The use of nonlinear filtering for a mechanical characterization of biological tissues

THE USE OF NONLINEAR FILTERING FOR A MECHANICAL CHARACTERIZATION OF BIOLOGICAL TISSUES

C. W. Oomens, M. A. N. Hendriks, G. W. M. Peters, J. D. Janssen, and J. J. Kok
Department of Engineering Fundamentals
Eindhoven University of Technology
Eindhoven, The Netherlands

ABSTRACT

Problems with regard to the mechanical characterization of biological tissues demand new methods to determine constitutive equations. Inhomogeneous properties, anisotropy and very complex time-dependent behaviour forces investigators to develop numerical procedures to solve the field equations and limit the use of classical mechanical tests. A new approach based on the measurement of strain distributions and nonlinear filtering techniques from system dynamics is suggested.

NOMENCLATURE

$\vec{\nabla}$	gradientoperator
ρ^{α}	apparent density of component α
c^{α}	mass production of component α from other component(s)
$\vec{\pi}^{\alpha}$	momentum supplied to component α by other omponent(s)
$\underline{\sigma}^{\alpha}$	partial stress in component α
v^{α}	velocity of component α
$\underline{F}^{\alpha}$	deformation tensor of component α
n^{α}	volume fraction of component α
p	hydrodynamic pressure
$\underline{K}$	permeability tensor
$\underset{\sim}{a}$	column
$\underset{\sim}{a}^{T}$	transposed column

INTRODUCTION

In essence the mechanical characterization of (biological) materials is an experimental problem. After preliminary structural and experimental research, ideas can be developed about models, which can be used for a description of the constitutive behaviour of some material. These models have to fit into a theoretical framework arising from a number of physical principles. Finally this will result in functional relationships between stress and strain, deformation rate, temperature, time etc. The parameters and/or functions that play a part in these equations have to be determined by means of experiments.

The present paper shortly reviews two research topics regarding tissue characterization. The first topic concerns the development of constitutive models and algorithms to solve the field equations. The second topic is about the measurement of stress and strain distributions. It will become clear from this review that both areas reveal problems with regard to the quantitative determination of material parameters. Because of this a new way to design experiments has to be developed.

The last section of this paper will be devoted to this subject.

CONSTITUTIVE MODELS

In the last three decades increasingly sophisticated models have been developed for biological tissues. Most of these models can be regarded as phenomenological models. Since the mid–seventies an increasing interest in structural models can be observed. The reason for this is obvious. Many biomechanical applications demand knowledge about what is happening inside the tissue (damage, physiological changes). Mixture models combine some of the advantages of phenomenological and structural models. Mow et al. (1984) started with the use of mixture models for articular cartilage and published a number of closed form solutions for boundary value problems. Because only for a limited number of problems closed form solutions can be derived, numerical procedures have been developed to solve the mixture equations (Simon et al. 1985; Oomens, 1985; Huyghe, 1986; Spilker et al. 1987). All of the mentioned models are based on the assumption that the tissue under consideration consists of a porous solid and a fluid and that solid and fluid do not chemically interact. For some of the tissues, for which mixture theory is used, it is questionable if these assumptions hold. The existence of pores in the tissues as well as the assumption that fluid can move freely through the tissue are points of discussion (Harrigan, 1987).

We assume that only a small part of the interstitial fluid is free to move through the tissue and that the remaining part is chemically tied to the solid. However, under certain conditions the solid is able to release fluid. So a mass exchange between solid and fluid is possible. When both components, solid as well as fluid, are considered to be intrinsically incompressible and isothermal conditions, absence of body forces and inertial forces are assumed, the following balance equation for a mixture with mass exchange can be derived (Oomens et al. 1987):

mass balance for component α: $$\vec{\nabla}\cdot\varphi^{\alpha}\vec{v}^{\alpha} + \frac{\partial\varphi^{\alpha}}{\partial t} = C^{\alpha} \qquad (1)$$

mass balance for the mixture: $$\sum_{\alpha} C^{\alpha} = 0 \qquad (2)$$

momentum balance for component α: $$\vec{\nabla}\cdot\underline{\sigma}^{\alpha} + \vec{\pi}^{\alpha} = \vec{0} \qquad (3)$$

momentum balance for the mixture: $$\sum_{\alpha} (\vec{\pi}^{\alpha} + C^{\alpha}\vec{v}^{\alpha}) = \vec{0} \qquad (4)$$

Typical constitutive equations for the solid stress $\underline{\sigma}^{s}$, the fluid stress $\underline{\sigma}^{f}$ and the momentum interaction $\vec{\pi}^{f}$ can be given as:

fluid stress: $$\underline{\sigma}^{f} = -n^{f} p \underline{I} \qquad (5)$$

solid stress: $$\underline{\sigma}^{s} = \underline{\sigma}_{eff} - n^{s} p \underline{I} \qquad (6)$$

momentum interaction: $$\vec{\pi}^{f} = -\vec{\pi}^{s} = p\vec{\nabla}n^{f} - (n^{f})^{2}\,\underline{K}^{-1}.(\vec{v}^{f} - \vec{v}^{s}) \qquad (7)$$

Equations (1) to (7) form a set of equations, which can be solved if the constitutive laws are known for $\underline{\sigma}_{eff}$ and C^{f}. When large displacements can occur and when the solid can be considered to be elastic, a relation between the Second–Piola–Kirchhoff stress $\underline{T}_{eff} = (\underline{F}^{s})^{-1}.\det(\underline{F}^{s})\,\underline{\sigma}_{eff}.(\underline{F}^{s})$ and the Green–Lagrange strain $\underline{E}$ is suitable. Oomens et al. (1987) used an exponential relationship between $\underline{T}_{eff}$ and $\underline{E}$ for skin. Finally constitutive equations for $\underline{K}$ and C^{α} have to be found.
It is questionable that ever closed form solutions for these highly nonlinear

equations can be found, even for the simplest geometries and loading conditions. Development of numerical procedures for these equations therefore is inevitable and fortunately possible (Snijders, 1986).
However this leads to considerable problems with regard to the determination of material parameters, because without some quantitative value for these parameters a numerical calculation cannot be performed. In the last section a procedure will be presented to solve this problem.

STRAIN DISTRIBUTION MEASUREMENTS

Peters (1987) has measured stress and strain distributions on biological tissues. He studied the influence of boundary conditions and of the proportion and texture of collagenous fibres on experimental results. The experiments were performed on rectangular samples of connective tissue. Collagenous fibres were mainly oriented in parallel fashion. One side of the tissue was gripped while on the other part four threads were mounted (fig. 1.)

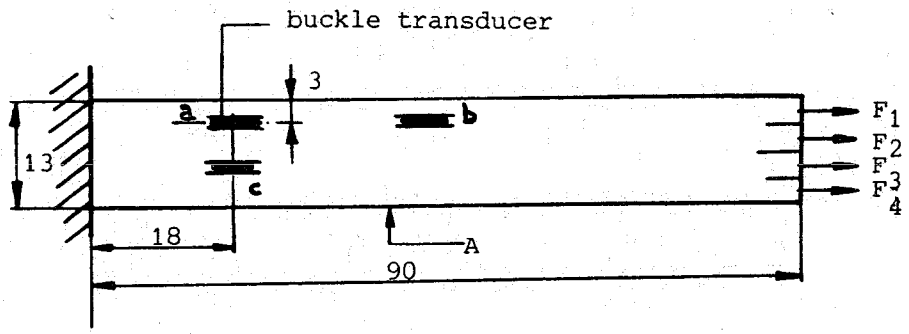

Fig. 1. Testing of the effect of anisotropy.

Buckle transducers (Peters, 1987) were used to measure forces at the positions a, b and c. Strains were measured by means of markers, attached to
the specimen, of which the positions were measured by means of a digital image technique (Peters, 1987).
Several experiments have been performed. In the load cases 1 to 4 only one thread was loaded with a force of 5 N, the other threads remained unloaded

(load case 1, $F_1 = 5$ N, $F_2 = 0$, $F_3 = 0$, $F_4 = 0$ etc.). In load case 5 the right hand side of the tissue was clamped and displaced in x–direction, thus performing a classical uni–axial strain test.

Imagine the specimen devided in 4 areas or strips. A buckle transducer is said to belong to one of the isolated loading strips when the tissue fibres passing along the transducer debouch on that strip. In that case the eccentricity is defined to be zero. Otherwise the excentricity e can be defined as the number of strips adjudged from the strip ascribed to the transducer. When the measured responses of transducers a, b and c for loading case 1 to 4 are given as a function of the excentricity e (fig. 2), inhomogeneous properties can be observed.

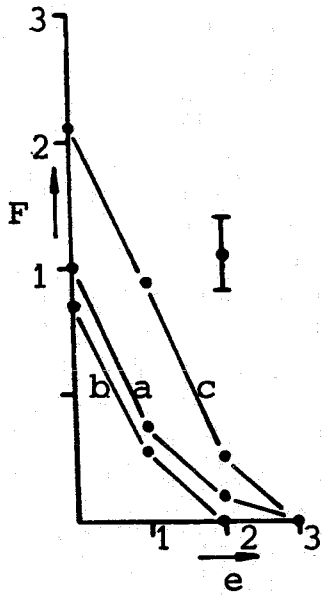

fig. 2. Measured respons of the transducers a, b and c as a function of the eccentricity e for $F_i = 5$ [N]

For a homogeneous material the curves a, b and c in fig. 2 should be the same. The result suggests inhomogeneous effects. These become more evident when strain measurements are considered. From the measured marker displacements strains can be calculated. Fig. 3 a and b show the measured positive principle strains for the load cases 1 and 4. Even without a quantitative analyses it is clear that large differences can be observed for the two situations. These are probably caused by

the fact that, although the fibres seem to be parallel distributed, in reality they are not.

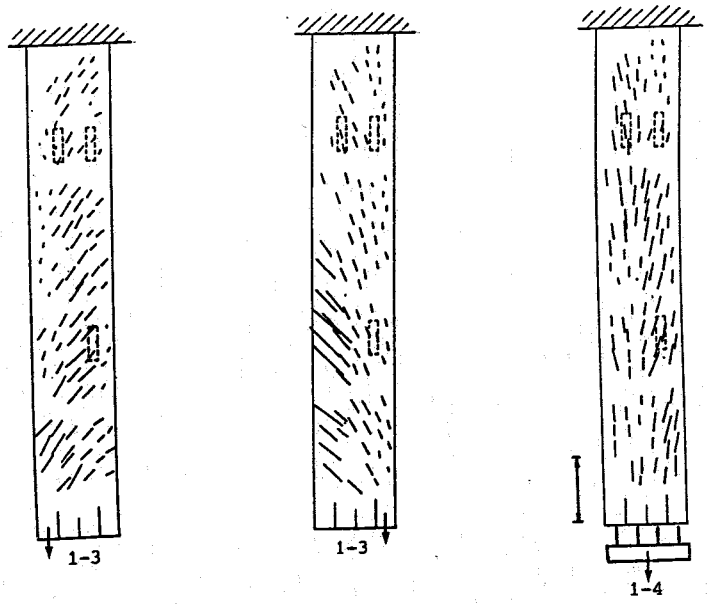

fig. 3. Strain distributions for:
a) load case 1, b) load case 4 c) a uniaxial test

Fig. 3c shows the results of a common uni-axial strain test and shows that inhomogeneous strains are also measured in this type of test.

Summarizing it can be said that due to inhomogeneous material properties, disruption of the connective tissue in small specimens and difficulties in creating homogeneous boundary conditions strongly inhibits the possibilities of classical material tests like uniaxial and biaxial tests. Thus it is necessary to develop alternative methods. The next section will be devoted to this subject.

NONLINEAR FILTERING AND TISSUE CHARACTERIZATION

A classical method to determine material parameters is to make specimens of simple shape and to load these in such a way that a homogeneous strain distribution can be measured in a part of the material. This strain and the load can

be measured and one or a few material parameters determined. This procedure leads to uni–axial and bi–axial strain tests, torsion tests etc. For biological materials this procedure has some disadvantages:

- One of the motives to pursue a homogeneous strain distribution is the resulting simple boundary value problem with usually closed form solutions. As described above constitutive laws for biological tissues are extremely complicated and numerical analyses become inevitable.
- Obtaining a homogeneous strain distribution is almost impossible when material properties are inhomogeneous, as usually is the case for biological materials.
- The number of parameters needed in the constitutive equations for biological materials is large. Many classical experiments would be necessary to be able to determine all these parameters.

We have decided to accept the facts that a strain distribution has to be measured and that a numerical analysis is necessary to find the parameters. An advantage of this approach is more freedom for the design of experiments. Earlier attempts of this kind (Kavanagh, 1973; Yettram and Vinson, 1979) were limited in their success because of computing times and the lack of accuracy for the measurement of the strain distribution. The prospects are a lot more favourable at the present time. Computers are faster and cheaper than before and digital image analysis techniques offer good prospects for strain distribution measurements.
The problem to be solved is now defined in the following way. Assume that some tissue is loaded in a well determined way and a number of resulting quantities, the outputs, are measured. Consider $\underset{\sim}{z}$ as a column of measured quantities (displacements, forces, strains). Assuming that already a constitutive law has been proposed (for example as a result of structural research or preliminary experiments), a column $\underset{\sim}{x}$ of unknown material parameters can be defined. The goal of the experiment is to determine
quantitative values for this column $\underset{\sim}{x}$. Usually a complicated, nonlinear functional $\underset{\sim}{y}$ image $\underset{\sim}{x}$ on $\underset{\sim}{z}$. The functional $\underset{\sim}{y}$ depends on the choice of the experiment. The problem to be solved is to find $\underset{\sim}{x}$ from the following equation:

$$\underset{\sim}{z} = \underset{\sim}{y}\,(\underset{\sim}{x}, \underset{\sim}{v}) + \underset{\sim}{w} \qquad (8)$$

The column $\underset{\sim}{v}$ represents model errors, $\underset{\sim}{w}$ represents measurement errors. The properties $\underset{\sim}{z}$, $\underset{\sim}{y}$, $\underset{\sim}{v}$ and $\underset{\sim}{w}$ may be time dependent, but the column $\underset{\sim}{x}$ is assumed to be constant.

The first step in modelling is to define a column of material parameters $\underset{\sim}{x}$, in other words, to find a suitable constitutive equation for the material under consideration. Assuming that a constitutive model has been found which –for proper values of the parameters $\underset{\sim}{x}$– enables a realistic representation of the mechanical behaviour, it is necessary to develop an algorithm to solve the field equations describing mechanical problems concerning these soft tissues, i.e. find the functional $\underset{\sim}{y}$. Due to the complexity of the constitutive equations it is usually not possible to obtain closed form solutions, so it is necessary to develop a numerical procedure, for example a finite element calculation. With such a procedure it is possible to calculate $\underset{\sim}{y}(\underset{\sim}{x})$ for a given column $\underset{\sim}{x}$. However the opposite is not true.
The problem to find $\underset{\sim}{x}$ from equation (8) when a column $\underset{\sim}{z}$ is available strongly resembles parameter estimation problems in the area of system dynamics. An extra complication in the problem of material characterization is the fact that $\underset{\sim}{y}$ is not explicitly known. Assuming that the errors $\underset{\sim}{v}$ and $\underset{\sim}{w}$ can be considered as white noise, then for a linear function $\underset{\sim}{y}(\underset{\sim}{x})$ a recursive estimation scheme for $\underset{\sim}{x}$ can be deduced which leads to an estimation $\hat{\underset{\sim}{x}}$ for which:

$$E[\underset{\sim}{x} - \hat{\underset{\sim}{x}})^T(\underset{\sim}{x} - \hat{\underset{\sim}{x}})] = \text{minimal} \qquad (9)$$

the estimate $\hat{\underset{\sim}{x}}$ for $\underset{\sim}{x}$ is called a minimum variance estimate. In system dynamics the same procedure is used to estimate the state of the system from output measurements. The recursive scheme for $\hat{\underset{\sim}{x}}$ is then called a Kalman filter (Norton, 1986). During the iteration process for each estimate $\hat{\underset{\sim}{x}}$ the difference between the measured column $\underset{\sim}{z}$ and the value of $\underset{\sim}{y}(\hat{\underset{\sim}{x}})$ is calculated and added in a weighted fashion to $\hat{\underset{\sim}{x}}$ to obtain a new estimate. The procedure also provides an updated estimate of the covariance of the column $\hat{\underset{\sim}{x}}$, thus giving an idea of the accuracy of the estimate $\hat{\underset{\sim}{x}}$.

In the case of material characterization the functional $\underset{\sim}{y}$ will usually be nonlinear. Assuming that an estimate $\hat{\underset{\sim}{x}}$ can be found which is sufficiently close to the actual value of $\underset{\sim}{x}$, the function $\underset{\sim}{y}$ can be linearized and a similar recursive scheme derived as for the linear case. In each iteration step Finite Element Calculations are to be evaluated to find $\underset{\sim}{y}(\hat{\underset{\sim}{x}})$ and $d\underset{\sim}{y}/d\hat{\underset{\sim}{x}}$.

DISCUSSION

The alternative approach suggested in this paper is not the ultimate solution with regard to the characterization of biological tissues. The presented ideas create more freedom with regard to the design of experiments and might give some guidelines of how to use this freedom, but cannot be used without much knowledge obtained from "classical" techniques. Of course the numerical analyses which has to be performed for each experiment is very costly and time consuming. The demands which have to be made upon the strain distribution measurements are high with regard to resolution and speed, and make the experiments expensive. Moreover, the path along which this technique has to be developed is full of pitfalls. However, the examples in the present paper illustrate that classical methods for tissue characterization have their limitations and it is hoped that an introduction of a new method from another discipline may lead to new ideas and shift the limits.

REFERENCES

Harrigen T.P. (1987): Cartilage is poroelastic but not biphasic. J. Biomechanics 20, 827–829.

Huyghe J.M.R.J. (1986): Non–linear finite element models of the beating left ventricle and the intramyocardial coronary circulation. Ph.D.–thesis Eindhoven University of Technology, the Netherlands.

Kavanagh K.T. (1973): Experiments versus analysis: computational techniques for the description of elastic solids. Int. J. Number, Meth. Engng., 5, 503–515.

Mow V.D., Holmes M.H., Lai W.M. (1984): Fluid transport and mechanical properties of articular cartilage: a review. J. Biomechanics 17, 377–394.

Norton J.P. (1986): An introduction to identification. Academic Press.

Oomens C.W.J., Van Campen D.H., Grootenboer H.J. (1987): A Mixture approach to the mechanics of skin. J. Biomechanics, 20, 9, 877–885.

Oomens C.W.J. (1985): A mixture approach to the mechanics of skin and subcutis. Ph.D–thesis, Twente University of Technolocy, The Netherlands.

Peters G.W.M. (1987): Tools for the measurement of stress and strain fields in soft tissue. Ph.D.–thesis Rijksuniversity of Limburg, The Netherlands.

Simon B.R., Wu J.S.S., Carlton M.W. (1985): Structural model for human spinal motion segments based on a poreelastic view of the intervertebral disk. J. Biomech. Engn. 107, 327–335.

Snijders H. (1986): The mechanical behaviour of soft biological tissues based on a mixture model (In Dutch). Internal Report WFW 86–011, Eindhoven University of Technology, Eindhoven, The Netherlands.

Spilker R.L., Suh J.K., Mow V.D. (1987): A finite element formulation for the linear biphasic representation of soft tissue. Proc. 1987 Biomech. Sym., ASME eds. D. Butler, P. Torzilli, M. Friedman., 13–16.

Yettram A.L., Vinson C.A. (1979): Orthotropic elastic moduli fo rleft ventricular mechanical behaviour. Med. Biol. Engng. Computing, 17, 25–30.