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Abstract

Control Structure Design (CSD) is a subproblem of control system design and is concerned with the selection of appropriate manipulated variables, measured variables, and decentralized control configurations. Due to the combinatorial nature of the selection problem, the number of candidate control structures may be huge and favorable candidates are easily overlooked. To circumvent this, CSD must be performed systematically. Despite its major importance for both the performance and the expenses of the system, relatively little attention has been paid to CSD.

Existing methods for CSD are surveyed and assessed. The main conclusion is that to date all methods show deficiencies; especially for nonlinear control systems, little progress has been made. The major shortcomings to be resolved in future research and a potential way towards a systematic CSD method for linear systems are discussed.

1 Introduction

The design of a control system involves six steps. First, the *control objectives* must be formulated, possibly in terms of time domain or frequency domain specifications. For example, a mechanical manipulator should follow a specified trajectory with a certain accuracy, while disturbances and measurement noise must be suppressed. Second, a *model* of the system to be controlled must be derived. Despite the development of advanced controllers, an accurate model is still a prerequisite for high performance. Third, *Control Structure Design* (CSD) must be performed, which is the focus of this paper and defined below. The fourth step in control system design is the design of the *controller*, which determines the control actions to be taken, based on information provided by the measurements. Some well-known controller design methods are PID control (Bueno *et al.* 1991), LQG control (Kwakernaak and Sivan 1972), adaptive control (Slotine and Li 1991), and $\mathcal{H}_\infty$-optimization (Francis 1987). Fifth, the control system's closed-loop behavior is *evaluated* by simulations or experiments. Often, iterative refinements of the preceding steps are necessary, *e.g.*, meeting the control objectives might call for a different control structure or controller design method. Finally, the sixth step is the *implementation* of the controller in the real system. Usually it will be necessary to tune the controller parameters due to differences between the model and reality.

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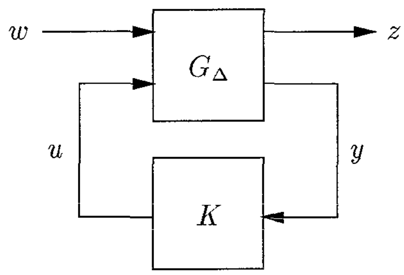

G_Δ	:	system to be controlled
K	:	controller
w	:	exogenous variables, (disturbances, measurement noise, reference signals)
z	:	controlled variables
u	:	manipulated variables (inputs)
y	:	measured variables (outputs)

Figure 1: Standard control system set-up

Consider the control system in Fig. 1. Control structure design involves two successive steps, which are Input Output (IO) selection and Control Configuration (CC) selection. In the IO selection phase, the number, the place, and the kind of inputs u ("actuators") and outputs y ("sensors") to be used for control is selected. It is emphasized that in this context "output" refers to *measured* variables y and not to controlled variables z. The latter are not necessarily equivalent to the measured variables and must be formulated beforehand. The CC selection phase must be performed when designing *decentralized* control systems and refers to the selection of the structural interconnections between the controller inputs y and the controller outputs u. This process of establishing which measured variables are used to determine each manipulated variable is often called "partitioning" (Braatz 1993, Reeves 1991). Note that a different option for CSD is to perform IO and CC selection *simultaneously*, although this approach is rarely encountered in literature.

In a decentralized control system, the actions of each subset of manipulated variables are determined by feedback from only a *subset* of measured variables. Thus, contrary to a centralized control system, there is now only a limited information flow through the controller, by which the performance may suffer. Nevertheless, especially in process control applications, decentralized control is very popular. Some of the advantages of decentralized control over centralized control are mentioned below (see also (Campo and Morari 1994, Hovd 1992)). In the first place, decentralized controllers are easier to design. In particular, the number of controller parameters to be specified is typically much smaller than for a centralized controller. In addition, individual subsystems can be (re)tuned on-line to accommodate the effects of changing process conditions. This is mainly a result of the fact that decentralized controllers are easy for operators to understand. Furthermore, the subsystems can be brought in or out of service individually. This flexibility allows the system to handle changing control objectives during different operating conditions, *e.g.*, start-up, shutdown, or temporary process modifications due to maintenance. Another important advantage is that the control system will be less expensive, since it is easier to implement, easier to maintain, and it avoids expensive communication links. Finally, tolerance for actuator and sensor failures is more easily incorporated in the design of a decentralized controller, which improves reliability: if an actuator or sensor fails, it might suffice to take only the involved subsystem out of service, without changes to other parts of the control system.

Contrary to modeling and controller design, only limited attention has been paid to the step in-between: CSD. Nevertheless, it is equally important. In the first place, an incorrect choice of the control structure may put fundamental limitations on the system's performance, which cannot be overcome by advanced controller design (Reeves 1991). For instance, a particular choice of measured and manipulated variables may introduce Right-Half-Plane (RHP) zeros, which impose restrictions on the achievable bandwidth, regardless of the type of controller that is used (Freudenberg and Looze 1985). In the second place, the control structure determines the number of inputs and outputs and the number of feedback interconnections between them. Restricting the number of inputs and outputs offers advantages with respect to hardware costs, maintenance, and possibly reliability. The advantages of limiting the number of feedback interconnections have already been discussed in the light of decentralized control.

Particularly for systems with a large number of candidate inputs and outputs, selection of the most appropriate control structure is usually far from trivial. Since the number of candidate control structures grows extremely rapidly with the number of candidate inputs and outputs, a systematic and efficient method for CSD is highly desirable. Suppose that there are L candidate outputs and M candidate inputs. During IO selection, a subset of m inputs and l outputs ("an $l \times m$ IO set") must be selected from this overall set. The number of distinct $l \times m$ IO sets is given by $\binom{L}{l}\binom{M}{m}$ with $\binom{X}{x} := \frac{X!}{x!(X-x)!}$. So, the total number of subsystems becomes $\sum_{l=1}^{L}\sum_{m=1}^{M}\binom{L}{l}\binom{M}{m}$. Suppose $L = M = 10$, which yields 1,046,529 distinct candidate IO sets. If the quality of one IO set can be assessed in 30 seconds, it would still take one year to assess all candidates. In the CC selection phase, the rate of growth of the number of candidate configurations is just as dramatic (Reeves 1991, Section 2.3). In conclusion, performing controller design and closed-loop evaluation for each candidate control structure might be the most effective CSD method, but it is not always feasible. This illustrates the need for an efficient CSD procedure, which is able to replace "brute force" approaches on the one hand and to replace approaches based on engineering heuristics on the other.

The main contributions of this paper are threefold. First, it provides a survey of methods for IO selection and CC selection (both for linear and nonlinear systems) which is believed to be rather complete. Second, a set of criteria is proposed which may serve as a basis for a preliminary qualification of existing CSD methods or newly developed ones. Third, these criteria are used to assess the various methods encountered in literature.

The structure of the paper is as follows. In Section 2, some applications of CSD are mentioned, revealing that CSD is important for a wide variety of control systems. Section 3 proposes some criteria for which a CSD method should account. In Section 4, various IO and CC selection concepts from literature are surveyed. Section 5 qualifies and compares the various methods, while the needs for future research are discussed in Section 6.

2 Applications of CSD

The purpose of this section is to illustrate that CSD is important for a wide variety of control applications. There is no attempt to give an exhaustive survey for each particular application. For instance, numerous papers focus on CC selection for distillation columns, but only the ones which are considered the most illustrative are mentioned here. For a more detailed survey of applications *and* the involved CSD methods, the reader is referred to (Van de Wal 1994).

Obviously, a systematic approach for CSD is particularly important for systems with a large number of candidate inputs, outputs, and CC's. As a consequence, it is not surprising that the greater part of literature on CSD stems from the field of process control. In this research area, CSD is related to, *e.g.*, the optimal placement of temperature sensors in distillation columns (Bequette and Edgar 1986, Braatz 1993, Lee *et al.* 1995, Lee and Morari 1991, Moore *et al.* 1987, Morari and Stephanopoulos 1980*a*) and tubular reactors (Kumar and Seinfeld 1978), the choice between temperature sensors and composition analyzers for distillation column control (Moore *et al.* 1987), measurement selection for a double-effect evaporator and a fluid catalytic cracking process, both in (Morari and Stephanopoulos 1980*a*), input selection for a heavy oil fractionator (Rivera 1989) and a boiler (Keller and Bonvin 1987), IO selection for double-effect evaporators (Morari and Stephanopoulos 1980*b*, Narraway and Perkins 1993), a mixer-blender (Morari and Stephanopoulos 1980*b*), and a distillation column (Rijnsdorp 1991, Chapter 14), CC selection for distillation columns (Chang and Yu 1994, Grosdidier and Morari 1986, Hovd and Skogestad 1992, Skogestad *et al.* 1990), boiler furnaces (Manousiouthakis *et al.* 1986, Reeves 1991), a coke oven battery (Fletcher *et al.* 1994), a continuous stirred tank reactor (Manousiouthakis and Nikolaou 1989), a mixing tank (Reeves and Arkun 1989), and for a system of heat-integrated reactors (Manousiouthakis *et al.* 1986), IO and CC selection for the fluid catalytic cracking process (Hovd and Skogestad 1993), for a distillation column (Yu and Luyben 1986), and for a heat-pump used in a distillation plant (Karlsmose *et al.* 1994). In (Govind and Powers 1982), IO selection is performed for a combination of a mixer, a divider, and a heat exchanger.

Applications of CSD as part of a plant-wide control problem are also encountered in literature, *e.g.*, the Tennessee Eastman plant (Banerjee and Arkun 1994, Reeves 1991) and a thermally integrated distillation sequence (Lin *et al.* 1994). In (Morari and Stephanopoulos 1980*b*), IO selection for the Williams-Otto plant is discussed.

In addition, CSD plays a crucial role in aircraft control, see (Geerts 1994, Reeves 1991) (attitude control), and (Samar and Postlethwaite 1994) (engine control). A proper placement of sensors and actuators is also essential for controlling flexible structures, see, *e.g.*, (Byeongsik *et al.* 1994, Hać 1995, Norris and Skelton 1989, Xu *et al.* 1994). The selection of actuator locations for satellite attitude control is discussed in (Müller and Weber 1972).

In (Van de Wal 1994), an active suspension control problem for a tractor-semitrailer combination was suggested as a representative example to evaluate CSD methods. Results for both IO and CC selection, obtained with the MATLAB Control Configuration Toolbox (Reeves *et al.* 1991) (see Sections 4.1.3 and 4.2.2), are reported in (Van de Wal 1995). A proper placement of actuators for an active vehicle suspension is also the focus of (Al-Sulaiman and Zaman 1994).

3 Important Aspects for CSD

In practice, CSD is often carried out in an intuitive and ad hoc fashion rather than systematically. Engineers use experience, simulation and trial and error to guide IO and CC selection. In general, these search techniques are impractical and therefore this survey focuses on *systematic* approaches. In order to assess the "quality" of existing tools for CSD, or newly developed ones, a set of criteria is proposed. Inevitably, the set is not exhaustive, but it is believed to represent the most important properties the "ideal" CSD procedure must possess. The abbreviations provided for each criterion will be used for the comparison in Section 5.

1. *Efficiency (Effic.):* Efficiency is related to the amount of analytical and computational effort. Since it must be possible to quickly and easily evaluate large numbers of candidate control structures, it is a very important property of a CSD method.

2. *Robust Performance (RP):* The control system must perform robustly, *i.e.*, it must remain stable *and* meet the performance specifications for a given level of uncertainty. As a consequence, a control structure must be selected for which it is possible to design a controller which achieves robust performance. Candidate control structures for which this is not possible must be rejected. Robust performance implies both *robust stability* and *nominal performance*, properties which can themselves serve as criteria for screening candidate control structures, see the next two points.

3. *Robust Stability (RS):* The control system must be robustly stable, *i.e.*, it must remain stable for a given level of uncertainty. The choice of the control structure should not endanger this property.

4. *Nominal Performance (NP):* With the control structure, it must at least be possible to meet the performance specifications in the absence of uncertainties.

5. *Controller Independence (CI):* A CSD method must provide a way to eliminate control structures for which *any* controller meeting the control objectives does not exist. Independence of the controller type, the controller design method, and the controller tuning is especially important for initial screening of a large number of candidates.

6. *Effectiveness (Effect.):* Effectiveness is related to the ability to eliminate nonviable candidates and maintain viable ones. Hence, effectiveness calls for necessary *and* sufficient conditions as viability tests, but such conditions often require the design of the controller and may therefore be inefficient for initial screening purposes (Reeves 1991, Section 2.4). Preferably, viability is addressed rigorously in order to generate a subset of viable control structures which is manageable for more detailed further analysis. For instance, viability is to a larger extent accounted for by controllability, NP, and RP respectively.

7. *Quantitative Nature (QN):* A CSD method must be based on quantitative measures to clearly distinguish between the prospects for the candidate control structures, *i.e.*, to address viability in a quantitative rather than in a qualitative way. For example, a quantitative measure for controllability of a system with a particular control structure provides more information on viability than the simple fact that the system is controllable.

8. *General Applicability (GA):* A CSD method should be suitable for a large class of control systems, *e.g.*, systems with an unequal number of inputs and outputs (nonsquare systems) and nonsquare subsystems under decentralized control. Moreover, for linear systems certain frequency ranges might be of special interest. Therefore, a CSD method should not be restricted to one particular frequency (range). Applicability to nonlinear systems is a special aspect of general applicability, see next point.

9. *Applicability to Nonlinear Systems (Nonl.):* Since all systems show nonlinearities, it is desirable that a CSD method is suitable for nonlinear control systems, or can be generalized for application to such systems.

10. *Control System Complexity (CSC):* It must be possible to impose the maximally allowable control system "complexity". A complexity definition at least covers the number of inputs, outputs, and feedback interconnections. In addition, a CSD method should be able to address *other* complexity aspects such as hardware and operating costs, reliability and maintainability, implementation effort and controller design effort, which is expected to be difficult.

11. *Directness (Dir.):* Usually, CSD is based on testing all individual candidates for a particular criterion, which is an indirect procedure. Candidates which do not satisfy the criterion are rejected. For the purpose of efficiency, a CSD method is desired which *directly* comes out with one, or with a few, favorable control structures. If the set of candidate control structures needs to be checked only partially, the method will be called semidirect. The design is iterative if accepted candidates are tested for different criteria in succession. Desirably, the number of iterations is small.

12. *Solid Theoretical Foundation (STF):* The theory behind a CSD method must be well-founded and complete, while a successful application should prove the method's practical relevance.

13. *Practical Applicability (PA):* Desirably, the implementation of the CSD method is not too complex or tedious and application of the method must be straightforward. Moreover, the method must be transparent, *i.e.*, bearing the fundamentals of the method in mind, the way in which the outcome of the CSD procedure is affected by a change in specifications must be interpretable.

It is emphasized that a CSD method which positively addresses all these aspects has not been encountered in literature, nor is it expected that such a method could easily be developed. Nevertheless, the list may serve as a guideline during the development of a CSD method.

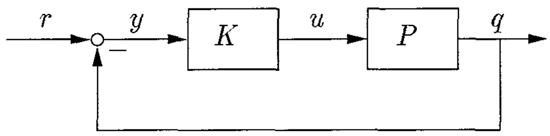

Figure 2: Control system based on measurement feedback

4 Concepts for CSD

Various concepts for IO selection (Section 4.1) and CC selection (Section 4.2) encountered in literature are discussed.

4.1 IO Selection Concepts

Assumptions with respect to y and z in Fig. 1 are often imposed in literature. First, it is sometimes assumed that all controlled variables z are measured, hence $z \subset y$. Second, it is frequently assumed that satisfactory control of the immeasurable variables z is possible via direct control of the measured variables y, by employing a known relationship between y and z. This approach is referred to as "inferential control," see, *e.g.*, (Stephanopoulos 1984) and the measurements are often called "secondary measurements." If it is impossible to transform the specifications for z into equivalent specifications for y, the assumption of inferential control is not justified. Unless noted otherwise, all IO selection concepts are developed under any of these two assumptions and the control system is commonly represented as in Fig. 2.

4.1.1 Structured Singular Value (μ)

The concept of structured singular value (Packard and Doyle 1993) allows uncertainty characterizations and performance specifications to be captured simultaneously. Tools for IO selection which employ the structured singular value, commonly abbreviated μ, are discussed in (Braatz 1993, Braatz *et al.* 1995) and (Lee 1991, Lee and Morari 1991) (secondary measurement selection). The tools suggested in (Lee 1991, Section 3.4) are tied to specific controller design methods, such as LQG control, Model Predictive Control (MPC), or tied to controllers with integral action, see also (Braatz 1993, Braatz *et al.* 1995). In addition, (Braatz 1993, Lee and Morari 1991) discuss screening tools for controllers designed by "robust loopshaping." This controller design method is based on bounds for the maximum singular value of Transfer Function Matrices (TFM's) which parametrize the controller and are of special interest for the particular problem. For instance, in the set-up of Fig. 2 the controller K could be parametrized in terms of the sensitivity $S = (I + PK)^{-1}$, which is a TFM that is of special interest for rejection of disturbances at the plant output q. The key idea for IO selection is to reject candidate IO sets for which a robustly performing controller cannot be designed by robust loopshaping. Application of the tools is restricted to square IO sets and steady-state. Future research should confirm our conjecture that the tools can be generalized to frequency ranges of special interest.

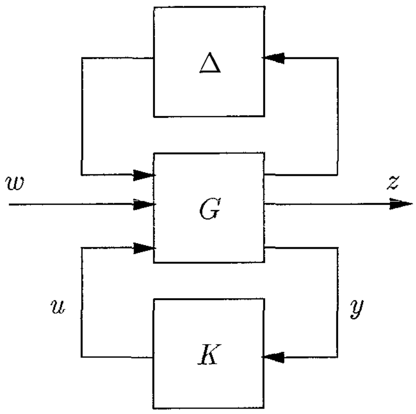

Figure 3: Standard control system set-up with uncertainties Δ separated

In (Rivera 1989), μ is employed in an IO selection approach involving three steps: the candidates are successively tested for satisfaction of constraints, robust stability, and combined satisfaction of constraints and robust stability. The proposed selection tools only apply at steady-state and assume maintenance of integral control in the presence of uncertainty, which is represented by $S(0) = 0$ for the uncertain system in Fig. 2.

4.1.2 Robust Performance (RP)

In (Lee *et al.* 1995), control systems are represented as in Fig. 3, where the structured uncertainties Δ are separated from the generalized plant G_Δ in Fig. 1, so G denotes the nominal generalized plant. One advantage of the IO selection in (Lee *et al.* 1995) is, that performance specifications and uncertainty characterizations are incorporated in G via weighting functions. Another advantage is, that the measured variables y and the controlled variables z are clearly distinguished.

Candidate IO sets are termed viable if there exists a Linear Time Invariant (LTI) controller achieving RP. This key idea is translated into a mathematical condition employing the structured singular value μ. Unfortunately, an algorithm to check the condition for a structured Δ-block is currently lacking. This is resolved by replacing μ with its upper-bound (which is usually very tight (Packard and Doyle 1993)) and dropping the causality requirement on the controller. Causality of the controller implies that its current and future inputs do not affect its past outputs. Hence, causality is required for physical realizability. The derived necessary condition for existence of a robustly performing, possibly acausal controller can only be evaluated for a few special cases, *e.g.*, for an RP problem with a full Δ-block. For more general IO selection purposes, the condition can be split up into two relaxed necessary conditions for the existence of a robustly performing, possibly acausal controller. These conditions are checked via convex optimization over all frequencies ω. A possibly acausal controller does not exist for IO sets disobeying either of these conditions *for some* ω, but even for IO sets which *do* pass both conditions, a possibly acausal controller may not exist.

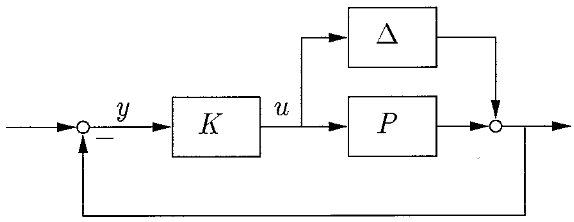

Figure 4: Control system with additive perturbations

Obviously, meeting the test conditions does not imply the existence of a *causal* controller achieving RP. In (Lee *et al.* 1995), it is noted that this drawback is expected to be significant only in the crossover region. However, with respect to performance and robustness properties this region is of special importance and evaluation of the IO sets in the crossover region is therefore recommendable. Another disadvantage is that the test conditions themselves do not provide a clear insight into the way in which the outcome of the IO selection can be affected by changing performance specifications and uncertainty characterizations. A clear insight would help to make an iterative IO selection less burdensome. The practical usefulness of the concept discussed in this section is currently under detailed investigation by the authors.

4.1.3 Combined Robust Stability and Nominal Performance (RSNP)

In (Reeves 1991, Chapter 3), a CSD method is proposed which has also been implemented in a MATLAB toolbox (Reeves *et al.* 1991). Consider the square control system in Fig. 4 with the LTI nominal plant P, the LTI controller K and the additive uncertainties Δ obeying $\overline{\sigma}(\Delta)/\overline{\sigma}(P) \leq \delta_{ra}$. A necessary and sufficient condition for RS serves as the basis for the derivation of an IO selection tool: there exists a controller which stabilizes all $P_\Delta = P + \Delta$ with the same number of RHP poles as P, if and only if

$$\overline{\sigma}(P)\overline{\sigma}(P^{-1}(I-S)) < \frac{1}{\delta_{ra}} \quad \forall\, \omega, \tag{1}$$

with $S = (I + PK)^{-1}$ the nominal sensitivity. In order to make (1) controller independent, a specification for S is invoked. This yields the following *necessary* condition for combined RS and NP, which is used for screening candidate IO sets: there exists a controller which 1) stabilizes all $P_\Delta = P + \Delta$ with the same number of RHP poles as P and which 2) achieves $\overline{\sigma}(S) \leq \sigma_S$ with $\sigma_S < 1\ \forall\, \omega \leq \omega_S$, only if

$$\kappa(P) < \frac{1}{\delta_{ra}(1-\sigma_S)} \quad \forall\, \omega \leq \omega_S, \tag{2}$$

with $\kappa(P) = \overline{\sigma}(P)/\underline{\sigma}(P)$ the Euclidean condition number of P. Since S is crucial for both tracking and disturbance rejection problems, its magnitude is employed as a measure for NP. Qualitatively, (2) implies that for IO sets associated with large condition numbers only small uncertainties (small δ_{ra}) are allowed and only limited performance (large σ_S) can be achieved. Candidate IO sets which do not meet (2) are rejected.

In (Reeves 1991), δ_{ra} is used as a *specification* for the allowable uncertainty associated with each IO set. With this specification and with the NP specification in σ_S, the right-hand-side of (2) is independent of the scaling of u and y, but $\kappa(P)$ is not. In order to make (2) scaling independent, a second IO selection tool is introduced in (Reeves 1991), which replaces $\kappa(P)$ by the scaling independent minimum condition number $\kappa^*(P)$, with $\kappa^*(P) \leq \kappa(P)$, see, *e.g*, (Nett and Manousiouthakis 1987).

In the examples in (Reeves 1991), the same values for δ_{ra} and σ_S are used for all candidates. Although this might result in a computationally *efficient* IO selection method, it can be made considerably more *effective* in the following way. First, it should be taken into account that distinct IO sets may ask for different sensitivity specifications σ_S. Second, knowledge of the expected uncertainty associated with the individual IO sets should be employed, *i.e.*, δ_{ra} should represent an uncertainty bound. Unfortunately, formulating sensitivity specifications and uncertainty bounds for each IO set separately is infeasible for a large number of candidates.

Another drawback is that each controlled variable must be represented by at least one of the measured variables y. This is due to the assumption of inferential control in the set-up of Fig. 4. Consequently, besides meeting (2), additional requirements are imposed on the number and type of measured variables to be preserved. This must be accounted for in advance, *e.g.*, on the basis of engineering insight (see the examples in (Reeves 1991)), which endangers the systematics of the IO selection.

4.1.4 Singular Value Decomposition (SVD)

The Singular Value Decomposition (SVD) of the plant is frequently encountered as an IO selection tool. In (Skogestad and Morari 1987*a*), an input selection method is proposed, based on the effectiveness of rejection of disturbances at the output q of the plant in Fig. 2. Inputs which require the smallest magnitude for disturbance rejection at steady-state are preferred. This is indicated by the "disturbance condition number," that is computed from the SVD. In (Keller and Bonvin 1987), the SVD of a scaled input matrix in the system's state space description is employed as a quantification of the strength and direction of a particular input set on the controlled variables. The approach is aimed at minimizing the number of inputs under preservation of the inputs which are most effective for control.

In (Bequette and Edgar 1986), a measurement selection method is presented which is based on a compromise between the measurements' sensitivity to manipulated variables on the one hand (preferably high) and the sensitivity of *controlled variables* to disturbances under perfect control of the measured variables on the other (preferably small). Steady-state SVD's are employed for this trade-off. Four related SVD-based steady-state measurement selection procedures are described in (Moore *et al.* 1987). The key idea is to find the best compromise for sensitivity of the measurements to the manipulated variables versus interdependence of the measurements.

4.1.5 Controllability and Observability (C&O)

In (Morari and Stephanopoulos 1980*b*), structural state controllability and structural state observability are applied as criteria for IO selection under multivariable "PI control" (augmented state-feedback). For this purpose, the system is represented in a *structural* model, which depends only on invariant aspects of the system. Since a *numerical* model also depends on the values of uncertain parameters, particular values may qualify the system as state uncontrollable or as state unobservable in the numerical sense. Hence, structural properties provide global information about the system, which is particularly useful if numerical values are poorly known. The goal for IO selection is to guarantee structural controllability and structural observability with a minimum number of inputs and outputs. In (Lin *et al.* 1994), *output* structural controllability is the key idea for an IO selection algorithm. Additional information on structural controllability and observability can, *e.g.*, be found in (Georgiou and Floudas 1989), where it is stated that the structural concepts are applicable to nonlinear systems as well.

A disadvantage of the structural properties is the impossibility to draw quantitative conclusions on controllability and observability, in the sense of the strength of the coupling between inputs and states and states and outputs respectively. In (Samar and Postlethwaite 1994), the Hankel Singular Values (HSV's) of the controllability and observability Gramian associated with a minimal balanced realization are used as a quantitative measure for joint state controllability and state observability. Candidate IO sets with large HSV's are preferred. Some other useful, quantitative measures are discussed in (Müller and Weber 1972).

4.1.6 Cost Functions for Control and Estimation (CF)

In (Kumar and Seinfeld 1978, Morari and Stephanopoulos 1980*a*), measurements are selected such that the accuracy of the state estimates is the best possible. For this purpose, particular cost functions are minimized with respect to the candidate measurements. Imperfect estimates might be caused by model uncertainties, disturbances, and inaccurate measurements. In the proposed screening tools, these error sources are treated from a stochastic point of view by introducing process noise and measurement noise in the system's state space description.

Minimization of a quadratic cost criterion that depends on the system's state and the control energy (via u^2) is the basis for selection of inputs and outputs for static output feedback control as discussed in (Xu *et al.* 1994). A related method is proposed in (Norris and Skelton 1989): noisy actuators and sensors, both with dynamics, are chosen to minimize a certain cost function under LQG control. For both methods, an optimization procedure is invoked to find the IO set that minimizes the cost criterion.

In (Al-Sulaiman and Zaman 1994), the input set which yields the minimum value of a quadratic cost function for control energy and control objectives is qualified to be the most appropriate one. Contrary to the approaches discussed above, the cost function is evaluated *after* a state feedback controller has been designed by pole placement and a closed-loop simulation has been run. Obviously, this method is inefficient.

4.1.7 Control Power and Speed (CPS)

In (Rijnsdorp 1991, Chapter 14) and (Bekkers and Rijnsdorp 1994, Rivera *et al.* 1993), control power and control speed are suggested as criteria for selection of inputs. Control power refers to the *static* influence of the inputs on the controlled variables: inputs associated with small steady-state gains between those variables are rejected. Control speed refers to the *dynamic* influence. Desirably, speed of reduction of a deviation in a controlled variable is high and hence inputs associated with large time constants and delays should be rejected (Rivera *et al.* 1993). In (Rijnsdorp 1991, Chapter 14) and (Bekkers and Rijnsdorp 1994), the resonance frequency of the control loop connecting one particular controlled variable with a candidate manipulated variable by a PID controller is used as a measure for control speed. For an objective comparison, the tuning of the PID controller is performed in a similar way for all candidate inputs. In fact, this approach simultaneously selects inputs *and* decides on the pairings between inputs and outputs in a diagonal control configuration.

A related concept for input selection is described in (Yu and Luyben 1986). Based on the work in (Morari 1983), the Morari Resiliency Index (MRI), defined as the minimum singular value of the TFM between the inputs and the controlled variables, is proposed as a measure of the plant's ability to move fast and smoothly from one operating point to another. For an objective comparison employing the MRI, the inputs and the variables to be controlled must be scaled to have the same order of magnitude. The input set with the largest MRI over the frequency range of interest is selected.

4.1.8 Cause-and-Effect Graphs (CEG)

A qualitative technique for generating alternative viable IO sets, based on the cause-and-effect graph of the system at steady-state, is discussed in (Govind and Powers 1982). Such a graph shows the relationships between state variables and manipulated, exogenous, measured, and controlled variables. The key idea for IO selection is, that a causal path must exist between the manipulated and the controlled variables on the one hand and the measured and the controlled variables on the other: with the manipulated variables it must be possible to affect the controlled variables, while with the measured variables it must be possible to obtain the values for the controlled variables. Unfortunately, a large number of candidate IO sets may termed to be viable and additional IO selection criteria must be invoked to address viability more rigorously. Since for *nonlinear* systems, cause-and-effect graphs can also be generated, the concept offers prospects for these systems as well.

4.1.9 Location of Right-Half-Plane Zeros (RHPZ)

It is well-known that Right-Half-Plane (RHP) zeros of the plant limit the achievable closed-loop performance in terms of the sensitivity and complementary sensitivity, regardless of the controller type, see, *e.g.*, (Freudenberg and Looze 1985) and (Maciejowski 1989, Sections 1.7 and 3.6). For instance, RHP zeros impose an upper-bound on the achievable bandwidth, which is disadvantageous for tracking and attenuation of output disturbances. For this reason, in (Hovd 1992, Chapter 3) and (Hovd and Skogestad 1993, Samar and Postlethwaite 1994) it is suggested that an IO set must be selected for which as few as possible RHP zeros occur and that they are as far away from the origin as possible.

4.1.10 Economic Optimality (EO)

In (Narraway and Perkins 1993), economic optimality of an IO set is related with the trade-off between instrumentation costs and operating benefits. The procedure is restricted to square, linear process models with dominant steady-state aspects and perfectly controlled measured variables. An optimization algorithm is invoked to generate a specified number of IO sets (size not specified) which are economically most viable, while the minimum plant condition number (preferably small, see Section 4.1.3) and the occurance and location of RHP zeros (preferably none or far away from the origin, see Section 4.1.9) are used as controllability indicators. Economics in conjunction with controllability is then used to decide on the best IO set.

4.2 CC Selection Concepts

The majority of CC selection methods encountered in literature employ a control system set-up similar to the one in Fig. 2, in which it is assumed that control of the measured variables y is the objective (note that this is contrary to Fig. 1, where the measured and controlled variables are treated separately). Unless noted otherwise, this will apply for all CC selection methods to be discussed. For more general control problems where this assumption does not hold, the methods of interest may be useless.

Usually, CC selection aims at minimizing the effects of interactions in the control system. In this context, interaction is related to the way in which the TFM of a particular subset of the manipulated variables and a particular subset of the controlled variables, *i.e.*, the TFM of a particular subsystem of the plant P, is affected by control of the other subsystems. Generally, the effect of manipulated variable actions on controlled variables which were not used to compute those actions will degrade the performance. In the sequel, $\overline{P}$ = block diag(P_{ii}) denotes the TFM whose configuration corresponds to the configuration of the *decentralized* controller K in Fig. 2. The TFM's $\overline{S} = (I + \overline{P}K)^{-1}$ and $\overline{T} = \overline{P}K(I + \overline{P}K)^{-1}$ are called the "ideal" sensitivity and the "ideal" complementary sensitivity respectively, since they are composed of non-interacting subsystems.

4.2.1 Structured Singular Value (μ)

The structured singular value μ is used to derive a dynamic interaction measure for square systems with (block) diagonal controllers in (Grosdidier and Morari 1986). The interactions are interpreted as additive uncertainties for $\overline{P}$ as in Fig. 4. The measure predicts the stability of the decentralized control system and the performance loss due to the decentralized configuration. In (Braatz 1993, Chapter 6) and (Braatz *et al.* 1995), the μ interaction measure is generalized to handle model uncertainties. For the purpose of CC selection, the interaction measure at steady-state is considered, giving rise to a screening tool based on a necessary condition for existence of a (robustly performing) decentralized control system supplied with integral action ($\overline{S}(0) = 0$). However, we expect that it is possible to make the screening tools suitable for application in frequency ranges of special interest, by specifying the desirable performance in terms of $\overline{S}$ or $\overline{T}$.

In (Braatz 1993, Chapter 6) and (Braatz *et al.* 1995) it is shown that the same tools for CC selection result if they are derived based on the concept of (robust) Decentralized Integral Controllability (DIC) (Campo and Morari 1994), see also Section 4.2.4. A plant P is said to possess (robust) DIC if there exists a *diagonal* controller with integral action in all channels, *i.e.*, $K(s) = \frac{1}{s}\hat{K}(s)$ with $\hat{K}(0)$ nonsingular, such that the closed-loop system is (robustly) stable if the gains of any combination of loops are reduced independently. This implies that the control loops can be detuned or taken out of service without endangering stability. Necessary conditions for non-existence of a control system achieving (robust) DIC are derived, which can be employed for CC selection. Also in (Rivera 1989), a μ-based test for nominal DIC is proposed as a CC selection tool. In addition, a steady-state CC selection tool is derived for *combined* satisfaction of constraints, robust stability (under integral control), and nominal DIC.

4.2.2 Combined Nominal Performance and Performance Degradation (NPPD)

In (Reeves 1991, Chapter 4) (also implemented in MATLAB (Reeves *et al.* 1991)), selection of block diagonal configurations for the system in Fig. 2 is based on the following necessary and sufficient condition for performance degradation due to decentralization: for the LTI nominal plant P there exists a LTI decentralized controller K which satisfactorily limits performance degradation, if and only if

$$\overline{\sigma}((T - \overline{T})\overline{T}^{-1}) \leq d_T \quad \forall\, \omega, \tag{3}$$

where d_T is a frequency dependent upperbound on the "difference" between the true complementary sensitivity T and the ideal complementary sensitivity $\overline{T}$, relative to $\overline{T}$. In order to make (3) controller independent, a specification for $\overline{T}$ is invoked. This yields the following *necessary* condition for combined nominal performance and limited performance degradation: for the LTI nominal plant P there exists a LTI decentralized controller K which 1) achieves $\overline{\sigma}(\overline{T}) \leq \sigma_{\overline{T}}$ with $\sigma_{\overline{T}} < 1\ \forall\, \omega \geq \omega_{\overline{T}}$ and which 2) achieves $\overline{\sigma}((T - \overline{T})\overline{T}^{-1}) \leq d_T$, only if

$$\frac{(1 - \sigma_{\overline{T}})\overline{\sigma}(V)}{1 + (1 - \sigma_{\overline{T}})\overline{\sigma}(V)} \leq d_T \quad \forall\, \omega \geq \omega_{\overline{T}}, \tag{4}$$

with $V = (P - \overline{P})P^{-1}$. According to (Reeves 1991), provision 1) accounts for nominal performance by imposing an upperbound on the ideal complementary sensitivity function $\overline{T}$, while provision 2) accounts for nominal performance *degradation*. Candidate configurations which do not meet (4) are termed nonviable. Note that performance is generally related to low and intermediate frequencies. As a consequence, it is doubtful if CC selection is physically meaningful at frequencies far above $\omega_{\overline{T}}$. Instead, it seems recommendable to perform CC selection in and just above the crossover region.

Criterion (4) depends on scaling due to the presence of $\overline{\sigma}(V)$; $\sigma_{\overline{T}}$ and d_T are specified under the assumption that the plant is properly scaled. This is the motivation in (Reeves 1991) to introduce a second CC selection tool, which replaces $\overline{\sigma}(V)$ by a scaling independent variable $\Psi(V)$ with $\Psi(V) \leq \overline{\sigma}(V)$, causing more candidates to pass the second criterion.

In analogy with the IO selection procedure in Section 4.1.3, in (Reeves 1991) the same specifications $\sigma_{\overline{T}}$ and d_T are used for all candidate CC's, which results in an *efficient* CC selection procedure. Because each configuration may ask for different specifications however, CC selection will be more *effective* if the specifications are formulated for each candidate individually. Unfortunately, in this way CC selection is tedious for a large number of candidates. Moreover, since performance and performance degradation are addressed with respect to the measured variables y, the CC selection criterion is only meaningful if specifications on the variables to be controlled can be transformed into equivalents for the measured variables.

Configurations which pass criterion (4) are not guaranteed to be stabilizing. For this reason, in (Banerjee and Arkun 1994) a necessary condition for stabilizability under block diagonal decentralized controllers with integral action (based on the Niederlinski index, see also Section 4.2.4) is used to reject nonviable configurations, followed by screening of the remaining configurations with (4).

4.2.3 Relative Gain and Related Concepts (RG)

Of all interaction measures for diagonal control configurations, the use of the Relative Gain Array (RGA) is certainly the most widespread. Originally, the RGA was defined and applied at steady-state (Bristol 1966), but it may easily be extended to higher frequencies, see, *e.g.*, (Hovd and Skogestad 1992, Skogestad and Hovd 1990). For the definition of the RGA, the control system in Fig. 2 with $r = 0$ is considered. If all other outputs are uncontrolled, *i.e.*, if all other loops are open, the gain from input u_j to output q_i is $P_{ij}(s)$. Furthermore, writing $u(s) = P^{-1}(s)q(s)$ it is concluded that the gain from u_j to q_i with all the other elements in q perfectly controlled ($q_j = 0 \; \forall \; j \neq i$) is $1/[P^{-1}(s)]_{ji}$. The relative gain is defined as the ratio of these open-loop and closed-loop gains. Preferably, this gain is close to 1, which indicates that only weak interactions occur and independent control of the loops is easier to achieve. A matrix of relative gains, the RGA Λ, can be computed at each frequency ($s = \mathrm{j}\omega$) using the formula

$$\Lambda(s) = P(s).*(P^{-1}(s))^T, \tag{5}$$

where ".*" denotes element-by-element multiplication. The assumption that all "other" loops are perfectly controlled is in practice only justified for a specific frequency (range). This is the main reason that for a long time the use of the RGA has been restricted to steady-state problems, for which perfect control can be achieved by integral action. In (Bristol 1966, Hovd and Skogestad 1992, Hovd and Skogestad 1993, Hovd and Skogestad 1994, Samar and Postlethwaite 1994, Skogestad and Hovd 1990, Skogestad and Morari 1987*b*), some rules for selecting the best diagonal configuration are proposed, see also (Van de Wal 1994, Section 4.4) for an overview. These rules, which address different aspects related to, *e.g.*, (robust) stability, tolerance for loop failures, performance degradation, and occurance of RHP zeros, can be used for a direct selection of the preferred diagonal configuration based on the RGA for a particular IO set.

One inadequacy of the RGA is that it only measures two-way interactions, *e.g.*, $\Lambda = I$ for a triangular plant P. Therefore, it may indicate that interactions are not a problem, even though significant one-way coupling may exist. The Performance Relative Gain Array (PRGA, (Hovd and Skogestad 1992, Hovd and Skogestad 1993, Skogestad and Hovd 1990)) resolves this shortcoming. It addresses the effect of a setpoint change r_i for q_i on the error in q_j and it is used to select the CC for which the NP requirements below the crossover frequency are most easily satisfied. In analogy, the Closed-Loop Disturbance Gain (CLDG, (Hovd and Skogestad 1992, Hovd and Skogestad 1993, Skogestad and Hovd 1990)) is developed, which addresses the effect of a disturbance on q_i on the error in q_j.

The Block Relative Gain (BRG) (Manousiouthakis *et al.* 1986, Nett and Manousiouthakis 1987) generalizes the RGA to *block diagonal* control configurations. Contrary to the RGA, each subsystem under decentralized control has its own BRG, which must be recomputed for different configurations. In analogy to the RGA, the BRG measures interactions via the "ratio" of two TFM's of the plant P evaluated for a particular frequency (range). In (Manousiouthakis *et al.* 1986), CC selection is based on eliminating configurations for which the associated BRG's are "not close" to identity matrices. A related concept is the *Dynamic* Block Relative Gain (DBRG) (Arkun 1986, Arkun 1987) of a particular subsystem of the plant. Contrary to the BRG, it does not rely on the assumption of perfect control in the other subsystems under decentralized control. However, it depends on the controllers associated with the other subsystems, by which the DBRG is less appropriate for screening a large number of candidate CC's.

To circumvent the disadvantages related to the BRG (assumption of perfect control) and the DBRG (dependence on controller data), the relative sensitivity is introduced in (Arkun 1988) as a closed-loop interaction measure for performance. It also accounts for one-way interaction, but application is tied to a specific controller type, namely decentralized Internal Model Control (IMC). The "relative sensitivity matrix" at $\omega = \infty$ is independent of the controller tuning and is therefore used to guide CC selection. The elements of this matrix indicate how much the j-th subsystem is excited relative to the response of the i-th subsystem for a setpoint change r_i.

By invoking the concept of pseudo-inverse of a matrix, the RGA and the (D)BRG are generalized to nonsquare systems in (Chang and Yu 1990) and (Reeves and Arkun 1989) respectively. With respect to the "nonsquare RGA" it is assumed that the number of inputs is smaller than the number of outputs, hence perfect control is impossible. In (Manousiouthakis and Nikolaou 1989), the (Dynamic) Nonlinear Block Relative Gain ((D)NBRG) is introduced as an interaction measure for decentralized *nonlinear* control systems. Contrary to the NBRG, the DNBRG depends on controller properties.

4.2.4 Nominal Stability and Integrity (NSI)

The so-called Niederlinski Index (NI) is often employed for CC selection, see, *e.g.*, (Banerjee and Arkun 1994, Chiu and Arkun 1990, Grosdidier and Morari 1986, Hovd and Skogestad 1994, Yu and Luyben 1986). It is a steady-state measure, which provides a necessary condition on the open-loop plant P for nominal stability under decentralized integral control ($\overline{S}(0) = 0$) with a particular (block) diagonal CC. In (Chiu and Arkun 1990), the NI is used in combination with steady-state BRG's for a CC selection method which is based on the following requirement: the control system must be stabilized by a stable decentralized controller with integral action and must possess "integrity." A control system demonstrates integrity if it maintains its nominal stability if any combination of controller blocks is out of service, see, *e.g.*, (Campo and Morari 1994, Hovd and Skogestad 1994).

A closely related concept for selection of diagonal configurations is based on the property of Decentralized Integral Controllability (DIC), see Section 4.2.1. In (Campo and Morari 1994) and (Morari and Zafiriou 1989, Chapter 14), (steady-state) conditions for DIC are derived that could be employed for CC selection. In (Campo and Morari 1994), also other conditions on the open-loop steady-state plant are derived for existence of a decentralized controller with integral action and particular closed-loop properties related to stability and integrity. These conditions might be useful for the purpose of CC selection as well.

4.2.5 Direct Nyquist Array (DNA)

In (Jensen *et al.* 1986), the combination of the Direct Nyquist Array (DNA) and Gershgorin bands is suggested to guide selection of *diagonal* configurations. The DNA is an array of polar plots of the elements of the plant TFM $P(\mathrm{j}\omega)$ (Fig. 2) and is interpreted as a graphical representation of the open-loop relationships of the inputs and outputs. The Gershgorin bands are formed by drawing circles with centers on the locus of P_{ii}. The radii of these circles are equal to the sum of the magnitudes of all off-diagonal elements of P in column i. Comparing the magnitude $|P_{ii}|$ at a given frequency with the radius of the Gershgorin circle at that frequency provides a measure of dynamic interactions. To minimize these interactions, the diagonal configuration corresponding with the largest DNA elements and the narrowest Gershgorin bands must be selected.

4.2.6 Relative Degree (RD)

The relative degree of a controlled variable with respect to a manipulated variable is used as a characterization of the dynamic interactions in a square control system in (Daoutidis and Kravaris 1990). Note that this differs from the more common notion of the relative degree as an integer computed for a controlled variable and the *vector* of manipulated variables, see, *e.g.*, (Isidori 1989, Chapter 5). Intuitively, the relative degree is related to the "physical closeness" between inputs and outputs, which is mentioned in (Morari 1983) as an important heuristic used for CSD. In this respect, the configuration associated with the smallest relative degrees offers the best prospects. The concept is directly suitable for *nonlinear* control systems and it could be employed for IO selection as well. Furthermore, the relative degree can be used to identify *groups* of inputs and outputs characterized by a weak structural coupling with the other ones, thus suggesting favorable candidates for *block diagonal* CC's. Computation of the relative degree does not require numerical information of the system. In (Daoutidis and Kravaris 1990), it is emphasized that other CC selection tools should be employed in addition, towards a more quantitative assessment of each candidate.

4.2.7 Interaction Potential (IP)

In (Huang *et al.* 1994), an approach is proposed for selecting diagonal configurations which involves two steps. First, as many nonviable configurations as possible are eliminated by applying the steady-state RGA (see Section 4.2.3). Second, preferable configurations are determined by using the "interaction potential matrix," whose elements provide a measure for the *possible* dynamic interactions from all other loops to one particular configured loop. This measure is developed under the assumption of IMC and the desired closed-loop performance and robustness are specified via $\overline{T}$. If desirable, weights can be attached to the elements of the interaction potential matrix. The (weighted) elements corresponding to a particular pairing are summed to a *total* interaction potential measure and a *preliminary* configuration is selected based on the smallest total interaction potential. To decide on the *optimal* configuration, an iterative procedure of controller design for the preliminary configuration, checking closed-loop stability, respecifying the closed-loop bandwidth, and recomputing the interaction potential matrix follows.

4.2.8 Numerical Invertibility (NInv.)

The numerical invertibility of the plant at steady-state is suggested as a measure for interaction analysis in (Mijares *et al.* 1986). This is based on the proposition that the diagonal configuration which most closely resembles a set of independent SISO systems is the best. The interactions are assessed by analyzing the effect of the off-diagonal elements of $P(0)$ on the difficulty of obtaining $P(0)^{-1}$ by an iterative procedure: with an increase in the dominance of the diagonal elements, the inverse is more easily to obtain. The rate of convergence of the iterative procedure provides a quantitative measure for invertibility and the diagonal configuration with the best invertibility is selected. Although the development of the criterion is based on purely algebraic properties, its physical meaning is shown by its relationship with stability characteristics.

4.2.9 Decentrally Fixed Eigenvalues (DFE)

Eigenvalues which cannot be shifted by a controller are called fixed eigenvalues. The uncontrollable and unobservable eigenvalues are called *centrally* fixed. If a decentralized controller is applied, additional *decentrally* fixed eigenvalues may occur (Wang and Davison 1973), which depend on the particular CC. Hence, configurations giving rise to unstable decentrally fixed eigenvalues (Hovd 1992), or otherwise undesirable eigenvalues, must be eliminated. Decentrally fixed eigenvalues can easily be identified by computing the closed-loop eigenvalues for an arbitrary constant output feedback matrix K with the particular configuration, since these eigenvalues are also fixed for dynamic output feedback. Closed-loop eigenvalues which coincide with open-loop eigenvalues and which are no centrally fixed eigenvalues correspond to the decentrally fixed eigenvalues with probability one. CC selection based on this concept does not require control of the measured variables.

4.2.10 Singular Value Decomposition (SVD)

In (Lau *et al.* 1985), singular values and singular vectors of P in Fig. 2 are used to find the diagonal configurations which are preferable for control. The associated loops which interact minimally with other ones are called "natural loops." An interaction measure is developed which quantifies the difference between the candidate CC's. Since the SVD on its own might not be able to indicate a unique pairing, in our opinion a combined study of the SVD and the interaction measure should be used to find the best pairing. In order to encompass both static and dynamic effects, the analysis should be carried out over the frequency range of interest.

5 Comparison

The various concepts for IO selection and CC selection are assessed, based on the 13 desirable properties listed in Section 3. The extent to which these properties are fulfilled is visualized in Tables 1 (IO selection) and 2 (CC selection), with the following meaning of the symbols:

+	:	The property is *positively* addressed.
0	:	The property is *not satisfactorily* addressed.
−	:	The property is *negatively* addressed or *not* addressed at all.
?	:	The property needs *further investigation* in order to address its prospects.

Since all concepts offer the possibility to impose the maximum number of inputs and outputs or feedback interconnections, only *other* complexity aspects, like those mentioned in Section 3, are considered to assess the ability to address complexity. It is emphasized that the assessment of the IO and CC selection concepts is purely based on the theory and the examples found in literature. For a thorough comparison, a more critical and more detailed analysis should be performed in combination with an evaluation for a representative example.

Table 1: The IO selection methods assessed

Section		Desirable Properties of a CSD Method												
		1:Effic.	2:RP	3:RS	4:NP	5:CI	6:Effect.	7:QN	8:GA	9:Nonl.	10:CSC	11:Dir.	12:STF	13:PA
4.1.1	μ	0	0	0	0	−	−	+	?	−	?	−	+	+
4.1.2	RP	−	+	+	+	+	−	+	+	−	?	−	+	−
4.1.3	RSNP	+	−	+	+	+	−	+	0	?	−	−	+	+
4.1.4	SVD	0	−	−	−	+	0	+	0	−	−	+	+	+
4.1.5	C&O	+	−	−	−	+	0	+	+	?	−	−	+	+
4.1.6	CF	−	−	−	+	−	+	+	+	?	?	0	+	0
4.1.7	CPS	−	−	−	0	−	+	+	−	−	−	−	0	+
4.1.8	CEG	+	−	−	−	+	−	−	+	+	−	−	+	+
4.1.9	RHPZ	+	−	−	0	+	0	0	0	?	−	−	0	+
4.1.10	EO	−	−	−	−	+	−	+	−	−	0	−	+	−

5.1 IO Selection Methods Compared

From Table 1, it is concluded that only one concept explicitly accounts for robust performance, while only one additional method accounts for robust stability. This is disappointing, since robustness is a major issue in modern control system design: stability and a guaranteed level of performance is always required, even in the presence of inevitable modeling errors. Ineffectiveness and computational inefficiency are two of the main disadvantages of the RP-based method, which is not known to have a nonlinear equivalent. Nevertheless, the facts that it is controller independent and generally applicable make this method the most promising one for linear systems. The prospects for addressing control system complexity merit further investigation. It is conjectured, that it is possible to address control system reliability in IO selection by employing μ for modeling actuator and sensor faults and failures (Braatz *et al.* 1994), (Braatz 1993, Chapter 5).

The method based on cause-end-effect graphs is the only one which is suitable for nonlinear systems. Unfortunately, it lacks many other desirable properties. Some concepts are known to have equivalents for nonlinear systems, but their usefulness should be studied in future research: (Nijmeijer and Van der Schaft 1990, Chapter 3) proposes equivalents for controllability and observability, (Nijmeijer 1990) discusses cost functions in optimal control of nonlinear systems, and (Isidori 1989, Section 4.3) explains the theory of unstable zero dynamics, which is a potential generalization of RHP zeros. Furthermore, $\mathcal{H}_\infty$ control theory has been extended to nonlinear systems, see, *e.g.*, (Isidori and Kang 1995, Van der Schaft 1992). So, future research aimed at an IO selection procedure for nonlinear systems based on either RS under unstructured uncertainties or NP is worthwhile.

In the method based on economic optimality, the costs associated with actuators and sensors as one aspect of complexity is accounted for. Like the RP-based method, IO selection employing the structured singular value might offer prospects to account for reliability. The IO

Table 2: The CC selection methods assessed

Section		Desirable Properties of a CSD Method												
		1:Effic.	2:RP	3:RS	4:NP	5:CI	6:Effect.	7:QN	8:GA	9:Nonl.	10:CSC	11:Dir.	12:STF	13:PA
4.2.1	μ	0	0	0	0	−	−	+	?	−	0	−	+	+
4.2.2	NPPD	+	−	−	+	+	−	+	0	−	−	−	+	+
4.2.3	RG	+	−	0	0	+	0	+	+	+	0	0	0	+
4.2.4	NSI	+	−	−	−	−	−	+	−	−	0	−	+	+
4.2.5	DNA	−	−	−	−	+	−	0	−	−	−	−	+	−
4.2.6	RD	+	−	−	−	+	−	0	0	+	−	+	0	+
4.2.7	IP	−	−	0	+	−	+	+	0	−	−	−	−	−
4.2.8	NInv.	+	−	−	−	+	?	+	−	−	−	−	0	+
4.2.9	DFE	+	−	−	−	+	−	0	+	−	−	−	0	+
4.2.10	SVD	0	−	−	−	+	0	+	0	−	−	−	−	+

selection based on minimizing cost functions might be made suitable for evaluating control system complexity by formulating additional weighting functions for the candidate actuators and sensors which represent their costs, reliability, and maintainability.

Only one IO selection approach is direct: the SVD-based method computes the SVD for the overall plant and the preferable inputs or outputs are directly visible. In the methods based on cost functions, optimization methods are invoked which assign the optimal set of inputs or outputs. Usually, the algorithms are iterative and the optimal variables are not obtained in one step. Since the optimization algorithm modifies a nonoptimal choice in a "smart" way, *not all* candidates need to be checked (which is in contrast with the methods qualified as "−") and the method is called semidirect.

5.2 CC Selection Methods Compared

From Table 2, it is concluded that to date there is no CC selection method which satisfactorily takes RP into account. Under integral action, the method based on μ only supplies steady-state conditions for RS and RP. Obviously, such conditions are inappropriate for, *e.g.*, tracking problems and dynamic disturbance rejection problems. For some special circumstances (*e.g.*, diagonal input uncertainty (Hovd and Skogestad 1992)), the RG-based methods provide "indications" when RS with a particular CC may be endangered (Arkun 1986, Chen *et al.* 1994, Hovd and Skogestad 1992, Skogestad and Morari 1987*b*). In the IP-based method, the bandwidth specification is not only based on the desired speed of response, but on modeling error considerations as well.

Since a lot of attention has been paid to the RGA after it was first introduced in 1966 (Bristol 1966), its application area has been extended considerably. The related BRG has been extended to nonlinear systems (Manousiouthakis and Nikolaou 1989), but since it requires nonlinear differential equations to be solved, efficiency will be low. Major disadvantages of the other CC selection concept which is suitable for nonlinear systems, the method based on relative degrees, are its ineffectiveness (many configurations might be qualified as viable) and its inability to account for performance or robustness. A common advantage of the RD-based method and the method based on the original RGA in equation (5) is their directness. A matrix of relative degrees and a matrix of relative gains respectively need to be computed only once for the particular IO set under consideration. From these matrices, the preferable configurations are directly visible.

Three methods offer the possibility to address reliability as one aspect of control system complexity. In the context of μ, a sufficient condition for a diagonal controller achieving (robust) DIC is derived (Braatz 1993, Chapter 6). In (Braatz *et al.* 1994) and (Braatz 1993, Chapter 5), μ is employed for modeling faults and failures in more general decentralized control systems. Future research must reveal if CC selection tools can be developed which rigorously address reliability aspects. The combination of RGA/NI and BRG/NI is used in (Hovd and Skogestad 1994) and (Chiu and Arkun 1990) respectively to address integrity in CC selection.

6 Needs for Future Research

The main conclusion which can be drawn from Section 5 is, that to date satisfactory methods for IO and CC selection do not exist (not even for linear control systems). Future research should be aimed at filling this gap, bearing in mind the desirable properties of a CSD method listed in Section 3.

While designing a control system and hence while designing the control structure, a wide variety of issues must be paid attention to. Inspired by the paradigm stated in (Reeves 1991, Chapter 1), the goal of control system design could be formulated as follows:

Minimize control system complexity, subject to the achievement of the control objectives. (I)

In (Reeves 1991, Section 5.4), it is noted that control system complexity is not a well-defined concept and that a simple definition of complexity in the context of CSD would be the sum of the number of inputs and outputs (determined during IO selection) and the number of feedback interconnections between them (determined during CC selection). Some additional aspects of complexity to be addressed during CSD are the following: costs of sensors, actuators, and communication links (hardware costs), operating costs, the control system's reliability and maintainability, the effort for controller design and implementation, and the controller order. In analogy to "complexity", different aspects must be covered by the "control objectives." A major aspect is to stabilize the control system and to meet the accuracy specifications in the face of uncertainty, *i.e.*, to achieve robust performance. Other important considerations should be made with respect to safety, product yield (process industry), marginal costs, and handling of constraints (*e.g.*, on manipulated and controlled variables).

However, at present the development of a CSD method which covers *all* these aspects of complexity and control objectives seems too ambitious. Instead, efforts could be aimed at developing a CSD method that focuses on the following more restricted version of goal (I):

Minimize the number of inputs and outputs and the number of interconnections between them, subject to the achievement of robust performance. (II)

Consequently, only control structure candidates for which it is possible to achieve the desired level of RP are termed viable. If the number of remaining control structures is small enough, the designer could subject them to an informal assessment for the other aspects of complexity and control objectives. The development of a systematic, formal way of assessing is an important topic for future research.

A recommendation is to develop a CSD method for *linear* systems based on goal (II). For this purpose, the control system set-up in Fig. 3 serves as the starting point for CSD. This allows (1) controlled variables z and measured variables y to be treated separately, (2) frequency-dependent performance specifications and uncertainty characterizations to be incorporated, and (3) nonsquare controllers K to be handled, while the majority of existing methods fails to deal with these desirable aspects, at least simultaneously. The IO selection approach discussed in Section 4.1.2 is based on this set-up, but the method's efficiency and effectiveness need considerable improvement.

Employing the set-up of Fig. 3 is commonly restricted to finite dimensional, linear, time-invariant control systems with L_∞-norm-bounded, stable, structured uncertainties Δ, though in (Braatz *et al.* 1994) it is stated that time-varying, linear and nonlinear uncertainties can be dealt with as well. With a given IO set, RP-analysis and RP-synthesis for *centralized* controllers in the set-up of Fig. 3 invoke μ, see, *e.g.*, (Balas *et al.* 1991, Packard and Doyle 1993, Stein and Doyle 1991). However, RP-synthesis of *decentralized* controllers in the μ-framework is an open problem, which has been paid only limited attention to in, *e.g.*, (Chiu and Arkun 1992, Ito *et al.* 1995, Lopez and Athans 1994, Skogestad and Morari 1989). As far as we know, CC selection in the set-up of Fig. 3 has not been paid attention to, but merits thorough future research in order to solve CSD aimed at goal (II). For CC selection, it is desirable that the subsystems under decentralized control are allowed to be nonsquare, while their inputs and outputs are allowed to "overlap", *i.e.*, are allowed to be present in more than one subsystem.

The key idea of an *indirect* RP-based CSD method is to eliminate candidate control structures for which there does not exist any controller K for which the desired level of RP can be met, *i.e.*, to eliminate nonviable candidates. Performing IO selection and CC selection in succession, each candidate IO set and candidate CC must be subjected to a mathematical condition for RP. In order for the selection to be effective, necessary and sufficient conditions for viability should be employed. Unfortunately, to date such conditions only exist for systems with centralized controllers and an unstructured Δ-block (Lee *et al.* 1995), *e.g.*, if RS for unstructured uncertainties is the focus. In (Reeves 1991, Section 2.4) it is argued, that sufficient or necessary and sufficient conditions are only useful for a small number of candidates, since such conditions must address *all* aspects of viability and hence must be employed in the context of controller design. Future research must reveal if controller independent, strong, necessary conditions for existence of a robustly performing controller can be derived and employed for an efficient and practical approach to CSD.

For *nonlinear* control system, there is still a long way to go towards a practical CSD method. As a starting point, it could be investigated if concepts for linear systems that have nonlinear equivalents (see also Section 5) are useful for CSD. At present, developing a CSD method for nonlinear systems aimed at goal II is rather ambitious.

In order to evaluate newly developed tools for CSD, a representative example is needed, *i.e.*, an example where uncertainties play an important role, where controlled and measured variables must be treated independently, and where nonsquare IO sets and decentralized CC's might be desired. In (Van de Wal 1994, Section 5.2), an active suspension application for a tractor-semitrailer is suggested as such an example. For this system, nonlinearities are easily incorporated in the linearized model, which makes it useful for evaluation of CSD methods for nonlinear control systems as well.

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