right]^2\cdot\left[1-2\cdot\beta\cdot r^2\right]+B\cdot r\cdot\left[\frac{h\cdot r}{2\cdot R_i}\right]\cdot\left[1-\beta\cdot r^2\right]\cdot\tan\alpha\right]\cdot r\cdot dr$$

$$\tan\alpha = \frac{h}{2\cdot R_i}$$

$$V_{out} = 2\cdot\Pi\cdot\int\left[B\cdot\left[\frac{h\cdot r}{2\cdot R_i}\right]^2\cdot\left[1-2\cdot\beta\cdot r^2\right]+B\cdot r^2\cdot\left[\frac{h}{2\cdot R_i}\right]^2\cdot\left[1-\beta\cdot r^2\right]\right]\cdot r\cdot dr$$

$$V_{out} = 2\cdot\Pi\cdot B\cdot\left[\frac{h}{2\cdot R_i}\right]^2\cdot\int\left[r^2\cdot\left[1-2\cdot\beta\cdot r^2\right]+r^2\cdot\left[1-\beta\cdot r^2\right]\right]\cdot r\cdot dr$$

Suitably rearranged:

$$V_{out} = 2\cdot\Pi\cdot B\cdot\left[\frac{h}{2\cdot R_i}\right]^2\cdot\int\left[2-3\cdot\beta\cdot r^2\right]\cdot r^3\cdot dr$$

$$V_{out} = 2\cdot\Pi\cdot B\cdot\left[\frac{h}{2\cdot R_i}\right]^2\cdot\left[\frac{2}{4}\cdot r^4-\frac{3}{6}\cdot\beta\cdot r^6\right]_0^{R_i}$$

$$V_{out} = 2\cdot\Pi\cdot B\cdot\left[\frac{h}{2\cdot R_i}\right]^2\cdot\left[\frac{2}{4}\cdot R_i^4-\frac{3}{6}\cdot\beta\cdot R_i^6\right]$$

$$V_{out} = \frac{1}{4}\cdot\Pi\cdot B\cdot h^2\cdot\left[R_i^2-\beta\cdot R_i^4\right]$$

$$V_{in} = V_{out}$$

$$\Pi\cdot\frac{Vo}{2}\cdot\left[R_n^2-R_i^2\right] = \frac{1}{4}\cdot\Pi\cdot B\cdot h^2\cdot\left[R_i^2-\beta\cdot R_i^4\right]$$

$$B = \frac{2\cdot V_o}{h^2}\cdot\frac{R_n^2 - R_i^2}{R_i^2\cdot(1 - \beta\cdot R_i^2)}$$

The velocity field for element III is equal to the velocity field as described in E4.4., billet through tube.

$$\dot{u}_z = C \cdot \left[1-\gamma \cdot r^2\right]$$

In the formula above, γ is a parameter describing the severity of the bulging.

Local volume invariance for axisymmetric processes gives :

$$\dot{u}_r = 0$$

As outlined in E4.4.,the constant C has to be determined with the aid of global volume invariance.

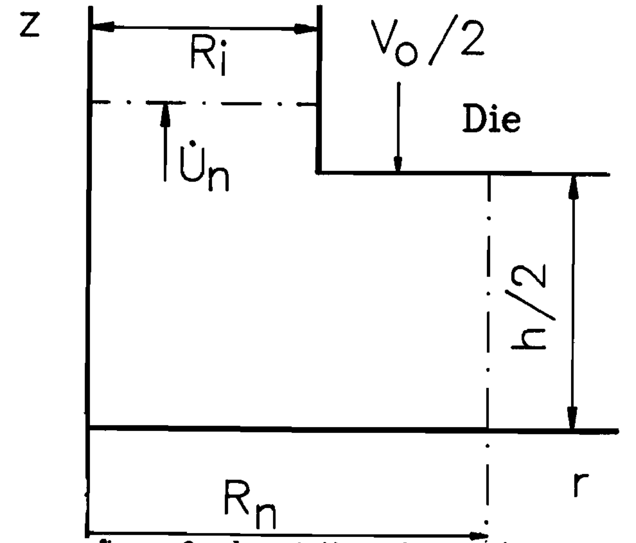


figure 2: description of geometry

The amount of material flowing into the forging at z = h/2, between r = Ri and r = Rn, equals the amount of flow through the tube.

$$V_{in} = \Pi \cdot \frac{Vo}{2} \cdot \left[R_n^2 - R_i^2 \right]$$

$$V_{out} = \int \dot{u}_n \cdot dA \qquad dA = 2 \cdot \Pi \cdot r \cdot dr$$

$$\dot{u}_n = \dot{u}_z$$

$$V_{out} = 2 \cdot \Pi \cdot \int C \cdot \left[1-\gamma \cdot r^2\right] \cdot r \cdot dr$$

$$V_{out} = 2 \cdot \Pi \cdot C \int \left[r-\gamma \cdot r^3\right] \cdot dr$$

$$V_{out} = 2 \cdot \Pi \cdot C \cdot \left[\frac{1}{2} r^2 - \frac{1}{4} \gamma \cdot r^4 \right]_0^{R_i}$$

$$V_{out} = \Pi \cdot C \cdot \left[R_i^2 - \frac{1}{2} \gamma \cdot R_i^4 \right]$$

$$V_{in} = V_{out}$$

$$\Pi \cdot \frac{Vo}{2} \cdot \left[R_n^2 - R_i^2 \right] = \Pi \cdot C \cdot \left[R_i^2 - \frac{1}{2} \gamma \cdot R_i^4 \right]$$

$$C = \frac{V_0}{2} \cdot \frac{R_n^2 - R_i^2}{R_i^2 \cdot (1 - \frac{1}{2} \cdot \beta \cdot R_i^2)}$$

E6.2. Slab method

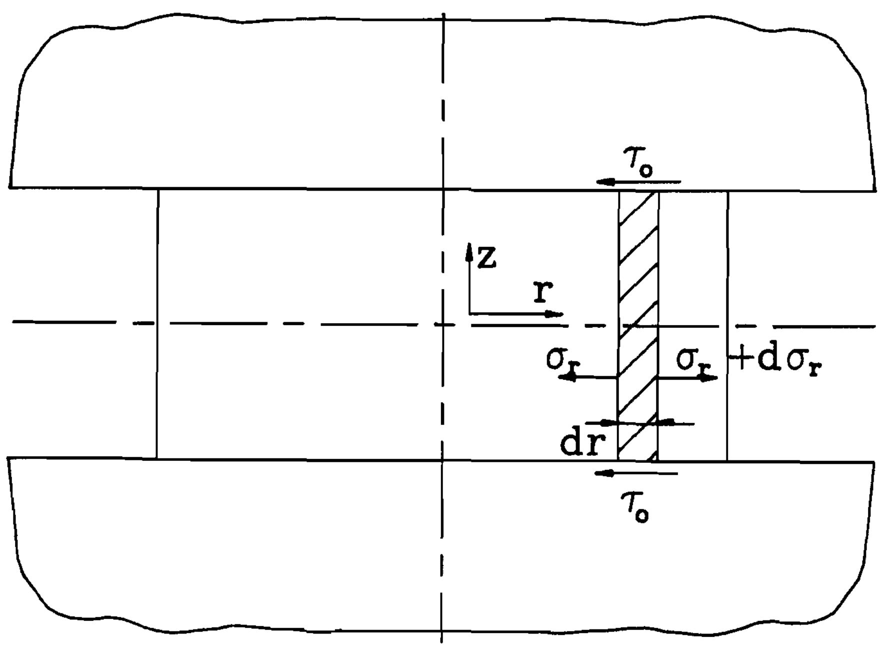


figure 3: description of geometry

In order to obtain flow into the pen, the ratio of the radial stress / yield stress has to be smaller than –1 at the position of the inner radius of the die in the pen extrusion process. The radial stress is analysed with the aid of the slab method for simple upsetting. Although this is arbritrately, the result proved to be satisfying.

Material model : ideal plastic

$$\sigma_f = \sigma_o$$

Friction model : von Mises

$$\tau_o = \frac{m}{\sqrt{3}} \cdot \sigma_f$$

Process model : $\epsilon_r = \epsilon_\varphi$

$\sigma_r = \sigma_\varphi$ (correct for this geometry, see Ramaekers et al. [45])

According to the Tresca yield criterion :

$$\sigma_r - \sigma_z = \sigma_f$$

Radial equilibrium of the slab gives :

$$-\sigma_r \cdot r \cdot d\varphi \cdot h + (\sigma_r + d\sigma_r)(r + dr) \cdot d\varphi \cdot h - 2 \cdot \sigma_\varphi \cdot \frac{d\varphi}{2} \cdot dr \cdot h - 2 \cdot \tau_o \cdot r \cdot d\varphi \cdot dr = 0$$

Suitably rearranged and negelection of the second order parts :

$$d\sigma_r = 2 \cdot \tau_o \cdot \frac{dr}{h}$$

Integration of the above equation with boundary condition :

$\sigma_r(r = R_o) = 0$ gives :

$$\sigma_r = 2 \cdot \tau_o \cdot \frac{r - R_o}{h}$$

Combined with the friction model :

$$\sigma_r = \frac{2}{\sqrt{3}} \cdot m \cdot \sigma_f \cdot \frac{r - R_o}{h}$$

Rearranged :

$$\frac{\sigma_r}{\sigma_f} = \frac{2}{\sqrt{3}} \cdot m \cdot \frac{r - R_o}{h}$$

In order to achieve inward flow at $r = R_i$, $\frac{\sigma_r}{\sigma_f}$ must be less than -1.

$$\frac{2}{\sqrt{3}} \cdot m \cdot \frac{R_i - R_o}{h} \leq -1$$

Suitably rearranged :

$$\frac{2}{\sqrt{3}} \cdot m \cdot \frac{R_o - R_i}{h} \geq 1$$

Note : As presented here, the figure 1 is an under estimation.