Capacities in inventory control

CAPACITIES IN INVENTORY CONTROL

J. Wijngaard

Eindhoven University of Technology
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1 Introduction

An important category of production situations is the category of situations characterized by repetitive manufacturing (see Hall [7]) and "Make to stock". The main elements in such a situation are products (in various stages of completion, including raw materials and components) and capacities. Coupled with these elements there are certain restrictions. On the product-side there are restrictions with respect to delivery patterns. These restrictions imply restrictions on the production pattern of intermediate products and on the procurement of raw materials and components. On the capacity-side there are restrictions on the production patterns of groups of products (capacity groups). These restrictions play also a role on the product-side since the throughputtimes per stage depend on the way the capacities are used. It is this interference of product-based restrictions and capacity-based restrictions which makes production control difficult. In a way one may interprete production control as the problem to structure this interference.

There are two important extreme ways to structure this interference, the product-oriented approach and the capacity-oriented approach. These two approaches will be explained briefly.

Product oriented approach: First, required delivery patterns (Master Production Schedule) are formulated, not taking into account the capacity restrictions. These delivery patterns are translated to production patterns by offsetting, using standard throughputtimes. Then these production patterns are coordinated, taking into account the capacity restrictions. Typically the horizon in this second planning step is smaller than in the first planning step. Uncertainties in required delivery patterns and capacity availability and interferences between products because of restricted capacities can be attacked by safety stocks and safety leadtime in the first planning step (so per product).
The MRP-I approach (see Orlicky [10]) and less extreme also the MRP-II approach are of this product oriented type.

Capacity oriented approach: First a capacity usage plan is made,

possibly combined with a capacity adjustment plan. This requires aggregation of delivery patterns and inventories to capacities. Then, using short term detailed information the capacity usage of the first period is distributed over the different products. This disaggregation can be based, for instance, on the run-out times of the individual products. Uncertainties in capacity availability and total required deliveries can be taken into account in the first planning step. Imbalances between the individual products resulting from this procedure, may also be estimated in an aggregate way. It is possible so to determine how much extra (aggregate) inventory is necessary because of these imbalances.
Such capacity oriented approaches are proposed by Van Beek [1] and Meal [8]. Both stress the capacity adjustment in the first step and assume in the second step that capacity usage and capacity availability are equal.

Both approaches are feasible. It is not clear, however, when to use what approach. It may be so that both approaches work well in certain situations, while in other situations only a mixture of both approaches is satisfying. The objective of this paper is to compare both approaches and to investigate their weak and strong points.

In the next section we will first illustrate the two approaches on hand of a simple one-stage case.

2 The one-stage deterministic case

In this section we consider the multi-product case with one production stage, periodic review, leadtime < 1, one capacity restriction, no production cost, completely predictable demand, backordering.

In the complete deterministic case it is optimal, or almost optimal to use a rolling-plan approach in which each period a detailed plan is made over a long horizon (T), taking into account as well the required deliveries as the capacity availability. So, each period a problem of the following type is solved:

$$\text{Choose } x_j(t) \geq 0,\ j=1, \ldots N,\ t=1, \ldots, T$$

$$\text{such that } \sum_{j=1}^{N} x_j(t) \leq C(t)$$

$$\text{and } \sum_{t=1}^{T} \sum_{j=1}^{N} h(I_j(t)) \text{ is minimal}$$

$$(\text{with } I_j(t+1) = I_j(t) + x_j(t+1) - d_j(t+1))$$

In this problem $x_j(t)$ represents the planned production for product j in period t, $d_j(t)$ the required delivery (demand) in period t and $I_j(t)$ the inventory of product j at the end of period t. The inventory cost is given by h(.), the available capacity by C(t).

The product oriented rolling-plan approach for the same deterministic case involvs each period the execution of a two-step procedure. The first step is to construct a plan for each product, assuming that the products do not interfere which each other (the whole capacity available for each product). That requires the solution of N problems of the following type

$$\text{Choose } 0 \leq y_j(t) \leq C(t),\ t=1, \ldots T$$

$$\text{such that } \sum_{t=1}^{T} h(I_j(t)) \text{ is minimal}$$

$$(\text{with } I_j(t+1) = I_j(t) + y_j(t+1) - d_j(t+1))$$

The next step is to coordinate the planned production $y_j(t)$, taking into account the restricted capacity. If only the first period is coordinated, this second step requires the solution of a problem of the following type

Choose $x_j(1) \geq 0,\ j=1, \ldots, N$

such that $\sum_{j=1}^{N} x_j(1) = \min \left\{ \sum_{j=1}^{N} y_j(1),\ C(1) \right\}$

and $\sum_{j=1}^{N} h(I_j(1))$ is minimal

(with $I_j(1) = I_j(0) + x_j(1) - d_j(1)$)

The capacity oriented rolling-plan approach also involves each period the execution of a two-step procedure. The first step is to construct an aggregate plan. This requires the solution of a problem of the following type:

Choose $0 \leq y(t) \leq C(t),\ t=1, \ldots, T$

such that $\sum_{t=1}^{T} h^*(I(t))$ is minimal

(with $I(t+1) = I(t) + y(t+1) - \sum_{j=1}^{N} d_j(t+1)$)

Here $h^*(\cdot)$ is the aggregate inventory cost function. We come back to the choice of h^*, but first we describe the second step, the disaggregation step. This step requires the solution of a problem of the following type:

Choose $x_j(1) \geq 0,\ j=1, \ldots, N$

such that $\sum_{j=1}^{N} x_j(1) = y(1)$

and $\sum_{j=1}^{N} h(I_j(1))$ is minimal

(with $I_j(1) = I_j(0) + x_j(1) - d_j(1)$)

The inventory cost function $h(\cdot)$ is in general convex. This implies that in the disaggregation step the (projected) inventories of the different products have to be made as equal as possible. If complete equality can be realized we get $\sum_{j=1}^{N} h(I_j(1)) = N.h(I(1)/N)$. So $h^*(I) := N.h(I/N)$ is a reasonable first choice. We come back to other possibilities.

In the complete deterministic case the product oriented and the capacity oriented approaches may be seen as simple heuristics to solve the detailed problem. It is also possible to construct more advanced heuristics in which both approaches are used. One can use for instance the aggregate model to determine a kind of "shadow"-prices for capacity usage and then use this "shadow"-prices in a product oriented approach. Such heuristics are mentioned by Billington et al., see [5] for references. Another possibility to solve the detailed problem is by starting with the first step of the product oriented approach (the construction of plans per product) and to let that follow by a coordination step in which not only one period is considered, but the whole horizon. Eisenhut [6] and Van Nunen/Wessels [9] for instance describe simple coordination procedures for multi-product Wagner-Whitin type problems with constrained capacity which lead to almost optimal solutions.
It is also possible to improve the disaggregation step in the capacity oriented approach and the coordination step in the product oriented approach by using the dynamic programming value functions of the 1-product problems (see Van Beek [1] and Wijngaard [12]).

In cases where demand and capacity are only partly predictable the straight forward detailed rolling-plan approach is not necessarily optimal or almost optimal. The problem to determine the optimal strategy is much more complex. The product oriented and capacity oriented approach may not be seen then as simple methods to approximate the optimal (detailed) strategy, but have to be interpreted as more or less independent approaches in controlling a complex stochastic dynamic system. In the next section the other extreme case, the pure stochastic case, will be discussed.

3 The one-stage stochastic case

We consider a system characterized in the following way:

* There are N products
* Customers arrive according to a stochastic process D. Each customer demands with probability 1/N one unit of product j.
* There is a stochastic process P generating production opportunities. Each production opportunity may be used to get a production run of size q of one of the products. The delivery is immediate.
* The performance criterion is the continuous time average of $\sum_{j=1}^{N} h(I_j(t))$, with h(.) convex.

This system has been chosen to analyze because it is the easiest system where products and capacity interfere. Notice that the assumption of immediate delivery is not essential. The system with leadtime ℓ and backordering can be transformed to this system by adjusting the penalty function h(.). In such a system all reasonable strategies differ only with respect to what production opportunities are used. That the products are identical and that h(.) is convex implies that it is anyway optimal to assign a production run to the product with the smallest inventory.

In the capacity oriented approach the decision whether or not to use a production run has to depend on the aggregate inventory. It is sufficient to consider capacity oriented strategies of the following type: Produce iff $I \leq I_c$ (some I_c).

Let $y_i := I_i - I_c/N$. The y_i are called the c-deviations.

Let $F(y_1,\ldots,y_N)$ be the steady state distribution of the c-deviations under a capacity oriented strategy. Notice that $F(y_1,\ldots,y_N)$ is independent of I_c.

The average cost is

$$\int_{y_1\ldots y_N} \sum_{i=1}^{N} h(I_c/N + y_i)\, dF(y_i,\ldots,y_N) \qquad (1)$$

The optimal capacity oriented strategy follows from minimization over I_c of this expression.

In the product oriented approach one has to decide for all products whether or not a production run is necessary. These decisions are based on the individual inventories.
A production opportunity is used as soon as one of the products requires a production run. So, it is sufficient to consider product oriented strategies of the following type: produce iff $\underset{j}{\text{Min}}\ \{I_j\} \leq I_p$ (some I_p).
Let $z_i := I_i - I_p$. The z_i are called the p-deviations.
Let $G(z_1,\ldots,z_N)$ be the steady state distribution of the p-deviations under a product oriented strategy. Notice that $G(z_1,\ldots,z_N)$ is independent of I_p.
The average cost is

$$\int_{z_1\ldots z_N} \sum_{i=1}^{N} h(I_p + z_i)\ dG(z_1,\ldots,z_N). \tag{2}$$

The optimal product oriented strategy follows from minimization over I_p of this expression.

Both the determination of the optimal capacity oriented approach and the optimal product oriented approach require the calculation of an N-dimensional distribution (F,G). So, the practical use of these optimal strategies is not very high, but the strategies are nevertheless interesting as reference cases. Bemelmans/Wijngaard [4] consider some specific systems and use simulation to find these strategies and the corresponding average cost. It turns out that, except in cases with a very loose capacity, the optimal capacity oriented strategy and the optimal product oriented strategy have about the same performance and are almost optimal. Notice that the minima of (1) and (2) may be seen as measures of the variations in the inventories under application of a capacity oriented strategy and under application of a product oriented strategy. The results indicate that the inventories vary in the same way under both types of strategies. That suggests that the choice whether to use a capacity oriented approach or a product oriented approach is not so important in the pure stochastic case as long as the average inventory is at the right level. The problem to choose the right level is considered in the next section.

4 Choosing the right level

To determine the optimal capacity oriented and product oriented strategy requires computation of the N-dimensional distributions $F(y_1,\ldots,y_N)$ and $G(z_1,\ldots,z_N)$.
We sketch some possibilities to get good capacity oriented and product oriented strategies in a 1-dimensional way.

First we consider capacity oriented strategies.
Expression (1) can be written

$$\int_y \int_{y_1\ldots y_N} \left\{ \sum_{i=1}^{N} h(I_c/N + y_i) \right\} d\, F_c\, (y_1,\ldots y_N | \Sigma\, y_i = y)\ d\, F\, (y)$$

where $F_c(y_1\ldots y_N | \Sigma\, y_i = y)$ is the steady state conditional distribution of $y_1,\ldots,y_N$ given that $\sum_{i=1}^{N} y_i = y$ and $F(y)$ is the steady state distribution of $\sum_{i=1}^{N} y_i$ $(= \sum_{i=1}^{N} I_i - I_c)$.

With

$$g(I_c,y) := \int_{y_1\ldots y_N} \left\{ \sum_{i=1}^{N} h(I_c/N + y_i) \right\} dF(y_1,\ldots,y_N | \Sigma\, y_i = y)$$

we get

$$\int_{y_1\ldots y_N} g(I_c,y)\ dF(y)$$

To determine $F(.)$ is a 1-dimensional problem. To get a good capacity oriented strategy is to get a good approximation of $g(I_c,y)$. The most trivial approximation results from assuming that it is possible to keep the inventories equal. This leads to the approximation

$$g^1(I_c,y) = N.h(I_c/N + y/N)$$

The resulting strategy is called the simple capacity oriented heuristic. Another approximation of $g(I_c,y)$ results from assuming that all y_i are uniformly distributed on the interval $[y/N - q/2,\ y/N + q/2]$. This leads to the approximation

$$g^2(I_c,y) = N\int_x h(I_c/N + y/N + x)\ d\ U\ (x)$$

where U (.) is the uniform distribution on the interval $[-q/2,\ + q/2]$.

Now the product oriented strategies.

Possible delays because of other products also needing a production run have to be taken into account in the critical level I_p. The most trivial approximation follows from assuming that the other products do not cause delays at all. To determine the right I_p under this assumption is a 1-dimensional problem. The resulting strategy is called the simple product oriented heuristic.

Another approximation results from the assumption that the delays are independent identically distributed stochastic variables, namely the stochastic variables representing the steady state waiting times in the corresponding queueing system. This is the approach proposed by Williams [11].

Results are given in Bemelmans/Wijngaard [4]. It turns out that in most cases the best of the two simple heuristics is close to optimal.

5 The partly predictable case

It is not possible to get a good insight in the value of both approaches without also considering the case with partly predictable demand and/or capacity. That is the most normal case in fact. The pure stochastic case and the pure deterministic case are only interesting as far as they contribute to the control of the partly predictable case.

We consider a situation with N products which are in average about identical. The treatment of the case with very different products is out of the scope of this paper. Therefore we refer to Bemelmans [2]. Even if the products are in average about identical there may be short term differences because of differences in demand forecasts. That means that the allocation of production to products is not a trivial problem now. It is not optimal now to assign a production run to the products with the smallest inventory. The effect of different allocation rules has been investigated in Bemelmans [3]. It turned out that even in the case with a rather high predictability, allocation to the product with the smallest inventory is close to optimal. That implies that we may restrict the attention to strategies according to which the production runs are allocated to the products with the smallest inventory. As in the pure stochastic case these strategies differ only with respect to which production opportunities are used.

For the pure stochastic case we know already that the pattern of inventory variations is not very sensitive for whether a capacity oriented strategy is used or a product oriented strategy. One may expect that this insensitivity extends to more complex mixed strategies and to the partly predictable case. This would imply that the quality of the optimal capacity oriented strategy and the optimal product oriented strategy is not influenced much by the predictability. Results in Bemelmans [3] seem to confirm that, but more thorough analyses could be useful.

Next point is how to get good capacity oriented and product oriented heuristics. It is possible to use the same type of heuristics as in the pure stochastic case.

The capacity oriented heuristics are based on approximations of the aggregate inventory cost function.

$$g^1(I) = N.h(I/N)$$

$$g^2(I) = N.\int_x h(I/N + x)\ d\ U\ (x)$$

Considering the insensitivity of the pattern of inventory variations one may expect that the quality of these approximations is about the same as in the stochastic case.

In the pure stochastic case one gets capacity oriented heuristics characterized by a critical inventory level. In this partly predictable case the available forecasts play also a role. The predictability severely increases the complexity of the aggregate inventory system. We will not discuss these difficulties here (see Bemelmans [3]). The point here is that one may expect that working capacity oriented can be as close to optimal as in the corresponding stochastic case.

The product oriented heuristics are based on the assumptions:

1 No interference between products

2 "Steady state" interference between products.

In both heuristics the N-product model is replaced by N coordinated one-product models in which the complete capacity is available for each of the products and the interference between the products is modelled as a stationary (stochastic) delay. Because of the low utilization rate in the one-product models, information about future demand is not very useful. That means that one may not expect that the performance of these heuristics can be improved by the available (imperfect) predictions. Recall that we restrict the attention here to strategies with myopic allocation (smallest inventory). This suggests that we may conclude that the capacity oriented approach extends better to the partly predictable case. But it is certainly necessary to get more numerical results to support this conclusion.

References

[1] Beek, P. van (1977), "An Application of Dynamic Programming and the HMMS rule on Two-level Production Control", Zeitschrift für Operations Research, 21, B133 - B141

[2] Bemelmans, R.P.H.G. (1984), "Capacity-oriented Production Planning in case of a Single Bottle-neck", report BDK/ORS/84/01, Eindhoven University of Technology

[3] Bemelmans, R.P.H.G., "A Single-Machine Multi-Product Planning Problem under Periodic Review", report BDK/ORS/83/02, Eindhoven University of Technology

[4] Bemelmans, R.P.H.G. and J. Wijngaard (1982), "Aggregation and Decomposition in case of a Stochastic Sinlge-Machine Multi-Product Planning Problem", report BDK/ORS/82/09, Eindhoven University of Technology

[5] Billington, P.J., J.O. McClain and L.J. Thomas (1983), "Capacity constrained MRP Systems", Management Science 29, 1126 - 1141

[6] Eisenhut, P.S. (1975), "A Dynamic Lot Sizing Algorithm with Capacity Constraints", AIIE-Transactions 7, 170 - 176

[7] Hall, R.W., "Driving the Productivity Machine", APICS, 1981

[8] Meal, H.C., "A Study of Multi-Stage Production Planning" Chapter 9 in "Studies in Operations Management", A.C. Hax (ed.), North-Holland, 1978

[9] Nunen, J.A.E.E. van and J. Wessels (1978), "Multi-item Lot Size Determination and Scheduling under Capacity Contraints", European Journal of Operations Research 2, 36 - 41

[10] Orlicky, J., "Material Requirements Planning", McGraw-Hill, 1975

[11] Williams, T.M. (1984), "Special Products and Uncertainty in Production/Inventory Systems", European Journal of Operations Research 15, 46 - 54

[12] Wijngaard, J. (1979), "Decomposition for Dynamic Programming in Production and Inventory Control", Engineering and Process Economics 4, 385 - 388