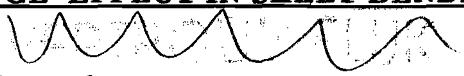
The edge-effect in sheet bending

THE EDGE–EFFECT IN SHEET BENDING



ir. S.M.Hoogenboom
ir. Guo Minghai

April 1991
IOPM WPA 1029

CONTENTS

I. INTRODUCTION

In sheet bending practice, it is often occured that the outline in the bending area is not straight, especially near the margin of the sheet material, there is a outwards curved contour at the both sides, the so called edge–effect(see fig.1).

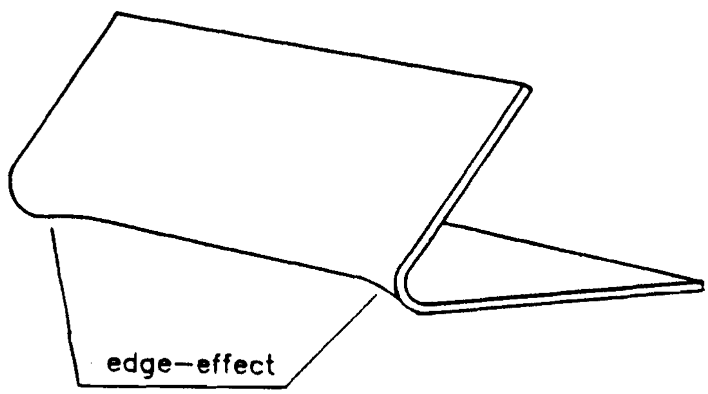


Fig.1 The Edge–Effect in Sheet Bending

In wide investigation on the bending products, the different widths, or the different curvatures and the different angles containing the curved areas, have been seen near the both sides with different bending curvatures and different sheet thicknesses. This means the shape of the edge–effect is influenced by the curvature of the deflection at the edge and the angle. It is also influenced by the factors such as material constants n, ϵ_0, and ϵ_β.

In this paper the influence of these factors on the shape of the edge–effect will be investigated by minimizing the deformation energy.

II. THE MODEL OF THE EDGE EFFECT

First, the following assumptions are applied:

— the curvature of the edge–effect area is constant, that is, the curved shape is part of a circle;

— the strain of the middle plane of the sheet material ϵ_β is a constant;

— strain hardening: $\overline{\sigma}=C(\overline{\epsilon}+\overline{\epsilon}_0)^n$;

— straight strain path.

The bending area can be considered as a combination of two different areas, one is in the intermediate part where the outline is assumed straight, and the other is at the edge where the outline is curved. We name the curved part as Area I, and the straight part as Area II (see fig.2.1). The geometry of Area I is the area

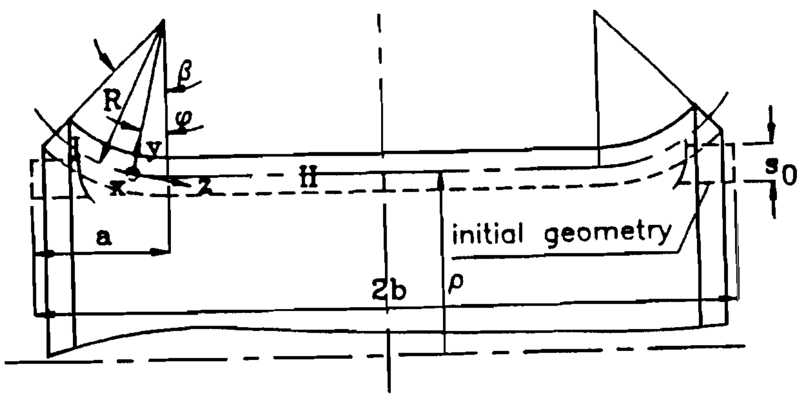


Fig.2.1 The model of the edge–effect in sheet bending

where the edge–effect is located, and the following parameters are introduced:

___ R — the curvature radius of the middle plane;

___ β — the angle establishes the width of Area I;

___ ϵ_β — the strain of the middle plane of Area I in the z–direction, which is assumed to be constant.

From Area II it is assumed that its outline is straight, and the strain in the z–direction being zero.

The task is to calculate the total deformation energy

$$W = W_1 + W_2 \qquad (2.1)$$

where W_1 is the deformation energy in Area I, W_2 in Area II, and the aim is to minimize W with respect to the free parameters R, β, and ϵ_β.

III. THE DEFORMATION ENERGY IN THE BENDING AREA

For convenience, the deformation energy in the bending area is considered in Area I and in area II separately.

3.1 The Deformation Energy in Area I

Taking an element from the Area I (see fig.2.1), the strain in the z–direction is

$$\begin{aligned}\epsilon_z &= \ln\frac{\beta(R-y)}{a} \\ &= \ln\frac{\beta R}{a} + \ln\frac{(R-y)}{R} \\ &= \epsilon_\beta + \ln(1-\frac{y}{R}) \qquad (3.1)\end{aligned}$$

where $\epsilon_\beta = \ln\frac{\beta R}{a}$, the strain of the middle plane of Area I.

The strain in the x–direction is

$$\epsilon_x = \ln\frac{\alpha[\rho+R-(R-y)\cos\varphi]}{\alpha\rho}$$
$$= \ln\left(1 + \frac{R-(R-y)\cos\varphi}{\rho}\right) \qquad (3.2)$$

where α is the bending angle.

Using the effective strain definition

$$\bar{\epsilon}_1 = \frac{2}{\sqrt{3}}\sqrt{\epsilon_x^2+\epsilon_z^2+\epsilon_x\epsilon_z} \qquad (3.3)$$

the deformation energy in Area I becomes

$$W_1 = \frac{C}{n+1}\int_{\varphi=0}^{\beta}\int_{y=-\frac{s_0}{2}}^{\frac{s_0}{2}}\left[(\bar{\epsilon}_1+\bar{\epsilon}_0)^{n+1}-\bar{\epsilon}_0^{\,n+1}\right]dv \qquad (3.4)$$

where C and n are the material constants, s_0 the thickness of the sheet material, and dv the volume of the element, which is

$$dv = \alpha[\rho+R-(R-y)\cos\varphi]\cdot(R-y)\,d\varphi\,dy \qquad (3.5)$$

3.2 The Deformation Energy in Area II

Assuming the plane strain in the z–direction being zero and the middle plane unstretched in x–direction, it follows

$$\epsilon_x = -\epsilon_y = \ln\frac{\alpha(\rho+y)}{\alpha\rho}$$
$$= \ln\left(1+\frac{y}{\rho}\right) \qquad (3.6)$$

and thus

$$\bar{\epsilon}_2 = \frac{2}{\sqrt{3}}\ln\left(1+\frac{y}{\rho}\right) \qquad (3.7)$$

The deformation energy in Area II becomes

$$W_2 = \frac{C}{n+1}(b-a)\int_{y=-\frac{s_0}{2}}^{\frac{s_0}{2}}\left[(\bar{\epsilon}_2+\bar{\epsilon}_0)^{n+1}-\bar{\epsilon}_0^{\,n+1}\right]\alpha(\rho+y)\,dy$$

$$(3.8)$$

where a is the initial width of Area I, which is

$$a = \frac{\beta R}{e^{\epsilon_\beta}} \tag{3.9}$$

Applying the following dimensionless quantities

$$y^* = \frac{y}{s_0} \tag{3.10a}$$

$$b^* = \frac{b}{s_0} \tag{3.10b}$$

$$R^* = \frac{R}{s_0} \tag{3.10c}$$

$$\rho^* = \frac{\rho}{s_0} \tag{3.10d}$$

$$W_1^* = \frac{W_1}{C\alpha\,\rho\,s_0 b} \tag{3.10e}$$

$$W_2^* = \frac{W_2}{C\alpha\,\rho\,s_0 b} \tag{3.10f}$$

and

$$\begin{aligned} W^* &= \frac{W}{C\alpha\,\rho\,s_0 b} \\ &= \frac{1}{C\alpha\rho s_0 b}(W_1 + W_2) \\ &= W_1^* + W_2^* \end{aligned} \tag{3.10g}$$

the total dimensionless deformation energy becomes

$$\begin{aligned} W^* =& \frac{1}{(n+1)b^*} \int_{\varphi=0}^{\beta} \int_{y^*=-\frac{1}{2}}^{\frac{1}{2}} [(\bar{\epsilon}_1+\bar{\epsilon}_0)^{n+1} - \bar{\epsilon}_0^{\,n+1}] \cdot \\ & \cdot [\rho^* + R^* - (R^* - y^*)\cos\varphi] \frac{R^* - y^*}{\rho^*} \cdot d\varphi \cdot dy^* + \\ & + \frac{1}{n+1}\left(1 - \frac{\beta R^*}{b^* e^{\epsilon_\beta}}\right) \int_{y^*=-\frac{1}{2}}^{\frac{1}{2}} [(\bar{\epsilon}_2+\bar{\epsilon}_0)^{n+1} - \bar{\epsilon}_0^{\,n+1}]\left(1 + \frac{y^*}{\rho^*}\right) dy^* \end{aligned} \tag{3.11}$$

where the front item concerns the energy in Area I, the rear one concerns Area II, and, $\bar{\epsilon}_1$, $\bar{\epsilon}_2$, ϵ_x, and ϵ_z can also be written with the dimensionless parameters as follows

$$\bar{\epsilon}_1 = \frac{2}{\sqrt{3}}\sqrt{\epsilon_x^2+\epsilon_z^2+\epsilon_x\epsilon_z} \qquad (3.12)$$

$$\epsilon_x = \ln\left(1 + \frac{R^*-(R^*-y^*)\cos\varphi}{\rho^*}\right) \qquad (3.13)$$

$$\epsilon_z = \epsilon_\beta + \ln\left(1 - \frac{y^*}{R^*}\right) \qquad (3.14)$$

$$\bar{\epsilon}_2 = \frac{2}{\sqrt{3}}\left|\ln\left(1 + \frac{y^*}{\rho^*}\right)\right| \qquad (3.15)$$

3.3 Derivation of the Bending Couple

Assuming that the length of the middle plane remains the constant l_0 gives

$$\alpha = \frac{l_0}{\rho} = \frac{l_0}{s_0}\cdot\frac{1}{\rho^*}$$

The bending couple has the relation with the bending energy, which is

$$M_B = \frac{\partial W}{\partial \alpha} = \frac{s_0}{l_0}\cdot\frac{\partial W}{\partial(1/\rho^*)} \qquad (3.16)$$

and with the dimensionless expression $M_B^* = M_B/(Cbs_0^2)$, Eq.(3.16) becomes

$$M_B^* = \frac{\partial W^*}{\partial(1/\rho^*)} \qquad (3.17)$$

Thus, from Eq.(3.11)

$$M_B^* = \frac{1}{(n+1)b^*}\int_{\varphi=0}^{\beta}\int_{y^*=-\frac{1}{2}}^{\frac{1}{2}}\Big\{[(n+1)(\bar{\epsilon}_1+\bar{\epsilon}_0)^n\frac{\partial\bar{\epsilon}_1}{\partial(1/\rho^*)}]\cdot$$

$$\cdot[\rho^*+R^*-(R^*-y^*)\cos\varphi]\frac{R^*-y^*}{\rho^*}$$

$$+[(\bar{\epsilon}_1+\bar{\epsilon}_0)^{n+1}-\bar{\epsilon}_0^{\,n+1}][R^*-(R^*-y^*)\cos\varphi](R^*-y^*)\Big\}\cdot d\varphi\cdot dy^*+$$

$$+\frac{1}{n+1}\left(1-\frac{\beta R^*}{b^*\,e^{\epsilon_\beta}}\right)\int_{y^*=-\frac{1}{2}}^{\frac{1}{2}}\Big\{[(n+1)(\bar{\epsilon}_2+\bar{\epsilon}_0)^n\frac{\partial\bar{\epsilon}_2}{\partial(1/\rho^*)}](1+\frac{y^*}{\rho^*})+$$

$$+[(\bar{\epsilon}_2+\bar{\epsilon}_0)^{n+1}-\bar{\epsilon}_0^{\,n+1}]y^*\Big\}dy^* \qquad (3.18)$$

where

$$\frac{\partial\overline{\epsilon}_1}{\partial(1/\rho^*)}=\frac{1}{\sqrt{3}}\cdot\frac{\epsilon_z+2\,\epsilon_x}{\sqrt{\epsilon_x^2+\epsilon_z^2+\epsilon_x\epsilon_z}}\cdot\frac{\rho^*\,[\,R^*-(R^*-y^*)\cos\varphi]}{\rho^*+R^*-(R^*-y^*)\cos\varphi} \qquad (3.19)$$

$$\frac{\partial\overline{\epsilon}_2}{\partial(1/\rho^*)}=\frac{2}{\sqrt{3}}\cdot\left|\frac{\rho^*\,y^*}{\rho^*+y^*}\right| \qquad (3.20)$$

IV. COMPUTATION OF THE MINIMAL ENERGY

According to the principle of the minimum deformation energy, the edge–effect in sheet bending must occur in the state with the minimum energy. This means that the dimensionless expression of the total deformation energy in Eq.(3.11) must be minimized when the edge–effect occurs. The effective way to obtain the minimal energy is to solve the equations which are the three partial derivatives with respect to the parameters R^*, β, and ϵ_β being zero, that is,

$$\begin{cases} \dfrac{\partial W^*}{\partial R^*}=0 \\ \dfrac{\partial W^*}{\partial \beta}=0 \\ \dfrac{\partial W^*}{\partial \epsilon_\beta}=0 \end{cases} \qquad (4.1)$$

But it is difficult for Eq.(3.11) to be integrated as an algebraic expression. In order to obtain these three partial derivatives the following formulae are applied,

$$\frac{d}{dy}\int_a^b f(x,y)dx=\int_a^b f_y'(x,y)dx \qquad [a]$$

$$\frac{d}{dt}\int_a^t f(x)dx=f(t) \qquad [b]$$

The partial derivative with respect to R^* is

$$\frac{\partial W^*}{\partial R^*}=\frac{1}{(n+1)b^*\rho^*}\cdot$$

$$\int_{\varphi=0}^{\beta}\int_{y^*=-\frac{1}{2}}^{\frac{1}{2}}\Big\{[(n+1)(\bar{\epsilon}_1+\bar{\epsilon}_0)^{n}\frac{\partial\bar{\epsilon}_1}{\partial R^*}][\rho^*+R^*-(R^*-y^*)\cos\varphi](R^*-y^*)+$$

$$+[(\bar{\epsilon}_1+\bar{\epsilon}_0)^{n+1}-\bar{\epsilon}_0^{\;n+1}][\rho^*+2R^*-y^*-(R^*-y^*)\cos\varphi]\Big\}\cdot d\varphi\cdot dy^*-$$

$$-\frac{1}{n+1}\cdot\frac{\beta}{b^*e^{\epsilon_\beta}}\int_{y^*=-\frac{1}{2}}^{\frac{1}{2}}[(\bar{\epsilon}_2+\bar{\epsilon}_0)^{n+1}-\bar{\epsilon}_0^{\;n+1}](1+\frac{y^*}{\rho^*})dy^*$$

(4.2)

where

$$\frac{\partial\bar{\epsilon}_1}{\partial R^*}=\frac{1}{\sqrt{3}}\cdot\frac{1}{\sqrt{\epsilon_x^2+\epsilon_z^2+\epsilon_x\epsilon_z}}[(2\epsilon_z+\epsilon_x)\frac{\partial\epsilon_z}{\partial R^*}+(\epsilon_z+2\epsilon_x)\frac{\partial\epsilon_x}{\partial R^*}]$$

(4.2a)

and

$$\frac{\partial\epsilon_z}{\partial R^*}=\frac{y^*}{R^*(R^*-y^*)} \qquad (4.2b)$$

$$\frac{\partial\epsilon_x}{\partial R^*}=\frac{1-\cos\varphi}{\rho^*+R^*-(R^*-y^*)\cos\varphi} \qquad (4.2c)$$

the expressions of ϵ_x and ϵ_z are mentioned in Eqs.(3.13) and (3.14).

The partial derivative with repect to β is

$$\frac{\partial W^*}{\partial\beta^*}=\frac{1}{(n+1)b^*\rho^*}\cdot$$

$$\int_{y^*=-\frac{1}{2}}^{\frac{1}{2}}\Big\{[(\bar{\epsilon}_1+\bar{\epsilon}_0)^{n+1}-\bar{\epsilon}_0^{\;n+1}][\rho^*+R^*-(R^*-y^*)\cos\beta](R^*-y^*)dy^*-$$

$$-\frac{1}{n+1}\cdot\frac{R^*}{b^* e^{\epsilon_\beta}}\int_{y^*=-\frac{1}{2}}^{\frac{1}{2}}[(\bar{\epsilon}_2+\bar{\epsilon}_0)^{n+1}-\bar{\epsilon}_0^{\,n+1}](1+\frac{y^*}{\rho^*})dy^*$$

(4.3)

where

$$\bar{\epsilon}_1=\frac{2}{\sqrt{3}}\sqrt{\epsilon_x^2+\epsilon_z^2+\epsilon_x\epsilon_z} \qquad (4.3a)$$

$$\epsilon_x=\ln(1+\frac{R^*-(R^*-y^*)\cos\beta}{\rho^*}) \qquad (4.3b)$$

$$\epsilon_z=\epsilon_\beta+\ln(1-\frac{y^*}{\rho^*}) \qquad (4.3c)$$

The partial derivative with respect to ϵ_β is

$$\frac{\partial W^*}{\partial\epsilon_\beta}=\frac{1}{(n+1)b^*\rho^*}\int_{\varphi=0}^{\beta}\int_{y^*=-\frac{1}{2}}^{\frac{1}{2}}\left\{[(n+1)(\bar{\epsilon}_1+\bar{\epsilon}_0)^n\frac{\partial\bar{\epsilon}_1}{\partial\epsilon_\beta}]\cdot\right.$$

$$\left.\cdot[\rho^*+R^*-(R^*-y^*)\cos\varphi](R^*-y^*)\right\}\cdot d\varphi\cdot dy^*+$$

$$+\frac{1}{n+1}\cdot\frac{\beta R^*}{b^* e^{\epsilon_\beta}}\int_{y^*=-\frac{1}{2}}^{\frac{1}{2}}[(\bar{\epsilon}_2+\bar{\epsilon}_0)^{n+1}-\bar{\epsilon}_0^{\,n+1}](1+\frac{y^*}{\rho^*})dy^*$$

(4.4)

where

$$\frac{\partial\bar{\epsilon}_1}{\partial\epsilon_\beta}=\frac{1}{\sqrt{3}}\cdot\frac{2\epsilon_z+\epsilon_x}{\sqrt{\epsilon_x^2+\epsilon_z^2+\epsilon_x\epsilon_z}} \qquad (4.4a)$$

the expressions of ϵ_x and ϵ_z are mentioned in Eqs.(3.13) and (3.14).

Substitution of Eqs.(4.2), (4.3) and (4.4) into Eq.(4.1) and solving them gives the solutions R^*_m, β_m, and $\epsilon_{\beta m}$ which are corresponding to the minimal energy,

$$W^*_m = W^*(R^*_m, \beta_m, \epsilon_{\beta m}) \qquad (4.5)$$

Theorically, the edge–effect in sheet bending will be in the shape with R^*_m, β_m, and $\epsilon_{\beta m}$.

The computation of the minimal energy is carried out after solving the equations in Eq.(4.1), however, these equations cannot be expressed as the obviously algebraic expressions because of the integrations, and this makes it more difficult to solve the problem. In this case, the numeral integration and the mathematical approach method have to be applied.

For applying the mathematical approach method, a new objective function is defined, which is

$$F(R^*, \beta, \epsilon_\beta) = (\frac{\partial W^*}{\partial R^*})^2+(\frac{\partial W^*}{\partial \beta})^2+(\frac{\partial W^*}{\partial \epsilon_\beta})^2 \qquad (4.6)$$

The value of $F(R^*, \beta, \epsilon_\beta)$ must be zero when the equations in Eq.(4.1) are satisfied. However, it seems too harsh for the approaches to reach the accuracy being zero. The practical way is to choose an appropriately small value ϵ, for example, 10^{-8}, as the identification of the approaches of the solutions. This means that the solutions of (4.1) are approximately obtained when

$$F(R^*_m,\beta_m,\epsilon_{\beta m}) \leq \epsilon \qquad (4.7)$$

Secondary, the first approximant F_1 of Eq.(4.6) is obtained by substituting the initial non–zero values of R^*, β, and ϵ_β into Eqs.(4.2), (4.3), and (4.4). Assuming that after k times of computation, the values of R^*, β, and ϵ_β are R^*_k, β_k, and $\epsilon_{\beta k}$, the objective of Eq.(4.6) is F_k.

The approximate solutions of Eq.(4.1) are obtained if $F_k \leq \epsilon$, and if not, the values R^*_k, β_k, and $\epsilon_{\beta k}$ will change into new values R^*_{k+1}, β_{k+1}, and $\epsilon_{\beta k+1}$ with their change ratio $\Delta F_k/\Delta R^*$ and so on, which

$$\frac{\Delta F_k}{\Delta R^*} = \frac{F(R^*_k+\Delta R^* \ ,\beta_k,\epsilon_{\beta k})-F_k}{\Delta R^*}$$

and so on, where $\Delta R^*=w\cdot R^*_k$, and w is a given small value (say, 10^{-5}). The excution mentioned above continues until the values of R^*, β, and ϵ_β satisfy

Eq.(4.7), and then they are regarded as R^*_m, β_m, and $\epsilon_{\beta m}$.

Due to the numerial integrations most of which are 2–dimensional, the computing time is considerable on a PC, so, it is important to choose the identifictional value ϵ and the initial values carefully.

V. RESULTS AND CONCLUSIONS

The results of the foregoing analysis are listed in Tab.4.1 and described in graphical form in Fig.4.1 until 4.4, where W^*_b is the dimensionless deformation energy and M_{Bb}^* is the corresponding couple in case of no edge effect occurs (a^*=0, or β=0). The values of the parameters describing the material behaviour are n=0.24 and $\bar{\epsilon}_0$=0.005.

The irregular shapes of some of the curves are caused by the deformation energy not being very sensitive to these parameters within the applied accuracy.

Tab.4.1 The Values of R^*, β, and ϵ_β when W^* Minimized

$1/\rho^*$	M_B^*	M_{Bb}^*	M_B^*/M_{Bb}^*	R^*	β	ϵ_β
0.0067	0.088842	0.089279	0.99510	485.0	1.400	–0.00012
0.01	0.092276	0.092613	0.99636	320.0	2.000	–0.00024
0.02	0.100484	0.100754	0.99732	160.0	3.000	–0.00051
0.05	0.116879	0.117179	0.99744	55.0	4.001	–0.00096
0.10	0.134129	0.134399	0.99800	28.0	6.004	–0.00230
0.20	0.155717	0.155910	0.99876	14.0	10.006	–0.00635
0.25	0.163529	0.163710	0.99889	10.8	12.010	–0.00908
0.50	0.189584	0.189811	0.99880	5.4	16.000	–0.02000

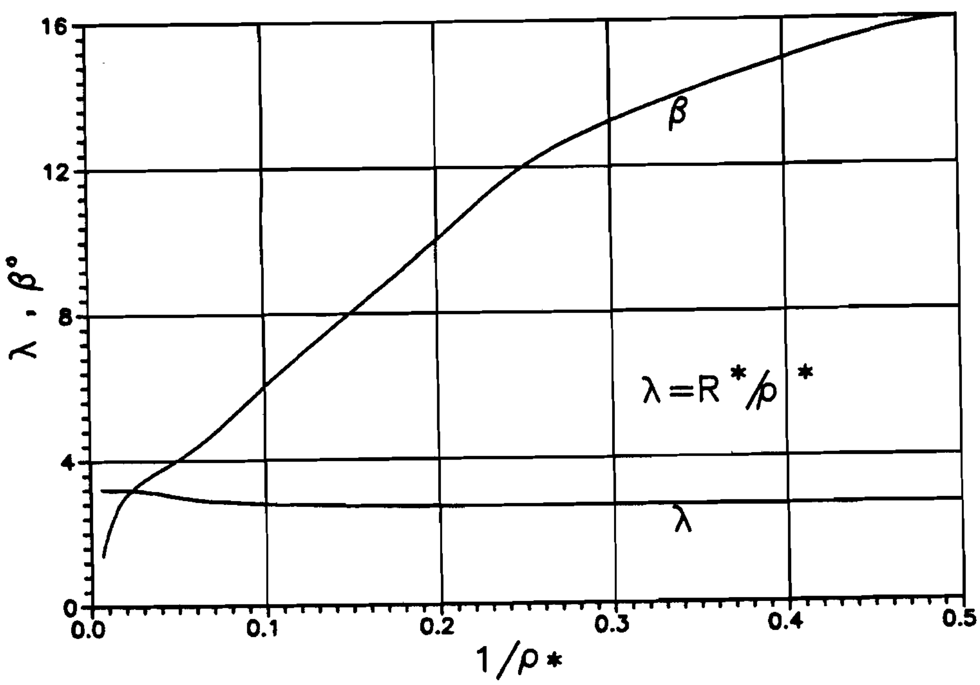


Fig.4.1 The Values of R_* and β when W_* Minimized

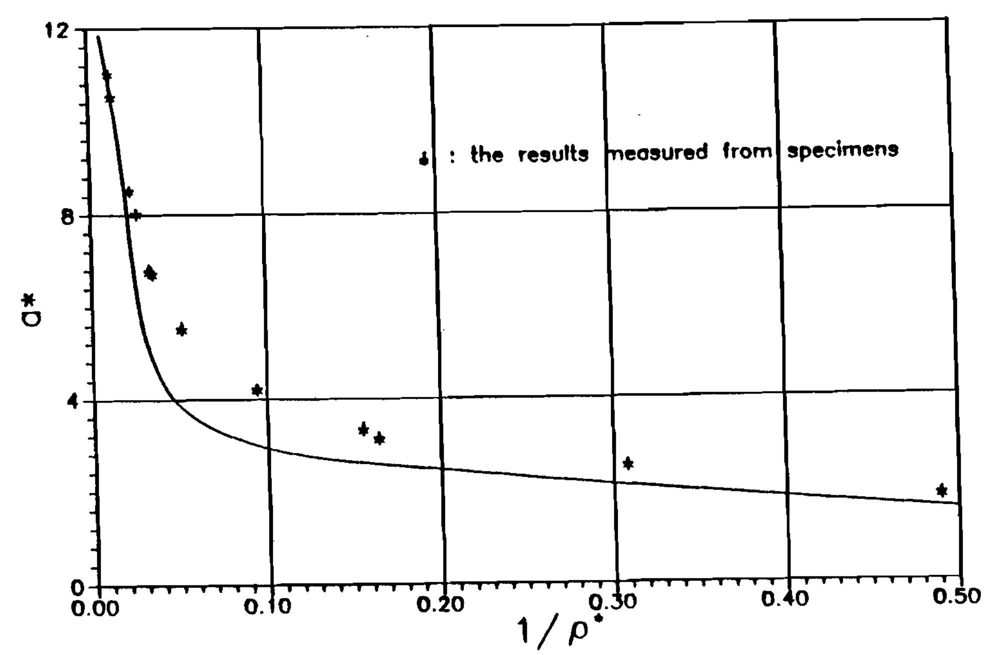


Fig.4.2 The Width of Edge–Effect Area

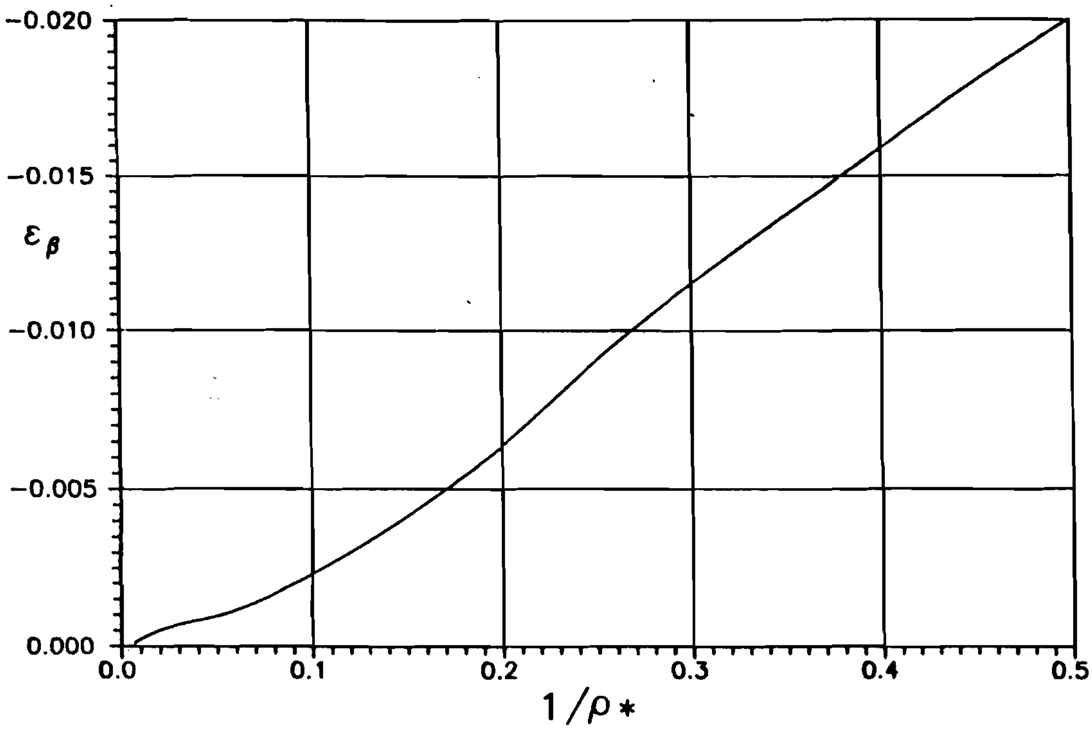


Fig.4.3 The Middle Plane Strain ϵ_β when $W*$ Minimized

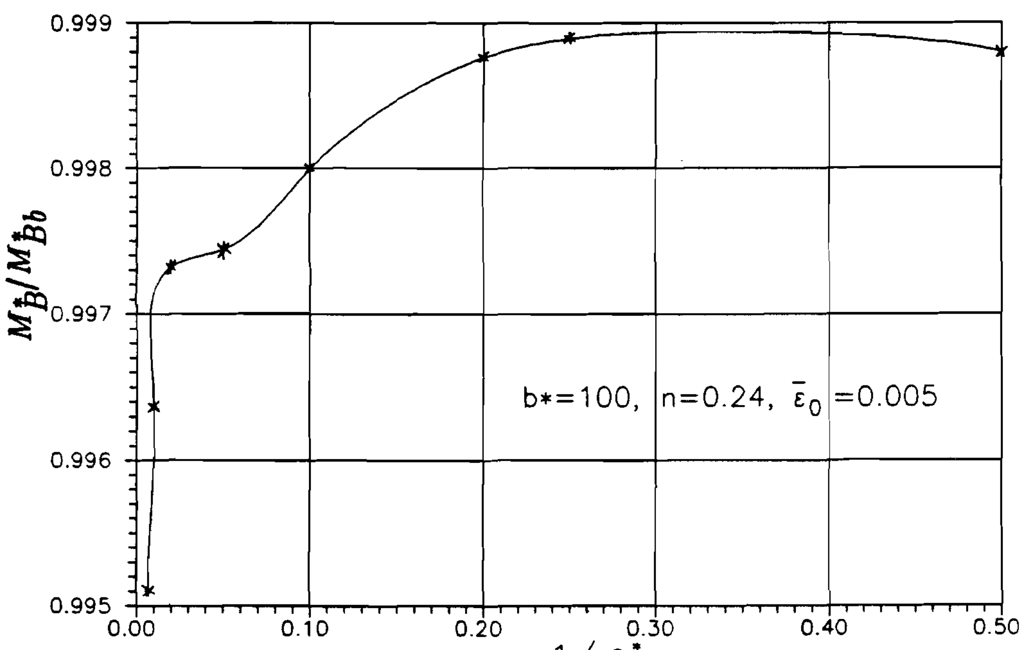


Fig.4.4 The Ratio of M_B^*/M_{Bb}^* when $W*$ Minimized

On basis of these results the following conclusions can be drawn.

— Although the parameters β and a^* vary strongly with the bending curvature ρ^* (see Fig.4.1 and 4.2), the quotient R^*/ρ^* appears to be a contant value (about 3) over a wide range of ρ^*;

— The assumption of a straight strain path is violated slightly because of the decreasing tendency of the edge area (see Fig.4.1), so the material has a different deformation history from the one given by the prestrain $\bar{\epsilon}_0$. However, the concerning strains are small because of the large value of R^* and may possiblly be neglected.

  An incremental analysis, applying a stepwise decreasing curvature ρ^*, in a way that the deformation history can be taken into account, should give a decisive answer about this;

— The edge geometry was among others decribed by a constant radius R. However, from experiments it appears that the radius depends on φ, so it is not useful to make a comparison between theory and experiment with respect to this parameter. A rather good measurable quantity is the parameter a. Fig.4.1 shows that there is good agreement between theory and experiment with regard to this parameter;

— The influence of the edge–effect on the bending couple is neglectible (<1%) from a technical point of view interesting domain ($1/\rho^*$>0.05, see Tab.4.1 and Fig.4.4);

— Applying different values for n gave just about the same results of the bending couple(see Tab.4.2).

Tab.4.2 The Ratio of Bending Couple with Different n
(ρ^*=150, b^*=100, $\bar{\epsilon}_0$=0.005)

n	M^*_B	M^*_{Bb}	M^*_B/M^*_{Bb}
0.05	0.225085	0.226052	0.995721
0.24	0.088842	0.089279	0.995102
0.50	0.024922	0.025065	0.994302

APPENDIX

Here are some figures which show the possiblly minimal deformation energy and help us to choose the initial values for the minimization. Figs.A.1a, b,c and A.2a, b, c respectively indicate the cases of ρ^*=4.0 and ρ^*=10.0 .

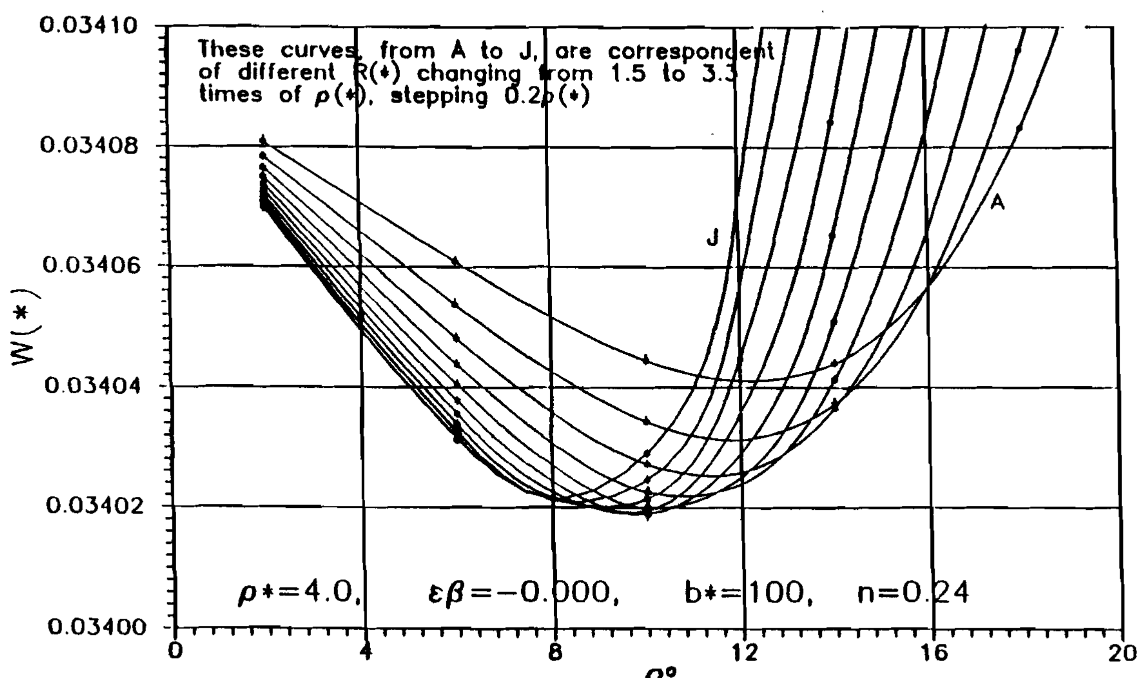


Fig.A.1a

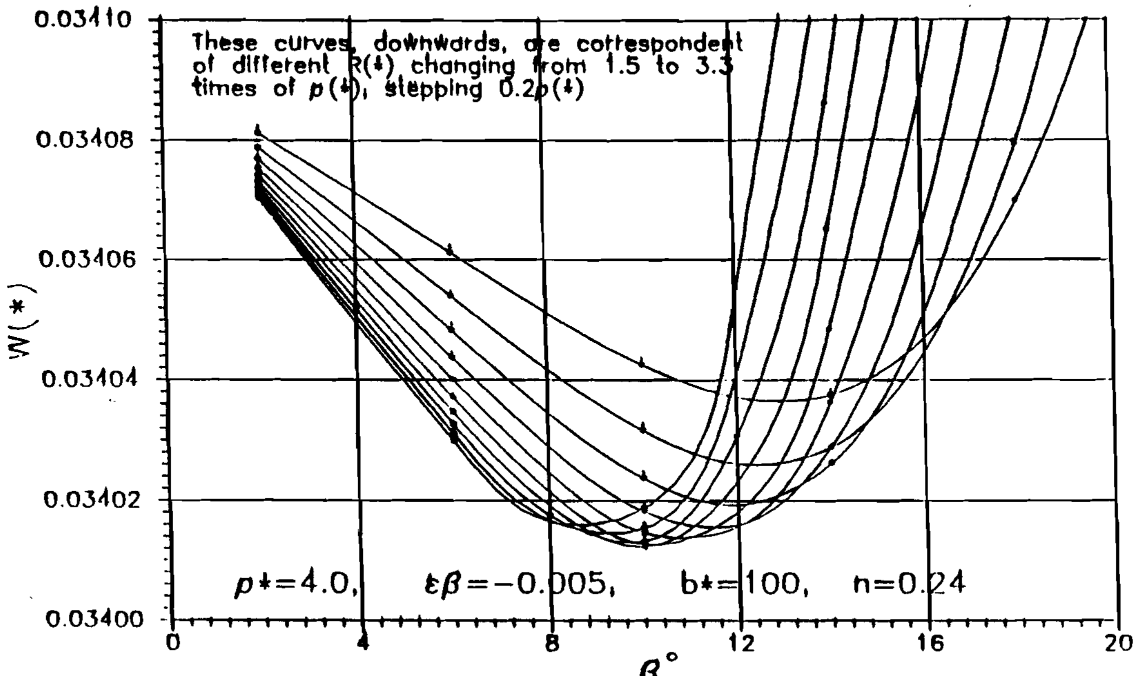


Fig.A.1b

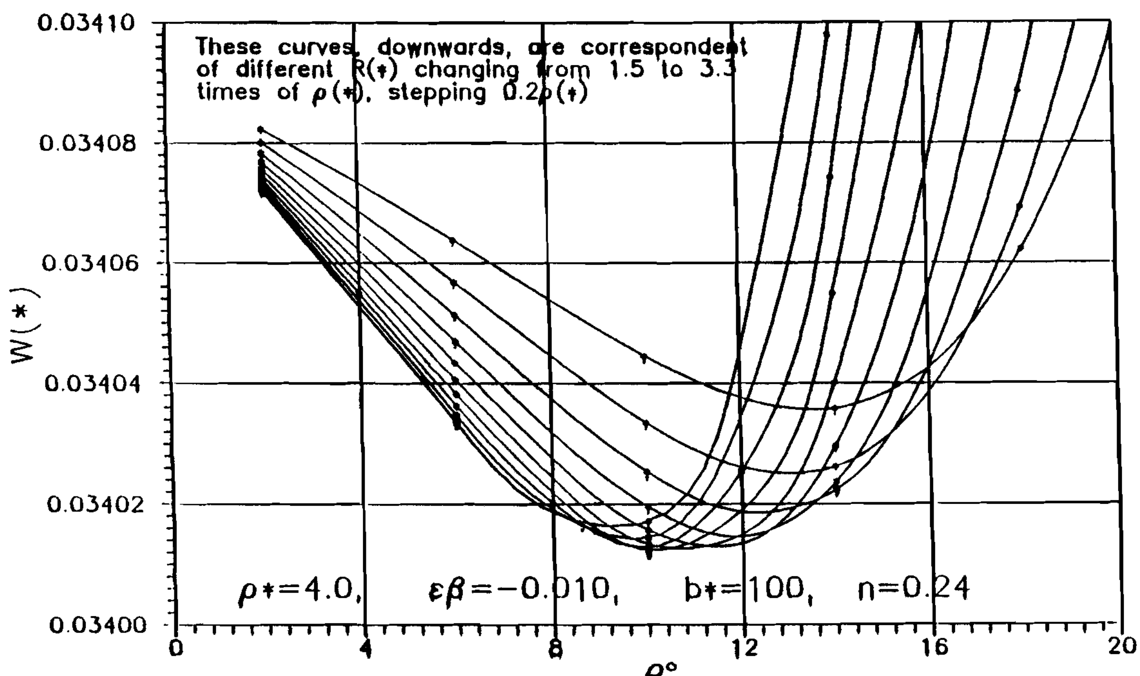


Fig.A.1c

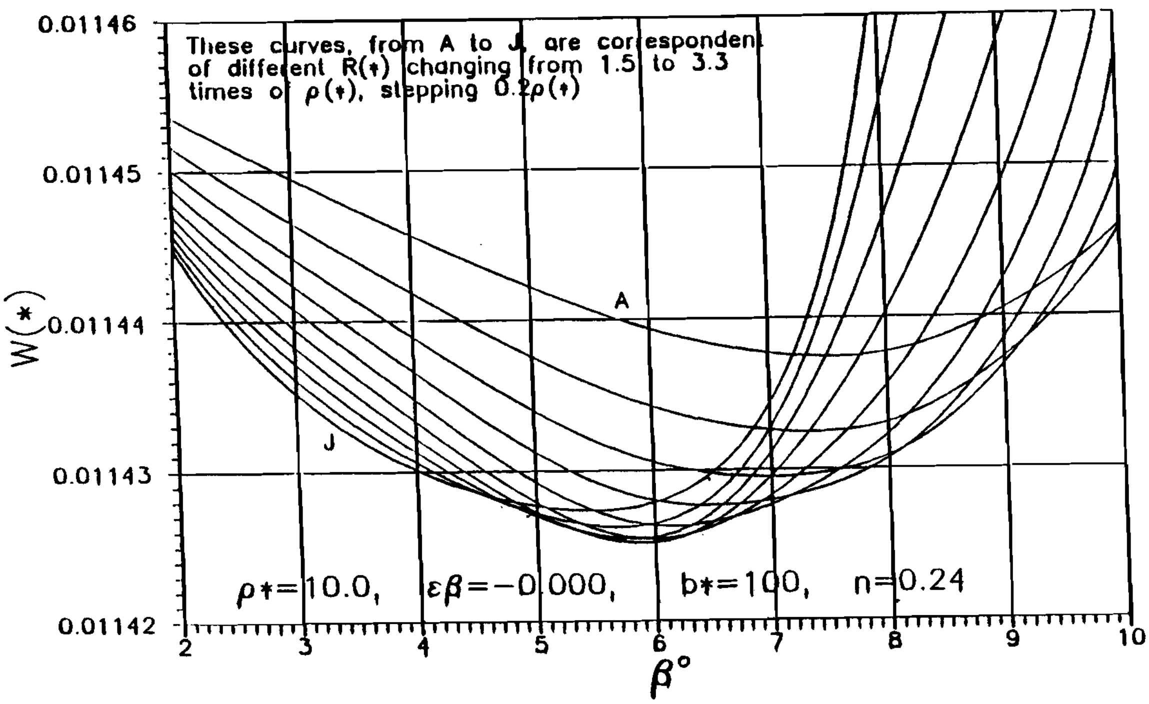


Fig. $A.2a$

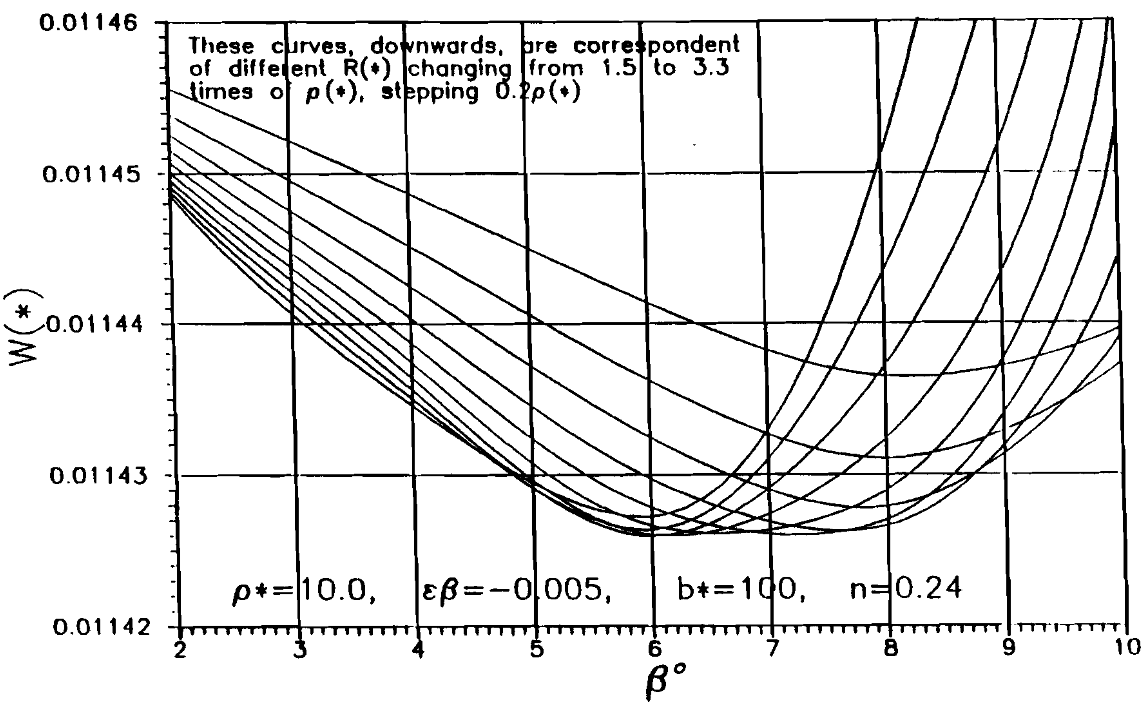


Fig. $A.2b$

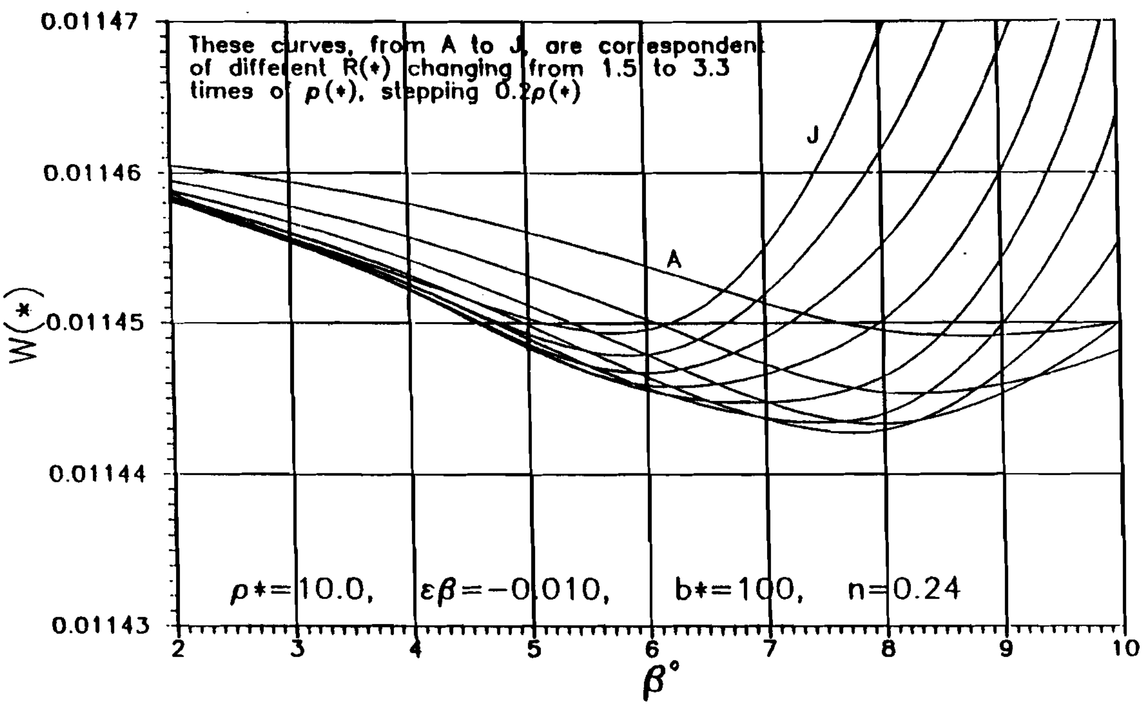

These curves, from A to J, are correspondent
of different R(*) changing from 1.5 to 3.3
times of ρ(*), stepping 0.2ρ(*)
J
A
ρ*=10.0, εβ=−0.010, b*=100, n=0.24
0.01147
0.01146
0.01145
0.01144
0.01143
W(*)
2
3
4
5
6
7
8
9
10
β°

Fig.*A*.2*c*