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A GENERAL PURPOSE TWO-DIMENSIONAL MESH GENERATOR

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INTRODUCTION

Composing and checking input for finite element programs is
very labour intensive; this is particular true for the divi-
sion of the area to be studied into elements. In the past
many programs, called mesh generators, have been developed in
order to automate this job. A survey of these programs is gi-
ven by Buell W.R. and Bush B.E. (1973). This paper deals with
a mesh generator for two-dimensional areas; the principal
characteristics of this mesh generator, named TRIQUAMESH, are:
1. a user oriented input language with debugging aid is pro-
vided; the user will only have to supply simple composable in-
put data.
2. both single and multiple coherent two-dimensional areas
with a complex geometry can be divided into triangular and/or
quadrilateral elements.
3. easy specification of the magnitude of the elements.
4. substructuring facilities have been incorporated.
5. the shape of the generated elements is optimised.
6. the mesh generator has some possibilities to reduce the
bandwidth of the assembled structural matrices.
7. the output of the mesh generator can be used directly as a
part of the input for three finite element programs, including
ASKA and MARC.

After introduction of some basic concepts, the method used in
TRIQUAMESH will be dealt with. Afterwards the use and possibi-
lities of TRIQUAMESH will be illustrated by means of some ex-
amples.

BASIC CONCEPTS

The area to be divided G can be divided into ns subareas G_1,
.... G_{ns} in order to specify element and material properties
and to define the substructuring of the area G. It is demanded

4

that subareas are simple coherent; n-fold coherent areas can be
made simple coherent by making at least n-1 cuts. The contour
of an area G is C, subarea G_{s} has C_{s} as contour and the overall
contour of all subareas is C^{*} (fig. 1).

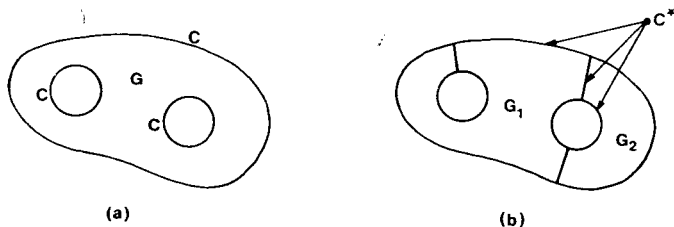(a)   (b)

Figure 1. An arbitrary two-dimensional area (a) and a division
thereof in simple coherent subareas (b).

In G a number of so-called basis points is fixed, by means of
numbers and coordinates. These points will supply a basis for
the geometrical description of contour C^{*} and for the determi-
nation of other user wishes, for example the desired element
size. With two or more basis points oriented elementary curves
can be defined, for example straight lines, arcs etc. (fig. 2a).
A contour part (identified by a number) is a non-branched coup-
ling of elementary curves and has an orientation. The geometric
description of the contour part consists of the coupling of
descriptions of the constituent elementary curves (fig. 2b).

3 RL 4   4 CM 6,5   3 RL 4 CM 6,5 RL 7 C_1: 3, 2
                                                       C_2: 1,-3

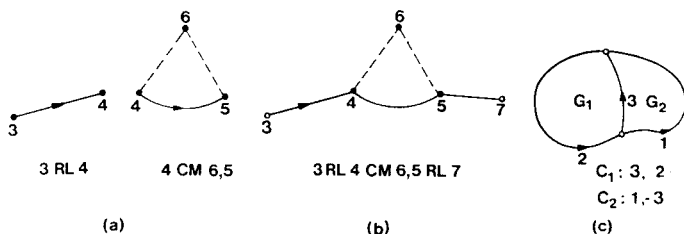(a)   (b)   (c)

Figure 2. Description of elementary curves (a), a contour part
(b) and subcontours (c).

Subcontour C_s consists of the coupling in a closed curve of one
or more contour parts. For reasons of univocality, C_s is des-
cribed by denoting the numbers of the anti-clockwise sequence
of joining contour parts. If a contour part is met in a direc-
tion opposite to its orientation, the number of this contour
part is denoted negatively (fig. 2c). A substructure is defined
by one or more subareas. All substructures together form one
structure: the area to be divided G. The concepts mentioned are
hierarchically ordered. The basis points define elementary
curves. These define contourparts etc.:

Basispoints → elementary curves → contourparts →
subareas → substructures → structure.
During the division of area G into elements, use is made of the
roughness function g2(x,y). It is postulated that in G, g2 is
directly proportional to the desired magnitude of the element
sides; the length ℓ of any element side between the points $(x_1,$
$y_1)$ and (x_2, y_2) therefore shall have to be "the best fit
possible":

$$\ell = \tfrac{1}{2}\{g2(x_1, y_1) + g2(x_2, y_2)\}.RI \tag{1}$$

The proportional constant RI has the dimension of length and
will be called the standard element side. Equation 1 is the de-
finition of the roughness concept.

DESCRIPTION OF THE METHOD

Globaly TRIQUAMESH has been developed as follows:
1. Checking and manipulation of input data.
2. Generation of nodal points on the total contour C^*.
3. Division of subareas into elements.
4. Post-processing, such as: optimization of element shape,
bandwidth reduction, transformation to elements with more nodal
points and output.
The aspects mentioned above will be described subsequently.

Checking and manipulation of input data
An user-oriented input language has been developed so that the
input can be interpreted simply and elaborate tests for errors
in the input data are possible. The program expects the input
to be delivered by means of punchcards or by means of a file
to be found on a disk-unit. From this input, arrays are deter-
mined which will serve as input parameters for the next steps.
During processing, the input is also checked for syntax and se-
mantic errors. Possible error messages are for instance:

1 TRIAX3 2,5)
            *   >>>> LEFT PARENTHESIS EXPECTED
        *   >>>> UNKNOWN SUBAREA

Generation of contour points
The user will have to supply values for the roughness function
g2(x,y) for each of the basis points in area G. Doing so, using
a chosen roughness behaviour along the contour, the roughness
on the total contour C^* is fixed. Starting from this, and to-
gether with the user given standard element side RI, the nodal
points to be generated on C^* will be determined. Because of the
assembly of a subcontour out of contour parts, which in turn
consist of elementary curves, it will only be necessary to ex-
plain the generation of nodal points on an elementary curve.
Consider an elementary curve K with length ℓ and curvilinear
coordinate s, $0 \le s \le \ell$; for reasons of simplicity a non essen-

tial simplification to a straight line is made. On curve K a
number of basis points are denoted, numbered locally with j,
j = 0,1,...,m, which divide the curve in m pieces, and, by
means of the user given values gk_j, a piecewise harmonic rough-
ness function is defined. (fig. 3):

$$gk(s) = \frac{gk_j + gk_{j+1}}{2} + \frac{gk_j - gk_{j+1}}{2} \cdot \cos \frac{s - s_j}{s_{j+1} - s_j} \cdot \pi \; ; \; j = 0..m \quad (2)$$
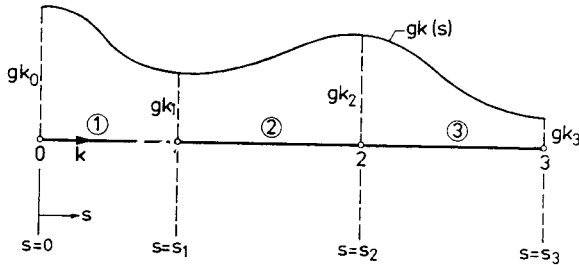

Figure 3. Harmonic roughness function on an elementary curve.

Starting form this function gk(s) nodal points are generated
on curve K, this is done in two steps: first the determination
of the number of nodal points and subsequently the computation
of the correct location. Suppose that n-1 nodal points will
have to be generated (and therefore n element sides) on the
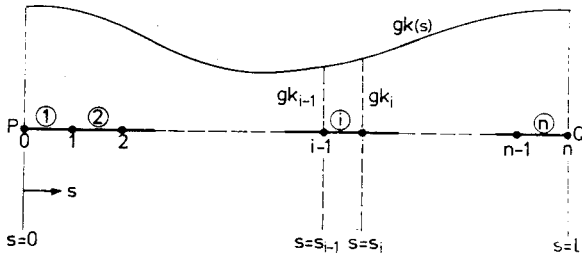elementary curve (fig. 4).

Figure 4. Nodal points and element sides on the curve PQ.

From equation 1 for element side i the following can be de-
rived:

$$\frac{s_i - s_{i-1}}{gk_i + gk_{i-1}} = \frac{s_{i+1} - s_i}{gk_{i+1} - gk_i} \quad (i = 1, 2, \ldots, n-1) \quad (3)$$

This can be met "as good as possible" for all element sides,
by computing n as follows:

$$n = \frac{1}{RI} \cdot \int_0^{\ell} \frac{\ell}{gk(s)} \, ds \quad (4)$$

After which n is rounded off to an integer in a suitable way. As soon as the curvilinear coordinates $s_1 \ldots s_{n-1}$ are known, one can easily determine the coordinates in the overall two-dimensional system.

Generating of elements in a subarea

A contour is defined by sequentially connected nodal points on this contour. The connection is made by straight lines (the element sides). Every subarea will have to be divided either in triangles or quadrilaterals, depending upon the user given element type. The nodal points on contour C_s of subarea G_s are numbered locally 1 ... ncp (fig. 5).
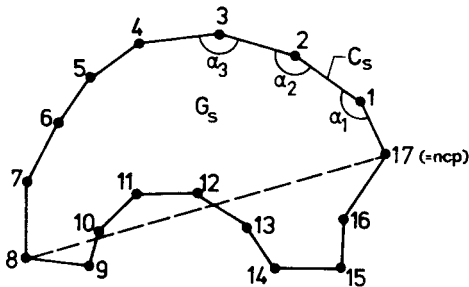

Figure 5. Local numbering of contourpoints on C_s.

Subarea G_s is concave whenever one of the angles enclosed by the contour $\alpha_i > \pi$. Let i and j, where $i \neq j$, be nodal points on C_s such that the interconnecting line between i and j lies completely within G_s. Whilst dividing G_s into elements such lines are frequently used, and an instantanuous check will have to be made to see whether this line is actually within G_s; for instance the connection between points 8 and 17 in fig. 5 is not acceptable. These checks for concave areas are quite complicated and therefor a concave subarea is split into two or more convex partial areas, after which these areas are divided into elements.

Splitting a concave subarea into convex partial areas
A nodal point is called concave if $\alpha_i > \pi$, (fig. 5). The splitting is done by the following steps:
1. Take a concave nodal point on the contour; call this point P. If no such point exists, the area will be convex.
2. Determine the accumulation V_1 of nodal points on the contour which are visible from P.
3. Determine out of V_1 that nodal point Q in such a way that, based on given criteria, PQ is the best splitting line.
4. Determine the accumulation V_2 of nodal points on the contour which are visible from Q.
5. Define on PQ a roughness function, based on the roughness values of the nodal points in V_1 and V_2.

6. Generate, using that roughness function, nodal points on PO.
7. Define two new areas separated by line PQ.
8. Continue with step 1 for both areas.

Explanation Step 1. Point P is chosen to be the most concave point on the contour or to be the middle point of a series of almost equally concave points. Steps 2 and 4. The determination of visible points is illustrated with an example (fig. 6a). Consider a continuous contour; β_k is defined as being the angle between line P-K and the tangent of the contour in P; Fig. 6b shows β_k as function of curvilinear coordinate s.

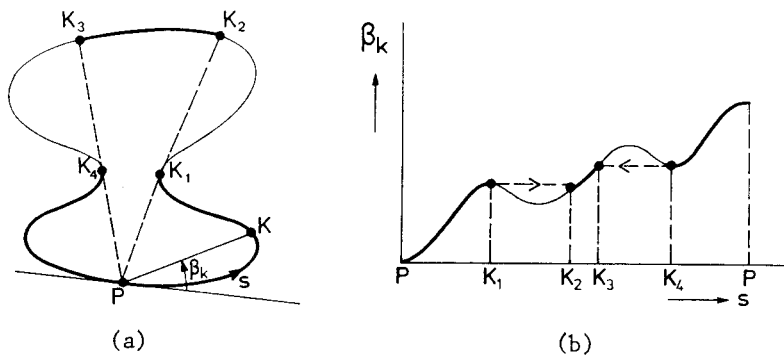
(a)   (b)

Figure 6. Determination of from P visible points.

Points of interest in the determination of visible points are those concave points L of the contour where β_L is a local extremum; not visible will be:
1. Points K with $s_K > s_L$ and $\beta_K < \beta_L$ if β_L is a local maximum; for example points between K_1 and K_2.
2. Points K with $s_K < s_L$ and $\beta_K > \beta_L$ if β_L is a local minimum; for example points between K_3 and K_4.
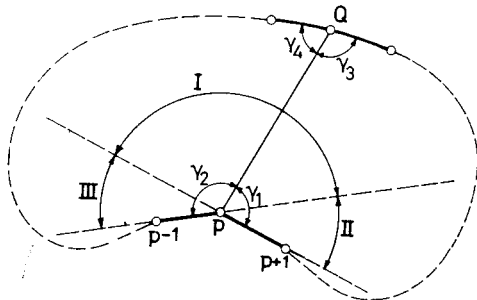

Figure 7. Search for a suitable splitting-line from point P.

Step 3. Consider fig. 7: The points P^{-1} and P^{+1} are the neighbouring points of P. The lines PP^{-1} and PP^{+1} divide the visible area in sectors I, II and III. At first the most suitable point

Q in sector I is searched for; if such a suitable point in
that sector exists, the concave angle in P can be eliminated.
However, if line PQ does not meet some minimum demands, the
searching-area is extended with sectors II and III.

Possible splitting lines are selected with respect to three
criteria: 1. the angles $\gamma_1 \ldots \gamma_4$ will have to fit as good as
possible multiples of $60°$ or $90°$ (respectively for triangles
and quadrilaterals); this results in a difference $\Delta\gamma$; 2. the
length of the splitting line, which should be as short as pos-
sible, yields a difference $\Delta\ell$; 3. the number of nodal points on
a splitting line is rounded off, which gives a $\Delta\eta$. From all
possible splitting lines that line is chosen for which $\Delta =
C_1 \cdot \Delta_\gamma + C_2 \cdot \Delta_\ell + C_3 \cdot \Delta_\eta$ reaches a minimum. The weighting factor
C_1, C_2 and C_3 have been empirially determined.
Step 5. A two-dimensional roughness polynomial g2 is determined
by means of a weighted residual method; let gk_k be the rough-
ness in contourpoint k, so it's weighting factor will be $1/gk_k$.
The result can be reduced to an one-dimensional polynomial g1
on the splitting line.

$$g2(x,y) = \sum_{i=0}^{n} \sum_{j=0}^{i} a_{i-j,j} x^{i-j} y^j \quad ; \quad g1(\nu) = \sum_{i=0}^{n} b_i \nu^i \qquad (5)$$

<u>Dividing a convex area into triangles</u> The contour of a convex
partial area is determined by n nodal points. For any of these
points, locally numbered **i**, i=1...n, the coordinates, the con-
tour-enclosed angle α_i and the roughness gk_i are known. An
angle α_i is called sharp whenever $\alpha_i < 80°$, because cutting
and splitting of $\alpha_i = 80°$ yields equal differences with res-
pect to $60°$; so $\alpha_i - 60° = 60° - \alpha_i/2$. The actual dividing of
a convex area can be described as follows:
1. If n = 3, one triangle is formed and the area is exhausted.
2. If a sharp angle is found, subsequent layers are cut from
the area as long as this is in accordance with given criteria.
These layers are divided into triangles.
3. Split the remaining area into two partial areas.
4. Define two contours for those partial areas and continue for
both areas with step 1.
<u>Explanation</u> Step 2. A cutting line i + j → i - j is acceptable
if it satisfies two conditions (fig. 8): 1. the angles γ_1, γ_2
γ_3 and γ_4 should be $> 40°$, and 2. if g_{i+j} and g_{i-j} are the
roughness values in the end points of the cutting line then
roughness g_ℓ on that line should satisfy:

$$\frac{2}{3} \cdot \min \{gk_{i+j}, gk_{i-j}\} < g\ell < \frac{3}{2} \cdot \max \{gk_{i+j}, gk_{i-j}\} \qquad (6)$$

This criterion reduces possible configurations as shown in
fig. 8b, which causes bad shaped elements. The nodal points
on an accepted cutting line are generated based upon the
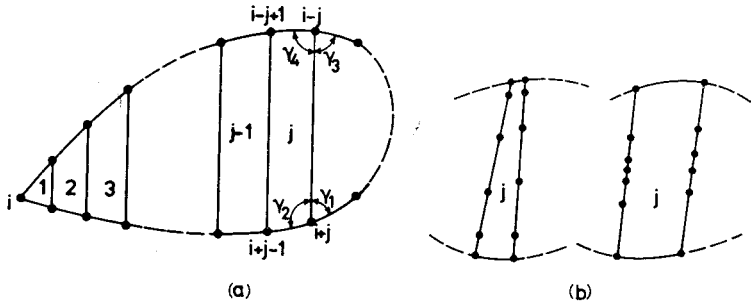roughness of all contour points in the original convex par-
tial area.

Figure 8. Cutting layers starting at sharp angle i.

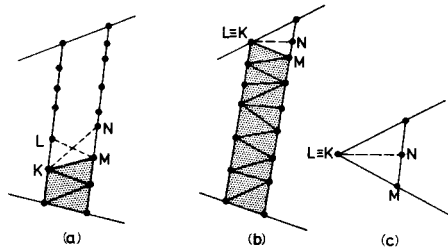Fig. 9 illustrates the dividing of a layer into elements; inside a layer no nodal points will be created.

Figure 9. Forming triangles within a layer.

If LM < KN (fig. 9a) then LKM will be the next triangle to be formed; otherwise KMN will be created. Fig. 9b shows a possible end situation and fig. 9c a similar situation for the first layer; the triangulation of such remaining areas is trivial. Step 3. The aspects with respect to which possible splitting lines are selected (i.e. angles, length and number of nodal points to be generated) are the same as for the splitting of concave areas.

Dividing a convex area into quadrilaterals   The method is main-ly analogous to the method for triangles. An angle α_i is now said to be sharp if $\alpha_i < 120°$. The actual division of a convex area into quadrilaterals is shown in the following steps:
1. If n = 4; one quadrilateral is formed, and the area is ex-hausted.
2. If n = 6; 2, 3, 4 or 5 quadrilaterals will be formed, and the area is exhausted.
3. Find a combination of two contour points i and j for which both α_i and α_j are "sharp" and the between i and j situated angles $\alpha_k = 180°$, which means that i and j are connected with

a straight line. Cut off layers, as long as this is possible with respect to given criteria, and divide these layers into quadrilaterals.
4. Split the remaining area into two partial areas.
5. Continue with step 1.
Explanation Step 2 is necessary because of the geometrically and topologically difficult element form. Typical geometries which will cause 2, 3, 4 or 5 quadrilaterals are shown below (fig. 10).
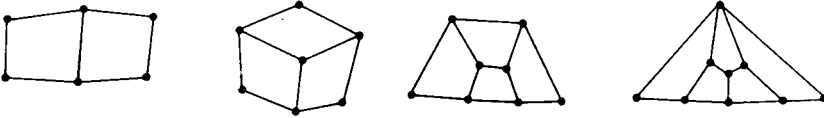

Figure 10. Dividing a hexagon into 2, 3, 4 or 5 quadrilaterals.

Step 3. Apart from the demands for the angles with the contour and the minimum and maximum roughness on the cutting-line as in the case of triangles, it will be a demand that the number of nodal points on the cutting-line is equal to the number on the previous cutting-line (or the original line between i and j). In that case, the division of the layer is trivial.

Post-processing
Shape improvement of the elements It has been found that the form of the elements joining at an internal nodal point k can be improved in an efficient way by giving point k new coordinates as the average of the coordinates of his nk neighbouring points (i.e. nodal points which are connected to k with an element side):

$$x_k := \frac{1}{nk} \sum_{i=1}^{nk} x_{ki} \quad \text{and} \quad y_k := \frac{1}{nk} \sum_{i=1}^{nk} y_{ki} \qquad (7)$$

The iteration process is stopped as soon as none of the internal nodal point changes significantly.

Bandwidth reduction The method which TRIQUAMESH uses for the dividing of an area G into elements causes that the coefficient-matrices of substructures defined in G, will have a bad or no band structure. Because many finite element programs only work efficiently by means of an acceptable band structure, some possibilities have been provided in order to reduce band-width by renumbering the nodal points in a suitable way. The user can, for each substructure, define more renumberings; finally that one is chosen which gives the best band structure. The available renumbering methods will be described subsequently.

CMK-Renumbering This method of renumbering was developed by Cuthill E. and Mckee J. (1969) and starts from the topology of
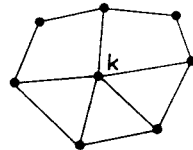
the element division of the substructure. Let e_1, e_2,...,e_n ($n \geq 1$) be the elements which contain nodal point k. The $m + 1$ nodal points k, k_1 ... k_m describe the elements e_1 ... e_n so that nodal point k has degree m and k_1 ... k_m are surrounding points of k. (fig. 11).
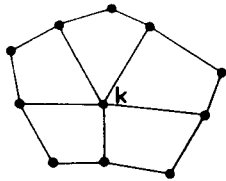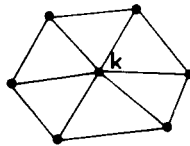
degree = 10   degree = 6   degree = 7

Figure 11. Degree of a nodal point k.

CMK-renumbering proceeds as follows:
1. Choose a starting point k_1 and give this number 1. The accumulation V of renumbered points now consists of the nodal point k_1 : $V = \{k_1\}$.
2. Number sequentially the not yet renumbered surrounding points of each nodal point in V, according to an increasing degree, with their new numbers.
3. Define V = {nodal points to which new numbers were given in step 2}.
4. If V is empty the process stops otherwise repeat step 2. Because of the fact that the CMK-renumbering uses no geometrical data, this method will be especially useful in the case of substructures with an irregular geometry.

Line-, Distance- and Angle-renumbering These methods are based on the geometry of the substructure (coordinates of nodal points), and are realised by means of a, user-given, new Cartesian coordinate-system $\bar{x}$, $\bar{y}$. In order to achieve this two basispoints P and Q will have to be specified. The origin of the new system will be P; $\bar{y}$ is measured along the line PQ and $\bar{x}$ along a line orthogonal to PQ. In this system a nodal point k can be characterised in two ways: the pair $(\bar{x}_k, \bar{y}_k)$ and the pair $(\bar{r}_k, \bar{\phi}_k)$. (fig. 12).
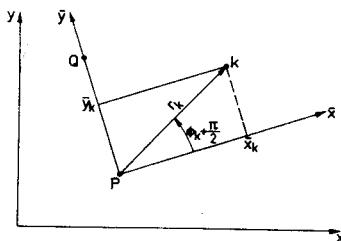
Figure 12. Characterisation of a nodal point in the $(\bar{x}, \bar{y})$-system.

By means of the nodal points k_1 with coordinates $(\bar{x}_1, \bar{y}_1)$ or $(\bar{r}_1, \bar{\phi}_1)$ and k_2 with coordinates $(\bar{x}_2, \bar{y}_2)$ or $(\bar{r}_2, \bar{\phi}_2)$, three renumbering methods can be specified:

k_1 will follow k_2 if:

1. $(\bar{x}_1 > \bar{x}_2) \vee ((\bar{x}_1 = \bar{x}_2) \wedge (\bar{y}_1 > \bar{y}_2))$ "Line"-renumbering

2. $(\bar{r}_1 > \bar{r}_2) \vee ((\bar{r}_1 = \bar{r}_2) \wedge (\bar{\phi}_1 > \bar{\phi}_2))$ "Distance"-renumbering

3. $(\bar{\phi}_1 > \bar{\phi}_2) \vee ((\bar{\phi}_1 = \bar{\phi}_2) \wedge (\bar{r}_1 > \bar{r}_2))$ "Angle"-renumbering

Fig. 13 shows some typical examples of element divisions which each in turn are typical for the different renumbering methods.
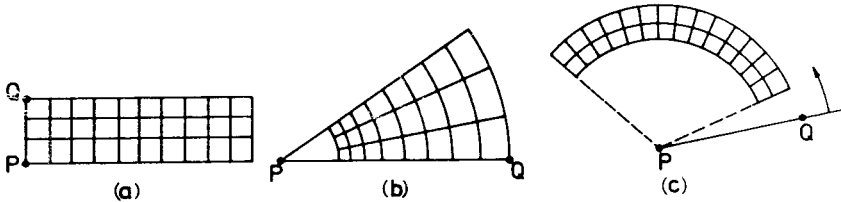

Figure 13. Typical element divisions for (a) Line-, (b) Distance- and (c) Angle-renumbering.

Generation of midpoints   The triangles and (or) quadrilaterals in which the area G has been divided, can be seen as elements with three or four nodal points. However, in many cases one wishes to use elements with more nodal points. Examples of these are the ASKA-elements TRIM6 and QUAM8; these have, apart from the nodal points on the corners, also nodal points on the edges of the elements (so called midpoints). In order not to lose the band structure which was obtained by renumbering the nodal points, for each new midpoint a number will have to be chosen which does not exceed the maximum or minimum number of the corner nodal points of that element. This will be achieved by giving a midpoint between k_1 and k_2 the exact or nearest "free" value of $(k_1 + k_2)/2$. It is evident that a spacing will have to be made in the original base of corner points; this is done by multiplying the numbers of the corner points by 4 (which is a quite arbitrary value). All lines defined by two corner points are gathered and for each of these lines a midpoint is generated by the method mentioned above. Afterwards a new topology is composed by the old topology, these lines and the newly created midpoints; at last the spaces left in the numbering will be eliminated.

The output of TRIQUAMESH

Using input parameters the user can obtain an output of the results of TRIQUAMESH on a lineprinter, card puncher, disc-pack or on a plotter. The characterisation of the generated division in elements, is presented in such a way on punch cards and (or) disc-pack, that it will match the demands of various finite element programs. At this moment it is possible to create output

which can serve as input for the systems ASKA, FEMSYS and MARC; extension for other systems is easily possible.

THE USE OF TRIQUAMESH

TRIQUAMESH has been programmed in BEA(Burroughs Extended Algol) and has been implemented on the Burroughs B7700 computer of the University of Technology Eindhoven. The program can be used both in batch and in the B7700 time sharing system. In the latter case an efficient use can be made of a CRT terminal (TEKTRONIX 4014) in order to obtain graphical (intermediate) output. The use of TRIQUAMESH will be illustrated by an example. Whilst composing the input-data the user makes a sketch of the area to be divided and specifies therein basispoints, contourpoints, subareas etc. (fig. 14a).

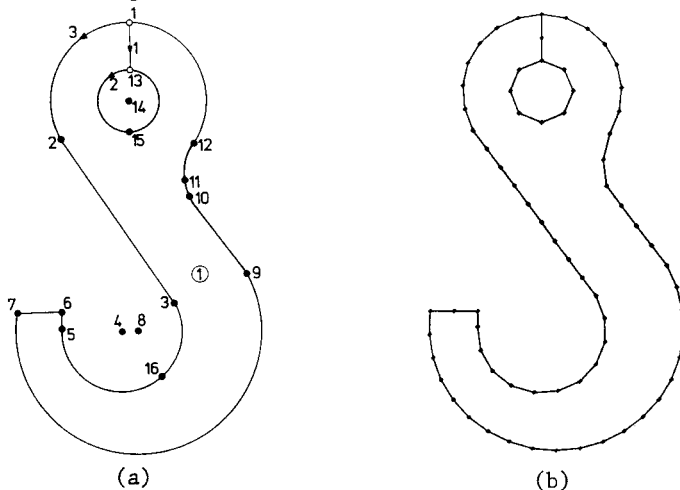(a)   (b)

Figure 14. Input sketch and generated contourpoints for a crane-hook.

The input for this problem is:

```
100 $INPUT "HOOKE"
110 $BASISPOINTS
120  1:0,200;    2:-43,125;   3:34.6,20;   4:0,0;
130  5:-40,0;    6:-40,10;    7:-70,10;    8:10,0;
140  9:79,40;   10:41,90;    11:39,100;   12:43,123;
150 13:0,170;   14:0,150;    15:0,130;    16:30,-30;
160 $CONTOURPIECES
170 1(1 RL 13)    2(13 CM 14,15 CM 14,13)
180 3(1 CM 14,2 RL 3 CB 16,5 RL 6 RL 7 CM 8,9
190 RL 10 CB 11,12 CM 14,1)
200 $SUBCONTOUR
210 1(3,1,-2,-1)
```

```
220 $GRADING
230 G 1 RI 15
240 $SUBSTRUCTURE
250 ASKA
260 1 TRIM3 (1)
270 $RENUMBER
280 PROFILE
290 (1 CMK 1)(1 CMK 7)(1 LINE 7,6)
300 $OUTPUT
310 1 COPRINT,TOPRINT,COCARD,TOCARD,PLOT(A2)+PONO;
320 $STOP
```

Explanation

$BASISPOINTS: contains the basis points and their coordinates.
$CONTOURPIECES: defines how each of the contour parts is composed.
$SUBCONTOUR: defines subcontours by means of contour parts.
$SUBSTRUCTURE: defines which subarea belong to which substructure and the element type of each subarea.
$GRADING: contains information about the desired magnitude of the elements.
$RENUMBER: control data for bandwidth-reduction.
$OUTPUT: defines the way output will have to be presented.

Fig. 14b shows the generated contour points and fig. 15a the division into elements.

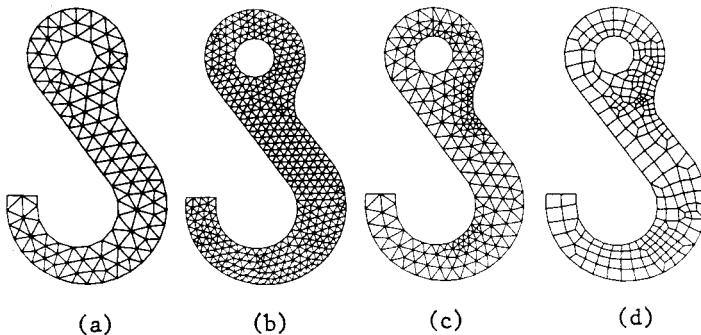|			
(a)	(b)	(c)	(d)

Figure 15. Element divisions for a crane-hook.

The divisions according to fig. 15b, c and d are created by making some slight alterations in the input above:
Fig. 15b: the complete division less coarse by a factor 2.
(RI = 7.5 at line 230).
Fig. 15c: Locally refinement in the basis points 10, 11, 12, 13 and 16 by a factor 2 (add GV 0.4 (10, 11, 12, 13, 16) after line 230).
Fig. 15d: as fig. 15c with quadrilaterals. (QUAM4 instead of TRIM3 at line 260).

The next example deals with a piston of a diesel engine which
is simplified to an axi-symmetric construction (fig. 16a). The
moulded aluminium piston is oil-cooled by means of a moulded-
in steel tube which serves as a coolant spiral; in a radial
plane one can see 4 cross-sections of the tube: the subareas
2, 3, 4 and 5. At the location of the upper two piston rings,
a considerably higher wear resistance material is used: a
moulded-in cast iron ring, the cross-section of which is given
by subarea 7.

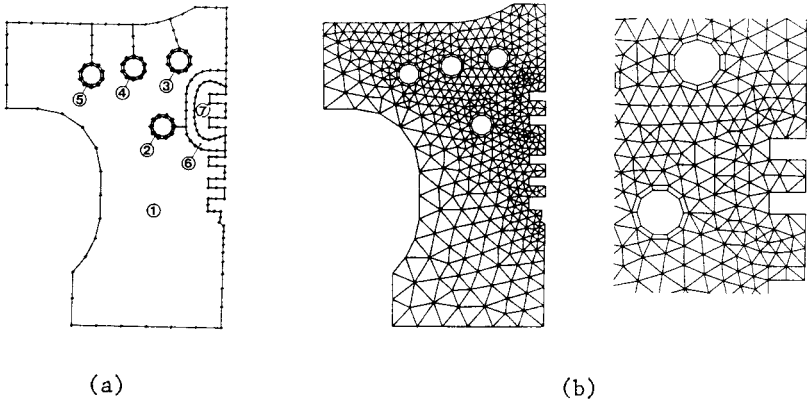(a)   (b)

Figure 16. Generated contourpoints (a) and element division (b).

In order to be able to make an optimal design of the form of
the cast iron ring, it is put into a seperate substructure to-
gether with its surroundings (subarea 6). Fig. 16b shows the
mesh generated for the piston.

DISCUSSION AND FUTURAL DEVELOPMENTS

The meshgenerator TRIQUAMESH has proved to be quite an useful
tool; the most revealing advantages of which are:
- user orientation; one tends to work with it quickly.
- freedom in choice of geometry and number of subareas.
- simple control of element size.
- possibility of quadrilateral elements.
Whilst using the program, a number of useful extensions have
become clear which shall be made in the near future; this will
be done in the frame work of a pre-processing system, with
which one can generate the complete input (mesh, kinematic and
dynamic boundary conditions etc.) for two-dimensional problems.

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