Inventory control : a cognitive human operator model

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INVENTORY CONTROL

A COGNITIVE HUMAN OPERATOR MODEL

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Report on a 4th-year project
performed from February until June 1981
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CONTENTS

Appendix D: Instructions for the experimental subject.

Appendix E: The system-arrays as a function of the period-number, for three random experiments.

Appendix F: Autocorrelation functions of the residuals, averaged over 10 experiments.

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SUMMARY

In this report the behaviour of a human operator in a specific control task is evaluated. The control task is an inventory control: the human operator has to determine the inventory for some product, based on information from the past. One part of this information is the demand, that is generated as first order filtered white noise; the second part is the profit rate: the profit in each period.

An attempt has been made to fit the behaviour of the human operator, as a cognitive system, into a zeroth order model, and this model is tested on its acceptability.

For this purpose a computer program has been written that is able to run experiments with experimental subjects.

This program evaluates the quality of the model by means of the runs test, and the determination of the autocorrelation function of the residual. The model parameters are estimated with a least-squares estimation procedure.

PREFACE

Many tasks imposed on a human operator are control tasks; in general these control tasks, and specifically cognitive tasks, can be separated into a few blocks (see also v.Bussel(1980)):

1. perceptual receiving of the value of some variables,
2. prediction of the future behaviour of one or more output variables that have to be controlled,
3. making a decision on adjusting the system to control the output variable(s).

Research on the behaviour of a human being in such a situation can be of use to incorporate the human operator into the description of a larger system; another result of these studies can be a quantification of the effects of learning, physical state etc.
To test the behaviour of a human operator, a simple control task has been arranged by the group "Funktieleer" of the Department of Psychology of the Tilburg University.
Based on the results of a.o. van den Hoven (1978) and Koenraads (1978) with experimental data, it seemed interesting to try to model the (cognitive) behaviour of the human operator into a model, that assumes the human operator to be a pure predictor.

The purpose of this project was to create a zero order model of the human operator in the specific inventory control task, and to evaluate its results.
Chapter 1 gives a description of the control task and explains the model of the human operator. In chapter 2 two tests will be introduced for evaluating the quality of the model. In chapter 3 the computer program is described that controls the PDP 11/60 computer in running the experiments and evaluating the results. Chapter 4 gives a description of the experiments and presents the results.

CHAPTER 1 : DESCRIPTION OF THE INVENTORY CONTROL SYSTEM

The inventory control task for a shopowner can be defined as the task to choose an inventory for a product every period in such a way that his profit will be maximal. Because of the fact that the shopowner does not know the exact demand of his clients for the following period, he has to make a prediction of this demand and base the inventory for the next period on this prediction.
Naturally his choice of an inventory is not only based on a prediction of the demand, but also on other variables as storage costs, costs of loss of goodwill, decay of his products during storage etc.
By way of counting his profit at the end of a period, the shopowner gets some kind of feedback on the decisions he made at the beginning of the period.

1.1 The processes in the inventory control system.

- First there is the demand process. (cf. figure 1)
  We assume this process to be first order filtered white noise;
  The demand in period i: $v(i) = v_0 + \alpha(v(i-1) - v_0) + \xi(i-1)$ (1)

  where α = autoregressive parameter, $-1 \leq \alpha \leq 1$
  ξ = gaussian white noise
  v_0 = a constant.

- The sale process:
  The sale in period i: $a(i) = \min(x(i), v(i))$ (2)

  where $x(i)$ = the inventory at the beginning of period i.

- The inventory at the beginning of period i depends on the products remaining from the previous period, on the decay, and on the purchase:
  $x(i) = (1-\lambda)\, y(i-1) + b(i-1)$ (3)

  where λ = decay parameter, $0 \leq \lambda \leq 1$
  $y(i)$ = inventory at the end of period i
  $b(i)$ = purchase order at the end of period i

  and $y(i) = x(i) - a(i)$ (4)

- The profit rate of period i will be calculated as follows:

$$w(i) = p_1\, a(i) - p_2\, b(i-1) - p_3\, y(i) - p_4\, (\, v(i) - a(i)\,) \qquad (5)$$

where p_1 = price of sale,
p_2 = price of purchase,
p_3 = storage costs,
p_4 = price of "loss of goodwill".

A block diagram of this system is given in fig -1-.

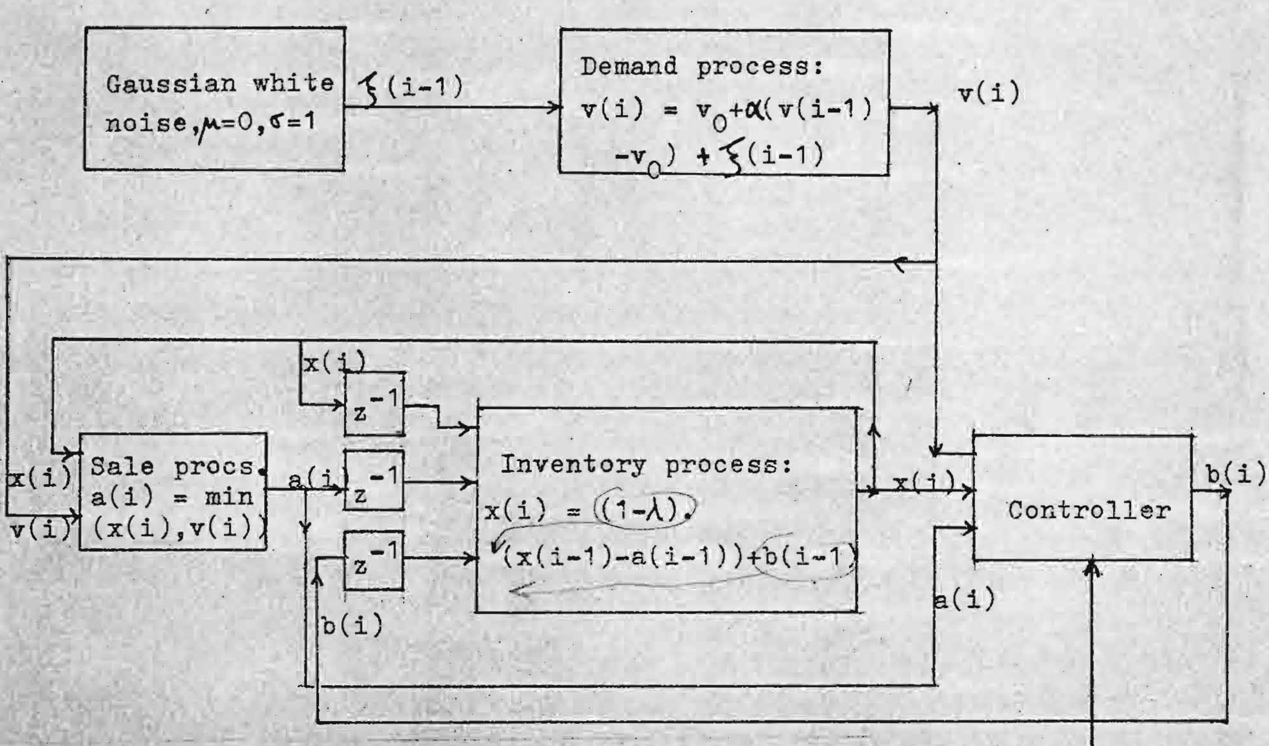

fig - 1 -
Block-diagram of the inventory control system.

Every period the shopowner has to make a choice with respect to the size of the purchase order for the next period. In this case this is the only choice he has to make.
Within the system, as drawn in fig -1-, there are a few parameters that can be chosen from outside the system.

These are:

- the parameters of the noise generator: μ, σ
- the parameters of the demand process: α , v_0
- the parameters of the inventory process: λ
- and the parameters that determine the profit rate: p_1, p_2, p_3, p_4.

As will become clear in the sequel, the only parameter that is important for studying the system is α. The other parameters only have to be chosen in a way that makes the inventory problem a realistic one.
Basing our choice on this criterion and not going further into specific arguments for the values, we have chosen the next values:

σ = 1.
v_0 = 5.
λ = 0.1
p_1 = 1.
p_2 = 0.5
p_3 = 0.05
p_4 = 0.1

α is the parameter that determines the character of the demand, and is therefore the most important parameter.

1.2 The optimal controller.

Knowing the way in which the demand is generated, an optimal controller can be constructed that optimizes the expected profit rate.
By using dynamic optimization techniques one can prove (Braakman, 1980) that, given the inventory x(i) at the beginning of a period i, and the demand v(i) during that period, an optimal choice for b(i) can be found that optimizes $\sum_{j=i}^{\infty} w(j)$.
This optimal b(i) can be written as:

$$b^*(i) = v_0 + \alpha(v(i) - v_0) - (1 - \lambda) y(i) + b_0 \qquad (6)$$

The optimal choice for b(i) can be transformed into an optimal choice for x(i) by using eq. (3):

$$x^*(i) = v_0 + \alpha(v(i) - v_0) + b_0 \qquad (7)$$

where b_0 can be determined by

$$P\left(\xi(i) \leq b_0\right) = \frac{p_1 - p_2 + p_4}{p_1 - (1-\lambda)p_2 + p_3 + p_4} \qquad (8)$$

and $\xi(i)$ is the noise sample in period i.

We can see that $b^*(i)$ is made up of three parts:

$v_0 + \alpha\,(v(i) - v_0)$,	a one step ahead predictor of the demand of the next period,
$(1 - \lambda)\,y(i)$,	the inventory at the beginning of the next period,
b_0,	an extra purchase that is dependent on the density function of the noise and the prices on which the profit rate is based. It is a result of relating the costs for possible loss of goodwill to the costs for possible extra storage.

Taking the p- and λ- parameters as mentioned before, b_0 will be given by $P\left(\xi(i) \leq b_0\right) = 0.857$.
If $\xi(i)$ is sampled white noise with zero mean and unit variance, as in our case, b_0 will equal 1.06.

1.3 The human operator.

As mentioned before, the human operator has to make a decision with respect to the size of the purchase b(i) for the next period, based on the inventory x, the sale a, and the demand v of previous periods.
Because of the fact that there is a unique relation between x, a and v: $a(i) = \min(v(i), x(i))$, and that a choice for the purchase b(i) comes to the same thing as a choice for the new inventory x(i+1) when the old inventory is known, we can write the process of the human operator as follows:

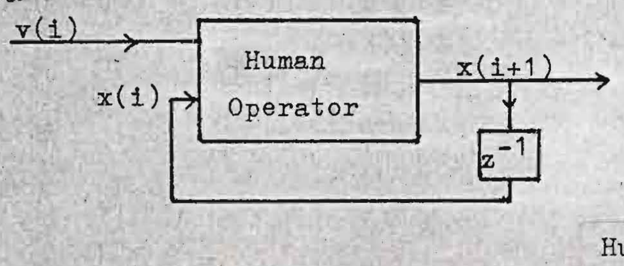

fig - 2 -
Human operator process.

So what first seemed to be a MISO process we now can write as a simple SISO system, considering the inventory x(i) as a state-condition. This interpretation leads to the following description of the process of the human operator:

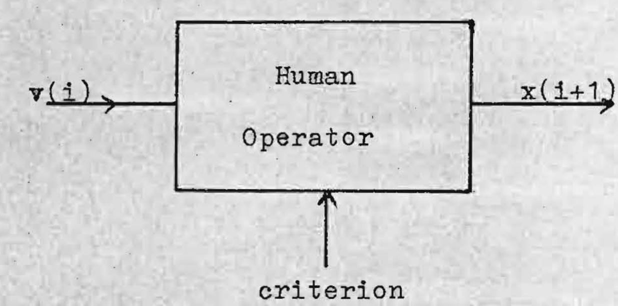

fig - 3 -

The human operator will, in control of some criterion, try to choose the inventory x(i+1) in such a way that his profit rate will become optimal. However the mathematical construction of the profit rate is not known to him, and if so would be (much) too complicated to base the right decisions upon.

- The Model of the human operator

To construct a model for this human operator-task, we have to visualise the way in which the human operator will make his decision.
He has got the following information:
- the demand v of period i and of previous periods,
- the value of the profit rate of period i and of previous periods.

Because of the construction of the profit rate, this function's only task is to give the human operator a view on the optimal strategy: taking an "over-inventory" to be able to serve all customers, or taking an "under-inventory" to be sure that all products can be sold.
The main point in the choice of the human operator will be his prediction of the demand in the next period. This prediction together with the effect of the profit rate, as mentioned above, will determine the inventory of the new period.
The value of the profit rate in each period is partly dependent on the demand, on which the human operator has no influence. In this way, he gets hardly any feedback on the optimality of his decision, and

therefore the control task can rather be considered as a prediction task.

The model of the human operator is now chosen to be as follows:

$$x(i+1) = E(v(i+1)) + \tilde{b}_0 \tag{9}$$

with $E(v(i+1))$ the prediction of the demand for the next period, and $\tilde{b}_0$ an extra inventory, based on the experience of the human operator in previous periods.

Theoretically the demand-function is given as:

$$v(i+1) = v_0 + \alpha(v(i) - v_0) + \xi(i-1)$$

Knowing the demand in period i, the right expectation of the demand in period i+1 is

$$E(v(i+1)) = v_0 + \alpha(E(v(i)) - v_0) + E(\xi(i))$$

$$E(v(i+1)) = v_0 + \alpha(v(i) - v_0) \tag{10}$$

The results of equation (9) and (10) can be combined into a theoretical model of the human operator:

$$x(i+1) = v_0 + \alpha(v(i) - v_0) + \tilde{b}_0 \tag{11}$$

Because the human operator will not make a distinction between v_0, α, and $\tilde{b}_0$, the model will become:

$$x(i+1) = A\, v(i) + B \tag{12}$$

with $A = \alpha$,
$B = (1-\alpha)\, v_0 + \tilde{b}_0$.

which is a moving average (MA) model.

- Determination of A and B.

After an experiment, in which the human operator has to choose the inventory for N successive periods, there are available i.a. two arrays: x(i) and v(i). Taking these two arrays as a starting point we can construct least squares estimators $\tilde{A}$ and $\tilde{B}$ for the A- and B-parameter. We want to determine $\tilde{A}$ and $\tilde{B}$ in such a way that

$$\sum_{i=0}^{N-1} (x(i+1) - \tilde{A}\, v(i) - \tilde{B})^2 \quad \text{becomes minimal.}$$

Therefor this expression has to be differentiated with respect to $\tilde{A}$ and $\tilde{B}$:

$$\begin{cases} -2 \sum_{i=0}^{N-1} \left(x(i+1) - \tilde{A}\, v(i) - \tilde{B} \right) v(i) = 0 \\ -2 \sum_{i=0}^{N-1} \left(x(i+1) - \tilde{A}\, v(i) - \tilde{B} \right) = 0 \end{cases}$$

Combining these two equations yields

$$N \sum_{i=0}^{N-1} x(i+1)\, v(i) - \sum_{i=0}^{N-1} x(i+1) \sum_{i=0}^{N-1} v(i) = \tilde{A} \left[N \sum_{i=0}^{N-1} v^2(i) - \left(\sum_{i=0}^{N-1} v(i) \right)^2 \right]$$

which results in:

$$\tilde{A} = \frac{\left(N \sum_{i=0}^{N-1} x(i+1)\, v(i) \right) - \left(\sum_{i=0}^{N-1} x(i+1) \sum_{i=0}^{N-1} v(i) \right)}{\left(N \sum_{i=0}^{N-1} v^2(i) \right) - \left(\sum_{i=0}^{N-1} v(i) \right)^2} \qquad (13)$$

$$\tilde{B} = \frac{\left(\sum_{i=0}^{N-1} x(i+1) \right) \left(\sum_{i=0}^{N-1} v^2(i) \right) - \left(\sum_{i=0}^{N-1} x(i+1)\, v(i) \right) \left(\sum_{i=0}^{N-1} v(i) \right)}{\left(N \sum_{i=0}^{N-1} v^2(i) \right) - \left(\sum_{i=0}^{N-1} v(i) \right)^2} \qquad (14)$$

The estimation of A and B according to the method described, does not introduce any bias in the results, because of the fact that the human operator model is a MA-model and the noise is assumed to be additive and independent of the demand (see also section 2.1).

1.4 Composition of the different systems.

Now we can compose (join) the different systems we have constructed in one scheme, resulting in an overview of the systems that are of interest:

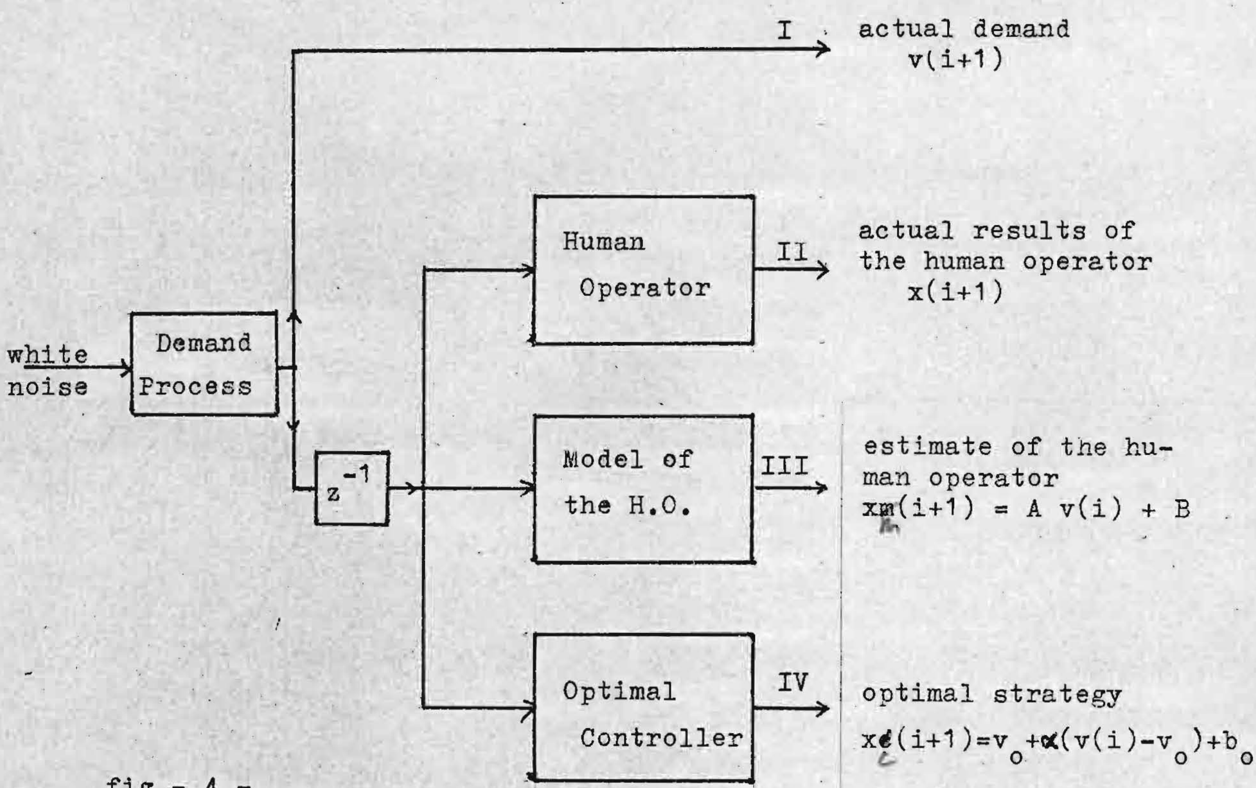

fig - 4 -

Comparison of the available systems.

For the experiments as described in chapter 3 and 4, the next four residuals are evaluated:

II - III	(H-M)	H = Human Operator
II - I	(H-D)	D = Actual Demand
II - IV	(H-C)	C = Optimal Strategy
IV - I	(C-D)	M = Model of the H.O.

The comparison of the four system-outputs is useful because all four systems are based on (a prediction of) the demand in the next period. The optimal strategy is in fact a prediction of this demand summed with a constant (see also section 1.2). Therefore, with the comparison of these four system outputs three different predictions of the demand are compared.

Evidently the comparison of the actual results of the human operator and the results of the other systems is of interest: comparison of II and III tells us something about the quality of the model; comparison of II and IV gives an idea of the optimality of the human operator decisions with respect to the profit rate. The residual of II minus I shows the prediction-capability of the human operator, while the residual of I minus IV gives information on the noise parameters and on b_o.

CHAPTER 2 : THE EVALUATION OF THE MODEL

2.1 Introduction.

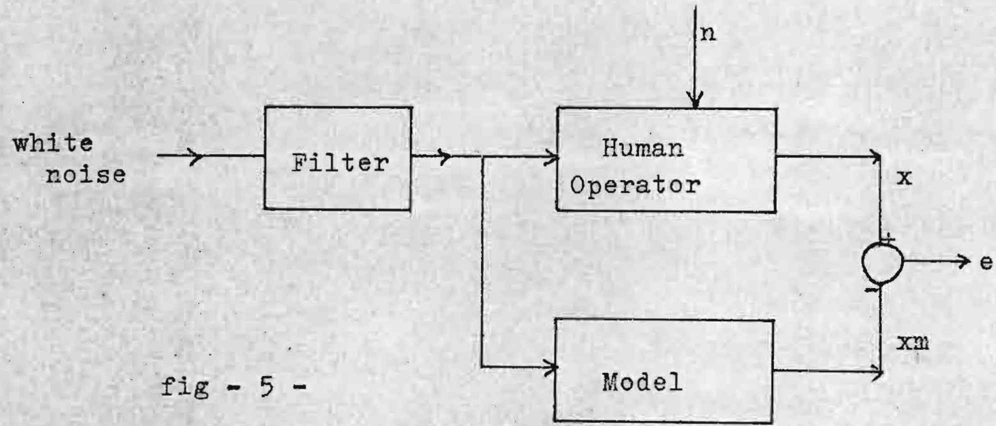

fig - 5 -

We assume that n is a signal of white noise samples, and that n is an additive noise which the human operator adds to the results. Any possible correlated noise is assumed to be an internal process in the human operator system. In view of this, the model has an optimal quality if there are no more deterministic factors in the residual e. If there should be any deterministic factors in e, they should be incorporated in the model, leaving e non-deterministic.
So the model is optimal if e equals white noise.
Tests on the quality of the model therefore can be tests on the whiteness of the residual e.

In this report two tests are used to examine the whiteness of e:

1. the runs test,
2. the determination of the autocorrelationfunction.

These two methods will be described respectively in the sequel of this chapter.

2.2 The runs test.

A statistical test for determining the whiteness of noise.
(see also Swed et al (1943) and Wald et al. (1940))

2.2.1 Explanation of the principle.

The crucial point of this test is a test on the hypothesis H that a sample-array can be regarded as sampled white noise.
For the test on this hypothesis H use will be made of the grouping of samples that have the same sign with respect to the median value

of the sample-array. According to this the samples with values exceeding the median will be regarded as + samples, the samples with values smaller than the median as - samples.
An uninterrupted part of the array with samples of the same sign is called a run; the total number of runs in the array is related to the probability that the sample-array can be regarded as white noise, and therefore to the acceptance/rejection of hypothesis H.

Let m be the number of samples with a + sign, and n the number of samples with a - sign; the total number of different arrangements of the + and - signs then equals $\binom{m+n}{n}$.

Let u be the number of runs in any one arrangement;
we then can state that:

$$P(\ u \leq u'\ ) \ = \ P(\ u=1\ ) + P(\ u=2\ ) + \ldots + P(\ u=u'\ )$$

About hypothesis H we now can say the following:
Assume that all possible arrangements are equally probable; the hypothesis H will then be rejected when

$$P(\ u \leq u'\ ) \quad \leq \beta \tag{15}$$

accepting this as a tendency for the distribution to be nonrandomly distributed.
For a given situation, a certain β results in a significant runlength u', the smallest integer u' for which the hypothesis holds.
β is called the level of significance, and can be chosen subjectively.

Because of the definition of the sign of a sample, one would expect that in all cases the equation $m = n$ would hold. The formulas to compute $P(\ u \leq u')$ would be much simpler then than in the general case.
The determination of $P(\ u \leq u'\ )$ is executed in appendix A of this report.
In this evaluation the general situation $m \neq n$ is assumed, for reasons that will become clear in chapter 3.

2.2.2. Application.

In this test, the level of significance β will be chosen 0.05; this choice leads to a significant runlength of 42. (see Swed and Eisenhart (1943)).

Assume a probability density function p(u) of the number of runs in an array of fixed length, as drawn in fig -5b-.

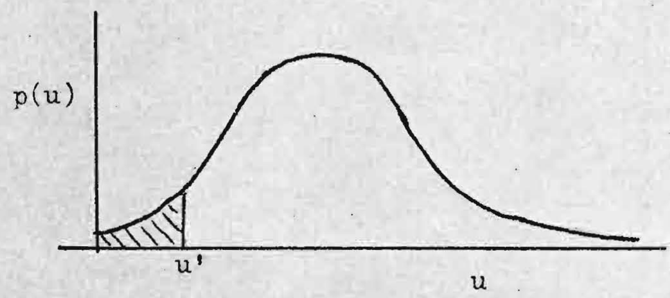

fig - 5b - Probability density function of the number of runs in a sample array of which all arrangements are equally probable.

There has been taken as a starting point that all possible arrangements of an array are equally probable. Therefore p(u) has the shape as drawn in fig -5b-.

Given a sample array, the number of runs can now be counted. If this number is so small that $\sum_{u=0}^{u=u'} p(u)$ is smaller than β (= 0.05), the statement that all arrangements are equally probable (and therefore the array is smapled white noise) will be rejected.

Concerning the usage use of a one-sided runs test in stead of a double-sided, attention is paid to this item in chapter 4.

2.3 Whiteness-test by means of the autocorrelationfunction.

2.3.1 Approximated autocorrelationfunction.

The autocorrelationfunction $\psi_{xx}(\tau)$ of a stationary discrete signal x is defined as:

$$\psi_{xx}(\tau) = \overline{x(i)\ x(i+\tau)}^{\text{ensemble}}$$

In case of ergodicity, as we will assume in our case, we can write:

$$\psi_{xx}(\tau) = \overline{x(i)\ x(i+\tau)}^{\text{time}} \quad (19)$$

When x(i) is an array of sampled white noise, $\psi_{xx}(\tau)$ will be a delta-function $\delta(\tau)$.

For the residuals, we want to investigate whether or not the computed autocorrelationfunction can be regarded as a delta-function.

Because of the fact that equation (19) is a mean value in the time domain, and there's only a finite number of samples available in the array x, we can only compute an approximated autocorrelationfunction, defined as:

$$\tilde{\Psi}_{xx}(k) = \frac{1}{N-k} \sum_{i=1}^{N-k} x(i) \cdot x(i+k) \qquad (20)$$

From this approximated autocorrelationfunction we are able to determine the estimation and the variance of Ψ, instruments to evaluate the delta-character of $\Psi_{xx}(k)$.

$$E\left[\tilde{\Psi}_{xx}(k)\right] = E\left[\frac{1}{N-k} \sum_{i=1}^{N-k} x(i)\ x(i+k)\right]$$

$$= \frac{1}{N-k} \sum_{i=1}^{N-k} E\left[x(i)\ x(i+k)\right]$$

$$= \frac{1}{N-k} (N-k)\, \Psi_{xx}(k)$$

$$\text{so} \qquad E\left[\tilde{\Psi}_{xx}(k)\right] = \Psi_{xx}(k) \qquad (21)$$

2.3.2 The variance of the approximated autocorrelationfunction.

$$\text{var}\left[\tilde{\Psi}_{xx}(k)\right] = E\left[\left\{\tilde{\Psi}_{xx}(k) - E(\tilde{\Psi}_{xx}(k))\right\}^2\right]$$

$$= E\left[(\tilde{\Psi}_{xx}(k) - \Psi_{xx}(k))^2\right]$$

$$= E\left[\tilde{\Psi}_{xx}^2(k)\right] + \Psi_{xx}^2(k) - 2\Psi_{xx}^2(k)$$

$$= E\left[\tilde{\Psi}_{xx}^2(k)\right] - \Psi_{xx}^2(k)$$

$$\text{var}\left[\tilde{\Psi}_{xx}(k)\right] = \frac{1}{(N-k)^2} \sum_{i=1}^{N-k} \sum_{j=1}^{N-k} E\left[x(i)\ x(j)\ x(i+k)\ x(j+k)\right] - \Psi_{xx}^2(k) \qquad (22)$$

On the assumption that x is of a normal distribution, we can write (Laning & Battin, 1956,p.162):

$$E(x_1\, x_2\, x_3\, x_4) = E(x_1\, x_2)\, E(x_3\, x_4) + E(x_1\, x_3)\, E(x_2\, x_4) + E(x_1\, x_4)\, E(x_2\, x_3).$$

Now we can write eq.(22) as:

$$\text{var}\left[\tilde{\Psi}_{xx}(k)\right] = \frac{1}{(N-k)^2} \sum_{i=1}^{N-k} \sum_{j=1}^{N-k} \left[\Psi_{xx}^2(j-i) + \Psi_{xx}^2(k) + \Psi_{xx}(j-i+k)\Psi_{xx}(i-j+k)\right] - \Psi_{xx}^2(k)$$

$$\operatorname{var}\left[\tilde{\Psi}_{xx}(k)\right] = \frac{1}{(N-k)^2}\sum_{i=1}^{N-k}\sum_{j=1}^{N-k}\left[\Psi_{xx}^2(j-i) + \Psi_{xx}(j-i+k)\Psi_{xx}(j-i-k)\right]$$

in which we have taken $\Psi_{xx}(i-j+k) = \Psi_{xx}(j-i-k)$.

The new expression for $\operatorname{var}\left[\tilde{\Psi}_{xx}(k)\right]$ is an even function of (j-i), so we can write:

$$\operatorname{var}\left[\tilde{\Psi}_{xx}(k)\right] = \frac{1}{(N-k)^2}\, 2\sum_{i=1}^{N-k}\sum_{\substack{j=1\\ j>i}}^{N-k}\left[\Psi_{xx}^2(j-i) + \Psi_{xx}(j-i+k)\Psi_{xx}(j-i-k)\right] +$$
$$+ \frac{1}{(N-k)^2}\sum_{i=1}^{N-k}\sum_{\substack{j=1\\ j=i}}^{N-k}\left[\Psi_{xx}^2(j-i) + \Psi_{xx}(j-i+k)\Psi_{xx}(j-i-k)\right]$$

We can take $j-i=\mu$ as a new argument for the correlationfunction:

$$\operatorname{var}\left[\tilde{\Psi}_{xx}(k)\right] = \frac{2}{(N-k)^2}\sum_{\mu=1}^{N-k-1}(N-k-\mu)\left[\Psi_{xx}^2(\mu) + \Psi_{xx}(\mu+k)\Psi_{xx}(\mu-k)\right]$$
$$+ \frac{1}{N-k}\left[\Psi_{xx}^2(0) + \Psi_{xx}^2(k)\right]$$

Conclusion:

$$\operatorname{var}\left[\tilde{\Psi}_{xx}(k)\right] = \frac{\Psi_{xx}^2(0)}{N-k} + \frac{\Psi_{xx}^2(k)}{N-k} + \\ + \frac{2}{N-k}\sum_{\mu=1}^{N-k-1}\left(1-\frac{\mu}{N-k}\right)\left[\Psi_{xx}^2(\mu) + \Psi_{xx}(\mu+k)\Psi_{xx}(\mu-k)\right] \quad (23)$$

When $\Psi_{xx}(k)$ has the character of a delta-function, the values of $\Psi_{xx}(k)$ for $k \neq 0$ will be much smaller than $\Psi_{xx}(0)$.

On this assumption, an approximation of the variance is:

$$\operatorname{var}\left[\tilde{\Psi}_{xx}(k)\right] = \frac{\Psi_{xx}^2(0)}{N-k} \quad (24)$$

2.3.3 Application.

With equations (21) and (24) we can test the delta-character of the approximated autocorrelationfunction. For this purpose we will follow the next procedure:

Given the sample-array x(i) we will construct the approximated autocorrelationfunction, devided by $\Psi_{xx}(0)$, which leads to a function with value 1 for k = 0 and value <1 for $k \neq 0$. (the normalized autocorrelationfunction).

Assuming again that $\Psi_{xx}(0) \gg \tilde{\Psi}_{xx}(k)_{k \neq 0}$ and considering $\Psi_{xx}(0)$ as a constant, we can state:

$$E\left[\frac{\tilde{\Psi}_{xx}(k)}{\Psi_{xx}(0)}\right] = \frac{1}{\Psi_{xx}(0)} \cdot \Psi_{xx}(k) \tag{25}$$

and

$$\operatorname{var}\left[\frac{\tilde{\Psi}_{xx}(k)}{\Psi_{xx}(0)}\right] = \frac{1}{\Psi_{xx}^{2}(0)} \; \frac{\Psi_{xx}^{2}(0)}{N-k} = \frac{1}{N-k}$$

so

$$\sigma\left[\frac{\tilde{\Psi}_{xx}(k)}{\Psi_{xx}(0)}\right] = \frac{1}{\sqrt{N-k}} \tag{26}$$

Considering $\Psi_{xx}(0)$ as a constant is allowed because of $\Psi_{xx}(0) \gg \tilde{\Psi}_{xx}(k)$ and var $\Psi_{xx}(0) \ll \Psi_{xx}(0)$.

In stead of the σ-value we can also work with the reliability interval, a more practical bound to test the function.

For a Gaussian distributed function the 95% reliability interval can be computed by

$$\Delta\Psi_{95\%} = 1.645 \cdot \sigma$$

so

$$\Delta\left[\frac{\tilde{\Psi}_{xx}(k)}{\Psi_{xx}(0)}\right]_{95\%} = \frac{1.645}{\sqrt{N-k}} \tag{27}$$

We will state that the evaluated autocorrelationfunction is a delta-function when all its values for $k \neq 0$ lie within the range

$$\pm \frac{1.645}{\sqrt{N-k}} \tag{28}$$

as defined in the equation above.

In case of a summation of n autocorrelationfunctions, the variance of this sum (σ_n) is given by $\sigma_n^{2} = 1/n \cdot \sigma^{2}$.

According to this, when we sum the autocorrelationfunctions over n experiments, the range as defined in (28) has to be multiplied by $1/\sqrt{n}$.

In this chapter two methods are described for testing the randomness of a sample array. These two methods are implemented in a computer program that runs experiments and analyses the results. The analysis with the runs test is done by computing P($u \leq u'$) and averaging this over the number of experiments.
The analysis with the autocorrelation function is done by computing this function for every experiment and averaging the results over the number of experiments. The result, an averaged autocorrelation function, is compared with a delta-function, taking into account the variance of the results.

CHAPTER 3 : THE COMPUTER PROGRAM

3.1 Structure of the program.

In view of the possibility to test the constructed model of the human operator in some experiments, a computer program has been written to run these experiments and to make the necessary computations on the results.

The program, called INV, is written in Fortran IV-Plus and implemented on a PDP 11/60 computer. To create the possibility of getting the results of the experiments in different forms (on screen, on lineprinter, or on plotter) the program is chosen to be of an interactive form. In this way one is also able to change any parameters of the system if required.

The tasks of the program are the following:

1. Communicating with the experimental subject, and running an experiment for N sample periods.
2. Computing the results after the experiment:
   - creating an optimal strategy for the choice of the inventory XC(i);
   - creating the model of the human operator, as explained in 1.3, XM(i);
   - evaluating the residuals that are to be investigated:
     1. human operator - model of the h.o.: XHM(i)
     2. human operator - optimal strategy : XHC(i)
     3. human operator - actual demand : XHD(i)
     4. optimal strategy - actual demand : XCD(i)
   - determining the mean squares of the four residuals, SHM,SHC,SHD, SCD;
   - evaluating the profit rate for the optimal controller and for the model of the human operator, WC and WM;
   - determining the normalized autocorrelationfunction of the four residuals: AUT(i,1-4);
   - determining the standard deviation of this function, AUT(i,5);
   - determining the results of the runs test: the probability $P(u \leq u')$, the number of runs u', the number of positive and negative samples, and the mean value of the array.

3. Printing the results on screen and/or on lineprinter. On demand plotting the four autocorrelationfunctions and/or the four system-arrays X(i), XM(i), XC(i), V(i).
4. Storing the results in files if the parameters chosen (such as λ, N, α) have the same values as in other experiments.
5. - On demand computing the results of all experiments, analysed for different values of α: $\alpha = 0.0$, $\alpha = 0.6$, and $\alpha = 0.8$.
   - Printing the total results on screen and/or on lineprinter; On demand plotting the four autocorrelationfunctions.

The program is written in an overlay structure, as drawn in fig -6-.

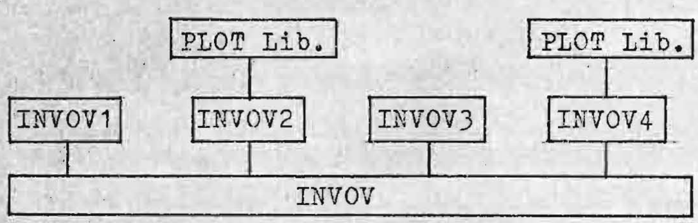

fig - 6 - Overlay structure of the program

In the second part of this project the overlay structure had to be introduced because of the comprehensive library of plotting routines that had to be used. This usage caused an overflow in memory allocation.

The four subroutines INVOV1 - INVOV4 execute the tasks of the program, as mentioned at the beginning of this chapter.
A more extensive description of these four subroutines and of the routines and functions they're using, is included in appendix B1 and B2.
A list of symbols and names of variables, used in this report and in the program listing, is added in appendix C.
The complete listing of the program is available in the archive of the group Measurement and Control.

3.2 Storage of the results.

The object of this part of the research was testing the quality of the model. To do so some experiments would be run for three different values of α : $\alpha = 0.0$, $\alpha = 0.6$, $\alpha = 0.8$.

In case of more than one experiment the results of the experiments have to be brought together. Therefore it is necessary to store the results of one experiment into one or more files that can be kept on floppy disk.
To be able to compute the total results of all experiments, the following results of one experiment have to be stored:

- first we have the parameters of the model: A, B, and the estimated value of b_o (the constant that the human operator adds to his prediction of the demand of the next period), computable ~~out of~~ from these values.
- then there are the values of the profit rates: one of the human operator (W), one of his model (WM), and one of the optimal strategy (WC).
- the mean squares of the four residuals have to be stored, to examine the residuals; SHM, SHC, SHD, SCD.
- the results of the runs test: the value of $P(u \leqslant u')$ for all residuals; P_{H-M}, P_{H-C}, P_{H-D}, P_{C-D}.
- the normalized approximated autocorrelationfunction of the four residuals: AUT(i,1-4);
  eventually (finally) the array of standard deviations of these functions, AUT(i,5).

Because of the required memory-space, the autocorrelationfunctions of the residuals of all experiments can not be stored separately. Therefore we have chosen to create files in which the functions from different experiments are summed. Because there are four residuals and one standard deviation-array, we have to store 5 arrays in a file.
These arrays have to be stored for 3 different values of α , so we get 3 storage-files for the autocorrelationfunctions; these files are called NAUTO.DAT, NAUT6.DAT, NAUT8.DAT.
For any value of α the number of experiments run with that α is recorded in the first record of each file.
The build-up (composition) of the files NAUT.DAT is drawn in fig -7-.

All ~~the~~ other variables are stored in one file: NVAR.DAT.
To separate the results of the experiments referring to different values of α , these variables are stored as drawn in fig -8-.

NAUTO.DAT, NAUT6.DAT, NAUT8.DAT:

recordnr.					
1	NT				
2	$\sum AUT(1)_{H-M}$	$\sum AUT(1)_{H-C}$	$\sum AUT(1)_{H-D}$	$\sum AUT(1)_{C-D}$	DEV(1)
3	$\sum AUT(2)_{H-M}$	$\sum AUT(2)_{H-C}$	$\sum AUT(2)_{H-D}$	$\sum AUT(2)_{C-D}$	DEV(2)
.					
.					
.					
N+1	$\sum AUT(N)_{H-M}$	$\sum AUT(N)_{H-C}$	$\sum AUT(N)_{H-D}$	$\sum AUT(N)_{C-D}$	DEV(N)

fig - 7 - Composition of the storage files NAUT.DAT for the autocorrelation functions.

NVAR.DAT

recordnr.	
1	NTOT
2	α
3	A b_0 B 100xW 100xWM 100xWC SHM SHC SHD SCD P_{H-M} P_{H-C} P_{H-D} P_{C-D}
4	α
5	A . . . .
.	
.	

fig - 8 - Composition of the storage file NVAR.DAT for the numerical results of the experiments.

The value of α in recordnr. i belongs to the results in record i+1.
NTOT is the total number of experiments.

3.3 Some remarks on the program.

- As mentioned in chapter 1, the optimal controller is constructed as follows: $XC(i) = v_0 + \alpha(v(i-1) - v_0) + b_0$

  In the program this optimal controller is computed assuming the correct values of v_0 and b_0. This means: given the value of v_0, used in the program, and given the distribution of the noise by which b_0 is known; It should be more correct to evaluate these two parameters during the experiment, based on the generated demand-function. Then a

fair comparison with the human operator is possible.
However, this adjustment will probably not change very much with respect to the results. On the other hand will this require much more complicated calculations and will cause an increase of computation time.

- In chapter 2 the principles of the runs test are explained. Two variables in this test are the number of samples above (m) and the number of samples beneath (n) the median. Because the value of the median in the array can appear more than once m does not have to equal n. For this reason the calculations in this test are done for the general case.

- The number of periods for which the program can be run is limited by the declaration of the required arrays. The maximum value of N equals 120.

- Because of the way of storage of the results in files, these files have to be created before the first storage of results takes place. For this purpose a program INIT is written that creates the three NAUT-files and the file NVAR, and that fills these files with zero's.

- To make the values of the profit rates more practical ones, we will be working with the variable INT(100xW) instead of W. The same holds for WM and WC.

CHAPTER 4 : EXPERIMENTS AND RESULTS

4.1 The Experiments.

There have been run 30 experiments with 30 different experimental subjects, who had never participated in a similar experiment before. Not one of them had any knowledge neither of the way of generating the demand, nor of any other crucial information.
The subjects were given a written instruction in which their task was described. This instruction is added in appendix D.
They were asked to enter the inventory for 100 successive periods. After each period the demand of the customers, and the cumulated profit rate after ~~the concerning~~ that period, were displayed on screen.
The subjects could take as much time as they wanted for the experiment, there was no time limit.
Although they knew the variables, determining the profit rate, they had no information on the exact construction of the profit rate: the prices of sale, purchase, storage and loss of goodwill were unknown, just as the DC component of the demand v_0, the decay-parameter λ , and evidently the autoregressive parameter α .
One experiment is defined as the action of one experimental subject, entering the inventories for 100 successive periods.

The experiments started with the generation of the demand in period 0; This value functioned as an indication of the size of the demand.
The expectation existed that the subjects would predict the demand of the next period, when ordering the inventory. In doing so they would probably add some extra inventory $\tilde{b}_0$ based on the information of the profit rate. This extra inventory causes a higher profit rate because loss of goodwill is chosen relatively more expensive than storage costs.

One remark that has to be made is that the experimental subject could not choose the inventory unlimited. There was one restriction: he was not allowed to choose the inventory for period i smaller than the inventory remaining from the previous period. In other words it was not allowed to sell products back to the wholesale dealer.
This restriction is incorporated in the program.

4.2 Results of the estimation of the modelparameters.

The results of the estimated modelparameters are listed in table -1- on the next page.

N = 10	α = 0.0		α = 0.6		α = 0.8	
	mean	σ	mean	σ	mean	σ
A	0.56	0.29	0.73	0.21	0.87	0.10
B	2.62	1.59	1.61	1.08	0.71	0.63
b_0	0.41	0.34	0.26	0.31	0.06	0.46

table - 1 - Results of the estimated modelparameters, for each value of α averaged over 10 experiments.

The results of the estimation of the A-parameter in the model vary considerably with different values of α; the standard deviation of the estimations decreases with increasing α, while in all cases A is overestimated with respect to α. For $\alpha = 0.6$ and $\alpha = 0.8$ the estimated parameters 0.73 and 0.87 are quite good estimators. In both cases the equation $\alpha - \sigma < A < \alpha + \sigma$ holds. Remarkable is the estimation of A for $\alpha = 0.8$: an estimation close to the real value, and with a small standard deviation.

For $\alpha = 0.0$ the estimator A is not as good (0.56); the difference $|A-\alpha|$ is larger than σ. In this case of complete white noise as demand, the human operator apparently wants to see some kind of correlation in the demand, although there is none. A is unlikely large, and therefore it may be reasonable to consider the test-situation very critically.

As mentioned in chapter 1, equation (12), $B = (1 - A)\, v_0 + \tilde{b}_0$; therefore $\tilde{b}_0$ can be derived from A and B.

$\acute{b}_0$, the parameter that leads to an optimal strategy, equals 1.06.
In all three cases the estimated value $\tilde{b}_0$ is far below this optimal one. Apparently the subjects have hardly or not given notice to the fact that an optimistic choice for the inventory leads to a higher profit rate than a pessimistic choice. In some way the information that the human operator should receive via the profit rate does not work out very well.
The next two points may have contributed to this underestimating of b_0:

1. the profit rate is presented to the human operator on screen in a cumulative form. In this way it is a stimulance for the subject to fulfil the task and to optimize the total profit rate. On the other hand this way of presenting the profit rate makes it hard for the subject to get information on the quality of his choice of the inventory. A profit rate, presented as an account in one period, should be a more direct way of giving feedback to the subject.

2. the profit rate, as calculated in this experiment, depends on the demand, which is not controllable by the subject. In other words: the level of the profit rate is dependent on the level of the demand, and therefore it is no direct measure for the performance of the subject. It would be more correct to relate this profit rate to the maximum profit rate that could have been achieved. In that way the subject gets direct information on his performance.

It is not certain that the clearly increasing $\tilde{b}_0$ with decreasing α also is a consequence of the previous remarks. One could say that the less deterministic the demand, the more the subject adds some constant level to his expectation. In other words: the more confidence the subject has in the demand v(i) as a predictor for v(i+1), the less he adds "external" components, such as $\tilde{b}_0$.

The profit rates were also calculated and averaged over any 10 experiments. The results are listed in table -2-.

N = 10	α = 0.0		α = 0.6		α = 0.8	
	mean	σ	mean	σ	mean	σ
W	216	5	222	14	210	33
WM	223	7	224	15	213	32
WC	233	6	232	13	225	29

table - 2 - Calculated values of the profit rates; for each value of averaged over 10 experiments.

Because of the fact that the noise generator is started before any experiment with a random number, and therefore with an unequal demand for the different experiments, the profit rates of the different experiments can not be compared. From these results we can draw a conclusion that in general the profit rate of the model approaches the one of the human operator from the upper side.

To give a picture of results of experiments as a function of time (or period) ~~there~~ are added some plots in appendix E. In these plots there are drawn the system-arrays: the inventory x(i), the demand v(i), the model xm(i), and the optimal strategy xc(i); they are plotted two and two and the plots are just an illustration of the results.
There has been taken an arbitrary experiment for any applied value of α.

Appendix E1 gives the results for $\alpha = 0.0$, E2 for $\alpha = 0.6$, and E3 for $\alpha = 0.8$.

4.3 Test on the quality of the model.

As mentioned before, two tests are applied on the four residuals as defined in fig -4-:

H-M : human operator - model of the h.o.

H-C : human operator - optimal strategy

H-D : human operator - actual demand

C-D : optimal strategy - actual demand.

In table -3- the results of the runs test $P(u \leq u')$ and the mean squares of the residuals are listed:

N = 10	$\alpha = 0.0$		$\alpha = 0.6$		$\alpha = 0.8$	
	mean	σ	mean	σ	mean	σ
SHM	0.88	0.66	0.24	0.17	0.27	0.26
SHC	1.78	0.74	1.08	0.63	1.56	0.90
SHD	2.63	0.93	1.41	0.28	1.45	0.58
SCD	2.15	0.28	2.07	0.30	2.12	0.33
P_{H-M}	0.13	0.24	0.03	0.07	0.06	0.11
P_{H-C}	0.22	0.26	0.00	0.01	0.04	0.07
P_{H-D}	0.79	0.30	0.37	0.37	0.52	0.36
P_{C-D}	0.59	0.27	0.50	0.32	0.66	0.25

table - 3 - Mean squares and the results of the runs test $P(u \leq u')$ of the four residuals; for each value of α averaged over 10 experiments.

The results for the approximated autocorrelationfunctions, summed over 10 experiments are added in appendix F1-F3. The scaling factor of these autocorrelationfunction equals the mean squares of the residuals, that are listed above. In the plots in appendix F also the standard deviation-array is plotted.

Some remarks on the results:

- In the optimal case the autocorrelation function of the residual of human operator - model is a delta-function.
  A tendency towards such a function can be seen in the drawings

in appendix F, although the values for small k are ~~out of~~ outside the reliability interval. This appears in all three cases. However there are some differences for different values of α : for $\alpha = 0.8$ only for $k \leq 4$ the values of $\psi(k)$ are beyond the 95%-reliability interval. For $\alpha = 0.6$ this holds for $k \leq 7$. For $\alpha = 0.0$ the model doesn't seem to fit very well, according to this picture.
Moreover one has to take into account that because of the fact that ψ is not purely Gaussian distributed, the 95%-reliability interval is larger than the assumed factor 1.645 multiplied by σ. (see eq. 27 pg.16)

- The residual of human operator - optimal strategy can tell us something about the learning effects of the subject. A tendency towards making choices in the optimal direction causes a decreasing of $\psi(k)$ for increasing k. A relatively small decrease of $\psi(k)$ can be seen in all pictures. The decrease in case of $\alpha = 0.8$ for $k > 50$ is the clearest one, although it is not very convincing.
Anyway, evaluation of this residual with window technics is a better strategy for getting knowledge about the learning effects.

- For the runs test there is chosen a level of significance $\beta = 0.05$. If $P(u \leq u')$ is smaller than 0.05, we therefore reject the hypothesis that the residual can be regarded as sampled white noise. According to table -3- this happens for the residuals H-M and H-C if $\alpha = 0.6$ and $\alpha = 0.8$.
These results are in contradiction with the results of the autocorrelation function. In that test the results for $\alpha = 0.0$ were worse than for $\alpha = 0.6$ and $\alpha = 0.8$.
One reason for this difference is the fact that the runs test is taken to be a one-sided test. There is assumed that the more runs in an array, the more chance that the array is sampled white noise. In principle this is not correct: very many runs in an array indicates the existence of relatively many high frequencies in the array.
The appearance of a few experiments with more than 50 runs in a residual array of 100 samples (and therefore $P(u \leq u') > 0.5$) can influence the results of the runs test essentially. Especially in case of $\alpha = 0.0$ (the demand-array has relatively more high frequencies in its spectrum then) this influence is not negligable.
For the residual H-D there are even situations where the number of runs

is higher than the double-sided critical value: $P(u \leq u') > 0.95$.
This happens four times when $\alpha = 0.0$.
In case of $\alpha = 0.6$ and $\alpha = 0.8$ introduction of a double sided runs test will have influence on the results for the residuals H-D and C-D.

- If we consider the remark above, we can conclude that the values of $\hat{\psi}(k)$ for small k affect the results of the runs test essentially with respect to the comparison of the human operator and his model. The results of this test only lead to the conclusion that the model is not correct.

CONCLUSIONS

The purpose of this project was to determine whether a simple zero order model could be a right description of the behaviour of a human operator in a specific control- c.q. prediction-task. To attain this purpose, a computer program is written that is able to test such a model, and to estimate its parameters in an experimental situation.
30 Experiments have been run in three groups of ten, and from the results of these experiments we can draw the ~~next~~ following conclusions:

- The zero order model can be a satisfying model to describe the main lines of the behaviour of the human operator.
- Extension of the model to a first- or maybe a second-order model probably ~~can~~ might improve the description of the behaviour.
- The profit rate as presented to the human operator in the test does not function very well as a feedback to the operator. It should not be presented in a cumulative form and it has to be considered whether it should be related to a maximum achievable profit rate.
- The human operator can, within fair limits, approximate the autoregressive parameter of the demand function. Only for $\alpha = 0.0$ the estimation differs from the real value. This can have been affected by the previous remark.

Based on these remarks the ~~next~~ following recommendations can be stated:

- Reconsidering the presentation of some profit rate,
- Changing the program in such a way that it is capable to apply the SATER-package to the experimental data,
- Applying an order-test to this data.

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APPENDIX A : THE RUNS TEST, DETERMINATION OF $P(U \leq U')$.

For determining $P(u \leq u')$ we will first evaluate $P(u=u')$.

A distinction can be made between two possible situations:

a) u' is even,

b) u' is odd.

In case a) the number of positive runs equals the number of negative runs. In case b), on the other hand, these numbers differ.
Let's call the number of positive runs e_+, the number of negative runs e_-. ($e_+ + e_- = u'$).
Let r_{+j} be the number of elements with sign + in the j^{th} run of this kind, and r_{-j} the number of elements with sign - in the j^{th} run of this kind.

a) u' is even $\longrightarrow$ $u' = 2\,k$, with $k \in \mathbb{N}$.

$e_+ = e_- = k$.

The first element v_1 of the array, together with the numbers $r_{+1}, r_{+2}, \ldots, r_{+k}, r_{-1}, \ldots, r_{-k}$ completely determine the array.

Let $v_1 = +$.

The number of sequences that conform to $\sum_{j=1}^{k} r_{+j} = m$ equals $\binom{m-1}{k-1}$ for: there are m characters of the same kind; these have to be separated into k parts. In other words: there are k-1 slashes that have to be distributed over m-1 locations; so there are $\binom{m-1}{k-1}$ possibilities.

For the same reason the number of sequences that conform to $\sum_{j=1}^{k} r_{-j} = n$ equals $\binom{n-1}{k-1}$.

As follows the number of possible arrangements in the situation $v_1=+$ is $\binom{m-1}{k-1}\binom{n-1}{k-1}$.

The same story holds for the case $v_1 = -$, so the total number of possible arrangements is

$$2\binom{m-1}{k-1}\binom{n-1}{k-1}.$$

The number of all possible arrangements is $\binom{m+n}{m}$.

These results lead to the conclusion that

$$P(u=2k) = \frac{2\binom{m-1}{k-1}\binom{n-1}{k-1}}{\binom{m+n}{m}} \qquad (16)$$

b) u' is odd $\longrightarrow$ $u' = 2k - 1$.

Let $v_1 = +$; then $e_+ = k$ and $e_- = k - 1$.

The number of sequences that conform to $\sum_{j=1}^{k} r_{+j} = m$ equals $\binom{m-1}{k-1}$.

The number of sequences that conform to $\sum_{j=1}^{k-1} r_{-j} = n$ equals $\binom{n-1}{k-2}$.

Therefore the number of possible arrangements is $\binom{m-1}{k-1}\binom{n-1}{k-2}$.

Let $v_1 = -$; then $e_+ = k - 1$ and $e_- = k$.

The number of sequences that conform to $\sum_{j=1}^{k-1} r_{+j} = m$ equals $\binom{m-1}{k-2}$.

The number of sequences that conform to $\sum_{j=1}^{k} r_{-j} = n$ equals $\binom{n-1}{k-1}$

Therefore the number of possible arrangements is $\binom{m-1}{k-2}\binom{n-1}{k-1}$.

These results lead to the conclusion that:

$$P(u = 2k - 1) = \frac{\binom{m-1}{k-1}\binom{n-1}{k-2} + \binom{m-1}{k-2}\binom{n-1}{k-1}}{\binom{m+n}{n}} \qquad (17)$$

Substituting k=1 in this formula gives $P(u=1) = 0$.

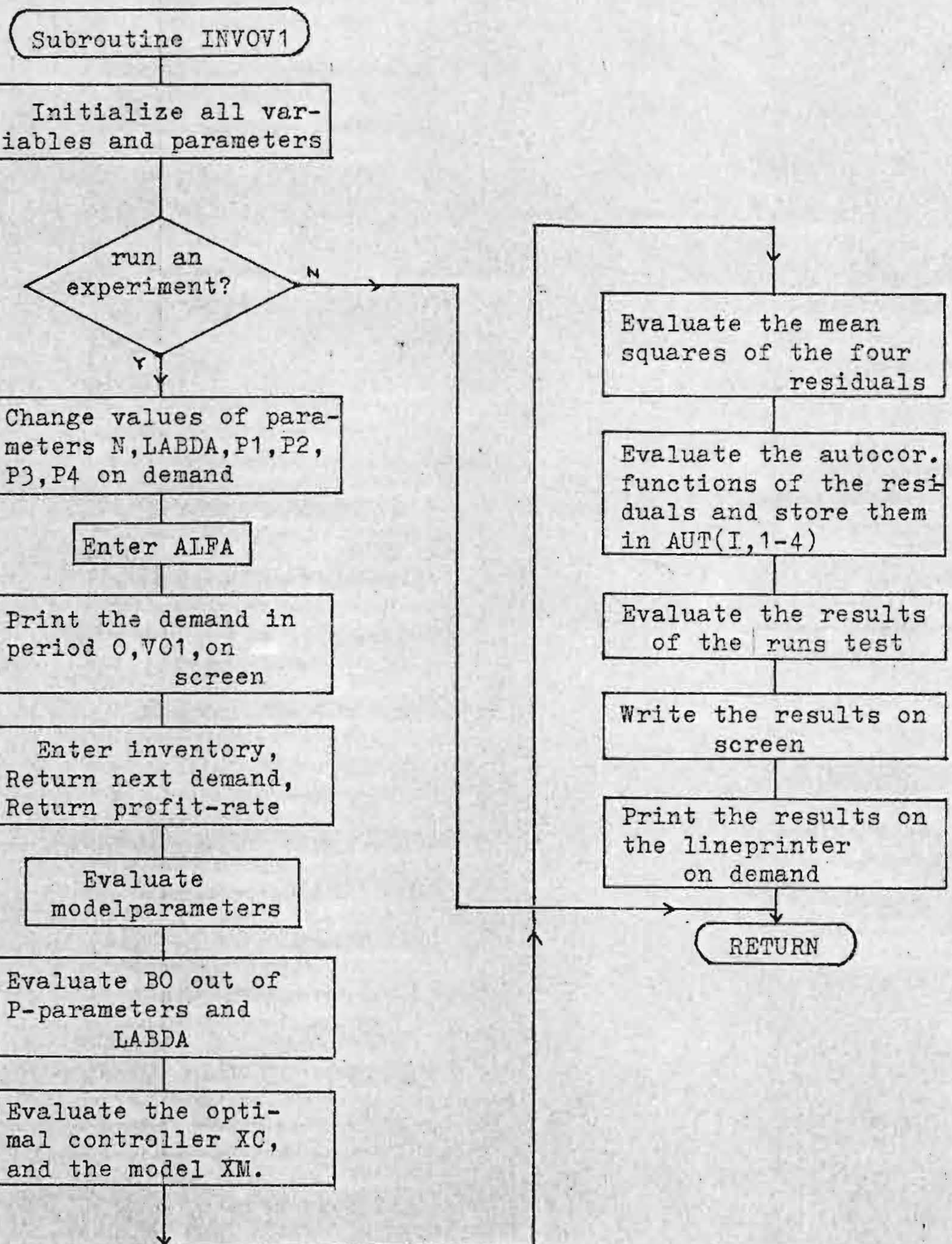
Out of a) and b) and the results of equations (16) and (17), we now can conclude:

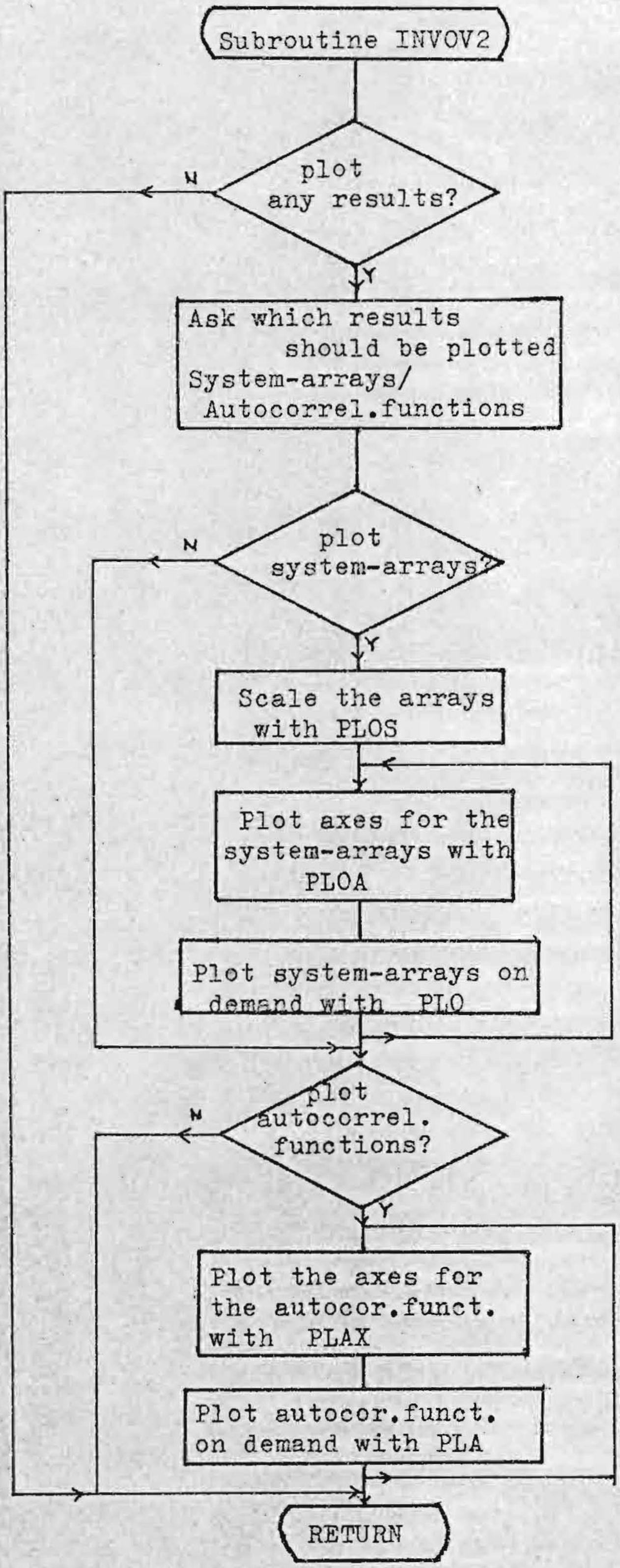
$$P(u \leq u') = \binom{m+n}{n}^{-1} \sum_{u=2}^{u'} f_u \qquad (18)$$

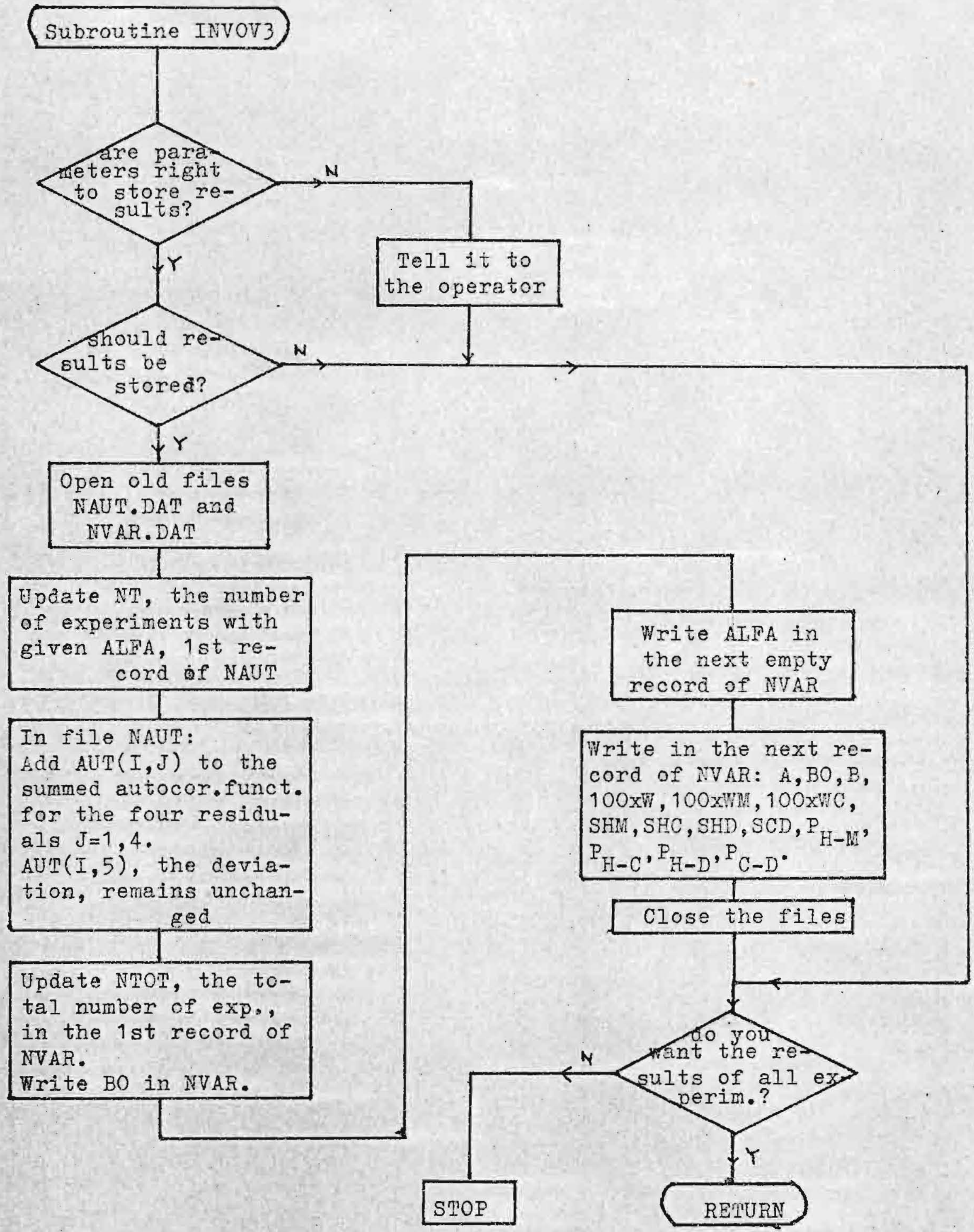
with $f_u = 2\binom{m-1}{k-1}\binom{n-1}{k-1}$ for $u = 2k$, $k \in \mathbb{N}$

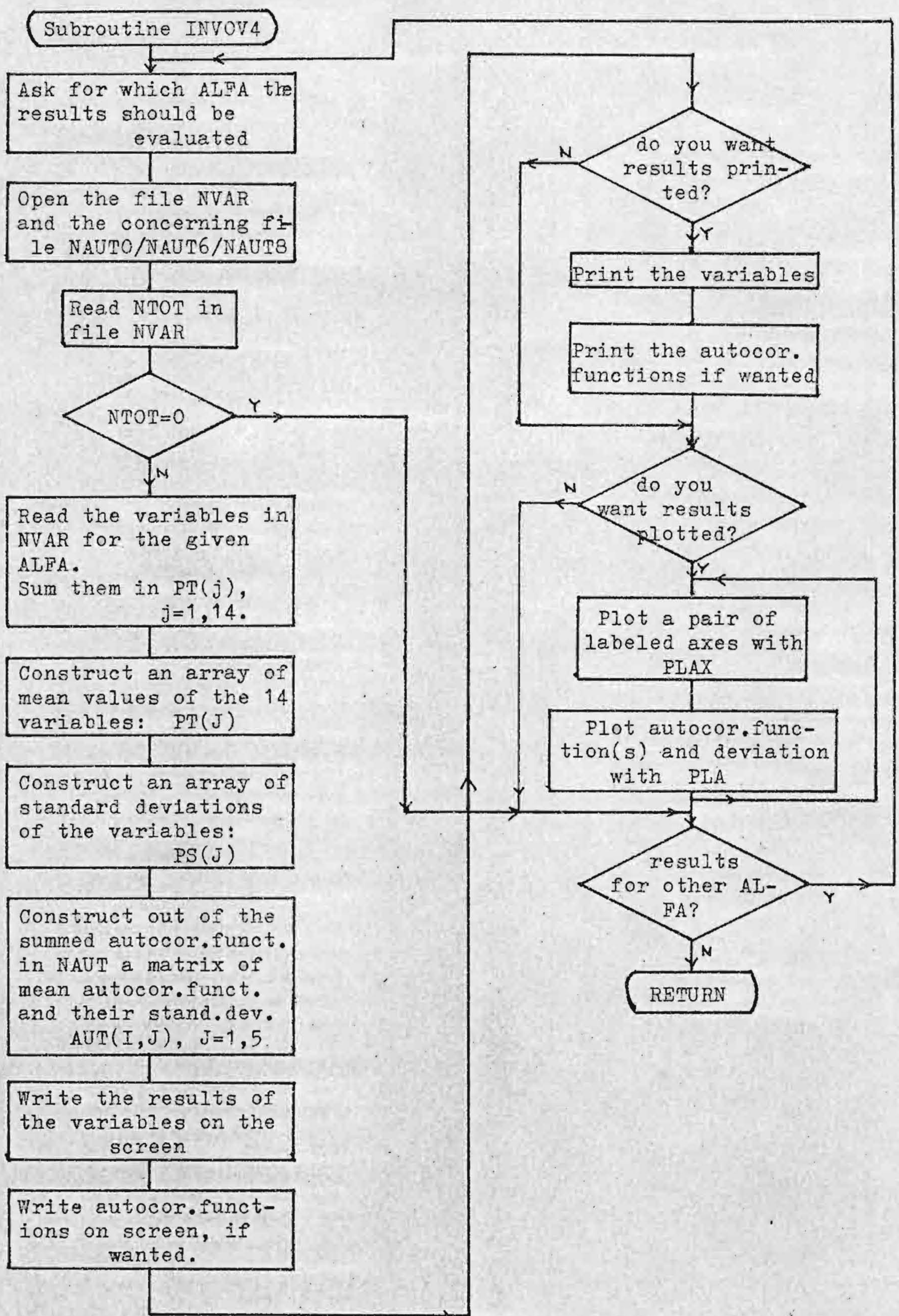
and $f_u = \binom{m-1}{k-1}\binom{n-1}{k-2} + \binom{m-1}{k-2}\binom{n-1}{k-1}$ for $u = 2k - 1$, $k \in \mathbb{N}$.

APPENDIX B1 : FLOW CHARTS OF THE SUBROUTINES.




Subroutine INVOV2
plot any results?
N
Y
Ask which results should be plotted System-arrays/ Autocorrel.functions
plot system-arrays?
N
Y
Scale the arrays with PLOS
Plot axes for the system-arrays with PLOA
Plot system-arrays on demand with PLO
plot autocorrel. functions?
N
Y
Plot the axes for the autocor.funct. with PLAX
Plot autocor.funct. on demand with PLA
RETURN


Subroutine INVOV3
are parameters right to store results?
N
Y
Tell it to the operator
should results be stored?
N
Y
Open old files NAUT.DAT and NVAR.DAT
Update NT, the number of experiments with given ALFA, 1st record of NAUT
In file NAUT: Add AUT(I,J) to the summed autocor.funct. for the four residuals J=1,4. AUT(I,5), the deviation, remains unchanged
Update NTOT, the total number of exp., in the 1st record of NVAR. Write BO in NVAR.
Write ALFA in the next empty record of NVAR
Write in the next record of NVAR: A,BO,B, 100xW,100xWM,100xWC, SHM,SHC,SHD,SCD,P_{H-M}, P_{H-C},P_{H-D},P_{C-D}.
Close the files
do you want the results of all experim.?
N
Y
STOP
RETURN


Subroutine INVOV4
Ask for which ALFA the results should be evaluated
Open the file NVAR and the concerning file NAUT0/NAUT6/NAUT8
Read NTOT in file NVAR
NTOT=0
Y
N
Read the variables in NVAR for the given ALFA. Sum them in PT(j), j=1,14.
Construct an array of mean values of the 14 variables: PT(J)
Construct an array of standard deviations of the variables: PS(J)
Construct out of the summed autocor.funct. in NAUT a matrix of mean autocor.funct. and their stand.dev. AUT(I,J), J=1,5
Write the results of the variables on the screen
Write autocor.functions on screen, if wanted.
do you want results printed?
N
Y
Print the variables
Print the autocor. functions if wanted
do you want results plotted?
N
Y
Plot a pair of labeled axes with PLAX
Plot autocor.function(s) and deviation with PLA
results for other ALFA?
Y
N
RETURN

APPENDIX B2 : SPECIFICATIONS OF SUBROUTINES AND FUNCTIONS.

INVOV1 : Subroutine that runs, on demand, an experiment and returns the results on screen or/and on lineprinter.
To start the experiment one has to enter values for the different parameters LABDA,P1,P2,P3,P4,N or leave them unchanged. The operator has to type the value of ALFA and to start the Gaussian noise generator.
INVOV1 calls the subroutines AUTO, RUNS and the functions SRAN, GAUSSN, PROB, OVER.

INVOV2 : This subroutine plots the results of one experiment: system-arrays in one or in separate pictures, and/or the autocorrelationfunction(s) of the residuals, also in one or in separate pictures.
INVOV2 calls the subroutines PLAX, PLA, PLOA, PLOS, PLO, and the plotting library.

INVOV3 : Subroutine that checks and asks if the results of one experiment can and should be stored. If the parameters have the right values, the results are stored on demand.

INVOV4 : Subroutine that evaluates the total results of all experiments. The results are written, printed and/or plotted. The evaluation is done for the value of ALFA that the operator has entered.
On demand the results can be displayed, printed and/or plotted.
INVOV4 calls the subroutines PLAX, PLA and the plotting library.

AUTO(XAU) : Subroutine that determines an approximation of the normalized autocorrelationfunction of the array XAU of length N; this function is returned in array PSI.
PSI(I) conforms to $\psi_{xx}(I-1)$.

OVER(K1,K2) : Real function, that determines the expression $\binom{K1}{K2}$.
If $K1 < K2$, OVER = 0.
If $K2 = 0$, OVER = 1.

RUNS(AR) : Subroutine that applies the runs test to the array AR, and returns a vector PRS(5) that contains:

PRS(1) : $P(u \leq u')$

PRS(2) : number of samples above the median.

PRS(3) : number of samples beneath the median.

PRS(4) : number of runs u'.

PRS(5) : mean value of the array-samples.

The number of + and - samples are calculated because the value of the median sample can appear more than once, so e_+ will not equal e_-.

RUNS calls the function OVER.

PROB(P) : Real function that determines BO out of the relation $P(\xi \leq BO) = P$, given a normal distribution of the noise ξ.

GAUSSN(RAN)
RANDS(RAN)
SRAN(IX) : Three real functions that together generate Gaussian noise. Calling GAUSSN(RAN) is enough to generate normally distributed noise with zero mean and unit variance.

The generator has to be initialized by the statement RAN=SRAN(IX), where IX can be chosen arbitrary

PLAX(NT) : Subroutine that plots a pair of labeled axes for plotting the autocorrelation-characteristics.

NT is the number of experiments runned, and is plotted in the heading. Also is plotted the array of standard deviations of the approximated autocorrelationfunctions.

PLA(NR) : Subroutine that plots the autocorrelationfunction of the residual with number NR:

NR = 1 : residual H-M,

NR = 2 : residual H-C,

NR = 3 : residual H-D,

NR = 4 : residual C-D.

PLOA : This subroutine plots a pair of labeled axes for the plot of the system-arrays.

PLOS : This subroutine scales the four system-arrays X, XM, XC, V, for plotting.
For this scaling the four system-arrays are arranged in one array. This array is scaled and afterwards the four arrays are separated again.

PLO(NS) : Subroutine that plots the system-array with number NS:
NS = 1 : array X(I),
NS = 2 : array XM(I),
NS = 3 : array XC(I),
NS = 4 : array V(I).

APPENDIX C : LIST OF SYMBOLS.

A	:	modelparameter; multiplicator of V(I-1).
A(I),a(i)	:	sale in period I.
α, ALFA	:	autoregressive parameter of the demand.
AUT(I,J)	:	- stored autocorrelationfunction of the residual XHM (J=1), XHC (J=2), XHD (J=3), XCD (J=4). - stored sum of autocorrelationfunctions.
AUT(I,5)	:	stored standard deviation of the autocorrelationfunction based on one experiment.
B	:	constant parameter in the model of the human operator.
B(I),b(i)	:	purchase order at the end of period I.
BO, b_0	:	extra purchase, determined by the optimal controller.
λ, LABDA	:	decay parameter.
N	:	number of periods.
NNEG	:	number of samples in an array with value beneath the median.
NPOS	:	number of samples in an array with value above the median.
P1	:	price of sale of the product.
P2	:	price of purchase.
P3	:	price of storage.
P4	:	price of "loss of goodwill".
PRS(I)	:	array with results of the runs test applied on one residual.
PRU(I,J)	:	matrix with results of the runs test of the four residuals.
PSI(I)	:	autocorrelationfunction with shifted number of samples is I-1.
SHM	:	mean square of XHM(I).
SHC	:	mean square of XHC(I).
SHD	:	mean square of XHD(I).
SCD	:	mean square of XCD(I).
u, u'	:	number of runs in a residual-array.
VO,v_0	:	the D.C. level of the demand.
VO1,v_{01}	:	the demand in period O.
V(I),v(i)	:	demand in period I.
W	:	profit rate for the human operator
WM	:	profit rate for the model of the human operator.
WC	:	profit rate for the optimal controller.
X(I),x(i)	:	inventory at the beginning of period I.
XM(I)	:	array of the model of the human operator.
XC(I)	:	array of the optimal controller.

XHM(I)	:	residual of human operator - model of the human operator.
XHC(I)	:	residual of human operator - optimal controller.
XHD(I)	:	residual of human operator - actual demand.
XCD(I)	:	residual of optimal controller - actual demand.
ξ(I)	:	array of sampled white noise, $\mu = 0$, $\sigma = 1$.
Y(I),y(i)	:	inventory at the end of period I.

APPENDIX D : INSTRUCTIONS FOR THE EXPERIMENTAL SUBJECT.

TEST INVENTORY CONTROL.

The problem is as follows:
You are a shopowner in a store, and you are selling one product: product X. You are asked to give the inventory for product X as you would choose it for 100 successive periods. Note that you are asked to give the inventory and not the purchase.
After choosing the inventory the machine in return will give the demand of the customers for product X in this period. Note that the machine returns the demand, and not the sale.
Record will be kept of your total profit rate. This amount appears in the last column at the end of every period. In this amount will be processed: the sale, the purchase, storage costs and costs of loss of goodwill when the inventory is too small.

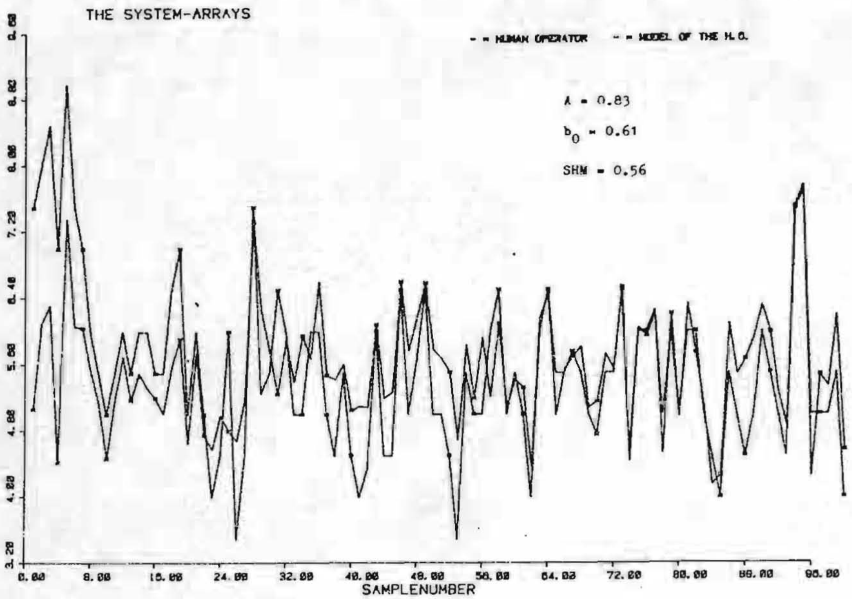
On the screen there appears:
in the first column: the number of the period,
in the second column: the inventory, to be typed in by you,
in the third column: the demand of the customers in this period,
in the fourth column: the profit rate.

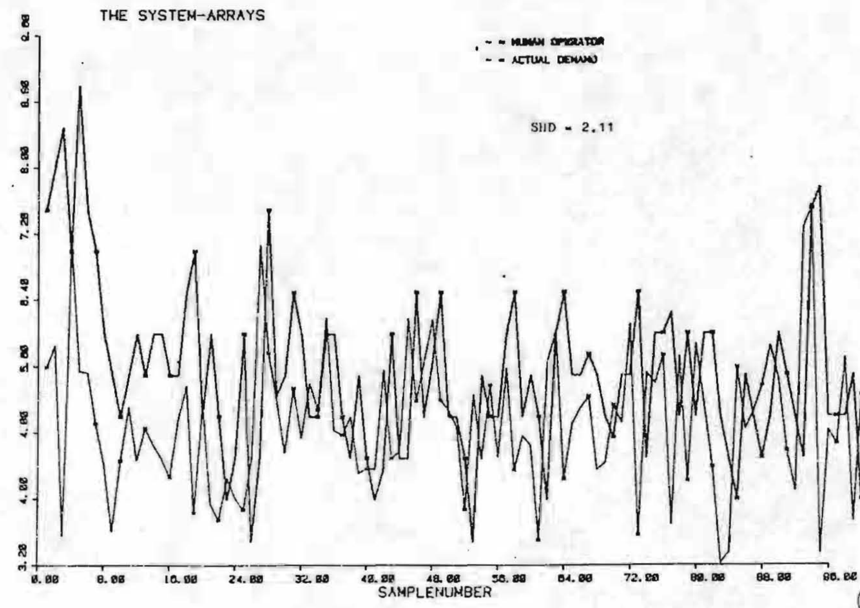
For typing the numbers you can use the keys on the upper row of the keyboard. After entering the inventory you have to push the RETURN - key. Corrections can be made by using the RUB - key.

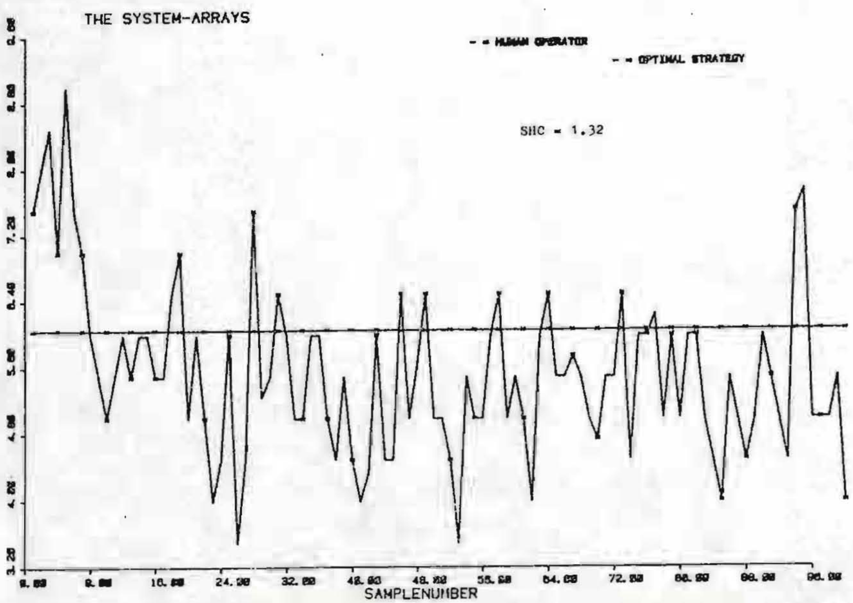
On the screen the text can appear:
"YOUR PURCHASE IS NEGATIVE, ENTER A NEW ONE".
This means that the entered inventory is smaller than the inventory remaining from the previous day. This situation is not permitted, so you will have to enter a new choice.

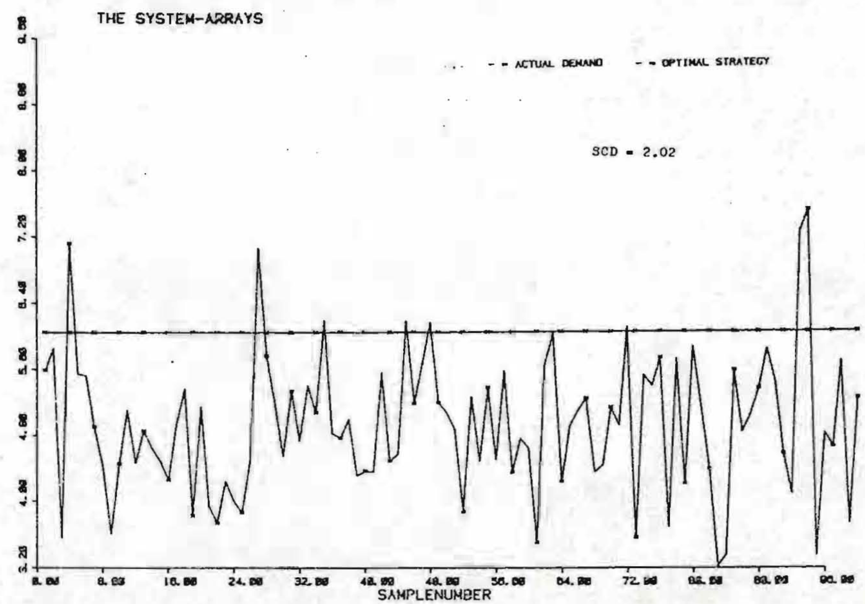
APPENDIX E : THE SYSTEM-ARRAYS AS A FUNCTION OF THE PERIOD-NUMBER.

An arbitrary experiment; $\alpha = 0.0$

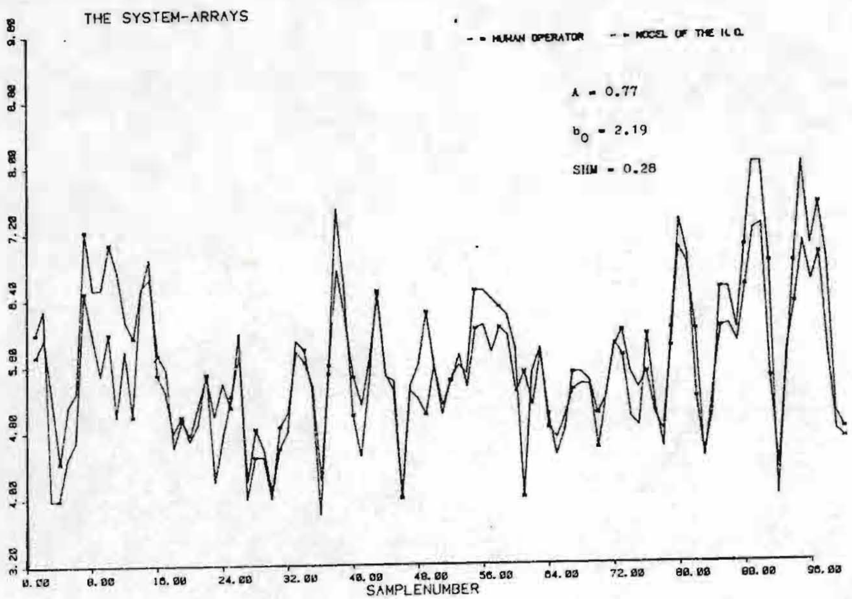


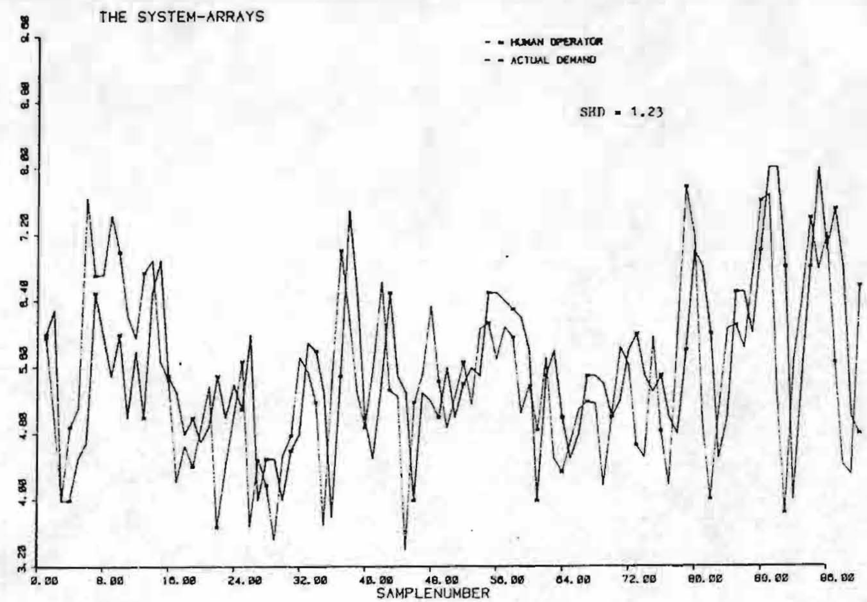


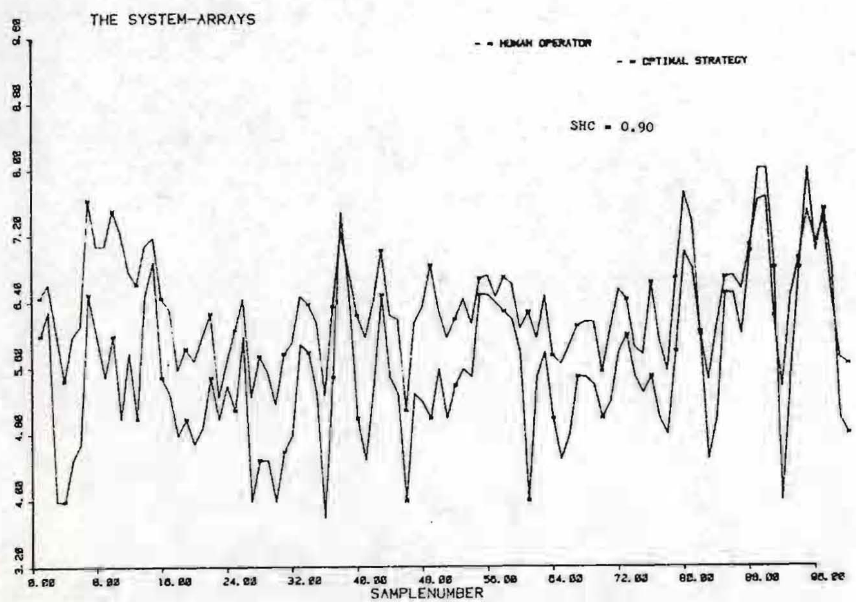


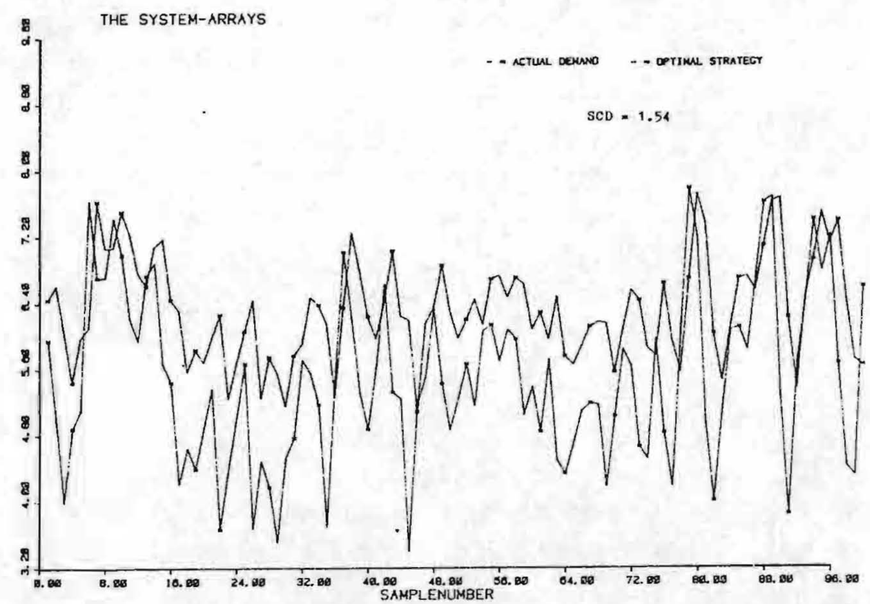


An arbitrary experiment; α = 0.6

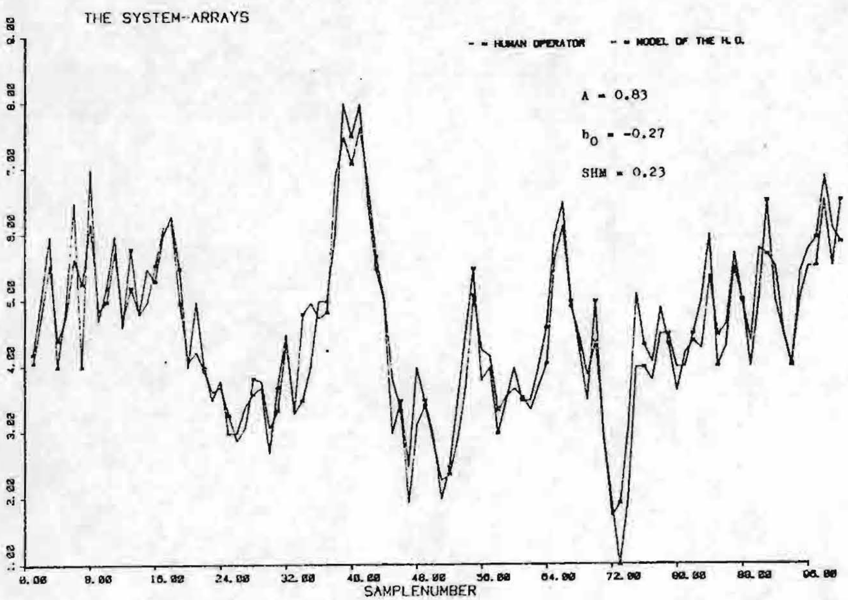


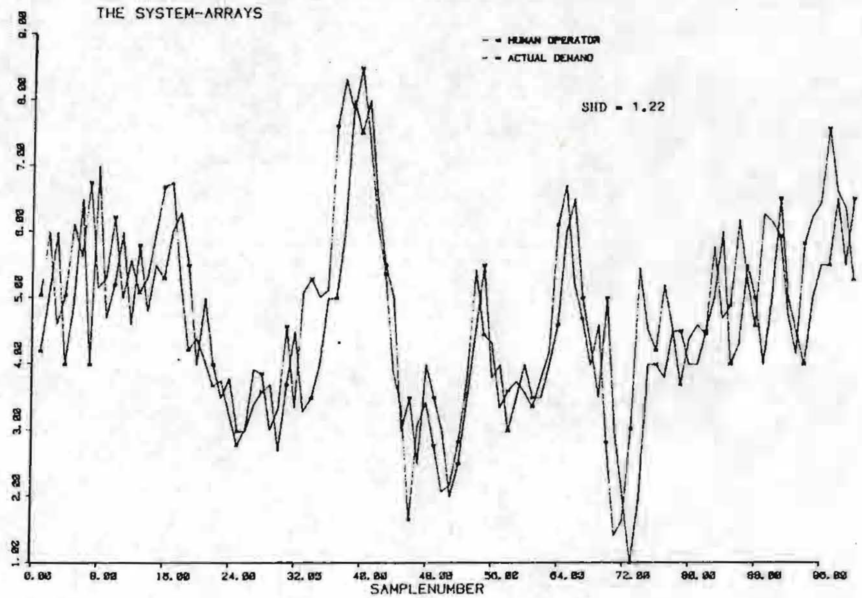


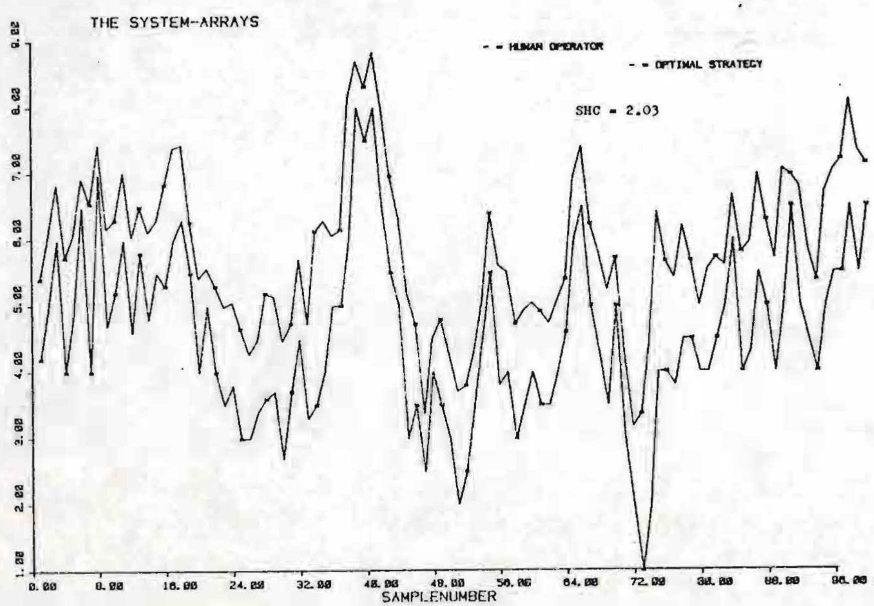


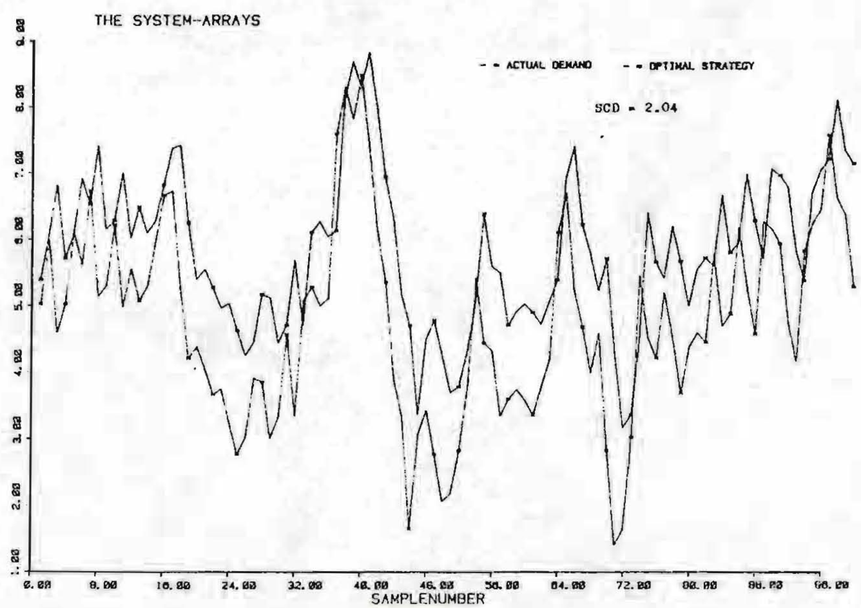


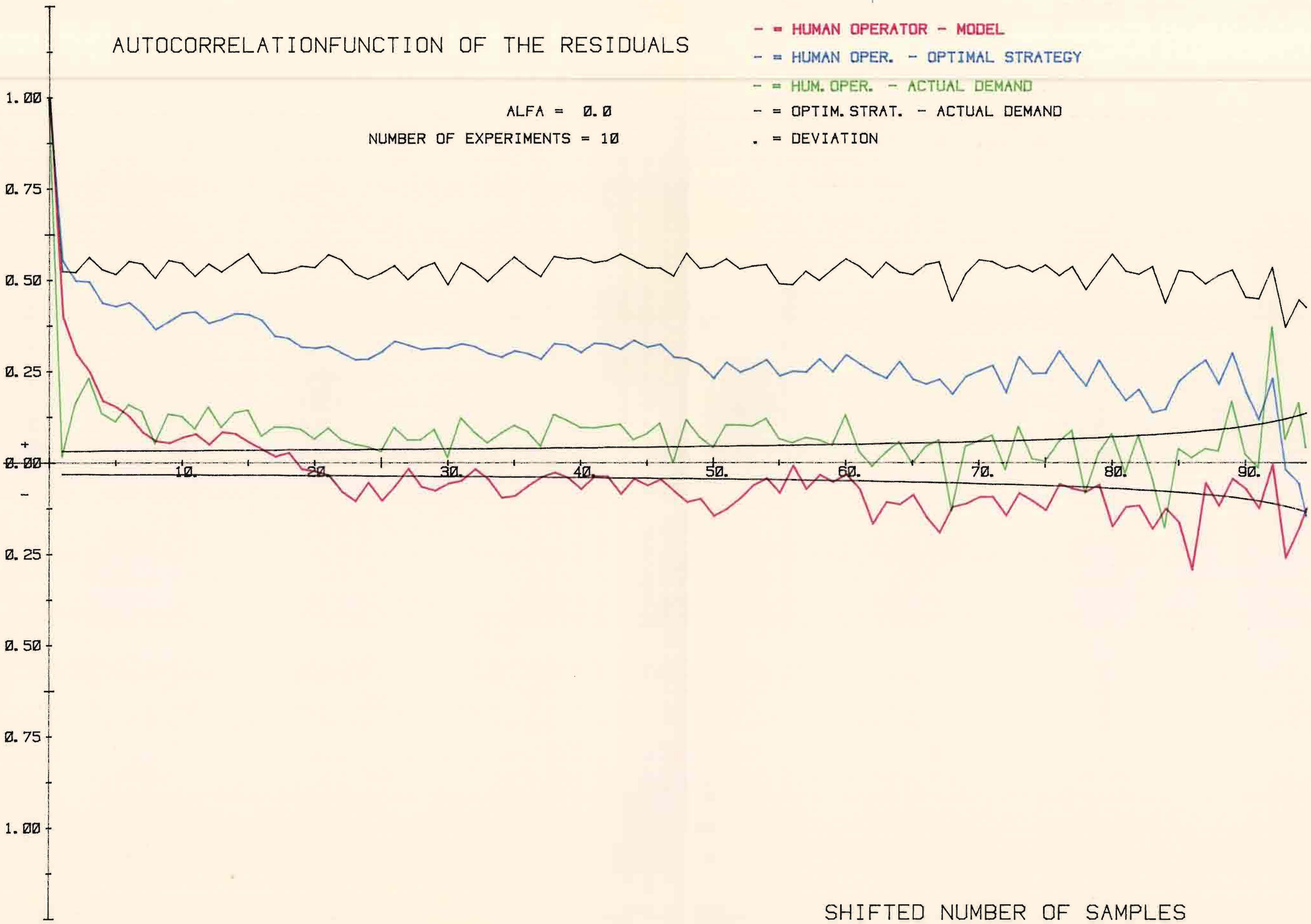
An arbitrary experiment; $\alpha = 0.8$







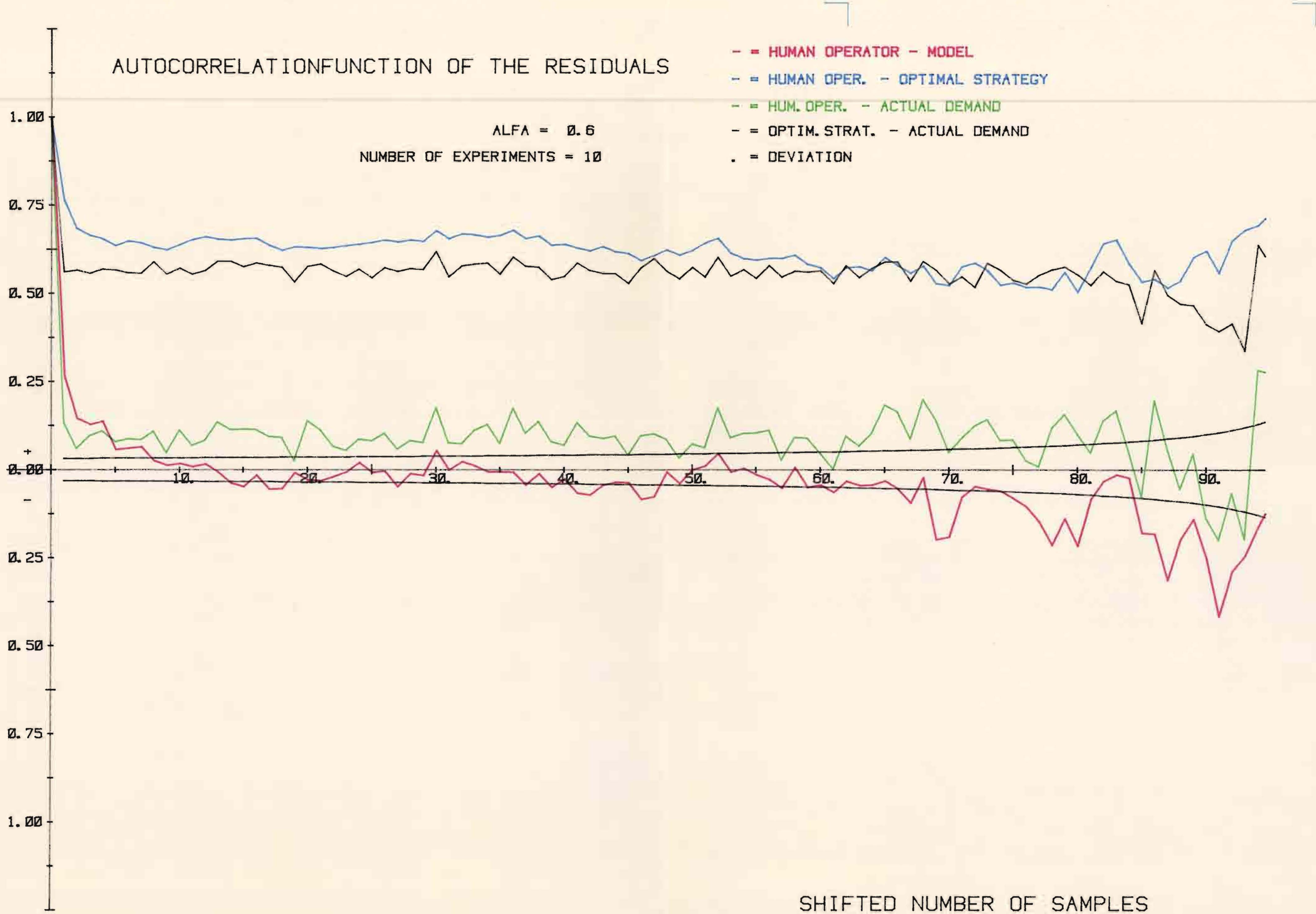




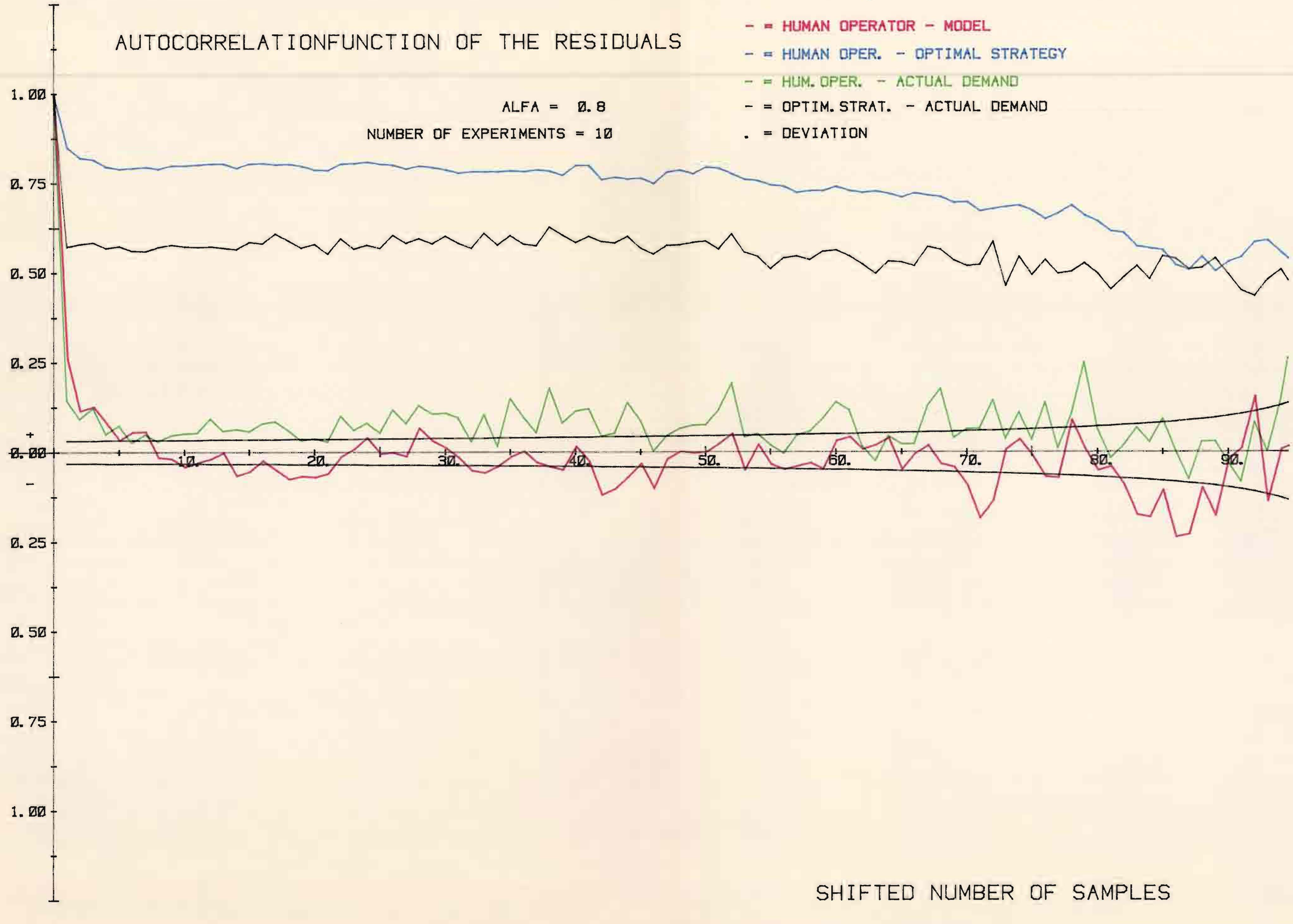
AUTOCORRELATIONFUNCTION OF THE RESIDUALS
ALFA = 0.0
NUMBER OF EXPERIMENTS = 10
- = HUMAN OPERATOR - MODEL
- = HUMAN OPER. - OPTIMAL STRATEGY
- = HUM. OPER. - ACTUAL DEMAND
- = OPTIM. STRAT. - ACTUAL DEMAND
. = DEVIATION
1.00
0.75
0.50
0.25
+
0.00
-
0.25
0.50
0.75
1.00
10.
20.
30.
40.
50.
60.
70.
80.
90.
SHIFTED NUMBER OF SAMPLES

APPENDIX F : AUTOCORRELATIONFUNCTIONS OF THE RESIDUALS, SUMMED OVER 10 EXPERIMENTS.

ALFA = 0.0


AUTOCORRELATIONFUNCTION OF THE RESIDUALS
ALFA = 0.6
NUMBER OF EXPERIMENTS = 10
- = HUMAN OPERATOR - MODEL
- = HUMAN OPER. - OPTIMAL STRATEGY
- = HUM. OPER. - ACTUAL DEMAND
- = OPTIM. STRAT. - ACTUAL DEMAND
. = DEVIATION
1.00
0.75
0.50
0.25
+
0.00
-
0.25
0.50
0.75
1.00
10.
20.
30.
40.
50.
60.
70.
80.
90.
SHIFTED NUMBER OF SAMPLES

ALFA = 0.6


AUTOCORRELATIONFUNCTION OF THE RESIDUALS
ALFA = 0.8
NUMBER OF EXPERIMENTS = 10
- = HUMAN OPERATOR - MODEL
- = HUMAN OPER. - OPTIMAL STRATEGY
- = HUM. OPER. - ACTUAL DEMAND
- = OPTIM. STRAT. - ACTUAL DEMAND
. = DEVIATION
1.00
0.75
0.50
0.25
0.00
0.25
0.50
0.75
1.00
10.
20.
30.
40.
50.
60.
70.
80.
90.
SHIFTED NUMBER OF SAMPLES

ALFA = 0.8