Scaling rules for flat and hollow fibre membrane oxygenators for total bypass

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Scaling rules for flat plate and hollow fiber membrane oxygenators for total bypass*

J. A. E. Spaan and J. M. M. Oomens

Department of Mechanical Engineering, Eindhoven University of Technology, Eindhoven, The Netherlands

INTRODUCTION

The membrane oxygenator is a blood-oxygenating device where the blood compartment is shielded from the gas phase by a porous or a nonporous membrane. Commonly, the geometry of the blood compartment exhibits a thickness of the blood layer which is much smaller than the surface dimensions of the membrane. Gas transfer through the membrane into the blood will always be hampered by the diffusion process in a blood boundary layer close to the membrane. In oxygenators where artificial mixing is provided, the boundary layer will be thin. However, in oxygenators without mixing the rate of gas exchange through the membrane is fully controlled by the diffusion process in the boundary layer.

The gas exchange in oxygenators of this type, being the flat plate and straight hollow fiber devices, can successfully be described in theoretical models. These models will be developed in this paper and will be applied to show the relations between construction and performance parameters and some control characteristics of membrane oxygenators used at total bypass. Although this study is of a theoretical nature, the conclusions obtained may be applied to a wide range of oxygenators available today.

NOTATIONS

A_m = membrane area of flat plate or hollow fiber oxygenator.
$A_{m,0}$ = A_m if $\measuredangle = 0$ when shunt is present.
d = gap width of the flat plate or tube diameter of hollow fiber oxygenator.

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d_m = membrane thickness.
D_b = oxygen diffusion coefficient for blood.
D_m = oxygen diffusion coefficient for membrane material.
f_m = fractional oxygen saturation increase.
f_{ox} = fractional oxygen saturation increase of oxygenator.
f_{ec} = fractional oxygen saturation increase of extracorporeal circuit.
H = ratio of the capacities of the blood to bind and to dissolve oxygen under the ruling boundary conditions.
h = oxygen-binding capacity of the blood.
H_o = value of H under conditions of rated oxygen transfer (ROT).
I_1, I_2 = integrals across the oxygenation boundary layer depending on the velocity profile.
I_3, I_4 = as I_1 and I_2 but also dependent on M.
K K=12 for the flat plate, K=32 for the hollow fiber.
K_1 = constant.
L = length of flat plate or hollow fiber oxygenator.
L* = dimensionless length.
L^*_0 = L* if $\alpha = 0$ when a shunt is present.
M = relative membrane resistance for O_2 transfer (eqs. 9a, 9b).
m m=1 for the flat plate, m=2 for the hollow fiber.
N = number of tubes of the hollow fiber oxygenator.
P_g = oxygen partial pressure in the gas phase.
P_i = oxygen partial pressure of the blood entering the oxygenator.
ΔP = pressure drop over the oxygenator.
Q = blood flow through the oxygenator.
Q_s = blood flow through the shunt.
Q_t = total blood flow through extracorporeal circuit.
r = radial coordinate for the hollow tube.
r_1 = fractional change of oxygen consumption.
r_2 = fractional change of blood flow.
r_3 = fractional change of oxygen binding capacity of the blood.
S_o = oxygen saturation at oxygenator outlet.
S_i = oxygen saturation at oxygenator inlet.
S_a = arterial saturation.
S_v = venous saturation.
S_{v0} = venous saturation if $\alpha = 0$ when shunt is present.
V = blood volume between the membranes.
v_m = mean velocity of blood.
$\dot{V}_{O_2}$ = oxygen consumption of the body.
x = coordinate perpendicular to the membrane.
y = coordinate within the length direction of oxygenator.
z = $(1-S_v)/(1-S_{v0})$ when shunt is present.

MODEL DEFINITION AND RELATED GAS TRANSFER PARAMETERS

The theoretical flat plate model presently studied simplifies the different flat plate oxygenators to two parallel rectangular membrane sheets having a length L and width W. The membranes form a blood compartment with a fixed gapwidth d. The blood is assumed to flow in the direction of the length (y). The theoretical hollow fiber model consists of N identical hollow fibers having a gas-permeable wall. The length of the fibers is L and the inner diameter d. As to the membrane area (A_m), the volume of the blood compartments (V) and the mean velocity of the blood (v_m) the following simple relations hold

flat plate		hollow fiber	
$A_m = 2\ W\ L$	(1a)	$A_m = \pi\ d\ L\ N$	(1b)
$V = \frac{1}{2}\ A_m\ d$	(2a)	$V = \frac{1}{4} A_m d$	(2b)
$v_m = \frac{Q}{d\ W}$	(3a)	$v_m = \frac{Q}{\frac{1}{4}\ \pi\ d^2\ N}$	(3b)

the quantity Q being the total blood flow through the oxygenator.

Conclusions on the oxygen transfer behavior of the model oxygenators follow from the analysis of the transfer process within a single tube on the one hand and similarly considering a cross-section in the y-direction through the flat plate model on the other. As to this analysis, information is wanted on the exact nature of the oxygen transfer in whole blood. However, in the present case the blood is assumed to be a newtonian homogenous fluid showing a parabolic velocity profile. Moreover, the diffusion of hemoglobin will be neglected.

It is preferred to present the results of the gas transfer analysis in terms of a fractional mixed average saturation increase f_m and a dimensionless length L^*. The parameters f_m and L^* are defined by

$$f_m = \frac{S_o - S_i}{1 - S_i} \quad (4), \text{ and } \quad L^* = \frac{L\ D_b}{\frac{1}{4}\ d^2\ v_m} \qquad (5)$$

where S_i, S_o are the oxygen saturation of the blood entering and leaving the oxygenator, respectively; D_b is the effective diffusion coefficient of oxygen in blood.

Normally the value of f_m will increase with an increasing value of L^*. For the hollow fiber as well as for the flat duct, numerical solutions of the transfer equations have been presented in the literature (Buckles et al., 1968; Mockros & Gaylor, 1975; Spaan, 1973). Approximate solutions may be obtained by means of the introduction of an 'Advancing Front' (Hill, 1929; Marx et al., 1960;

Lightfoot, 1968; Dorson et al., 1971; Lautier et al., 1971; Spaan, 1973). The 'Advancing Front' approach is illustrated in Figure 1.

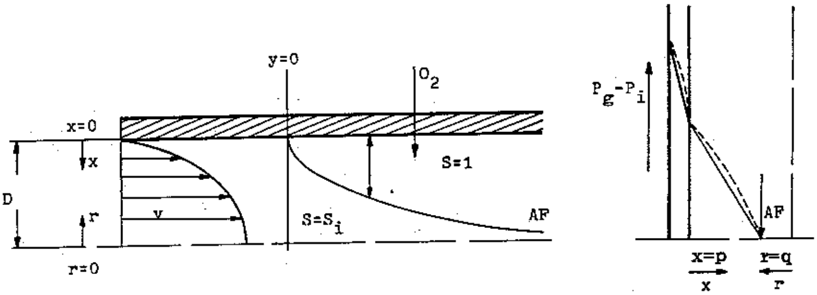


Fig. 1. Schematic of axial section of a flat duct or hollow fiber showing the parabolic velocity profile, oxygenation boundary layer and assumed oxygen partial pressure profile in membrane and blood.

Oxygen diffuses from a gaseous atmosphere having an oxygen partial pressure P_g through a membrane into the blood flowing through the duct. The blood close to the membrane is fully oxygenated. This layer of oxygenated blood is referred to as the oxygenation boundary layer. The thickness of this layer increases with increasing y. Oxygen diffuses through the boundary layer and will react with hemoglobin at the 'Advancing Front'. The mass balance of oxygen diffusing into the duct and the oxygen carried away by convection delivers mathematical formulas giving p as a function of y* and f_m as a function of p. For the two oxygenator models studied, the 'Advancing Front' equations (Oomens & Spaan, 1975) result in (see Table I):

$$L^* = H(I_2+M\ I_1)+I_3 \tag{6}$$

$$f_m = m\left(I_1+ \frac{1}{H}\ I_4\right) \tag{7}$$

where

$$H = \frac{h\ (1-S_i)}{\alpha_b\ (P_g-P_i)} \tag{8}$$

$$M = \frac{\alpha_b\ D_b}{\alpha_m\ D_m}\frac{d_m}{\frac{1}{2}\ d} \quad \text{for the flat plate, and} \tag{9a}$$

$$M = \frac{\alpha_b\ D_b}{\alpha_m\ D_m}\ln\ \left(1+\frac{d_m}{\frac{1}{2}d}\right) \quad \text{for the tube,} \tag{9b}$$

where m is a constant, m=1 for the flat plate, m=2 for the hollow fiber, I_1, I_2, I_3, I_4 are integrals over the oxygenation boundary layer determined by the velocity

TABLE I

Definition of terms for the advancing front equations

$L^* = \frac{L\,D_b}{\frac{1}{4}d^2 v_m}$	$H = \frac{h(1-S_i)}{\alpha_b(P_g-P_i)}$	$f_m = \frac{S_o-S_i}{1-S_i}$	$\left.\begin{matrix} x^* = p \\ r^* = q \end{matrix}\right]$ position of the advancing front

functions that belong to the general AF model	FLAT PLATE	HOLLOW FIBER
velocity	$v=v_y(x)$	$v=v_y(r)$
mean velocity	$v_m=\frac{1}{d}\int_0^d v_y(x)dx$	$v_m=\frac{8}{d^2}\int_0^{\frac{1}{2}d} v_y(r)rdr$
dimensionless velocity function	$v^*(x^*)=v_y(x)/v_m$	$v^*(r^*)=v_y(r)/v_m$
saturated flow	$I_1=\int_0^p v^*(X^*)\,dx^*$	$I_1=-\int_1^q r^*\,v^*(r^*)dr^*$
saturated flow moment	$I_2=\int_0^p x^*v^*(x^*)dx^*$	$I_2=\int_1^q r^*\ln(r^*)v^*(r^*)dr^*$
influence of $[O_2]$ in the plasma on L^*	$I_3=\int_0^p \frac{I_2+MI_1}{x^*+M}dx^*$	$I_3=\int_1^q \frac{I_2+MI_1}{r^*\ln(r^*)+M}dr^*$
first order mixing term of f_m	$I_4=\frac{1}{p+M}(pI_1+I_2)$	$I_4=\frac{I_1\ln(q)+I_2}{\ln(q)-M}$
membrane resistance	$M=\frac{\alpha_b D_b}{\alpha_m D_m}\frac{d_m}{\frac{1}{2}d}$	$M=\frac{\alpha_b D_b}{\alpha_m D_m}\ln\left(+\frac{d_m}{\frac{1}{2}d}\right)$

profile alone (I_1 and I_2) or both by the velocity profile and M (I_3 and I_4) (see Appendix); H is the ratio of the capacity of the blood to bind and to dissolve oxygen under the governing boundary conditions (P_g, P_i, S_i); h is the maximum oxygen binding capacity of the blood; α_b, α_m are the solubilities of oxygen in the blood and membrane; D_b, D_m are the diffusion coefficients of oxygen in the blood and membrane; P_g, P_i is the oxygen partial pressure in the gas phase and of the blood entering the duct, respectively.

'Advancing Front' equations as published until now in the literature neglect one or more terms constituting equations (6) and (7).

Equation (7) may result in a value for f_m greater than unity. However, this, is a matter of definition. In the oxygenation boundary layer, oxygen is physically dissolved and simultaneously chemically bound to hemoglobin. The blood will be mixed when leaving the oxygenator and the oxygen dissolved will mainly be bound by desoxyhemoglobin, resulting in an extra oxygen saturation increase. However, if more oxygen is dissolved than can be bound to hemoglobin, the oxygen saturation will amount to 1 and consequently the value of (f_m-1) $h(1-S_i)$ will be approximately equivalent with the amount of oxygen dissolved.

Equations (8) and (9) give the simple relationship between f_m and L^*. Parameters in this relationship are H and M. In order to obtain a more general solution the relation between f_m and L^*/H is shown in Figure 2. Obviously in the case of the flat plate, straight lines are obtained when plotting log (f_m) versus log

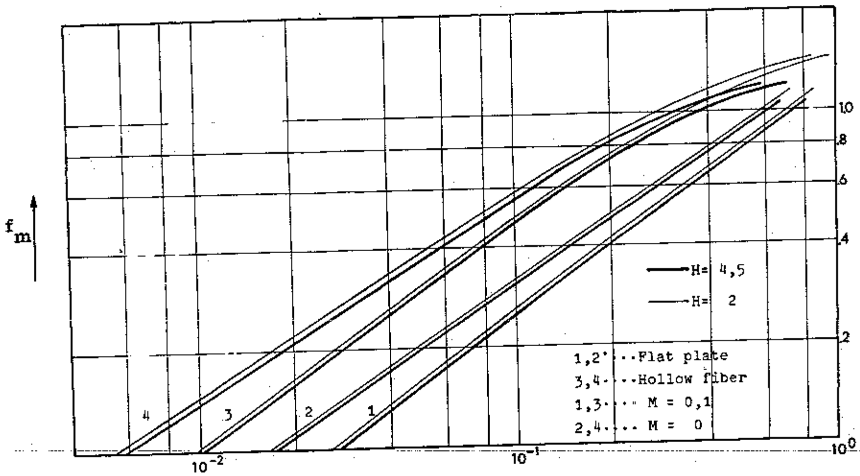


Fig. 2. Fractional saturation increase versus L^*/H for the flat plate and hollow fiber calculated by means of the advancing front equations.

(L^*/H). However, in the range of relatively large values of M, a slight deviation of this behavior can be observed. In the case of the hollow fiber, the curvature is always clearly present. There is only little dependence on the values of H. The influence of the membrane resistance shifts the curves to the right, which indicates that the dimensionless lengths necessary to obtain a fixed value of f_m are to be increased. This numerical increase of the value L^*/H relative to the value $(L^*/H)_{M=0}$ was studied as function of M and results are shown in Figure 3. This relationship clearly proves to be linear for a fixed value of f_m.

The variable L^*/H collects quite a few relevant parameters in one single dimensionless number. These parameters may be divided in parameters related either to the geometry of the design (d, L), or to physical constants (D_b, α_b), or to the operating conditions of the oxygenator (P_g, P_i, h, S_i, S_0). The design parameter M influences the relation between f_m and L^*/H. Thus the equations (6) and (7) form the basis for the derivation of scaling rules and permit studies of the membrane oxygenator performance under varying conditions.

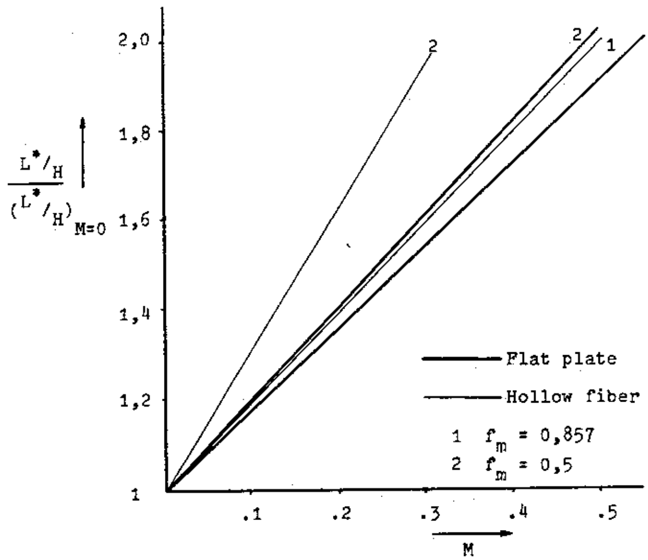


Fig. 3. The influence of the membrane resistance on L*/H at fixed conditions for f_m.

SCALING RULES

Standard test conditions

A membrane oxygenator must oxygenate per unit time a defined amount of blood having a given oxygen-binding capacity and venous saturation up to a desired level of arterial saturation. In order to introduce some standardization Galletti et al. (1971) have defined conditions as follows: temperature 37°C, hematocrit 40 (h = .187 cc 0_2/cc blood), venous saturation 65% and arterial saturation 95%. The blood flow capacity of the oxygenator under these conditions is known as rated blood flow (RBF) and the amount of oxygen transferred as rated oxygen transfer (ROT) (see also pp. 22–24).

Membrane area

Oxygen transfer requirements may be characterized by the values of the parameters previously defined: f_m, h $(1-S_i)$, Q and T. Assuming M to be zero the value of f_m results in a value for L*/H. By means of equations (1) and (2) this ratio may be rewritten as

$$\frac{L^*}{H} = \frac{2}{m}\,\frac{A_m\,D_b}{d\,Q}\,\frac{\alpha_b\,(P_g - P_i)}{h\,(1 - S_i)} = \frac{2}{m}\,\frac{A_m\,D_b}{d\,V_{O_2}}\,\frac{\alpha_b\,(P_g - P_i)}{f_m} \tag{10}$$

where V_{O_2} is the amount of oxygen to be transferred.

The membrane area required to meet the chosen conditions of oxygen transfer is proportional both to the amount of oxygen to be transferred and ,
which quantity represents either the gap width of the flat plate oxygenator or the inner diameter of the fibers of the hollow fiber oxygenator. The membrane area is inversely proportional to the solubility and the effective diffusion coefficient of oxygen in the blood (dependent on T) and also inversely proportional to the oxygen partial pressure of the gas blown through the oxygenator. It is important to note that the ratio of L and W is of no importance. For example, the surface area may be realized by a large number of short fibers as well as by a smaller number of fibers with greater length.

Influence of membrane resistance

From Figure 3 one can see that there is a linear relationship between $(L^*/L^*_{M=0})$ and M at a constant value of H. Therefore it is valid for the plate (cf. eq. 10)

$$\frac{A_m}{Q} = \frac{L^*_{M=0}}{2}\left(\frac{d}{D_b} + \epsilon\,\frac{\alpha_b\,d_m}{\alpha_m\,D_m}\right) \tag{11}$$

where ϵ is an empirical constant.

For $D_b \to \infty$ the oxygen transfer in the oxygenator will be limited by the membrane. From a balance of oxygen transfer through the membrane and convective removal by the blood the next equation is derived

$$\left(\frac{A_m}{Q}\right)_{D_b \to \infty} = f_m\,H\,\frac{\alpha_b\,d_m}{\alpha_m\,D_m} \tag{12}$$

and therefore

$$\epsilon = \frac{2\,H\,f_m}{L^*_{M=0}} \tag{13}$$

Equation (11) therefore results in

$$\frac{A_m}{Q} = \left(\frac{A_m}{Q}\right)_{M=0} + \left(\frac{A_m}{Q}\right)_{D_b \to \infty} \tag{14}$$

The same formula may be derived for the hollow fiber oxygenator. Thus, the membrane area per unit of blood flow required to achieve a certain value of f_m

is the sum of A_m/Q calculating purely blood limitation to oxygen transfer and A_m/Q calculating purely membrane limitation.

Pressure drop over the oxygenator

In most flat plate and hollow fiber oxygenators the driving force for blood flow is a pressure difference over the oxygenator. Assuming that the blood is homogeneous and newtonian, the velocity profile will be parabolic. The pressure drop over the oxygenator is related to the average velocity, length and height of the channel or fiber radius by

$$\Delta P = K \frac{\eta}{d^2} v_m L \qquad (15)$$

where K=12 for the flat plate; K=32 for the hollow fiber; η=viscosity. This can be rewritten when using equation (3) as

$$\Delta P = \frac{4K \eta \ D_b}{L^*} \frac{L^2}{d^4} \qquad (16)$$

In these formulas η and D_b are physical constants independent of the oxygenator system chosen. The value of L* is determined by the geometry of the flow channel, the velocity profile and the required value for f_m. So in fact ΔP is determined by the ratio L^2/d^4. In terms of oxygen transfer it has been shown not to be important how the membrane area is designed. However, the aspect ratio, or length—number of fibers ratio is very important for the pressure drop across the oxygenator. Some numerical results concerning membrane area, influence on membrane resistance, and pressure drop have been given in the Appendix.

Membrane area and shunt

In most extracorporeal circuits the membrane oxygenator is shunted by a blood line. Within this shunt the flow may be parallel to the flow in the oxygenator and is applied at the beginning or the end of the perfusion. Sometimes recirculation of blood through the oxygenator is necessary to increase the oxygen transfer. The shunt (Fig. 4a) may influence the membrane area needed to meet the standard conditions. Figure 4b lists numerical results for the model flat plate oxygenator in terms of the shunt fraction α (Fig. 4a).

As may be expected, recirculation leads to a smaller membrane area required, parallel flow to a larger area. However, the positive influence of recirculation is not very large. The flow through the oxygenator should amount to twice the flow through the body to obtain a membrane area reduction of 10%.

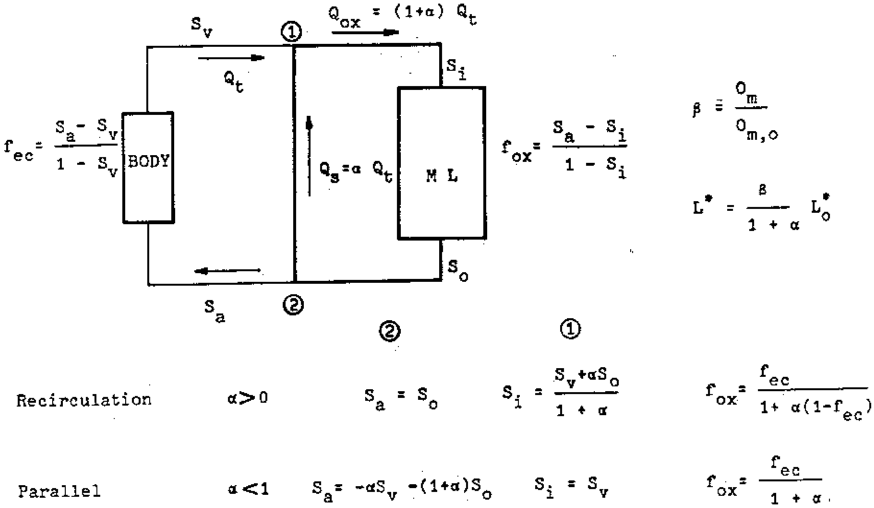
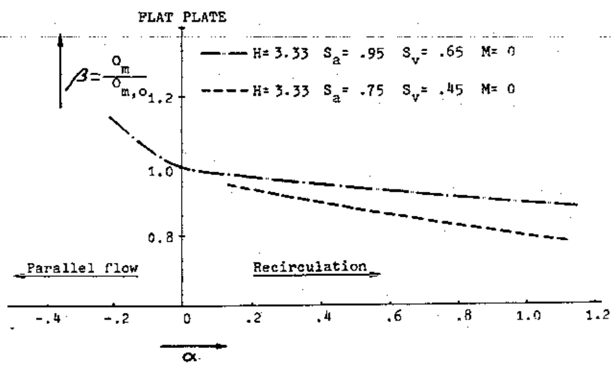


Fig. 4a. Diagram of the membrane lung with shunt coupled to the patient. α is the shunt fraction, f_{ox} and f_{ec} the increase of fractional saturations of the oxygenator and the total extracorporeal systems, respectively, $A_{m,o}$ and L^*_o are membrane area and dimensionless length of the oxygenator, respectively, for $\alpha=1$.



b. Influence of shunt on membrane area needed for the flat plate oxygenator.

MEMBRANE OXYGENATOR PERFORMANCE UNDER VARYING CONDITIONS

The notions RBF and ROT are difficult to handle in the clinic when using membrane oxygenators. The conditions of S_a, S_i, h, T and Q are fixed, but this is not exactly the case during surgery. Temperature most often decreases, hemodilation is frequently applied, and the flow Q is set particularly to maintain acceptable arterial and venous pressures and not to realize optimal oxygen transfer in the oxygenator. Realizing the relation between f_m and L^*/H, it becomes possible to predict oxygen transfer performance under varying conditions. This can be shown for an oxygenator designed to operate under the standard conditions of RBF and ROT. The influence of S_a, S_v and S_a—S_v can be studied by varying conditions of O_2 consumption, flow and oxygen-binding capacity of the blood. Therefore, the next parameters are introduced

$$r_1 = \frac{\dot{V}_{O_2}}{\dot{V}_{\bigcirc}}\,; \qquad r_2 = \frac{Q}{Q_{\bigcirc}}\,; \qquad r_3 = \frac{h}{h_{\bigcirc}} \tag{17}$$

where $\dot{V}_{\bigcirc}$, $Q_{\bigcirc}$ and $h_{\bigcirc}$ define the standard conditions. It is now assumed that the relation between f_m and L^*/H can be described by

$$f_m = K_1 \left(\frac{L^*}{H}\right)^{\gamma} \tag{18}$$

where K_1 and γ are constants.
Equation (18) can be rewritten as

$$f_m = K_1 \left(\frac{L^*_o}{H_o}\,\frac{1}{zr_2r_3}\right)^{\gamma} \tag{19}$$

where L^*_o and H_o are the values of L^* and H under standard conditions and

$$z = \frac{1 - S_v}{1 - S_{v,o}} \tag{20}$$

$S_{v,o}$ being the standard venous saturation.
A second relation between S_a and S_v is determined by the oxygen uptake of the body

$$S_a - S_v = \frac{r_1\, \Delta_o S}{r_2\, r_3} \tag{21}$$

where Δ_o S is the arterial—venous saturation difference under standard conditions.
Equations (19) and (21) result in

$$S_a = 1 - \frac{r_1}{r_2\, r_3}^{\,1/(1-\gamma)} (1 - S_{v,o}) + \frac{r_1}{r_2\, r_3}\, \Delta_o S \tag{22}$$

$$S_v = 1 - \frac{r_1}{r_2\, r_3}^{\,1/(1-\gamma)} (1 - S_{v,o}) \tag{23}$$

In the case that the oxygen consumption equals the designed oxygen transfer of the oxygenator equations (22) and (23) reduce to

$$S_a = 1 - \frac{1}{r_2\, r_3}\,(1 - S_{a,o}) \tag{24}$$

$$S_v = 1 - \frac{1}{r_2\, r_3}\,(1 - S_{v,o}) \tag{25}$$

From equations (24) and (25) it follows that with changing flow and with hemodilution the arterial saturation is less affected than the venous saturation. If the standard arterial saturation had been 1, S_a would keep this value. Equations (22) and (23) show that r_2 and r_3 have similar influences and therefore hemodilution may be compensated by increasing the flow.

Numerical results for the flat plate oxygenator as obtained from the 'Advancing Front' equations (and not from the approximation of equation 18), are given in Fig. 5. Figure 5a shows the influence of changing oxygen consumption. To a

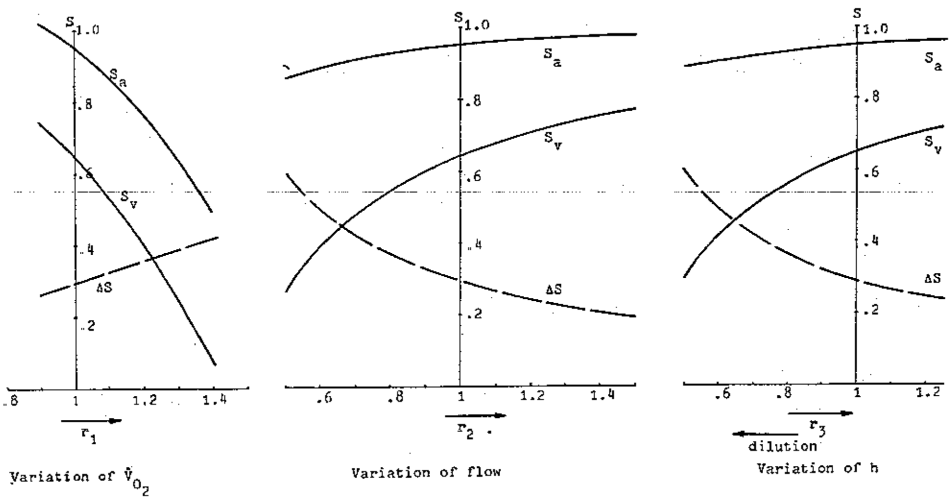


Fig. 5. Parameter analysis of flat plate oxygenator at total bypass. The behavior of S_a, S_v and S_a—S_v are shown as function of fractional change in O_2 consumption (5a), blood flow (5b) and hematocrit (5c).

certain extent the membrane oxygenator may have a greater oxygen transfer than it has been designed for. However, it will only do so under worse conditions of oxygen saturation.

DISCUSSION

It is almost classical to describe oxygen transfer performance of oxygenators by the relation f_m versus L*/H. Graphical representation of this relation is known as the 'Graetz plot'. Curtis & Eberhart (1974) showed the strength of this approach by comparing Graetz type analysis of several different oxygenator designs. Although the designs were far from the theoretical model, the performance of these oxygenators could be described in terms of f_m versus L*/H. One of the aims of presenting the theoretical model here is to base the Graetz analysis on a more solid theoretical basis. For example, the Graetz plot should be given in a double log graphical representation rather than in a semi-log representation as is done by Curtis & Eberhard (1974).

The 'Advancing Front' model results in a number of simple relations giving the membrane area as function of blood film thickness and the influence of membrane resistance. Moreover, it shows the relation of the pressure drop over the oxygenator and the geometrical parameters. These relations do not take into account the flow resistance of layer manifolds, the extra priming volume of manifolds etc. For good examples of analysis of this kind of effects on the design of membrane oxygenators have been published (Mockros & Gaylor, 1975; Gaylor & Mockros, 1975).

Application of the theoretical oxygenator models to a dynamic analysis of actual membrane oxygenator performance leads to some interesting conclusions. Changing the operational conditions in terms of flow and hemodilution will affect above all the venous saturation. Changes in the arterial saturation during bypass will be a result of changing oxygen consumption of the body or changing oxygen transfer capacity of the oxygenator. The latter may be caused for example by clots in the oxygenator or water condensation in the gas compartments.

Considering control mechanisms, the conclusion from Fig. 5 can be formulated in a different manner. If the oxygen consumption stays constant and equal to the predicted oxygen transfer of the oxygenator, the changes in arterial saturation are small and hence the oxygenator has ideal control mechanisms for variations in flow and hematocrit. However, there is virtually no adaptation mechanism for an increase in oxygen consumption. The body has to adapt its consumption to the capacity of the lung. This adaptation is made possible by decreasing the temperature of the body. From equations (22) and (23) it follows that changes in S_a and S_v with $\dot{V}_{O_2}$ are smaller when γ is smaller and thus when the f_m versus L*/H plot is less steep.

Study of Fig. 5 also may induce a new definition of oxygen transfer characteristics of a membrane oxygenator. If the oxygen transfer capacity of the membrane oxygenator is such as to produce an arterial saturation of 95% the clinician is sufficiently informed to decide if this device is appropriate for his purpose. The notion 'rated oxygen transfer' may still be applied but now only the arterial saturation requires an exact definition. The notion ROT per square meter membrane area is then adequate to define the efficiency of the oxygen transfer of a particular device. It is clear that this new definition can only be applied if a Graetz analysis is relevant. From the above-mentioned analysis of Curtis this definition might be applied to most of the present available oxygenators.

APPENDIX

Some numerical values

Several expressions for the dimensionless length are

$$L^* = \frac{L\ D_b}{\frac{1}{4}\ d\ v_m} = \frac{2}{m}\frac{A\ D_b}{d\ Q} = \frac{1}{m^2}\frac{A^2\ D_b}{V\ Q} \tag{1.1}$$

The standard conditions were defined as

$S_v = .65$ $S_a = .95$
HCT=40 so $h = .183$ cc 0_2/cc blood (hb=13.6 g%)
$P_g - P_l = 660$ mm Hg.
Therefore $f_m = .86$; $H = 3.33$.
From Fig. 2 we find the following values for L^* when $M = 0$:
$L^*_p = 1.52$ (flat plate); $L^*_f = .694$ (hollow fiber).

From these numbers and equation (1.1) it follows that the ratio of the blood volumes of the flat plate and hollow fiber, respectively, at equal membrane area amounts to 1.8.

To calculate the surface area needed to oxygenate a blood flow of 1 liter per minute one needs a value for D_b, which is taken here 1.4 10^{-5} cm²/sec. We now find for the flat plate with a gap width of 100 μm

$$\frac{A}{Q} = .9\ m^2/(L/min)$$

and for a hollow fiber device with a fiber diameter of 200 μm

$$\frac{A}{Q} = 1.64\ m^2/(L/min).$$

If the fiber length is 10 cm, one needs per liter blood flow an amount of 2.6 10^4 fibers. In this case the pressure drop over the fiber oxygenator will be 35 mm Hg.
A Silicone rubber membrane of 100 μm has a transfer capacity of approximately 360 cc O_2/m²/min/ato.
So for this membrane $(A/Q)_{D_b \to \infty} = .17$ m²/(L/min).

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