The system plate-pillar in the computer analysis of presses and die-sets

THE SYSTEM PLATE-PILLAR IN THE COMPUTER ANALYSIS OF PRESSES AND DIE-SETS.

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0. SUMMARY

In the analysis of presses and die-sets with two dimensional beam-type finite element computer programs, it is not possible to replace plate-pillar systems by realistic beam elements when the height and the diameter of the pillar are of the same order of magnitude as the thickness of the plate. Still it is advantageous to use these programs rather than large sophisticated finite element programs. It is shown that one can get correction factors by a one time three-dimensional analysis of the plate-pillar system mentioned above. These factors enable us to use simple beam-type elements with high accuracies even in extreme situations.

1. INTRODUCTION

The use of computer programs with two dimensional beam-type finite elements in the analysis of presses and die-sets [1], [2] gives reasonable results. With these programs one can correlate in a simple way the horizontal deflections of the cutters of a die-set with the parameters of the punching process and the dimensions of the press and the die-set. The results of this analysis in combination with a technological criterion for the maximum misalignment of die and punch, can be used to establish the optimum design of a die-set for a specific product.

In transforming the press and the die-set into a topological model one often meets a plate-pillar system as shown in Fig. 1. When the quantities z, D and H are of the same magnitude, it is not realistic to replace the pillar and the plate simply by beam elements. Actually, there are two problems. First of all, it is not possible to establish the effective length of the pillar and, secondly, the angular deflection of the surface of the plate in the neighbourhood of the pillar is not only determined by the magnitude of the load and the second moment of area with respect to the Y-axis of the plate, but also by the diameter of the pillar.

Of course, it is possible to get an exact analysis of the mentioned plate-pillar system by means of a large finite-element

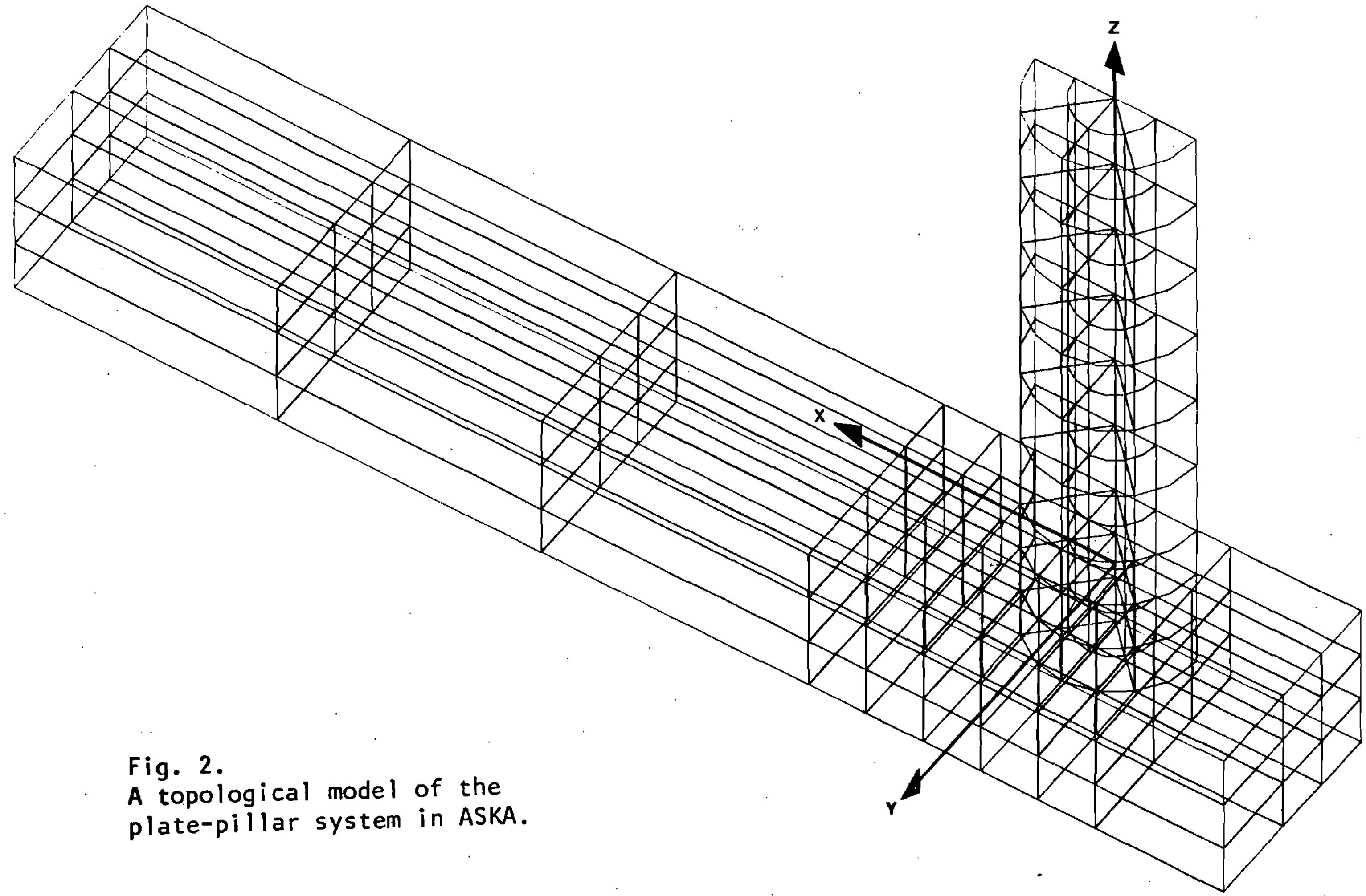


Fig. 2.
A topological model of the
plate-pillar system in ASKA.

program using sophisticated element-types. Fig. 2 shows for example a topological model for the plate and the pillar when applying the ASKA-system. For symmetrical reasons only half of the model is shown. The elements used are PENTAC 18 (54 degrees of freedom), HEXEC 27 and HEXE 27 (81 degrees of freedom). It is obvious that using this kind of analysis to get a proper design of the die-set will cost a tremendous lot of preparation- and computer-time and will be financially unacceptable in most cases.

In this article a method will be described how correction factors ψ and λ can be obtained by a one-time analysis of the plate-pillar system with a sophisticated program-system like ASKA. The factor ψ concerns the correction of the stiffness of the pillar, while λ corrects the angular deflection of the surface of the plate in the neighbourhood of the pillar. Including these correction factors in the stiffness matrix of a two dimensional beam element makes it possible to carry out an analysis with simple elements and high accuracy even when one combines a short pillar with a thick plate.

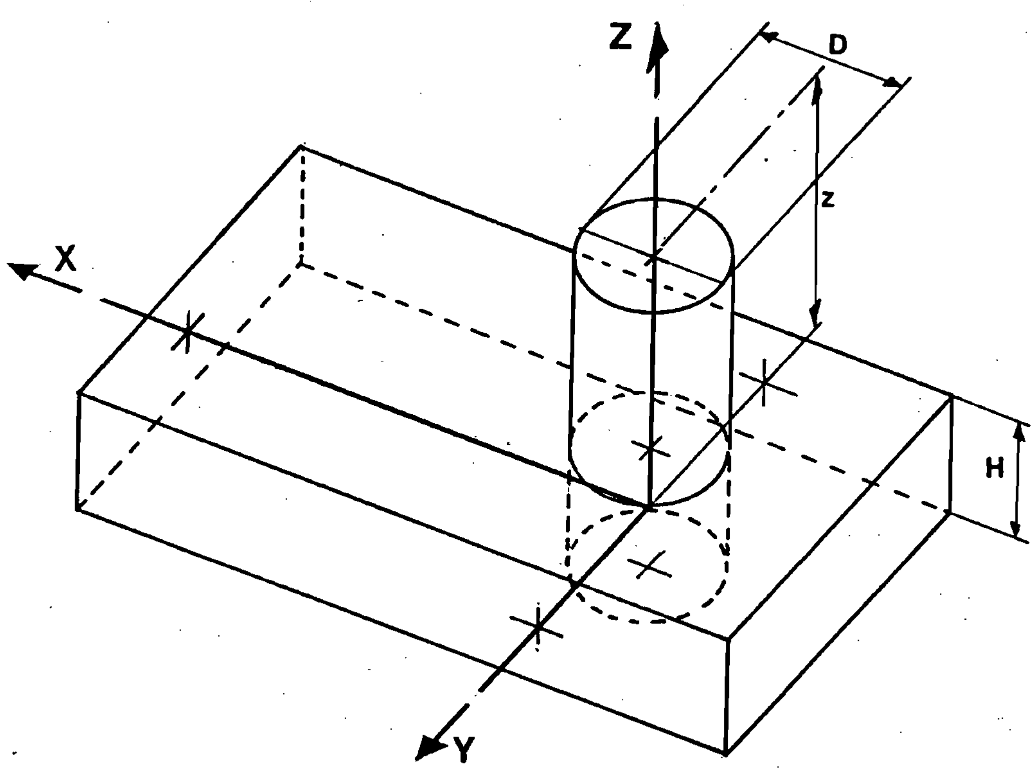


Fig. 1. The plate-pillar system.

2. THE ANALYSIS WITH THE ASKA-SYSTEM

Even a one-time analysis of the structure shown in Fig. 2 by means of ASKA is very costly, mainly because of the complicated elements for the pillar with the circular cross sectional area. For this reason the circular pillar is replaced by an equivalent square one (see Fig. 3), under the condition that the bending stiffness of both pillars is the same. From this it follows:

$$h = D_{eq} = 0.876 D \qquad (1)$$

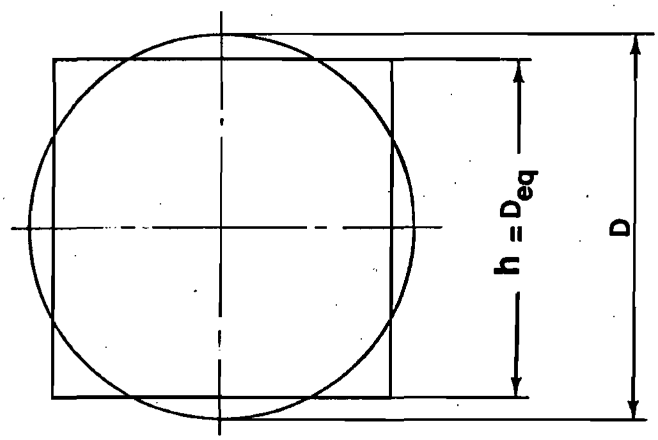
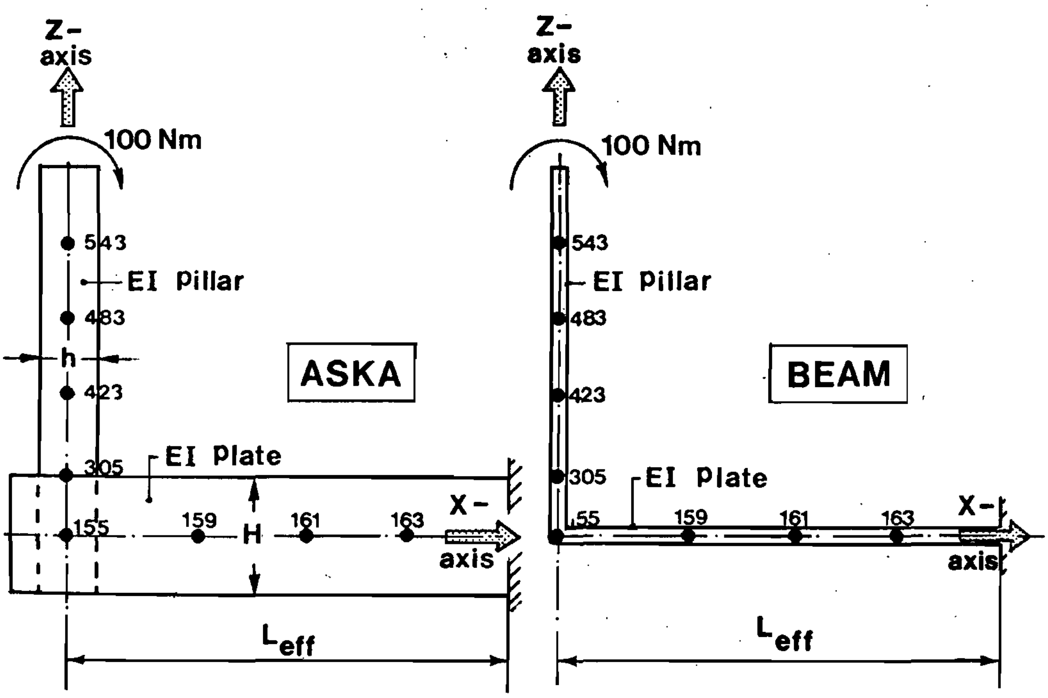


Fig. 3. Circular and square cross sectional area of pillar.

3. A TYPICAL EXAMPLE

In order to give an idea of the magnitude of the errors one can make in using beam-type finite elements in the topological model for plate-pillar systems of die-sets, the typical example of Fig. 4 is examined.



F.g. 4. Typical example of plate-pillar system. Plate thickness H = .03 m, plate width PW = .05 m, equivalent pillar diameter $h = D_{eq} = .017522$ m, $L_{eff} = .12$ m.

The pillar is loaded by a moment of 100 Nm. Tables 1 and 2 give the results of the calculations for the deflections of the plate in Z-direction and the deflections of the pillar in X-direction respectively.

PLATE				
Nodal point	X-coord. [m]	Z-deflection ASKA [m]	Z-deflection BEAM [m]	Error %
155	.000000	$.300870*10^{-4}$	$.3048*10^{-4}$	1.3
159	.036571	$.142840*10^{-4}$	$.1473*10^{-4}$	3.1
161	.064380	$.063010*10^{-4}$	$.6547*10^{-5}$	3.9
163	.092190	$.015612*10^{-4}$	$.1637*10^{-5}$	4.8

Table 1. Deflections of plate in Z-direction.

PILLAR				
Nodal point	Z-coord. [m]	X-deflection ASKA [m]	X-deflection BEAM [m]	Error %
305	.015	$.072580*10^{-4}$	$.144*10^{-4}$	98.4
423	.035	$.331965*10^{-4}$	$.549*10^{-4}$	65.4
483	.055	$.834500*10^{-4}$	$1.196*10^{-4}$	43.3
543	.075	$1.590950*10^{-4}$	$2.086*10^{-4}$	31.1

Table 2. Deflections of pillar in X-direction.

As far as the X-deflections of the pillar are concerned (see Table 2), one has to be very careful with the results of the beam solution particularly in the neighbourhood of the plate. Considering the ASKA solution to be exact, the example shows that errors of about 100% will occur at the surface of the plate. Going along the pillar this error gradually decreases. The error in the Z-deflections of the plate (see Table 1) is less and does not exceed 5% in this example. In most practical cases this figure will be admissible.

4. THE CORRECTION FACTOR ψ FOR THE LENGTH OF THE PILLAR

From the example of Fig. 4 it is clear that it is not allowed to consider the pillar as a beam element up to the center of the plate. Therefore a part ζ of the pillar will be assumed to be rigid (Fig. 5). The plate-pillar system can be loaded by a moment M as well as a force F. For both kinds of loads the function

$$\psi_{\square} = \psi_{\square}\ (H,\ D,\ z) \qquad (2)$$

where

$$\psi_{\square} = \frac{\zeta}{H/2} \qquad (3)$$

can be derived.

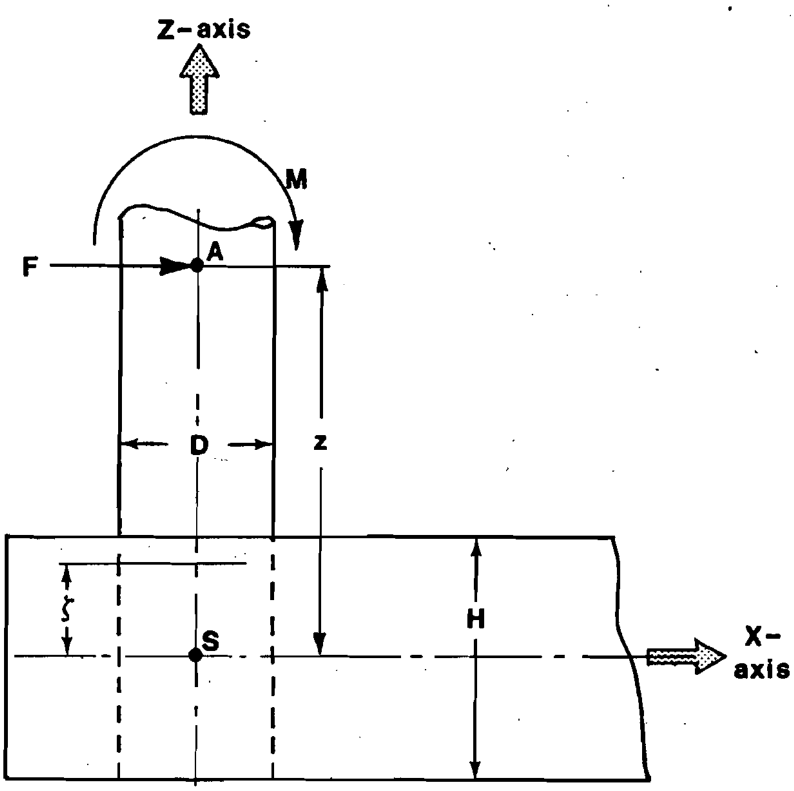


Fig. 5. Plate-pillar system loaded by moment M and force F.

When loaded by a moment M, the deflection of point A in the X-direction does consist of three parts viz.:
- the deflection due to the bending of that part of the pillar for which holds $z > \zeta$,
- the deflection due to the angular deflection ϕ of the plate in point S,

- the deflection due to the displacement x_o of the origin S in X-direction.

Hence, the deflection of point A in X-direction can be written as:

$$x(z) = \frac{M(z-\zeta)^2}{2\,EI_{pillar}} + \phi z + x_o \tag{4}$$

Using Eqs. (3) and (4), we find:

$$\psi_{\square} = \frac{2z}{H} - \sqrt{(x(z) - x_o - \phi z)\,\frac{8\,EI_{pillar}}{MH^2}} \tag{5}$$

Eq. (5) is equivalent with Eq. (2), since each term between the brackets in Eq. (5) is proportional with M/E.

In order to find the function $\psi_{\square}$, ASKA runs for a variety of plate-pillar systems have been carried out. For obvious reasons we replace the variable z by:

$$\ell = z - \frac{H}{2} \tag{6}$$

After each ASKA-run, we try to fit the results in the model function:

$$\psi_{\square}(\ell) = A\, e^{-B\ell} + C \tag{7}$$

Some typical results of this procedure are listed in Table 3.

D [m]	H [m]	$\frac{H}{D}$	modelfunction $\psi_{\square}(\ell)$	ℓ^* [m]	$\frac{\ell^*}{D}$
.03	.02	.67	.37 exp (-42ℓ) + .842	.0517	1.72
.02	.02	1.00	.11 exp (-35ℓ) + .851	.0271	1.35
.02	.03	1.50	.15 exp (-93ℓ) + .847	.0136	.68
.02	.015	.75	.42 exp (-91ℓ) + .871	.0250	1.25
.02	.04	2.00	.07 exp (-63ℓ) + .872	.0075	.37

Table 3. Some typical results with the model function of Eq. (7).

As we can see from this table, the C-value stabilizes at .85 ± .025, in spite of the different D- and H-values. This is true for all ASKA-runs carried out. Furthermore, from Table 2 it can be concluded that $A\,e^{-B\ell}$ is small with respect to C if the length ℓ exceeds a certain value.

Considering ε to be the relative error in the C-value, we define the distance ℓ^* above the plate, where the relative difference between $\psi_{\square}$ and C is equal to ε. Thus:

$$\frac{\psi_{\square}(\ell^*) - C}{C} = \varepsilon \tag{8}$$

For $\varepsilon = .05$ the corresponding values of ℓ^* and $\frac{\ell^*}{D}$ are listed in Table 3.
In Fig. 6 $\frac{\ell^*}{D}$ is plotted as a function of $\frac{H}{D}$. In the upper area it is allowed to use $\psi_\square = C$, while in the lower area, $\psi_\square = Ae^{-B\ell} + C$ must be used in order to avoid errors > 5%.

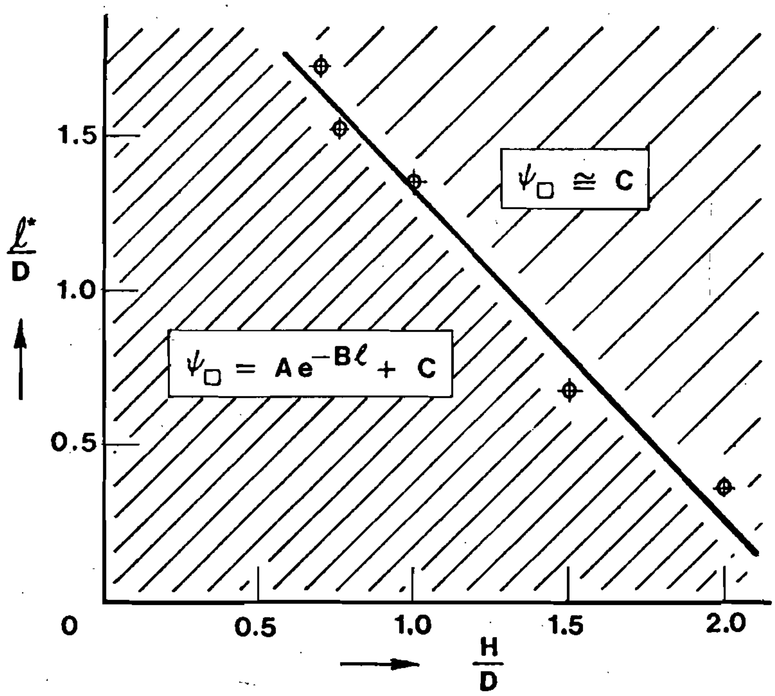


Fig. 6. The value $\frac{\ell^*}{D}$ as a function of $\frac{H}{D}$ for a relative error $\varepsilon = .05$.

For the design of die-sets only the upper area is of importance. We can conclude, that for an equivalent square pillar we may use:

$$\psi_\square = .85 \tag{9}$$

Carrying out the same analysis for the circular pillar of Fig. 2, ASKA analysis shows that an equivalent square pillar gives a stiffer fixing point than the corresponding circular pillar. Therefore, the correction factor in the case of an equivalent square pillar $\psi_\square$ will be larger. From computer analysis it can be shown that systematically holds:

$$\psi_{\square} = 1.33\ \psi_0 \qquad (10)$$

This result can also be made plausible starting from the Boussinesque-theory for a force on a semi-infinite solid body [3]. As far as the other correction factor λ (see Sec. 5) is concerned, there is no significant difference between a circular and an equivalent square pillar.

Eq. (12) gives us the opportunity to carry out the ASKA-analysis with equivalent square pillars and to use the results for circular pillars.
So referring to Eq. (10), we may conclude that for a circular pillar will hold

$$\psi_0 = .75 \qquad (11)$$

5. THE CORRECTION FACTOR λ FOR THE ANGULAR DEFLECTION OF THE PLATE

The correction factor λ deals with the angular deflection ϕ which appears in point S of the loaded plate-pillar system (see Fig. 5). For a certain plate width PW and effective length of the plate L_{eff}, the error in ϕ_{BEAM} with respect to ϕ_{ASKA} only depends upon the ration $\frac{H}{D}$ as can be seen in Table 4 for a typical example.

$\frac{H}{D}$	$\frac{\phi_{ASKA}-\phi_{BEAM}}{\phi_{BEAM}}$ * 100%
.400	-12.2
.500	- 8.2
.667	- 4.2
.750	- 1.8
1.000	.4
1.333	3.5
1.500	4.2
2.000	8.9

Table 4. Typical example for the deviation of ϕ_{BEAM} as a function of $\frac{H}{D}$.
Plate thickness H = .02 m, plate width PW = .05 m, L_{eff} = .12 m.

For ratios $\frac{H}{D} > 1$ it follows that $\phi_{ASKA} > \phi_{BEAM}$, from which we may conclude that in this ratio range the local deformations determine the angular deflection in point S. For ratios H/D < 1, the pillar contributes to the stiffness of the plate in point S, hence $\phi_{BEAM} > \phi_{ASKA}$.
Along the Y-axis of the plate the displacements in Z-direction

are large in the vicinity of point S and smaller onto the side of the plate. These differences in Z-displacements increase strongly with increasing PW. For relative large values of PW the deformation of the plate in the neighbourhood of the pillar has to be considered as a local phenomenon. From this, it turns out that ϕ_{BEAM} in the origin S will gradually lag behind ϕ_{ASKA} with increasing PW-values. Table 5 shows an example of this effect.

PW [m]	ϕ_{BEAM} [rad]	ϕ_{ASKA} [rad]	$\frac{\phi_{ASKA}-\phi_{BEAM}}{\phi_{BEAM}} * 100\%$
.03	$28.57*10^{-4}$	$28.33*10^{-4}$	-.8
.05	$17.14*10^{-4}$	$17.20*10^{-4}$	.4
.08	$10.71*10^{-4}$	$11.08*10^{-4}$	3.5
.12	$7.14*10^{-4}$	$7.83*10^{-4}$	9.7
.20	$4.29*10^{-4}$	$5.49*10^{-4}$	28.0

Table 5. Typical example for the deviation of ϕ_{BEAM} for a load M = 100 Nm and variation of PW. Plate thickness H = .02 m, pillar diameter D = .02 m, L_{eff} = .12 m.

Defining in general p as the percentage that ϕ_{ASKA} deviates from ϕ_{BEAM}, we may write:

$$p = p\,(H,\ D,\ L_{eff},\ PW) \tag{12}$$

and also for the correction factor:

$$\lambda = \frac{p}{100} * L_{eff} \tag{13}$$

Eq. (13) is especially convenient for the implementation of the correction factor in the stiffness matrix of the beam element as exposed in Sec. 6.
From a large number of ASKA-runs, the following model function for p satisfies best:

$$\begin{aligned} p = & -5.9153 \frac{PW}{H} + 28.8222 \frac{PW}{L_{eff}} - 74.9665 \frac{D}{L_{eff}} + 5.2008 \frac{PW}{D} \\ & -5.2219 \frac{D}{PW} + 0.1449 \frac{D}{H} + 1.3611 \frac{L_{eff}}{H} - 0.6476 \frac{L_{eff}}{D} \\ & -1.6262 \frac{L_{eff}}{PW} - 9.0037 \frac{H}{D} + 9.9213 \frac{H}{L_{eff}} + 33.8696 \frac{H}{PW} \\ & -2.2375 \end{aligned} \tag{14}$$

If we neglect second-order effects the following - more simple - model function for p also satisfies:

$$p = 23.7987 \frac{PW}{L_{eff}} - 127.2420 \frac{D}{L_{eff}} + 66.6015 \frac{H}{L_{eff}} \qquad (15)$$

If Eq. (14) is used in order to calculate λ with the aid of Eq. (13) the correction factor λ still depends upon the effective length of the plate L_{eff}. From this it follows, that we need an estimation of L_{eff} from the design drawing of the die-set when using the rather complicated formula of Eq. (14). In order to get an idea of the error in p which can be made in following this procedure, this error has been calculated with the aid of the partial derivative $\partial p/\partial L_{eff}$ from Eq. (14) for a number of cases. On an average an estimation error of 10% of L_{eff} results in an error of 0.6% in p, which is really a second-order effect. When using the Eqs. (13) and (15) we do not have this problem, because in that case λ does not depend upon L_{eff}.

6. IMPLEMENTATION OF THE CORRECTION FACTORS ψ AND λ IN THE STIFFNESS MATRIX OF A BEAM ELEMENT

In Fig. 7 a part of a die-set is shown. In order to compute the deflections of this construction with high accuracy, the correction factors ψ for the pillar and λ for the plates must be used. There are a number of ways to introduce these correction factors, but to our opinion the solution described below is most convenient for the user.

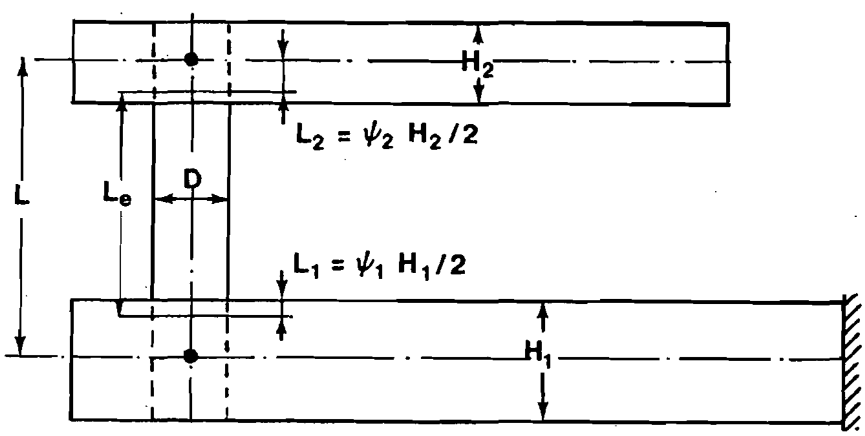


Fig. 7. An example of a pillar-like element.

$$
\begin{matrix}
0 & 0 & -\frac{AE}{L} & 0 & 0 \\
\frac{12EIA}{AL_e^3 + SL_e} & \frac{-6EIA(L_e+2L_1)}{AL_e^3 + SL_e} & 0 & \frac{-12EIA}{AL_e^3 + SL_e} & \frac{-6EIA(L_e+2L_2)}{AL_e^3 + SL_e} \\
\frac{-6EIA(L_e+2L_1)}{AL_e^3 + SL_e} & \frac{EIA(4L_e^2+12L_eL_1+12L_1^2) + EIS}{AL_e^3 + SL_e} & 0 & \frac{6EIA(L_e+2L_1)}{AL_e^3 + SL_e} & \frac{EIA(2L_e^2+6L_eL_1+6L_eL_2+12L_1L_2) - EIS}{AL_e^3 + SL_e} \\
0 & 0 & \frac{AE}{L} & 0 & 0 \\
\frac{-12EIA}{AL_e^3 + SL_e} & \frac{6EIA(L_e+2L_1)}{AL_e^3 + SL_e} & 0 & \frac{12EIA}{AL_e^3 + SL_e} & \frac{6EIA(L_e+2L_2)}{AL_e^3 + SL_e} \\
\frac{-6EIA(L_e+2L_2)}{AL_e^3 + SL_e} & \frac{EIA(2L_e^2+6L_eL_1+6L_eL_2+12L_1L_2) - EIS}{AL_e^3 + SL_e} & 0 & \frac{6EIA(L_e+2L_2)}{AL_e^3 + SL_e} & \frac{EIA(4L_e^2+12L_eL_2+12L_2^2) + EIS}{AL_e^3 + SL_e}
\end{matrix} \right]
$$

Fig. 8. Stiffness matrix for a pillar like beam element; where $S = 24\ \kappa I(1+\nu)$.

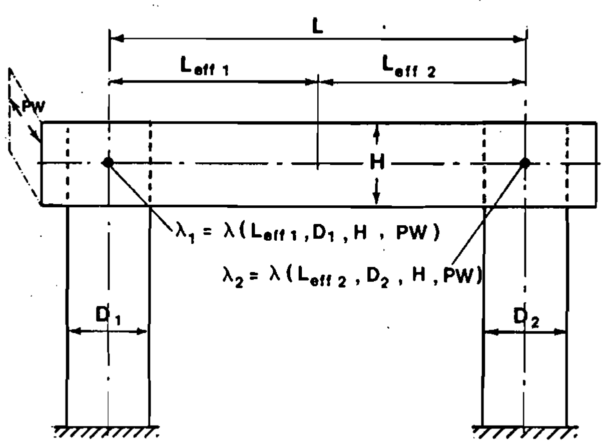


Fig. 9. An example of a plate like element.

$$
\begin{bmatrix}
\frac{AE}{L} & 0 & 0 & -\frac{AE}{L} & 0 & 0 \\
0 & \frac{12EIA(L+\lambda_1+\lambda_2)}{S} & \frac{-6EIA(L+2\lambda_2)L}{S} & 0 & \frac{-12EIA(L+\lambda_1+\lambda_2)}{S} & \frac{-6EIA(L+2\lambda_1)L}{S} \\
0 & \frac{-6EIA(L+2\lambda_2)L}{S} & \frac{4EI(AL^2+3AL\lambda_2+6\kappa I(1+\nu))L}{S} & 0 & \frac{6EIA(L+2\lambda_2)L}{S} & \frac{EI(2AL^2-24\kappa I(1+\nu))L}{S} \\
-\frac{AE}{L} & 0 & 0 & \frac{AE}{L} & 0 & 0 \\
0 & \frac{-12EIA(L+\lambda_1+\lambda_2)}{S} & \frac{6EIA(L+2\lambda_2)L}{S} & 0 & \frac{12EIA(L+\lambda_1+\lambda_2)}{S} & \frac{6EIA(L+2\lambda_1)L}{S} \\
0 & \frac{-6EIA(L+2\lambda_1)L}{S} & \frac{EI(2AL^2-24\kappa I(1+\nu))L}{S} & 0 & \frac{6EIA(L^2+2\lambda_1)L}{S} & \frac{4EI(AL^2+3AL\lambda_1+6\kappa I(1+\nu))L}{S}
\end{bmatrix}
$$

Fig. 10. Stiffness matrix for a plate like beam element; where
$S = AL^2(L^2+4L\lambda_1+4L\lambda_2+12\lambda_1\lambda_2) + 24\kappa LI(1+\nu)(L+\lambda_1+\lambda_2)$.

- Design a stiffness matrix for a pillarlike beam element.
  The length L of the pillar of Fig. 7 can be devided into the parts $L_1 = \psi_1.H_1/2$, $L_2 = \psi_2.H_2/2$ and $L_e = L-L_1-L_2$.

  The parts L_1 and L_2 are rigid. Now, it is possible to assemble the stiffness matrix for the pillar of length L. In Fig. 8 this matrix is shown. For $L_1=L_2=0$ the original stiffness matrix for a two dimensional beam element will be found. In the case of a pillar fixed at one end only, one of the values L_1 or L_2 equals zero.
- Design a stiffness matrix for a plate-like beam element.
  For a plate, connected with a pillar, the stiffness at the connection points is lower. The factor λ (Eq. 13) describes the extra flexibility with respect to the rotation in this connection point. The value of λ still depends on an effective bending length L_{eff}. This length L_{eff} must be estimated from the drawing and needs not necessarily to be equal to the length L of the plate. In Fig. 9 an example is shown of a plate connected with two pillars. When the flexibilites λ_1 and λ_2 of the plate element are inserted into the stiffness matrix of that plate the assembled matrix of Fig. 10 will be found.
  In the case $\lambda_1=\lambda_2=0$, we get back the original stiffness matrix of a beam element.

7. DISCUSSION

With the aid of the method described in this article a plate-pillar system can be analyzed by a beam program with an inaccuracy of about 5%. In doing this, it is allowed that the characteristic quantities D, H and z are of the same order of magnitude. However, the inaccuracy will be more than 5% if the condition D = H = z is reached.
In some cases the accuracy can be increased by using a more accurate model function for the correction factor λ.
The costs of this computer analysis amounts only a fraction of the costs when applicating a large computer system with sophisticated elements.

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